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Neutrino Oscillation Global Data Fitting with GAMBIT

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What is GAMBIT?

The Global And Modular Beyond-the-Standard Model Inference Tool (**GAMBIT**) *[1]* is an **open-source** software framework for performing global fitting of particle, astroparticle, and cosmological models.

GAMBIT is designed to tackle the task of analysing the vast BSM parameter space, and is highly **modular** and **flexible** for including new physics models and parameters.

GAMBIT & neutrino

Previous neutrino topics explored using the GAMBIT module, **NeutrinoBit**

- **Neutrino mass** (2020, *[2]*): using both cosmological and terrestrial data, the upper limit of the mass of the lightest neutrino is set at 0.037 eV (NO) and 0.042 eV (IO) at 95% confidence.
- **Right-handed neutrinos** (2020, *[3]*): A frequentist global analysis on 60 MeV to 500 MeV right-handed neutrinos, utilising data from both direct and indirect searches

The next topic to explore is the tremendously diverse field of **neutrino oscillation**!

Essential ingredients

To perform a neutrino global data fit, several ingredients are needed:

- **Neutrino experiment information** and **data**
- A set of **module functions** representing the observational results
- A parameter space **sampler**. In this study the `Diver` sampler is used.

Scanner: `Diver`

`Diver` is an open-source **differential evolution** fast parameter sampler that utilises the population-based self-adaptive λjDE algorithm (rand-to-best/1/bin).

All DE algorithms consist of three main steps: **mutation**, **crossover**, and **selection**. An example of the vanilla DE (rand/1/bin):

- **Mutation:** Starting with a population of N parents $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$. For each parent $\mathbf{x}_i$, three other members $\mathbf{x}_{i1}, \mathbf{x}_{i2}, \mathbf{x}_{i3}$ are chosen uniformly to form a mutation vector $\mathbf{u}_i$. In DE/rand/1/bin,

$$\mathbf{u}_i = \mathbf{x}_{i1} + \beta\left(\mathbf{x}_{i2} - \mathbf{x}_{i3}\right) \qquad \beta = \text{'learning rate'}$$

- **Crossover:** The parent $\mathbf{x}_i$ and the mutation vector $\mathbf{u}_i$ are recombined to form the offspring $\mathbf{x}'_i$. In binomial crossover, each element of the offspring inherited from the parent has a probability of P to mutate:

$$x'_i[j] = \begin{cases} \mathbf{u}_i[j] & \text{if } r \le P \\ \mathbf{x}_i[j] & \text{if } r > P \end{cases}, \quad \text{random number } 0 < r < 1$$

- **Selection:** The parent $\mathbf{x}_i$ and the child $\mathbf{x}'_i$ compete to survive and become a member of the next generation.

In the λjDE/rand-to-best/1/bin algorithm, β and P change as the population evolve, and the mutation vector $\mathbf{u}_i$ takes a new form of

$$\mathbf{u}_i = \lambda \mathbf{x}_{\text{best}} + (1-\lambda)\mathbf{x}_{i1} + \beta_i\left(\mathbf{x}_{i2} - \mathbf{x}_{i3}\right)$$

Neutrino oscillation global fits with GAMBIT

Aim

- Combine neutrino oscillation experimental results to set stricter constraints. Some features include
  - publicly available experiment information and data
  - experiment-specific, physics-motivated systematic uncertainty treatments
  - modern statistical and computational methods
- Establish an open-source framework by which new physics models can be scrutinised

Method

Each experimental result is represented by a **module function**, which, for a given hypothesis and a set of nuisance parameters, **generates a GAMBIT counterpart** of the experiment result and **computes the likelihood**. Based on the likelihood, one can then perform statistical inferences and construct confidence intervals.

To generate a GAMBIT counterpart of an experimental result
A model of the experiment is constructed based on the available experimental information and data to predict the observation. For example, the T2K likelihood function predicts the far detector neutrino energy spectrum using the formula below. For bin *i*,

$$N_i^{\alpha} = N_{\text{bkg},i} + \int_{E_i}^{E_{i+1}} dE_{\text{rec}} \int_0^{\infty} dE_{\nu}\, R\left(E_{\text{rec}}|E_{\nu}\right) \frac{d\Phi}{dE_{\nu}} \sigma_{\alpha}(E_{\nu}) \epsilon(E_{\nu}) P_{\nu_{\mu}\to\nu_{\alpha}}(E_{\nu})$$

Reconstruction function (R); Neutrino beam flux ($d\Phi/dE_\nu$); Total cross section (σ_α); Selection efficiency (ϵ)

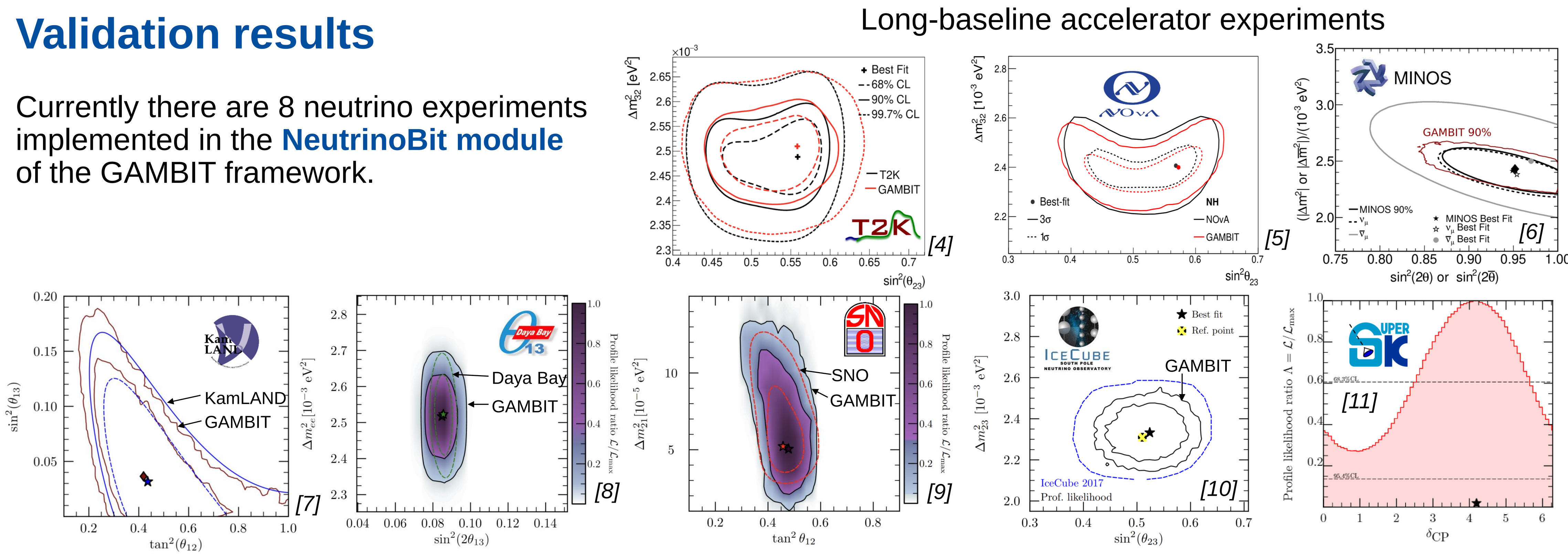
To compute the likelihood
For a given set of neutrino oscillation and nuisance parameters, the module function compares the GAMBIT predicted result to the experimental observation and computes the likelihood using

$$\ln(L) = \ln\left(\prod_{i=1}^{N_{\text{bin}}} \frac{e^{-m_i} m_i^{n_i}}{n_i!}\right) = \sum_{i=1}^{N_{\text{bin}}} \left[-m_i + n_i \ln(m_i) - \ln(n_i!)\right],$$ n_i = (observed events)$_i$ and m_i = (expected signal)$_i$ + (expected background)$_i$.

Validation results

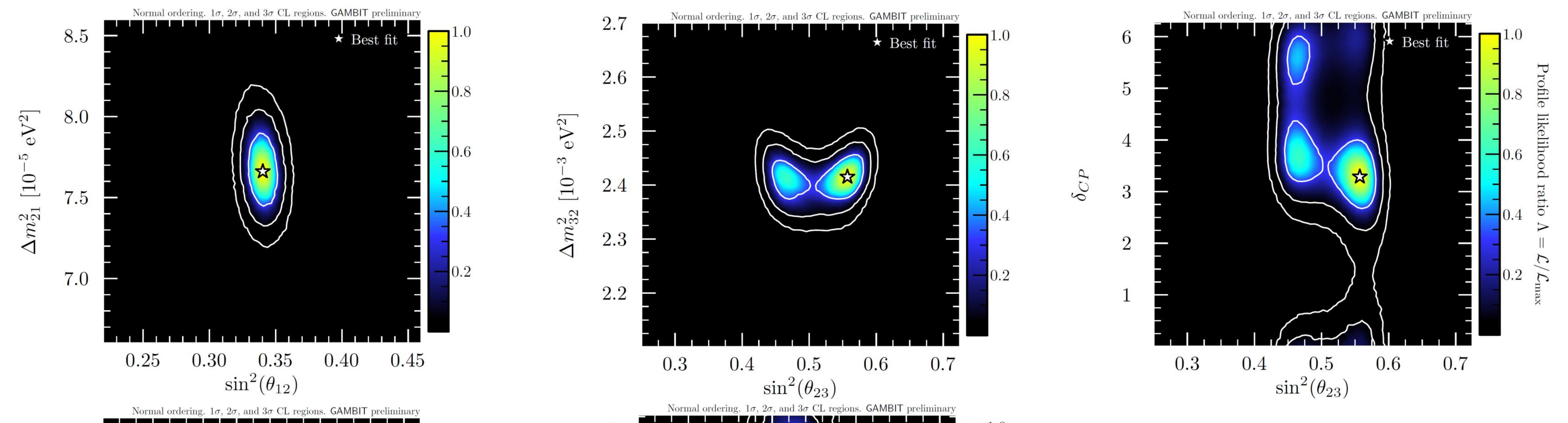
Currently there are 8 neutrino experiments implemented in the **NeutrinoBit module** of the GAMBIT framework.

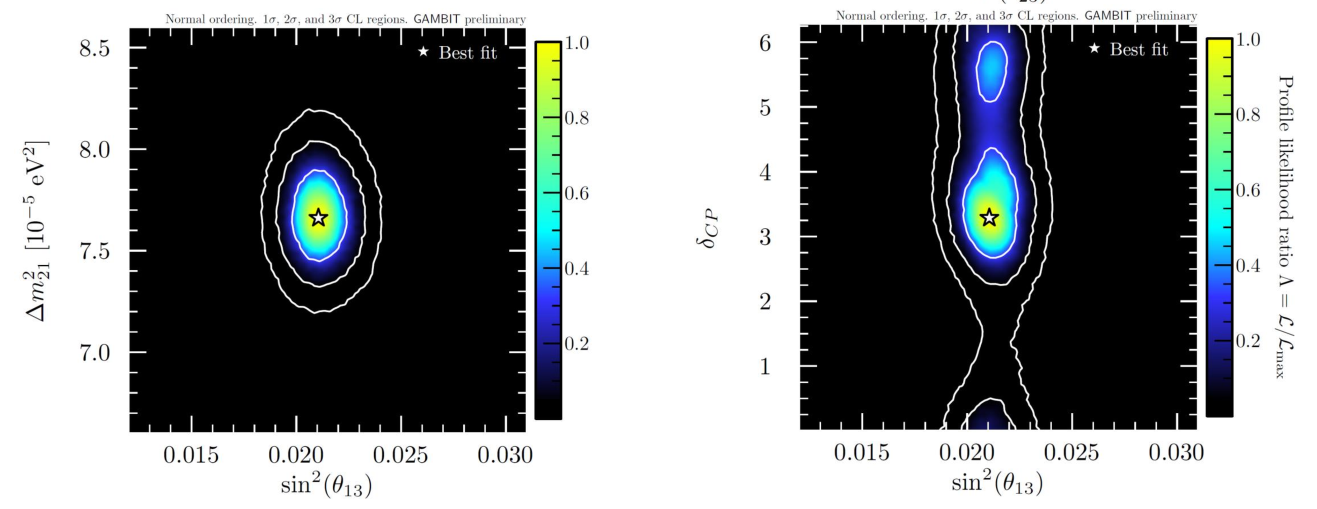
Long-baseline accelerator experiments



Reactor experiments — Solar exp. — Atmospheric experiments

Preliminary global fit results





	Normal ordering ($\pm 1\sigma$)
θ_{12} [°]	35.7 ± 0.5
θ_{13} [°]	8.34 ± 0.25
θ_{23} [°]	$48.3^{+1.0}_{-1.4}$
$\sin^2\theta_{12}$	0.340 ± 0.009
$\sin^2\theta_{13}$	$0.0211^{+0.0013}_{-0.0012}$
$\sin^2\theta_{23}$	$0.557^{+0.018}_{-0.024}$
Δm^2_{21} [10^{-5} eV^2]	$7.66^{+0.15}_{-0.14}$
Δm^2_{32} [10^{-3} eV^2]	2.41 ± 0.03
δ_{CP} [° /π rad]	188^{+27}_{-23} / $1.05^{+0.15}_{-0.13}$

[1] GAMBIT main paper. https://arxiv.org/abs/1705.07908
[2] GAMBIT neutrino mass paper. https://arxiv.org/abs/2009.03287
[3] GAMBIT RH neutrino paper. https://arxiv.org/abs/1908.02302
[4] T2K 2020 result. https://doi.org/10.5281/zenodo.3959558
[5] NOvA 2020 result. https://doi.org/10.5281/zenodo.4142045
[6] MINOS 2013 paper. https://doi.org/10.1103/PhysRevLett.110.251801
[7] KamLAND 2011 paper. https://doi.org/10.1103/PhysRevD.83.052002
[8] Daya Bay 2018 paper. https://doi.org/10.1103/PhysRevLett.121.241805
[9] SNO paper. https://arxiv.org/abs/1109.0763
[10] IceCube 2019 paper. https://doi.org/10.1103/PhysRevD.99.032007
[11] Super-K paper. https://arxiv.org/abs/1710.09126

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