Seesaw Effective Field Theories at One-loop Level

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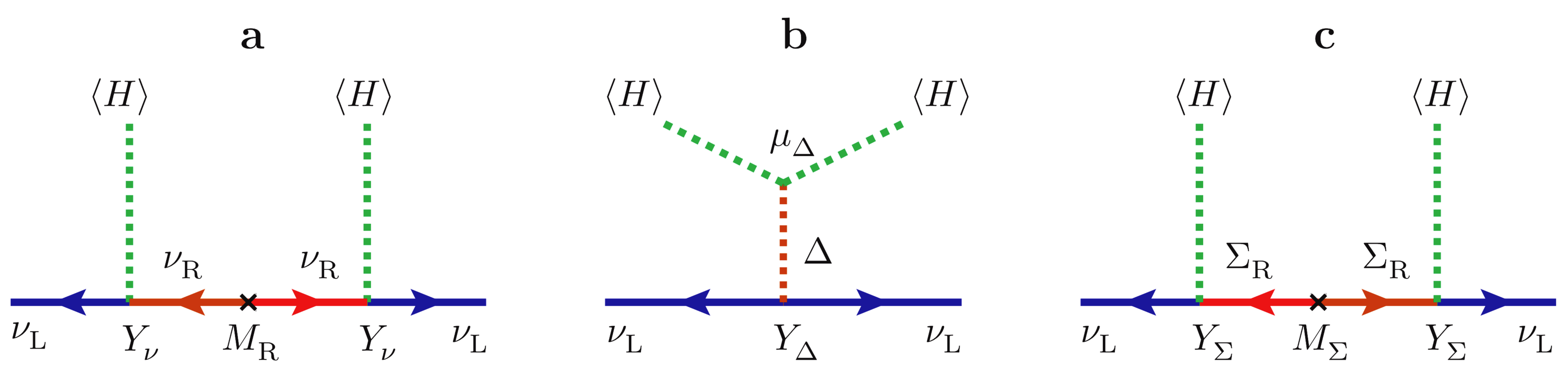
Introduction

The discovery of neutrino oscillations indicates that **neutrinos are massive particles** and the origin of neutrino masses calls for **new physics** beyond the Standard Model (SM) of elementary particles.

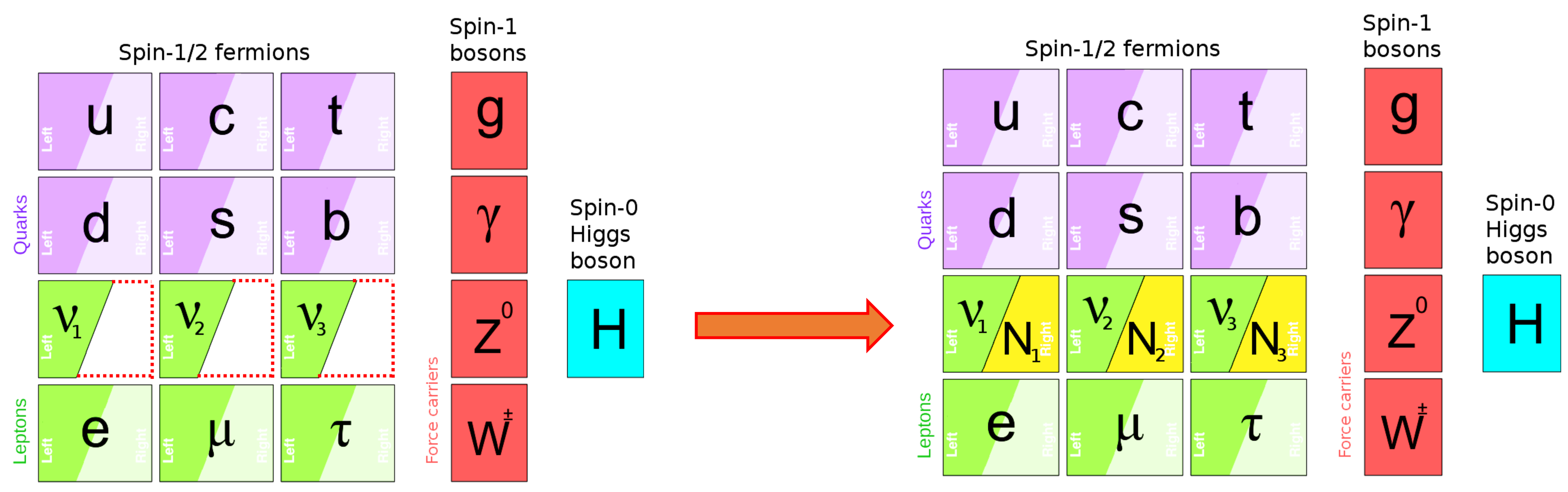
Canonical seesaw models



$M_\nu = -\langle H\rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T}$ $\quad$ $M_\nu = \langle H\rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$ $\quad$ $M_\nu = -\langle H\rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^{\rm T}$

In canonical seesaw models, tiny Majorana neutrino masses and the observed baryon number asymmetry in the Universe can be naturally accommodated.

The minimal extension is to add three right-handed neutrinos into the SM and introduce a Majorana mass term for them



Tiny Majorana masses of ordinary neutrinos can be attributed to the existence of three heavy Majorana neutrinos, whose masses are not subject to the electroweak gauge symmetry breaking

$$M_\nu = -\langle H\rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T}$$

$\mathcal{O}(0.1~{\rm eV})$ — Majorana nature: 0νββ decays

Electroweak Scale $\mathcal{O}(10^2~{\rm GeV})$

$\mathcal{O}(10^{14}~{\rm GeV})$ — Matter-antimatter asymmetry: leptogenesis

One-loop Matching

Below the seesaw scale, we can integrate out three heavy Majorana neutrinos from the ultraviolet (UV) complete theory and construct the low-energy EFT of the seesaw model, which will be called **Seesaw Effective Field Theories** (SEFTs).

$$Z_{\rm UV}[J_\Phi, J_\phi] = \int [D\Phi][D\phi]\, e^{i\int d^dx (\mathcal{L}_{\rm UV}[\Phi,\phi] + J_\Phi \Phi + J_\phi \phi)}$$

sources for heavy & light fields

$\phi = \phi_{\rm b} + \phi' \qquad \Phi = \Phi_{\rm b} + \Phi'$

Background field method

$$\frac{\delta \mathcal{L}_{\rm UV}}{\delta \Phi}[\Phi_{\rm b}, \phi_{\rm b}] + J_\Phi = 0$$

$$\frac{\delta \mathcal{L}_{\rm UV}}{\delta \phi}[\Phi_{\rm b}, \phi_{\rm b}] + J_\phi = 0$$

EoMs for the background fields

Quantum fields to be integrated

$$\mathcal{L}_{\rm UV}[\Phi,\phi] + J_\Phi\Phi + J_\phi\phi = \mathcal{L}_{\rm UV}[\Phi_{\rm b},\phi_{\rm b}] + J_\Phi\Phi_{\rm b} + J_\phi\phi_{\rm b}$$

Gaussian integral $\quad -\frac{1}{2}\left(\Phi'^{T}~\phi'^{T}\right) \mathcal{Q}_{\rm UV}[\Phi_{\rm b},\phi_{\rm b}] \begin{pmatrix}\Phi' \\ \phi'\end{pmatrix}$

$$\begin{pmatrix} -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\Phi^2}[\Phi_{\rm b},\phi_{\rm b}] & -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\Phi\delta\phi}[\Phi_{\rm b},\phi_{\rm b}] \\ -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\phi\delta\Phi}[\Phi_{\rm b},\phi_{\rm b}] & -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\phi^2}[\Phi_{\rm b},\phi_{\rm b}] \end{pmatrix}$$

$$\int d^dx\, \mathcal{L}_{\rm EFT}^{\rm 1\text{-}loop}[\phi] = \frac{i}{2}\log\det \mathcal{Q}_{\rm UV}[\Phi_{\rm c}[\phi],\phi] - \frac{i}{2}\log\det \mathcal{Q}_{\rm EFT}^{\rm tree}[\phi]$$

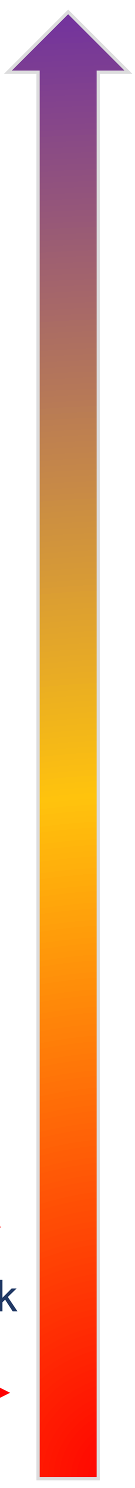
While there are only two dim-6 operators in SEFT-I at the tree level in the Warsaw basis of the SMEFT, we obtain 31 dim-6 operators at the one-loop level.

X^2H^2		ψ^2DH^2		Four-quark	
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}H^\dagger H$	$\mathcal{O}_{HQ}^{(1)\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L})(H^\dagger i\overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{QU}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L})(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\lambda\rm R})$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}H^\dagger H$	$\mathcal{O}_{HQ}^{(3)\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu\tau^I Q_{\beta\rm L})(H^\dagger i\overleftrightarrow{D}^I_\mu H)$	$\mathcal{O}_{QU}^{(8)\alpha\beta\gamma\lambda}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu T^A Q_{\beta\rm L})(\overline{U_{\gamma\rm R}}\gamma_\mu T^A U_{\lambda\rm R})$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\tau^I H)$	$\mathcal{O}_{HU}^{\alpha\beta}$	$(\overline{U_{\alpha\rm R}}\gamma^\mu U_{\beta\rm R})(H^\dagger i\overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{Qd}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L})(\overline{D_{\gamma\rm R}}\gamma_\mu D_{\lambda\rm R})$
H^4D^2		$\mathcal{O}_{Hd}^{\alpha\beta}$	$(\overline{D_{\alpha\rm R}}\gamma^\mu D_{\beta\rm R})(H^\dagger i\overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{Qd}^{(8)\alpha\beta\gamma\lambda}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu T^A Q_{\beta\rm L})(\overline{D_{\gamma\rm R}}\gamma_\mu T^A D_{\lambda\rm R})$
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{H\ell}^{(1)\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(H^\dagger i\overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{QUQd}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{Q^a_{\alpha\rm L}}U_{\beta\rm R})\epsilon^{ab}(\overline{Q^b_{\gamma\rm L}}D_{\lambda\rm R})$
$\mathcal{O}_{HD}$	$(H^\dagger D_\mu H)^*(H^\dagger D^\mu H)$	$\mathcal{O}_{H\ell}^{(3)\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu\tau^I \ell_{\beta\rm L})(H^\dagger i\overleftrightarrow{D}^I_\mu H)$	Four-lepton	
H^6		$\mathcal{O}_{He}^{\alpha\beta}$	$(\overline{E_{\alpha\rm R}}\gamma^\mu E_{\beta\rm R})(H^\dagger i\overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{\ell\ell}^{\alpha\beta\gamma\beta}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{\ell_{\gamma\rm L}}\gamma_\mu \ell_{\lambda\rm L})$
$\mathcal{O}_H$	$(H^\dagger H)^3$	ψ^2H^3		$\mathcal{O}_{\ell e}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{E_{\gamma\rm R}}\gamma_\mu E_{\lambda\rm R})$
ψ^2XH		$\mathcal{O}_{UH}^{\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}\widetilde{H}U_{\beta\rm R})(H^\dagger H)$		
$\mathcal{O}_{eB}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R})HB_{\mu\nu}$	$\mathcal{O}_{dH}^{\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}HD_{\beta\rm R})(H^\dagger H)$		
$\mathcal{O}_{eW}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R})\tau^I HW^I_{\mu\nu}$	$\mathcal{O}_{eH}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}HE_{\beta\rm R})(H^\dagger H)$		
Semi-leptonic					
$\mathcal{O}_{\ell Q}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{Q_{\gamma\rm L}}\gamma_\mu Q_{\lambda\rm L})$	$\mathcal{O}_{\ell U}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\lambda\rm R})$	$\mathcal{O}_{\ell edQ}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}E_{\beta\rm R})(\overline{D_{\gamma\rm R}}Q_{\lambda\rm L})$
$\mathcal{O}_{\ell Q}^{(3)\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu\tau^I \ell_{\beta\rm L})(\overline{Q_{\gamma\rm L}}\gamma_\mu\tau^I Q_{\lambda\rm L})$	$\mathcal{O}_{\ell d}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(D_{\gamma\rm R}\gamma_\mu D_{\lambda\rm R})$	$\mathcal{O}_{\ell eQU}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{\ell^a_{\alpha\rm L}}E_{\beta\rm R})\epsilon^{ab}(\overline{Q^b_{\gamma\rm L}}U_{\lambda\rm R})$

Phenomenological implications of all these dim-6 operators need to be further explored.

The Seesaw Scale

The seesaw scale is characterized by the masses of heavy Majorana neutrinos, but not constrained within the theory itself



Planck 10^{19} GeV

GUT 10^{16} GeV

10^3 TeV

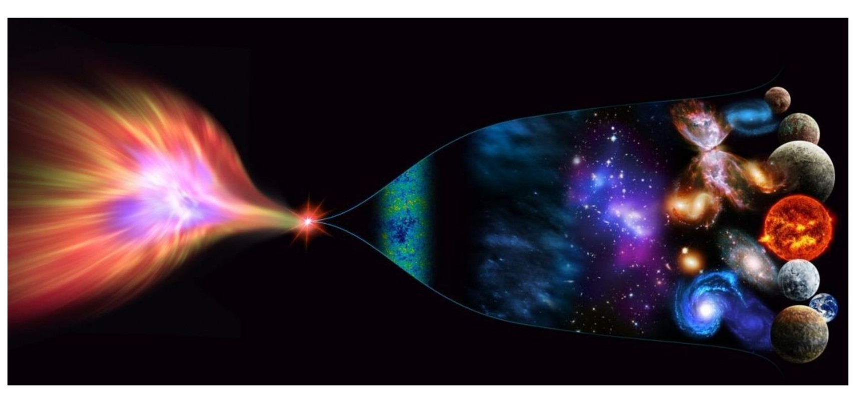
Seesaw?

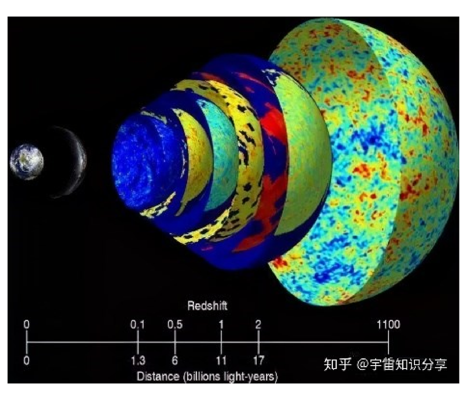
10^2 TeV

LHC 10^3 GeV

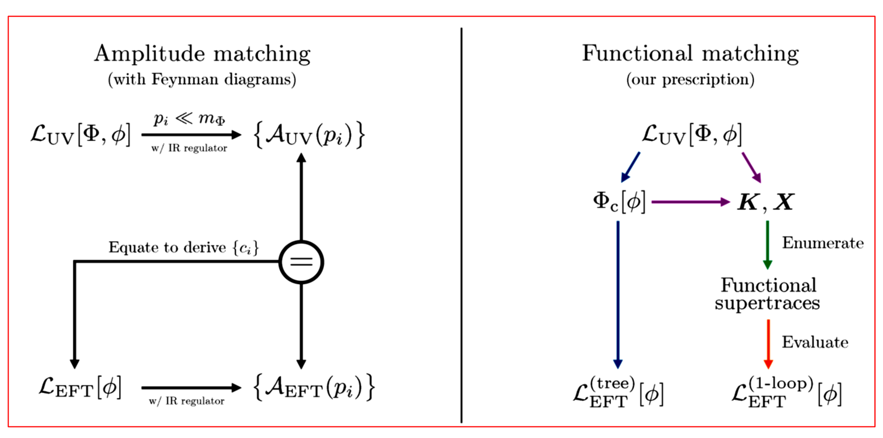
Electroweak 10^2 GeV

Cosmic Frontier: indirect signals from cosmological observations



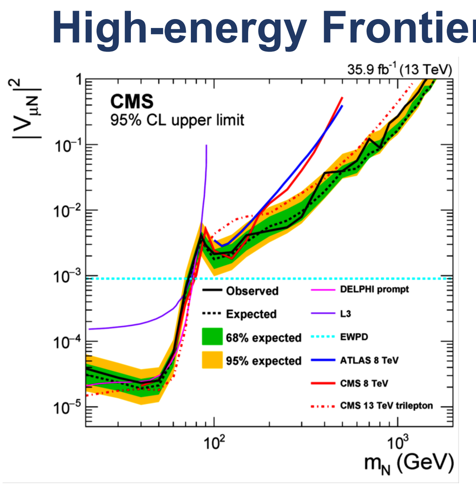


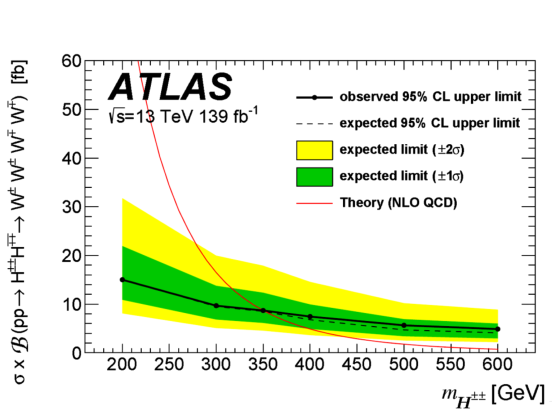
High-intensity Frontier: EFT approach vs. precision measurements

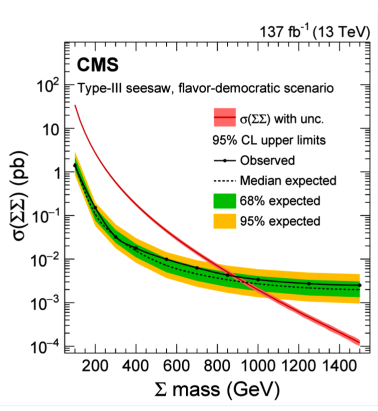


Construct the effective field theories (EFTs) of neutrino mass models & compare theoretical predictions with data to test the models

High-energy Frontier: direct searches in the collider experiments

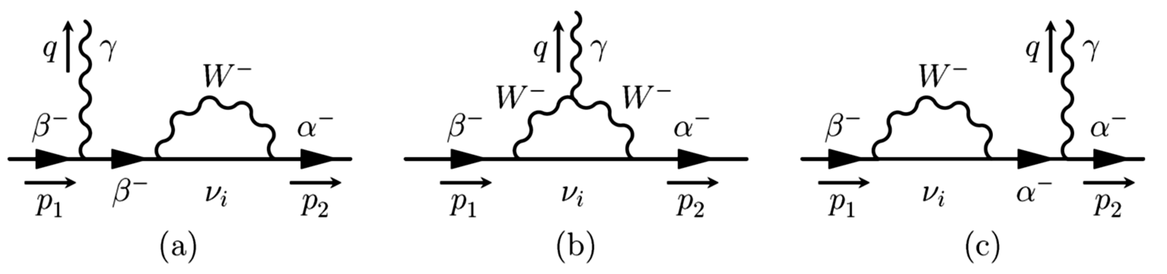






Lepton-Flavor-Violating Decays

One immediate consequence of massive neutrinos is the lepton-flavor-violating decays of charged leptons. In the SEFT-I at the tree level, one dim-5 operator and two dim-6 operators after gauge symmetry breaking lead to $\beta^- \to \alpha^- + \gamma$ decays.



$\mathcal{B}(\mu^- \to e^- + \gamma) < 4.2 \times 10^{-13}$
$\mathcal{B}(\tau^- \to e^- + \gamma) < 3.3 \times 10^{-8}$
$\mathcal{B}(\tau^- \to \mu^- + \gamma) < 4.4 \times 10^{-8}$

The decay rate can be calculated in the mass basis with a non-unitary mixing matrix

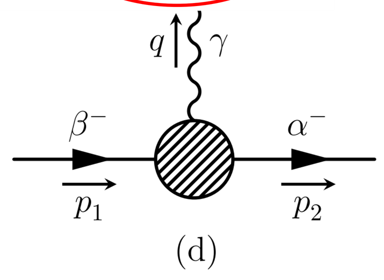
$$\Gamma(\beta^- \to \alpha^- + \gamma) \simeq \frac{\alpha_{\rm em} G_{\rm F}^2 m_\beta^5}{128\pi^4}\left|\sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^* \left(-\frac{5}{6} + \frac{m_i^2}{4M_W^2}\right)\right|^2$$

The correct rate can be obtained only by taking account of two dim-6 dipole operators arising from the one-loop matching

$$\Gamma(\beta^- \to \alpha^- + \gamma) \simeq \frac{\alpha_{\rm em} G_{\rm F}^2 m_\beta^5}{128\pi^4}\left|\sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^* \left(-\frac{5}{6} + \frac{m_i^2}{4M_W^2}\right) - \frac{1}{3}(RR^\dagger)_{\alpha\beta}\right|^2$$

A self-consistent calculation in the SEFT at one-loop requires a matching at the same level

ψ^2XH	
$\mathcal{O}_{eB}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R})HB_{\mu\nu}$
$\mathcal{O}_{eW}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R})\tau^I HW^I_{\mu\nu}$



References

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