Flows Mitigate Gaussian Error Approximations in Neutrino Cross-Section Measurements

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Companion paper

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Introduction

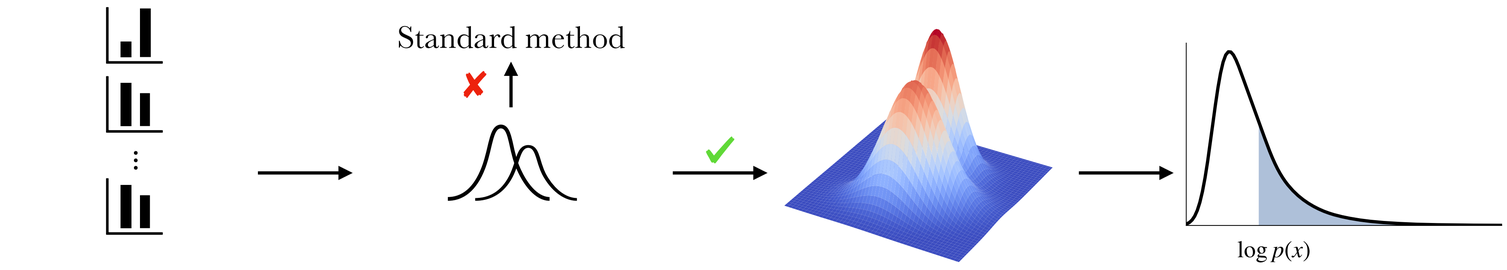
Neutrino-nucleus cross-section results are typically presented as a binned measurement.

To quantify uncertainties, ensembles of measurements (or toys) are produced, varying parameters controlling systematic uncertainties (e.g detector simulation).

In experimental data releases, the toys are usually assumed to follow a Gaussian, which may lead to inaccurate uncertainty estimation in systematics-limited measurements.

We show ways of making neutrino interaction model predictions without relying on Gaussian approximations by learning the underlying distribution with flows.

Overview



Vary uncertainties of the measurement

1. Check for non-Gaussianity using classifier test

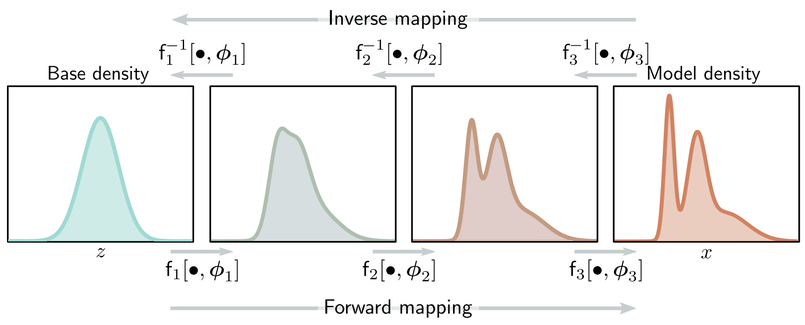
2. Estimate density with normalizing flows

3. Make ν-interaction model predictions

Method

Classifier Test
A supervised classifier estimates the overlap between two distributions. Use as a test of normality of toys and overlap with generated data.

Density Estimation
Learn the distribution of the toys using normalizing flows [1, 2].



Neutrino Model Predictions
Use the (log) density to directly compare neutrino interaction model predictions:

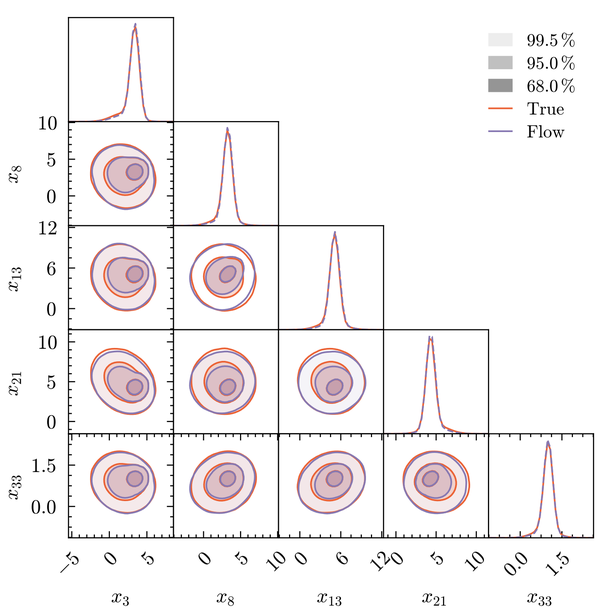
$$\log p(x_1) > \log p(x_2)$$

To make neutrino model selection for a model $\hat{x}$ given N toys:

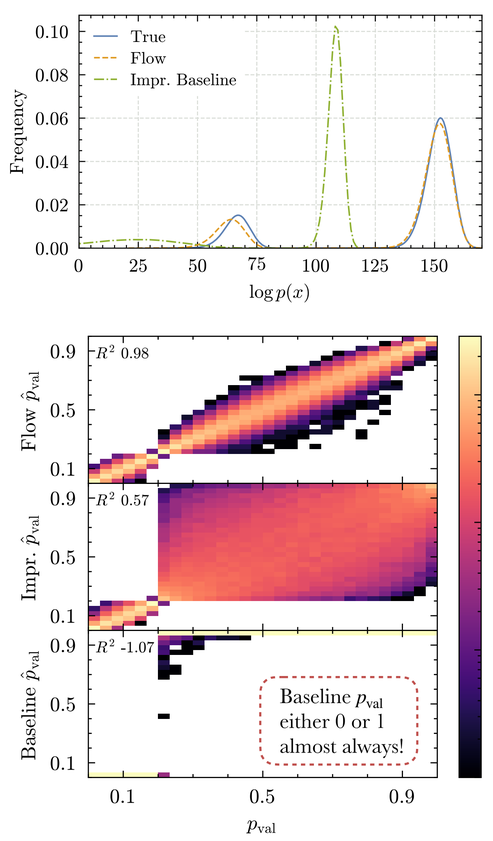
$$p_{\text{val}} \approx \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}(\log p(x_i) \geq \log p(\hat{x}))$$

50D Gaussian Mixture

We test our method with a set of quasi-realistic toys with known likelihood.

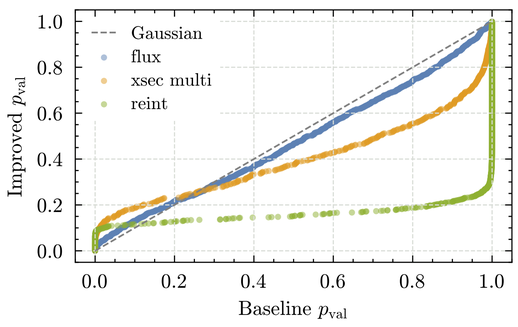


The normalizing flow achieves excellent high-dimensional density estimation!



Since we know the true PDF we can compare p-value estimates.

Realistic Measurements

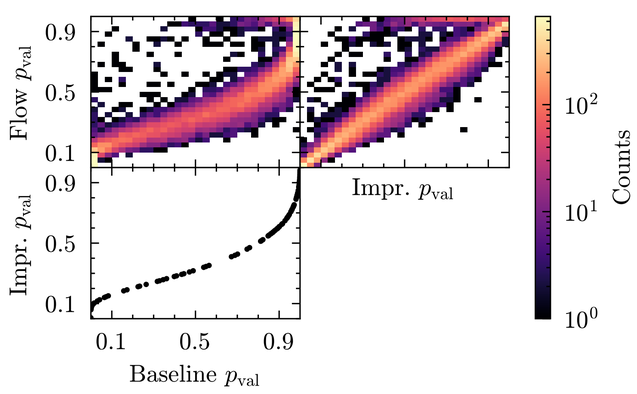


Classifier test accuracy (C2ST) on toys from MicroBooNE (wrt a Gaussian). Higher CS2T indicates non-Gaussian toys.

Type	N_{toys}	C2ST	p_{val}
Cross-Section	500	66.3 ± 4.6	0.01
Re-interaction	1000	92.4 ± 1.1	0.00
Flux	1000	52.2 ± 5.0	0.33

Using toys from a recent result by MicroBooNE [3], we see that that uncertainties cannot be accounted by a Gaussian.

We also use toys from a T2K-like simulation projected to 5 times current statistics. In this case we also find that the Gaussian approximation does not hold. We further show the normalizing flow can be trained to account for non-Gaussian features.



Conclusion

We show a way of doing neutrino interaction model comparisons to cross-section measurements without relying on Gaussian approximations.

To do this we train normalizing flows to estimate the distribution of the toys and use the estimated log-probability as a robust way of making model comparisons or model selections.

We encourage experiments to publish their toys (as recently done for the first time by MicroBooNE).

Visit our project page for our preprint and upcoming software package: https://radiradev.github.io/nu-flows/

References

[1] Tabak, Esteban G., and Cristina V. Turner. "A family of nonparametric density estimation algorithms." *Communications on Pure and Applied Mathematics* 66.2 (2013): 145-164.
[2] Prince, Simon JD. Understanding Deep Learning. MIT press, 2023.
[3] Abratenko, P., et al. "Measurement of double-differential cross sections for mesonless charged-current muon neutrino interactions on argon with final-state protons using the MicroBooNE detector." *arXiv preprint arXiv:2403.19574* (2024)