Neutron star matter: learning from collective modes

Constança Providência

Universidade de Coimbra, Portugal

CELEBRATING MARCELLO BALDO'S 80TH BIRTHDAY
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Motivation

The thermal evolution of neutron stars depends on the elementary excitations affecting the stellar matter. In particular, the low-energy excitations, whose energy is proportional to the transferred momentum, can play a major role in the emission and propagation of neutrinos.

`in Baldo & Ducoin, PRD84,035806,2011`

Works of Marcelo Baldo and Camille Ducoin

- Elementary excitations in homogeneous neutron star matter （Phys.Rev.C 79 (2009) 035801）
- Elementary excitations in homogeneous superfluid neutron star matter: Role of the proton component （Phys.Rev.C 84 (2011) 035806）
- Elementary excitations in homogeneous superfluid neutron star matter: role of the neutron-proton coupling （Phys.Rev.C96 (2017) 2, 025811）
- Coupling between superfluid neutrons and superfluid protons in the elementary excitations of neutron star matter （ Phys.Rev.C 99 (2019) 2, 025801）

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Motivation

What information can we get from the collective modes of nuclear matter?

- defining the crust core transition
- nuclear modes of infinite matter
- coupling to the plasmon mode

EOS: relativistic mean field description

RMF Lagrangian for stellar matter

- Lagrangian density: causal Lorentz-covariant Lagrangian (baryon densities and meson fields)

$$\mathcal{L}_{NLWM} = \sum_{B=baryons} \mathcal{L}_B + \mathcal{L}_{mesons} + \mathcal{L}_l + \mathcal{L}_\gamma,$$

- Baryonic contribution: $\mathcal{L}_B = \bar{\psi}_B \left[\gamma_\mu D_B^\mu - M_B^*\right] \psi_B,$
  $D_B^\mu = i\partial^\mu - g_{\omega B}\omega^\mu - \frac{g_{\rho B}}{2}\boldsymbol{\tau} \cdot \mathbf{b}^\mu$
  $M_B^* = M_B - g_{\sigma B}\sigma$

- Meson contribution

$$\mathcal{L}_{mesons} = \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{non-linear}$$

- Lepton contribution: homogeneous matter
  $\mathcal{L}_l = \sum_l \bar{\psi}_l \left[\gamma_\mu i\partial^\mu - m_l\right] \psi_l$

- Electromagnetic contribution: $\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

- Electron contribution: $\mathcal{L}_e = \bar{\psi}_e \left[\gamma_\mu \left(i\partial^\mu + eA^\mu\right) - m_e\right] \psi_e$

EOS: relativistic mean field description

Density dependence of the EOS determined by introducing

- non-linear meson terms (Boguta&Bodmer 1977, Mueller&Serot 1996)
- NL3, NL3$\omega\rho$, TM1, TM1$\omega\rho$, FSU, FSU2, FSU2R

$$\mathcal{L}_{non-linear} = -\frac{1}{3} b g_\sigma^3 (\sigma)^3 - \frac{1}{4} c g_\sigma^4 (\sigma)^4 + \frac{\xi}{4!} (g_\omega \omega_\mu \omega^\mu)^4 + \Lambda_\omega g_\varrho^2 \boldsymbol{\varrho}_\mu \cdot \boldsymbol{\varrho}^\mu g_\omega^2 \omega_\mu \omega^\mu ,$$

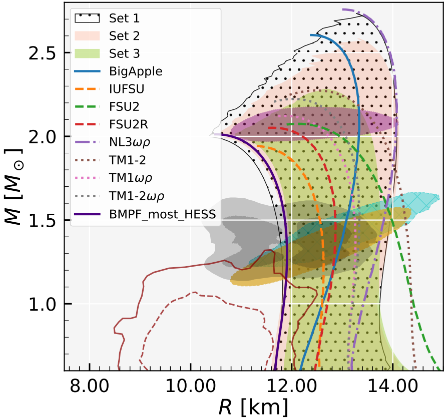
  - Parameters: $g_i (i = \sigma, \omega, \rho)$, b, c, ξ, Λ_ω (Malik arxiv:2301.08169)
- Bayesian estimation of model parameters

Spanning the full range of NS properties with a microscopic model

Malik ApJ930 17, Malik arxiv: 2301.08169

Constraints				
Quantity		Value/Band	Ref	DDB
NMP (MeV)	ρ_0	0.153 ± 0.005	Typel & Wolter (1999)	✓
	ϵ_0	-16.1 ± 0.2	Dutra et al. (2014)	✓
	K_0	230 ± 40	Todd-Rutel & Piekarewicz (2005); Shlomo et al. (2006)	✓
	$J_{\mathrm{sym},0}$	32.5 ± 1.8	Essick et al. (2021a)	✓
PNM (MeV fm^{-3})	$P(\rho)$	$2 \times \mathrm{N^3LO}$	Hebeler et al. (2013)	✓
NS mass ($M_\odot$)	$M_{\max}$	>2.0	Fonseca et al. (2021)	✓

NS properties: full posterior NL



- **Observations:** GW170817, NICER J0740 and J0030, HESS
- **RMF models:** NL3$\omega\rho$,FSU2, FSU2R, IUFSU, BigApple, TM1-2($\omega\rho$)
- **Bayesian study Left**: Set 1 ($\xi < 0.004$), 2, 3 ($\xi > 0.015$)

Density modes

Small perturbation of the nuclear system

- Semiclassical description: Vlasov equation for the distribution functions determine time evolution.
- Small oscillations around an equilibrium are considered
- Equilibrium state characterized by: P_{Fn}, P_{Fp}, P_{Fe}
- Charge neutrality: $P_{Fe} = P_{Fp}$
- Perturbed fields: $F_i = F_{i0} + \delta F_i$,
- Perturbed distribution function: $f = f_0 + \delta f$,

$$\delta f_i = \{S_i, f_{0i}\}$$

- Generating function: $S(\mathbf{r}, \mathbf{p}, t) = \mathrm{diag}\,(S_p, S_n, S_e)$,

Linearized Equations of Motion

- The time evolution of f_i is described by the Vlasov equation

$$\frac{\partial f_i}{\partial t} + \{f_i, h_i\} = 0, \qquad i = p,\ n,\ e$$

- The linearized relativistic Vlasov equation

$$\frac{d\mathcal{S}_i}{dt} + \{\mathcal{S}_i, h_{0i}\} = \delta h_i$$

$$\delta h_e = -e\left[\delta A_0 - \frac{\mathbf{p}\cdot\delta\mathbf{A}}{\epsilon_{0e}}\right],$$

$$\delta h_i = -g_s\delta\phi\frac{M^*}{\epsilon_0} + \delta\mathcal{V}_{0i} - \frac{\mathbf{p}\cdot\delta\boldsymbol{\mathcal{V}}_i}{\epsilon_0}, \quad i = p, n$$

- Longitudinal fluctuations:

$$\begin{pmatrix} S_i & \delta F_j & \delta\rho_i & \delta h_i \end{pmatrix} = \begin{pmatrix} S_{\omega,i}(x) & \delta F_{\omega,j} & \delta\rho_{\omega,i} & \delta h_{\omega,i} \end{pmatrix} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$$

$x = \cos(\mathbf{p}\cdot\mathbf{q})$,
$i = e, p, n$,
$j = \sigma, \omega, \rho, \gamma$

Dispersion relations

- In terms of the transition densities: $\delta\rho_i = \frac{3}{2}\frac{k}{P_{Fi}}\rho_{0i}A_{\omega i}$,

$$\begin{pmatrix} 1+F^{pp}L_p & F^{pn}L_p & C_A^{pe}L_p \\ F^{np}L_n & 1+F^{nn}L_n & 0 \\ C_A^{ep}L_e & 0 & 1-C_A^{ee}L_e \end{pmatrix} \begin{pmatrix} A_{\omega p} \\ A_{\omega n} \\ A_{\omega e} \end{pmatrix} = 0,$$

- Lindhard function, speed of sound:

$L(s_i) = 2 - s_i \ln\left(\frac{s_i+1}{s_i-1}\right)$, with $(s_i = \omega/\omega_{oi} = \omega/(k\,V_{Fi})$, $V_{F_i} = \frac{P_{F_i}}{\epsilon_{F_i}}$

$F^{ij} = C_s^{ij} - C_v^{ij} - \tau_i\,\tau_j\,C_\rho^{ij} - C_A^{ij}\delta_{ip}\delta_{jp}, \quad i,j = n,p,$

- dispersion relation

$$[1-C_A^{ee}L_e]\,[1+L_pF^{pp}+L_nF^{nn}+L_pL_n(F^{pp}F^{nn}-F^{pn}F^{np})]$$
$$-C_A^{ep}C_A^{pe}L_eL_p(1+L_nF^{nn}) = 0.$$

Dispersion relations

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- dispersion relation

$$[1 - C_A^{ee}L_e]\,[1 + L_pF^{pp} + L_nF^{nn} + L_pL_n\,(F^{pp}\,F^{nn} - F^{pn}\,F^{np})]$$
$$-C_A^{ep}C_A^{pe}L_eL_p(1 + L_nF^{nn}) = 0.$$

- Equivalent to $\Delta = det[Re(1 - \Pi_0 \mathbf{v}_{res}] = 0$

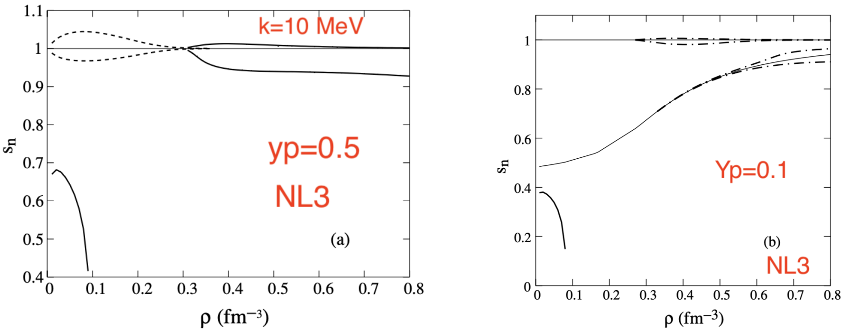
Solutions of dispersion relation

- pair of branches associated with each fluid if the interaction between particles is repulsive enough, and interfluid interaction not too strong
- pair of branches are above and below the $\omega = qV_{Fi}$, the lower branch is strongly damped
- a single branch identifies an unstable mode: it defines the spinodal region

Collective modes

NL3

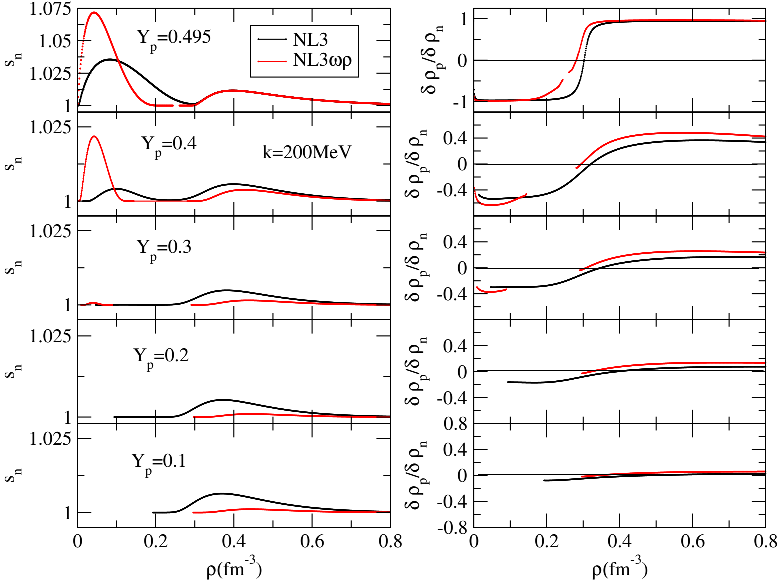
- NL3, $y_p = 0.5, 0.1$



$$s_n = v_s / v_{Fn}$$

Collective modes in asymmetric pn matter

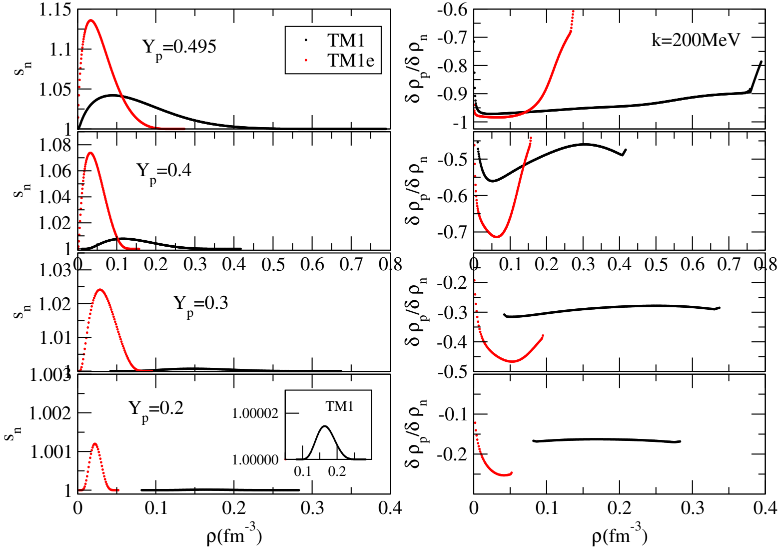
NL3



$$s_n = v_s / v_{Fn}$$

Collective modes in asymmetric pn matter

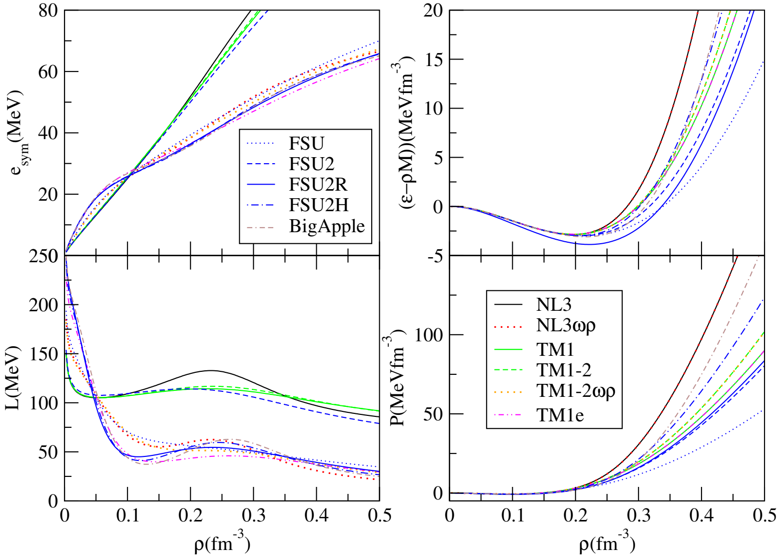
TM1



see also: Haensel, NPA301,53; Greco *et al* PRC67, 015203

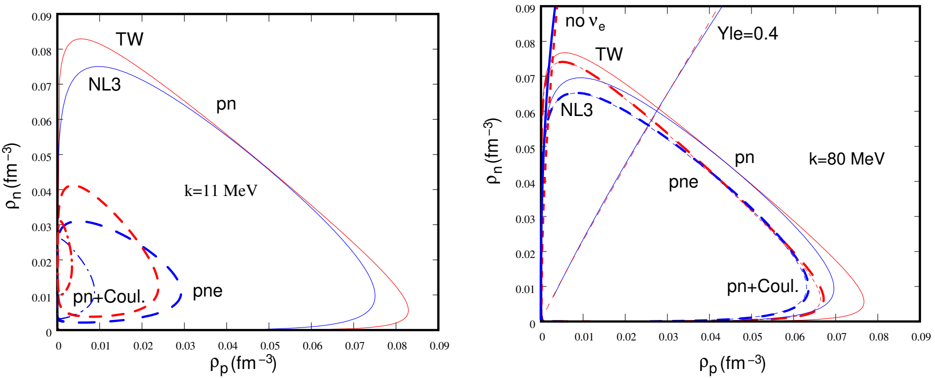
Collective modes in asymmetric pn matter

EoS behavior



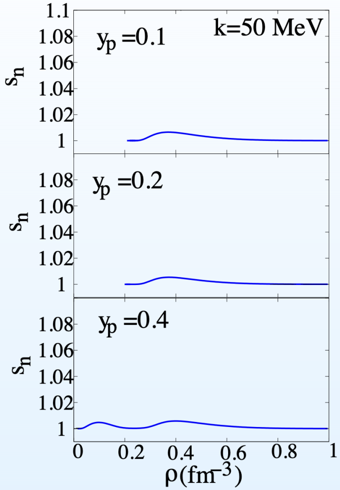
Unstable mode: dynamical Spinodal (pne vs pn)

NL3 versus TW, k=11 MeV versus 80 MeV



Collective models (pn)

- Sound velocities of the Collective Modes for different isospin asymmetries
- np matter



(Providência *et al* PRC74,045802(2006))

Collective models (pne)

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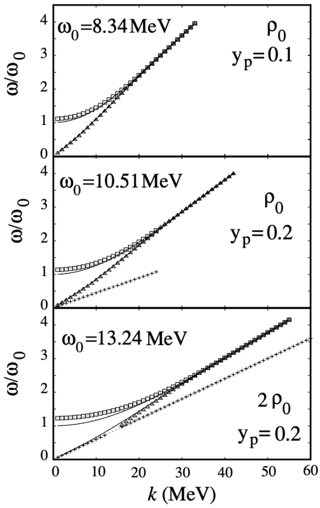
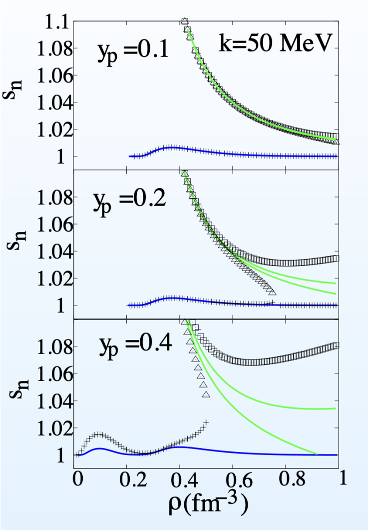




plasmon frequency: $\omega_0 = \sqrt{\frac{e^2 \rho_e}{\epsilon_{Fe}}}$

Multifluid RPA equations

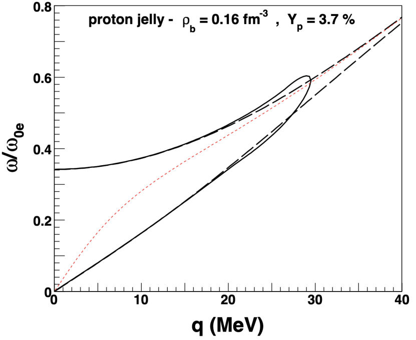
- RPA equations for N species in terms of polarization propagator

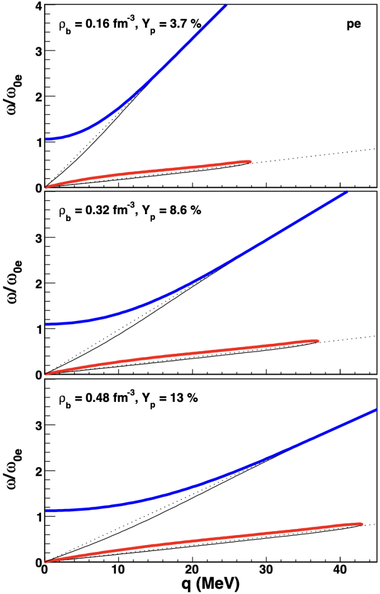
$$\Pi^{ik}(q,\omega) = \Pi_0^i(q,\omega)\left[\delta_{ik} + \sum_j v_{res}^{ij}\Pi^{jk}(q,\omega)\right], \quad i,k = 1,2,...,N$$

- Branches of the dispersion relation: Poles of the response function
- The strength function (-Im $\Pi^{ik}(q,\omega)$) gives information on the collectivity of the modes
  - Collective modes correspond to peaks of strength function
  - width of peak gives damping rate
- Micro: BHF with 2B Argonne v_{18} plus 3B Urban IX
- Skyrme: NRAPR and SLy230a

Collective modes (pe)

(Baldo&Ducoin PRC79,035801)



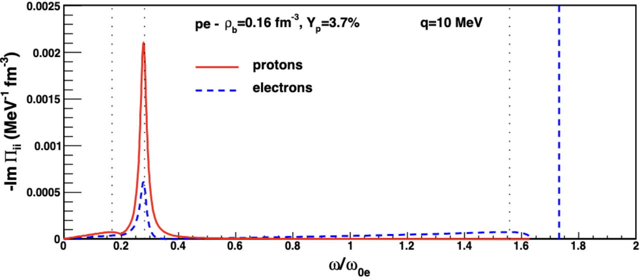


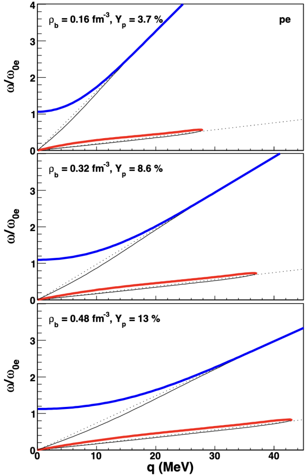
electron plasmon

electron-proton system

Collective modes (pe): spectral function

(Baldo&Ducoin PRC79,035801)

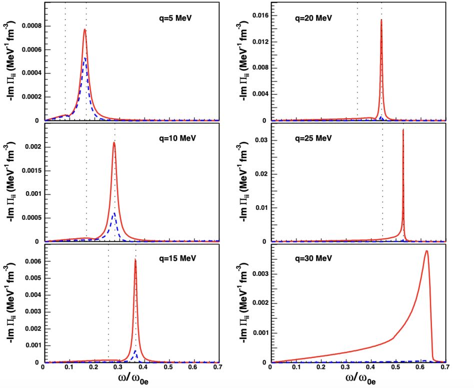


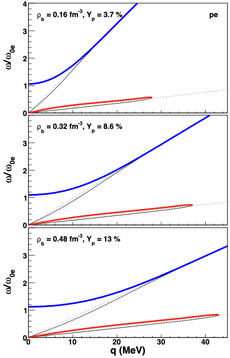


$det[Re(1 - \Pi_0 \mathbf{v}_{res}] = 0$ dotted lines $\omega = qV_{Fi}$

Collective modes (pe): spectral function

(Baldo&Ducoin PRC79,035801)

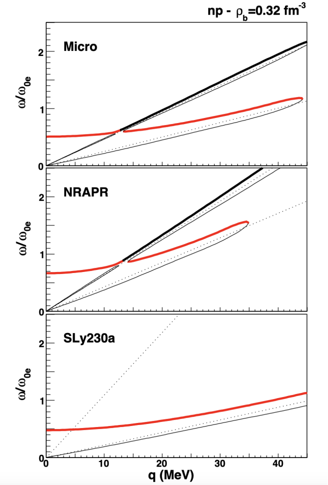




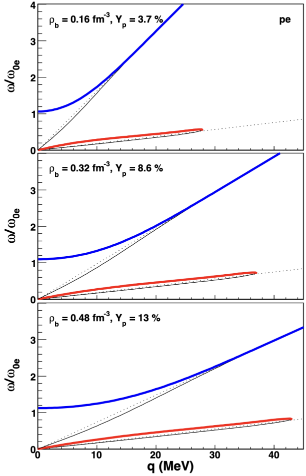
electron-proton system: spectral function

Collective modes (pn)

(Baldo&Ducoin PRC79,035801)



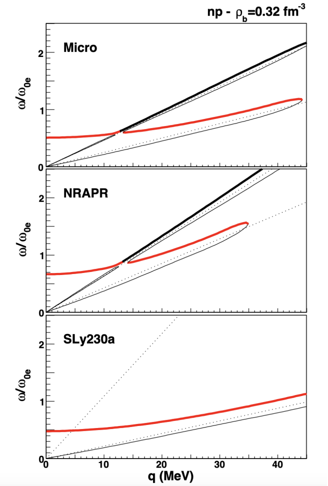
proton-neutron

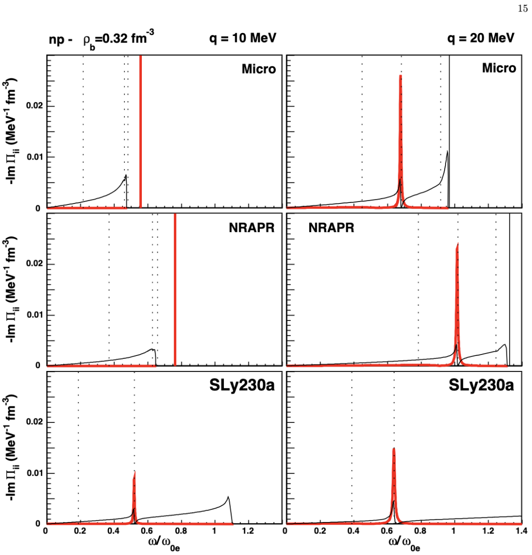


proton-electron

Collective modes (pn): spectral function

(Baldo&Ducoin PRC79,035801)

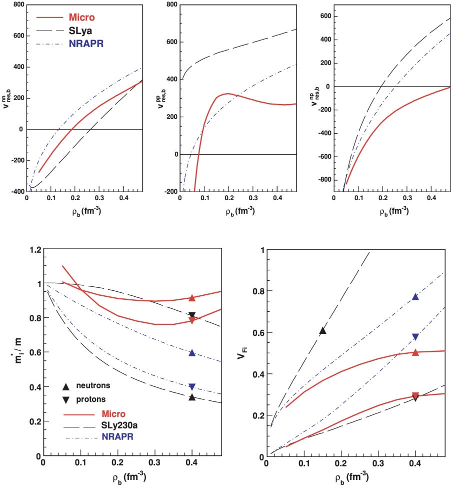




Model behavior is determined by quantities as the effective masses and residual interactions

Collective modes (pn): models

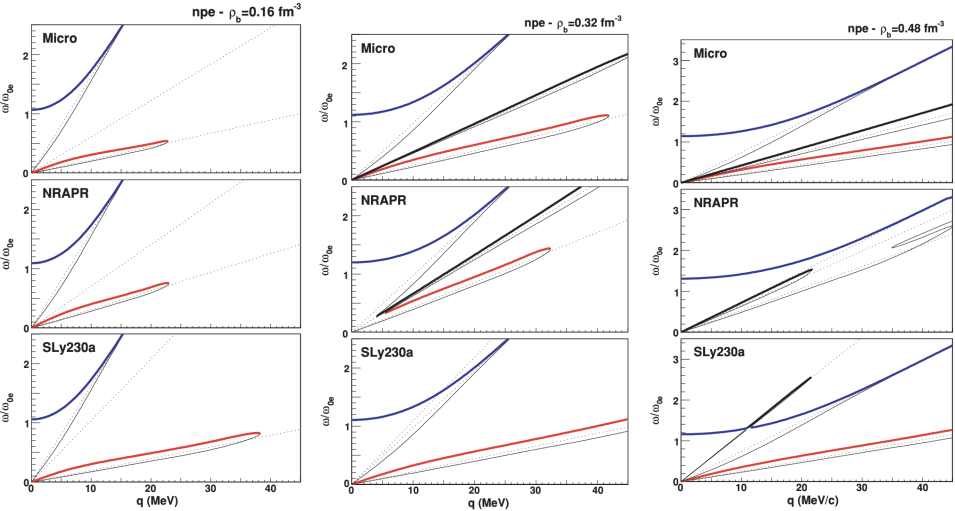
(Baldo&Ducoin PRC79,035801)



Model behavior is determined by quantities as the effective masses and residual interactions

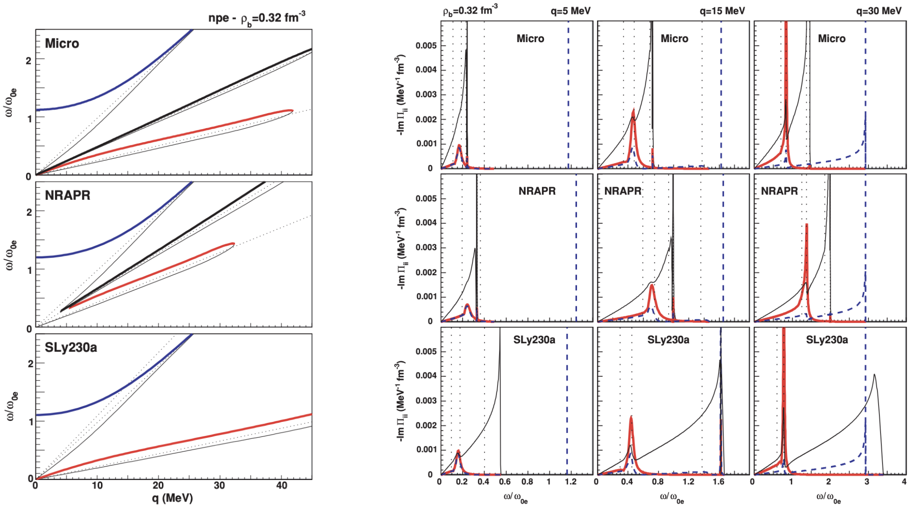
Collective modes (pne)

(Baldo&Ducoin PRC79,035801)



Collective modes (pne): spectral function

(Baldo&Ducoin PRC79,035801)



Conclusions

- the RPA formalism is essential to identify the collective modes, and their damping
- density modes in npe matter are extremely model dependent:
  - knowledge of high density behavior is necessary
  - can we learn about the high density behavior from the excitations?
- proton plasmon mode:
  always present due to repulsive Coulomb
  reduced to a sound like mode by electrons
  gets stronger for larger k - less e^- screening
  may merge with neutron mode
- Effect of superfluidity: PRC84, PRC96, PRC99
- Effect of magnetic fields?

Congratulations !