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# Professor Based ReWeight for GENIE Generator

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## 1. Why We Need ReWeight?

- A Prediction is made of multiple models convoluted together:

Mass ordering, mixing angle, CPV.

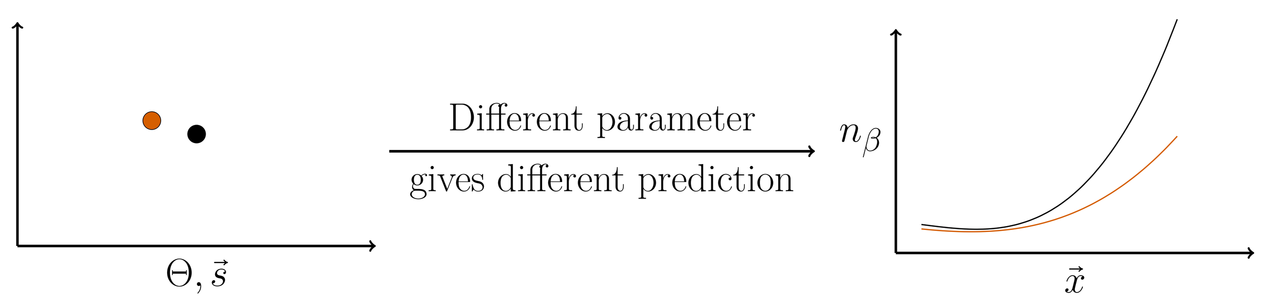
| Neutrino Flux Model | Oscillation Parameters | Interaction Model | Detector Model |
|---|---|---|---|
| $\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})$ | $P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})$ | $\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})$ | $R(\vec{x_0},\vec{x};\vec{s_R})$ |

Interaction Generator Internal Models

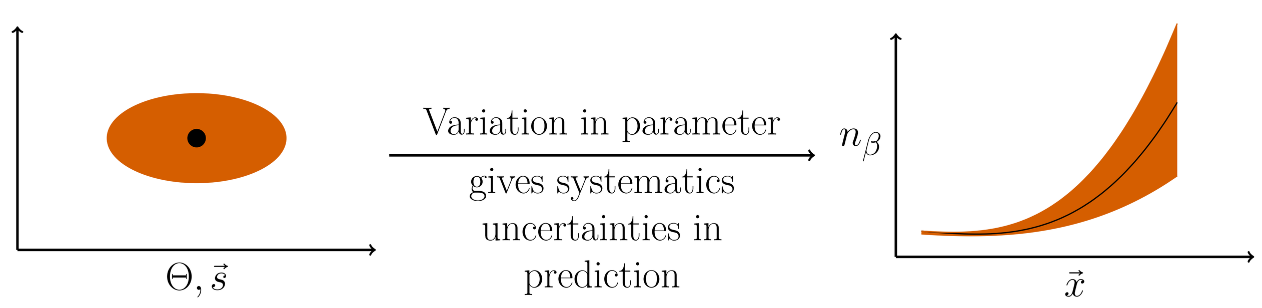
$$\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma}) = \sum\!\!\!\!\!\!\int_{c_i,\vec{y}} \mathcal{M}_1(E_\nu,\vec{x_0},c_i,\vec{y};\vec{s_\sigma})\cdots\mathcal{M}_n(E_\nu,\vec{x_0},c_i,\vec{y};\vec{s_\sigma})$$

$$n_\beta(\vec{x};\Theta,\vec{s}) = \sum\!\!\!\!\!\!\int_{\alpha,\vec{p_\nu},\vec{x_0}} \Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\,\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})\,R(\vec{x_0},\vec{x};\vec{s_R})$$

- It is near impossible to work analytically with convolution chain: **Monte Carlo** method is used to evaluate the prediction.
- Model prediction is parameter dependent




- Parameters comes with uncertainties, the uncertainties will be propagated to prediction




- **Being able to propagate uncertainties from parameters to prediction is crucial to understanding the systematics of the prediction.**
  CPU time and storage is expensive, we can't do full Monte Carlo for each parameter set: **ReWeight** is an approach to get model prediction for different parameter set using existing Monte Carlo samples.

## 2. The Mathematics Behind ReWeight

Mathematics tells us, Monte Carlo samples in "permitted phase space" $\lambda = (\vec{p_\nu},\vec{x_0}) \in \Omega$ to calculate convolution:

$$n_\beta(\vec{x};\Theta,\vec{s}) = \sum_\alpha \int \mathrm{d}\vec{p_\nu} \int \mathrm{d}\vec{x_0}\,\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\,\underbrace{\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})}_{\int_{\vec{y}}\mathcal{M}_1(E_\nu,\vec{x_0},\vec{y})\cdots\mathcal{M}_n(E_\nu,\vec{x_0},\vec{y})}\,R(\vec{x_0},\vec{x};\vec{s_R})$$
$$\approx \frac{V}{N}\sum_{i=1}^{N} f(\lambda_i)$$

where $V = \int \mathrm{d}\vec{p_\nu} \int \mathrm{d}\vec{x_0}$ is the volume of the phase space, $N$ is the number of samples. $f$ is defined as

$$f = \frac{\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\,\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})\,R(\vec{x_0},\vec{x};\vec{s_R})}{p(\lambda)}$$

$p(\lambda)$ is the probability density function we used to sample $\lambda$.

Usually, we start with "equal weight" sample (which is the usual case for GENIE [1] and NuWro):

$$p(\lambda) = k\cdot\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\,\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})\,R(\vec{x_0},\vec{x};\vec{s_R})$$
$$f = k^{-1}$$

If samples are already generated for given parameter set $s$, then the prediction for an altered parameter set $s'$ can be obtained by reweighting the samples:

$$n_\beta(\vec{x};\Theta,\vec{s}') \approx \frac{V}{N}\sum_{i=1}^{N}\frac{\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi}')\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta}')\,\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma}')\,R(\vec{x_0},\vec{x};\vec{s_R}')}{k\cdot\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})\,P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\,\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})\,R(\vec{x_0},\vec{x};\vec{s_R})}$$
$$= \frac{V}{N}\sum_{i=1}^{N} f(\lambda_i;s)\,w(\lambda,s,s')$$

with additional factor $w(\lambda,s,s') = \frac{\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi}')P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta}')\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma}')R(\vec{x_0},\vec{x};\vec{s_R}')}{\Phi_\alpha(\vec{p_\nu};\vec{s_\Phi})P_{\alpha\beta}(\vec{p_\nu};\vec{\Theta})\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})R(\vec{x_0},\vec{x};\vec{s_R})}$, that accounts for the sample $\{\lambda_i\}$ being sampled from $s$ and being reweighted to $s'$.

- But for interaction model, we need to rely on the internal model of the interaction generator, which is another convolution chain ($\mathcal{M}_i$)
  When only variation of $\vec{s_\sigma}$ is considered, the weight ideally should be

$$w(E_\nu,\vec{x_0};s,s') = \frac{\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma}')}{\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma})}$$

But the practical way is (since we can't get the cross section analytically):

$$w(E_\nu,\vec{x_0},\vec{y};s,s') = \frac{\mathcal{M}_1(E_\nu,\vec{x_0},\vec{y};\vec{s_\sigma}')\cdots\mathcal{M}_n(E_\nu,\vec{x_0},\vec{y};\vec{s_\sigma}')}{\mathcal{M}_1(E_\nu,\vec{x_0},\vec{y};\vec{s_\sigma})\cdots\mathcal{M}_n(E_\nu,\vec{x_0},\vec{y};\vec{s_\sigma})}$$

## 3. Limitations of Current Generator ReWeight

- **Weight calculation is highly dependent on internal models**:
  - Manpower needed to implement reweight code per each model;
  - Continuous maintaining effort needed to keep the reweight code up-to-date with the models.
- **"Internal phase space" variables $\vec{y}$ are needed to calculate the per-model weight**:
  - Need to save those information on event generation: hard or nearly-impossible for models like cascade and hadronization.
- **Can not reweight missing phase space into existence**:
  - Missing visible phase space ($\vec{x_0}$): hard cut on final state kinematics, this is intrinsically un-weightable.
  - **Missing internal phase space** ($\vec{y}$): this is a limitation of the traditional reweight.

## 4. Professor Based ReWeight

But we don't essentially need to get the weight with the internal models: Monte Carlo itself can give us information about $\sigma_\beta(E_\nu,\vec{x_0};\vec{s_\sigma}) = \frac{\mathrm{d}^n\sigma_\beta(E_\nu|\vec{s_\sigma})}{\mathrm{d}\vec{x}^n}$.

Considering:
- Auto-correlation of $\vec{x}$ (e.g. conservation law);
- Some varied parameter only affect a subset of models (e.g. changes in QEL $M_A$ only affect the $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}$);

we could replace the differential cross section from the "final state" $\vec{x}$ to the "observable" $\vec{K}$:

$$\frac{\mathrm{d}^n\sigma_\beta(E_\nu\mid\vec{s_\sigma})}{\mathrm{d}\vec{x}^n} = \frac{\mathrm{d}^N\sigma_\beta(E_\nu\mid\vec{s_\sigma})}{\mathrm{d}\vec{K}^N}f(\vec{K};\vec{x})$$

where $f(\vec{K};\vec{x})$ contains the Jacobian of the transformation and the contribution from integrated out unrelated components of $\vec{x}$, which would be cancelled when we calculate the ratio to get the weight.

**With those in mind, we can propose a new reweight method based on parameterized differential cross section in terms of observable $\vec{K}$:**
- Scan different $\{s_{\sigma,i}\} \in S$, generate Monte Carlo for each $s_{\sigma,i}$ scanned;
- Process the generated sample, represent the estimation of $\frac{\mathrm{d}^N\sigma_\beta(E_\nu|\vec{s_\sigma})}{\mathrm{d}\vec{K}^N}$ by a $N+1$ dimensional histogram binned by $\vec{K}$ and $E_\nu$ for each sample;
- For each given $(\vec{K},E_\nu)$ bin $j$, consider the value of $\frac{\mathrm{d}^N\sigma_\beta(E_\nu|\vec{s_\sigma})}{\mathrm{d}\vec{K}^N}$ on different $s_{\sigma,i}$ as a function of $s_\sigma$, use polynomial interpolation to get polynomial function $P_j(s_\sigma)$;

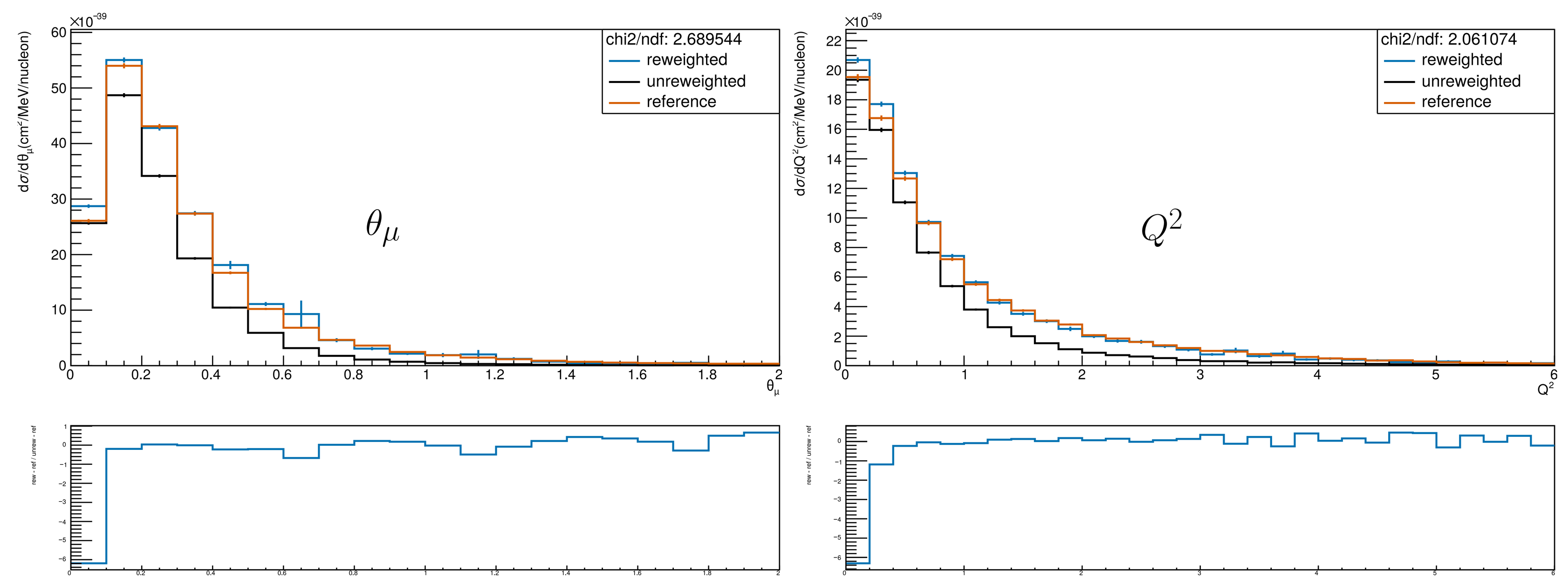
**Professor2 [3] is a tool to provide essential support to scan the parameter space and do the interpolation on the large database generated from the Monte Carlo.** Many tools in GENIE tunning [4] are reused to parameterize the differential cross section.
All $P_j(s_\sigma)$ together, make up a full parameterized differential cross section. When the ReWeight code handles an event:
- For each event, calculate the $\vec{K}$ and $E_\nu$, locate the corresponding bin $j$;
- Use the polynomial function $P_j(s_\sigma)$ to get the differential cross section for the event;
- Weight is calculated as the ratio of the differential cross section

$$w = \frac{P_j(s'_\sigma)}{P_j(s_\sigma)}$$

**In this way we can remove dependence on internal model details and the internal phase information of the ReWeight process.**

## 5. Simple Proof of Concept for New ReWeight

Starting with GENIE Tune G18_10a_02_11b, observable $\vec{K}$ is chosen as $(\vec{p_\mu},W,E_\nu)$.
ReWeight from the default ($M_A^{\mathrm{QE}} = 0.994989$, $M_A^{\mathrm{RES}} = 1.088962$) to $M_A^{\mathrm{QE}} = 0.77$, $M_A^{\mathrm{RES}} = 1.64$:




**The reweight can get the distribution of $\theta_\mu$ and $Q^2$ correct.**

## 6. Summary

- New Professor2 based reweight would extend the capability of current reweight tool.
  - **Less maintenance effort needed**
  - **More systematics will become reweightable**
- Currently under active development: planed to be **next major upgrade** for GENIE ReWeight.
- Cooperate with tunning efforts [2, 4], this will enable full propagation of errors from **experimental data to model** and **model to prediction**

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[3] VA Muradyan, NZ Akopov, HM Ghumaryan, and GA Karyan. "professor2" package for tuning the parameters of fragmentation process in $e^+e^-$ annihilation. *Journal of Contemporary Physics (Armenian Academy of Sciences)*, 56(2):85–90, 2021.
[4] Júlia Tena-Vidal et al. Neutrino-nucleon cross-section model tuning in GENIE v3. *Phys. Rev. D*, 104(7):072009, 2021.