Understanding gravitationally induced decoherence parameters in neutrino oscillations using a microscopic quantum mechanical model

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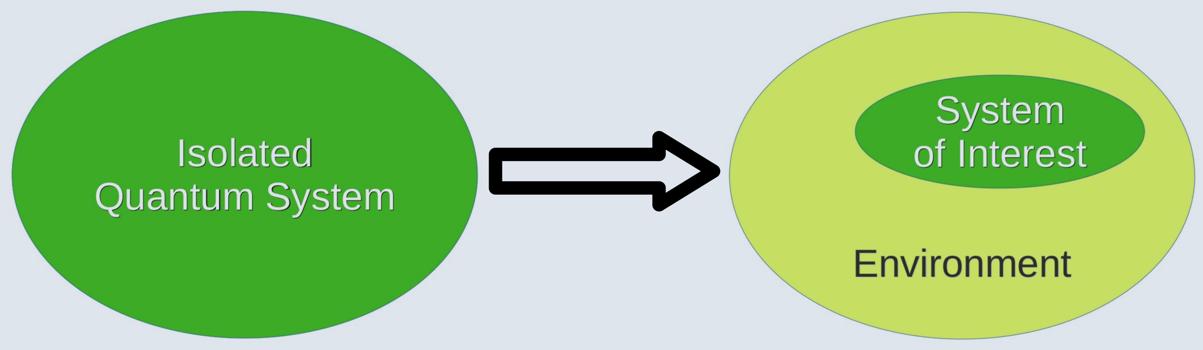
Neutrinos and Gravity

- Neutrinos interact with gravity $\Longrightarrow$ influenced by gravitational waves $\Longrightarrow$ changes oscillation behaviour (gravitationally induced decoherence)
- Determining this influence...
  - ... helps to better understand neutrino oscillations
  - ... yields information on stochastic gravitational waves that are not directly detectable, produced by several sources after the Big Bang
- Many works using phenomenological models (e.g. [1, 7, 8, 10]) $\longleftrightarrow$ connection to underlying microscopic physics not always immediate

  $\Longrightarrow$ Here: Study a microscopic quantum mechanical model to connect to phenomenological models

Neutrinos as an open quantum system



- Investigate effective dynamics of a neutrino (system of interest $\hat{\rho}_S$) interacting with gravitational waves (environment) without the necessity to track their detailed dynamics
- Master equation for effective evolution of a neutrino:

$$\frac{\partial}{\partial t}\hat{\rho}_S(t) = -\frac{i}{\hbar}[\hat{H}_S + \hat{H}_{add}, \hat{\rho}_S(t)] + \mathcal{D}[\hat{\rho}_S(t)]$$

  $\Longrightarrow$ Red terms have to be postulated (phenomenological models) or derived

- New processes compared to an isolated quantum system encoded in red terms:
  - **Energy shifts/renormalisation** of the energies of $\hat{\rho}_S$
  - **Dissipation** (energy flux from the system into the environment)
  - **Decoherence** (information flux into the environment)

    $\Longrightarrow$ diagonalisation ("classicalisation") of $\hat{\rho}_S$ in a certain basis

Microscopic quantum mechanical model

- Based on [11], extended for neutrinos in [3]
- System of interest: Neutrino, Hamiltonian $\hat{H}_S$
- Environment: Gravitational waves modeled by a bath of Harmonic oscillators with frequencies ω_i, positions (configuration variables) $\hat{q}_i$ and canonically conjugated momenta $\hat{p}_i$
- Coupling: motivated by General Relativity, where the energy-momentum tensor of matter couples to the metric (= configuration variable) of the gravitational field; coupling constants g_i
- Total Hamiltonian of the quantum mechanical model:

$$\hat{H} = \underbrace{\hat{H}_S^{(0)} + \hat{H}_S^{(C)}}_{\hat{H}_S} \otimes \hat{1}_{\mathcal{E}} + \hat{1}_S \otimes \underbrace{\frac{1}{2}\sum_{i=1}^{N}\left[\hat{p}_i^2 + \omega_i^2\hat{q}_i^2\right]}_{\hat{H}_{\mathcal{E}}} - \underbrace{\hat{H}_S \otimes \sum_{i=1}^{N} g_i\hat{q}_i}_{\hat{H}_{\text{int}}}$$

with counter term $\hat{H}_S^{(C)}$ that renormalises a Lamb-shift-like contribution to the master equation

Derivation of the master equation

- Assumptions:
  - Interaction weak compared to the free evolution of the neutrino
  - Gravitational waves follow a Bose-Einstein distribution moderated by a temperature parameter T
  - Correlation functions in the environment decay on time scales much shorter than the state of the neutrino varies (Markov assumption; holds here, see [3]) $\Longrightarrow$ coarse-graining, send $t_0 \to -\infty$
- Master equation (Lindblad form):

$$\frac{d}{dt}\hat{\rho}_S(t) = -\frac{i}{\hbar}\left[\hat{H}_S^{(0)}, \hat{\rho}_S(t)\right] + \frac{8\eta^2}{\hbar^2}\frac{k_B T}{\hbar}\left(\hat{H}_S^{(0)}\hat{\rho}_S(t)\hat{H}_S^{(0)} - \frac{1}{2}\left\{(\hat{H}_S^{(0)})^2, \hat{\rho}_S(t)\right\}\right)$$

- Free parameters: T, η

Neutrino setup

- Consider neutrinos that propagate through the Earth
- System Hamiltonian in vacuum mass basis:

$$\hat{H}_S^{(0)} = \hat{H}_{vac} + \hat{U}^{\dagger}\hat{H}_{mat}\hat{U},$$

  with

$$\hat{H}_{vac} = E\mathbb{1}_3 + \frac{c^4}{6E}\begin{pmatrix} -\Delta m_{21}^2 - \Delta m_{31}^2 & 0 & 0 \\ 0 & \Delta m_{21}^2 - \Delta m_{32}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 + \Delta m_{32}^2 \end{pmatrix}, \qquad \hat{H}_{mat} = \pm\sqrt{2}G_f N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Three bases:
  - Flavour basis
  - Vacuum mass basis: $\hat{H}_S^{(0)}$ is diagonal for $N_e = 0$, i.e. for vacuum. Connected to flavour basis by PMNS matrix $\hat{U}$.
  - Effective mass basis: $\hat{H}_S^{(0)}$ is diagonal (with eigenvalues $\widetilde{H}_i$) for a fixed N_e, i.e. for a specific layer of density of the Earth. Connected to vacuum mass basis by $\widetilde{V}(N_e)$.
- Solution of the master equation in effective mass basis:

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \frac{4\eta^2 k_B T}{\hbar^3}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t}$$

  and in vacuum mass basis:

$$\rho_{ij}(t) = \rho_{mn}(0)\widetilde{V}_{km}^{\dagger}\widetilde{V}_{nl}\widetilde{V}_{ik}\widetilde{V}_{lj}^{\dagger}e^{-\frac{i}{\hbar}\left(\widetilde{H}_k - \widetilde{H}_l\right)t - \frac{4\eta^2 k_B T}{\hbar^3}\left(\widetilde{H}_k - \widetilde{H}_l\right)^2 t}$$

Results

- Connection to phenomenological models:
  - Phenomenological models: specific Lindblad form for dissipator $\mathcal{D}[\hat{\rho}_S(t)]$

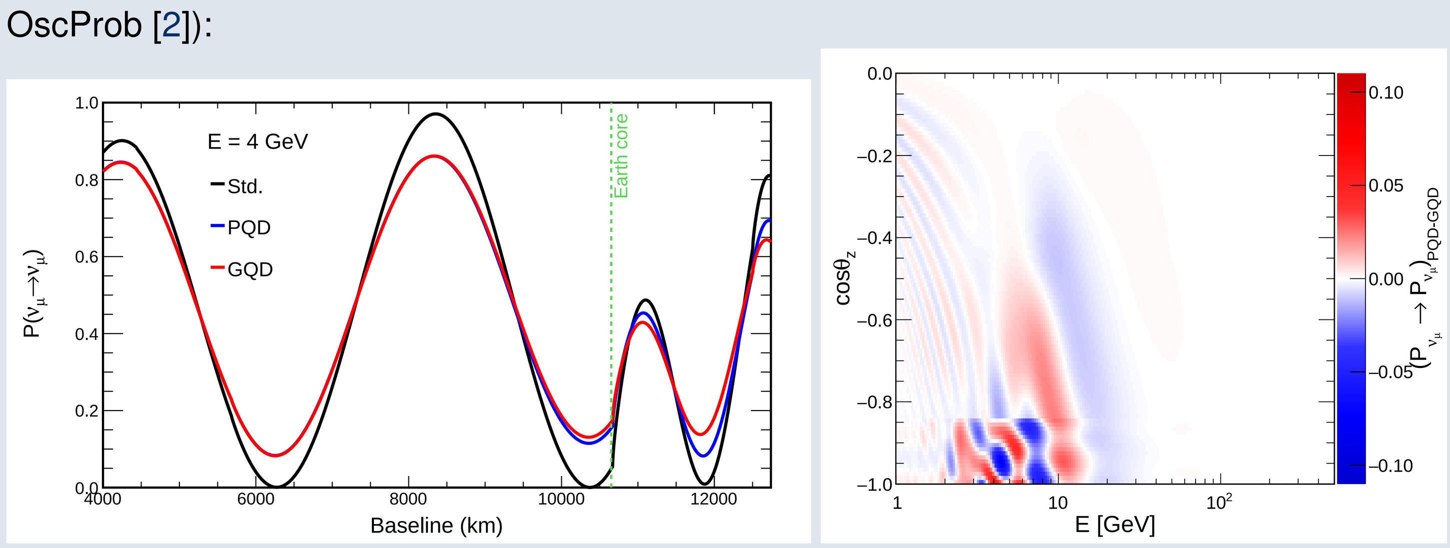
    $\Longrightarrow$ Solution of the master equation:

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(\widetilde{H}_i - \widetilde{H}_j\right)t - \Gamma_{ij}t} \qquad \text{with } \Gamma_{ij} = \gamma_{ij}E^n$$

    Then restrict number of parameters by considering specific cases where γ_{ij} are zero or equal to each other
  - Implications from our quantum mechanical model in vacuum:

$$\gamma_{ij} = \frac{\eta^2 c^8 k_B T}{\hbar^3}(\Delta m_{ij}^2)^2 \qquad n = -2$$

  - In matter: No match possible, as for the phenomenological models γ_{ij} is constant while in the quantum mechanical model it depends on the matter density N_e $\Longrightarrow$ different oscillation probabilities (for $T = 0.9K, \eta = 10^{-8}s$, $n = -2$ and fitting values for γ_{ij} using PREM [4] and OscProb [2]):



  $\Longrightarrow$ **Analyses using the phenomenological ansatz from above can only constrain the free parameters in vacuum, not in matter**

- Lamb-shift contribution: Renormalised using a counter term. Without renormalisation: dependence on unphysical arbitrary cutoff-frequency Ω of gravitational waves background, diverges for $\Omega \to \infty$

  $\Longrightarrow$ **Interpretation of the Lamb-shift contribution without a renormalisation problematic**

- Free parameters:
  - T: "Temperature" parameter characterising the gravitational waves environment
  - η: Coupling strength between neutrino system and gravitational environment. Should be determined by the coupling of matter to gravity by General Relativity. Using a field-theoretical model from ([6, 9]), a naive comparison (see [3, 5]) yields a direct relation to the Planck length, in particular $\eta \approx 10^{-42}s$.

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