Complete one-loop renormalization-group equations in the seesaw effective field theories

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Alexander von HUMBOLDT STIFTUNG

I. Seesaw Effective Field Theories

Seesaw mechanisms are the simplest and the most natural ways to explain **tiny neutrino masses** and may also elegantly account for **the matter-antimatter asymmetry of the Universe**.

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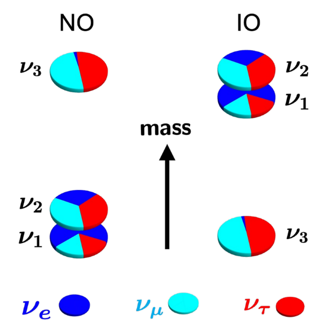
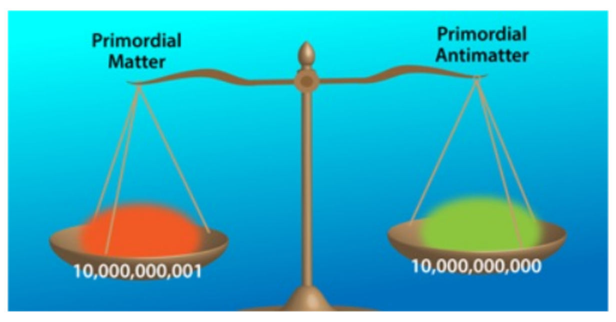
Seesaw Mechanisms





$\Lambda_{\rm SS}$ — Matching — $C_i(\Lambda_{\rm SS})$

Seesaw EFTs — RGE Running

- Resummation of large logs
- Bridge among parameters at different scales

$\Lambda_{\rm EW}$ — $C_i(\Lambda_{\rm EW})$ — Mapping → **Low-energy Observables**

To achieve the **complete** one-loop RGEs up to $\boldsymbol{O(1/\Lambda_{\rm SS}^2)}$ in the **seesaw EFT** induced by the **type-I seesaw mechanism**

II. Matching at the Tree Level

The type-I seesaw mechanism: Three singlet right-handed neutrinos

$$\mathcal{L}_{\rm SS} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i}\partial\!\!\!/ N_{\rm R} - \left(\frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + {\rm h.c.} \right)$$

Integrating out heavy right-handed neutrinos at the **tree level**

The tree-level seesaw EFT up to $\boldsymbol{O(1/\Lambda_{\rm SS}^2)}$:

$$\mathcal{L}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(C_5^{\alpha\beta} \mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.} \right) + C^{(1)\alpha\beta}_{H\ell} \mathcal{O}^{(1)\alpha\beta}_{H\ell} + C^{(3)\alpha\beta}_{H\ell} \mathcal{O}^{(3)\alpha\beta}_{H\ell}$$

Dim-5 $\mathcal{O}^{(5)}_{\alpha\beta} = \overline{\ell_{\alpha{\rm L}}} \widetilde{H} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta{\rm L}}$ The Weinberg operator — Neutrino masses

Dim-6
$$\mathcal{O}^{(1)}_{\alpha\beta} = \left(\overline{\ell_{\alpha{\rm L}}} \gamma^\mu \ell_{\beta{\rm L}} \right) \left(H^\dagger {\rm i} \overleftrightarrow{D}_\mu H \right)$$
$$\mathcal{O}^{(3)}_{\alpha\beta} = \left(\overline{\ell_{\alpha{\rm L}}} \gamma^\mu \sigma^I \ell_{\beta{\rm L}} \right) \left(H^\dagger {\rm i} \overleftrightarrow{D}^I_\mu H \right)$$

Unitarity violation of the lepton flavor mixing

The corresponding **Wilson coefficients** at the matching scale $\boldsymbol{\mu_M \sim \Lambda_{\rm SS} = O(M_R)}$

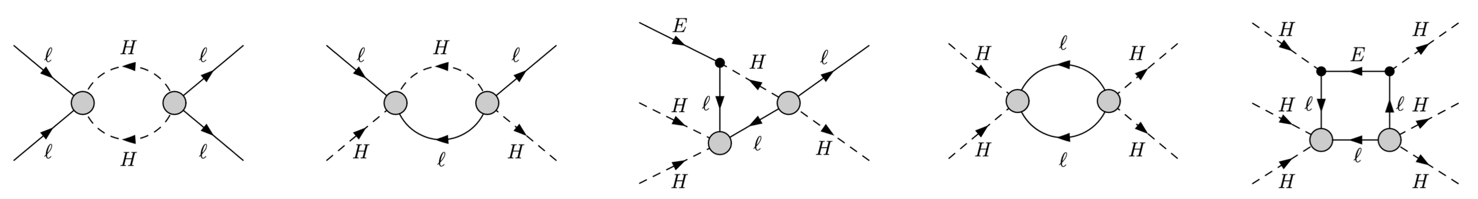
$$C_5\left(\mu_{\rm M}\right) = Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T} \qquad C^{(1)}_{H\ell}\left(\mu_{\rm M}\right) = -C^{(3)}_{H\ell}\left(\mu_{\rm M}\right) = \frac{1}{4} Y_\nu M_{\rm R}^{-2} Y_\nu^\dagger$$

III. One-loop RGEs in the Seesaw EFT

The general structure of the RGEs up to $\boldsymbol{O(1/\Lambda_{\rm SS}^2)}$

$$16\pi^2 \mu \frac{{\rm d}C^{(5)}_i}{{\rm d}\mu} = \gamma'_{ij} C^{(5)}_j \qquad 16\pi^2 \mu \frac{{\rm d}C^{(6)}_i}{{\rm d}\mu} = \gamma_{ij} C^{(6)}_j + \widetilde{\gamma}^i_{jk} C^{(5)}_j C^{(5)}_k$$

- Single insertion of the dim-5 and dim-6 operators (Too many to show)
- All double insertions of the dim-5 operator



1PI diagrams → UV divergences (counterterms) → RGEs

a) Results for the RGEs of the SM couplings $T \equiv {\rm tr}\left(Y_l Y_l^\dagger + 3 Y_{\rm u} Y_{\rm u}^\dagger + 3 Y_{\rm d} Y_{\rm d}^\dagger \right)$

$$16\pi^2 \mu \frac{{\rm d}g_1}{{\rm d}\mu} = \frac{41}{6} g_1^3 , \qquad 16\pi^2 \mu \frac{{\rm d}Y_l}{{\rm d}\mu} = \left[-\frac{15}{4} g_1^2 - \frac{9}{4} g_2^2 + T + \frac{3}{2} Y_l Y_l^\dagger - 2m^2 \left(C^{(1)}_{H\ell} + 3C^{(3)}_{H\ell} \right) \right] Y_l ,$$

$$16\pi^2 \mu \frac{{\rm d}g_2}{{\rm d}\mu} = -\frac{19}{6} g_2^3 , \qquad 16\pi^2 \mu \frac{{\rm d}Y_{\rm u}}{{\rm d}\mu} = \left[-\frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8 g_s^2 + T + \frac{3}{2}\left(Y_{\rm u} Y_{\rm u}^\dagger - Y_{\rm d} Y_{\rm d}^\dagger \right) \right] Y_{\rm u} ,$$

$$16\pi^2 \mu \frac{{\rm d}g_s}{{\rm d}\mu} = -7 g_s^3 . \qquad 16\pi^2 \mu \frac{{\rm d}Y_{\rm d}}{{\rm d}\mu} = \left[-\frac{5}{12} g_1^2 - \frac{9}{4} g_2^2 - 8 g_s^2 + T - \frac{3}{2}\left(Y_{\rm u} Y_{\rm u}^\dagger - Y_{\rm d} Y_{\rm d}^\dagger \right) \right] Y_{\rm d} ,$$

$$16\pi^2 \mu \frac{{\rm d}\lambda}{{\rm d}\mu} = 24\lambda^2 - 3\lambda\left(g_1^2 + 3g_2^2 \right) + \frac{3}{8}\left(g_1^2 + g_2^2 \right)^2 + \frac{3}{4} g_2^4 + 4\lambda T - 2{\rm tr}\left[\left(Y_l Y_l^\dagger \right)^2 + 3\left(Y_{\rm u} Y_{\rm u}^\dagger \right)^2 + 3\left(Y_{\rm d} Y_{\rm d}^\dagger \right)^2 \right] + m^2 {\rm tr}\left(2C_5 C_5^\dagger - \frac{8}{3} g_2^2 C^{(3)}_{H\ell} + 8 C^{(3)}_{H\ell} Y_l Y_l^\dagger \right) .$$

III. One-loop RGEs in the Seesaw EFT

b) Results for the RGEs of higher-dimensional operators

Dim-5
$$16\pi^2 \mu \frac{{\rm d}C_5}{{\rm d}\mu} = \left(-3g_2^2 + 4\lambda + 2T \right) C_5 - \frac{3}{2}\left[Y_l Y_l^\dagger C_5 + C_5 \left(Y_l Y_l^\dagger \right)^{\rm T} \right]$$

Dim-6
$$16\pi^2 \mu \frac{{\rm d}C^{(1)}_{H\ell}}{{\rm d}\mu} = -\frac{3}{2} C_5 C_5^\dagger + \frac{2}{3} g_1^2 {\rm tr}\left(C^{(1)}_{H\ell} \right) \mathbb{1} + \left(\frac{1}{3} g_1^2 + 2T \right) C^{(1)}_{H\ell} + \frac{1}{2}\left[\left(4C^{(1)}_{H\ell} + 9C^{(3)}_{H\ell} \right) Y_l Y_l^\dagger + Y_l Y_l^\dagger \left(4C^{(1)}_{H\ell} + 9C^{(3)}_{H\ell} \right) \right]$$

$$16\pi^2 \mu \frac{{\rm d}C^{(3)}_{H\ell}}{{\rm d}\mu} = C_5 C_5^\dagger + \frac{2}{3} g_2^2 {\rm tr}\left(C^{(3)}_{H\ell} \right) \mathbb{1} + \left(-\frac{17}{3} g_2^2 + 2T \right) C^{(3)}_{H\ell} + \frac{1}{2}\left[\left(3C^{(1)}_{H\ell} + 2C^{(3)}_{H\ell} \right) Y_l Y_l^\dagger + Y_l Y_l^\dagger \left(3C^{(1)}_{H\ell} + 2C^{(3)}_{H\ell} \right) \right]$$

Referring to our paper [JHEP 05 (2023) 044] for more results for

1) the RGEs of the other **17 dim-6 operators** absent at the tree level
2) the seesaw EFTs induced by the **type-II** and **type-III** seesaw mechanisms

IV. One-loop RGEs of the Flavor Mixing Parameters

After spontaneous symmetry break (in the lepton mass basis)

$$\mathcal{L}^{\rm CC}_{\rm SEFT} = \left(\frac{g_2}{\sqrt{2}} \overline{l_{\rm L}} \gamma^\mu V \nu_{\rm L} W^-_\mu + {\rm h.c.} \right) \quad \text{Non-unitary lepton flavor mixing}$$

$$\mathcal{L}^{\rm NC}_{\rm SEFT} = \frac{g_2}{2c_{\rm W}} \overline{\nu_{\rm L}} \gamma^\mu N^\dagger N \nu_{\rm L} Z_\mu - \frac{g_2}{2c_{\rm W}} \overline{l_{\rm L}} \gamma^\mu \left[\left(1 - 2s^2_{\rm W} \right) + \left(\eta' - 2\eta \right) \right] l_{\rm L} Z_\mu + \frac{g_2}{c_{\rm W}} s^2_{\rm W} \overline{l_{\rm R}} \gamma^\mu l_{\rm R} Z_\mu$$

FCNC

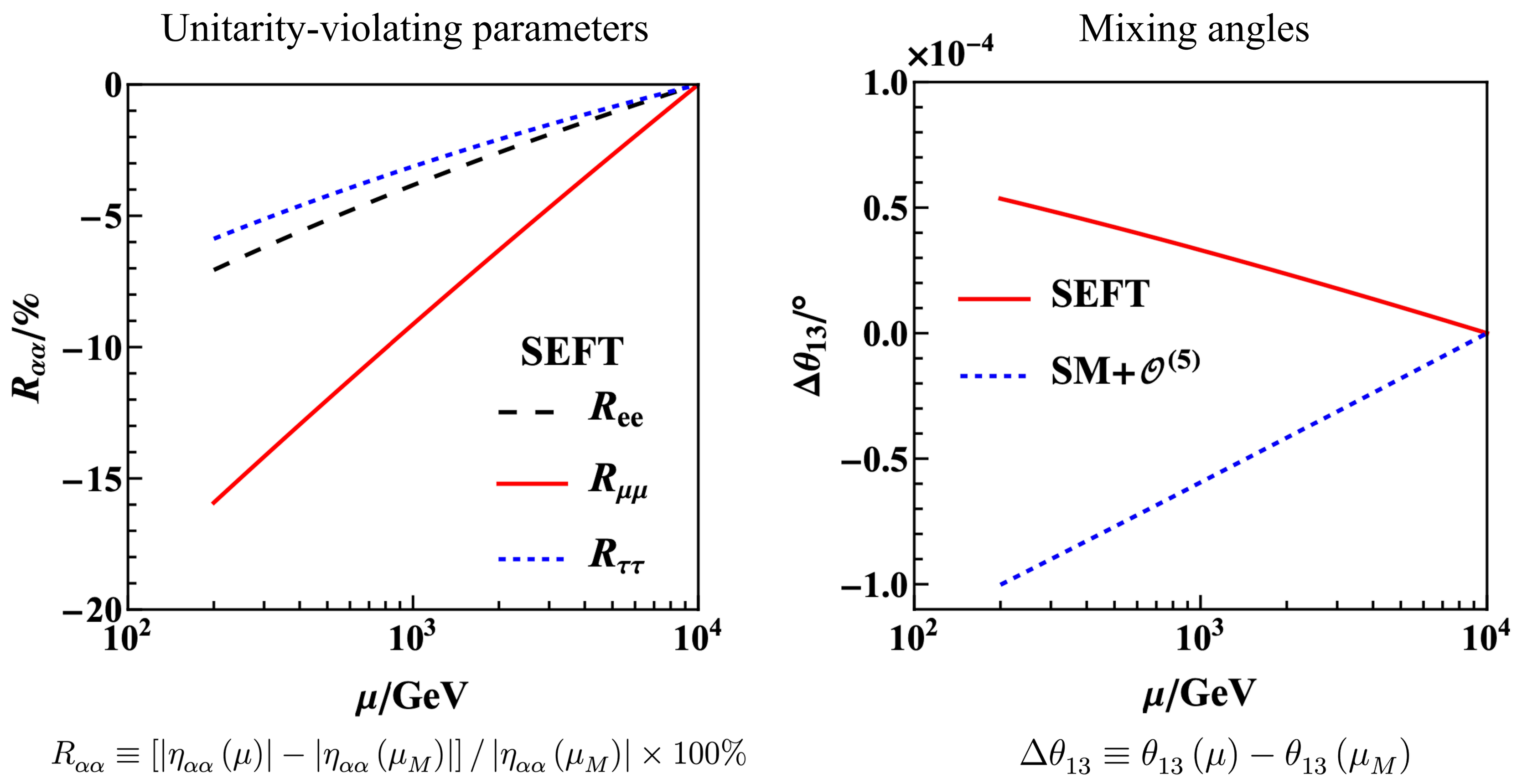
PMNS matrix $V \equiv (\mathbb{1} - \eta) \cdot U \cdot Q$ $\qquad \eta \equiv P^\dagger U_l^\dagger \left(-C^{(3)}_{H\ell} v^2 \right) U_l P$ Unitarity violation

$$N \equiv (\mathbb{1} - \eta'/2) \cdot U \cdot Q , \quad \eta' \equiv P^\dagger U_l^\dagger \left[\left(C^{(1)}_{H\ell} - C^{(3)}_{H\ell} \right) v^2 \right] U_l P , \quad V' \overset{\rm def}{\equiv} U_l^\dagger U_\nu \overset{\rm para}{=} P \cdot U \cdot Q$$

From the RGEs in Sec. III, one can achieve RGEs of

1) Mixing angles and Dirac phase in U
2) Majorana phases in Q
3) **Unitarity-violating parameters in η**
4) FCNC parameters in η'

V. Examples for Numerical Results



$R_{\alpha\alpha} \equiv \left[|\eta_{\alpha\alpha}(\mu)| - |\eta_{\alpha\alpha}(\mu_M)| \right] / |\eta_{\alpha\alpha}(\mu_M)| \times 100\%$ $\qquad \Delta\theta_{13} \equiv \theta_{13}(\mu) - \theta_{13}(\mu_M)$

- All running behaviors of physical parameters can be well understood with the help of their analytical results
- Unitarity-violating parameters can significantly affect the running of mixing angles and CP-violating phases

VI. Summary

- We derive the **complete** set of **one-loop RGEs** for the SM couplings and Wilson coefficients of operators up to dim-6 and $\boldsymbol{O(1/\Lambda_{\rm SS}^2)}$ in seesaw EFTs
- Besides two tree-level-generated dim-6 operators, **17** dim-6 operators can be generated by the **one-loop** RGEs in the **type-I** seesaw EFT
- We give the **explicit expressions** of the RGEs of all the **physical parameters** involved in the charged- and neutral-current interactions of leptons
- With the one-loop matching results at $\Lambda_{\rm SS}$, these one-loop RGEs establish a **self-consistent** framework to investigate **low-energy phenomena** of seesaw models up to $\boldsymbol{O(1/\Lambda_{\rm SS}^2)}$ at the **one-loop level**

This poster is based on: Y. Wang, D. Zhang, and S. Zhou, JHEP 05 (2023) 044