Energy Dependence of Angular-Driven Flavor Instabilities in Dense Astrophysical Environments

UNICAMP

Pedro Dedin Neto[1,2], Irene Tamborra[2], Shashank Shalgar[2]

[1]Instituto de Física Gleb Wataghin, UNICAMP, Campinas-SP, Brazil
[2]Niels Bohr Institute, University of Copenhagen, Denmark

Introduction

In core-collapse supernovae (CCSN) and neutron star mergers (NSM), neutrino flavor conversion remains an unsolved mystery. Inside these environments, free-streaming neutrinos obey the following equation of motion:

$$i\left(\partial_t + \vec{v}\cdot\vec{\nabla}_{\vec{x}}\right)\rho(t,\vec{p},\vec{x}) = [H, \rho(t,\vec{p},\vec{x})]. \quad (1)$$

The Hamiltonian contain the usual vacuum component $H_{\text{vac}} \sim \omega = \Delta m^2/2E$, the potential due to forward scattering on electrons $H_{\text{matt}} \sim \lambda = \sqrt{2}G_F n_e$, and a neutrino-neutrino potential [1, 2]

$$H_{\nu\nu} = \mu\int \mathrm{d}\vec{p}'(1-\hat{p}\cdot\hat{p}')[\rho(\vec{p}') - \bar{\rho}(\vec{p}')], \quad \mu = \sqrt{2}G_F n_\nu. \quad (2)$$

This term leads to different collective phenomena, usually classified as **slow**, when dependent on ω, and **fast** when not ($\mu \gg \omega$). The last is strictly driven by the directional angular distribution of neutrinos, in which the vacuum term is usually ignored. In this work [3], we analyze how non-vanishing vacuum mixing ($\omega \neq 0$) can affect the onset of angular-driven flavor conversions, showing that **the assumption $\omega = 0$ may miss sizable conversion rates** for realistic ω/μ ratios.

Neutrino System

- **Simplifications:** Homogeneous, axially-symmetric, and mono-energetic neutrino gas;
- In a two neutrino families approximation(ν_e, $\nu_x = \nu_\mu, \nu_\tau$):

$$\rho(t,v) = \begin{pmatrix}\rho_{ee}(t,v) & \rho_{ex}(t,v)\\ \rho_{ex}^*(t,v) & \rho_{xx}(t,v)\end{pmatrix} \quad \text{and} \quad \bar{\rho}(t,v) = \begin{pmatrix}\bar{\rho}_{ee}(t,v) & \bar{\rho}_{ex}(t,v)\\ \bar{\rho}_{ex}^*(t,v) & \bar{\rho}_{xx}(t,v)\end{pmatrix}, \quad (3)$$

where $v \equiv \cos\theta$ represents the projection of the velocity $\vec{v} = \vec{p}/E$ along the axis of symmetry.

Linear Stability Analysis

If $|\rho_{ex}| \ll |\rho_{ee} - \rho_{xx}|$ (e.g., at the neutrino production), one can linearize the equations of motion at first order in $|\rho_{ex}|$, such that the solutions will be plane waves:

$$\rho_{ex}(t,v) = Q_v e^{-i\Omega t} \quad \text{and} \quad \bar{\rho}_{ex}(t,v) = \bar{Q}_v e^{-i\Omega t}. \quad (4)$$

Unstable solutions will have eigenfrequencies $\Omega = \gamma + i\kappa$ with $\kappa > 0$. To find these solutions, one needs to solve the following **eigenvalue problem** ($\Omega' = \Omega - D_0^z - \lambda$):

$$\begin{vmatrix} I_0 - 1 & -I_1 \\ I_1 & -I_2 - 1\end{vmatrix} = 0, \quad I_n(\Omega') = \mu\int \mathrm{d}v v^n\left[\frac{\bar{g}_v}{\Omega' + \mu v D_1^z + \omega^c} - \frac{g_v}{\Omega' + \mu v D_1^z + \omega^c}\right], \quad (5)$$

where

$$g_v \equiv \rho_{ee}^0(v) - \rho_{xx}^0(v), \quad \bar{g}_v \equiv \bar{\rho}_{ee}^0(v) - \bar{\rho}_{xx}^0(v), \quad D_n^z \equiv \int_{-1}^{+1} v^n(g_v - \bar{g}_v). \quad (6)$$

In this work, we adopt forward peaked Gaussian for the angular distributions, characterized by a standard deviation σ_{ν_β} and a normalization α_{ν_β}.

$$\rho_{\beta\beta}^0(v) = \alpha_{\nu_\beta}\mathcal{G}(v;1,\sigma_{\nu_\beta}) \quad \text{normalized such that} \quad \int_{-1}^{+1}\mathrm{d}v\rho_{\beta\beta}^0(v) = \frac{n_{\nu_\beta}}{n_{\nu_e}} = \alpha_{\nu_\beta}, \quad (7)$$

Fast Limit - Vanishing Vacuum Mixing ($\omega = 0$)

In this limit, the flavor stability will depend only on the angular distribution **electron lepton number (ELN)**:

$$I_n(\Omega') = \mu\int \mathrm{d}v v^n \frac{(\bar{g}_v - g_v)}{\Omega' + \mu v D_1^z} = \mu\int \mathrm{d}v v^n\frac{(\bar{\rho}_{ee}^0 - \rho_{ee}^0)}{\Omega' + \mu v D_1^z}, \quad (\bar{\rho}_{xx}^0 = \rho_{xx}^0). \quad (8)$$

This system is completely stable if there is no ELN zero-crossing ($\zeta = 0$) or if D_0^z and D_1^z have opposite signs [4].

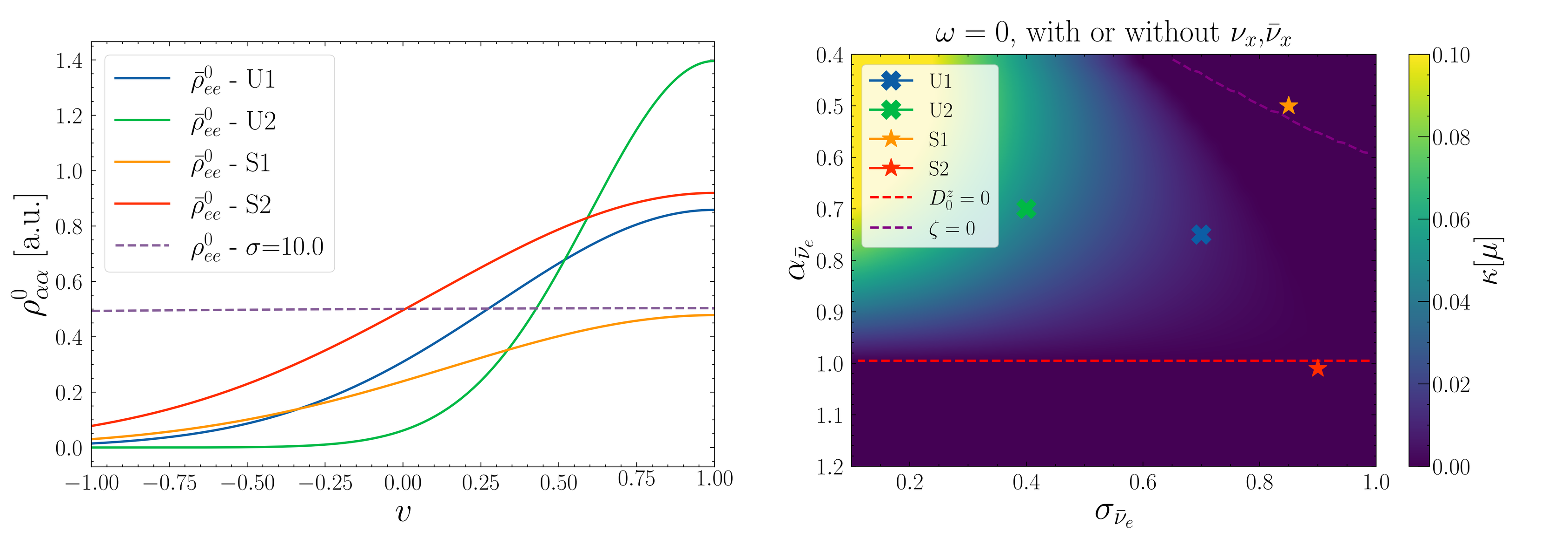

Figure 1. (Left) Initial angular distributions for the four benchmark cases adopted in this work. (Right) Contour plot of the growth rate κ for vanishing vacuum mixing in the plane spanned by $\sigma_{\bar{\nu}_e}$ and $\alpha_{\bar{\nu}_e}$.

Non-Vanishing Vacuum Mixing ($\omega \neq 0$)

1.Perturbative Expansion

Considering $\mu \gg \omega$, one can do the following expansion:

$$I_n(\Omega') = \mu\sum_{k=0}^{\infty}\int \mathrm{d}v v^n\frac{\bar{g}_v - (-1)^k g_v}{\Omega' + \mu v D_1^z}\left(\frac{\omega^c}{\Omega' + \mu v D_1^z}\right)^k. \quad (9)$$

- **Even powers** will depend on flavor lepton number (FLN) angular distributions, i.e. $\bar{\rho}_{\alpha\alpha}^0(v) - \rho_{\alpha\alpha}^0(v)$;
- **Odd powers** will depend on flavor particle number (FPN) angular distributions, i.e. $\bar{\rho}_{\alpha\alpha}^0(v) + \rho_{\alpha\alpha}^0(v)$.

2.Benchmark Scenarios

- **Fast unstable configurations** (e.g. scenario U1) show only a first-order correction ($\sim\omega$) with slope depending on the angular distribution of $\nu_x, \bar{\nu}_x$;
- **Fast stable configurations** (e.g. scenario S2) develop instabilities depending on the sign of ω and on the angular distribution of $\nu_x, \bar{\nu}_x$.

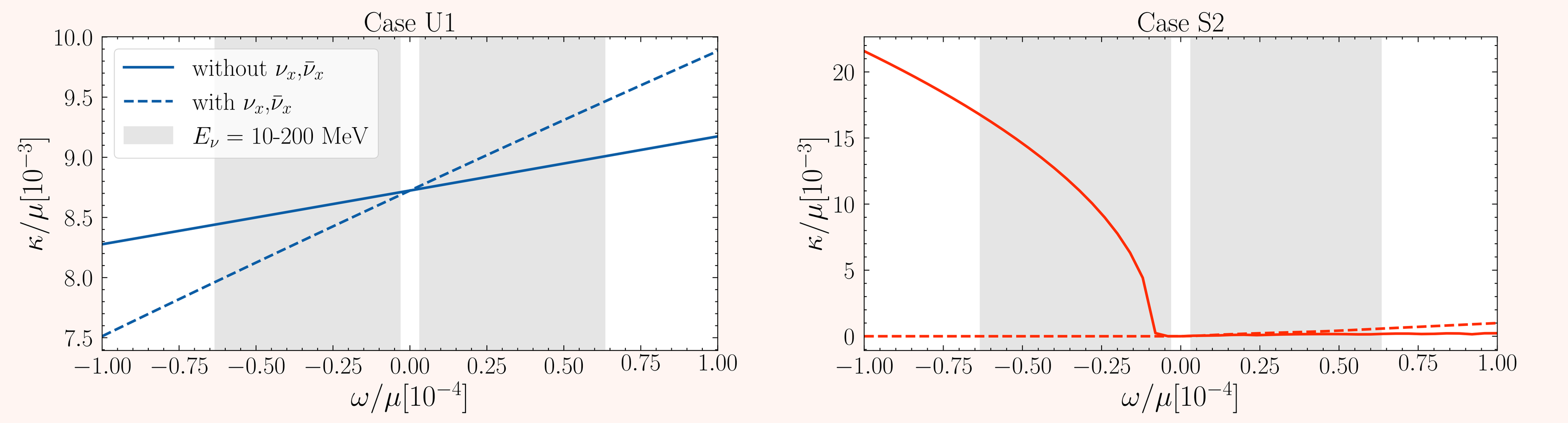

Figure 2. Growth rate κ as a function of ω with (dashed) and without (solid) non-electron neutrinos. (Left) Fast unstable benchmark scenario U1. (Right) Fast stable benchmark scenario S2.

3.Angular Configuration Space $\alpha_{\bar{\nu}_e} \times \sigma_{\bar{\nu}_e}$

One can see that instabilities tend to appear around the edge of total stability for the fast system (Fig. 1), i.e. $D_0^z = 0$ and $\zeta = 0$.

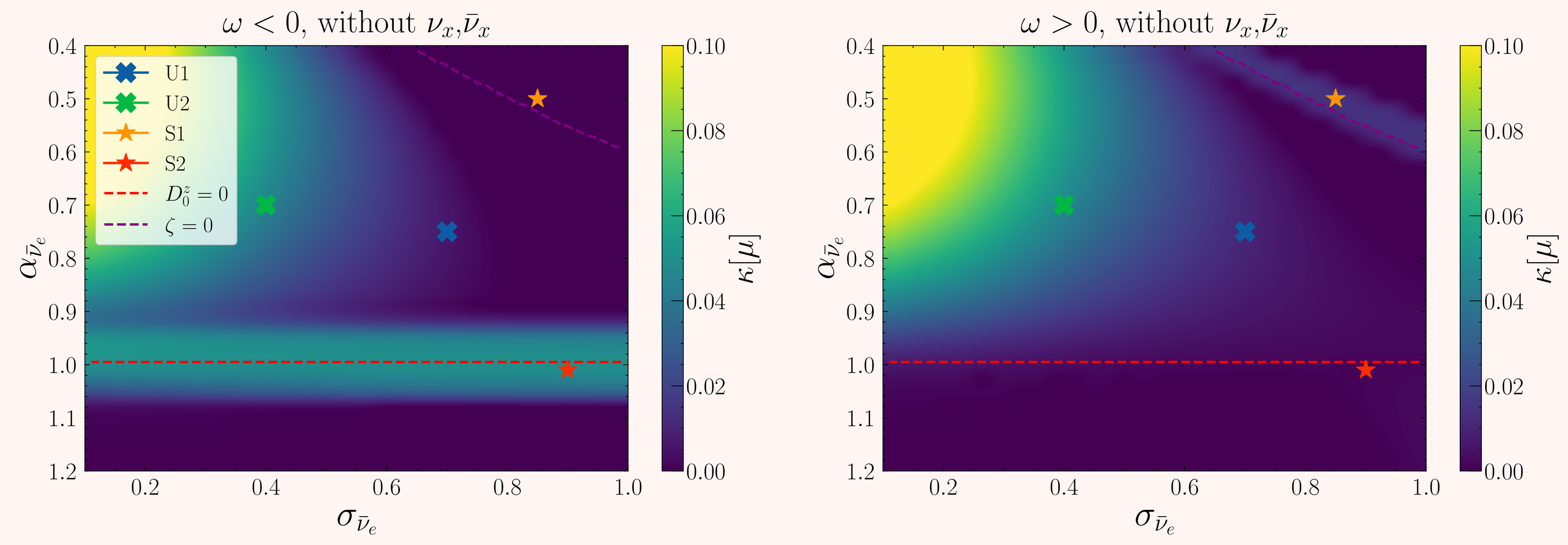

Figure 3. Contour plot of the growth rate κ in the plane spanned by $\sigma_{\bar{\nu}_e}$ and $\alpha_{\bar{\nu}_e}$ for the scenario without $\nu_x, \bar{\nu}_x$ and $\omega = -5\times 10^{-4}\mu$ (left) and $\omega = 5\times 10^{-4}\mu$ (right).

Conclusions

Although $\mu \gg \omega$ deep inside astrophysical environments, the effect of vacuum mixing is not negligible in realistic scenarios, in which it can induce sizable flavor instabilities. Using a perturbative approach, we have shown that $\omega \neq 0$ induces a dependency on FPN in addition to the usual FLN from the fast limit. We have also explored numerically where these new instabilities tend to appear in the space of angular configurations. Finally, we highlight that stability conditions developed in the fast limit $\omega = 0$ (e.g., ELN zero-crossing) do not fully capture instabilities in a realistic scenario. Therefore, one should be careful when using these instabilities criteria, e.g., in CCSN and NSM simulations.

References

[1] G. Sigl and G. Raffelt. "General kinetic description of relativistic mixed neutrinos". In: *Nucl. Phys. B* 406 (1993), pp. 423–451. DOI: 10.1016/0550-3213(93)90175-O.

[2] Pedro Dedin Neto and Ernesto Kemp. "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies". In: *Mod. Phys. Lett. A* 37.08 (2022), p. 2250048. DOI: 10.1142/S0217732322500481. arXiv: 2111.11480 [hep-ph].

[3] Pedro Dedin Neto, Irene Tamborra, and Shashank Shalgar. "Fast Conversion of Neutrinos: Energy Dependence of Flavor Instabilities". In: (Dec. 2023). arXiv: 2312.06556 [astro-ph.HE].

[4] Ian Padilla-Gay, Irene Tamborra, and Georg G. Raffelt. "Neutrino Flavor Pendulum Reloaded: The Case of Fast Pairwise Conversion". In: *Phys. Rev. Lett.* 128.12 (2022), p. 121102. DOI: 10.1103/PhysRevLett.128.121102. arXiv: 2109.14627 [astro-ph.HE].

Acknowledgements

This project has received support from the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP, Project No. 2022/01568-0 and No. 2022/09421-8), the Danmarks Frie Forskningsfond (Project No. 8049-00038B), the European Union (ERC, ANET, Project No. 101087058), and the Deutsche Forschungsgemeinschaft through Sonderforschungbereich SFB 1258 "Neutrinos and Dark Matter in Astro- and Particle Physics" (NDM).

dedin@ifi.unicamp.br