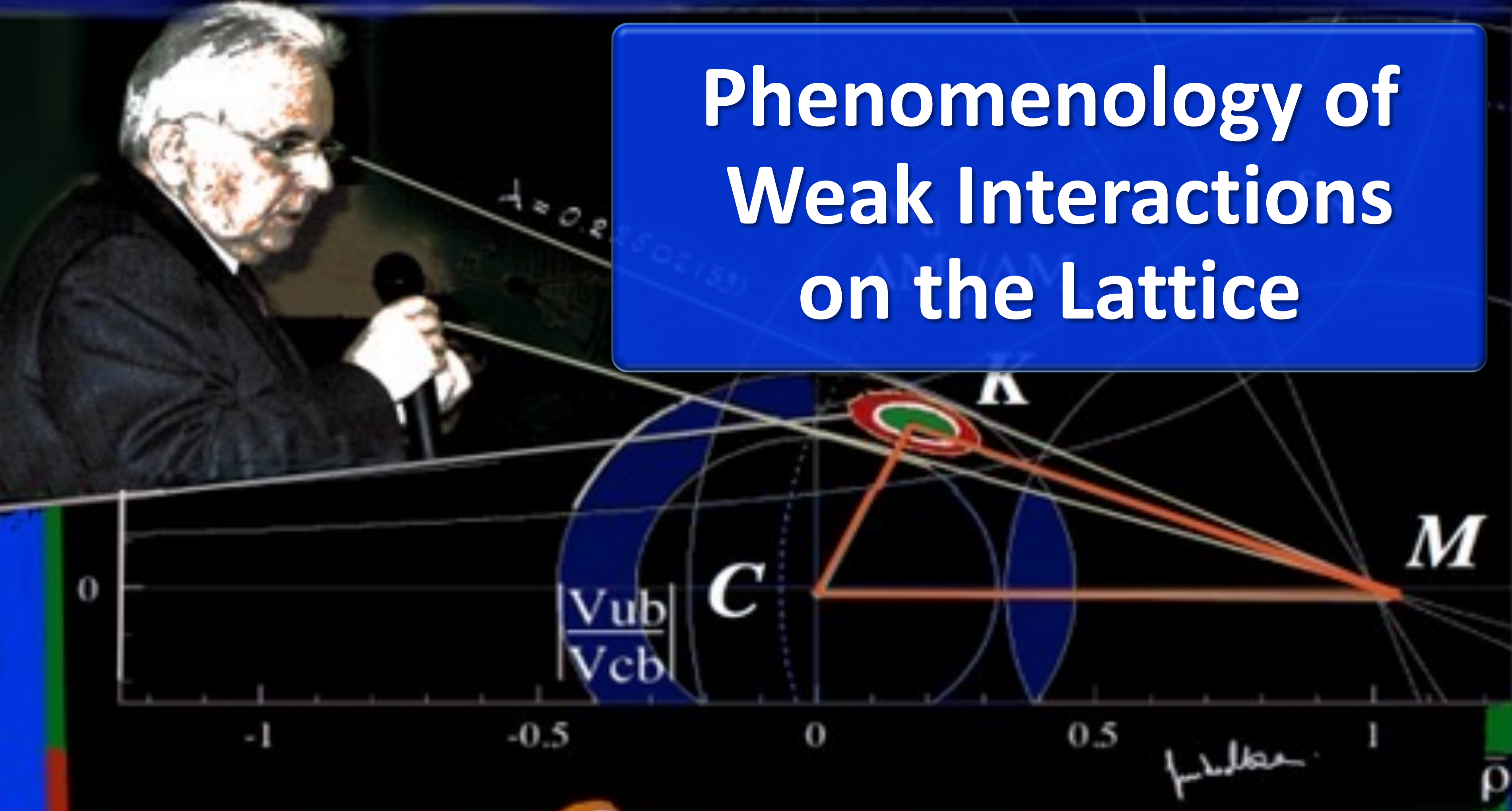
Phenomenology of Weak Interactions on the Lattice



Vittorio Lubicz

Roma, Accademia dei Lincei - December 4, 2023

Cabibbo's legacy

VOLUME 10, NUMBER 12 PHYSICAL REVIEW LETTERS 15 JUNE 1963

UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the V-A theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \to n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-

This is the paper we are celebrating today, which paved the way to FLAVOR PHYSICS

Cabibbo's legacy

but in this talk I want to start from another paper, which also paved a way

Nuclear Physics B244 (1984) 381–391

1984

WEAK INTERACTIONS ON THE LATTICE

N. CABIBBO

Dipartimento di Fisica, II Università di Roma "Tor Vergata", INFN, Sezione di Roma, Roma, Italy

G. MARTINELLI

INFN, Laboratori Nazionali di Frascati, Frascati, Italy

R. PETRONZIO[1]

CERN, Geneva, Switzerland

Received 5 December 1983
(Revised 16 April 1984)

We show that lattice QCD can be used to evaluate the matrix elements of four-fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations are also presented.

Cabibbo's legacy

WEAK INTERACTIONS ON THE LATTICE

We show that lattice QCD can be used to evaluate the matrix elements of four-fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations are also presented.

$K^0 - \bar{K}^0$ MIXING

14 configurations $10^3 \times 20$ lattice

$\beta_W = 6$, and $K = 0.150$ and 0.155

$M_\pi \simeq 1.2$ GeV

$$(O_{\pm}^{LL})^{\text{cont}} = \left[1 + \frac{g^2}{16\pi^2} Z_{\pm}(r)\right](O_{\pm}^{LL})^{\text{latt}} + \frac{g^2}{16\pi^2} r^2 Z^*(r)\left[O_{\pm}^{STP} + O_{\pm}^{VA} + O_{\pm}^{SP}\right]^{\text{latt}}$$

Renormalization and mixing

$$\frac{\langle K_0|(\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma_\mu(1-\gamma_5)d)|\bar{k}_0\rangle}{m^2} = (4a^2)(10 \pm 1)10^{-2} \quad [\sim 7.7 \times 10^{-2}].$$

"In good agreement in sign and magnitude with the VIA"

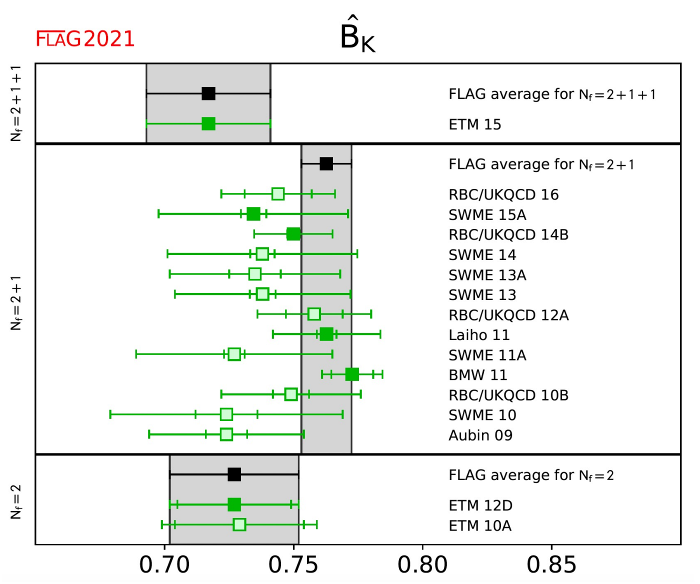
$B_K \simeq 1$

$B_K \simeq 0.33$ from PCAC + SU(3)

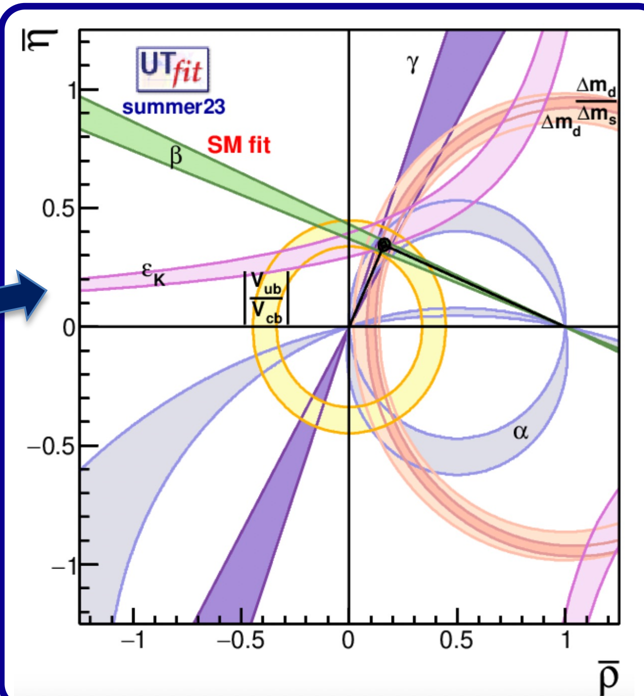
Cabibbo's legacy

WEAK INTERACTIONS ON THE LATTICE

We show that lattice QCD can be used to evaluate the matrix elements of four-fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations are also presented.



$B_K = 0.756(16)$ **2%**



See talk by L. Silvestrini

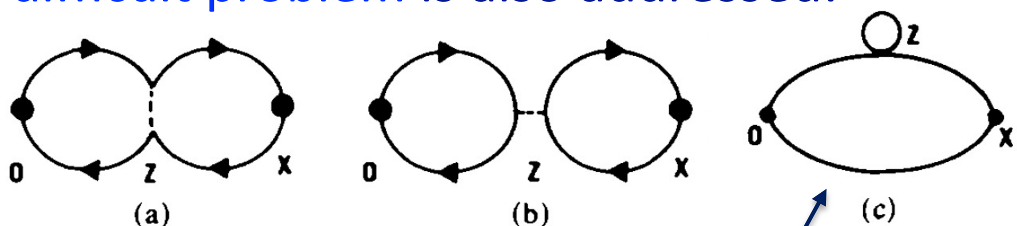
Cabibbo's legacy

WEAK INTERACTIONS ON THE LATTICE

We show that lattice QCD can be used to evaluate the matrix elements of four-fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations are also presented.

In the same paper, a much more difficult problem is also addressed:

$K \to \pi\,\pi$, $\Delta I = 1/2, 3/2$



$$\frac{\langle \pi^+ | (\bar{u}\gamma^\mu_L u)(\bar{s}\gamma^L_\mu d) | K^+ \rangle}{m^2} = (4a^2)(3.7 \pm 0.3)10^{-3} \quad [2.4 \times 10^{-3}],$$

$$\frac{\langle \pi^+ | (\bar{s}\gamma^\mu_L u)(\bar{u}\gamma^L_\mu d) | K^+ \rangle}{m^2} = (4a^2)(8.8 \pm 0.7)10^{-3} \quad [7.2 \times 10^{-3}],$$

$$\langle \pi^+ | (\bar{s}\gamma^\mu_L d)(\bar{u}\gamma^R_\mu u) | K^+ \rangle = (16a^4)(-4.2 \pm 0.9)10^{-2} \quad [-1.20 \times 10^{-2}],$$

$$\langle \pi^+ | (\bar{s}t^A\gamma^\mu_L d)(\bar{u}t^A\gamma^R_\mu u) | K^+ \rangle = (16a^4)(-6.8 \pm 0.8)10^{-2} \quad [-1.63 \times 10^{-2}],$$

[Shifman, Veinshtein, Zakharov (1976)]

Not computed

The penguin operators Q_5 and Q_6 couldn't be really evaluated

It was too early...

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

Performing the lattice study of $K \to \pi\pi$ decays required more than 30 years

PRL **110**, 152001 (2013) PHYSICAL REVIEW LETTERS week ending 12 APRIL 2013

Emerging Understanding of the $\Delta I = 1/2$ Rule from Lattice QCD

P. A. Boyle,[1] N. H. Christ,[2] N. Garron,[3] E. J. Goode,[4] T. Janowski,[4] C. Lehner,[5] Q. Liu,[2] A. T. Lytle,[4] C. T. Sachrajda,[4] A. Soni,[6] and D. Zhang[2]

PHYSICAL REVIEW D **91**, 074502 (2015)

$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit

T. Blum,[1,2] P. A. Boyle,[3] N. H. Christ,[4] J. Frison,[3] N. Garron,[5,6] T. Janowski,[7] C. Jung,[8] C. Kelly,[2] C. Lehner,[8] A. Lytle,[9] R. D. Mawhinney,[4] C. T. Sachrajda,[7] A. Soni,[8] H. Yin,[4] and D. Zhang[4]

PHYSICAL REVIEW D **102**, 054509 (2020)

Editors' Suggestion | Featured in Physics

Direct *CP* violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the standard model

R. Abbott,[1] T. Blum,[2,3] P. A. Boyle,[4,5] M. Bruno,[6] N. H. Christ,[1] D. Hoying,[3,2] C. Jung,[4] C. Kelly,[4] C. Lehner,[7,4] R. D. Mawhinney,[1] D. J. Murphy,[8] C. T. Sachrajda,[9] A. Soni,[4] M. Tomii,[2] and T. Wang[1]

(RBC and UKQCD Collaborations)

[Only one collaboration so far]

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

Many difficult theoretical and technical problems had to be solved:

- Contribution of 10 operators (7 independent) to the effective $\Delta S=1$ weak Hamiltonian and calculation of 48 Wick contractions
- Use of anti-p … 1/2) boundary conditions to … ate, solving the problem …
- Use non-per … chniques to control the a … rs and subtraction of power divergences (for non precisely matched kinematics)

I will not discuss the details,
but I want to give you the idea
of the complexity of the calculation

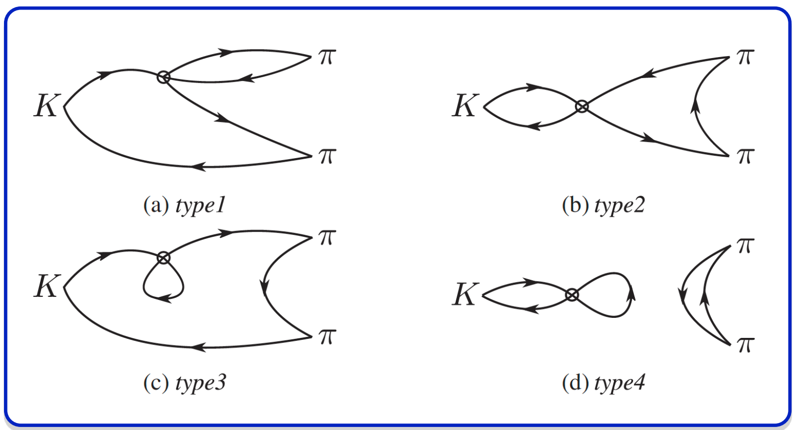
- Subtraction of finite volume corrections which, due to the presence of two pions in the final state, decrease as inverse powers of the volume
- Use of multi-state and multi-operator techniques to control unexpectedly large excited state contamination

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

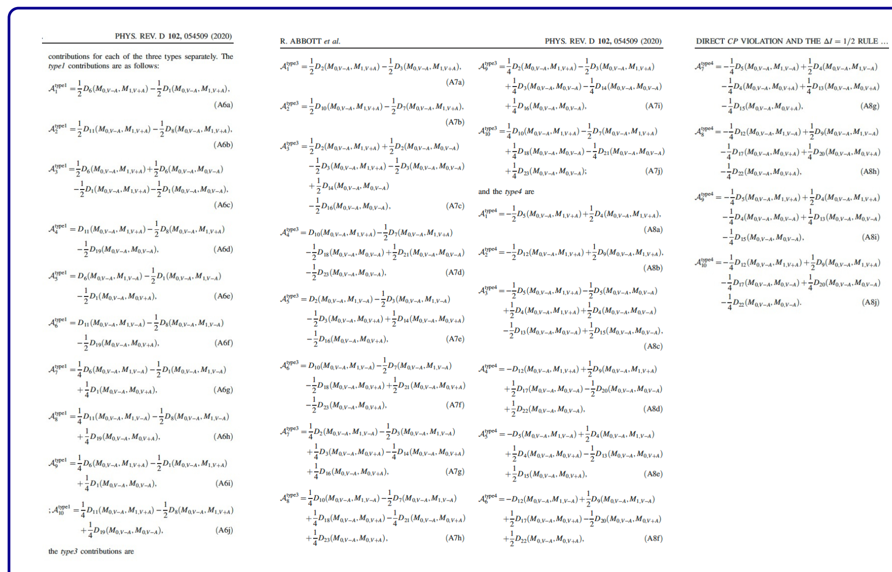
Many difficult theoretical and technical problems had to be solved:

- Contribution of 10 operators (7 independent) to the effective ΔS=1 weak Hamiltonian and calculation of 48 Wick contractions

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu)$$



(a) *type1* (b) *type2* (c) *type3* (d) *type4*


PHYS. REV. D 102, 054509 (2020)

R. ABBOTT et al. PHYS. REV. D 102, 054509 (2020)

DIRECT CP VIOLATION AND THE ΔI = 1/2 RULE ...

\+ very large statistics: 5000 MD trajectories, 1300 measurements

$K \rightarrow \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

Many difficult theoretical and technical problems had to be solved:

- Contribution of 10 operators (7 independent) to the effective $\Delta S=1$ weak Hamiltonian and calculation of 48 Wick contractions
- Use of anti-periodic (for $\Delta I=3/2$) and sophisticated G-parity (for $\Delta I=1/2$) boundary conditions to match the physical two-pion state with the ground state, solving the problem posed by the Maiani-Testa theorem

Volume 245, number 3, 4 PHYSICS LETTERS B 16 August 1990

1990

Final state interactions from euclidean correlation functions ☆

L. Maiani
Dipartimento di Fisica, Università di Roma "La Sapienza", and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, I-00185 Rome, Italy

and

M. Testa
Dipartimento di Fisica, Università di Lecce, and Istituto Nazionale di Fisica Nucleare, Sezione di Lecce, I-73100 Lecce, Italy

Received 11 May 1990

PHYSICAL REVIEW D **101,** 014506 (2020)

Lattice simulations with *G*-parity boundary conditions

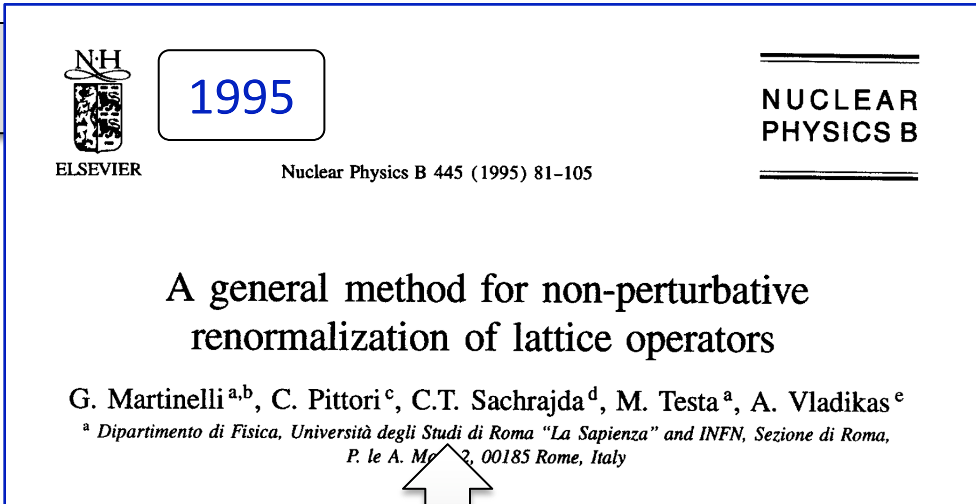
N. H. Christ, C. Kelly, and D. Zhang

(RBC and UKQCD Collaborations)

Physics Department, Columbia University, New York, New York 10027, USA

(Received 23 August 2019; published 21 January 2020)

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε


1995

NUCLEAR PHYSICS B

Nuclear Physics B 445 (1995) 81–105

ELSEVIER

A general method for non-perturbative renormalization of lattice operators

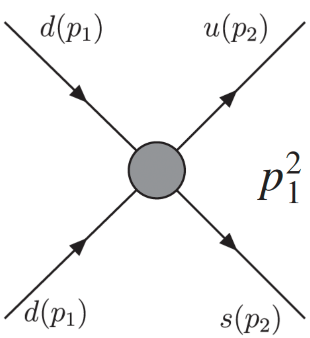
G. Martinelli[a,b], C. Pittori[c], C.T. Sachrajda[d], M. Testa[a], A. Vladikas[e]

[a] Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza" and INFN, Sezione di Roma, P. le A. Mo... 2, 00185 Rome, Italy

...l problems had to be solved:

- ... to the effective ΔS=1 weak ...actions
- ...ated G-parity (for ΔI=1/2) boundary ...te with the ground state, solving the problem posed ... the Maiani-Testa theorem

- Use non-perturbative renormalization methods and step-scaling techniques to control the accuracy of renormalization and mixing of the 7 operators and subtraction of power divergences (for non precisely matched kinematics)



RI-SMOM scheme

$$p_1^2 = p_2^2 = (p_1 - p_2)^2 \equiv \mu^2$$

$Z^{RI \leftarrow LAT}(4.0\ \mathrm{GeV})$

0.40845(42)	0	0	0	0	0	0
0	0.485(23)	−0.114(20)	−0.012(10)	0.0077(63)	0	0
0	−0.0908(93)	0.5248(89)	−0.0089(37)	0.0061(26)	0	0
0	−0.051(70)	−0.067(58)	0.432(30)	−0.003(19)	0	0
0	0.021(37)	0.025(35)	−0.073(15)	0.574(10)	0	0
0	0	0	0	0	0.47514(49)	−0.01786(21)
0	0	0	0	0	−0.04460(26)	0.55914(99)
0.42011(43)	0	0	0	0	0	0
0	0.422(38)	−0.207(36)	−0.005(13)	0.0084(77)	0	0
0	−0.094(24)	0.570(24)	−0.0120(83)	0.0059(47)	0	0
0	−0.14(14)	−0.15(12)	0.424(44)	0.013(26)	0	0
0	−0.030(63)	−0.073(66)	−0.106(23)	0.620(15)	0	0
0	0	0	0	0	0.47715(49)	−0.02113(24)
0	0	0	0	0	−0.05960(55)	0.6030(14)

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

Many difficult theoretical and technical problems had to be solved:

Commun. Math. Phys. 219, 31 – 44 (2001)

2001

Communications in **Mathematical Physics**

Weak Transition Matrix Elements from Finite-Volume Correlation Functions*

Laurent Lellouch[1,**], **Martin Lüscher**[2,***]

[1] LAPTH, Chemin de Bellevue, B.P. 110, 74941 Annecy-Le-Vieux Cedex, France
[2] CERN, Theory Division, 1211 Geneva 23, Switzerland

Received: 29 March 2000 / Accepted: 10 April 2000

Dedicated to the memory of Harry Lehmann

- … e effective ΔS=1 weak … s
- … G-parity (for ΔI=1/2) boundary … n the ground state, solving
- … step-scaling techniques to … of the 7 operators and … matched kinematics)

- Subtraction of finite volume corrections which, due to the presence of two pions in the final state, decrease as inverse powers of the volume

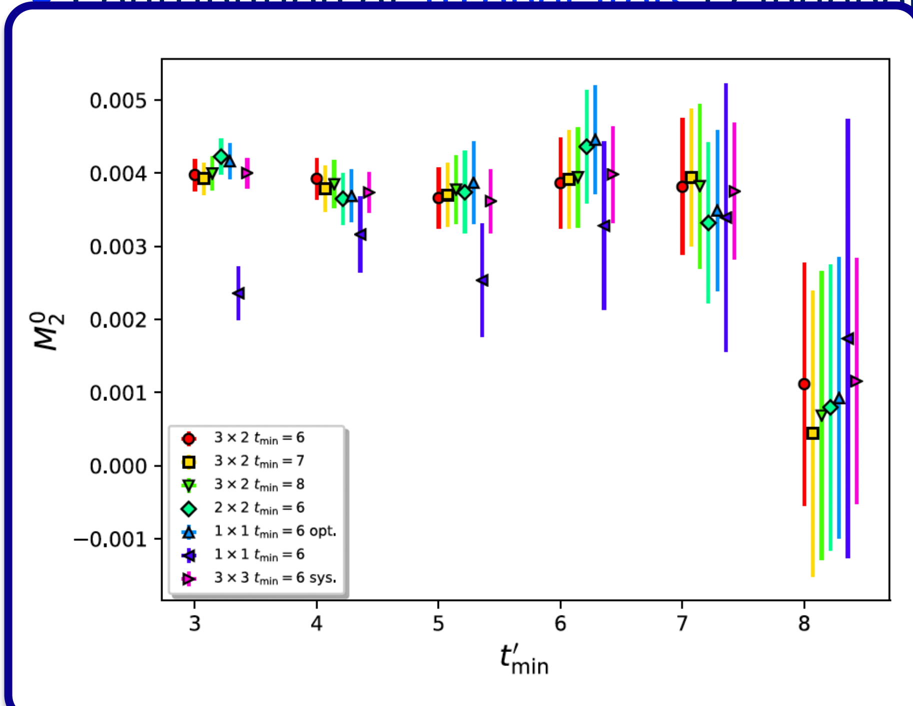
$$F^2 = \frac{4\pi m_K E_{\pi\pi}^2}{k^3}\left(k\frac{d\delta_0}{dk} + q\frac{d\phi}{dq}\right) , \quad k^2 = \left(\frac{E_{\pi\pi}}{2}\right)^2 - m_\pi^2 , \quad q = \frac{Lk}{2\pi}$$

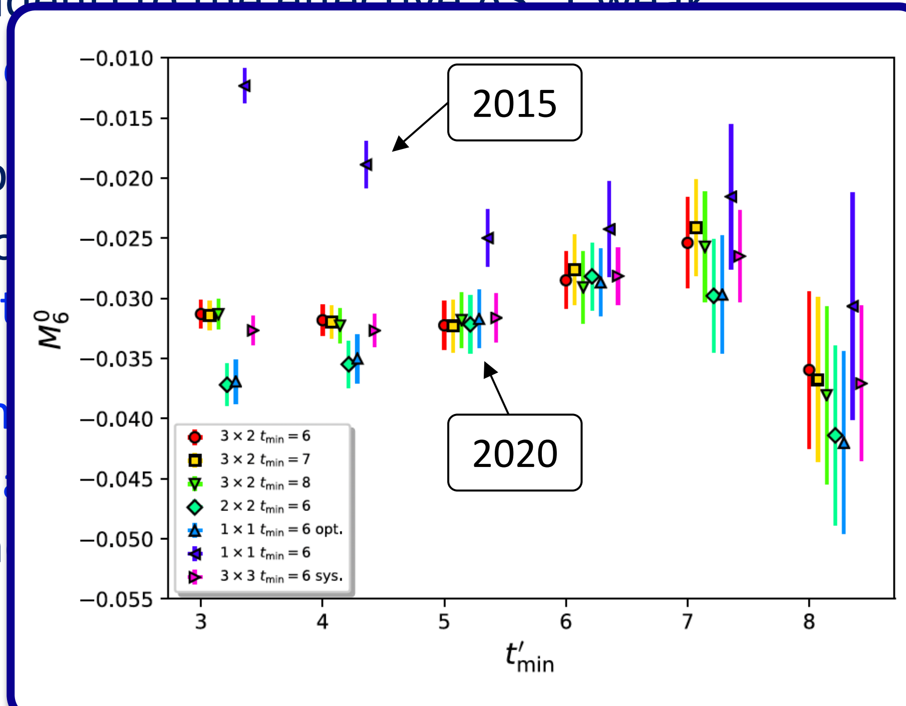
$$F = 26.696(52)$$

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

Many difficult theoretical and technical problems had to be solved:

- Contribution of 10 operators (7 independent) to the effective ΔS=1 weak





in the final state, decrease as inverse powers of the volume

- Use of multi-state and multi-operator techniques to control unexpectedly large excited state contamination

$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

RESULTS

Main errors due to finite lattice spacing and Wilson coefficients

$$\mathrm{Re}(A_0) = 2.99(0.32)(0.59) \times 10^{-7}\ \mathrm{GeV}$$

$Q_2 \simeq 97\,\%$

$Q_1 \simeq -Q_6 \simeq 13\,\%$

Expt: $\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}\ \mathrm{GeV}$

Significant cancelation by the two dominant contributions

$$\mathrm{Re}(A_2) = 1.50(4)(14) \times 10^{-8}\ \mathrm{GeV}$$

Expt: $\mathrm{Re}(A_2) = 1.4787(31) \times 10^{-8}\ \mathrm{GeV}$

Important ! (see next)

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} = 19.9(2.3)(4.4)$$

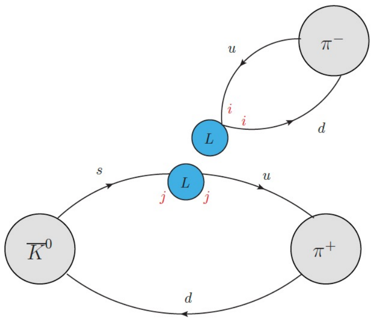
Expt $= 22.45(6)$

The $\Delta I = 1/2$ rule is explained

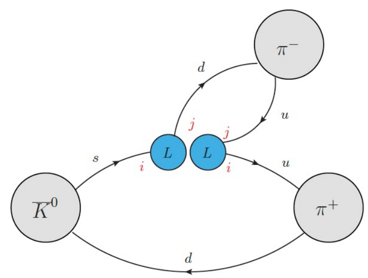
$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

RESULTS

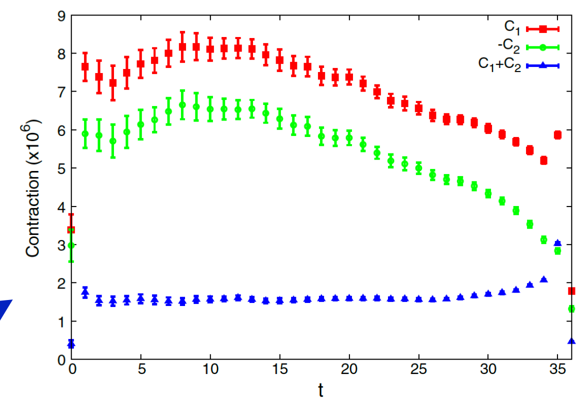
- In the calculation of $\mathrm{Re}(A_2)$ a surprising cancellation has been found



C_1



C_2



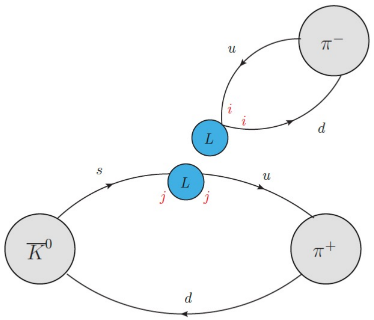
$\mathrm{Re}(A_2) = C_1 + C_2$ and $C_2 \simeq -C_1$

In the VIA: $C_2 \simeq +1/3\ C_1$

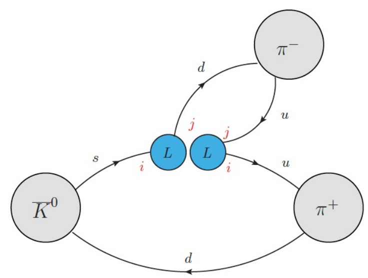
$K \to \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

RESULTS

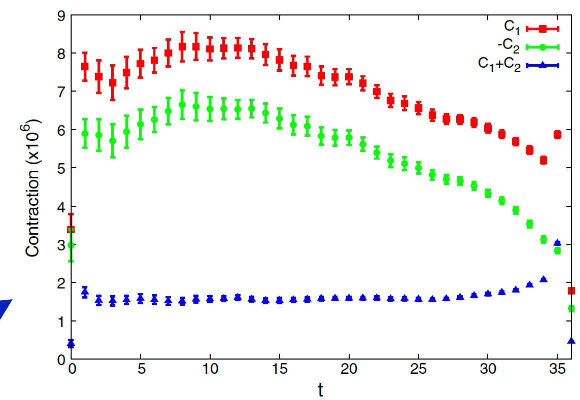
- In the calculation of Re(A_2) a surprising cancellation has been found



C_1



C_2

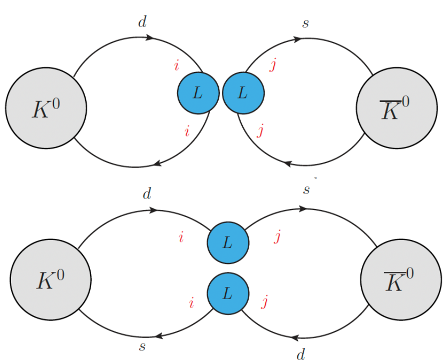


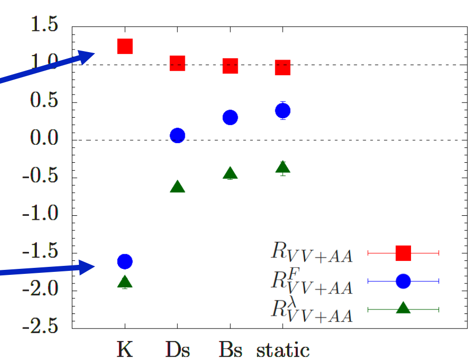
$\mathrm{Re}(A_2) = C_1 + C_2$ and $C_2 \simeq -C_1$

In the VIA: $C_2 \simeq +1/3\, C_1$

A similar result observed in K-$\overline{\text{K}}$, but not in D-$\overline{\text{D}}$ and B-$\overline{\text{B}}$

[N. Carrasco, VL, L.Silvestrini, PLB 736, 2014]





$K \rightarrow \pi\pi$, $\Delta I = 1/2$ rule and ε'/ε

RESULTS

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

$$\mathrm{Im}(A_0) = -6.98(0.62)(1.44)\times 10^{-11}\ \mathrm{GeV}$$

← $Q_6 \simeq 123\,\%$
$Q_4 \simeq -17\,\%$

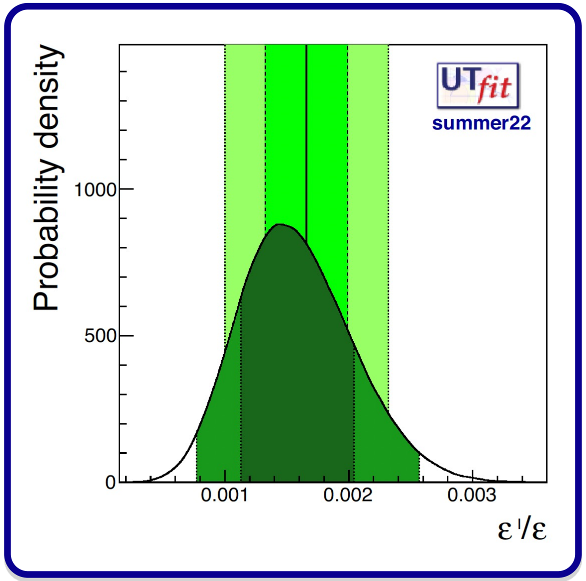
$$\mathrm{Im}(A_2) = -6.99(20)(84)\times 10^{-13}\ \mathrm{GeV}$$

Isospin breaking correction
V.Cirigliano *et al.*, JHEP 02, 2020

$$\mathrm{Re}(\varepsilon'/\varepsilon) = 2.17(26)(62)(50)\times 10^{-3}$$

Expt: $\mathrm{Re}(\varepsilon'/\varepsilon) = 1.66(23)\times 10^{-3}$

Direct CP violation is explained in the SM

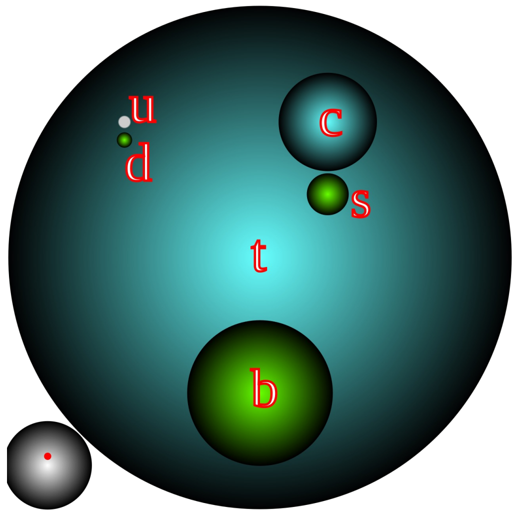


From LINCEI CELEBRATIVE ESSAYS

The Standard Model parameters

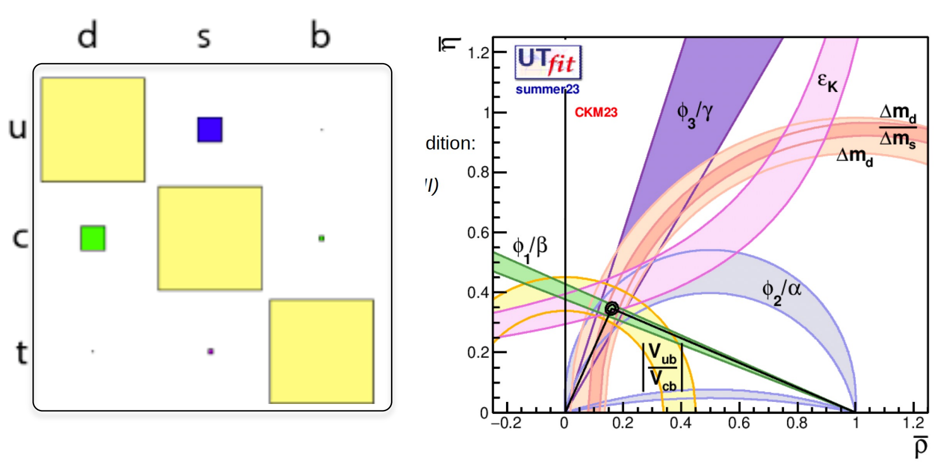
A crucial task for Lattice QCD is to determine the Standard Model fundamental parameters in the quark sector

1 Quark masses



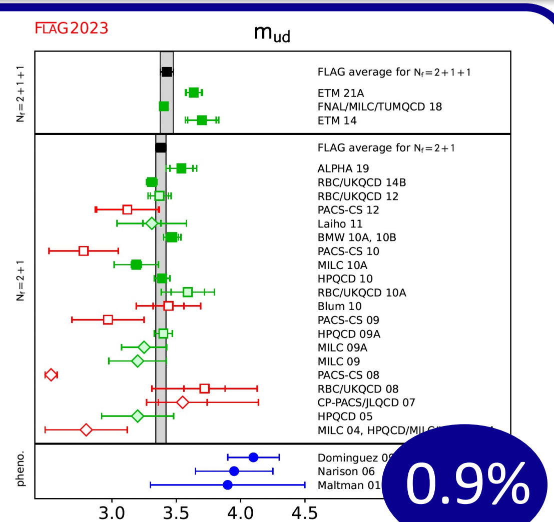
2 CKM matrix elements

Cabibbo-Kobayashi-Maskawa



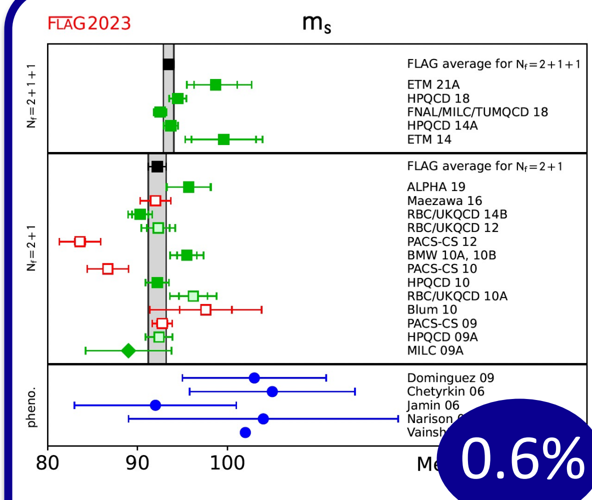
6 quark masses, 4 CKM parameters ⇨ 10 fundamental parameters

Quark masses



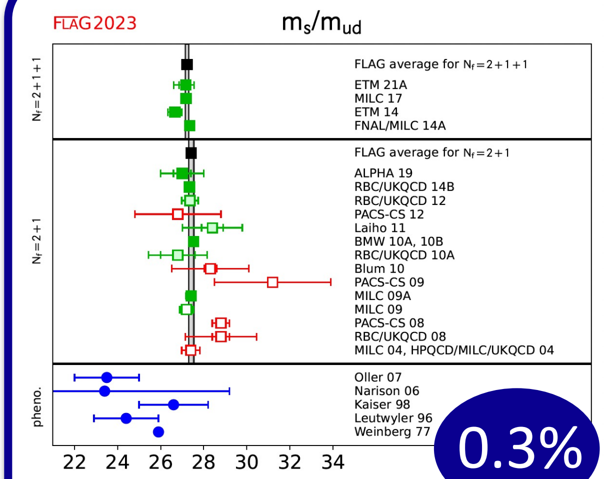
0.9%

$\overline{m}_{ud}$ = 3.399(31) MeV



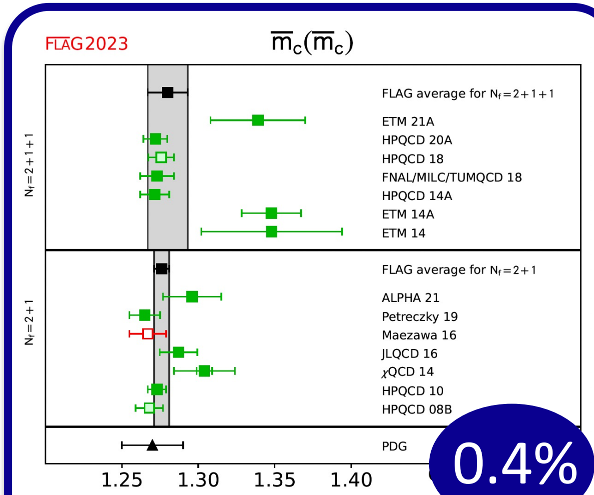
0.6%

$\overline{m}_s$ = 93.14(55) MeV



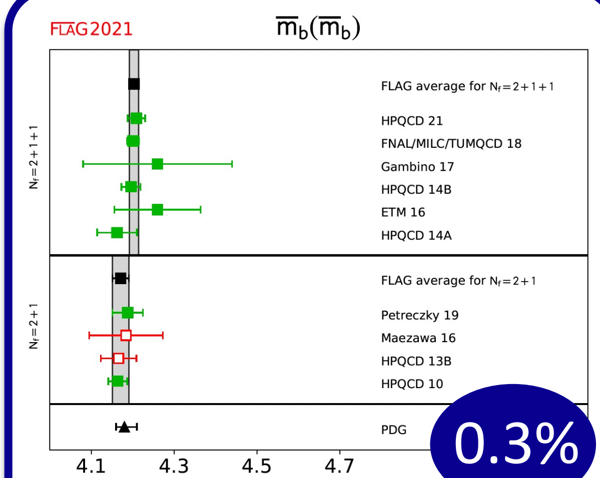
0.3%

$\overline{m}_s / \overline{m}_{ud}$ = 27.287(89)



0.4%

$\overline{m}_c$ = 1.2917(48) GeV



0.3%

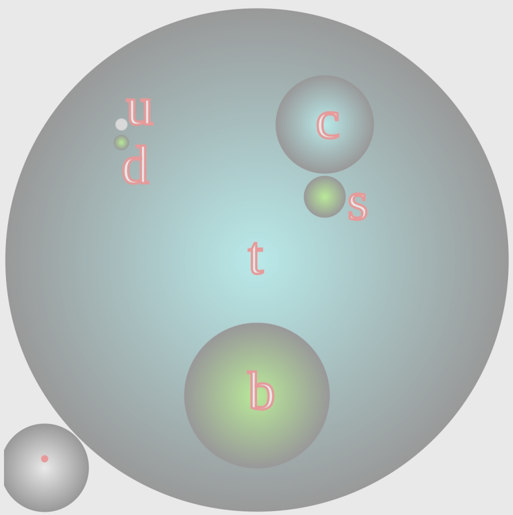
$\overline{m}_b$ = 4.196(14) GeV

The accuracy is at the permille level !

(Unthinkable without lattice ...)

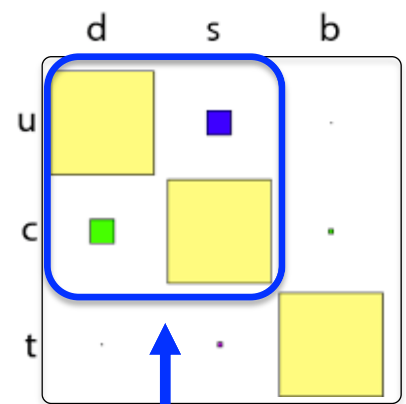
The Cabibbo angle

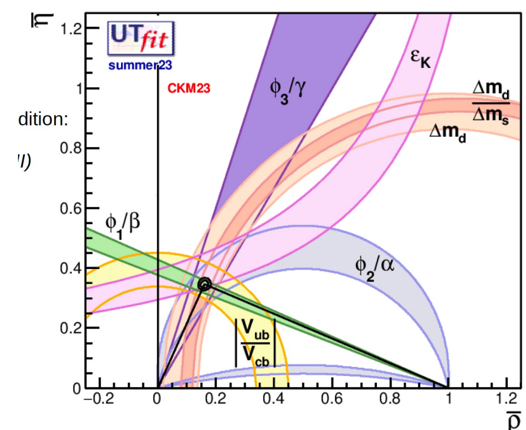
1 Quark masses



2 CKM matrix elements

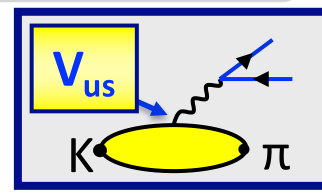
Cabibbo-Kobayashi-Maskawa

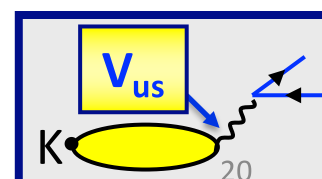




I will concentrate on the Cabibbo angle

$$\sin\vartheta_C = \lambda \simeq V_{us}$$

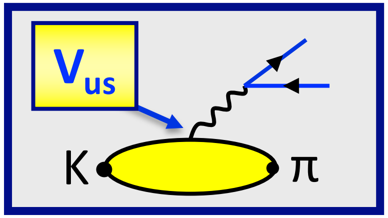




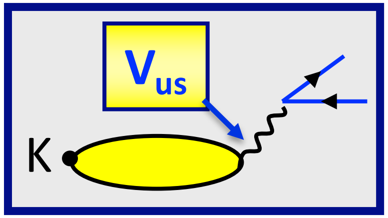
The Cabibbo angle

$$\sin\vartheta_C = \lambda \simeq V_{us}$$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) \propto (|V_{us}| f_+^{K\pi}(0))^2 (1 + \delta R_{K\pi}^{\ell})$$



$$\frac{\Gamma(K \to \ell \nu_\ell(\gamma))}{\Gamma(\pi \to \ell \nu_\ell(\gamma))} \propto \left(\frac{|V_{us}| f_K}{|V_{ud}| f_\pi}\right)^2 (1 + \delta R_{K\pi})$$

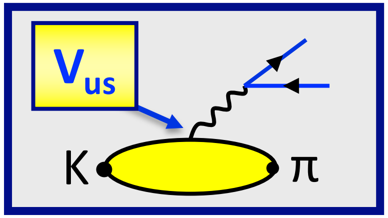


The most precise determinations of V_{us} come from leptonic and semileptonic kaon decays

The Cabibbo angle

$$\sin\vartheta_C = \lambda \simeq V_{us}$$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) \propto (|V_{us}| f_+^{K\pi}(0))^2 (1 + \delta R_{K\pi}^{\ell})$$



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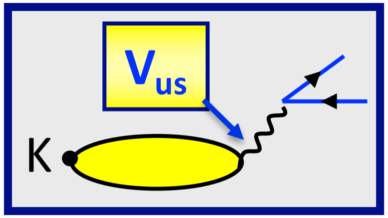


Experiments

$$\frac{|V_{us}|\, f_K}{|V_{ud}|\, f_\pi} = 0.2760(4)$$

PDG, arXiv:1509.02220

< 0.2%

$$|V_{us}|\, f_+^{K\pi}(0) = 0.21654(41)$$

M. Moulson, arXiv:1704.04104

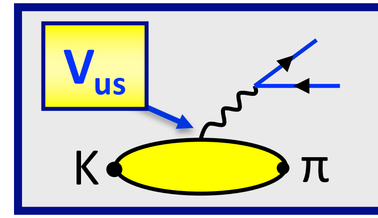
The first row unitarity test: $|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 = 1$

See talk by V. Cirigliano

The Cabibbo angle

$$\sin\vartheta_C = \lambda \simeq V_{us}$$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) \propto (|V_{us}| f_+^{K\pi}(0))^2 (1 + \delta R_{K\pi}^{\ell})$$



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Nuclear Physics B 705 (2005) 339–362

NUCLEAR PHYSICS B

The $K \to \pi$ vector form factor at zero momentum transfer on the lattice

D. Bećirević [a], G. Isidori [b], V. Lubicz [c,d], G. Martinelli [e], F. Mescia [b,c], S. Simula [d], C. Tarantino [c,d], G. Villadoro [e]

2004

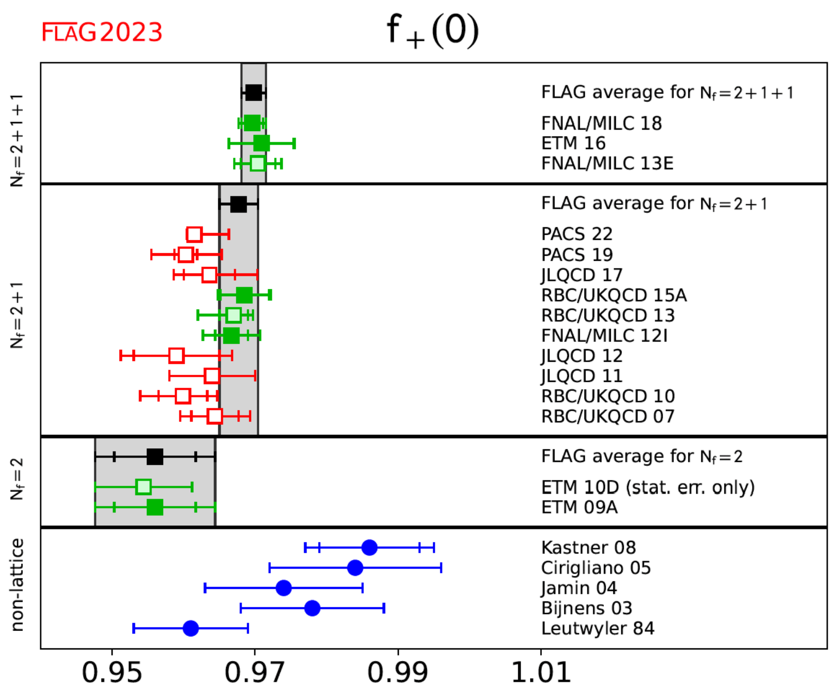
The first lattice study of $f_+^{K\pi}(0)$ with the required accuracy

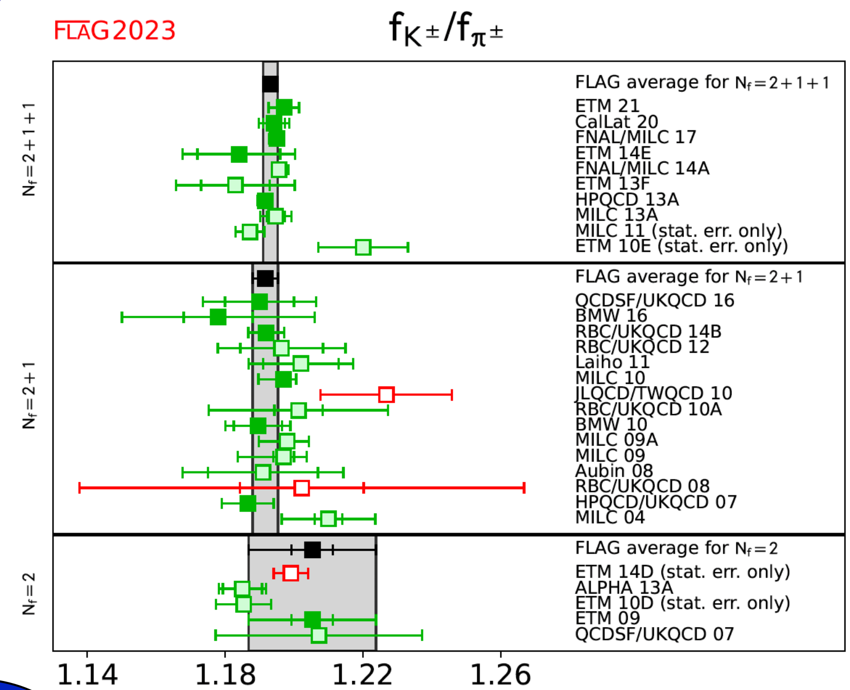
$$f_+^{K\pi}(0) = 0.960(9)$$

$f_+^{K\pi}(0) = 0.961(8)$ Quark model, 1984 Leutwyler and Roos

$f_+^{K\pi}(0) = 0.970(2)$ Lattice QCD today

The Cabibbo angle





$f_+^{K\pi}(0) = 0.9692(14)$

0.14%

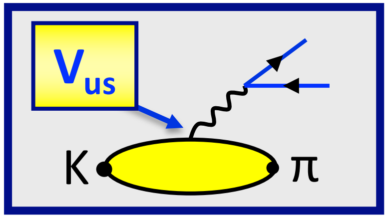
$f_{K^\pm}/f_{\pi^\pm} = 1.1930(17)$

The accuracy is comparable with the experimental one

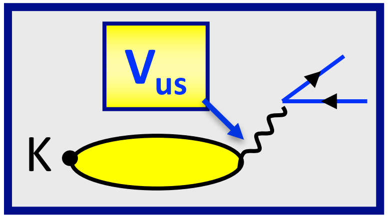
To achieve better than 1% final accuracy, however, an important issue must also be addressed

Isospin breaking effects

$$\Gamma\big(K \to \pi \ell \nu_\ell(\gamma)\big) \propto \left(|V_{us}| f_+^{K\pi}(0)\right)^2 \left(1 + \delta R_{K\pi}^{\ell}\right)$$



$$\frac{\Gamma\big(K \to \ell \nu_\ell(\gamma)\big)}{\Gamma\big(\pi \to \ell \nu_\ell(\gamma)\big)} \propto \left(\frac{|V_{us}| f_K}{|V_{ud}| f_\pi}\right)^2 \left(1 + \delta R_{K\pi}\right)$$



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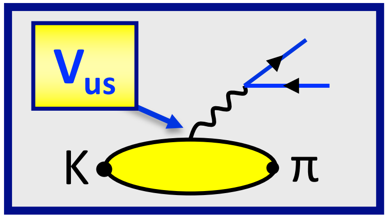
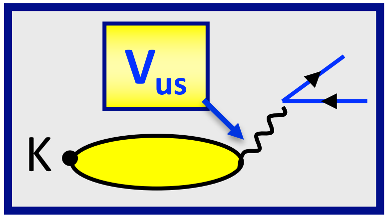




Isospin breaking effects are induced by:

$m_u \neq m_d$: $\mathcal{O}[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$ Strong

$Q_u \neq Q_d$: $\mathcal{O}(\alpha_{em}) \approx 1/100$ Electromagnetic

For many observables in flavor physics IB effects cannot be neglected

Isospin breaking effects

A strategy for Lattice QCD: the isospin breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ α_{em}

2011

JHEP
Published for SISSA by Sprin
Received: November 7,
Revised: March 16,
Accepted: April 2,
Published: April 26,

Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration
G.M. de Divitiis,[a,b] P. Dimopoulos,[c,d] R. Frezzotti,[a,b] V. Lubicz,[e,f] G. Martinelli,[g,d] R. Petronzio,[a,b] G.C. Rossi,[a,b] F. Sanfilippo,[c,d] S. Simula,[f] N. Tantalo[a,b] and C. Tarantino[e,f]

2013

PHYSICAL REVIEW D 87, 114505 (2013)

Leading isospin breaking effects on the lattice

G. M. de Divitiis,[1,2] R. Frezzotti,[1,2] V. Lubicz,[3,4] G. Martinelli,[5,6] R. Petronzio,[1,2] G. C. Rossi,[1,2] F. Sanfilippo,[7] S. Simula,[4] and N. Tantalo[1,2]

(RM123 Collaboration)

[1]Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy
[2]INFN, Sezione di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy
[3]Dipartimento di Matematica e Fisica, Università Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy
[4]INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy
[5]SISSA, Via Bonomea 265, 34136 Trieste, Italy
[6]INFN, Sezione di Roma, Piazzale Aldo Moro 5, I-00185 Rome, Italy
[7]Laboratoire de Physique Théorique (Bâtiment 210), Université Paris Sud, F-91405 Orasay-Cedex, France

(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

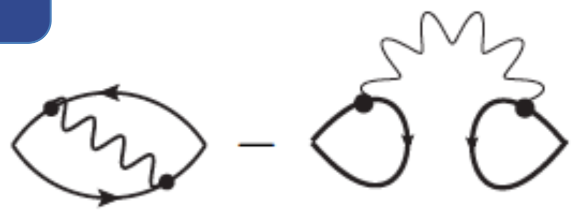
For present accuracy, only the leading order of the expansion is required

IB corrections to the hadronic spectrum

Two examples:

$M_{\pi^+} - M_{\pi^0}$

Only two QED diagrams at the leading order



$M_{\pi^+} - M_{\pi^0} = 4.622(95)\ \mathrm{MeV}$

Frezzotti, Gagliardi, VL, Martinelli, Sanfilippo, Simula, PRD 106 (2022) 1

$M_{\pi^+} - M_{\pi^0} = 4.534(60)\ \mathrm{MeV}$

Feng, Jin, Riberdy, PRL 128 (2022) 052003

Expt: $= 4.5936(5)\ \mathrm{MeV}$

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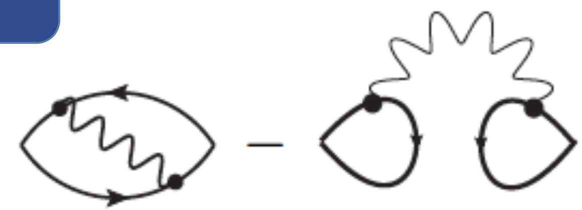

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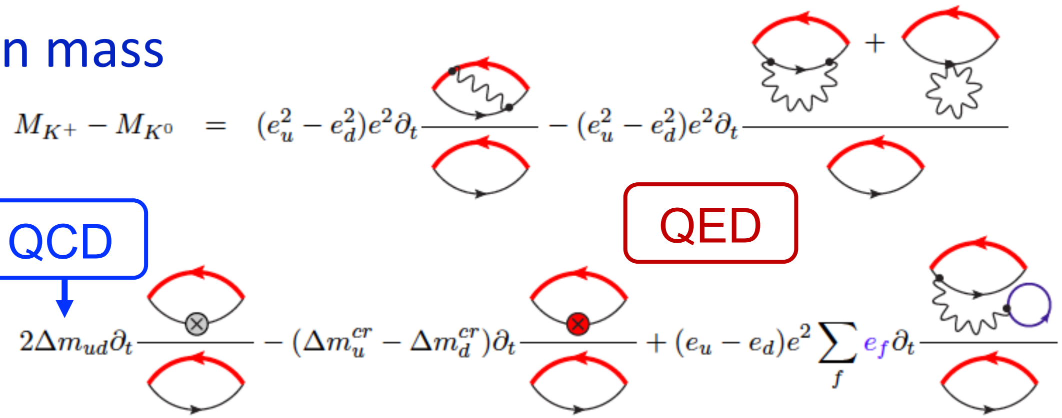
Feng, Jin, Riberdy, PRL 128 (2022) 052003

Expt: $= 4.5936(5)\ \mathrm{MeV}$

m_u - m_d

From the kaon mass difference



$$m_\mathrm{u}/m_\mathrm{d} = 0.513(30)$$

$$\overline{m}_\mathrm{u} = 2.50(17)\ \mathrm{MeV}$$

$$\overline{m}_\mathrm{d} = 4.88(20)\ \mathrm{MeV}$$

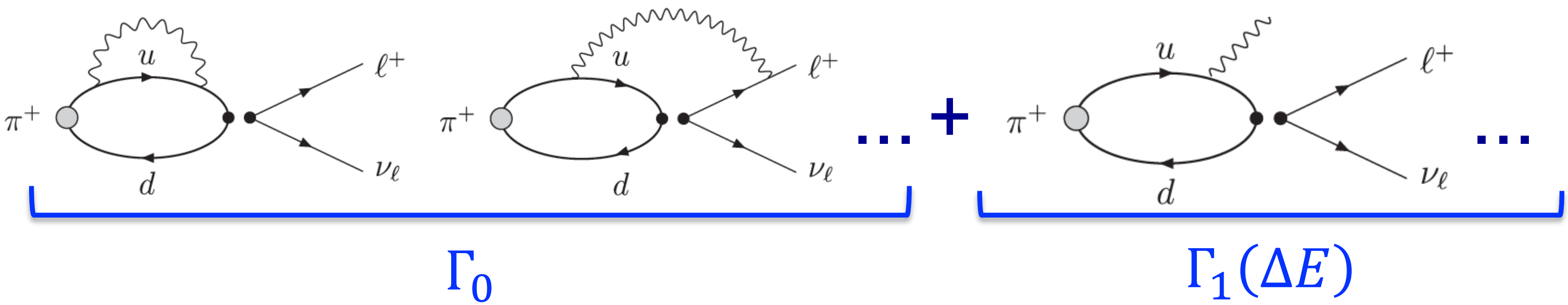
[RM123 Collaboration, D.Giusti *et al.*, PRD 95 (2017) 1]

QED corrections to hadronic processes

MORE CHALLENGING:

Infrared divergences appears in the intermediate step of the calculation

For instance, at O(α) for leptonic pion decays, one has to consider:



$$\Gamma(\Delta E) = \Gamma(\pi \to \ell \nu_\ell) + \Gamma\big(\pi \to \ell \nu_\ell \gamma(\Delta E)\big) \equiv \Gamma_0 + \Gamma_1(\Delta E)$$

with $0 \leq E_\gamma \leq \Delta E$. The sum is infrared finite [Bloch and Nordsieck, 1937]

QED corrections to hadronic processes

RM123+Southampton (RM123S) Collab.

PHYSICAL REVIEW D **91**, 074506 (2015)

QED corrections to hadronic processes in lattice QCD

N. Carrasco,[1] V. Lubicz,[1] G. Martinelli,[2] C. T. Sachrajda,[3] N. Tantalo,[4,5] C. Tarantino,[1] and M. Testa[6]

PHYSICAL REVIEW D **95**, 034504 (2017)

Finite-volume QED corrections to decay amplitudes in lattice QCD

V. Lubicz,[1] G. Martinelli,[2,3] C. T. Sachrajda,[4] F. Sanfilippo,[4] S. Simula,[5] and N. Tantalo[6]

PHYSICAL REVIEW LETTERS **120**, 072001 (2018)

First Lattice Calculation of the QED Corrections to Leptonic Decay Rates

D. Giusti, V. Lubicz, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo, C. Tarantino

PHYSICAL REVIEW D **100**, 034514 (2019)

Editors' Suggestion

Light-meson leptonic decay rates in lattice QCD+QED

M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo

cont.

QED corrections to hadronic processes

RM123+Southampton (RM123S) Collab.

PHYSICAL REVIEW D **103**, 014502 (2021)

First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons

A. Desiderio,[1] R. Frezzotti,[1] M. Garofalo,[2] D. Giusti,[3,4] M. Hansen,[5] V. Lubicz,[2] G. Martinelli,[6] C. T. Sachrajda,[7] F. Sanfilippo,[4] S. Simula,[4] and N. Tantalo[1]

PHYSICAL REVIEW D **103**, 053005 (2021)

Comparison of lattice QCD + QED predictions for radiative leptonic decays of light mesons with experimental data

R. Frezzotti,[1] M. Garofalo,[2,3] V. Lubicz,[2] G. Martinelli,[4] C. T. Sachrajda,[5] F. Sanfilippo,[6] S. Simula,[6] and N. Tantalo[1]

RBC-UKQCD Collaboration

JHEP 2023

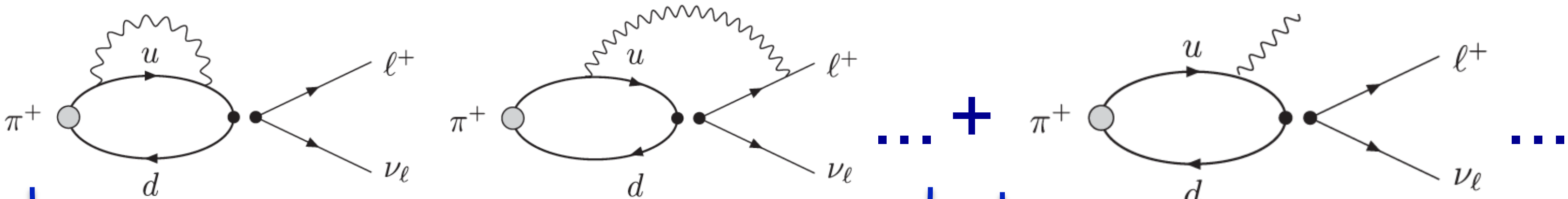
PUBLISHED FOR SISSA BY SPRINGER

Isospin-breaking corrections to light-meson leptonic decays from lattice simulations at physical quark masses

Peter Boyle,[a,b] Matteo Di Carlo,[b] Felix Erben,[b] Vera Gülpers,[b] Maxwell T. Hansen,[b] Tim Harris,[b] Nils Hermansson-Truedsson,[c,d] Raoul Hodgson,[b] Andreas Jüttner,[e,f] Fionn Ó hÓgáin,[b] Antonin Portelli,[b] James Richings[b,e,g] and Andrew Zhen Ning Yong[b]

JHEP02(2023)242

QED corrections to hadronic processes



Γ_0 $\Gamma_1(\Delta E)$

THE STRATEGY

RM123S Collab.

$$\Gamma(\Delta E) = \lim_{\Lambda_{IR}\to 0}\left(\Gamma_0 + \Gamma_1(\Delta E)\right) =$$

1 $$= \lim_{L\to\infty}\left(\Gamma_0(L) - \Gamma_0^{pt}(L)\right)$$ ⇐ Universal 1/L corrections also cancel in the difference

2 $$+ \lim_{m_\gamma\to 0}\left(\Gamma_0^{pt}(m_\gamma) + \Gamma_1^{pt}(m_\gamma, \Delta E)\right)$$ ⇐ Computed in perturbation theory

3 $$+ \lim_{L\to\infty}\left(\Gamma_1(\Delta E) - \Gamma_1^{pt}(\Delta E)\right)$$ ⇐ $= \Gamma_1^{SD}(\Delta E) + \Gamma_1^{INT}(\Delta E)$

Structure dependent

QED corrections to hadronic processes

RESULTS

$$\frac{\Gamma(K \to \ell\nu_\ell(\gamma))}{\Gamma(\pi \to \ell\nu_\ell(\gamma))} \propto \left(\frac{|V_{us}|f_K}{|V_{ud}|f_\pi}\right)^2 (1 + \delta R_{K\pi})$$

$$\delta R_{K\pi} = \delta R_K - \delta R_\pi$$

Lattice (RM123S Collaboration, M. Di Carlo *et al.*, PRD 100, 2019)	ChPT (V. Cirgliano, H. Neufeld, PLB 700, 2011)
$\delta R_\pi = 0.0153(19)$	$0.0176(21)$
$\delta R_K = 0.0024(10)$	$0.0064(24)$
$\delta R_{K\pi} = -0.0126(14)$	$-0.0112(21)$

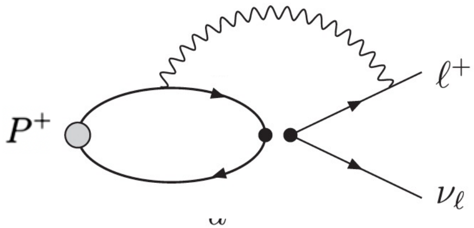
$\delta R_{K\pi} = -0.0086(41)$

RBC-UKQCD Collaboration
P. Boyle *et al.*, JHEP 02, 2023

Lattice results in GOOD AGREEMENT with ChPT, with smaller uncertainties

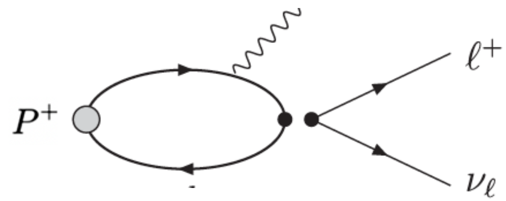
For heavy mesons only the lattice approach is available

QCD+QED on the lattice



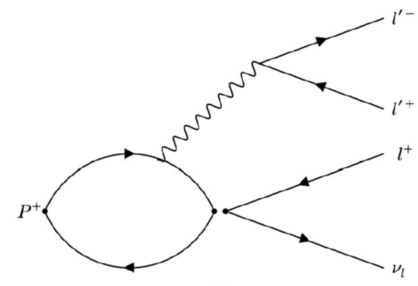
Inclusive leptonic decays ($P=\pi, K$)

RM123S, RBC-UKQCD.



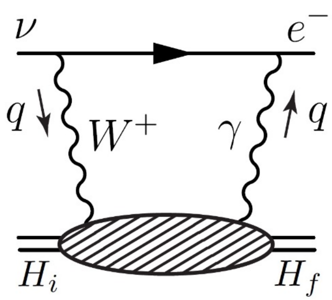
Radiative leptonic decays ($P=\pi, K, D_s$)

RM123S, RBC-UKQCD



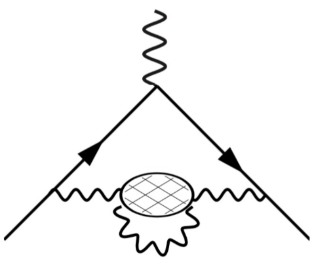
Virtual photon emission in leptonic decays ($P=K$)

Tuo, Feng, Jin, Wang; RM123S



Electroweak box contribution to semileptonic decays ($H=\pi, K, N$)

Ma, Feng, Gorchtein, Jin, Seng *et al.*



Isospin breaking corrections to the HVP contribution to $g_\mu - 2$

BMW, ETM, Mainz, RBC-UKQCD

+ ...

Inclusive hadronic τ decay on the lattice

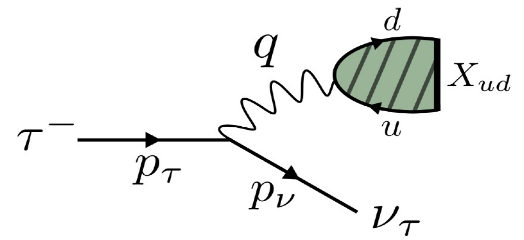
NEW!

The hadronic inclusive decay rate of the τ-lepton can be written as

$$R_{ud}^{(\tau)} = \frac{\Gamma(\tau \to X_{ud}\,\nu_\tau)}{\Gamma(\tau \to e\,\bar{\nu}_e\,\nu_\tau)} = 6\pi\, S_{EW}\, |V_{ud}|^2 \int_0^1 ds\,(1-s)^2 \left[(1+2\,s)\,\rho_T(s) + \rho_L(s)\right]$$

where $\rho_T(s)$ and $\rho_L(s)$ are the form factors of the spectral density

$$\rho_{ud}^{\alpha\beta}(q) = \sum_X \langle\, 0\,|\, J_{ud}^{\alpha}(0)\,|\, X_{ud}(q)\,\rangle \langle\, X_{ud}(q)\,|\, J_{ud}^{\beta}(0)^\dagger\,|\,0\,\rangle$$
$$= \langle 0| J_{ud}^{\alpha}(0)\,(2\pi)^4 \delta^4(\mathbb{P} - q)\, J_{ud}^{\beta}(0)^\dagger |0\rangle$$



It is related to the Euclidean lattice correlator by:

$$C_{\mathrm{I}}(t) \equiv \int_0^\infty \frac{dE}{2\pi}\, e^{-Et}\, E^2 \rho_{\mathrm{I}}(E^2)$$

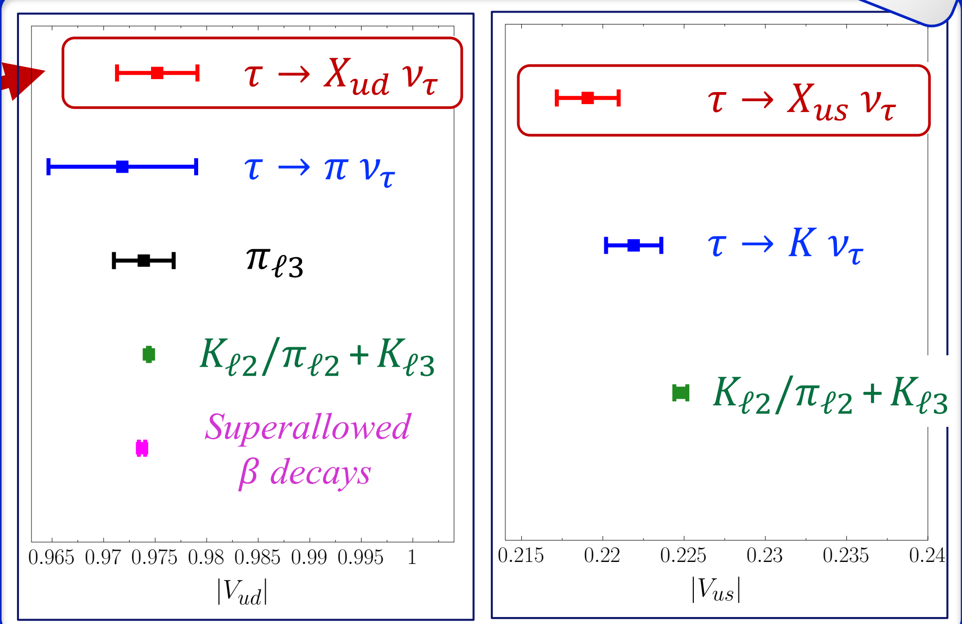
The inverse problem can be solved with the HLT method [Hansen, Lupo, Tantalo, PRD 99 2019]

Inclusive hadronic τ decay on the lattice

NEW!

RESULTS

Thanks to G.Gagliardi



PHYSICAL REVIEW D **108,** 074513 (2023)

Editors' Suggestion

Inclusive hadronic decay rate of the τ lepton from lattice QCD

A. Evangelista, R. Frezzotti, and N. Tantalo
Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma, Italy

G. Gagliardi, F. Sanfilippo, and S. Simula
Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

V. Lubicz
Dipartimento di Matematica e Fisica, Università Roma Tre and INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

(Extended Twisted Mass Collaboration)

$$R^{(\tau)}_{ud}/|V_{ud}|^2 = 3.650(28) \quad \Rightarrow \quad |V_{ud}|^2 = 0.9752(37)_{\text{lat}}(10)_{\text{exp}}\ [39]_{\text{TOT}}$$

$$R^{(\tau)}_{us}/|V_{us}|^2 = 3.403(18) \quad \Rightarrow \quad |V_{us}|^2 = 0.2191(6)_{\text{lat}}(18)_{\text{exp}}\ [19]_{\text{TOT}}$$

Preliminary

Dominated by the EXP error

The R-ratio on the lattice

LETTERE AL NUOVO CIMENTO VOL. IV, N. 1 4 Luglio 1970

1970

Hadron Production in e^+e^- Collisions (*).

N. Cabibbo
Istituto di Fisica dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

G. Parisi and M. Testa
Istituto di Fisica dell'Università - Roma

(ricevuto il 30 Maggio 1970)

1. – The simple properties of deep inelastic electron-proton scattering has suggested models where these processes arise as interactions of virtual photons with an «elementary» component of the proton. These as yet unspecified elementary components of the proton have been given the name of «partons» by Feynman (1). The model has been studied by Bjorken and Paschos (2) and successively by Drell, Levy and Tung Mow Yan (3) who gave a field-theoretical treatment of the parton model, and were able to recover some of the experimentally observed properties of this process. In this letter we wish to extend the method of ref. (3) to the study of the total cross-section of electron-positron annihilation into hadrons.

This treatment leads to an asymptotic (very high cross-section c.m. energy, $2E$) of the form

$$\sigma \to \frac{\pi\alpha^2}{12E^2}\left[\sum_{\text{spin }0}(Q_i)^2 + 4\sum_{\text{spin }\frac{1}{2}}(Q_i)^2\right], \qquad (1)$$

where Q_i is the charge of the i-th parton in units of e. This is simply the sum of the contributions of the single partons considered as pointlike (4). Each parton contributes a different kind of events to the total cross-section. The typical high-energy event should consist in the production of a pair of virtual partons, each of which develops into a jet of physical hadrons.

PHYSICAL REVIEW LETTERS **130**, 241901 (2023)

2023

Probing the Energy-Smeared R Ratio Using Lattice QCD

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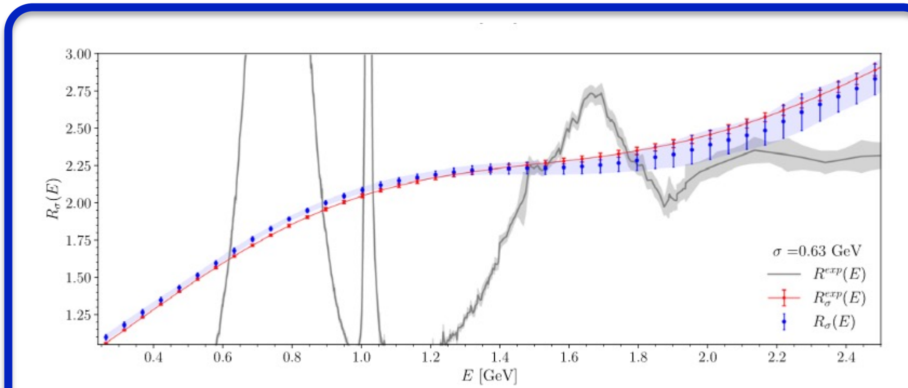
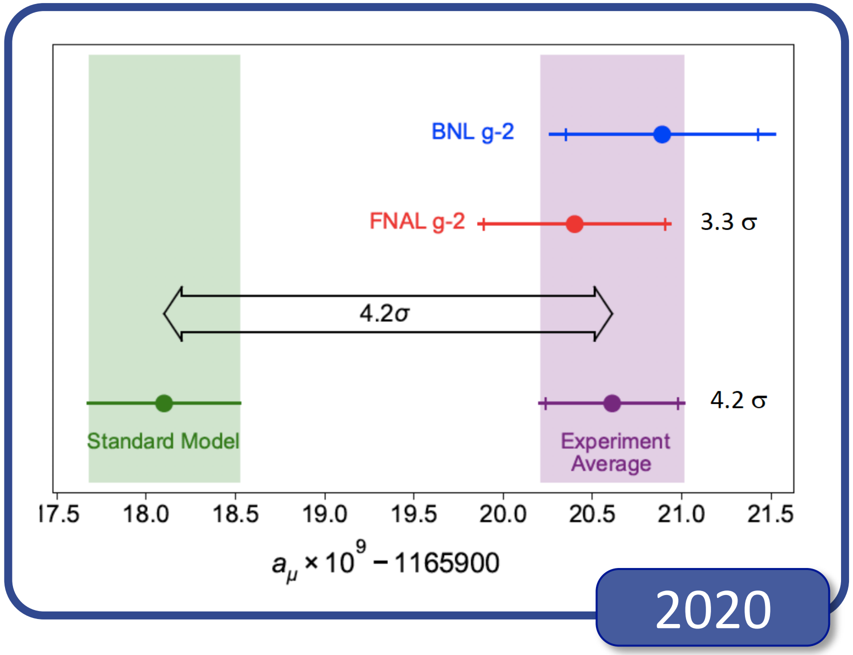
(Extended Twisted Mass Collaboration (ETMC))

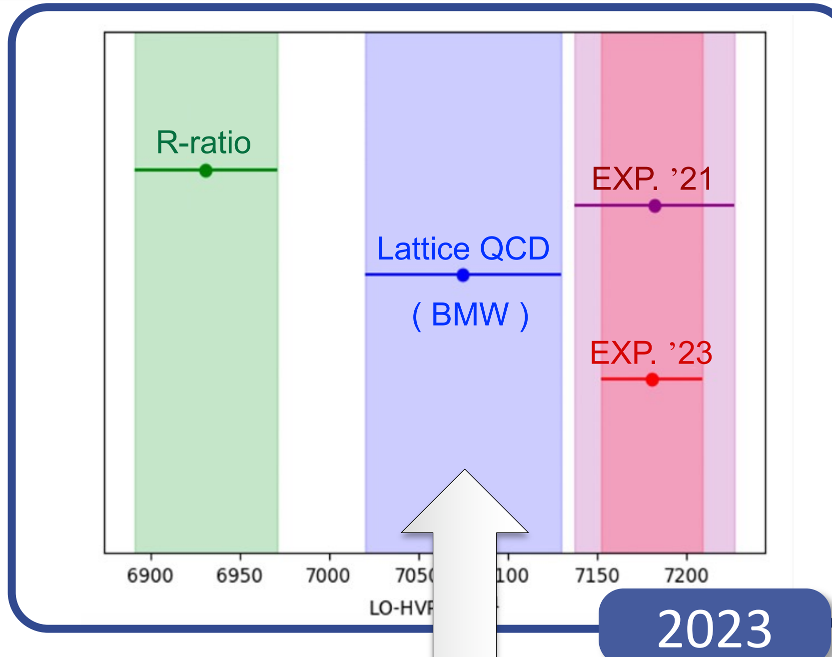
Using the same HLT method to solve the inverse problem

$$V(t) = \frac{1}{12\pi^2}\int_0^\infty d\omega\,\omega^2 R(\omega) e^{-t\omega}.$$

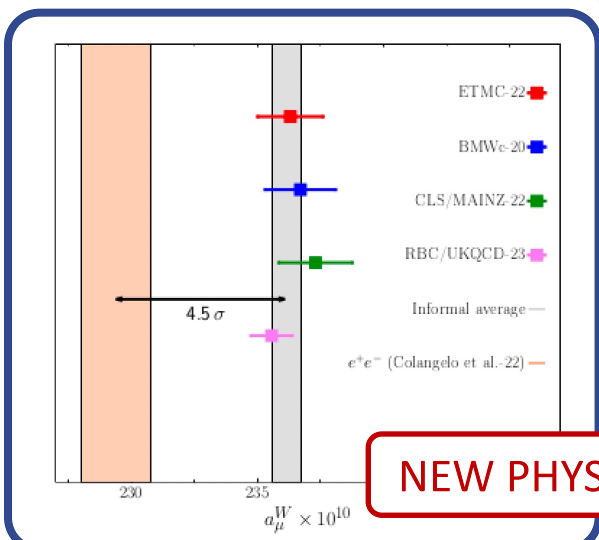


The muon anomalous magnetic moment





An essential contribution from Lattice QCD (+QED)



NEW PHYSICS ?

Agreement between lattice and muon g-2.

But 4.5 σ discrepancy in the *window* between lattice and e^+e^- data (except for CMD-3)

Cabibbo 60

- A lot of progress has been done since the first paper of weak interactions on the lattice

- I have tried to summarize some of the most recent and interesting results

Nuclear Physics B244 (1984) 381–391

WEAK INTERACTIONS ON THE LATTICE

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Received 5 December 1983
(Revised 16 April 1984)

We show that lattice QCD can be used to evaluate the matrix elements of four-fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations are also presented.

$\Delta I = 1/2$ rule and ε'/ε

The Cabibbo angle

Isospin breaking effects

Inclusive τ decays

The muon g-2

Radiative decays

- For most of these quantities, lattice predictions have the accuracy of precision physics

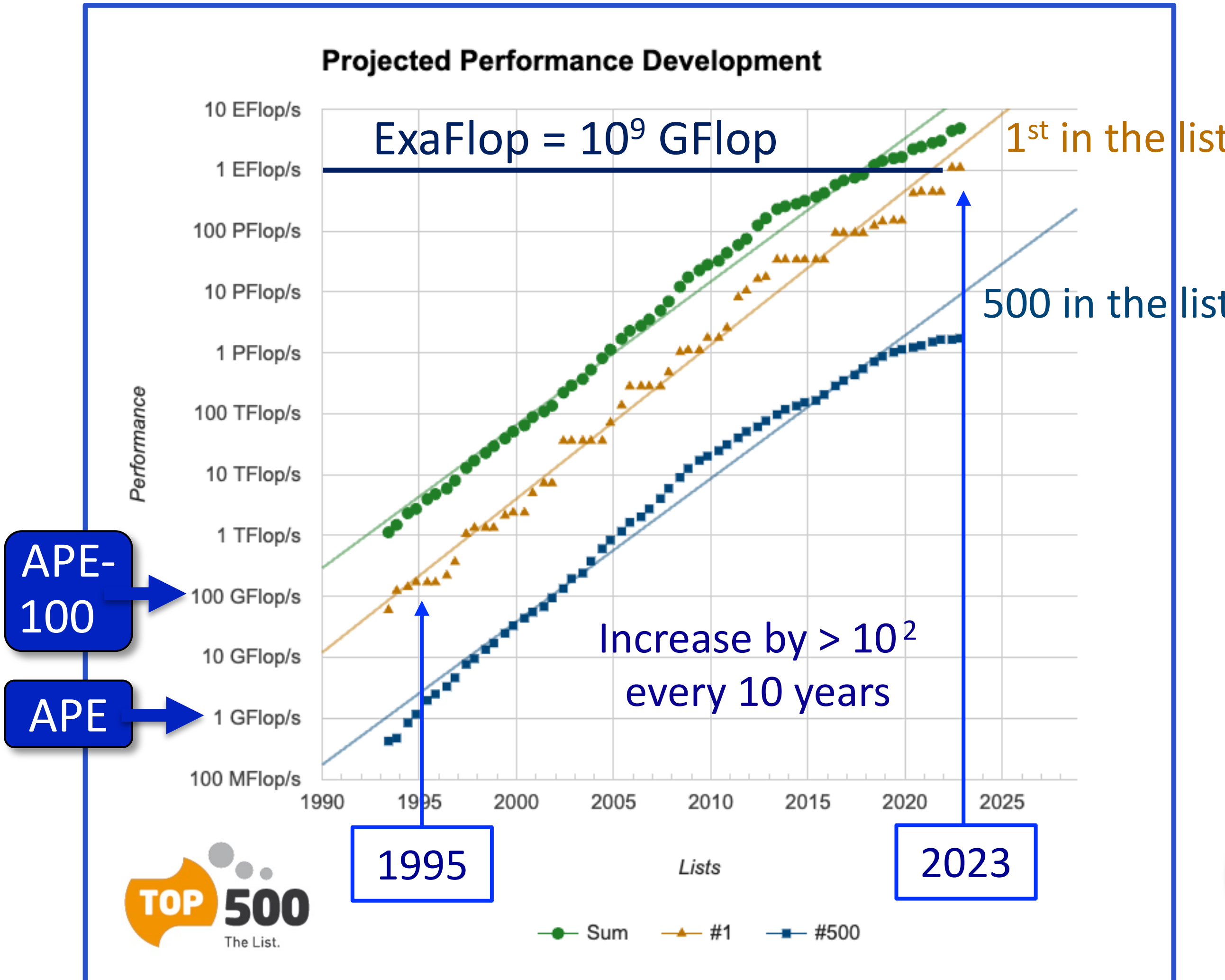
Cabibbo 60

Grazie Nicola!

BACKUP SLIDES

The precision era of Lattice QCD

The increasing of computational power



Rank	Site
1	DOE/SC/Oak Ridge National Laboratory United States Frontier Rpeak = 1,680 PFlop/s
2	RIKEN Center for Computational Science Japan Fugaku Rpeak = 537 PFlop/s
3	EuroHPC/CSC Finland LUMI Rpeak = 429 PFlop/s
4	EuroHPC/CINECA Italy Leonardo Rpeak = 305 PFlop/s

Quark masses

ELSEVIER

Nuclear Physics B 431 (1994) 667–685

NUCLEAR PHYSICS B

Quark masses from lattice QCD at the next-to-leading order

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1994

The first study of quark masses on the lattice at NLO

$$m^{\overline{\rm MS}}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\pi/a)}\right)^{\gamma^{(0)}/2\beta_0}\left[1 + \frac{\alpha_s(\mu)-\alpha_s(\pi/a)}{4\pi}\left(\frac{\gamma^{(1)}}{2\beta_0} - \frac{\gamma^{(0)}\beta_1}{2\beta_0^2}\right) + \frac{\alpha_s(\pi/a)}{4\pi}K_m\right] m(a)$$

The renormalized quark mass

The bare quark mass fixed from the hadronic spectrum

In the last 30 years, many lattice calculations have been performed.

NPR methods have proven to be an essential ingredient for high accuracy

Inclusive τ decay on the lattice

THE HLT METHOD

Hansen, Lupo, Tantalo,
PRD 99 2019

- $$R_{ud}^{(\tau,\mathrm{I})}(\sigma) \propto \int_{m_h}^{\infty} \frac{dE\, E^2}{m_\tau^3}\, K_\mathrm{I}^\sigma\left(\frac{E}{m_\tau}\right) \rho_\mathrm{I}(E^2)$$

$$C_\mathrm{I}(t) \equiv \int_0^\infty \frac{dE}{2\pi}\, e^{-Et}\, E^2 \rho_\mathrm{I}(E^2)$$

$$K_I^\sigma(x) = f(x)\,\Theta_\sigma(x)\,,$$ where $$\Theta_\sigma(x) \equiv \frac{1}{1+e^{-x/\sigma}}\,, \quad \lim_{\sigma\mapsto 0}\Theta_\sigma(x) = \theta(x)$$

- One looks for an approximation of the kernel in terms of a basis function

$$K_\mathrm{I}^\sigma\left(\frac{E}{m_\tau}\right) \simeq \sum_{n=1}^{n_{max}} g_\mathrm{I}(n,\sigma,am_\tau)\, e^{-aEn}$$

- The coefficients are obtained by minimizing: $$W_\mathrm{I}^\alpha[\boldsymbol{g}] \equiv \frac{A_\mathrm{I}^\alpha[\boldsymbol{g}]}{A_\mathrm{I}^\alpha[\boldsymbol{0}]} + \lambda B_\mathrm{I}[\boldsymbol{g}]$$ where

$$A_\mathrm{I}^\alpha[\boldsymbol{g}] = \int_{E_{min}}^{E_{max}} dE\, e^{aE\alpha} \left| \sum_{n=1}^{T/(2a)} g(n) e^{-aEn} - K_\mathrm{I}^\sigma\left(\frac{E}{m_\tau}\right)\right|^2\,, \quad B_\mathrm{I}[\boldsymbol{g}] = B_\mathrm{norm} \sum_{n_1,n_2=1}^{T/(2a)} g(n_1)\, g(n_2)\, \mathrm{Cov}_\mathrm{I}(an_1, an_2)$$

- Then: $$R_{ud}^{(\tau,\mathrm{I})}(\sigma) \propto \sum_{n=1}^{T/(2a)} g_\mathrm{I}(n,\sigma,am_\tau)\, C_\mathrm{I}(na)$$

Inclusive τ decay on the lattice

Table 13: HFLAV 2021 τ branching fractions to strange final states.

Branching fraction	HFLAV 2021 fit (%)
$K^-\nu_\tau$	0.6957 ± 0.0096
$K^-\pi^0\nu_\tau$	0.4322 ± 0.0148
$K^-2\pi^0\nu_\tau$ (ex.K^0)	0.0634 ± 0.0219
$K^-3\pi^0\nu_\tau$ (ex.K^0,η)	0.0465 ± 0.0213
$\pi^-\overline{K}^0\nu_\tau$	0.8375 ± 0.0139
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.3810 ± 0.0129
$\pi^-\overline{K}^0 2\pi^0\nu_\tau$ (ex.K^0)	0.0234 ± 0.0231
$\overline{K}^0 h^- h^- h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0155 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\overline{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi(K^+K^-)\nu_\tau$	0.0022 ± 0.0008
$K^-\phi(K^0_S K^0_L)\nu_\tau$	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex.K^0,ω)	0.2924 ± 0.0068
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex.K^0,ω,η)	0.0387 ± 0.0142
$K^-2\pi^-2\pi^+\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9076 ± 0.0478

HFLAV compilations