Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU

UNIVERSITAT DE BARCELONA

@HadNucAtUB

An excursion into dense matter with Marcello

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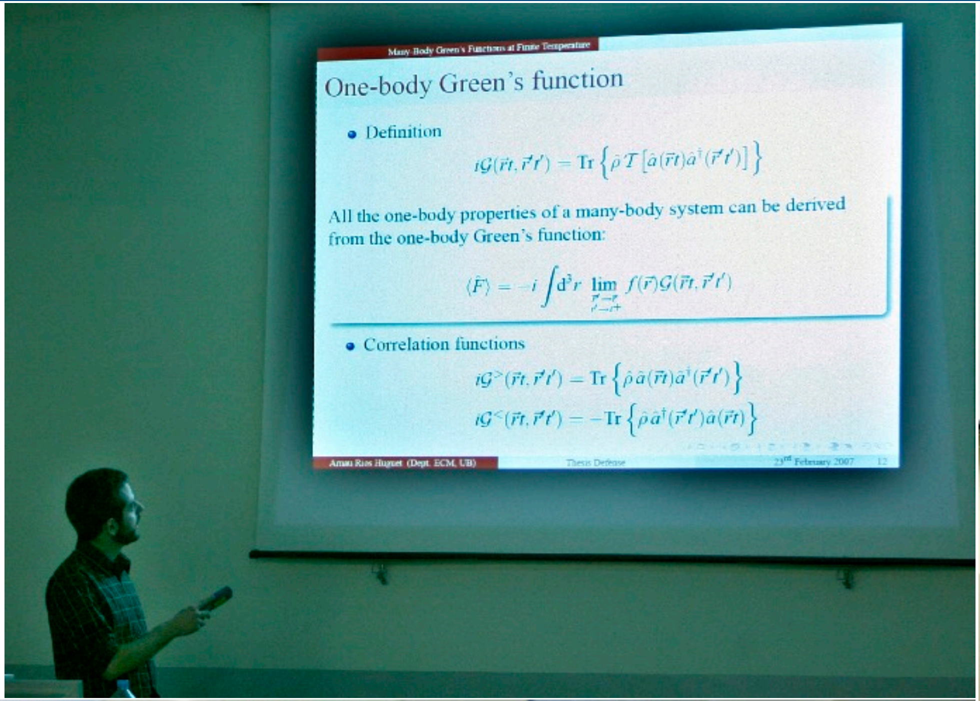
arnau.rios@fqa.ub.edu
@RiosArnau

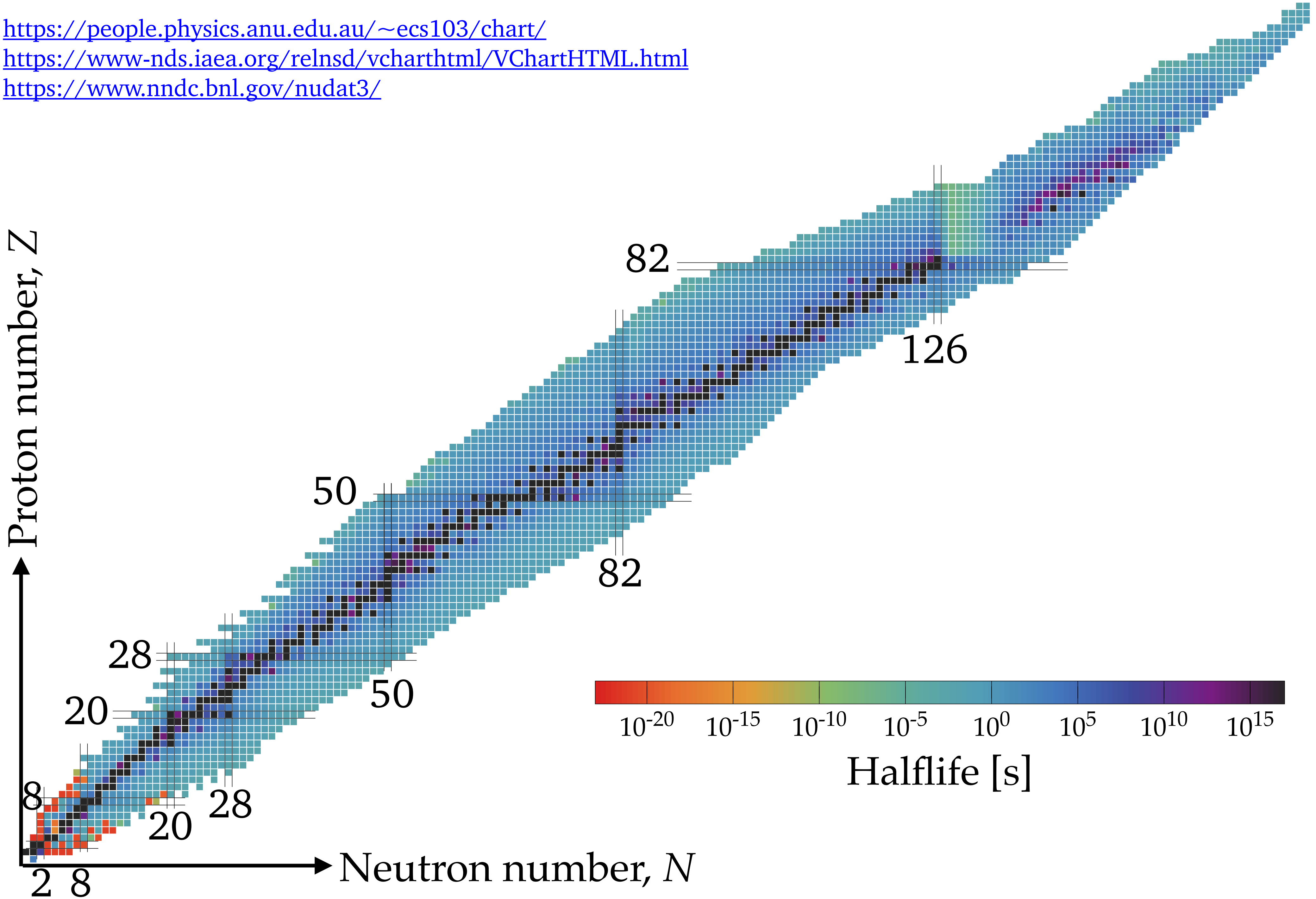
Celebrating Dr. Marcello Baldo's 80th Birthday
16 October 2023

23 February 2007



Isotope chart

https://people.physics.anu.edu.au/~ecs103/chart/
https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html
https://www.nndc.bnl.gov/nudat3/



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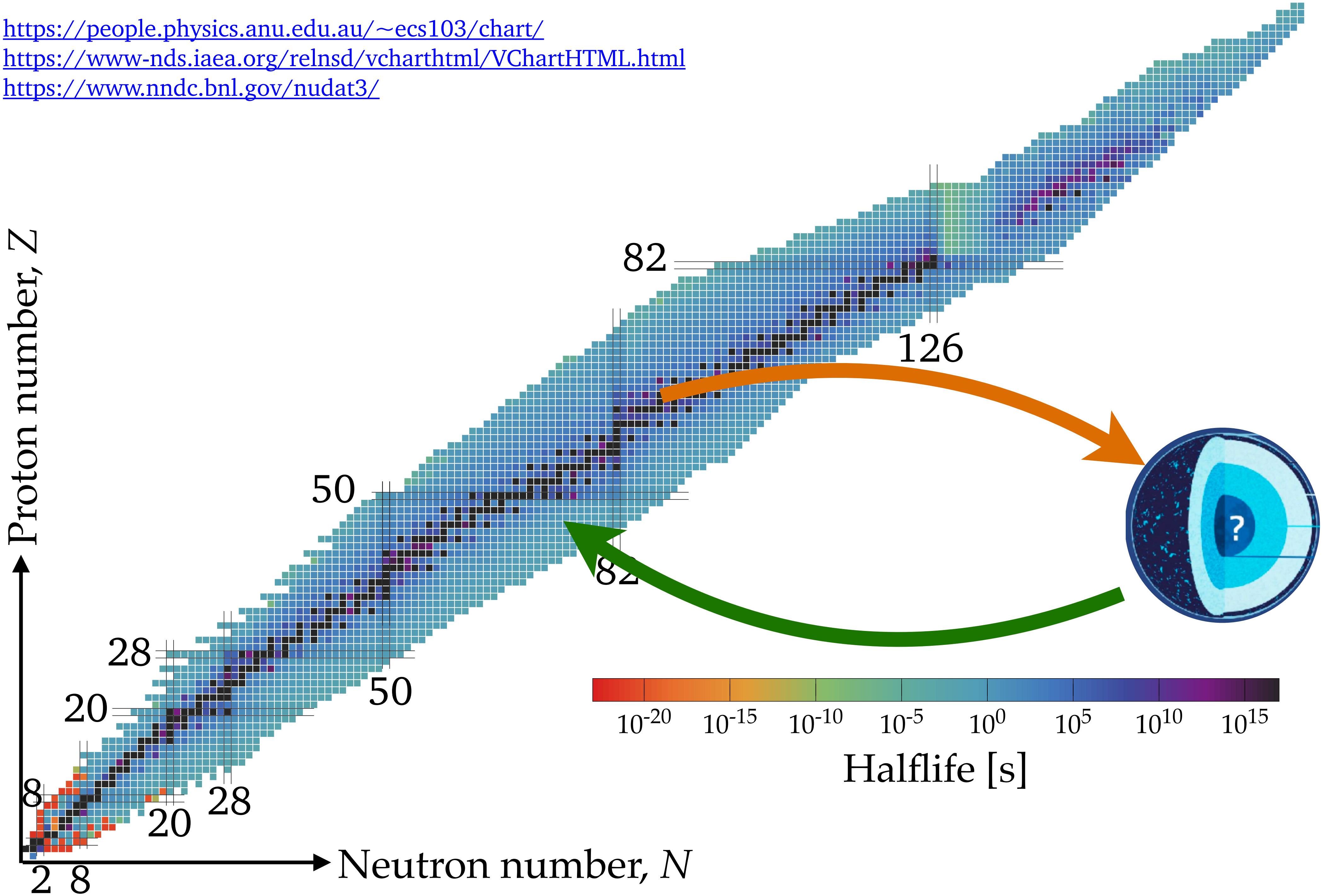

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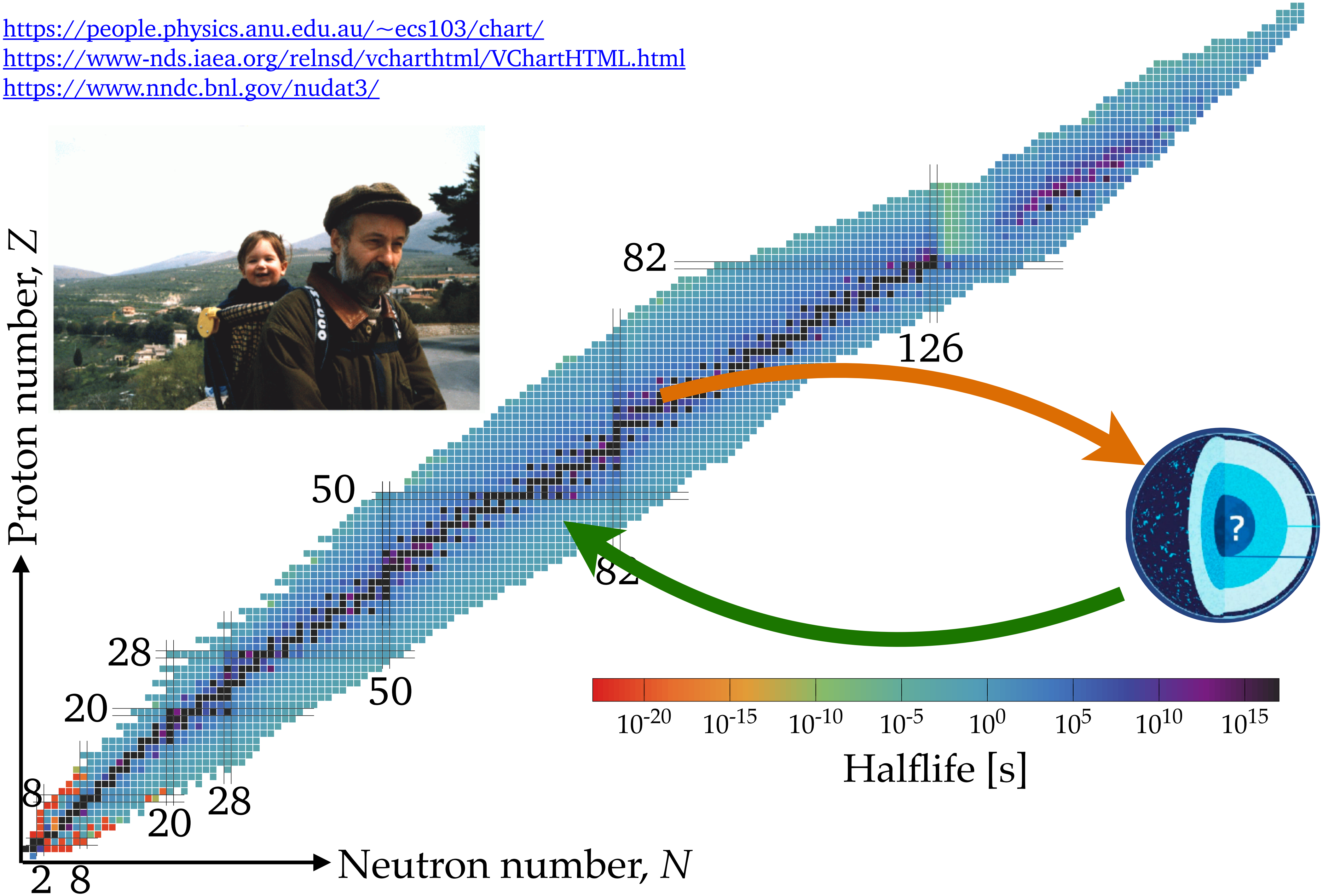

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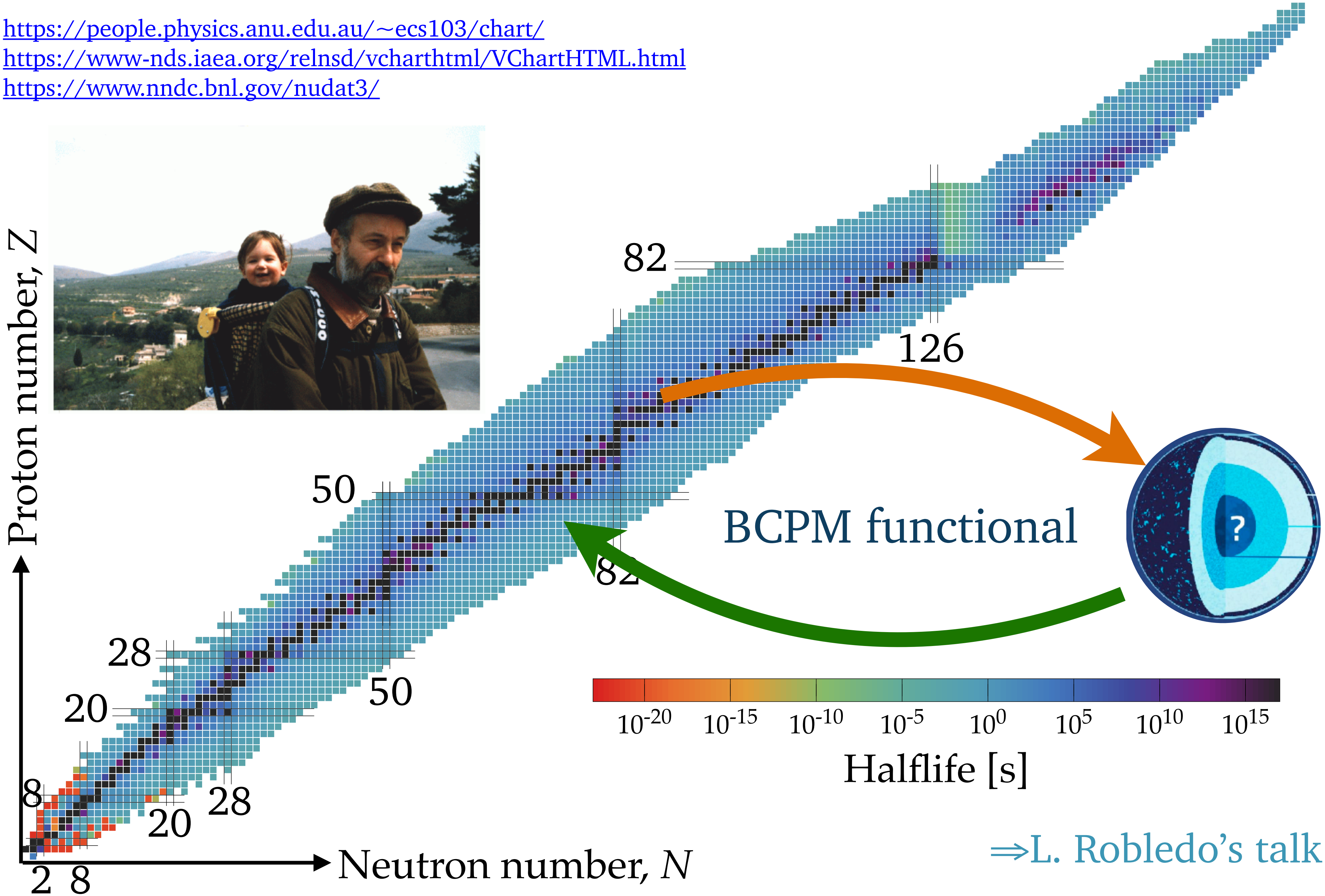


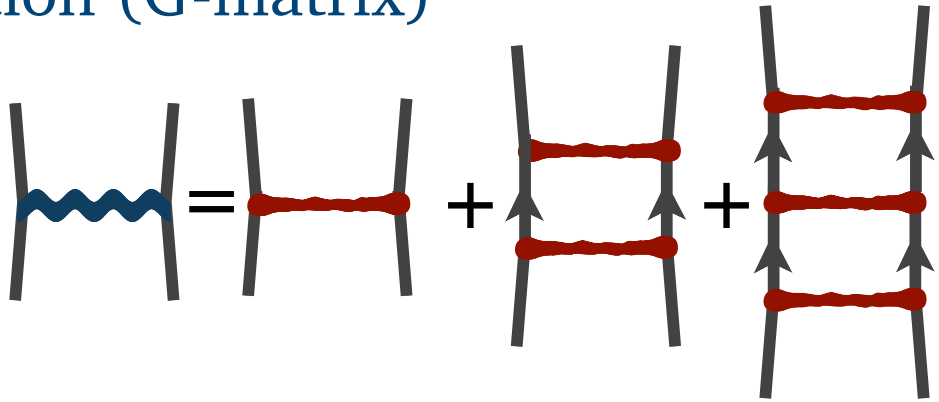
⇒L. Robledo's talk

Brueckner-Bethe-Goldstone

- Infinite resummation of two-hole line diagrams (energy expansion)
- pp Pauli blocked in-medium interaction (G-matrix)

Effective interaction

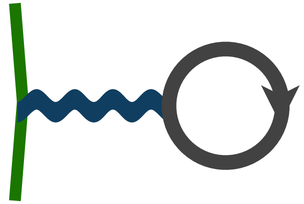
$$G(\omega) = V + V \frac{Q}{\omega - \epsilon - \epsilon' + i\eta} G(\omega)$$



Single-particle spectrum

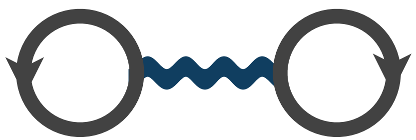
$$U(k) = \sum_{|\vec{k}'|<k_F} \langle \vec{k}\vec{k}' | G(\omega = \epsilon(k) + \epsilon(k')) | \vec{k}\vec{k}' \rangle_A$$

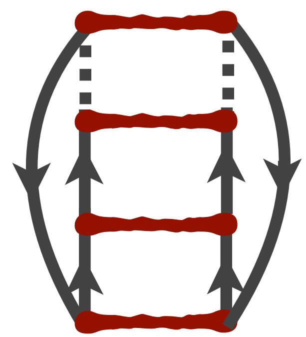
$$\epsilon(k) = \frac{\hbar^2 k^2}{2m_\tau} + Re[U(k)]$$



Energy

$$\frac{E}{A}(\rho,\beta) = \frac{1}{A} \sum_\tau \sum_{|\vec{k}|<k_{F_\tau}} \left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} Re[U_\tau(\vec{k})] \right)$$





B. D. Day, Rev. Mod. Phys. **39** 719 (1967)

3-hole line "epics"

Three hole-line contribution in nuclear matter revisited

H.Q. Song [a,b,c], M. Baldo [d,1], G. Giansiracusa [e], U. Lombardo [e]

[a] Institute of Nuclear Research, Academia Sinica, Shangai 201800, China
[b] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
[c] T.D. Lee Physics Laboratory, Fudan University, Shangai 200433, China
[d] INFN, Sezione di Catania, 57 Corso Italia, I-95129 Catania, Italy
[e] Dipartimento di Fisica and INFN, 57 Corso Italia, I-95129 Catania, Italy

Song, Baldo, Giansiracusa, Lombardo,
Phys. Lett. B **411** 237 (1997)

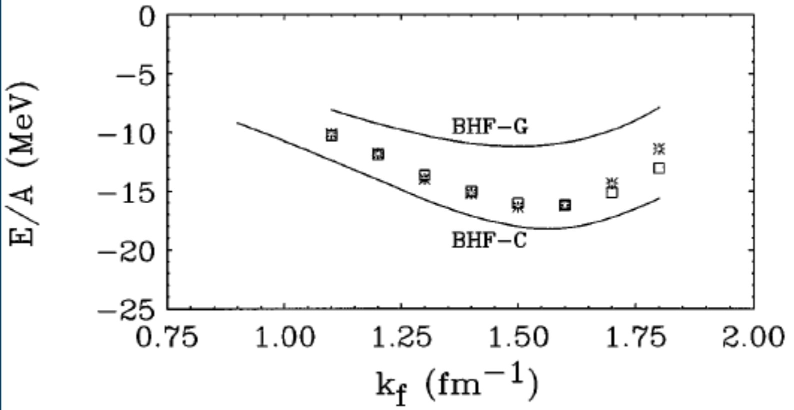
1, NUMBER 8 PHYSICAL REVIEW LETTERS 24 AUG

Bethe-Brueckner-Goldstone Expansion in Nuclear Matter

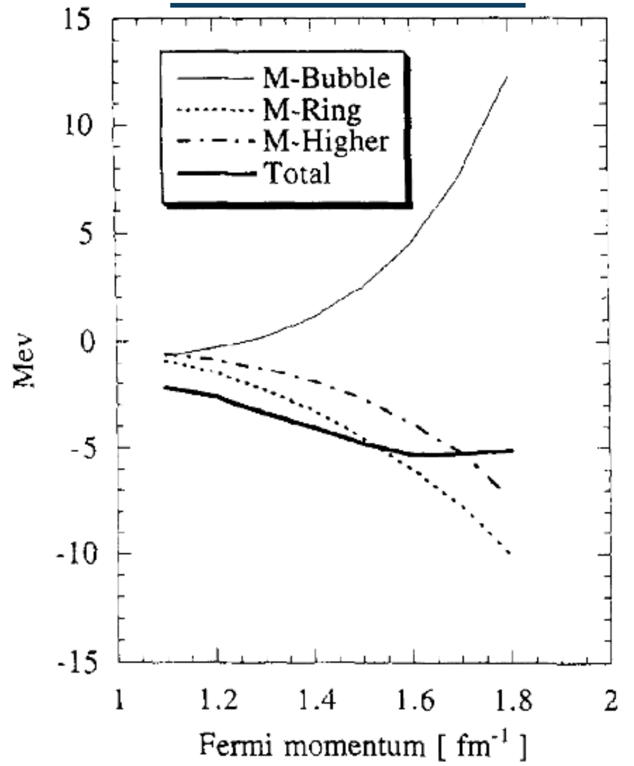
H. Q. Song,[1,2,3] M. Baldo,[4,*] G. Giansiracusa,[5] and U. Lombardo[5]
[1] Institute of Nuclear Research, Academia Sinica, Shangai 201800, China
[2] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
[3] T. D. Lee Physics Laboratory, Fudan University, Shangai 200433, China
[4] INFN, Sezione di Catania, 57 Corso Italia, I-95129 Catania, Italy
[5] Dipartimento di Fisica and INFN, 57 Corso Italia, I-95129 Catania, Italy

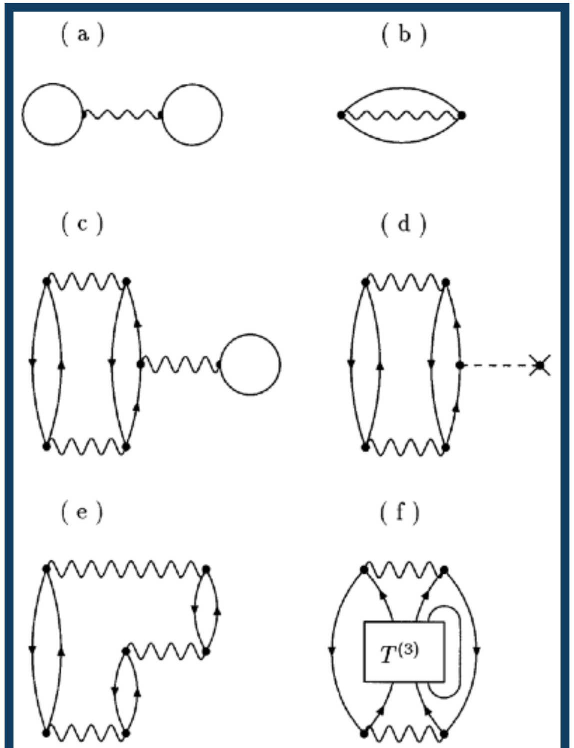
Song, Baldo, Giansiracusa, Lombardo,
Phys. Rev. Lett. **81** 1584 (1998)

Total EoS

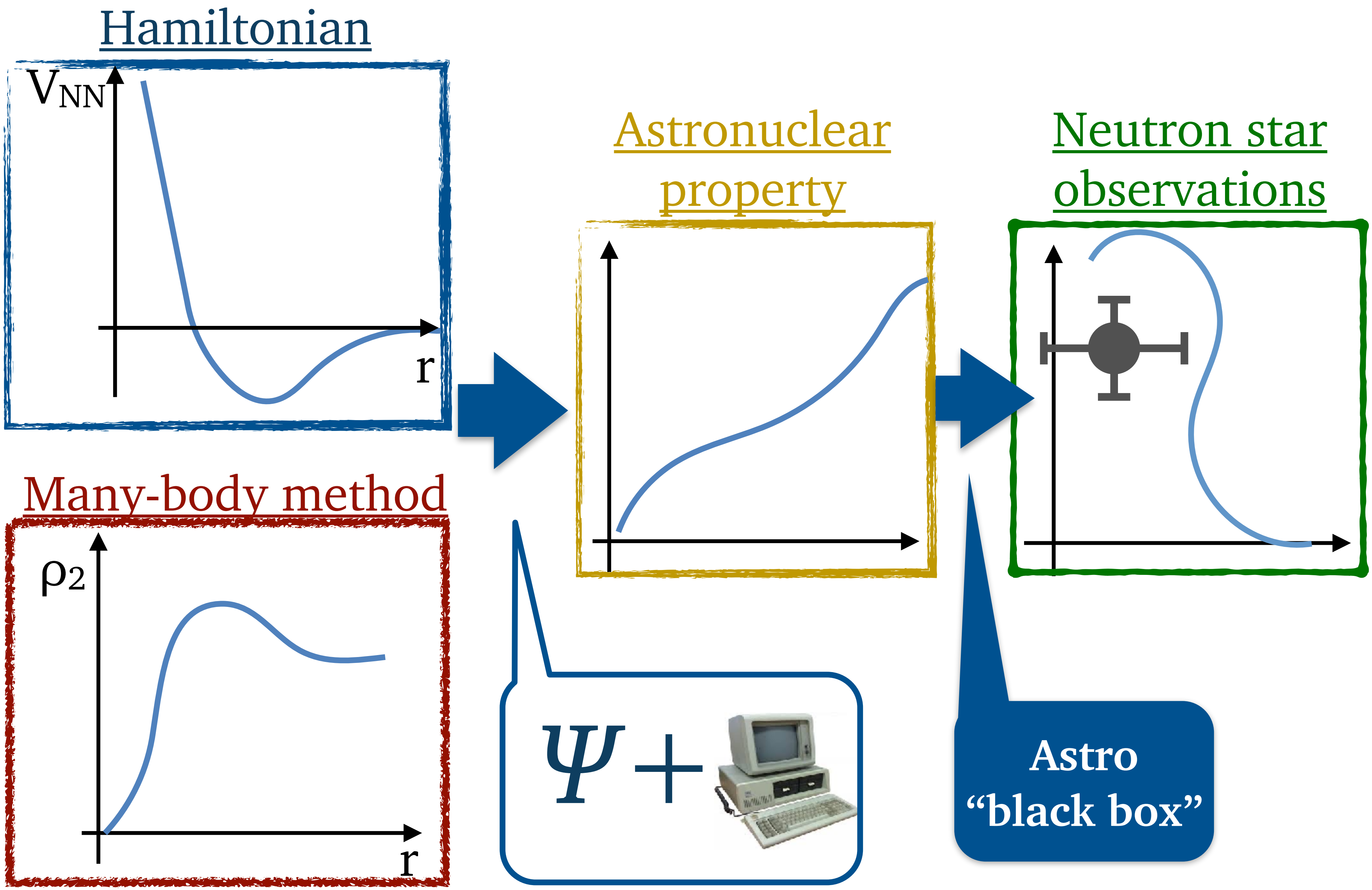


3-hole line

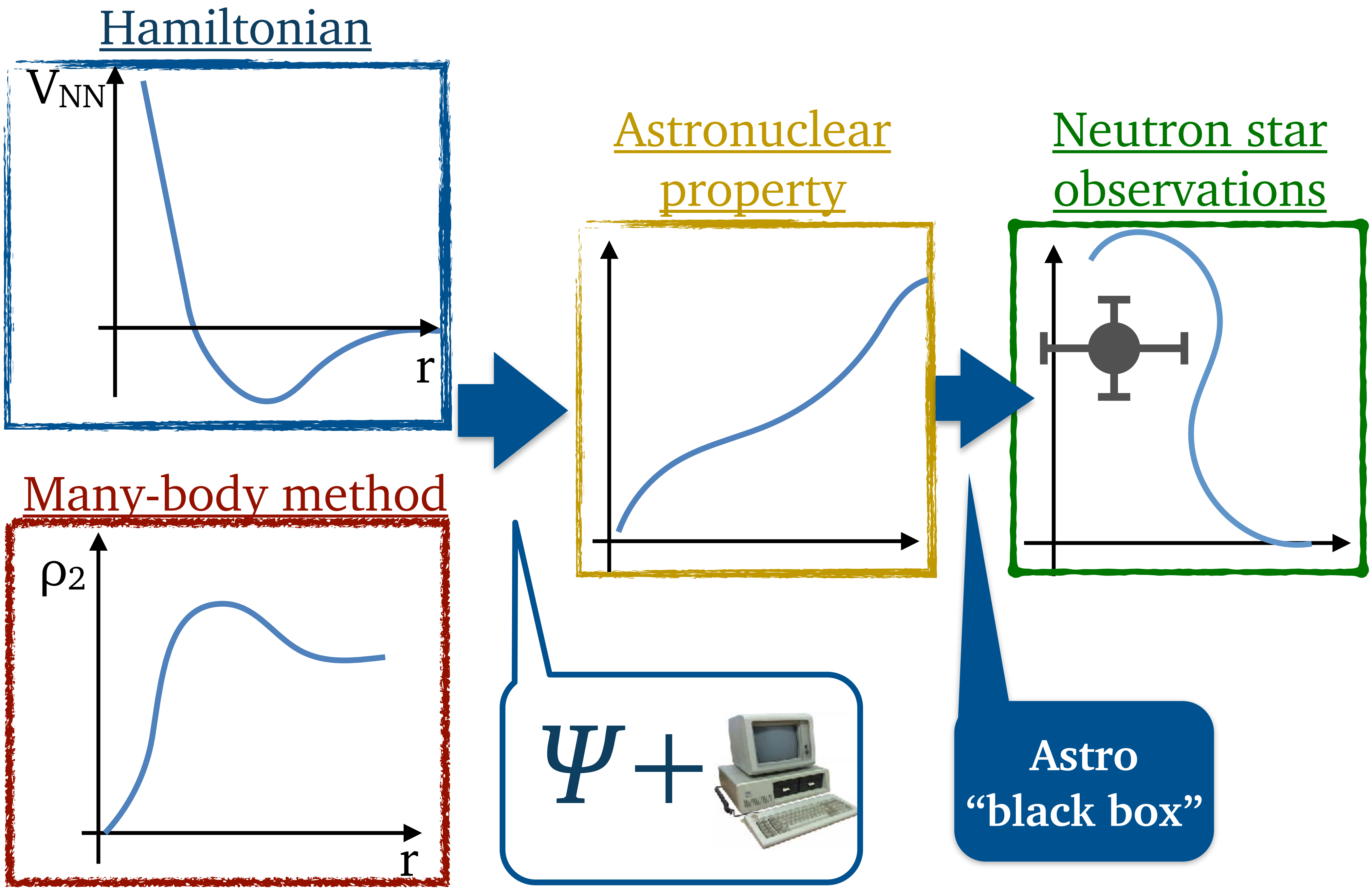




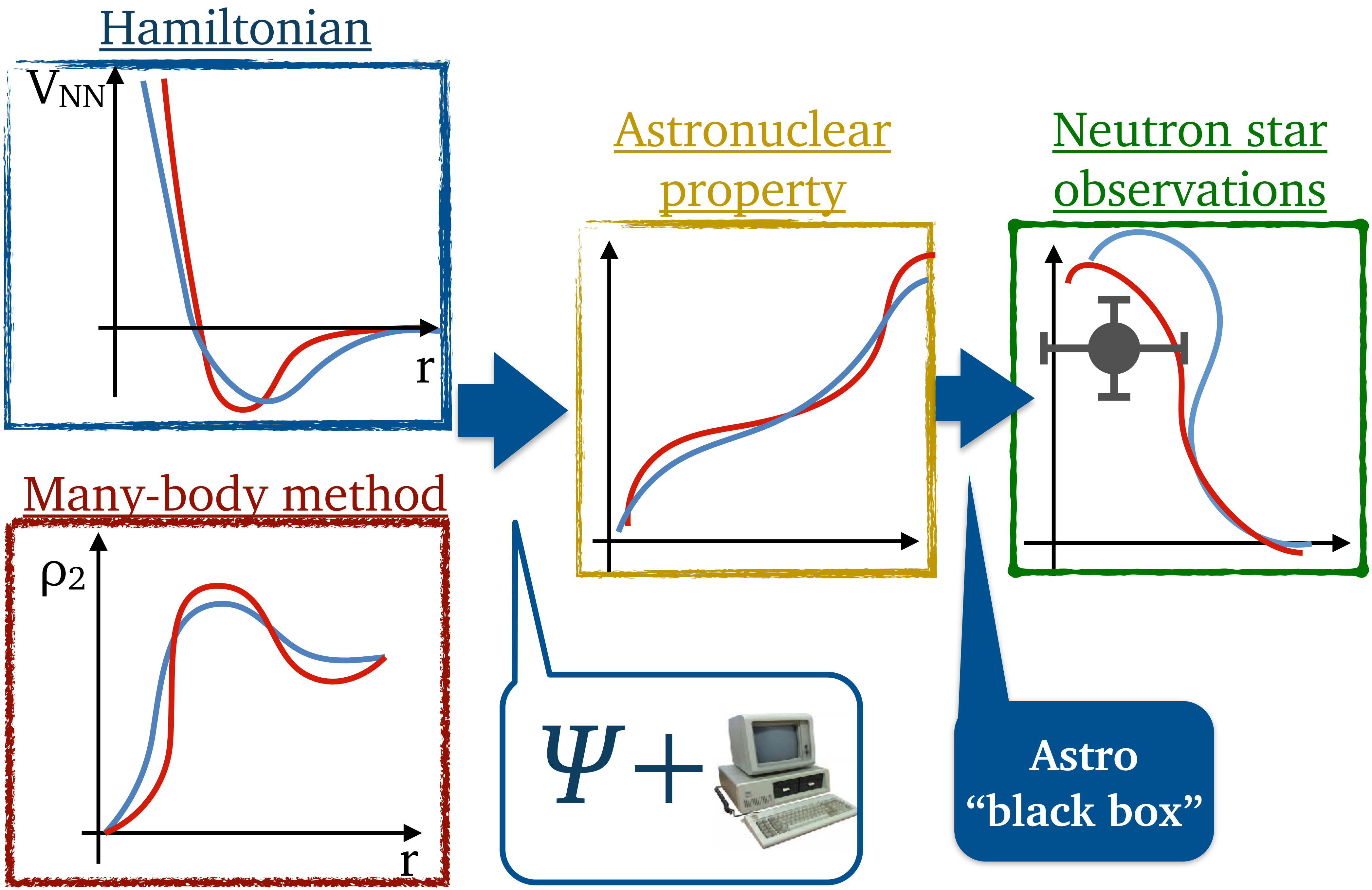
Nuclear predictions 19xx style



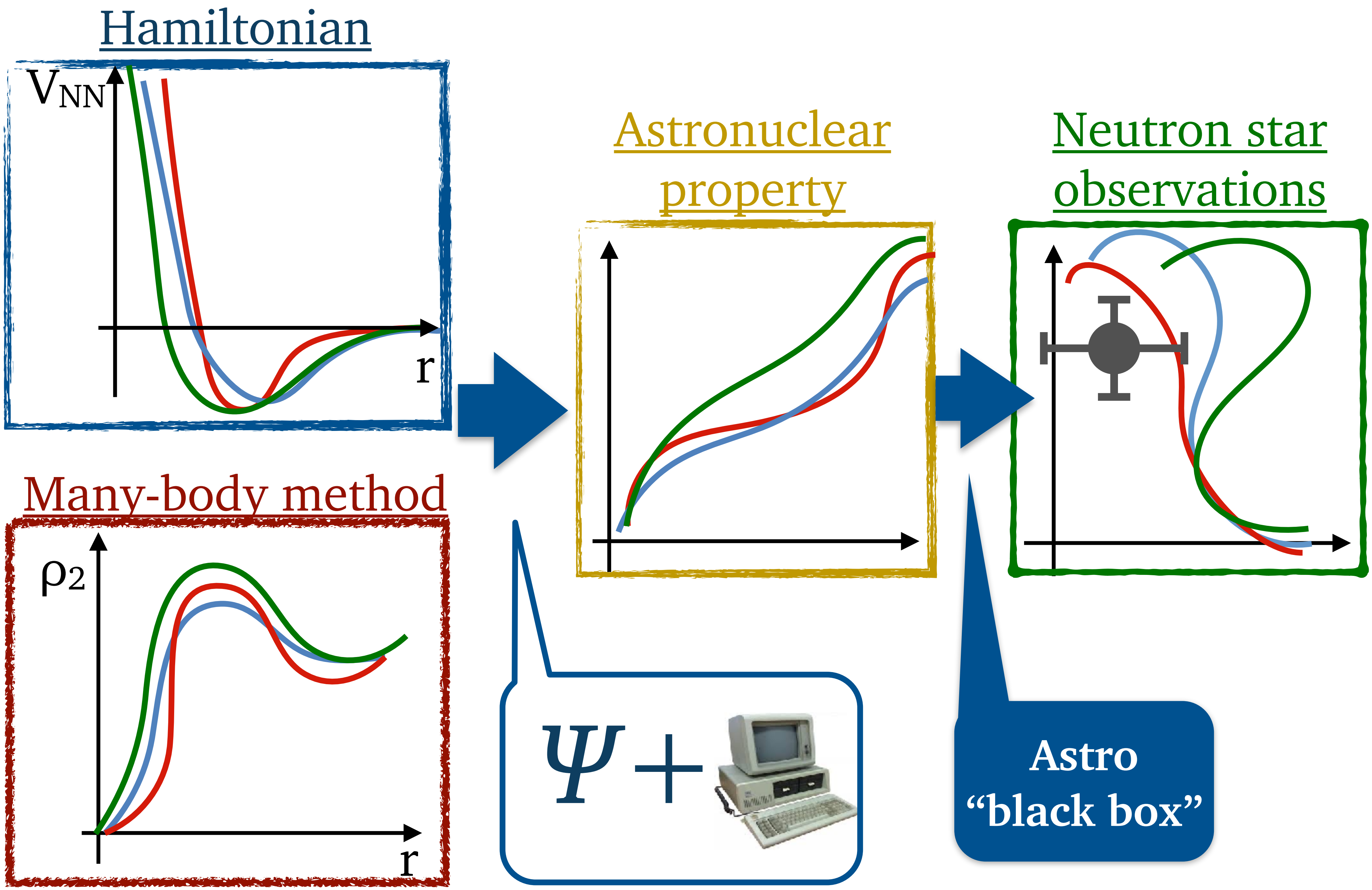
Nuclear predictions 19xx style



Nuclear predictions 19xx style



Nuclear predictions 19xx style

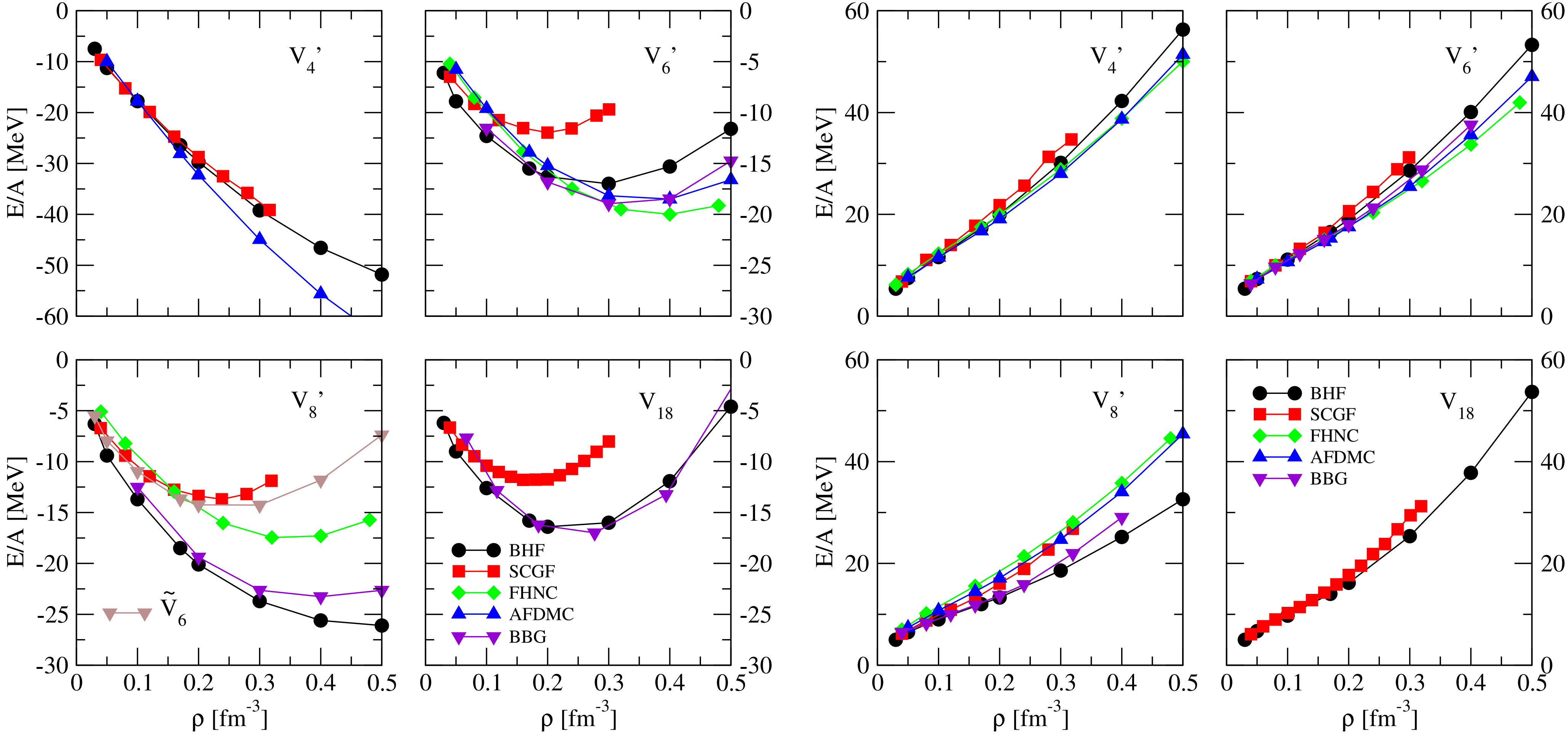


Our common paper

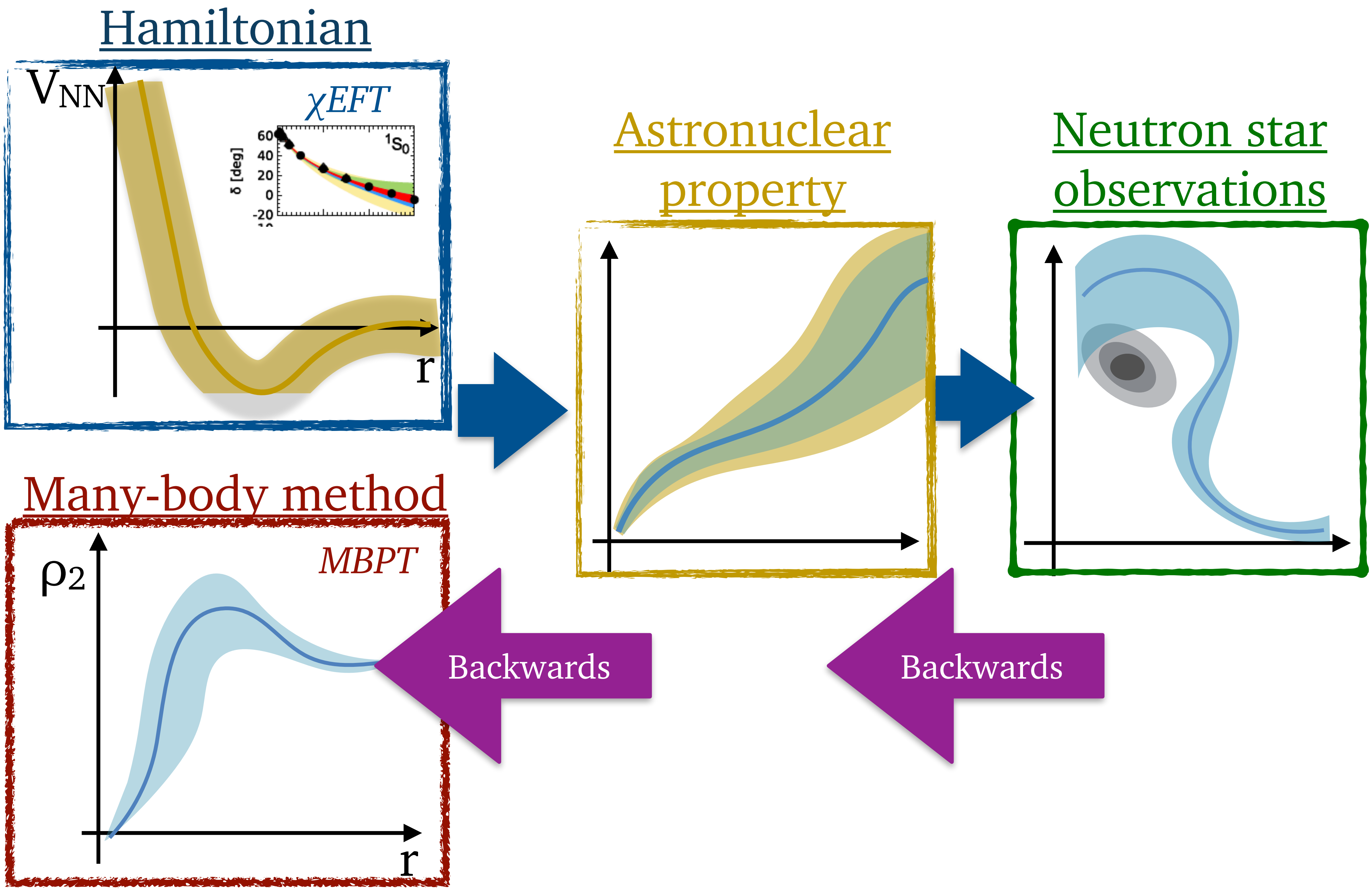
PHYSICAL REVIEW C **86**, 064001 (2012)

Comparative study of neutron and nuclear matter with simplified Argonne nucleon-nucleon potentials

M. Baldo,[1] A. Polls,[2] A. Rios,[3] H.-J. Schulze,[1] and I. Vidaña[4]
[1]*INFN Sezione di Catania, Dipartimento di Fisica, Via Santa Sofia 64, I-95123 Catania, Italy*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, E-08028 Barcelona, Spain*
[3]*Department of Physics, Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom*
[4]*Centro de Física Computacional, Department of Physics, University of Coimbra, PT-3004-516 Coimbra, Portugal*
(Received 26 July 2012; revised manuscript received 26 October 2012; published 3 December 2012)



Nuclear error quantification



History matching

nature physics ARTICLES
https://doi.org/10.1038/s41567-022-01715-8

OPEN

Ab initio predictions link the neutron skin of ^{208}Pb to nuclear forces

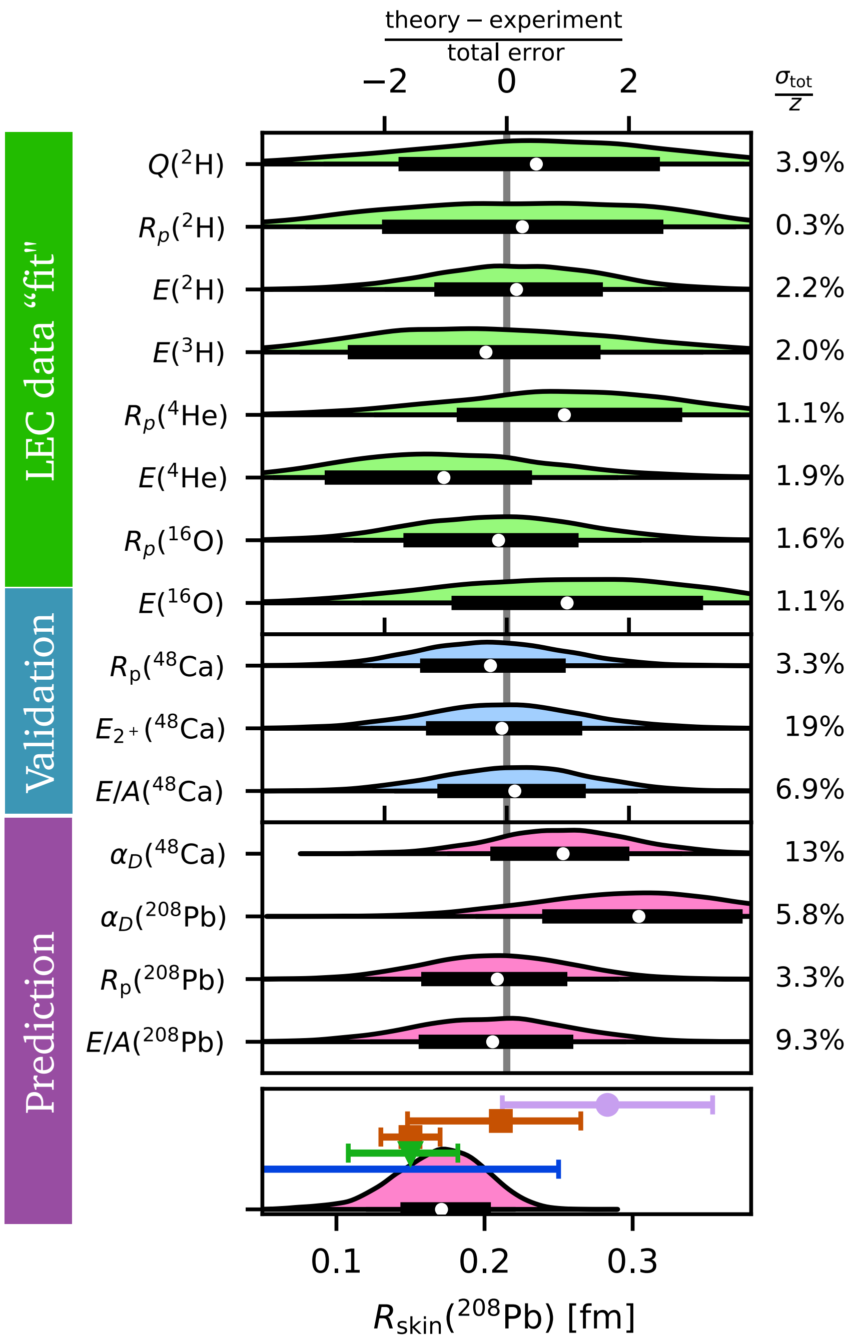
Baishan Hu[1,11], Weiguang Jiang[2,11], Takayuki Miyagi[1,3,4,11], Zhonghao Sun[5,6,11], Andreas Ekström[2], Christian Forssén[2], Gaute Hagen[1,5,6], Jason D. Holt[1,7], Thomas Papenbrock[5,6], S. Ragnar Stroberg[8,9] and Ian Vernon[10]

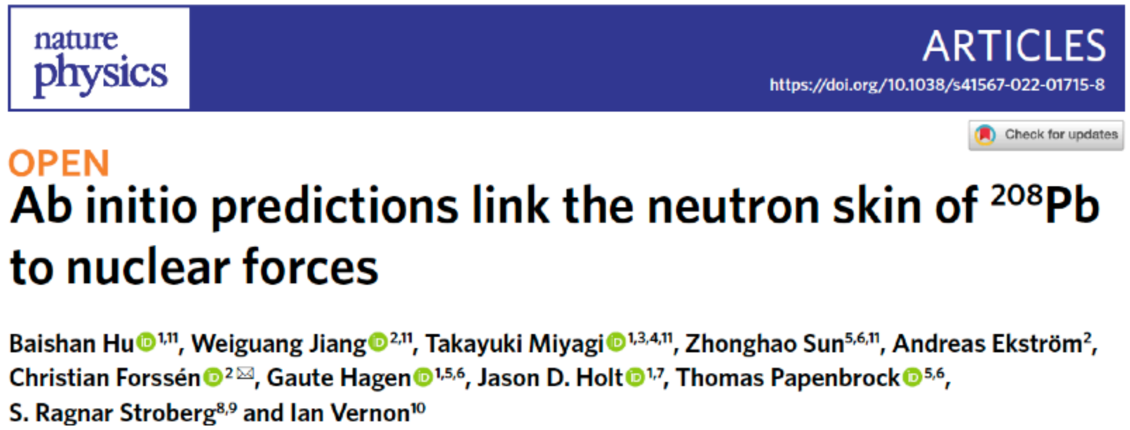
- 17 LECs from χEFT
- Start from **10^9** realisations
- **History matching** reduces to **34**

Hu, Jiang, Miyagi, Holt, *Nature Physics* **18** 1196 (2022)

Comment: Rios, *A historic match for nuclei and neutron stars, Nature Physics* **18** 1137 (2022)

History matching





- 17 LECs from χEFT
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- Validate data in ^{48}Ca (importance weights)
- Predict ^{208}Pb & nuclear matter

Hu, Jiang, Miyagi, Holt, *Nature Physics* **18** 1196 (2022)

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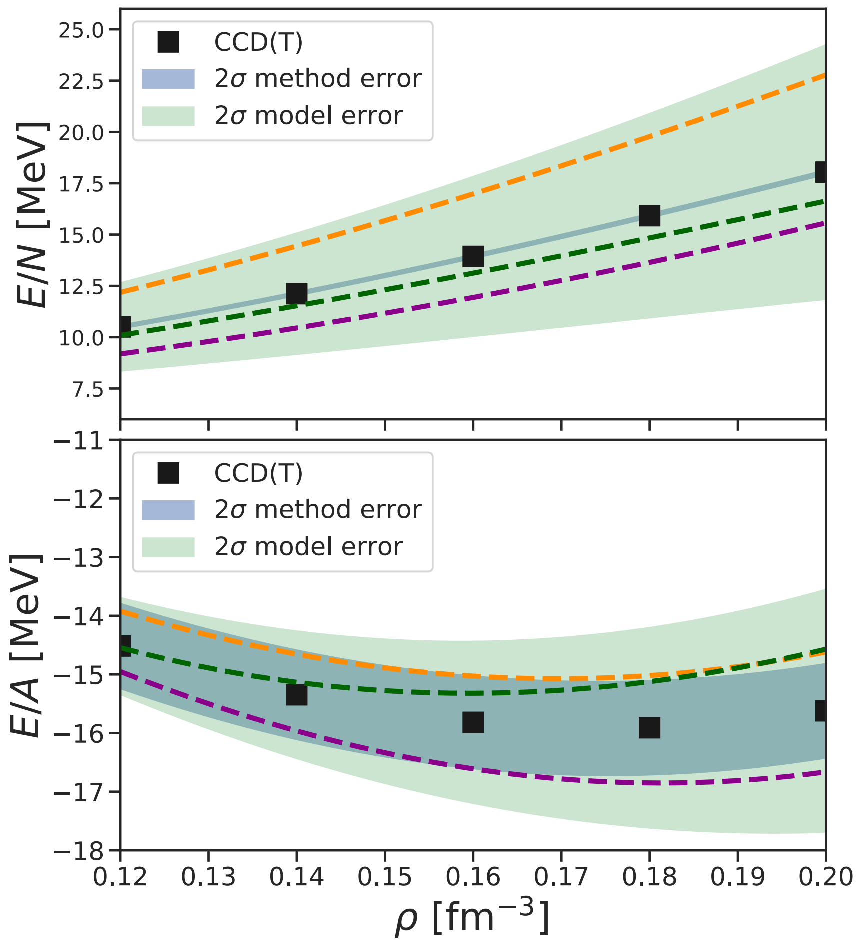

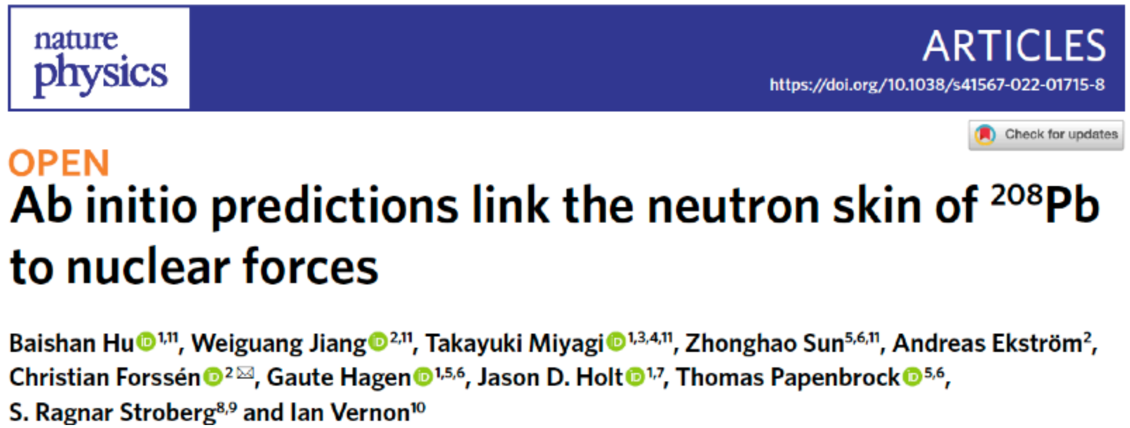

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https://doi.org/10.1038/s41567-022-01715-8

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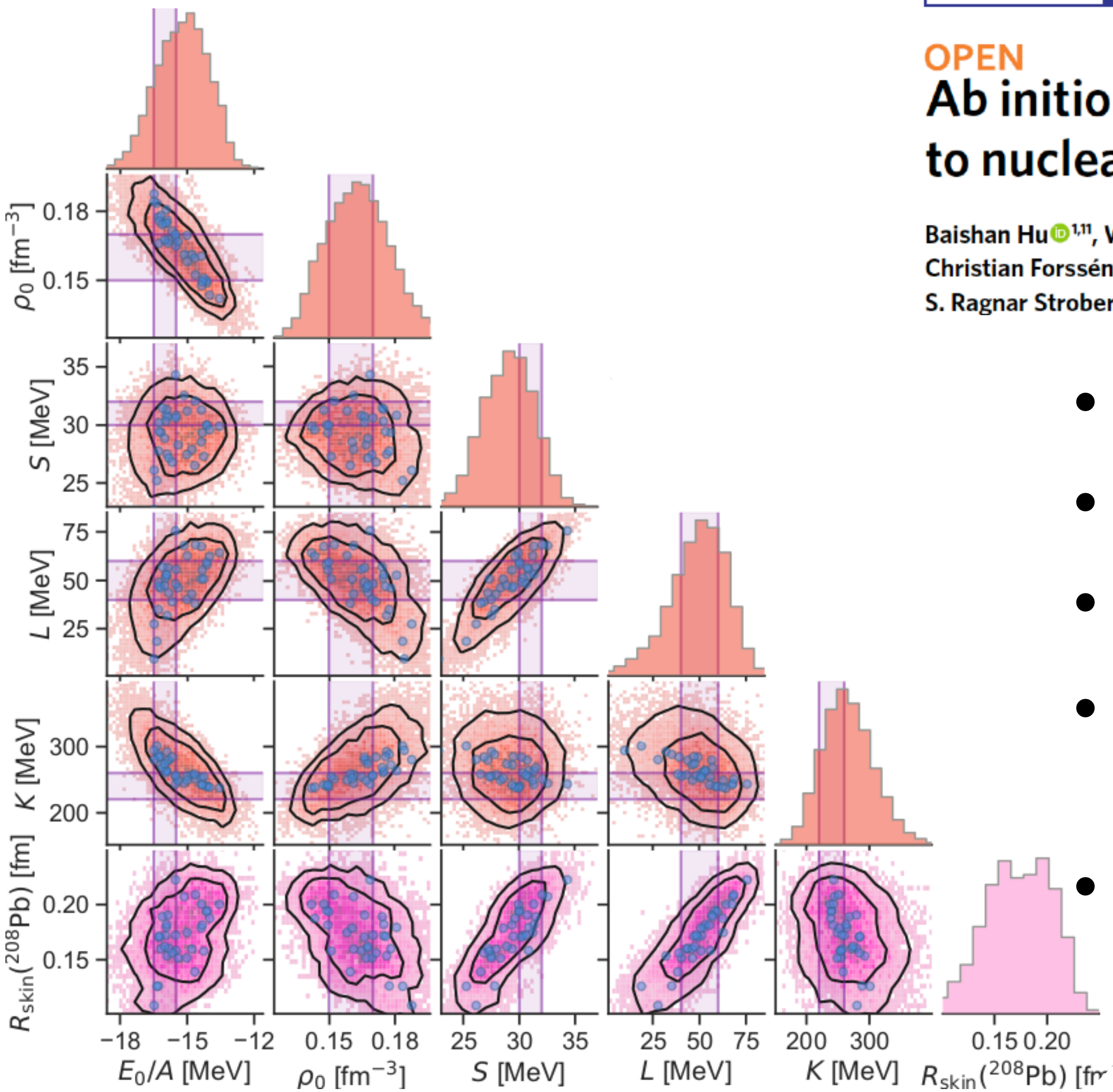

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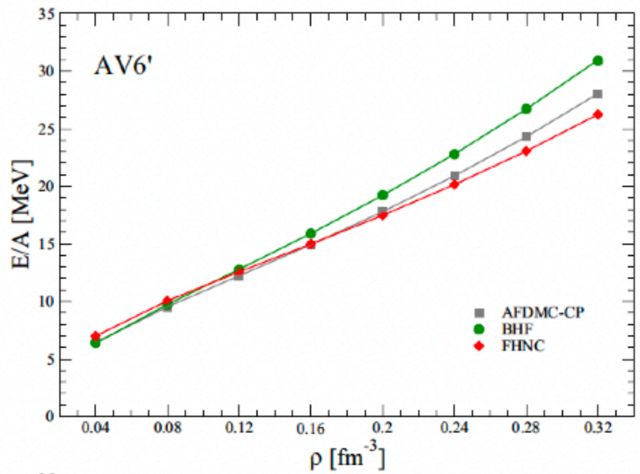
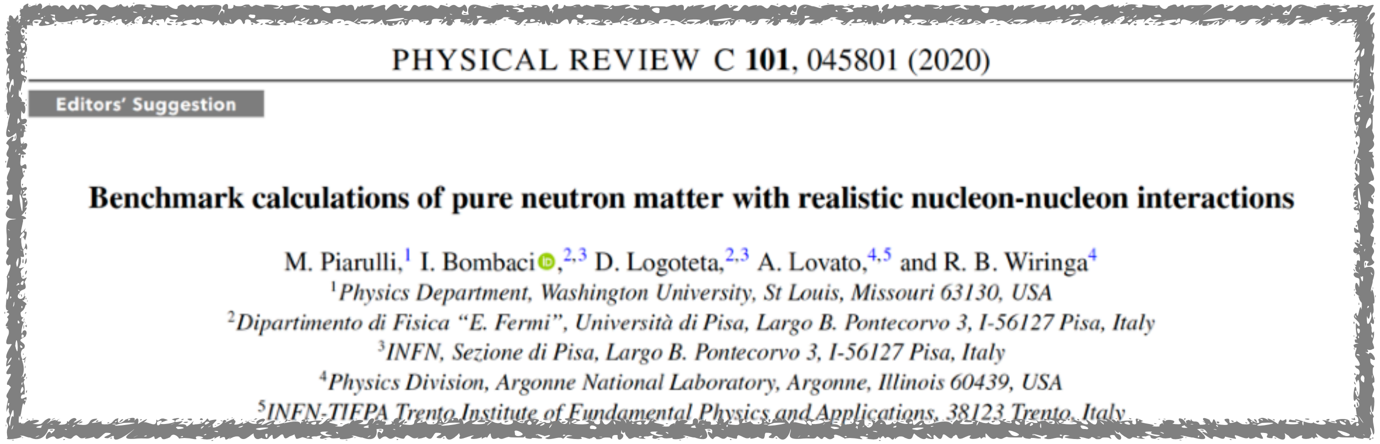
An idea that lasted!

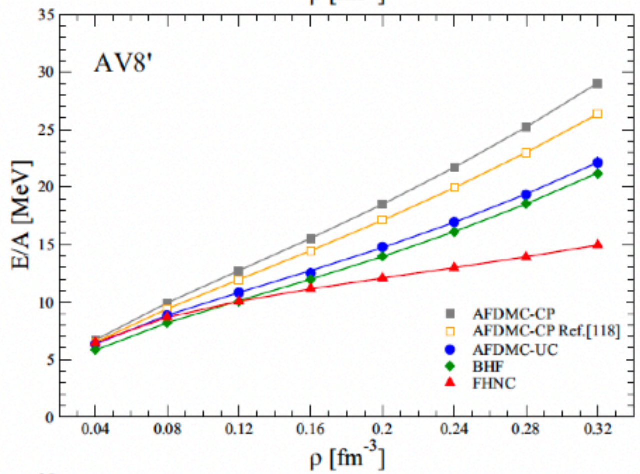


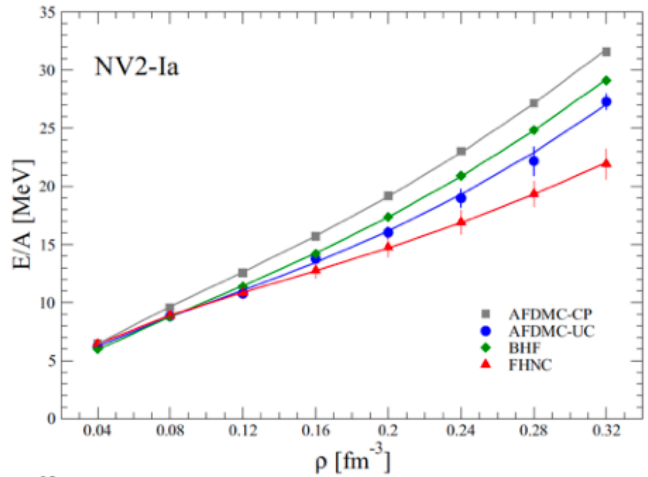

PHYSICAL REVIEW C **101**, 045801 (2020)

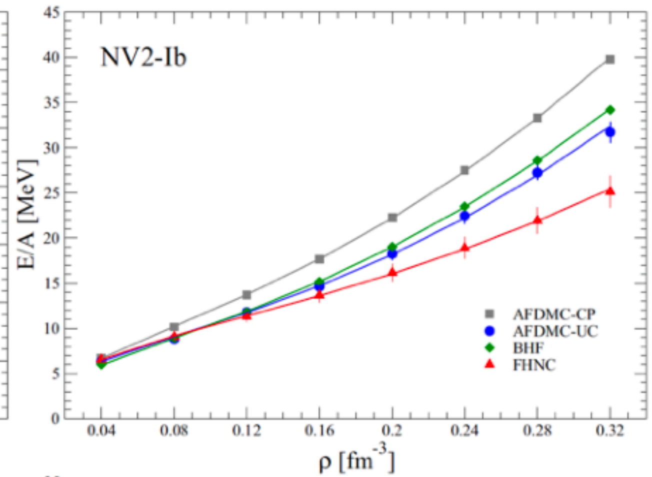
Editors' Suggestion

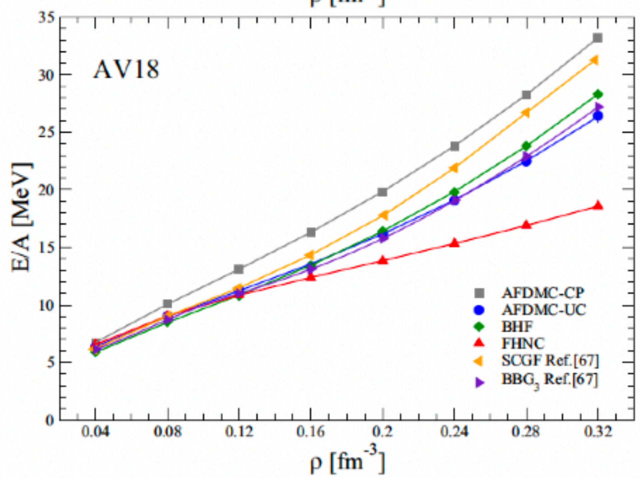
Benchmark calculations of pure neutron matter with realistic nucleon-nucleon interactions

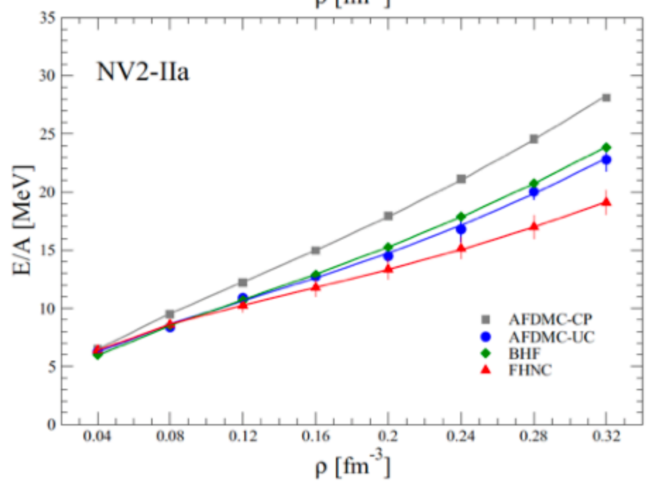
M. Piarulli,[1] I. Bombaci,[2,3] D. Logoteta,[2,3] A. Lovato,[4,5] and R. B. Wiringa[4]
[1] *Physics Department, Washington University, St Louis, Missouri 63130, USA*
[2] *Dipartimento di Fisica "E. Fermi", Università di Pisa, Largo B. Pontecorvo 3, I-56127 Pisa, Italy*
[3] *INFN, Sezione di Pisa, Largo B. Pontecorvo 3, I-56127 Pisa, Italy*
[4] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[5] *INFN-TIFPA Trento Institute of Fundamental Physics and Applications, 38123 Trento, Italy*

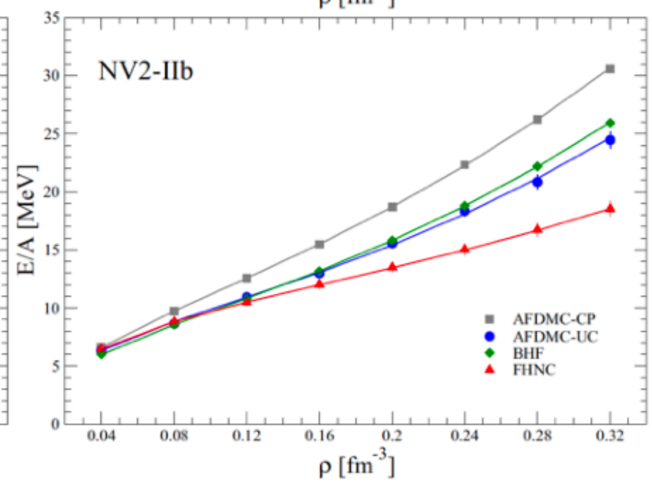




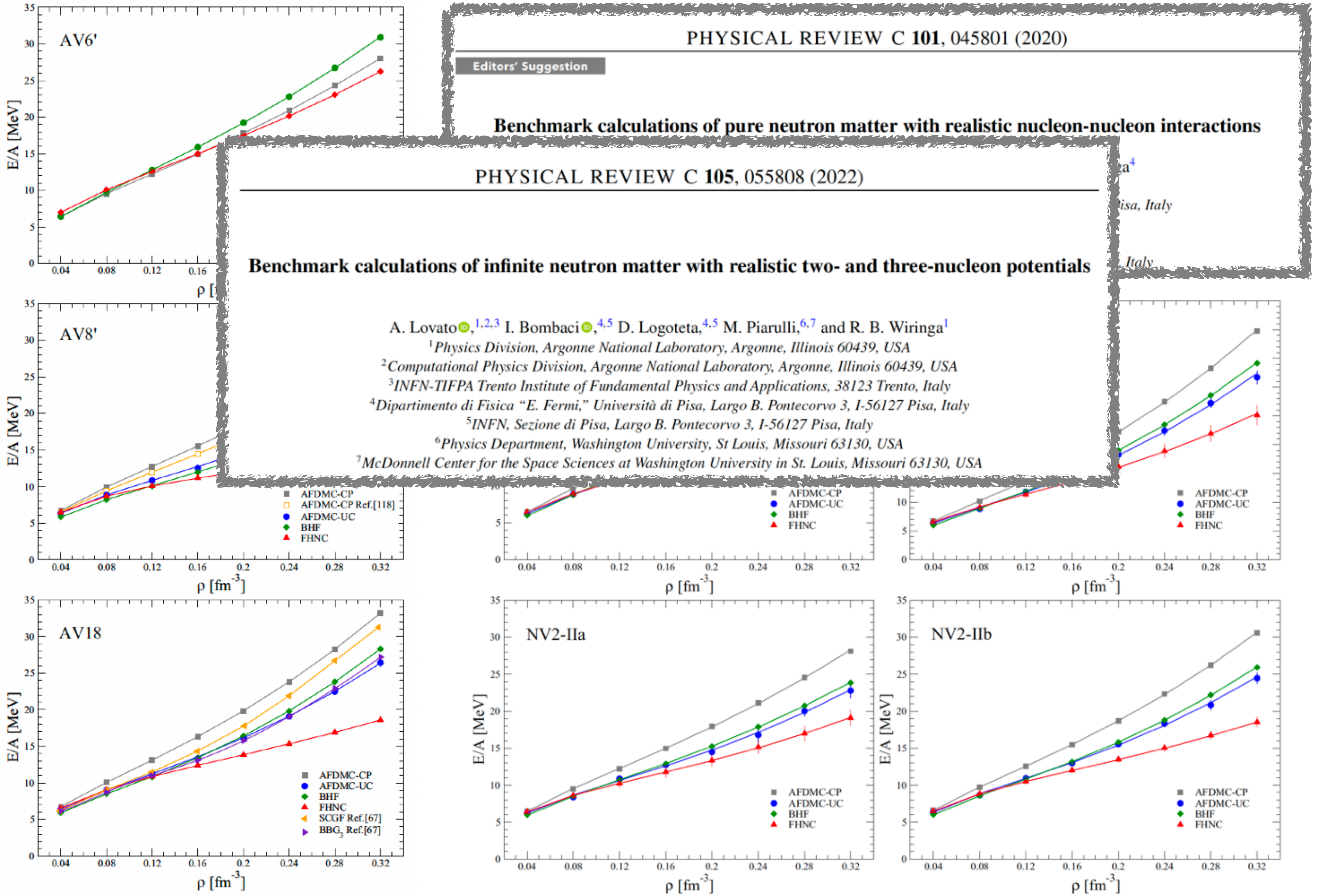




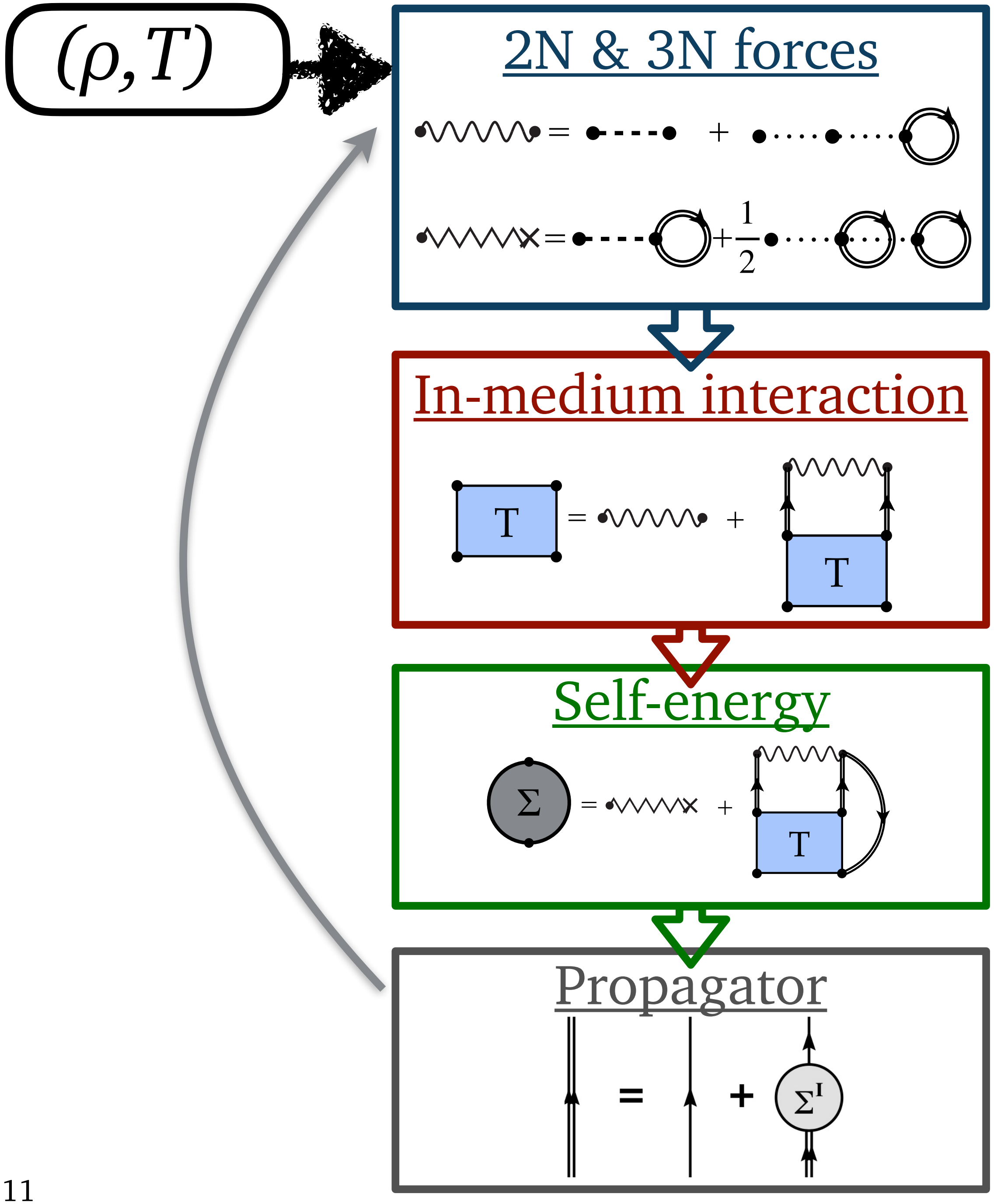




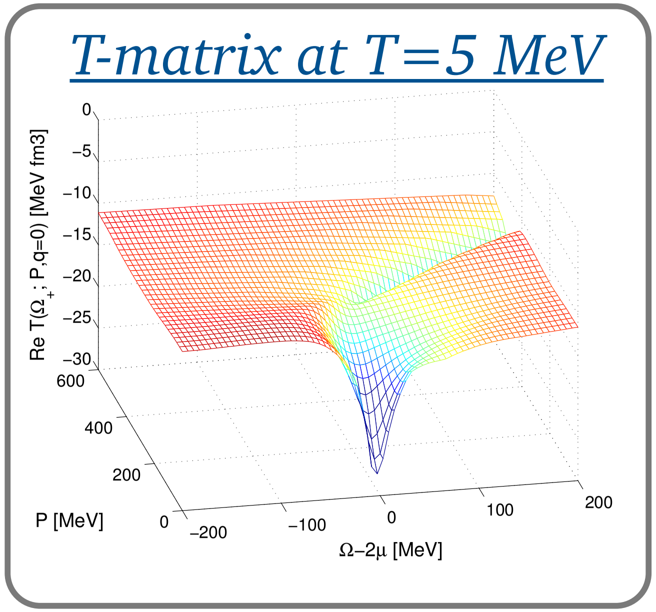
An idea that lasted!



Self-Consistent Green's Functions

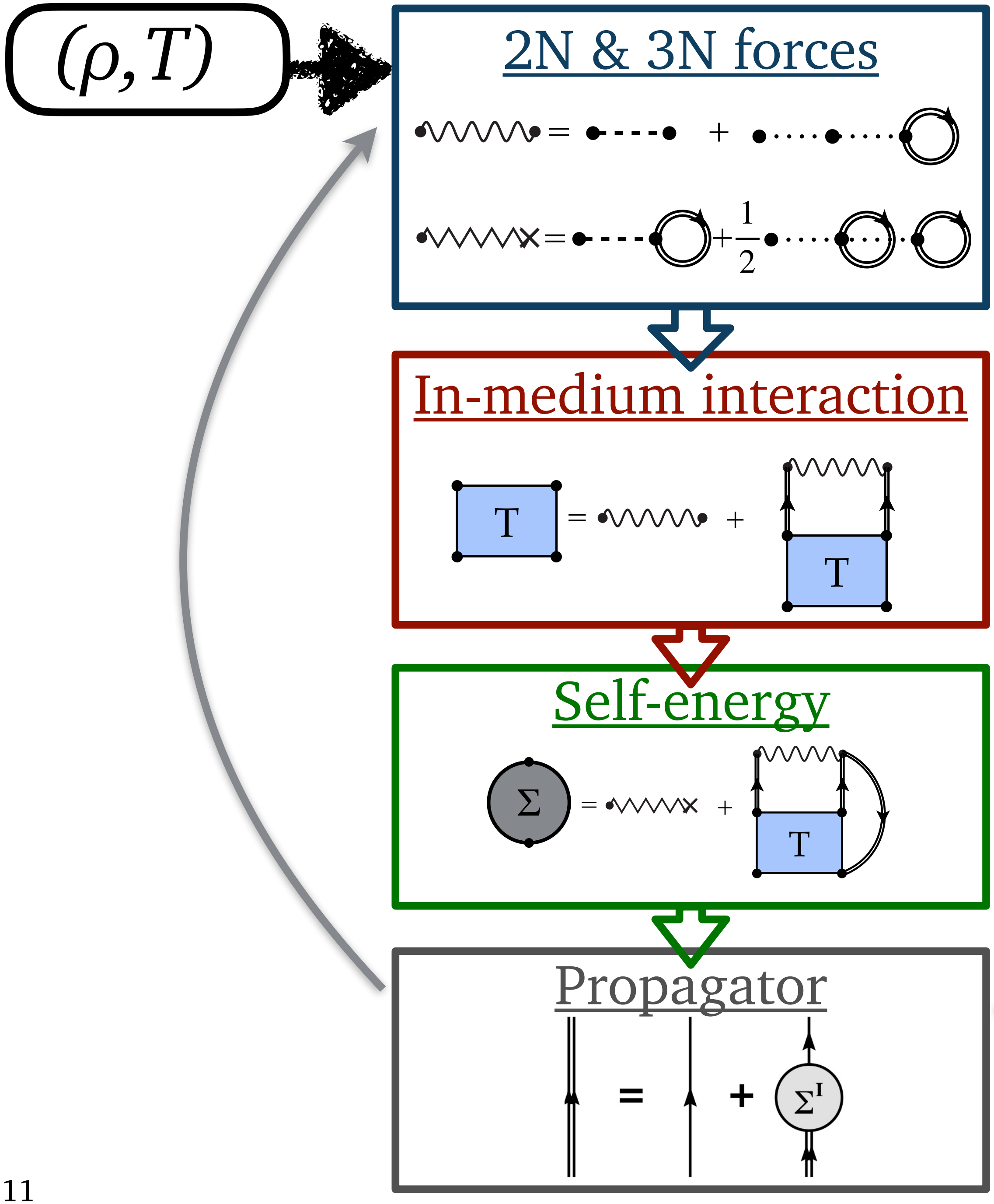


Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis



Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

Self-Consistent Green's Functions



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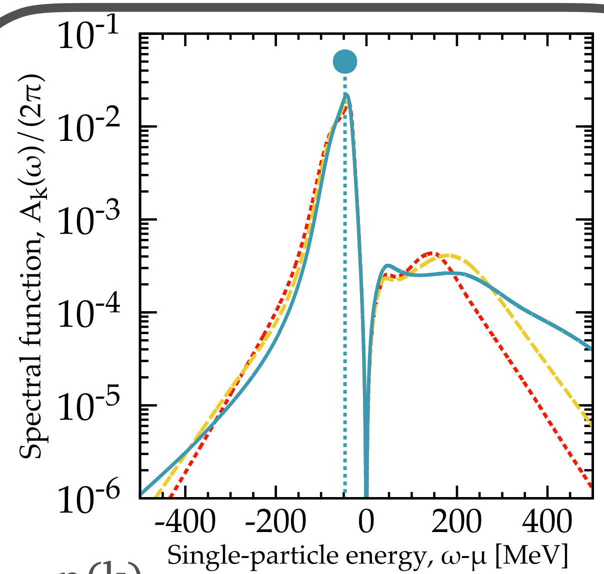

n(k)
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
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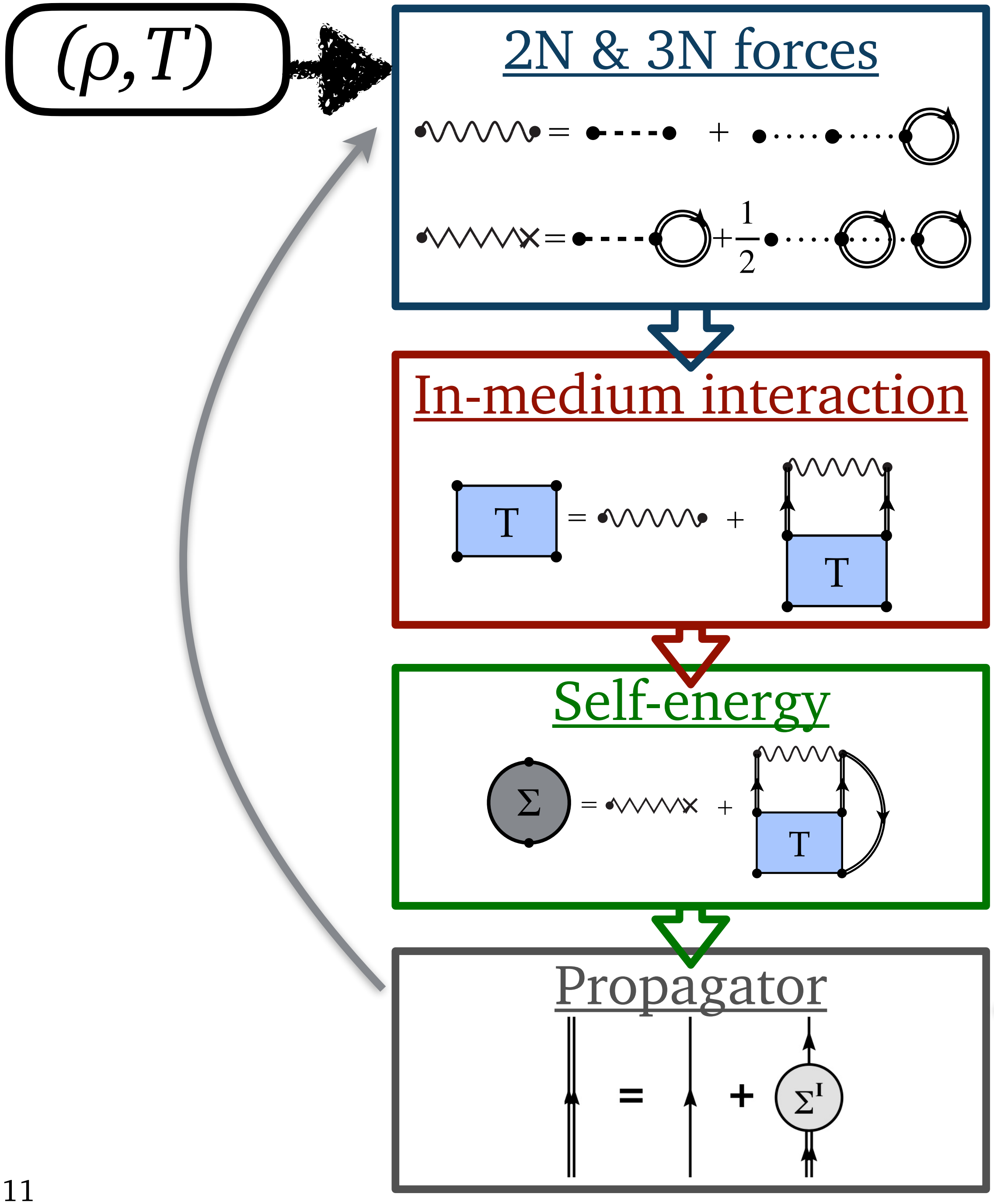
Self-Consistent Green's Functions



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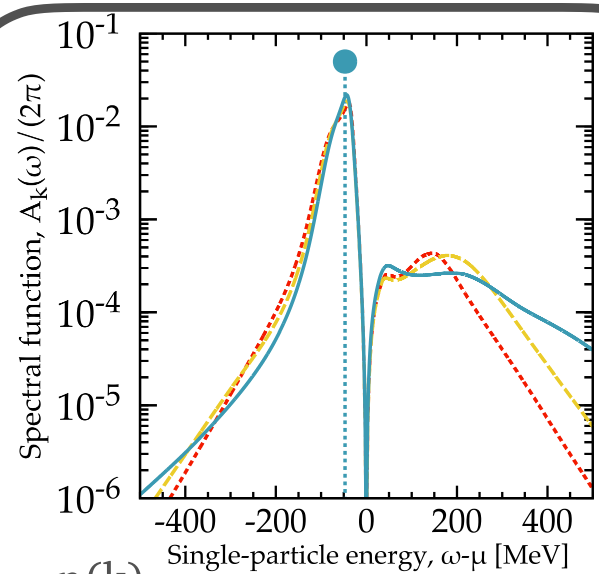
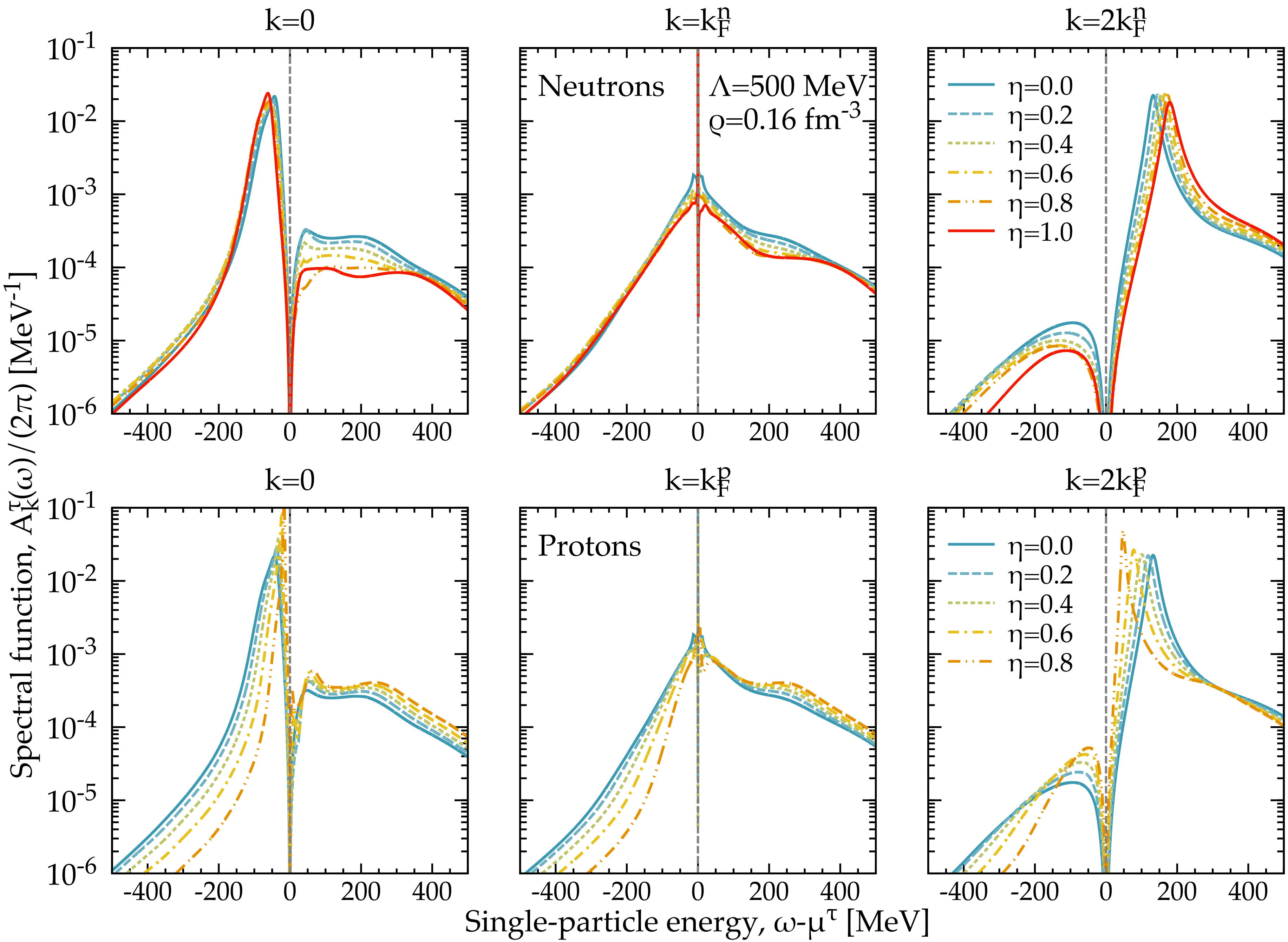


n(k)
Thermodynamics & EoS
Transport

- Off-shell ✔
- Matsubara formalism ✔
- Φ-derivable ✔

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

Asymmetry dependence in spectral functions



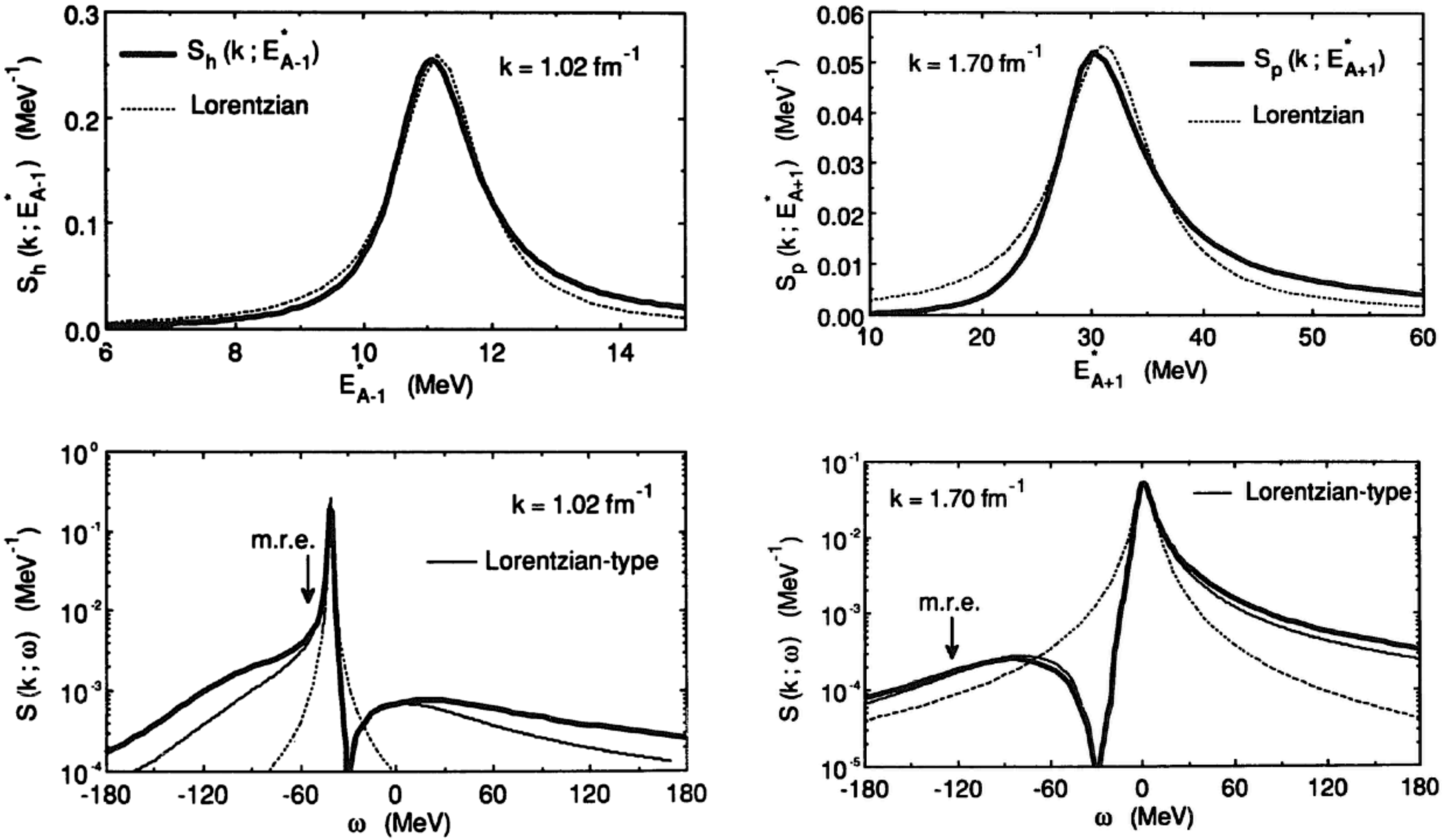
Spectral functions à la Baldo

Off-the-energy-shell properties of the mass operator and spectral functions in nuclear matter

M. Baldo, I. Bombaci, G. Giansiracusa and U. Lombardo
Dipartimento di Fisica, Università di Catania, and Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Corso Italia 57, I-95129 Catania, Italy

C. Mahaux and R. Sartor
Institut de Physique B5, Université de Liège, B-4000, Liège 1, Belgium

Baldo *et al*, Nucl Phys A **545** 741 (1992)



Finite temperature

Nuclear liquid-gas phase transition

M. Baldo[1] and L. S. Ferreira[1,2]
[1]Istituto Nazionale di Fisica Nucleare, Sezione di Catania and Dipartimento di Fisica, Universitá di Catania
Corso Italia 57, I-95129 Catania, Italy
[2]Centro de Física das Interaccoẽs Fundamentais and Departámento de Física, Instituto Superior Técnico,
Av. Rovisco Pais, 1096 Lisboa, Portugal
(Received 8 June 1998)

Baldo & Ferreira, Phys Rev C **59** 682 (1999)

Cylindrical diagrams!

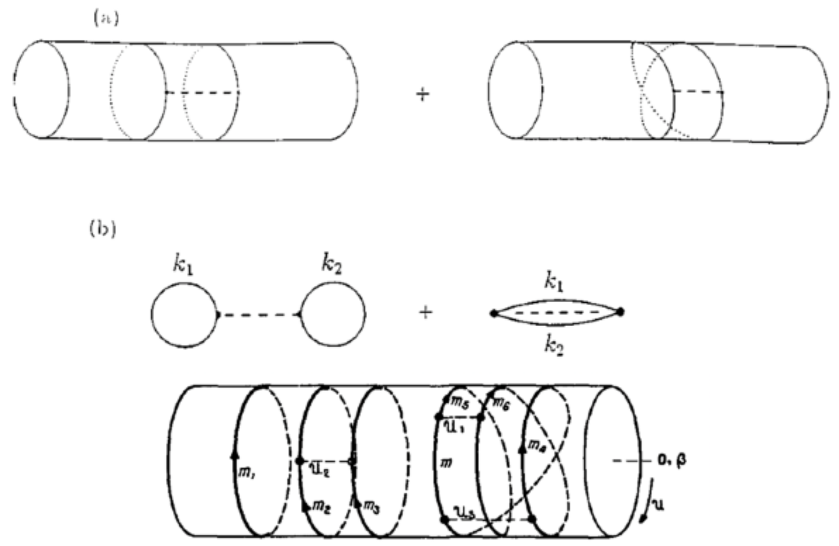
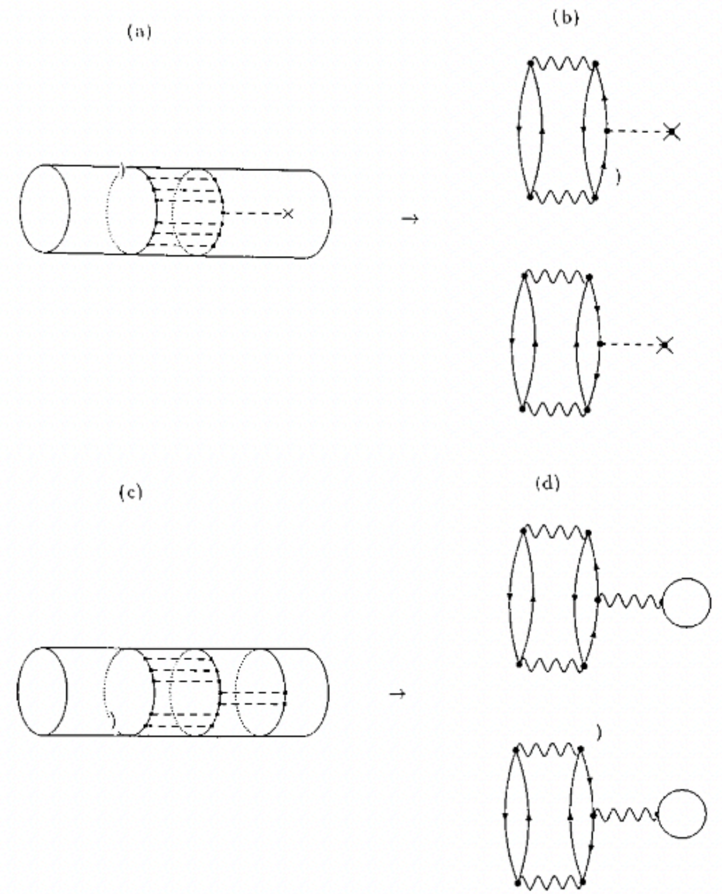


Fig. 2. Le diagramme de la fig. 1 dessiné sur un cylindre.

Bloch, Nucl. Phys. **7** 451 (1958)
Bloch and De Dominicis, Nucl. Phys. **7** 459 (1958)
Nucl. Phys. **10** 181 (1959)
Nucl. Phys. **10** 509 (1959)



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[1]*Istituto Nazionale di Fisica Nucleare, Sezione di Catania and Dipartimento di Fisica, Universitá di Catania, Corso Italia 57, I-95129 Catania, Italy*

[2]*Centro de Física das Interaccoẽs Fundamentais and Departámento de Física, Instituto Superior Técnico, Av. Rovisco Pais, 1096 Lisboa, Portugal*

(Received 8 June 1998)

Baldo & Ferreira, Phys Rev C **59** 682 (1999)

Cylindrical diagrams!

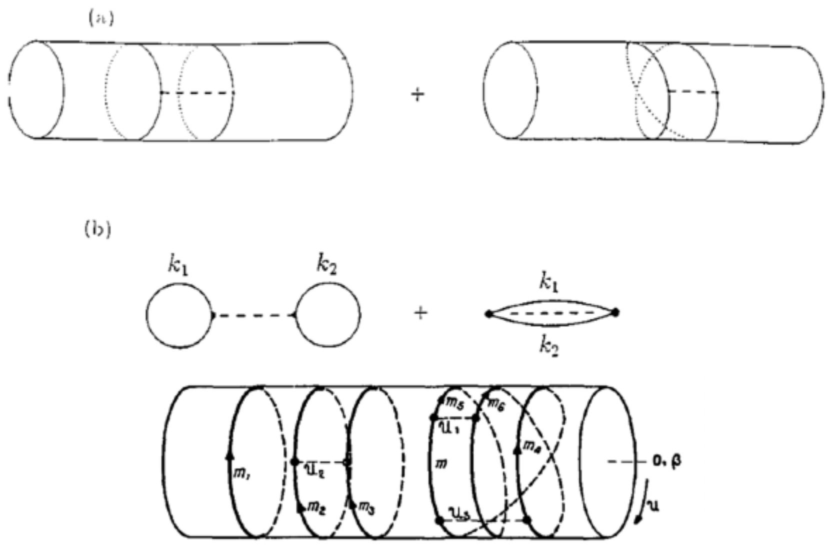
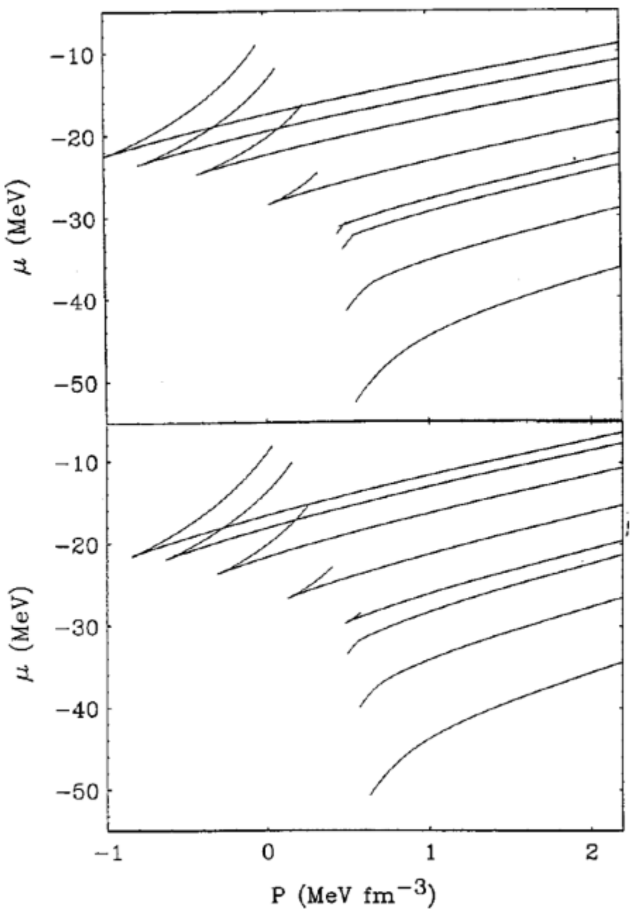
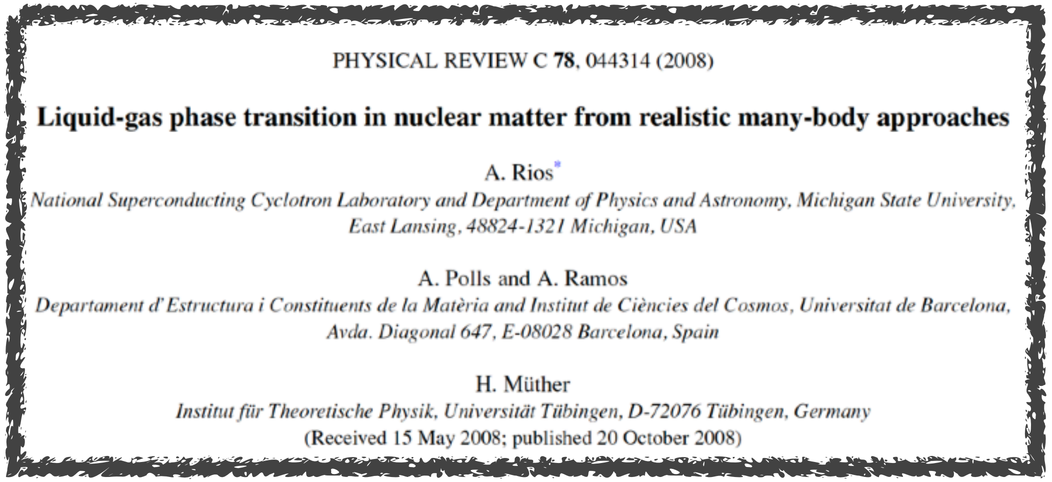


Fig. 2. Le diagramme de la fig. 1 dessiné sur un cylindre.

Bloch, Nucl. Phys. **7** 451 (1958)
Bloch and De Dominicis, Nucl. Phys. **7** 459 (1958)
Nucl. Phys. **10** 181 (1959)
Nucl. Phys. **10** 509 (1959)



Standing on the shoulders…



PHYSICAL REVIEW C **78**, 044314 (2008)

Liquid-gas phase transition in nuclear matter from realistic many-body approaches

A. Rios*
National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, 48824-1321 Michigan, USA

A. Polls and A. Ramos
Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Avda. Diagonal 647, E-08028 Barcelona, Spain

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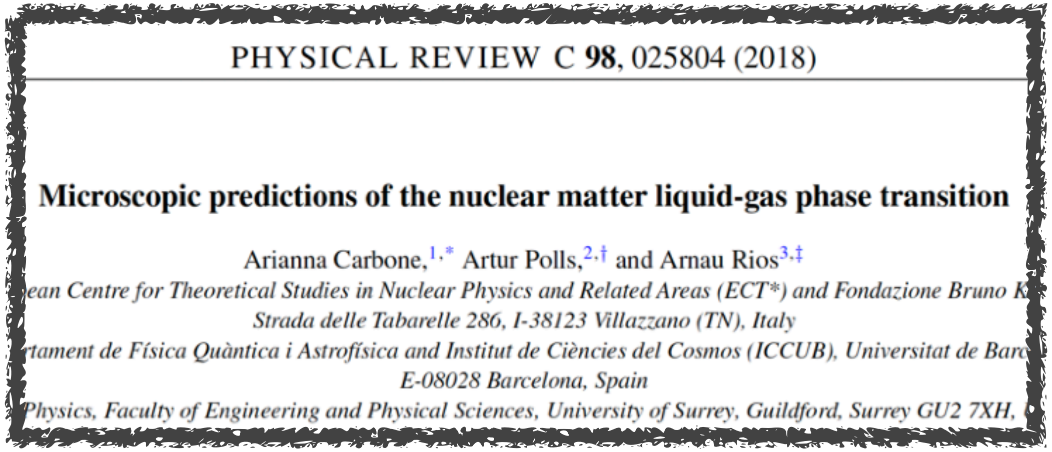
Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany

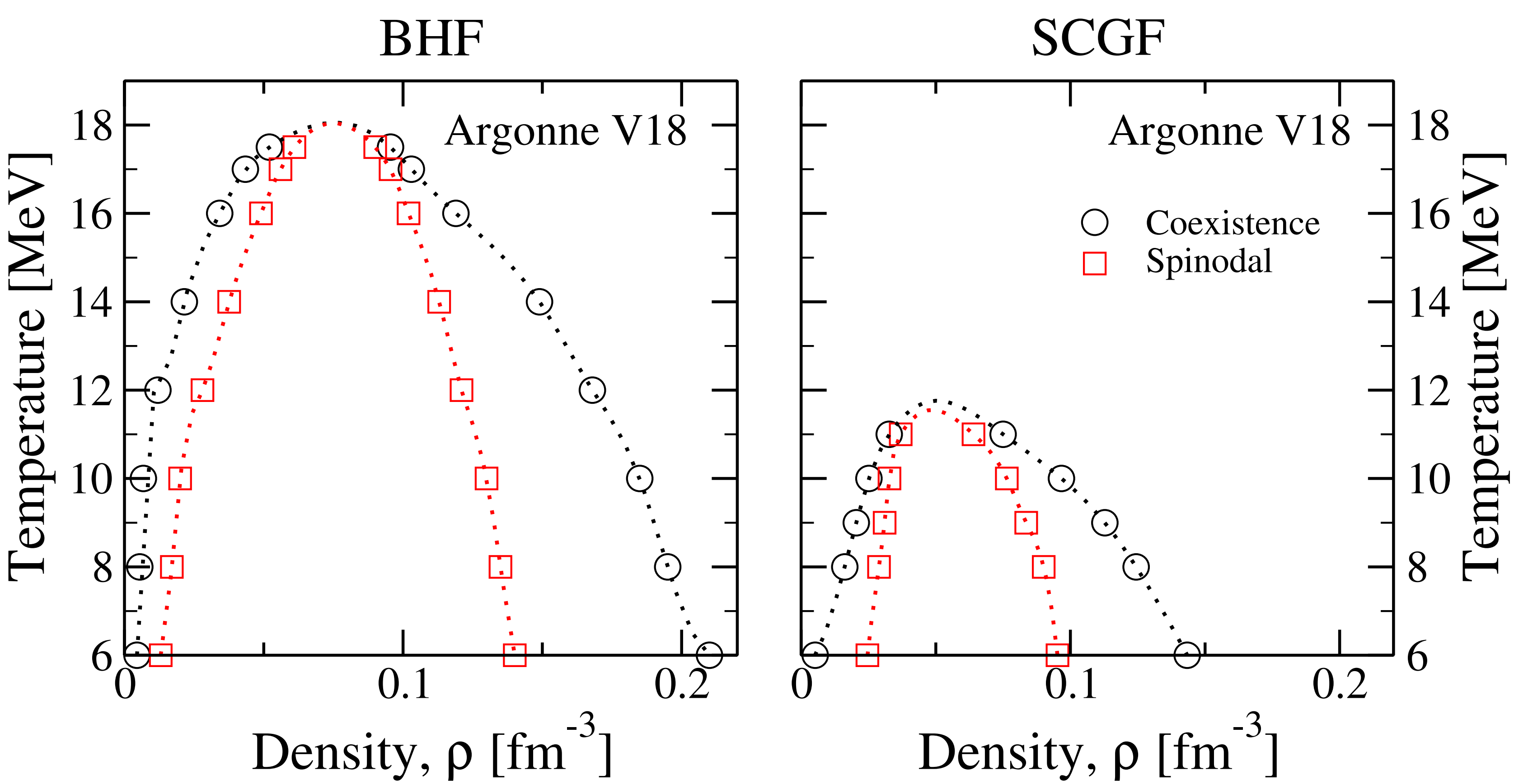
(Received 15 May 2008; published 20 October 2008)



PHYSICAL REVIEW C **98**, 025804 (2018)

Microscopic predictions of the nuclear matter liquid-gas phase transition

Arianna Carbone,[1,*] Artur Polls,[2,†] and Arnau Rios[3,‡]
ean Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT) and Fondazione Bruno K*
Strada delle Tabarelle 286, I-38123 Villazzano (TN), Italy
rtament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos (ICCUB), Universitat de Barc
E-08028 Barcelona, Spain
Physics, Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, Surrey GU2 7XH,



Standing on the shoulders…

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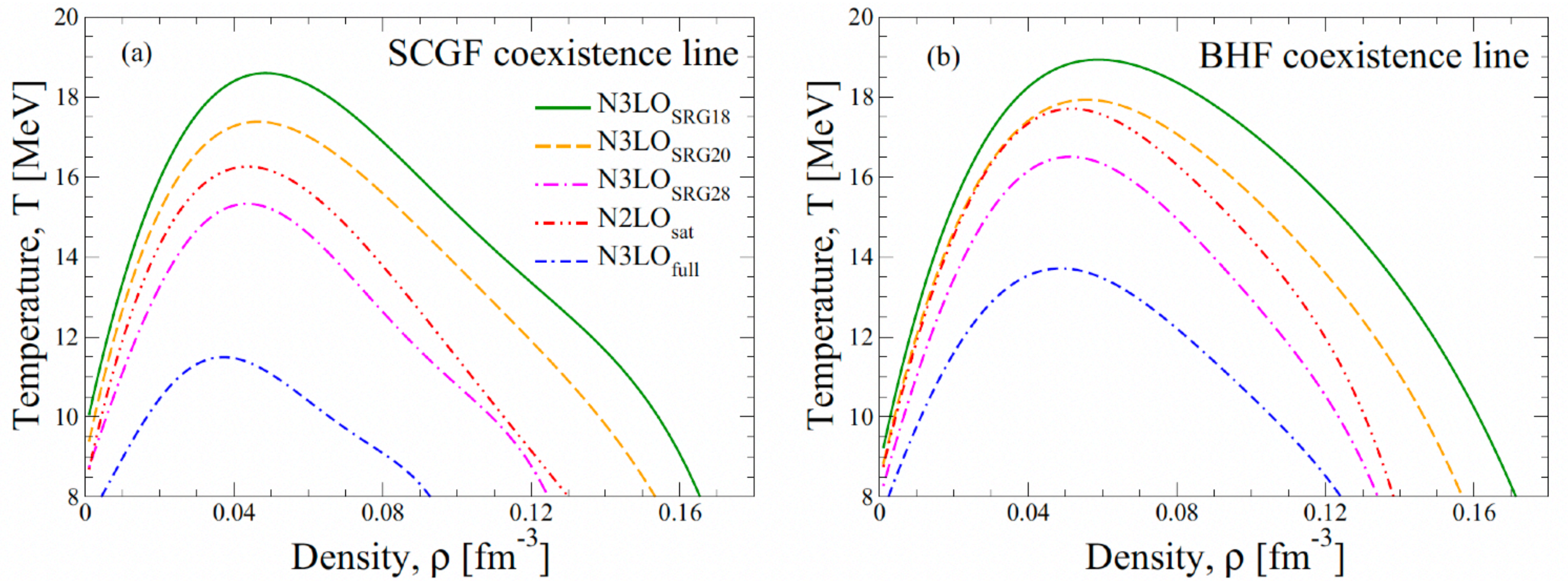

[Carbone, Polls, Rios, Phys. Rev. C **98** 025804 (2018)](#)
[arXiv:2006.10610]

Hyperons

Physics Letters B 355 (1995) 21–26

Hypernuclear matter in the Brueckner-Hartree-Fock approximation

H.-J. Schulze[a], A. Lejeune[a], J. Cugnon[a], M. Baldo[b], U. Lombardo[b]
[a] *Université de Liège, Institut de Physique B5, B-4000 Liège 1, Belgium*
[b] *Dipartimento di Fisica, Università di Catania, Corso Italia, 57, I-95129 Catania, Italy*

Received 25 April 1995
Editor: G.F. Bertsch

VOLUME 57, NUMBER 2

Hyperonic nuclear matter in Brueckner theory

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(Received 3 July 1997)

PHYSICAL REVIEW C, VOLUME 61, 055801

Hyperon stars in the Brueckner-Bethe-Goldstone theory

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Istituto Nazionale di Fisica Nucleare, Sezione di Catania and Dipartimento di Fisica, Università di Catania, Corso Italia 57, I-95129 Catania, Italy

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Description of hyperonic matter and hypernuclei within the Brueckner–Hartree–Fock theory

Isaac Vidaña Haro

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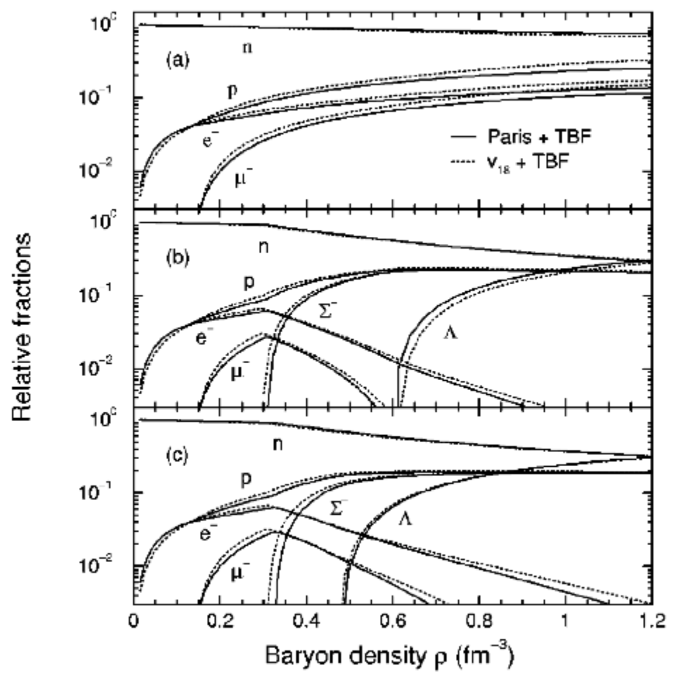
Universitat de Barcelona



Hyperons

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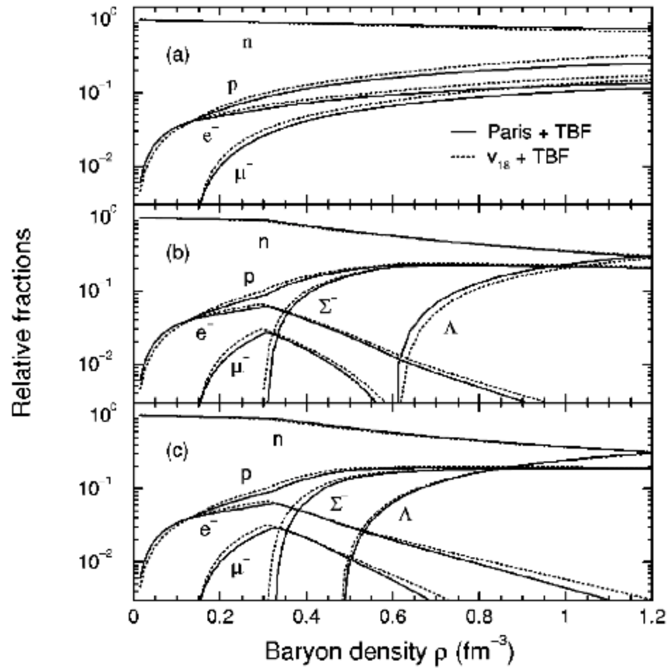

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Universitat de Barcelona

⇒D. Blaschke's talk

Reviews

IOP PUBLISHING
Rep. Prog. Phys. 75 (2012) 026301 (53pp)

Properties of the nuclear medium

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Review

The nuclear symmetry energy

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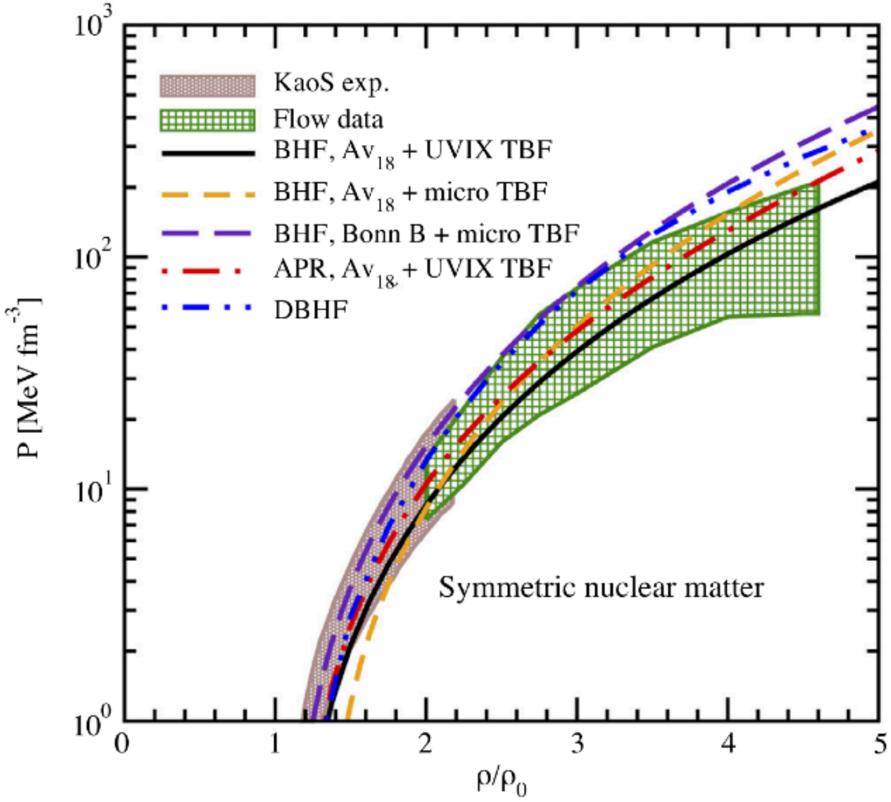
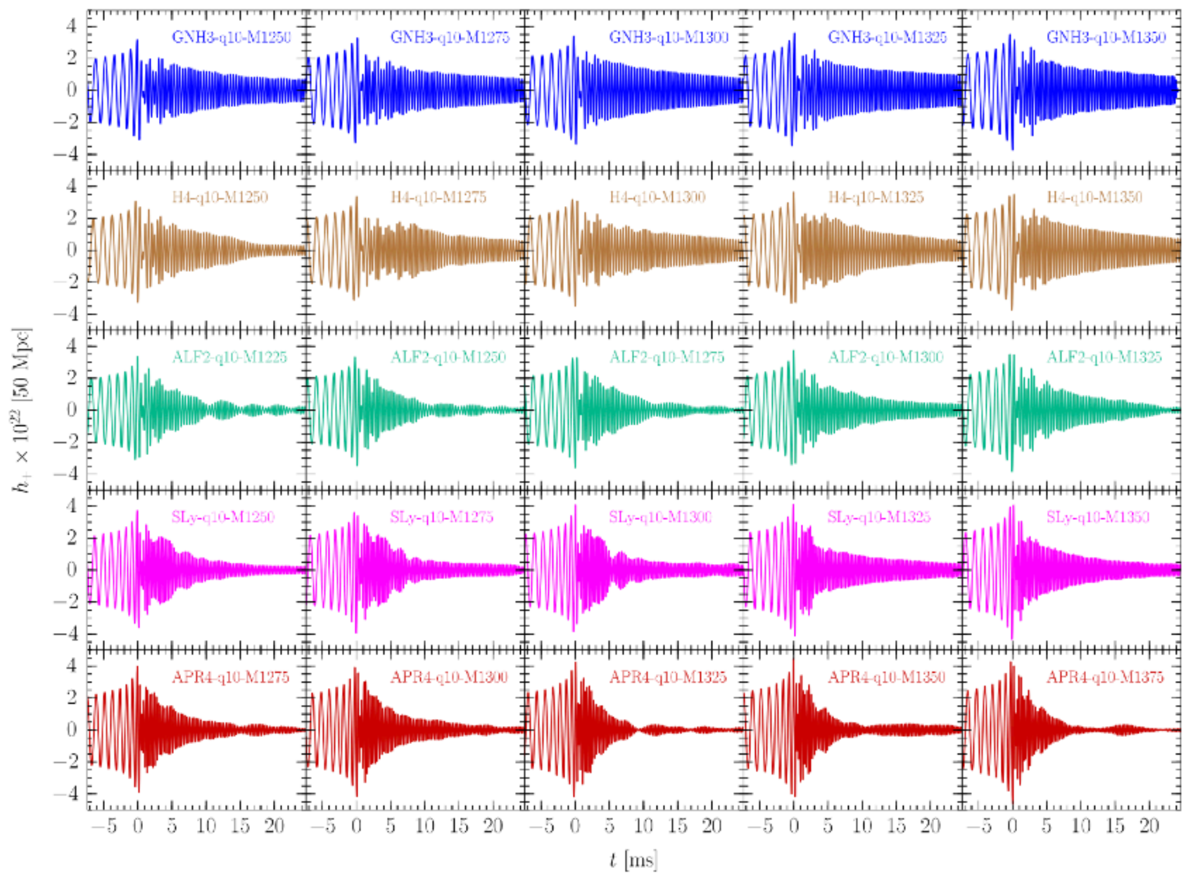
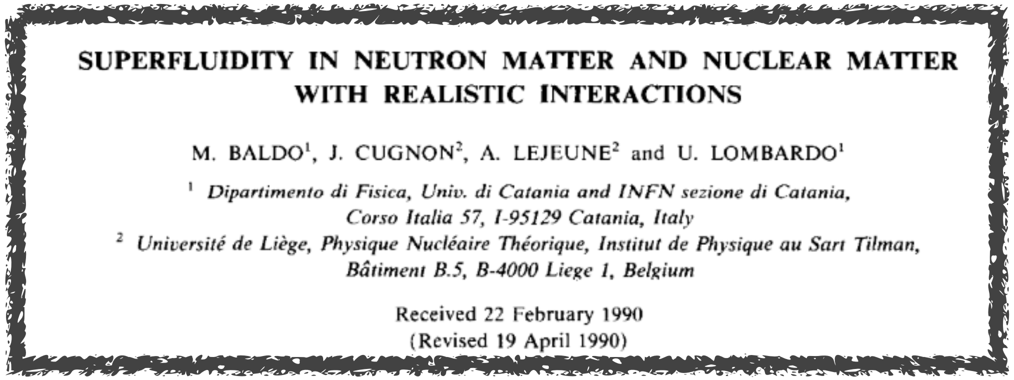




Baldo & Burgio, *Prog. Part. Nucl. Phys.* **91** 203–258 (2016)

Beyond-BCS pairing: overview



SUPERFLUIDITY IN NEUTRON MATTER AND NUCLEAR MATTER WITH REALISTIC INTERACTIONS

M. BALDO[1], J. CUGNON[2], A. LEJEUNE[2] and U. LOMBARDO[1]

[1] *Dipartimento di Fisica, Univ. di Catania and INFN sezione di Catania, Corso Italia 57, I-95129 Catania, Italy*

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Received 22 February 1990
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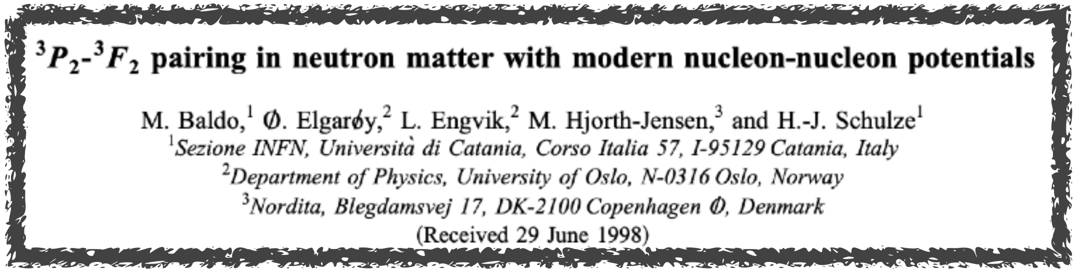

$^3P_2$-$^3F_2$ pairing in neutron matter with modern nucleon-nucleon potentials

M. Baldo,[1] Ø. Elgarøy,[2] L. Engvik,[2] M. Hjorth-Jensen,[3] and H.-J. Schulze[1]

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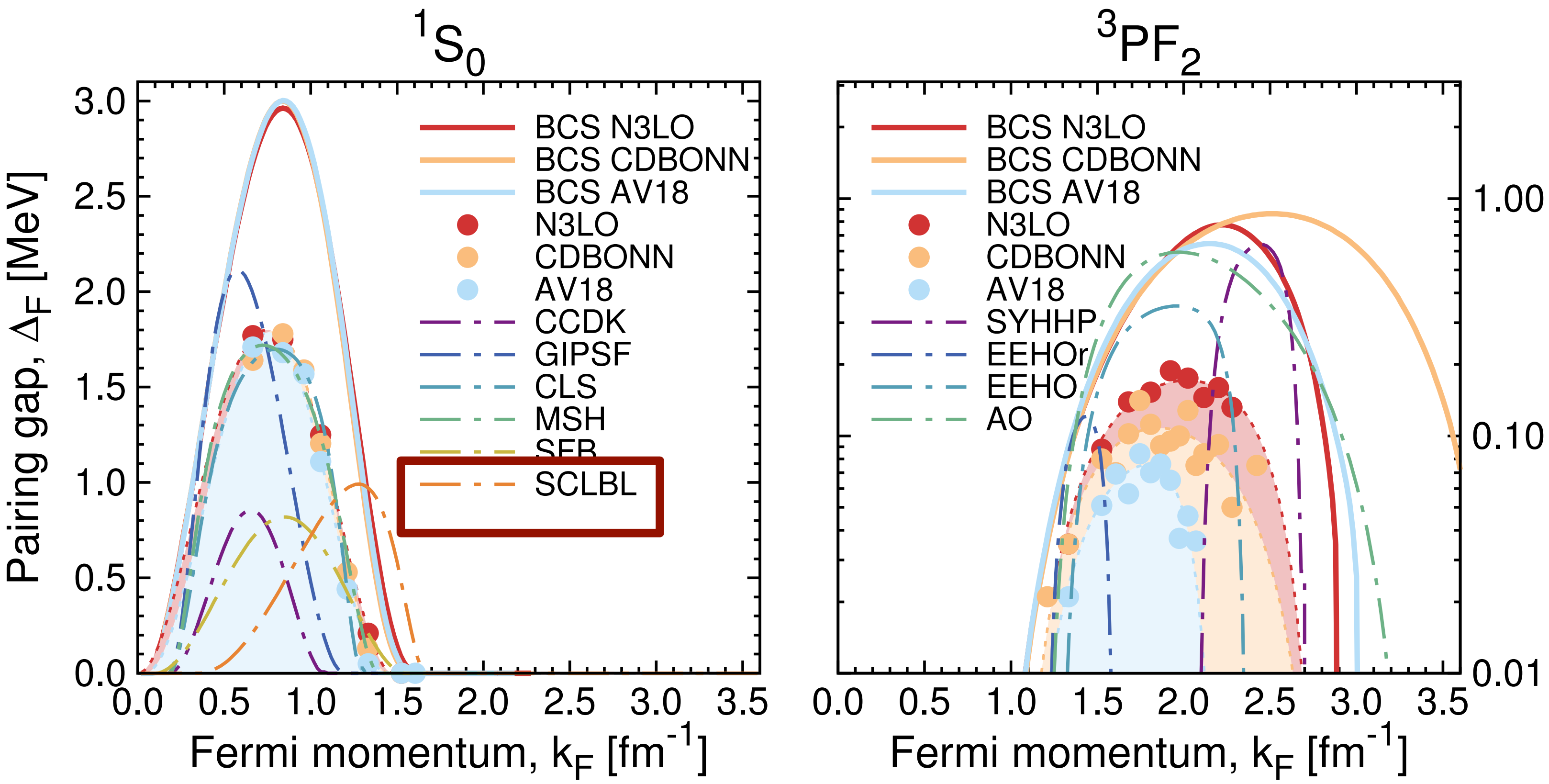
(Received 29 June 1998)

Baldo et al, Phys. Rev. C **58** 1921 (1996)



SCLBL=Schulze, Cugnon, Lejeune, Baldo & Lombardo, Phys. Lett. B 375 1 (1996)

Beyond-BCS pairing: overview

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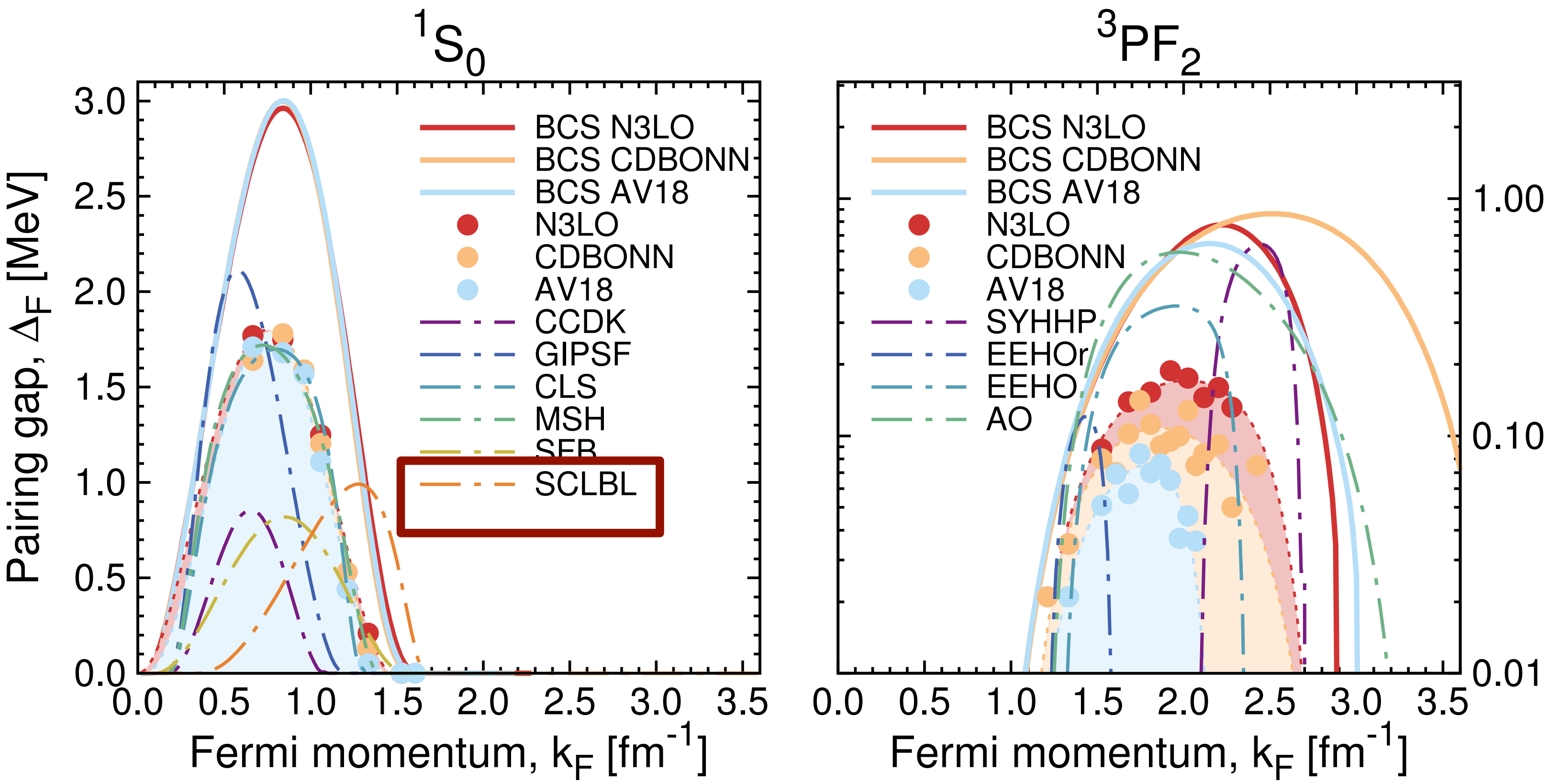
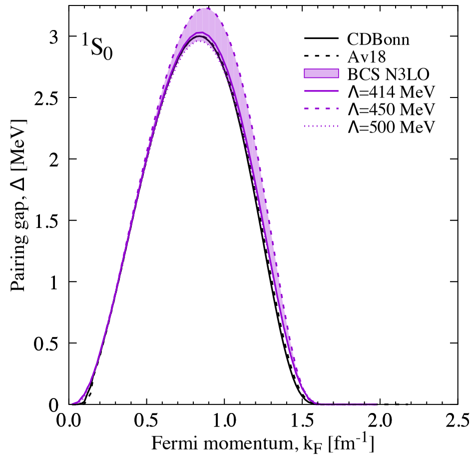


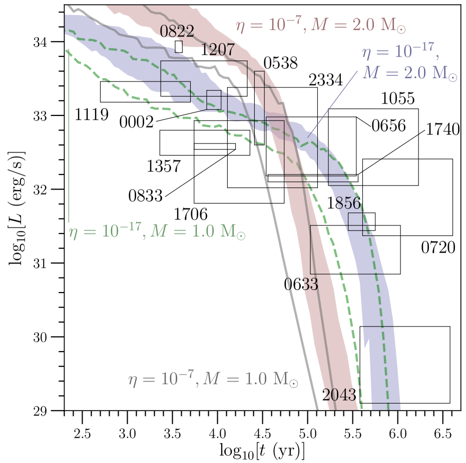
SCLBL=Schulze, Cugnon, Lejeune, Baldo & Lombardo, Phys. Lett. B 375 1 (1996)

⇒U. Lombardo's talk

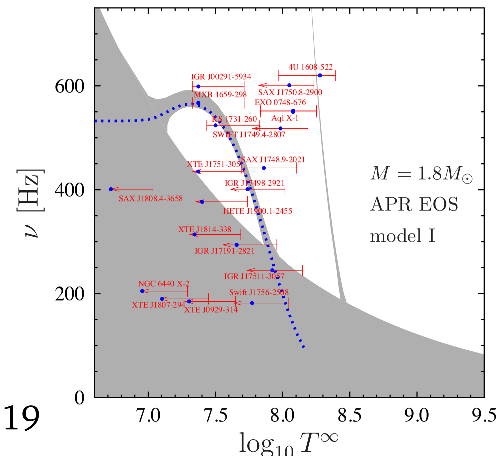
Why are pairing gaps necessary?



- Characterisation of **superfluidity**
  - Neutron superfluid, proton superconductor
  - Phase transitions



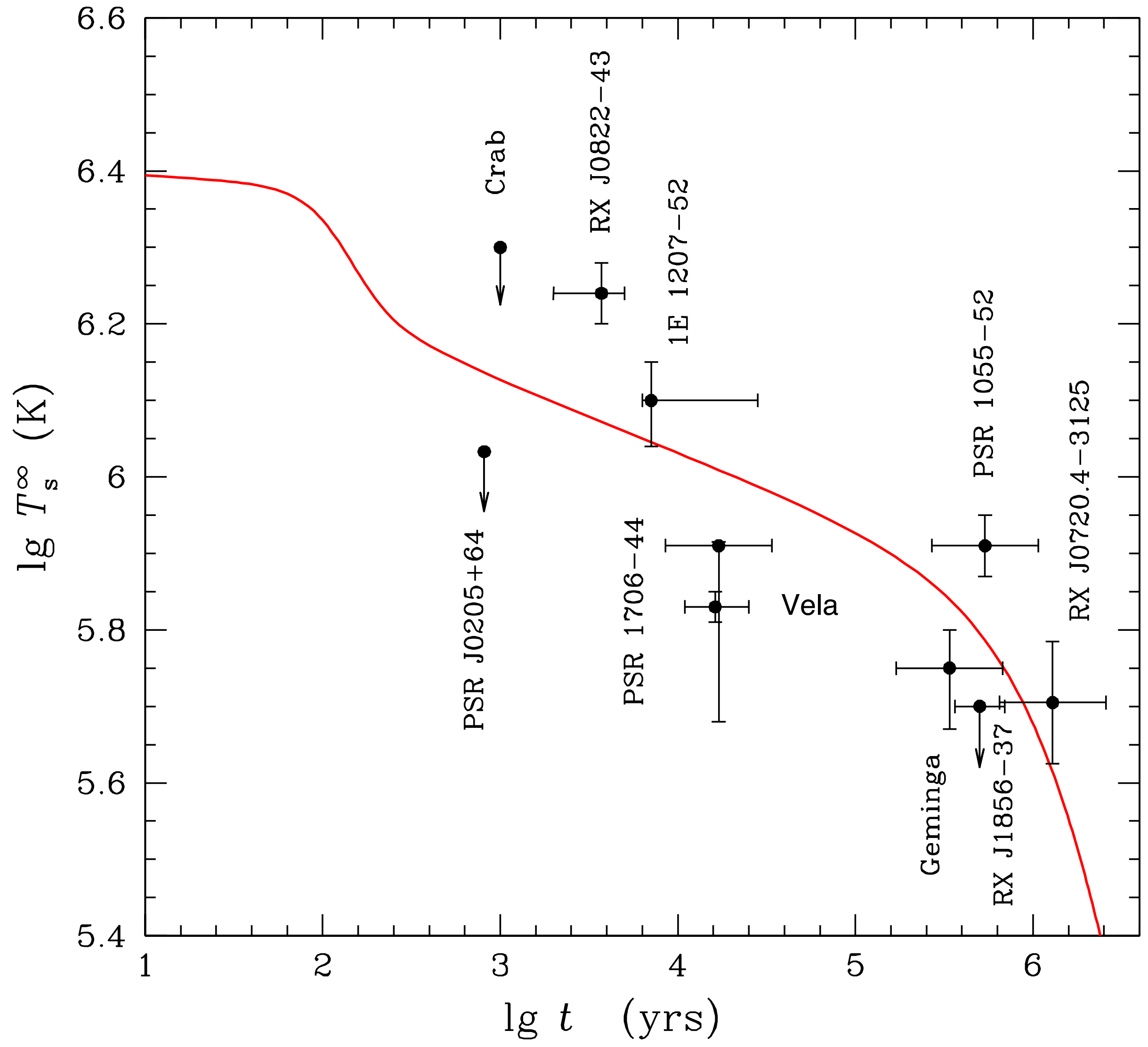
- Neutron star **cooling**
  - Neutrino rates through pair-breaking



- **r-mode** coupling
  - Superfluidity & viscosity

Kantor et al. Phys. Rev. Lett. **125** 151101 (2020)

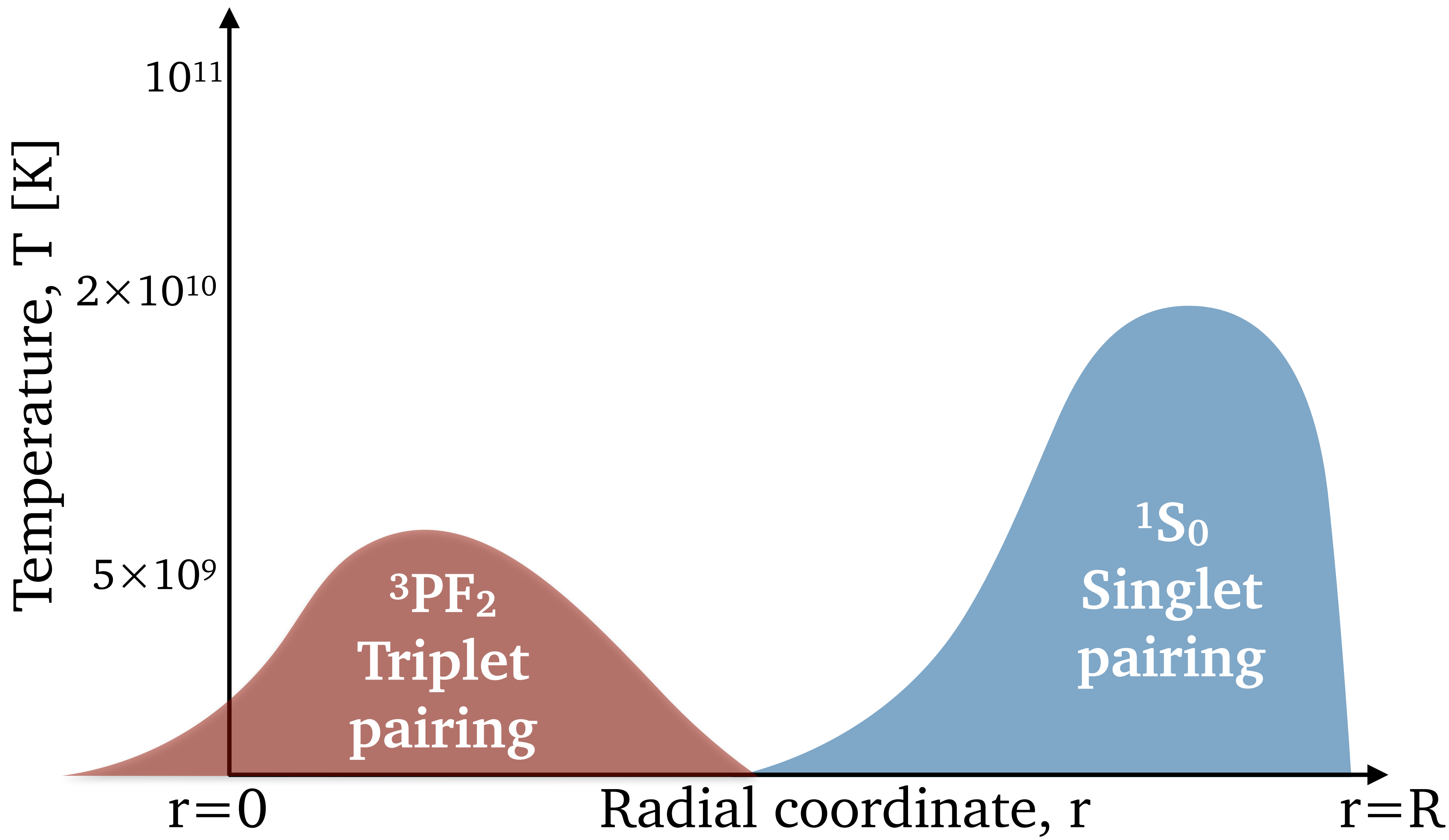
Cooling curve of neutron stars



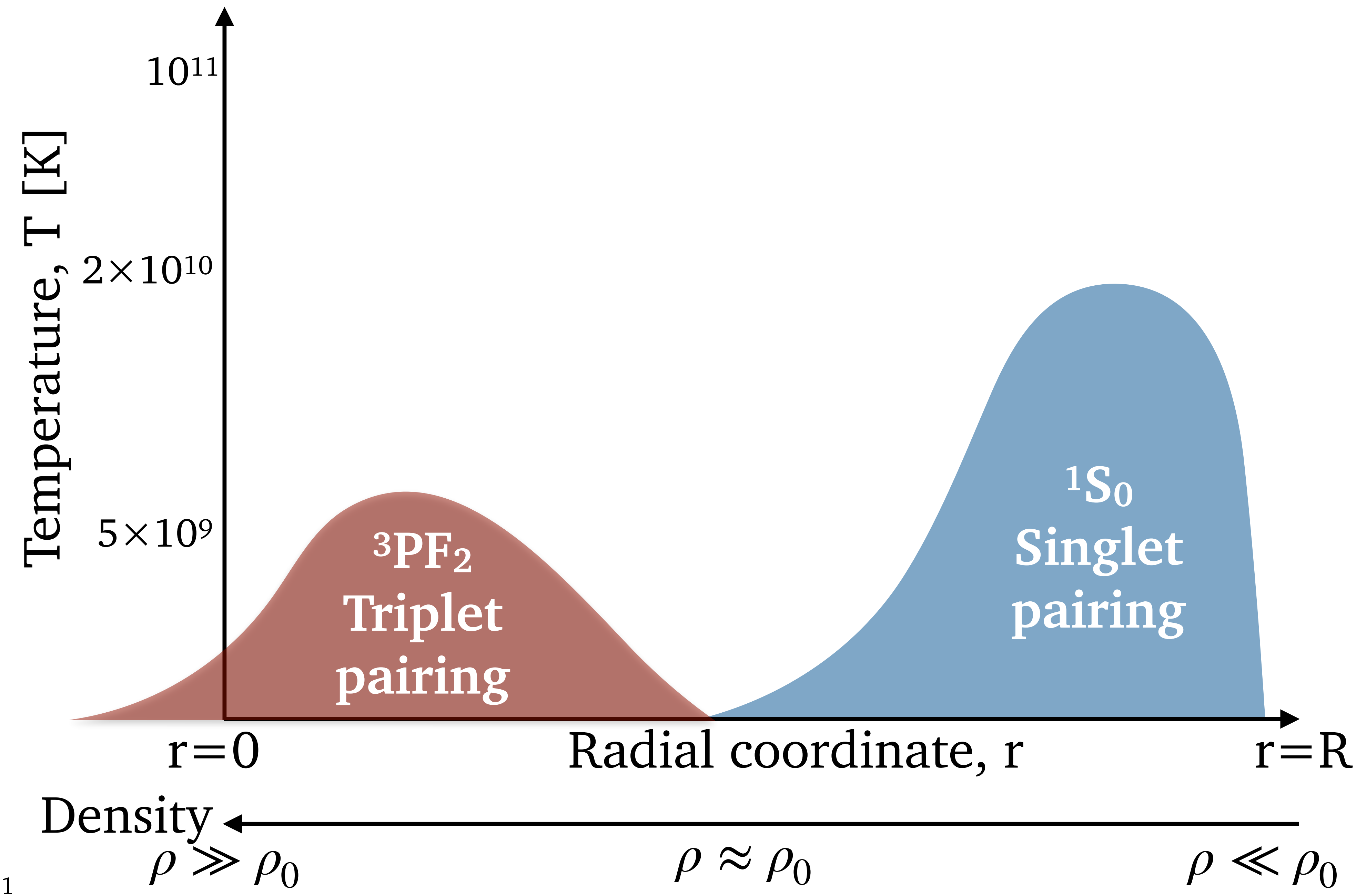
- Observational data available for a handful of NS
- Sensitive to **interior** physics (mostly **pairing**)

Yakovlev & Pethick, ARAA **42** 169 (2004)

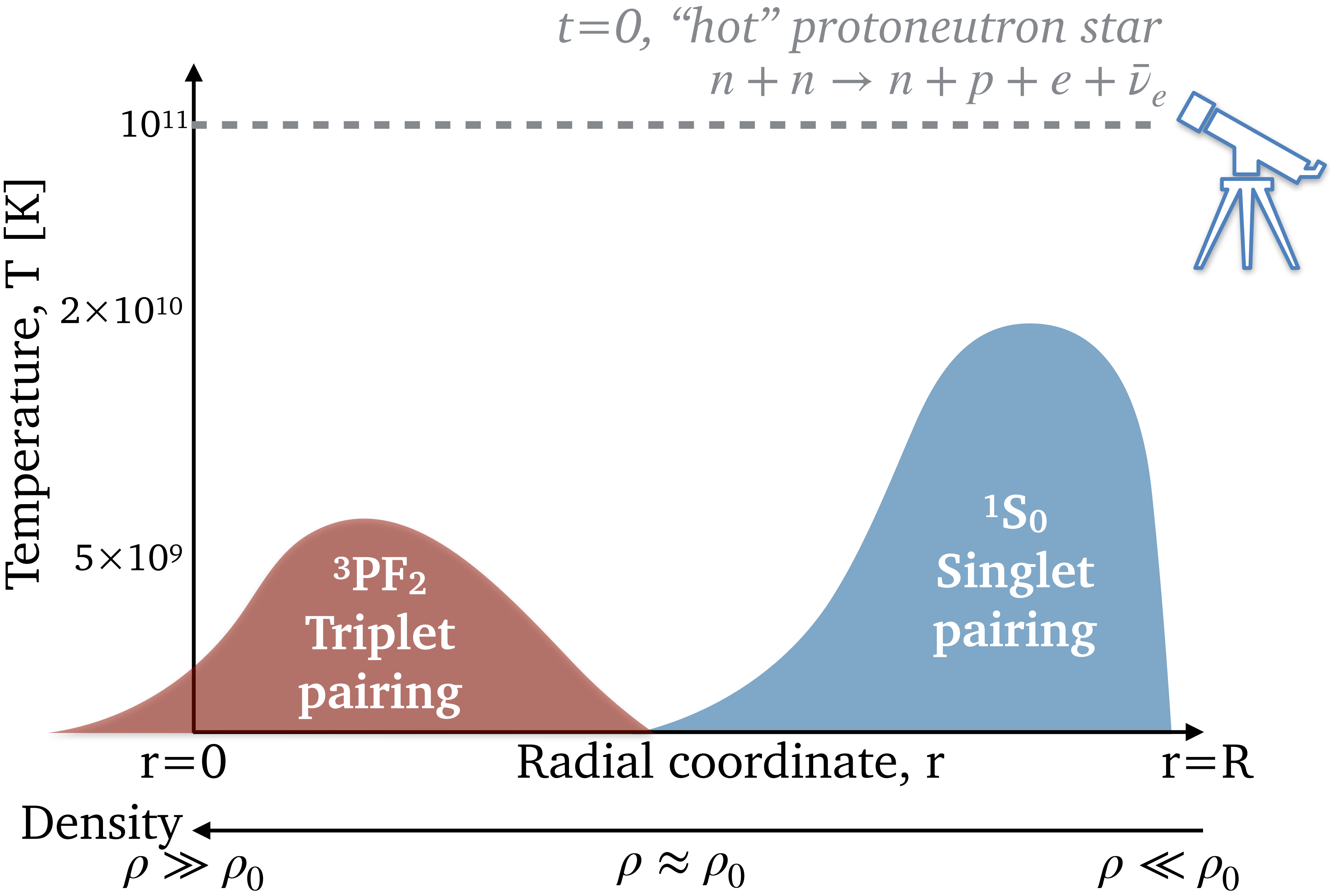
Pairing gaps & cooling



Pairing gaps & cooling



Pairing gaps & cooling



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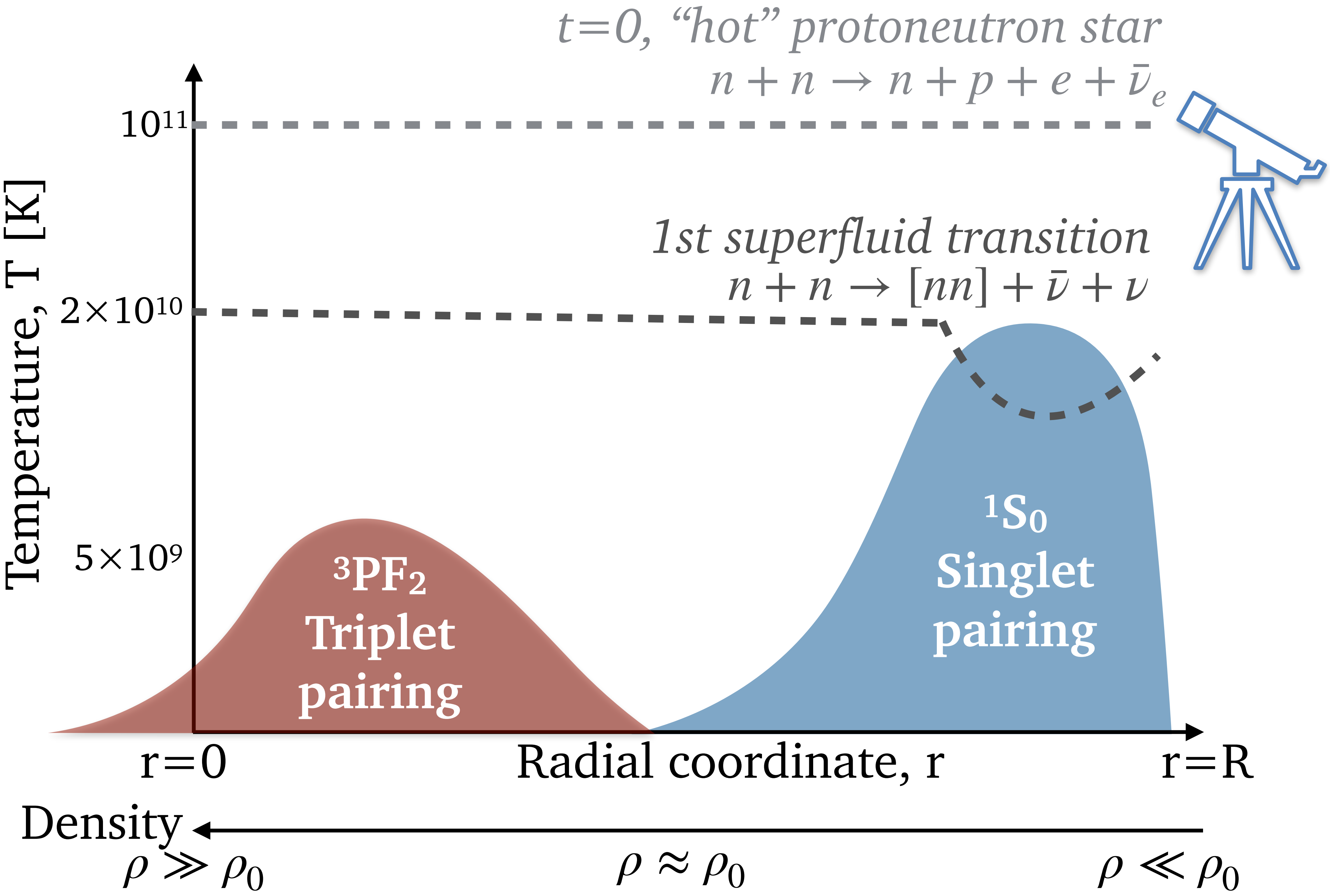

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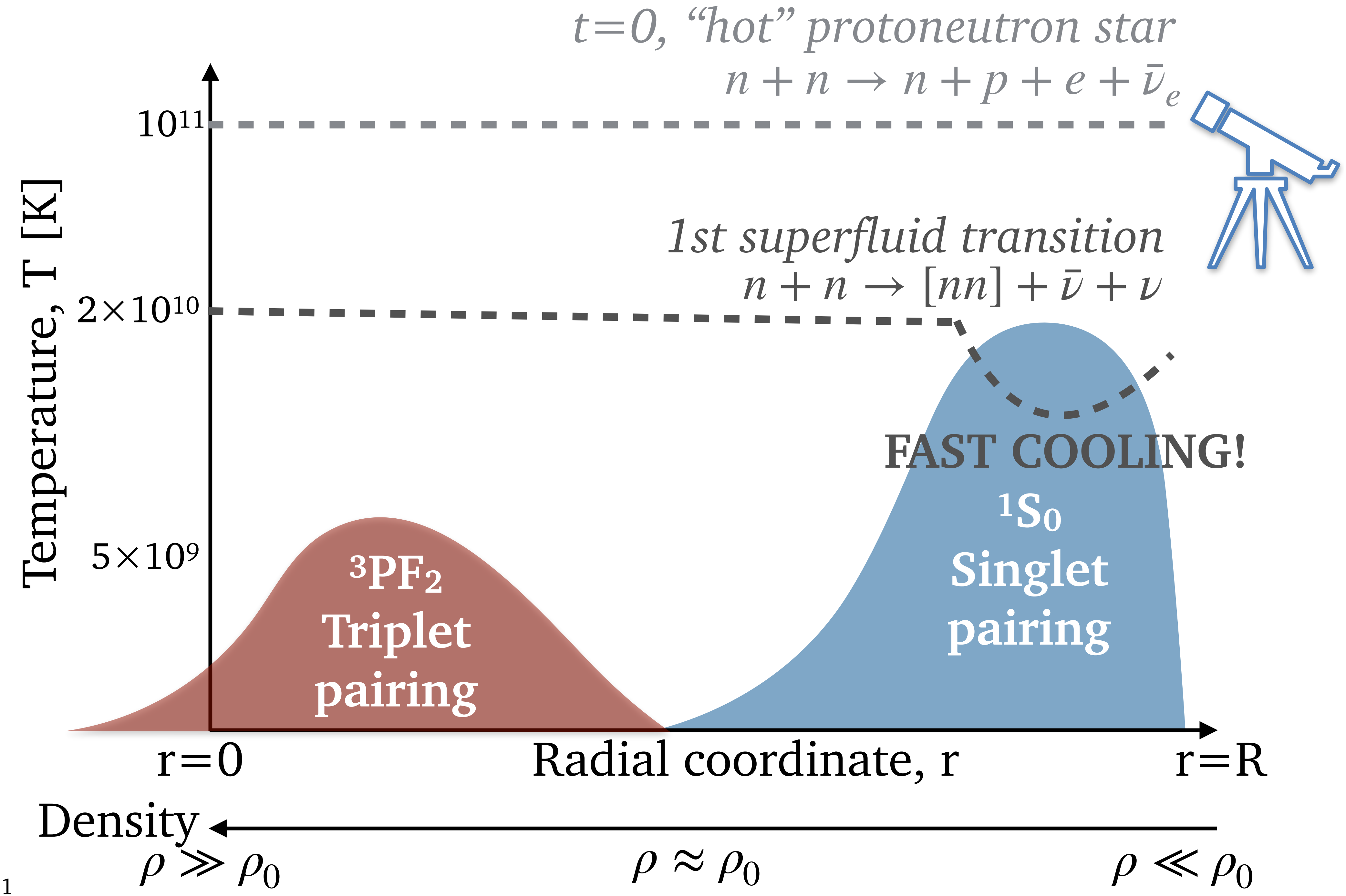
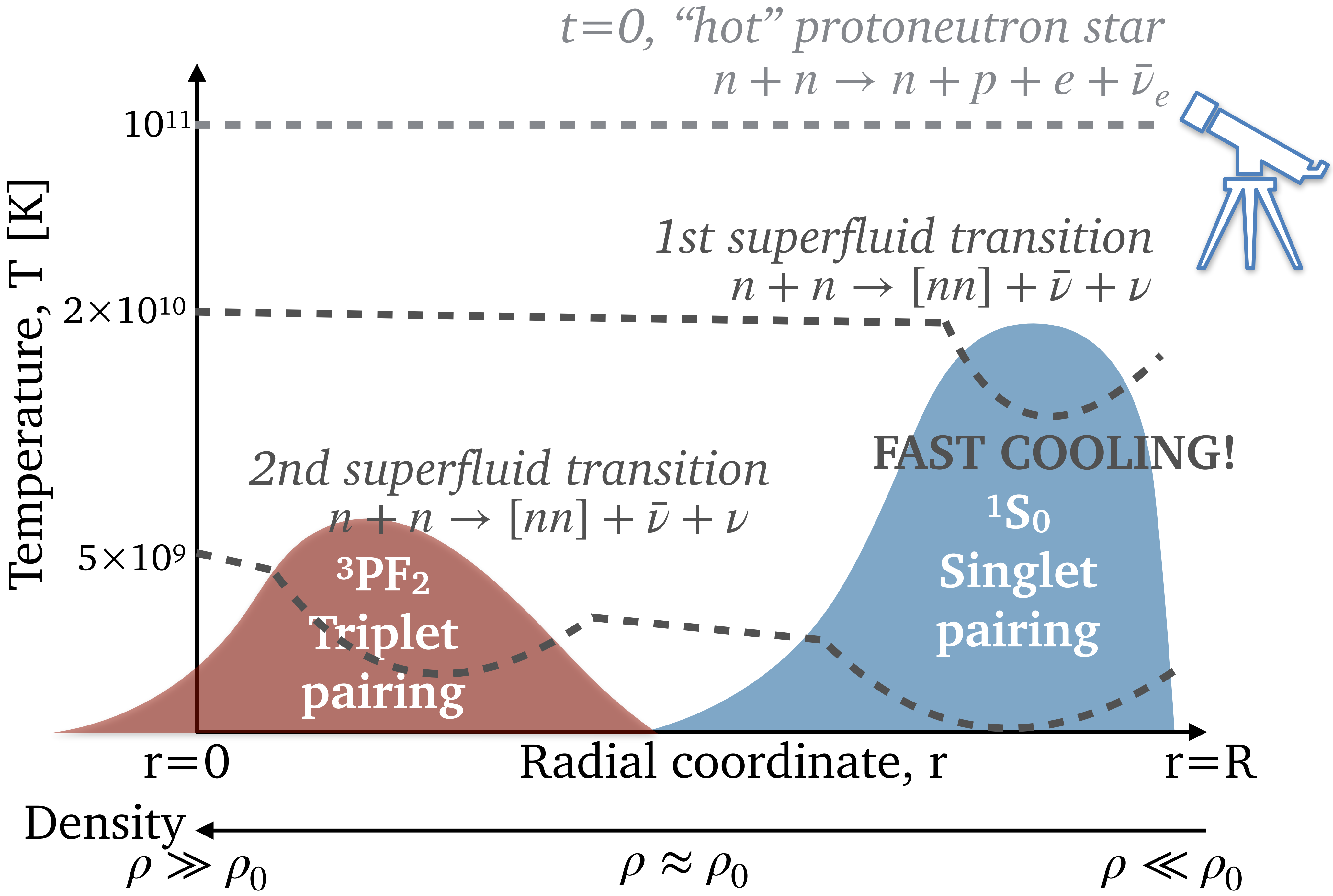
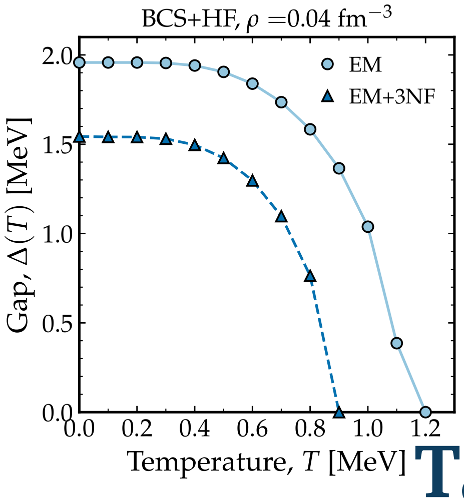


Pairing gaps & cooling

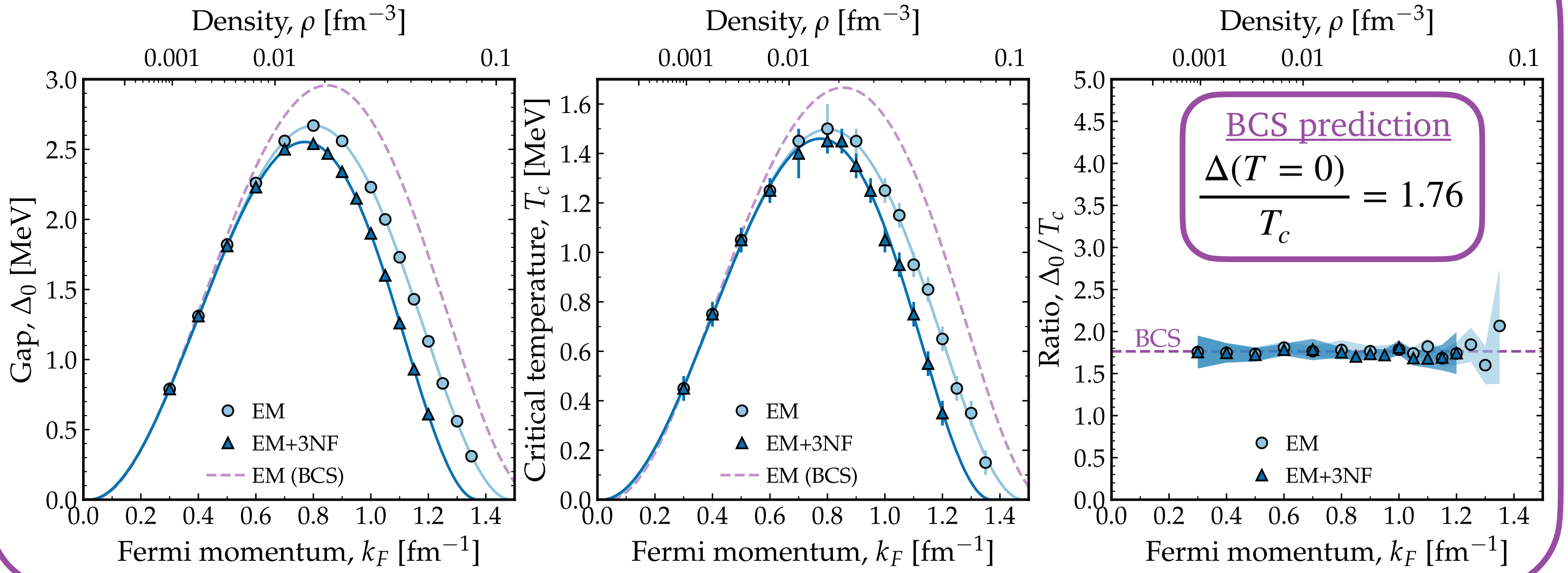


Why does it matter?



T_c

BCS+HF



Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

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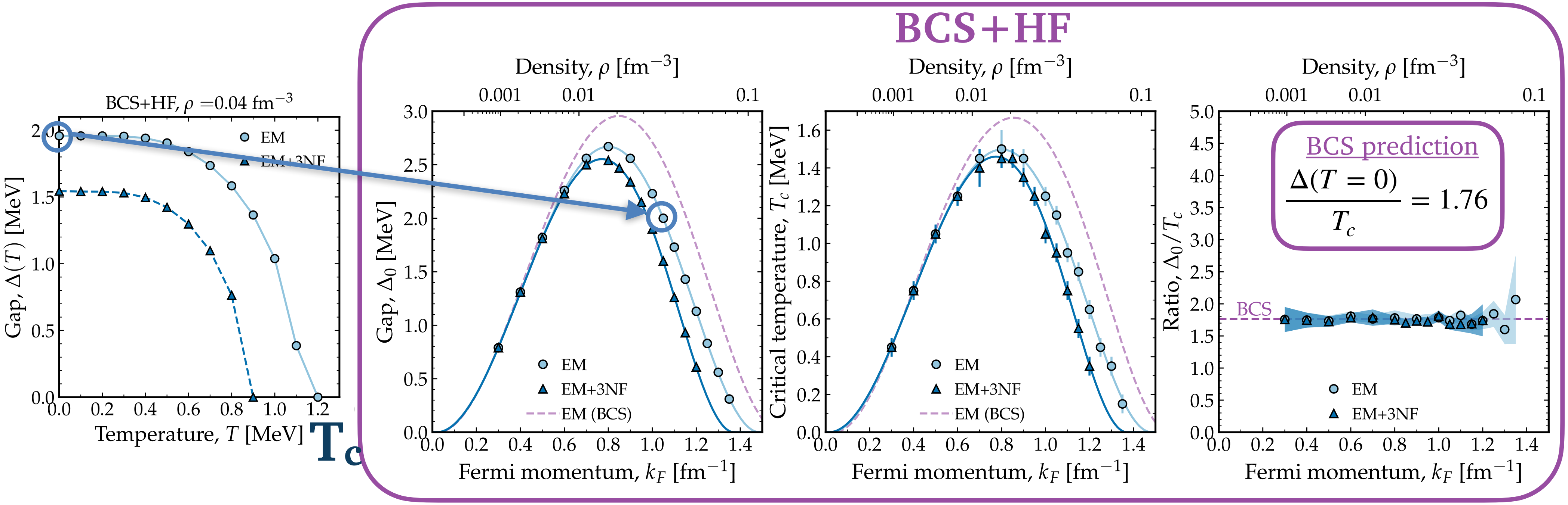

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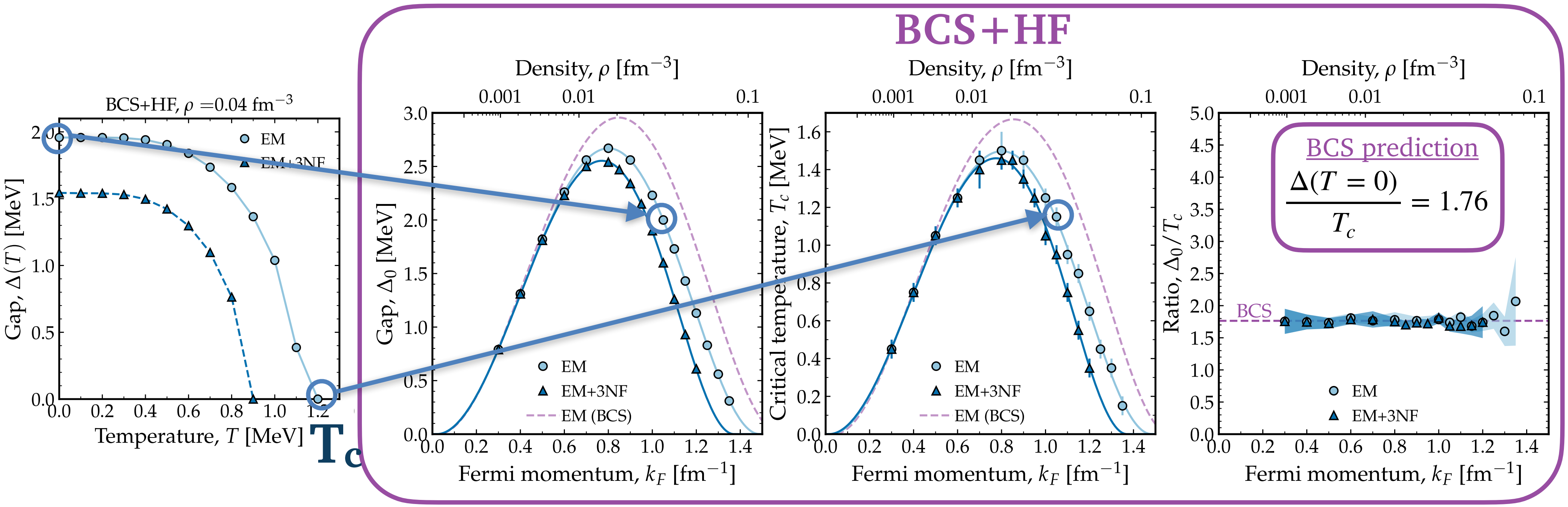

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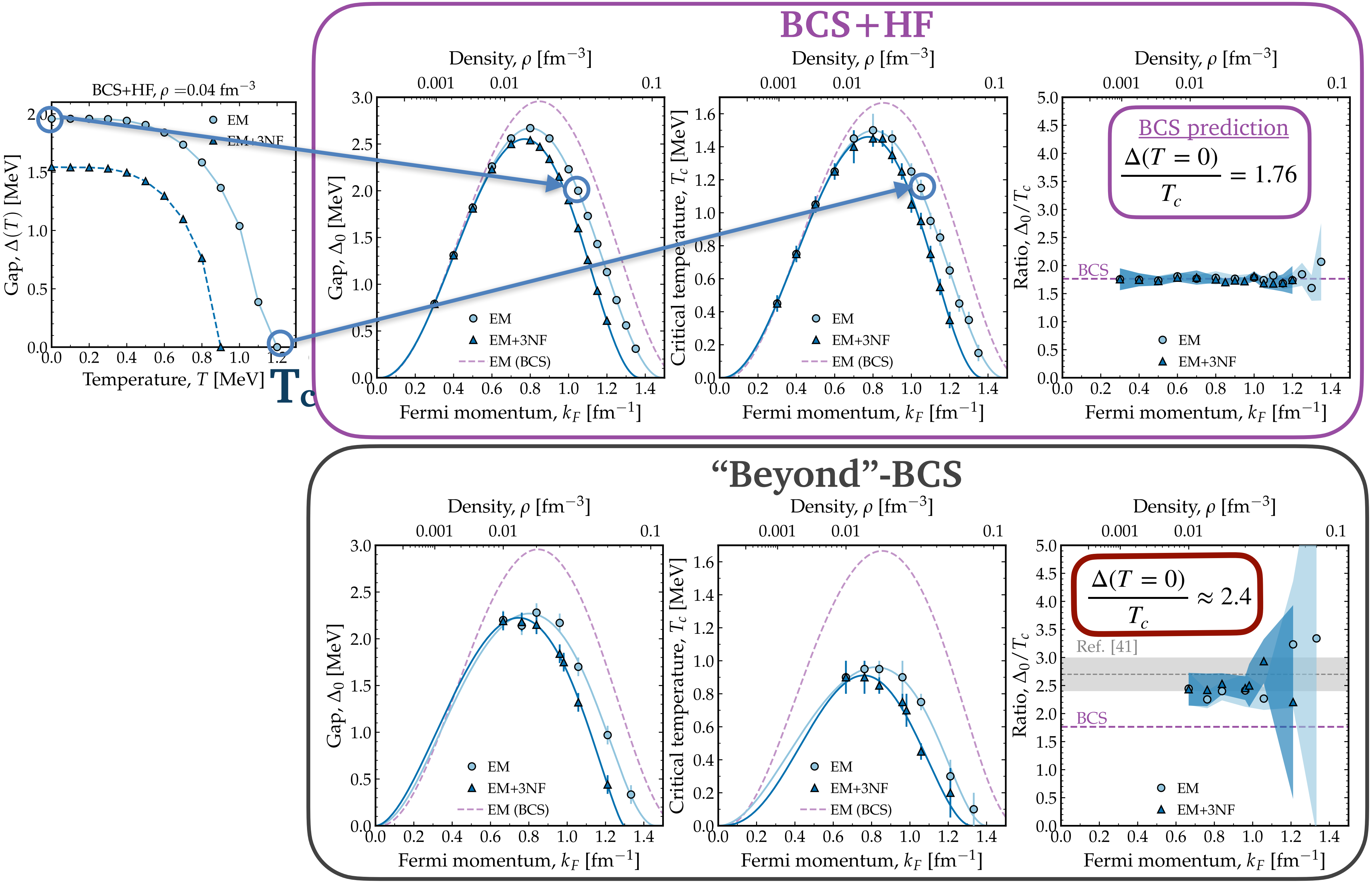
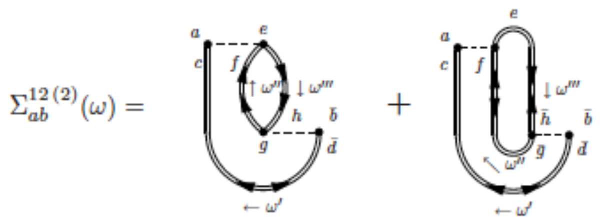
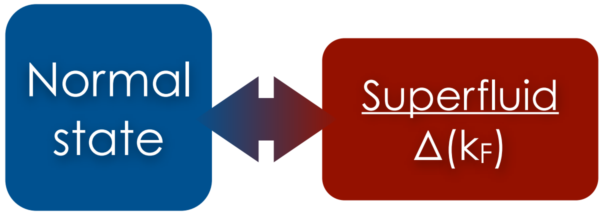


Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

How to go beyond?





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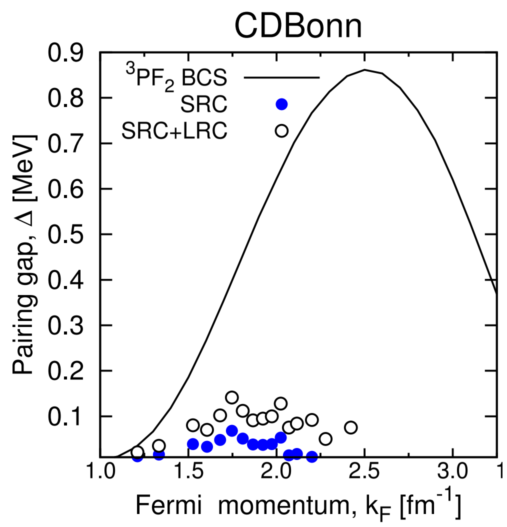

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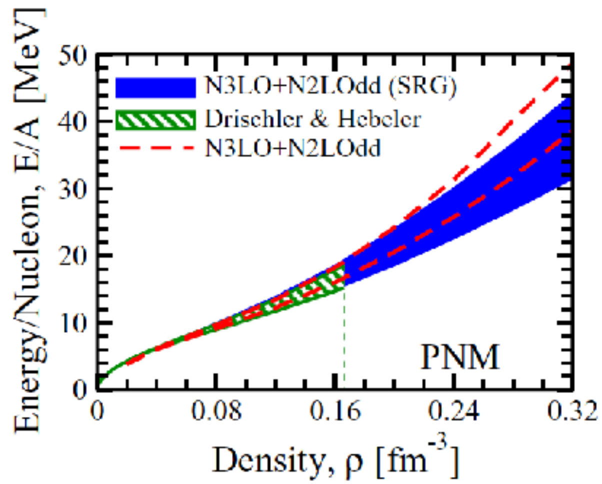
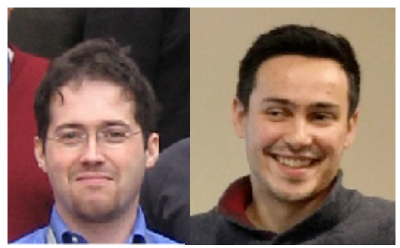
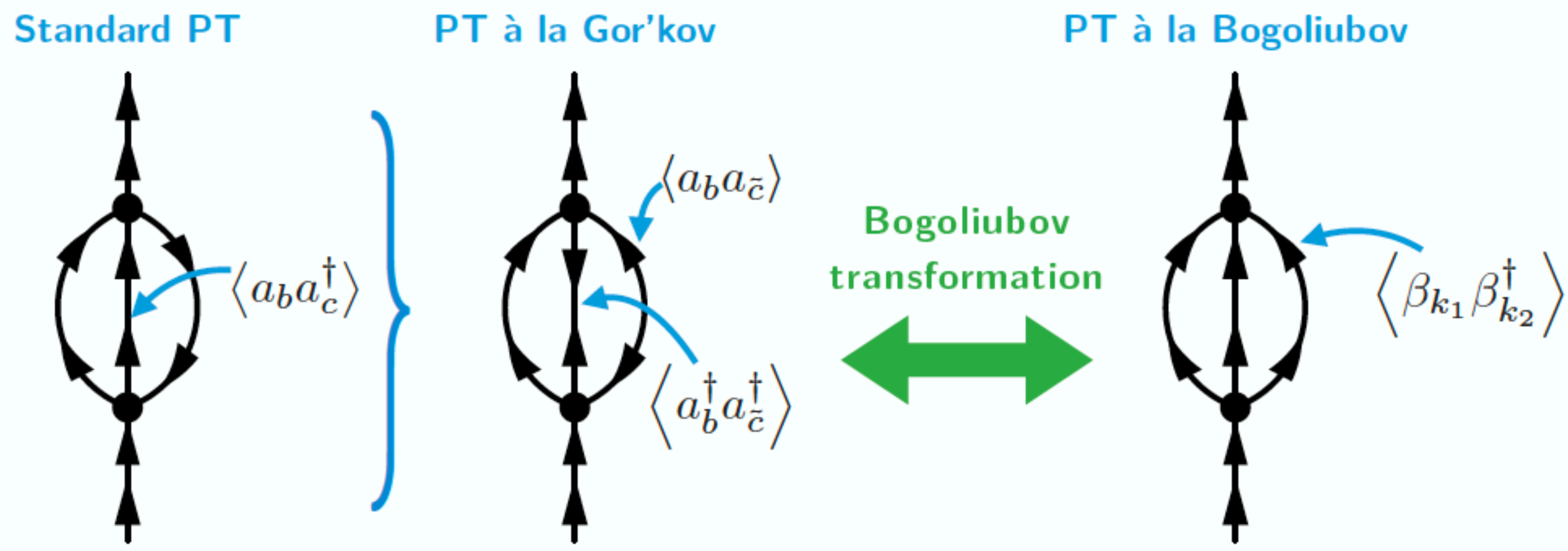


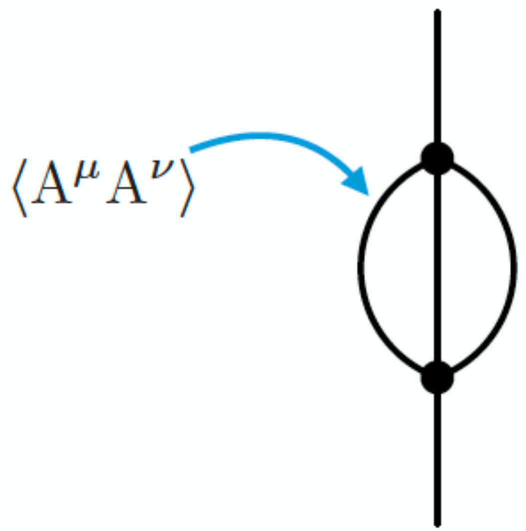
- **Existing frameworks** difficult to generalise
- **Nambu-covariant SCGF technique**
  - Symmetry breaking ✔
  - Finite temperature ✔
  - Systematic expansion w diagrams ✔
  - 3 nucleon forces ✔



Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

What was the issue before?





Reformulation

- Based on Nambu fields
- Propagators transform contravariantly
- Vertices transform covariantly
- Un-oriented diagrammatic

Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Nambu-Covariant Perturbation Theory

Double dimension H space

$$\mathscr{H}_1^e \equiv \mathscr{H}_1 \times \mathscr{H}_1^\dagger$$

Basis

$$\mathcal{B}^e \equiv \mathcal{B} \cup \bar{\mathcal{B}}$$

$|b\rangle$

$\langle \bar{b}|$

Elements

$$\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}$$

$$\mu = (b, l) \qquad \bar{\cdot} : 1 \mapsto \bar{1} = 2$$

$$\bar{\mu} \equiv (b, \bar{l}) \qquad 2 \mapsto \bar{2} = 1$$

$$|b,1\rangle \equiv \begin{pmatrix} |b\rangle \\ 0 \end{pmatrix} \qquad |b,2\rangle \equiv \begin{pmatrix} 0 \\ \langle \bar{b}| \end{pmatrix}$$

Product & metric tensor

$$g\left(\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}, \begin{pmatrix} |\Psi_2\rangle \\ \langle \Psi_2'| \end{pmatrix} \right) \equiv \langle \Psi_2' \,|\, \Psi_1 \rangle + \langle \Psi_1' \,|\, \Psi_2 \rangle$$

$$g_{\mu\nu} \equiv g\left(|\mu\rangle, |\nu\rangle \right) = \delta_{\mu\bar{\nu}}$$

Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Nambu fields

Nambu fields

$$\mathrm{A}^{(b,1)} \equiv a_b \,, \qquad \mathrm{A}^{(b,2)} \equiv \bar{a}_b \,, \qquad \bar{\mathrm{A}}_{(b,1)} \equiv \bar{a}_b \,, \qquad \bar{\mathrm{A}}_{(b,2)} \equiv a_b \,.$$

$$\mathrm{A}^{\mu} \equiv A^{(b,g)} = \begin{pmatrix} a_b \\ a_{\bar{b}}^{\dagger} \end{pmatrix}$$

$$\bar{\mathrm{A}}_{\mu} \equiv \bar{A}_{(b,g)} = \begin{pmatrix} a_{\bar{b}}^{\dagger} & a_b \end{pmatrix}$$

$$\bar{a}_b = a_{\bar{b}}^{\dagger} \neq a_b^{\dagger}$$

Commutator relations

(On extended indices!)

$$\{\, \mathrm{A}^{\mu}, \mathrm{A}^{\nu} \,\} = g^{\mu\nu} \,,$$

$$\{\, \mathrm{A}^{\mu}, \bar{\mathrm{A}}_{\nu} \,\} = g^{\mu}{}_{\nu} \,,$$

$$\{\, \bar{\mathrm{A}}_{\mu}, \mathrm{A}^{\nu} \,\} = g_{\mu}{}^{\nu} \,,$$

$$\{\, \bar{\mathrm{A}}_{\mu}, \bar{\mathrm{A}}_{\nu} \,\} = g_{\mu\nu}$$

Co- or contravariant

$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu\nu} \, \mathrm{A}^{\nu} \,,$$

$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu}{}^{\nu} \, \bar{\mathrm{A}}_{\nu} \,,$$

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Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Why Bogoliubov tensor algebra?

Tensor product: $r^{\mu_1}{}_{\nu_1}{}^{\mu_2\mu_3} = s^{\mu_1}{}_{\nu_1} \; t^{\mu_2\mu_3}$

Tensor contraction: $r^{\mu}{}_{\nu} = \sum_{\alpha} s^{\mu}{}_{\alpha} \; t^{\alpha}{}_{\nu}$

- Co(contra-)variance under Bogoliubov transforms provide **invariant** expressions **in any basis**
- Potential to **optimise** the extended basis
- **Tensor-network** structure becomes **transparent**
- Leads to **diagrammatic** expansion (à la de Dominicis-Martin or Haussmann)
- Other **formalisms** through specific basis or metric

Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Perturbative expansion

Hamiltonian partitioning

$$\Omega = \Omega_0 + \Omega_1$$

$$\Omega_0 = \frac{1}{2} \sum_{\mu\nu} U_{\mu\nu} \mathrm{A}^\mu \mathrm{A}^\nu$$

$$\Omega_1 = \sum_{k=1}^{n} \frac{1}{(2k)!} \sum_{\mu_1 \dots \mu_{2k}} v^{(k)}_{\mu_1 \dots \mu_{2k}} \mathrm{A}^{\mu_1} \dots \mathrm{A}^{\mu_{2k}}$$

Covariant **k-body** vertices

Fully antisymmetric vertex

- **Definition**

$$v^{(k)}_{[\mu_1\, \mu_2\, \dots\, \mu_{2k-1}\, \mu_{2k}]} \equiv \frac{1}{(2k)!} \sum_{\sigma \in S_{2k}} \epsilon(\sigma)\, v^{(k)}_{\mu_{\sigma(1)}\, \mu_{\sigma(2)}\, \dots\, \mu_{\sigma(2k-1)}\, \mu_{\sigma(2k)}}$$

- **Antisymmetrisation** defines a new *(0,2k)*-tensor
- ***Not*** the case in a *mixed* representation

Green's functions

- Contravariant **k-body** Green's function

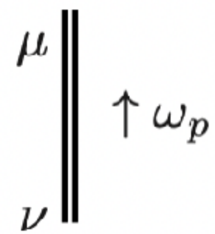
$$(-1)^k\, \mathscr{G}^{\mu_1 \dots \mu_{2k}}(\tau_1, \dots, \tau_{2k}) \equiv \left\langle \mathrm{T}\left[\mathrm{A}^{\mu_1}(\tau_1)\ \dots\ \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right]\right\rangle$$

with $\langle\, .\, \rangle = \mathrm{Tr}\left(\, .\, \rho\right)$ and $\rho \equiv \dfrac{e^{-\beta\Omega}}{\mathrm{Tr}\left(e^{-\beta\Omega}\right)}$

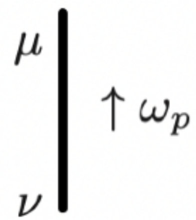
- Unperturbed case: $\Omega \longleftrightarrow \Omega_0$

Propagators

$$-\mathscr{G}^{\mu\nu}(\omega_p) =$$



$$-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$$



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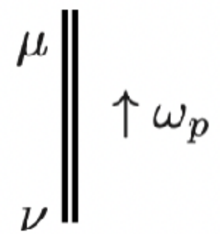
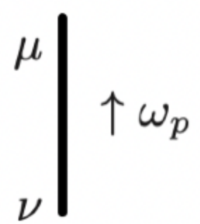
$-\mathscr{G}^{\mu\nu}(\omega_p) =$ 

$-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$ 

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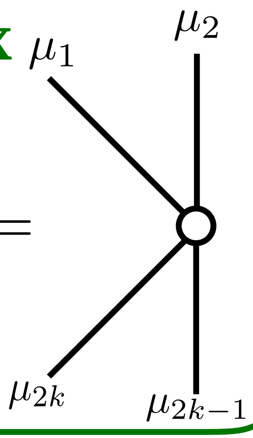
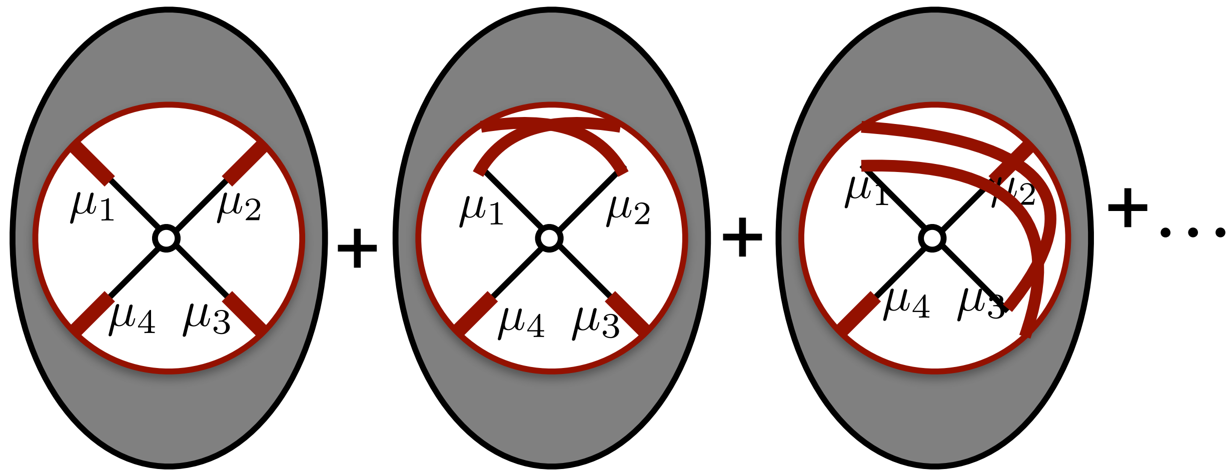
Why antisymmetric vertices?

Un-symmetrised vertex

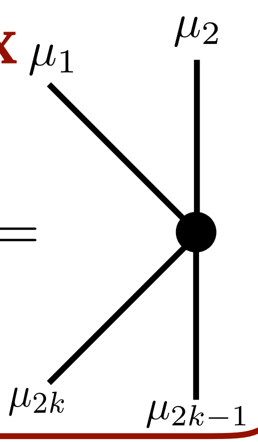
$$v^{(k)}_{\mu_1\mu_2\ldots\mu_{2k-1}\mu_{2k}} =$$





Antisymmetrized vertex

$$v^{(k)}_{[\mu_1\mu_2\ldots\mu_{2k-1}\mu_{2k}]} =$$



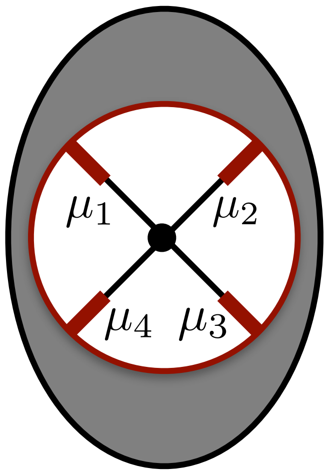


Diagram factorisation

- Derivations rely on
  - Wick theorem ⇒ sum over pairing
  - Sum over single-particle and Nambu indices

➡ Extends Hugenholtz antisymmetrisation

- Antisym is a **one-off pre-computing** cost

Drissi, Rios & Barbieri, Paper I, arxiv:2107.09759

Perturbative expansion

Order n graphical rules

- Draw all topologically distinct **connected unlabelled** diagrams
  - with ***2k* external** legs
  - with ***n* vertices** (for order n contributions)

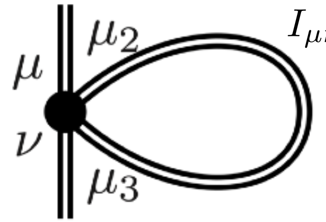
Feynman rules

1. Label vertices from *1* to n
   - S is the number of vertex labels permutations leaving the diagram invariant
2. For each line multiply by $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)$
3. For each k-body vertex multiply by $v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]}$
4. Sum over each internal μ index and each independent ω_e frequency
5. Multiply by $\dfrac{(-1)^{n+L}}{S\times 2^T\ \prod_{l=2}^{l_{\max}}(l!)^m}$

Gaudin rules

- These simplify **Matsubara sums**
- Require **spanning trees**

Tadpoles are exceptional



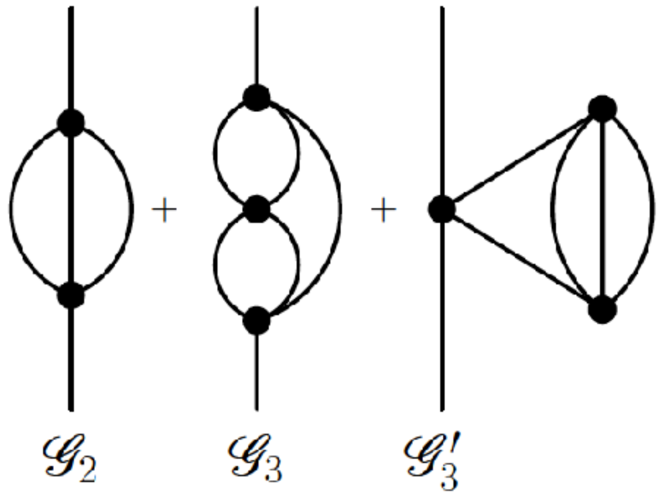
$$I_{\mu\nu} = \sum_{\mu_2\dots\mu_{2k-1}} \frac{(-1)^k}{2^{k-1}(k-1)!}\, v^{(k=2)}_{[\mu\ \dot\mu_2\ \dot\mu_3\ \nu]} \times \frac{1}{\beta}\sum_{\omega_e} -\mathcal{G}^{\mu_2\mu_3}(\omega_e)\, e^{-i\omega_e\eta_p}$$

- **Partially antisymmetrized** vertices needed:

$$v^{(k)}_{[\mu_1\dots\dot\mu_x\dots\dot\mu_y\dots\mu_{2k}]} \equiv \frac{2^p p!}{(2k)!} \sum_{\sigma\in S_{2k}/S_2^p\times S_p} \epsilon(\sigma)\, v^{(k)}_{\mu_{\sigma(1)}\dots\dot\mu_x\dots\dot\mu_y\dots\mu_{\sigma(2k)}}$$

- p internal lines are **fixed**
- ***k*-body generalisation** works

HFB partitioning 3rd order

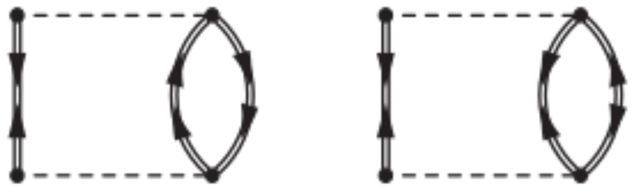


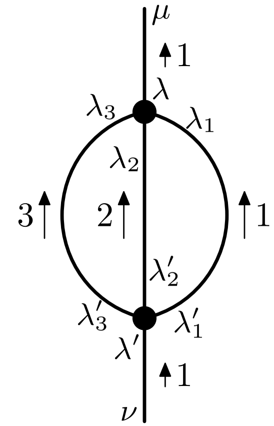
Advantages vs Gorkov

Gorkov GF | NCGF

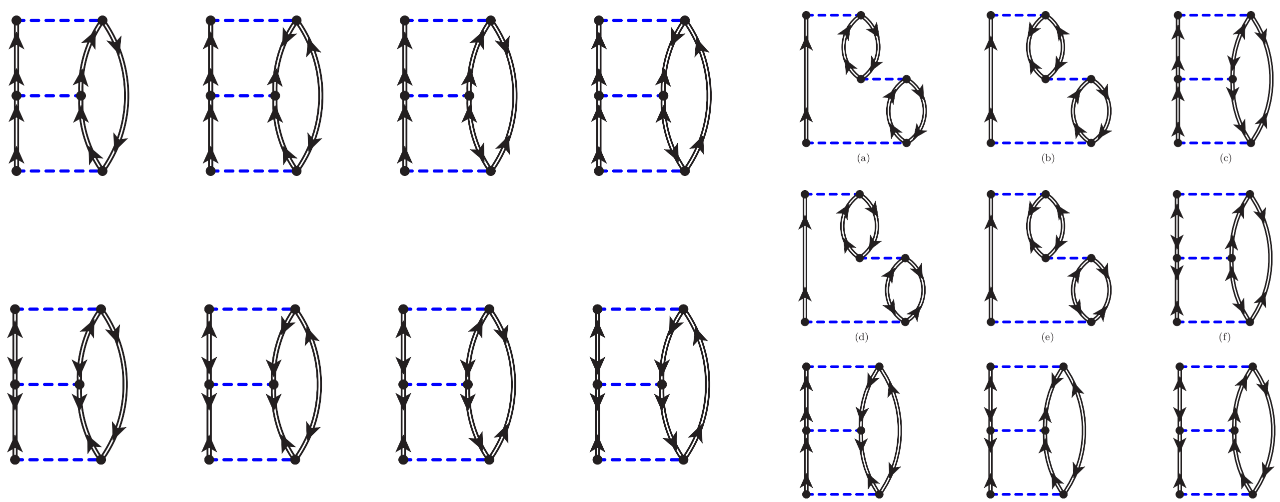
Order 2

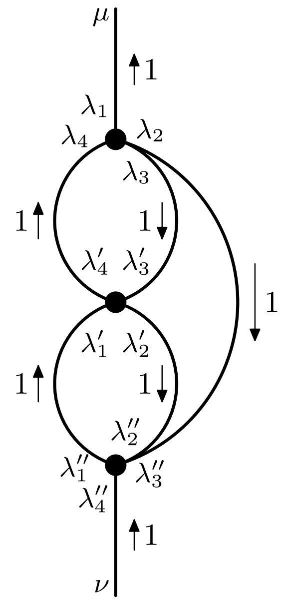
$$\Sigma^{11\,(2)}_{ab}(\omega) =$$





Order 3





Self-consistent Green's function resummation

Dyson equation

Partitioning considered

$$\Omega = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\ A^{\mu}A^{\nu}}_{\Omega_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\ A^{\alpha}A^{\beta}A^{\gamma}A^{\delta}}_{\Omega_1}$$

Dyson equation

$$\mathscr{G}^{\mu\nu}(\omega_n) = \mathscr{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathscr{G}^{(0)\mu\lambda_1}(\omega_n)\ \Sigma_{\lambda_1\lambda_2}(\omega_n)\ \mathscr{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion of $\Sigma_{\mu\nu}(\omega_n)$

with unperturbed propagators

$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{J}_{\mu\nu}(\omega_n) - \mathscr{J}_{\nu\mu}(-\omega_n)}{2}$$

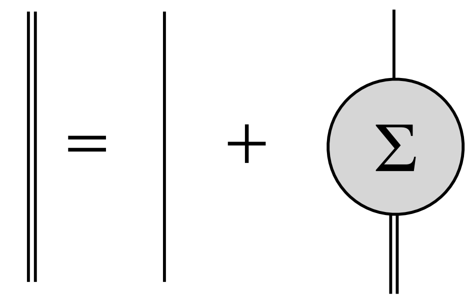
$$\mathscr{J}_{\mu\nu}(\omega_n) = \sum \text{1PI diagrams with } \mathscr{G}^{(0)}$$

with self-consistent propagators

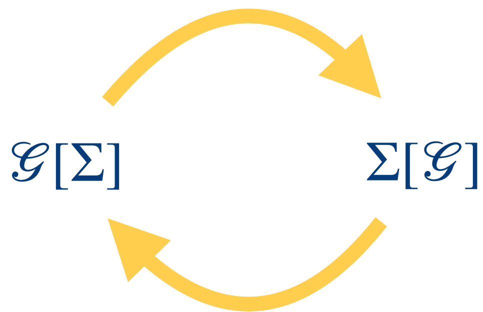
$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{J}_{\mu\nu}(\omega_n) - \mathscr{J}_{\nu\mu}(-\omega_n)}{2}$$

$$\mathscr{J}_{\mu\nu}(\omega_n) = \sum \text{2PI diagrams with } \mathscr{G} \left(= \mathscr{J}_{\mu\nu}(\omega_n)\right)$$

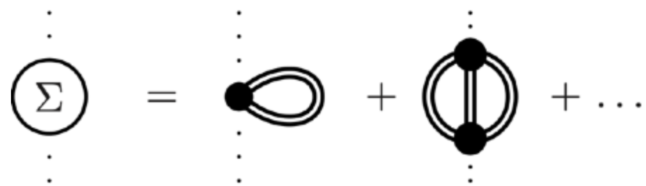
Diagrammatic representation



SCGF cycle



Self-energy expression

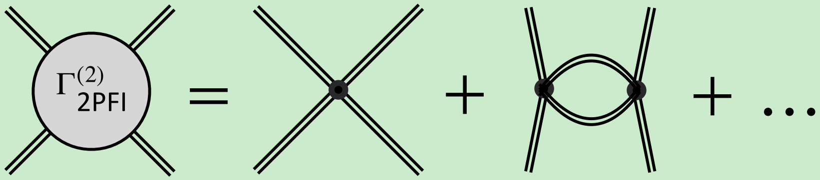


Drissi, Rios & Barbieri, Paper II, arxiv:2107.09763

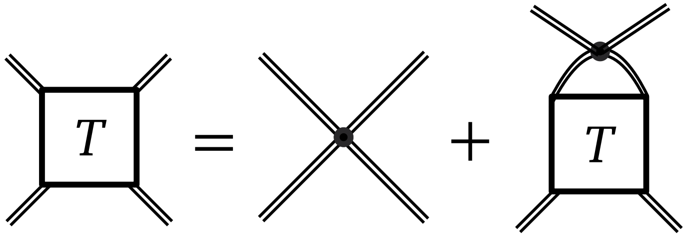
T-matrix: ladders

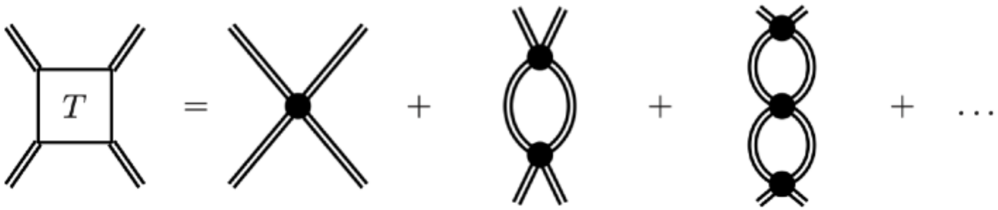
Approximations on $\Gamma^{(2)}_{\mathrm{2PFI}}$

- Sum of all possible rungs



T-matrix $\equiv \Gamma^{(2)}$ in ladder approximation





Ladder approximation

- Analytic/Retarded/Advanced/Sp function ⇒ as usual
- <u>T-matrix equation</u>

$$T_{MN}(Z) \;=\; V^{(2)}_{MN} \;+\; \frac{1}{2}\sum_{LL'} V^{(2)}_{ML}\,\Pi^{LL'}(Z)\,T_{L'N}(Z)$$

where $V^{(2)}_{MN} \equiv v^{(2)}_{[\mu_1\mu_2\nu_1\nu_2]}$, $M \equiv (\mu_1,\mu_2)$ & $N \equiv (\nu_1,\nu_2)$

Solving the ladder

- <u>Spectral representation</u>

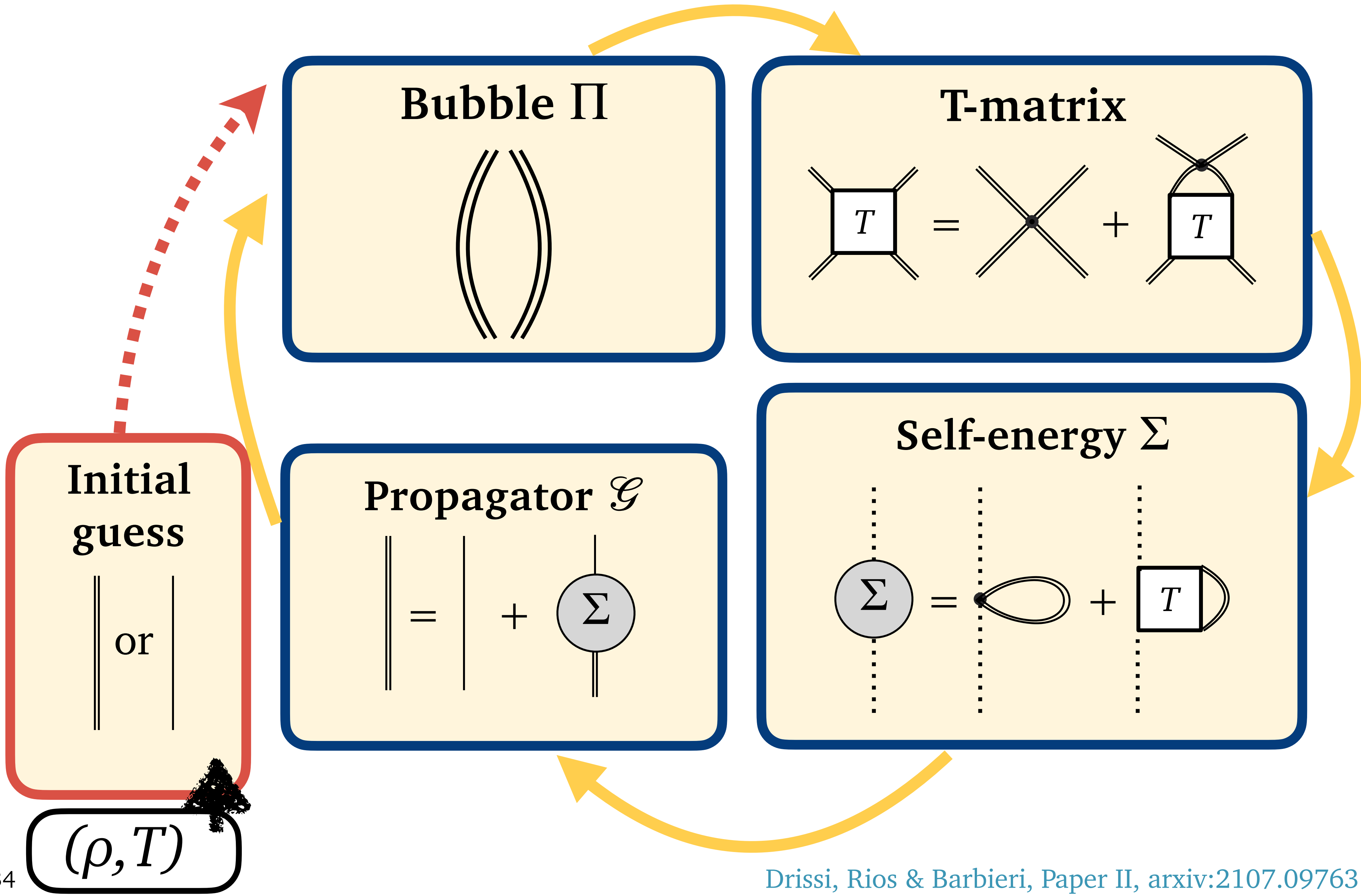
$$T_{MN}(Z) \equiv V^{(2)}_{MN} + \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi}\,\frac{T_{MN}(\Omega)}{Z-\Omega}$$

- <u>Solution</u>

$$\mathcal{T}(\Omega) = iV^{(2)}\left\{\left(gg - \frac{1}{2}\Pi^{R}(\Omega)V^{(2)}\right)^{-1} - \left(gg - \frac{1}{2}\Pi^{A}(\Omega)V^{(2)}\right)^{-1}\right\}$$

Drissi, Rios & Barbieri, Paper II, arxiv:2107.09763

Nambu-Covariant Ladders



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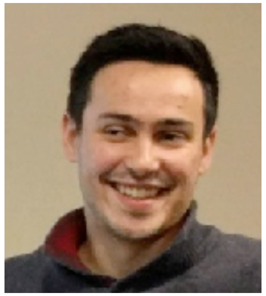

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