JYU. Since 1863.

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Kimmo Kainulainen [a,b], **Harri Parkkinen** [a,b]

[a] Department of Physics, University of Jyväskylä, Finland
[b] Helsinki Institute of Physics, University of Helsinki, Finland
Email: harri.h.parkkinen@jyu.fi

Quantum Transport Theory for mixing Neutrinos

ABSTRACT

We derive quantum kinetic equations for mixing neutrinos including consistent forward scattering terms and collision integrals for coherent neutrino states. Our derivation is valid for arbitrary neutrino masses and kinematics, it includes the local coherence effects, and a comprehensive set of generalized Feynman rules for computing the coherent collision integrals. We also discuss the importance of helicity coherence and particle-antiparticle coherence in the case of adiabatic background fields using field theoretical methods. Our results can be used, for example, to model neutrino distributions accurately in hot and dense environments and to study the production and decay of heavy neutrinos in colliders.

QUANTUM KINETIC EQUATIONS

Coherently mixing neutrinos can be described by a set of Kadanoff-Baym (KB) equations for real-time valued correlation functions. These KB equations are manifestly non-local and feature a direct coupling between the statistical functions and the pole functions. In order to reduce them to a single quantum kinetic equation (QKE), one must both localize them and decouple the pole equations from the statistical equations. In the decoupling problem the key idea is to split the statistical function into a background, which is strongly coupled to the pole functions, and to a perturbation part whose equation formally decouples. In turn, from Wigner-space point of view the localization task is to curtail the infinite expansion in gradients which can be justified by assumption of adiabaticity. The resulting local and decoupled QKEs, whose solutions are characterized by eigenfrequencies, read [1]

$$\boxed{\begin{aligned}\partial_t f^{<ee'}_{\boldsymbol{k}hij} + (\mathcal{V}^{e'e}_{\boldsymbol{k}hij})_{aa'}\boldsymbol{k}\cdot\nabla f^{<aa'}_{\boldsymbol{k}hij} &= -2i\Delta\omega^{ee'}_{\boldsymbol{k}ij} f^{<ee'}_{\boldsymbol{k}hij} + \overline{\mathcal{C}}^{<ee'}_{\mathrm{H},\boldsymbol{k}hij}\\ &\quad - i(\mathcal{W}^{ee'}_{\boldsymbol{k}hij})^l_a f^{<ae'}_{\boldsymbol{k}hlj} + i[(\mathcal{W}^{e'e}_{\boldsymbol{k}hji})^l_a]^* f^{<ea}_{\boldsymbol{k}hli},\end{aligned}}$$

where $f^{<ee'}_{\boldsymbol{k}hij}$ are neutrino distribution functions, and sum over the repeated indices a (energy sign) and l (flavor) is understood. We defined the oscillation frequency as

$$2\Delta\omega^{ee'}_{\boldsymbol{k}ij} \equiv e\omega_{\boldsymbol{k}i} - e'\omega_{\boldsymbol{k}j},$$

the forward scattering coefficient tensor is given by

$$(\mathcal{W}^{ee'}_{\boldsymbol{k}hij})^l_a \equiv \mathrm{Tr}\Big[P^{e'e}_{\boldsymbol{k}hji}\bar{\Sigma}^{\mathrm{H}}_{\mathrm{eff}\boldsymbol{k}il}P^{ae'}_{\boldsymbol{k}hlj}\Big],$$

and

$$(\mathcal{V}^{e'e}_{\boldsymbol{k}hij})_{aa'} = \delta_{a'e'}\mathcal{V}^{eae'}_{\boldsymbol{k}hij}, + \delta_{ae}\mathcal{V}^{a'e'e}_{\boldsymbol{k}hji},$$

with

$$\mathcal{V}^{abc}_{\boldsymbol{k}hij} \equiv \frac{1}{2}N^{ac}_{\boldsymbol{k}ij}N^{bc}_{\boldsymbol{k}ij}\Big(\frac{1}{\omega_{\boldsymbol{k}i}}\Big[\frac{a}{(N^{bc}_{\boldsymbol{k}ij})^2} + \frac{b}{(N^{ac}_{\boldsymbol{k}ij})^2}\Big] - \frac{c}{\omega_{\boldsymbol{k}j}}\delta_{a-b}\Big).$$

These QKEs describe both flavor and particle-antiparticle oscillations for arbitrary neutrino masses with arbitrary interactions in backgrounds that are only constrained to be adiabatic in space. This generality results to the complex tensor structures in the QKEs.

In the UR-limit when neglecting the particle-antiparticle oscillations, our QKEs reduce to a familiar form of density matrix evolution equation:

$$\boxed{\partial_t f^e_{\boldsymbol{k}h} + \frac{1}{2}\{\bar{\boldsymbol{v}}\cdot\nabla, f^e_{\boldsymbol{k}h}\} = -i[H^e_{\boldsymbol{k}h}, f^e_{\boldsymbol{k}h}] + \overline{\mathcal{C}}^e_{\boldsymbol{k}h},}$$

with $\bar{\boldsymbol{v}} \equiv \delta_{ij}\boldsymbol{k}/\omega_{\boldsymbol{k}i}$.

COLLISION INTEGRALS

Computation of the collision term with flavor and particle-antiparticle mixing for arbitrary neutrino masses and kinematics has been an unsolved problem so far. However, using our formalism this task is straightforward, and the collision term can be written simply as [2]

$$\overline{\mathcal{C}}^{<ee'}_{\mathrm{H},\boldsymbol{k}hij} = \sum\nolimits_Y \frac{1}{2\bar{\omega}^{aa'}_{\boldsymbol{k}lj}}\int \mathrm{dPS}_3 \times\Big[(\mathcal{M}^2)^{e'e}_{\boldsymbol{k}hij\{\boldsymbol{p}_i,Y\}}\Lambda_{\boldsymbol{k}hj\{\boldsymbol{p}_i,Y\},x} + h.c.\Big],$$

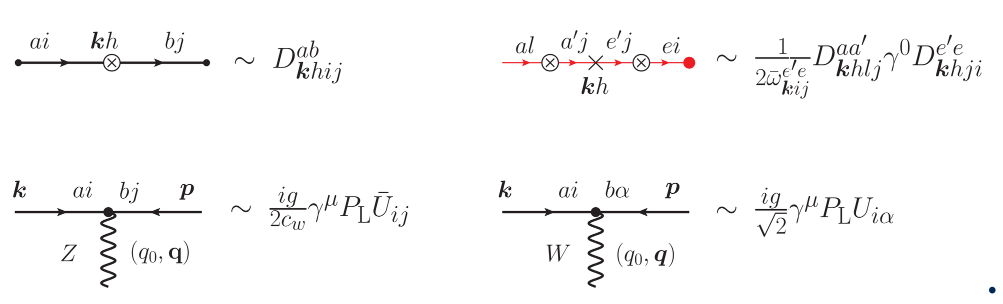
where we collected all summed indices into curly brackets, $Y = \{\mathrm{X}_i, h', a, a', l\}$, defined a shorthand notation $A_{\mathrm{X}_i} \equiv A^{a_i a'_i}_{h_i l_i l'_i}$, the factor containing all particle distribution functions is $\Lambda \equiv \Lambda^> - \Lambda^<$ with

$$\Lambda^{<,>}_{\boldsymbol{k}hj\{\boldsymbol{p}_i,Y\},x} = f^{<,>}_{\mathrm{X}_1\boldsymbol{p}_1}(x)\, f^{>,<}_{\mathrm{X}_2\boldsymbol{p}_2}(x)\, f^{<,>}_{\mathrm{X}_3\boldsymbol{p}_3}(x)\, f^{>,<aa'}_{\boldsymbol{k}h'lj}(x),$$

the phase space factor is defined as

$$\int \mathrm{dPS}_3 \equiv \int\Big[\prod_{i=1,3}\frac{\mathrm{d}^3\boldsymbol{p}_i}{(2\pi)^3 2\bar{\omega}_{\boldsymbol{p}_i l_i l'_i}}\Big] \times(2\pi)^4\delta^4(k^a_i + p^{a_2}_{2\,l_2} - p^{a'_1}_{1\,l'_1} - p^{a'_3}_{3\,l'_3}),$$

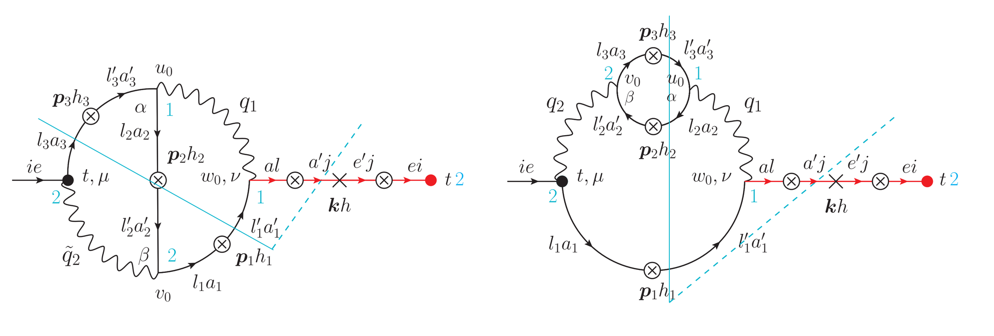
and the (effective) invariant matrix element squared $(\mathcal{M}^2)^{e'e}_{\boldsymbol{k}hij\{\boldsymbol{p}_iY\}}$ contains all the diracology which is computed using the following set of generalized Feynman rules:



$$\sim D^{ab}_{\boldsymbol{k}hij} \qquad \sim \frac{1}{2\bar{\omega}^{e'e}_{\boldsymbol{k}ij}}D^{aa'}_{\boldsymbol{k}hlj}\gamma^0 D^{e'e}_{\boldsymbol{k}hji}$$

$$\sim \frac{ig}{2c_w}\gamma^\mu P_{\mathrm{L}}\bar{U}_{ij} \qquad \sim \frac{ig}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}U_{i\alpha}.$$

Here $D^{ab}_{\boldsymbol{k}hij} \equiv 2\bar{\omega}^{ab}_{\boldsymbol{k}ij}P^{ab}_{\boldsymbol{k}hij}\gamma^0 = ab\hat{N}^{ab}_{\boldsymbol{k}ij}P_{\boldsymbol{k}h}(\not{k}^a_i + m_i) \times(\not{k}^b_j + m_j)$ with $(k^a_i)^\mu \equiv (a\omega_{\boldsymbol{k}i}, \boldsymbol{k})$, $\hat{N}^{ab}_{\boldsymbol{k}ij} \equiv N^{ab}_{\boldsymbol{k}ij}\bar{\omega}^{ab}_{\boldsymbol{k}ij} \times(2\omega^a_{\boldsymbol{k}i}\omega^b_{\boldsymbol{k}j})^{-1}$, $2\bar{\omega}^{ab}_{\boldsymbol{k}ij} \equiv a\omega_{\boldsymbol{k}i} + b\omega_{\boldsymbol{k}j}$, and U_{ij} is the mixing matrix. The red propagator is called the dependent momentum propagator, and is of special interest since it depends on the momentum related to which the QKEs are solved. In contrast to the usual neutrino QKE's, both direct and interference terms of s, t and u channels contribute to the collision term in our formalism:



In environments where collisions between neutrinos are important, these flavor off-diagonal collisions should be taken into account.

WEIGHT FUNCTIONS

We can never have complete and exact information about a system. This means that the physical quantities that one can study are some smeared-out objects which carry information about the preparation of the system into the theory. Quantitatively this means that the correlator (i.e. Wightman function $S_{ij}(\bar{k}, \bar{x})$), which consists of physically measurable quantities $\bar{k}$ and $\bar{x}$, is weighted average of the original correlator:

$$S_{ij}(\bar{k}, \bar{x}) \equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{\mathrm{d}^4 x}{(2\pi)^4}\mathcal{W}(\bar{k}, \bar{x}; k, x)S_{ij}(k, x),$$

where $\mathcal{W}$ is the weight function encoding the observationally accessible information about the system. Weight functions generalize our formalism and give an adjustable and quantitative way to take the effects of neutrino production and detection processes into account when describing neutrino evolution.

For example, the QKEs presented here corresponds to using the wieght function

$$\mathcal{W}(\bar{k}, \bar{x}; k, x) = (2\pi)^3\delta^3(\bar{\boldsymbol{k}} - \boldsymbol{k})\,\delta^4(\bar{x} - x).$$

This setup is suitable for studying problems including particle-antiparticle mixing, such as particle production, where the particle-and antiparticle mixing shells are widely separated.

COHERENCE EFFECTS

From our formalism it is evident that neither the particle-antiparticle coherence nor the helicity coherence is relevant for most of the neutrino physics, including e.g. supernova physics. That is, the collective phenomena for neutrino-antineutrino or helicity coherence are negligible. However, the flavor structure of neutrino-neutrino collisions is more rich than considered in the literature.

CONCLUSIONS

- Derived QKEs include flavor and particle-antiparticle xoherence, and are valid for arbitrary neutrino masses and kinematics.
- Generalized Feynman rules provide systematic and simple way to compute collision terms with flavor and particle-antiparticle coherences.
- Weight functions carry prior information of the system into the theory, giving quantitative way to study the effect of neutrino production and detection processes to neutrino evolution.

REFERENCES

[1] K. Kainulainen and H. Parkkinen, *Quantum transport theory for neutrinos with flavor and particle-antiparticle mixing*, JHEP **02** (2024) 217 [arXiv:2309.00881 [hep-ph]].

[2] K. Kainulainen and H. Parkkinen, *Coherent collision integrals for relativistic quantum transport theory*, In preparation .