My relationship with Marcello

in short

I have known Marcello for more than 50 years

we published together over 55 papers

26 in the Nuclear Matter and Brueckner Theory

25 BCS theory and NM Superfluidity

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ICTP (Trieste, 1969)

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80° Marcello’s Anniversary

Marcello’s Contributions to the Theory of Nuclear-Matter Superfluidity

Cottage *Le Muse*, 2022

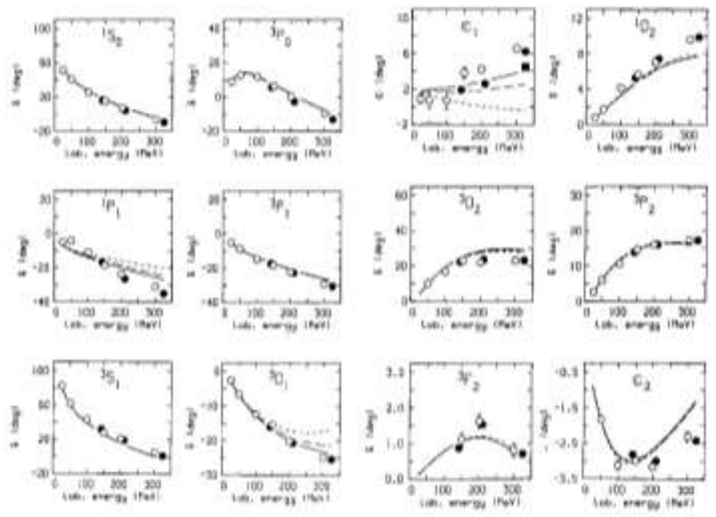
Outline:

- BCS calculations with realistic interactions
- Selfenergy and corrections
- Transition to Bose condensate
- from Homogeous to Inhomogeneus Matter:
- Superfluid States of Nuclear Matter in Astrophysics

BCS Calculations with Realistic Interactions

degenerate Fermi Systems

Inter.Coll.: J. Cugnon, A. Lejeune (1990-96)



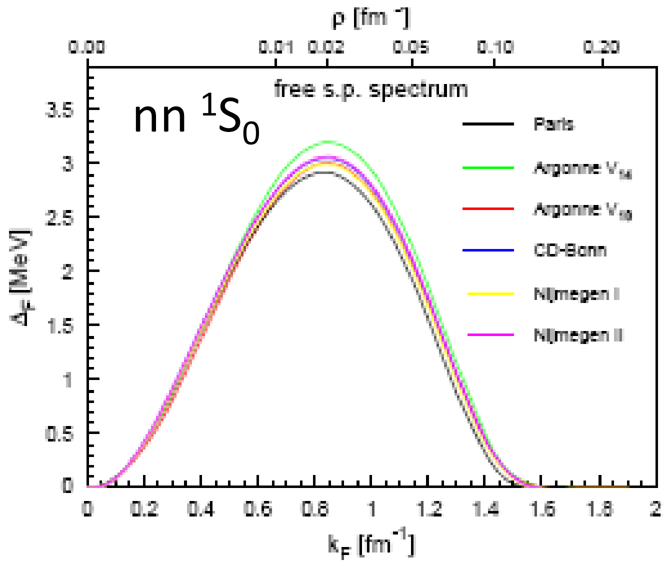
$$\Delta_k = \sum V_{kk'} \frac{\Delta_{k'}}{2\sqrt{(e_{k'} - e_F)^2 + \Delta_{k'}^2}}$$

$$\rho = \sum_k \theta(e_F - e_k)$$

$V_{kk'}$ bare interaction (from exp phase shifts)

$e_k = k^2/2m + U_k$ from BHF

- independent of interaction
- overestimate the experimental values in nuclei
- peacked at low densities



Querelle with J. Cugnon: $V_{kk'}$ or $G_{kk'}$?

$$\Delta_k = \sum V_{kk'} \frac{\Delta_{k'}}{2\sqrt{(e_{k'} - e_F)^2 + \Delta_{k'}^2}}$$

L. Cooper et.al. (PRC 1959)
''…the true two-body potential v must be used rather than the g-matrix, since we are looking for a state which arises from strong two-body interactions and hence corresponds to iteration of v… ''

Marcello was right !

Querelle with J. Cugnon: $V_{kk'}$ or $G_{kk'}$?

$$\Delta_k = \sum V_{kk'} \frac{\Delta_{k'}}{2\sqrt{(e_{k'} - e_F)^2 + \Delta_{k'}^2}}$$

Etna volcano, 2900m a.s.l., 1990

L. Cooper et.al. (PRC 1959)

"…the true two-body potential v must be used rather than the g-matrix, since we are looking for a state which arises from strong two-body interactions and hence corresponds to iteration of v… "

analytical demonstration

Gap Equation : $\boldsymbol{\Delta} = \mathrm{V}\, \mathbf{G}\mathbf{G}_s \boldsymbol{\Delta}$

Splitting the (e,p)-space into two sub-spaces, P and Q, such that

Q : $\boldsymbol{\Delta} << |e - e_F|$ $n_p^2 << n_p$

the gap equation splits

$$\boldsymbol{\Delta} = \tilde{\mathrm{V}}\, \mathbf{G}\mathbf{G}_s \boldsymbol{\Delta}$$

$$\tilde{\mathrm{V}} = \mathrm{V} + \mathrm{V}\, \frac{Q}{2e_F - e_p - e_{p'}}\, \tilde{\mathrm{V}}$$ (Bethe-Goldostone like Eq.)

$Q = 1 - n_p - n_{p'} \approx$ Pauli operator

Medium Dispersive Effects

quasi-degenerate Fermi Systems

Self-energy corrections

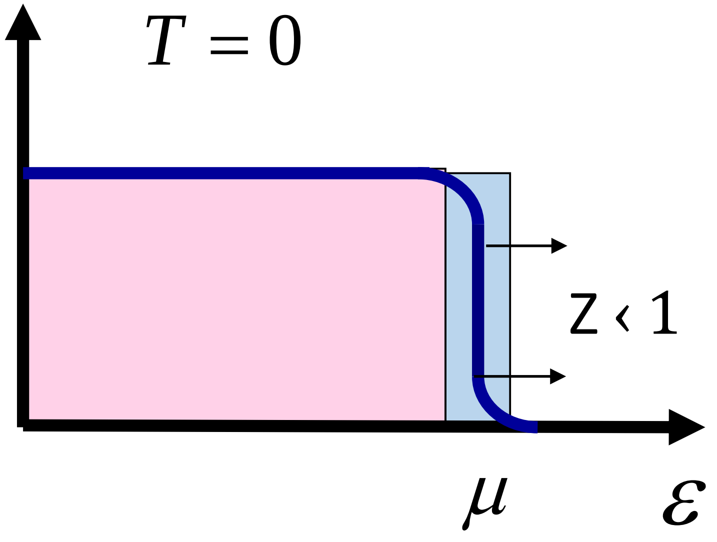
$\mathbf{G}^{-1}(\mathcal{E}, p) = \mathbf{G}_0^{-1} - \boldsymbol{\Sigma}(\mathcal{E}, p) = Z^{-1}(\mathcal{E} - (p^2 - p_F^2)/2m^*)$

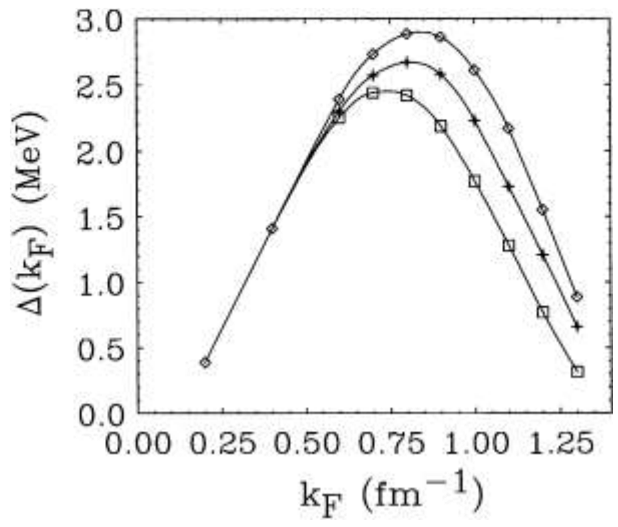
$\mathbf{G}^{-1}_S(\mathcal{E}, p) = \mathbf{G}_0^{-1}(\mathcal{E}, p) + \mathbf{G}_0(-\mathcal{E}, -p)\,|\Delta|^2$

quasi-particle strength $Z(p)^{-1} = \left(1 - \frac{\delta\Sigma(p,\omega)}{\delta\omega}\right)_F$

$\mathcal{E} = (p^2 - p_F^2) / 2m^*$; $\frac{m}{m^*} = Z(p)\left(1 - \frac{\delta\Sigma(p,\omega)}{\delta p}\right)_F$

$$\Delta_k = \sum V_{kk'} \frac{Z^2\,\Delta_{k'}}{2\sqrt{(e_{k'} - e_F)^2 + \Delta_{k'}^2}}$$





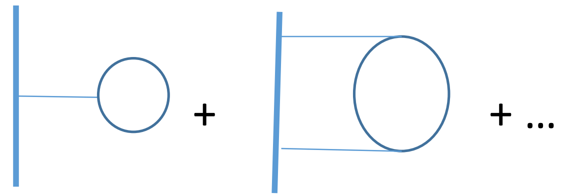
Baldo & Grasso PLB, 2000

Medium Polarization Effects

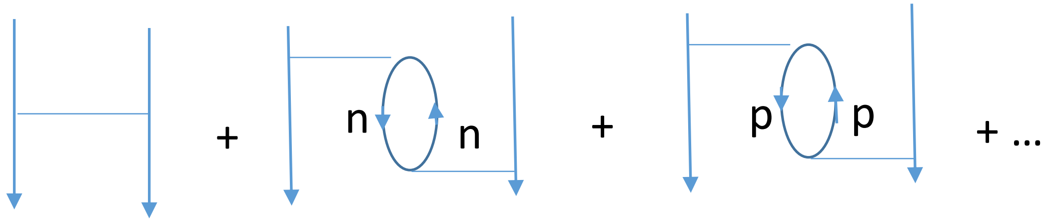
quasi-degenerate Fermi Systems

Inter.Coll.: , H.-J. Schulze,
J. Cugnon, A. Lejeune

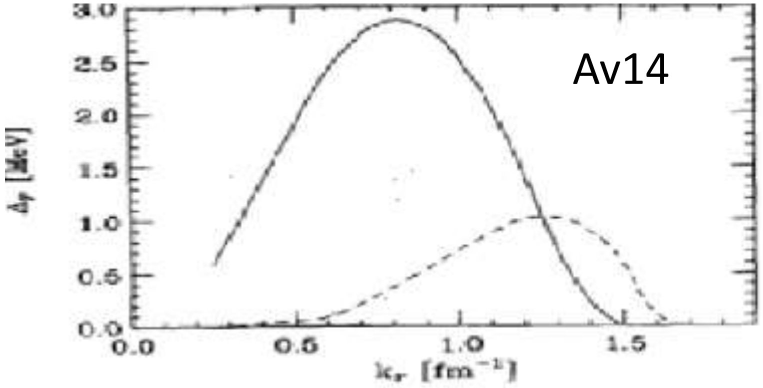
Self-Energy Polarization

$\Sigma =$ 

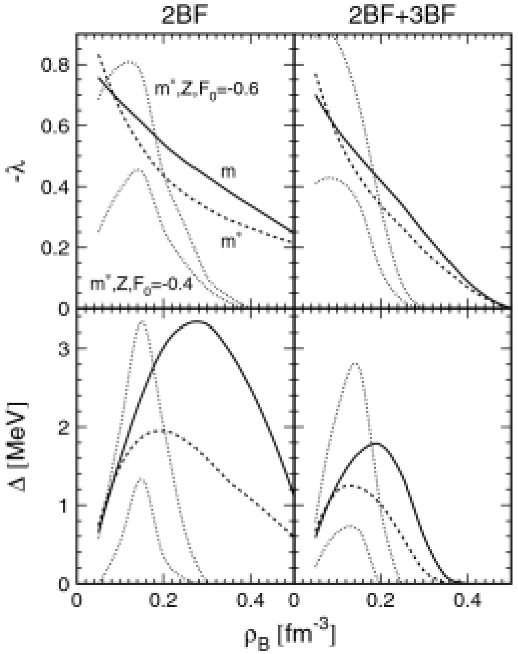
Vertex Polarization

$V =$ 

neutron matter



Nuclear Matter (β-stable)

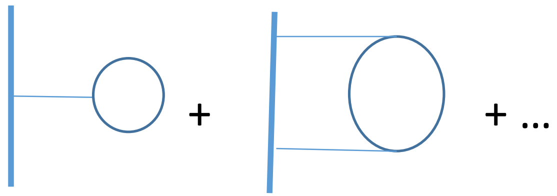


Medium Polarization Effects

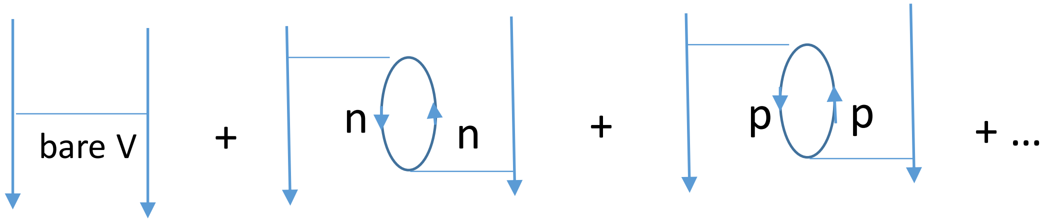
quasi-degenerate *Fermi Systems*

Inter.Coll.: , H.-J. Schulze,
J. Cugnon, A. Lejeune

Self-Energy Polarization

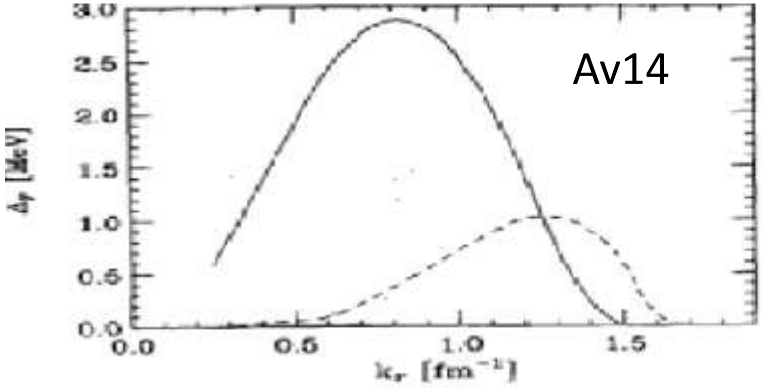
$\Sigma =$ 

Vertex Polarization

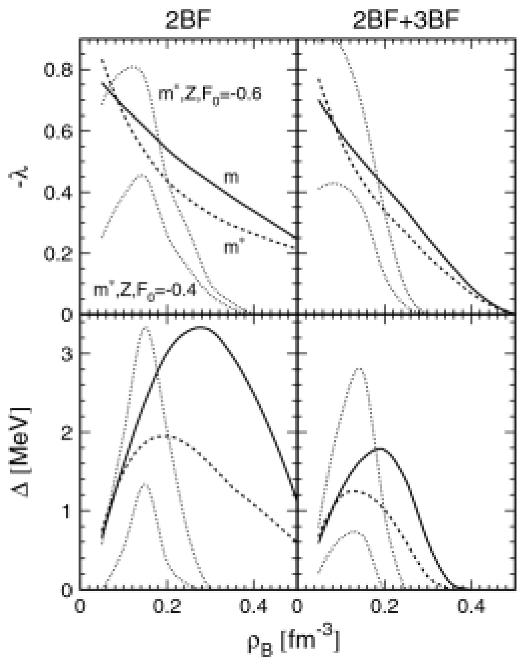
$V =$ 

Rostock,2009

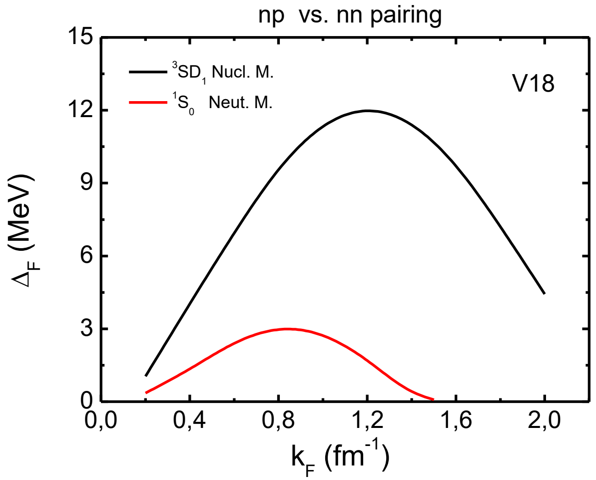
neutron matter



Nuclear Matter (β-stable)



Spin-triplet n-p Pairing



A.M. Lane (Nuclear Theory, Benjamin 1964) :

"The neglect of the neutron-proton interaction is the major weakness of the pairing force theory. This interaction is just as strong as that between a pair of like nucleons. In fact in the T=0 state is stronger."

Investigation keywords:

- Crossover from n-p Fermi system to deuteron Bose system
- Suppression in nuclei (N=Z: Bertsch (20109), Sagawa & Colò (2014))
- Searching experimental evidence (low energy HIC, NS cooling,…)

Inter.Coll.: P. Schuck

Low-density Crossover to Deuteron Bose Condensate

Inter.Coll.: P. Schuck

$$2(\varepsilon_p - \mu)\varphi_p + (1 - 2n_p)\,\Sigma_{p'} V(p,p')\varphi_{p'} = 0$$

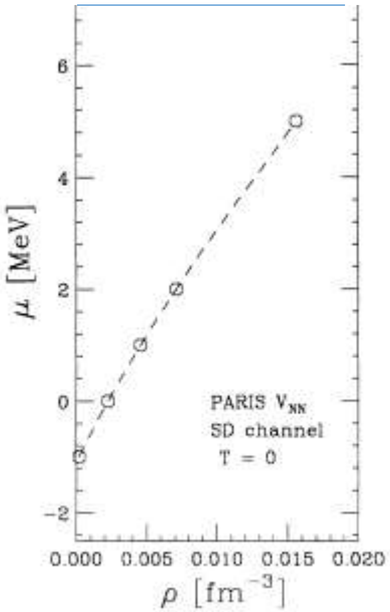
$$\rho = \frac{N}{V} = \Sigma_p\, \tfrac{1}{2}\,(1 - \frac{\varepsilon_p - \mu}{E})$$

$\varphi = \Delta/2E$ (pairing correlation fnct)

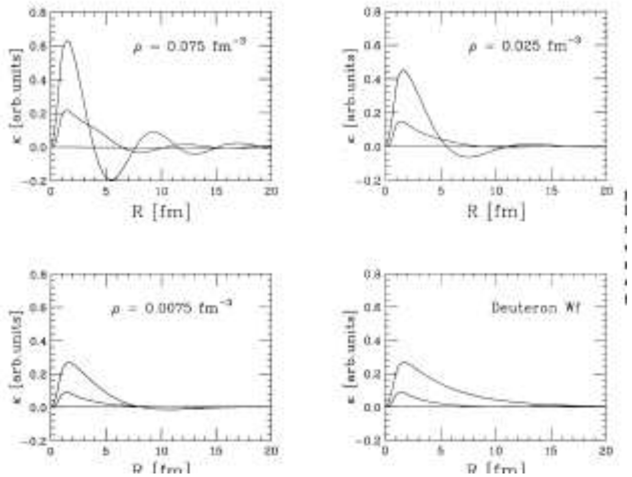
$2n = 1 - (\varepsilon - \mu)/E$ (occupation number)

$E = [(\varepsilon - \mu)^2 + \Delta^2)]^{1/2}$

at home, 1999



zero density 2μ -> - 2.2 MeV



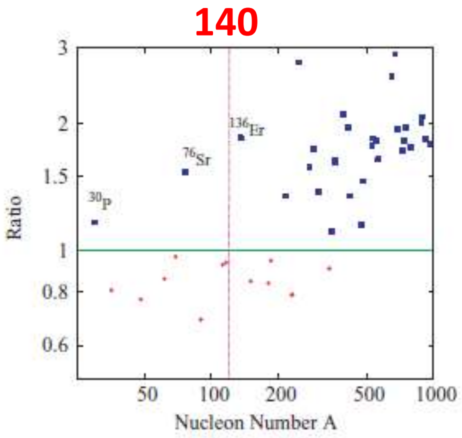
The puzzle of the missing neutron-proton pairing in nuclei
short history

- *A.M. Lane . (Nuclear Theory, Benjamin 1964)*

« The neglect of the neutron-proton interaction is the major weakness of the pairing force theory. This interaction is just as strong as that between a pair of like nucleons. In fact in the T=0 state is stronger»

G.F. Bertsch et al (PRC 2010)

Study the effect of ***spin-orbit splitting*** *on the pairing in N=Z nuclei and predict a crossover*



from spin-singlet to spin-triplet pairing at A ~ 140

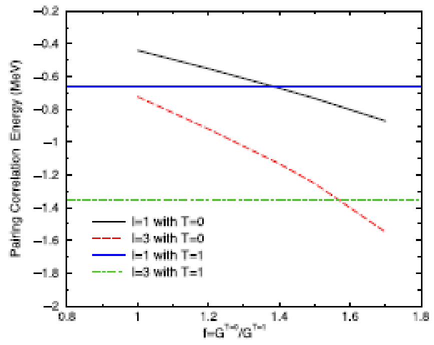
N >> Z Fermi energy splitting prevents np pairing

moving from lighter to heavier nuclei the pairing force quenches down due to the surface dependence of spin-orbit force

$$H_{sp} = \frac{p^2}{2m} + V_{WS}\, f(r) + \vec{\ell} \cdot \vec{s}\, V_{so} \frac{1}{r} \frac{df(r)}{dr}$$

H. Sagawa et al (Physica Scripta,2014)

Study interplay between S=1 np and nn S=0 pairing in pf-shell of N=Z nuclei , based on the pairing ***w.f. projection on the jj coupling***

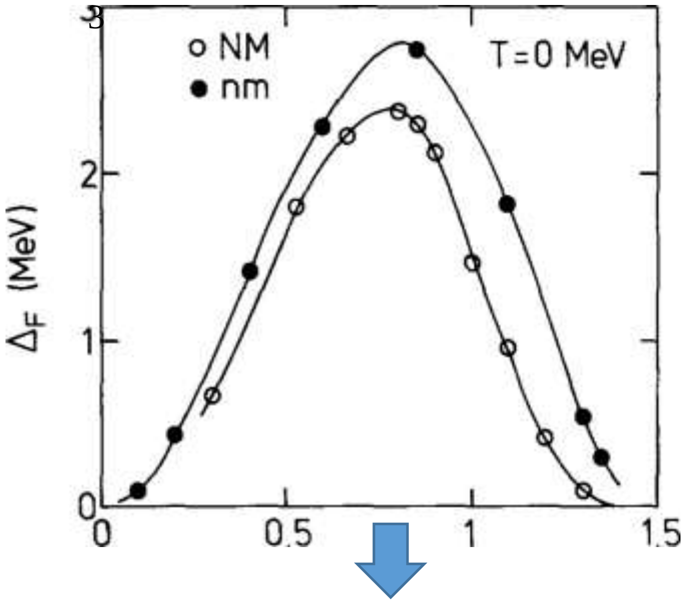


On the surface nature of the nuclear pairing

from low density pairing in nuclear matter to surface pairing in nuclei

Int.Coll.: E.Saperstein et al.

Moscow, 1993

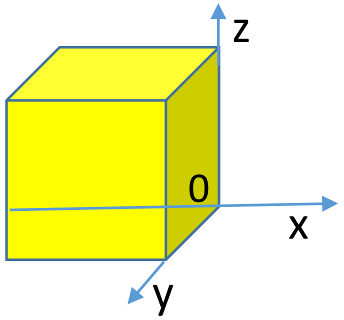


0,041 fm^{-3}

Guess LDFA : pairing located on the nuclear surface

1D-Inhomogeneous Nuclear Matter

Semi-infinite slab

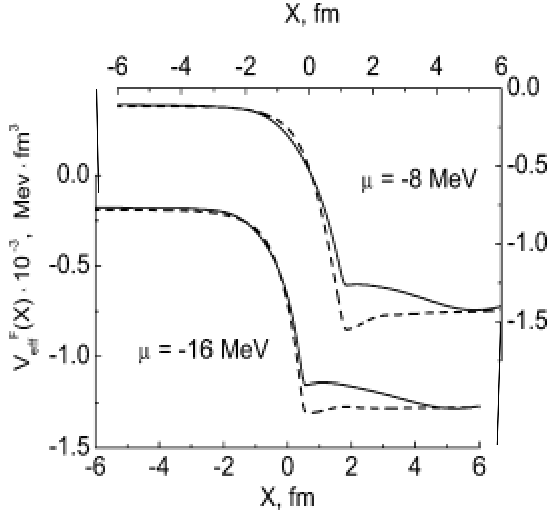


$$\boldsymbol{\Delta}(\mathrm{x}) = \tilde{\mathrm{V}}(\mathrm{x})\, \mathbf{G}\mathbf{G}_{\mathrm{s}}\, \boldsymbol{\Delta}(\mathrm{x})$$

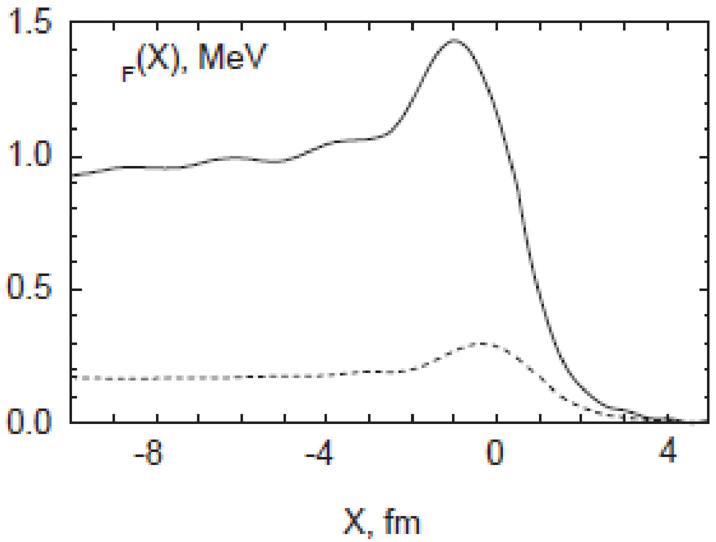
$$\tilde{\mathrm{V}}(\mathrm{x}) = \mathrm{V} + \mathrm{V}\ \frac{Q}{2eF - ep - ep,}\ \tilde{\mathrm{V}}(\mathrm{x})$$

from LDA to LPA

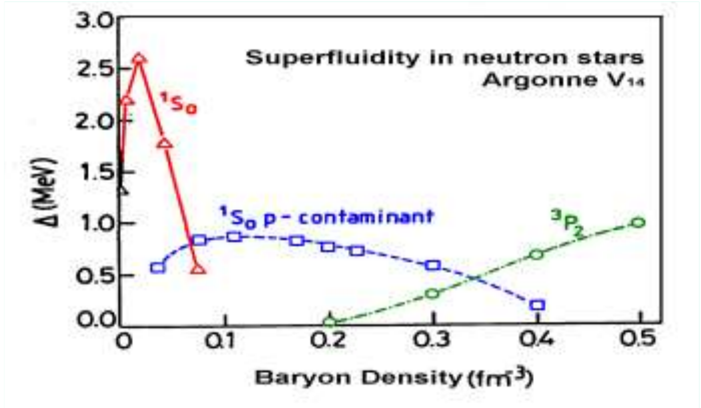
Pairing interaction $\tilde{\mathrm{V}}$ (x)



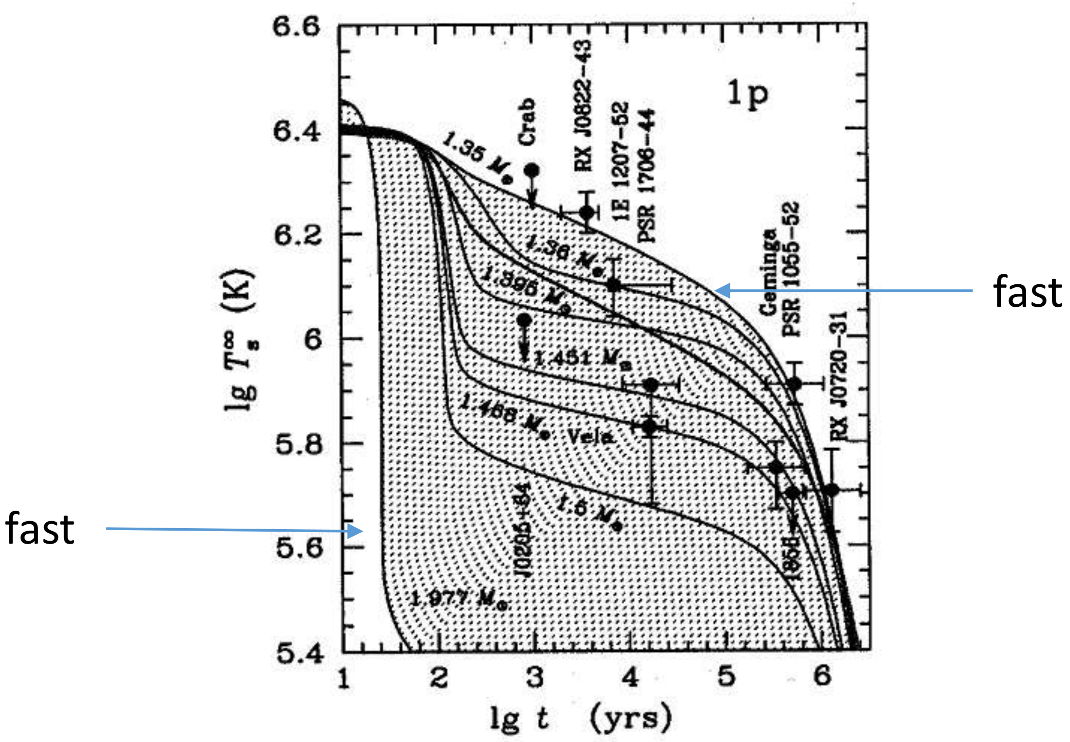
Energy Gap $\boldsymbol{\Delta}(\mathrm{x})$

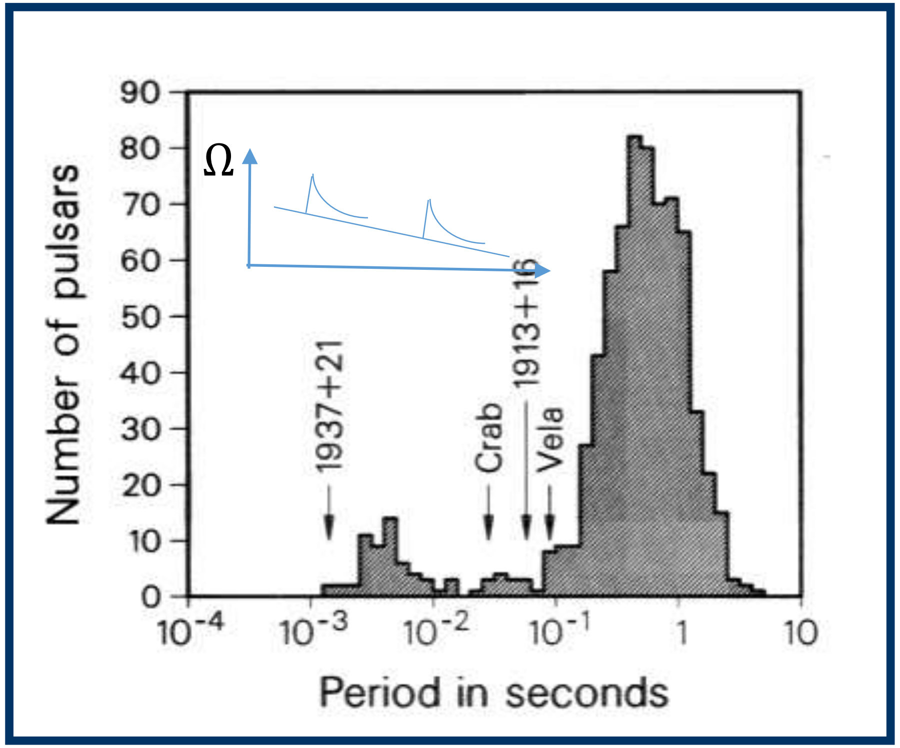


Superfluidity in Neutron Stars



cooling





Anomalously large post-glitche relaxation time

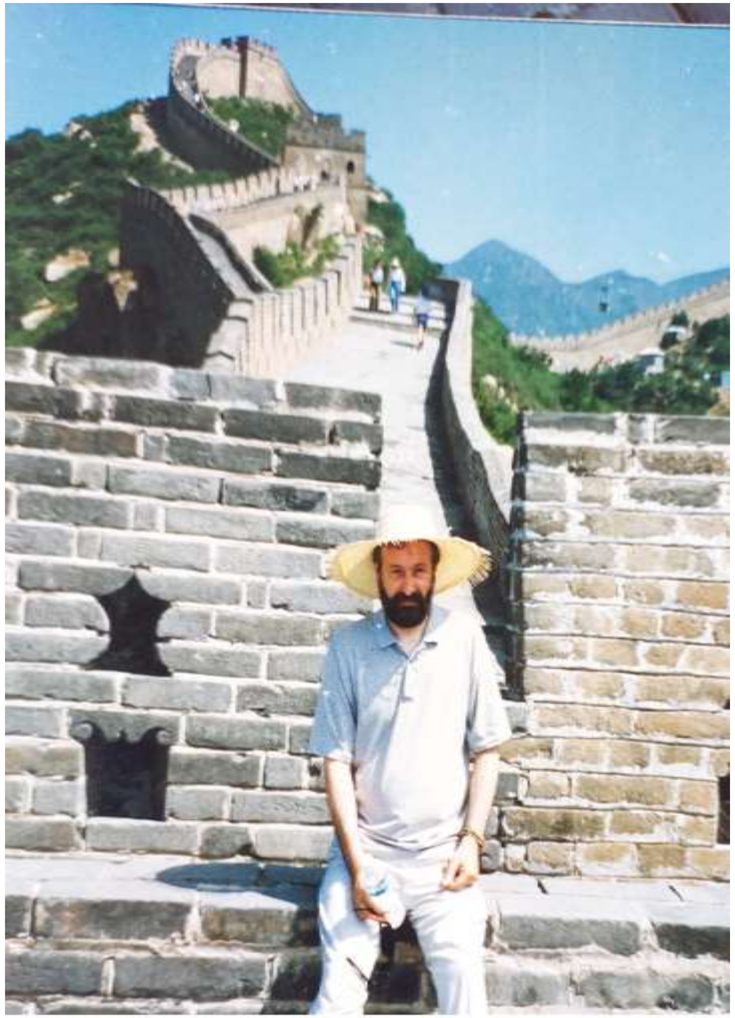
Superfluidity in Neutron Stars

- selected papers -

The role of superfluidity in the structure of the neutron star crust,
M. Baldo, U. Lombardo, E.E. Saperstein and S.V. Tolokonnikov,
Nucl. Phys. A 750 (2005) 409.

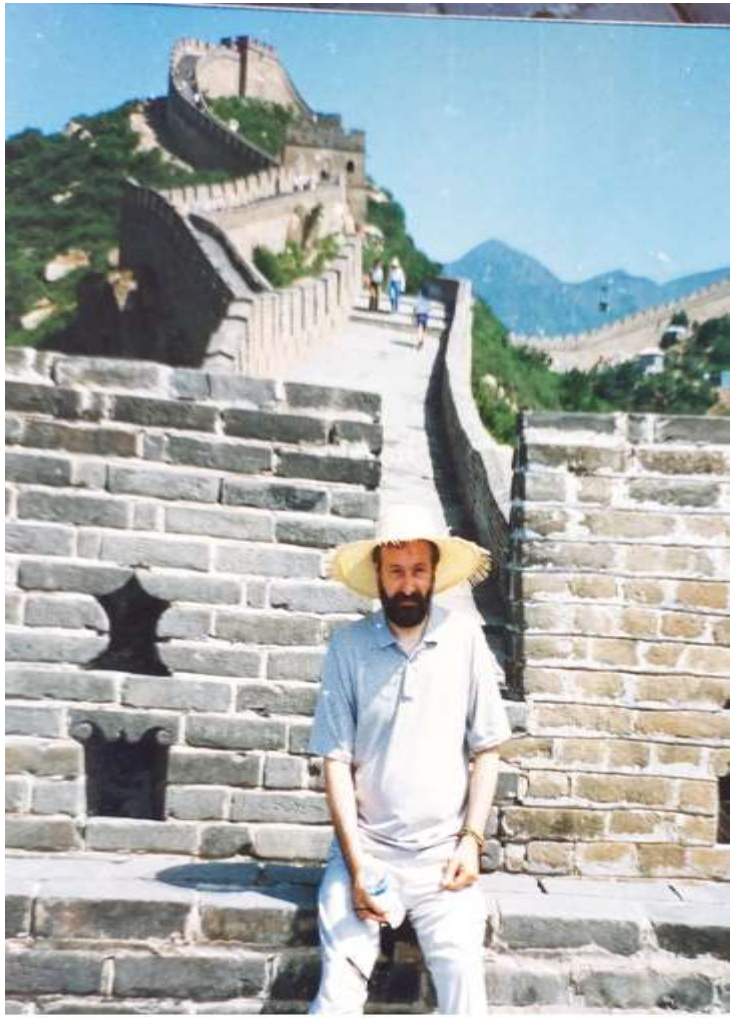
Elementary excitations in homogeneous superfluid neutron star matter :
M. Baldo and C. Ducoin
Phys. Rev. C84, 035806 (2011); C96, 025811 (2017); C99, 025801 (2019)

The neutron star in Cassiopeia A : equation of state, superfluidity and Joule heating,
A. Bonanno, M. Baldo, G.F. Burgio and V. Urpin,
Astronomy & Astrophysics 561, L5 (2014)



Great Wall 1993

A well-deserved rest after a long journey



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A well-deserved rest after a long journey

Long Live Marcello!