# The tune of Love and a near-zone symmetry resolution

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## Spherically symmetric and non-spinning compact body

- $\vec{\nabla}^2 \Phi_{\rm N} = 4\pi G \rho$

$$\Phi_{\rm N}\left(\omega, \vec{x}\right)\Big|_{R_\oplus \leq r} = -\frac{G M_\oplus}{r}$$

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Spherically symmetric and non-spinning compact body perturbed by pure quadrupolar tidal source

- $\vec{\nabla}^2\Phi_{\mathrm{N}} = 4\pi G\rho\,, \quad \bar{\mathcal{E}}^{☾}_{ij} = -3\frac{GM_{☾}}{(L^{☾})^5}L^{☾}_{\langle i}L^{☾}_{j\rangle}$

$$\Phi_{\mathrm{N}}\left(\omega,\vec{x}\right)\Big|_{R_\oplus\leq r\ll L^{☾}} = -\frac{GM_\oplus}{r} + \underbrace{\frac{1}{2}\bar{\mathcal{E}}^{☾}_{ij}\left(\omega\right)x^ix^j}_{\text{“source”}}$$

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$$\Phi_{\rm N}(\omega,\vec{x})\Big|_{R_\oplus\leq r\ll L^{☾}} = -\frac{GM_\oplus}{r} + \underbrace{\frac{1}{2}\bar{\mathcal{E}}^{☾}_{ij}(\omega)\,x^ix^j}_{\text{“source”}} \underbrace{-\frac{3}{2}G\frac{\delta Q^{\oplus}_{ij}(\omega)\,x^ix^j}{r^5}}_{\text{“response”}}$$

## Love numbers in Newtonian gravity A.E.H. Love (1912), Poisson & Will (2014)

Spherically symmetric and non-spinning compact body perturbed by pure quadrupolar tidal source

- Assuming weak, slowly-varying source and no hysteresis:

$$\delta Q_{ij}^{\oplus}(\omega) = -\frac{1}{3}\frac{R_{\oplus}^{5}}{G} k_2^{\oplus}(\omega)\, \bar{\mathcal{E}}_{ij}^{☾}(\omega) + \mathcal{O}\left(\bar{\mathcal{E}}^{☾\,2}\right)$$

- $k_2(\omega)$: Quadrupolar Tidal Response Coefficient

$$\delta\Phi_{\mathrm{N}}(\omega, \vec{x})\Big|_{R_{\oplus} \leq r \ll L^{☾}} = \frac{1}{2}\left[1 + k_2^{\oplus}(\omega)\left(\frac{R_{\oplus}}{r}\right)^{5}\right]\bar{\mathcal{E}}_{ij}^{☾}(\omega)\, x^i x^j$$

More background multipole moments, e.g. mountains, spin (multi-index notation: $L \equiv i_1 \ldots i_\ell$, $x^L \equiv x^{i_1} \ldots x^{i_\ell}$)

$$\delta\Phi_{\mathrm{N}}\left(\omega, \vec{x}\right)\Big|_{r\geq R} = \sum_{\ell,\ell'} \frac{(\ell-2)!}{\ell!}\left[\underbrace{\delta_{L,L'}}_{\text{“source”}} + \underbrace{k_{LL'}\left(\omega\right)\left(\frac{R}{r}\right)^{2\ell+1}}_{\text{“response”}}\right]\bar{\mathcal{E}}^{L'}\left(\omega\right)x^L$$

$$k^{\text{Love}}_{LL'}\left(\omega\right) \equiv k^{\text{cons.}}_{LL'}\left(\omega\right) \subset k_{LL'}\left(\omega\right)$$

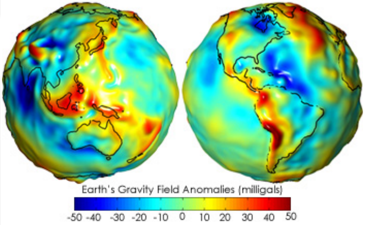
e.g. shape deformation Vs tidal locking

Axisymmetric spinning body with no $\ell$-mode mixing

$$k^{\text{Love}}_{\ell m}\left(\omega\right) = \mathrm{Re}\left\{k_{\ell m}\left(\omega\right)\right\}$$

$$k^{\text{diss}}_{\ell m}\left(\omega\right) = \mathrm{Im}\left\{k_{\ell m}\left(\omega\right)\right\}$$

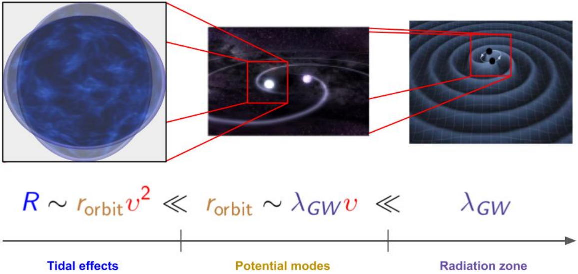
(analogous to $\mathbb{C}$-valued susceptibility in EM)




GRACE data, 2004

Goldberger et al. (2020), Chia (2020), CDI (2021a)

# Love numbers in GR - Worldline EFT Goldberger&Rothstein (2006), Porto (2006)




$$S_{\text{EFT}}^{\text{1-body}}\left[x_{\text{cm}}, h, A, \Phi\right] = S_{\text{bulk}}\left[\eta + h, A, \Phi\right] - M \int d\tau + S_{\text{finite-size}}\left[x_{\text{cm}}, h, A, \Phi\right]$$

**Love numbers in GR $\equiv$ Wilson coefficients in worldline EFT**

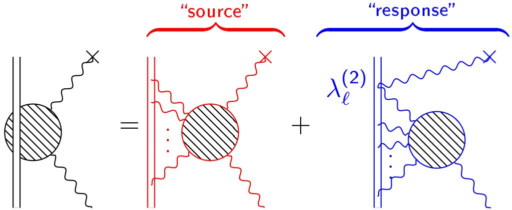
$$S_{\text{finite-size}} \supset \sum_{s=0}^{2} \frac{\lambda_\ell^{(s)}}{2\ell!} \int d\tau \, \mathcal{E}_L^{(s)}\left(x_{\text{cm}}(\tau)\right) \mathcal{E}^{(s)\,L}\left(x_{\text{cm}}(\tau)\right) + \left(\mathcal{E} \leftrightarrow \mathcal{B}\right)$$

$$\mathcal{E}_L^{(0)} = \partial_{\langle i_\ell} \ldots \partial_{i_1\rangle} \Phi \ , \quad \mathcal{E}_L^{(1)} = \partial_{\langle i_\ell} \ldots \partial_{i_2} F_{i_1\rangle 0} \ , \quad \mathcal{E}_L^{(2)} = \partial_{\langle i_\ell} \ldots \partial_{i_3} R_{i_2|0|i_1\rangle 0}$$

## Newtonian matching Kol & Smolin (2011), Hui et al. (2021), CDI (2021a)

- Put a pure $2^\ell$-pole background with source moments $\bar{\mathcal{E}}_L$ at large distances and match 1-pt function:

$\langle h_{00} \rangle_{\text{EFT}} =$




$$= \bar{\mathcal{E}}_L x^L \left[ \left(1 + a_1 \frac{GM}{r} + \dots \right) + \frac{\lambda_\ell^{(2)}}{r^{2\ell+1}} \left(1 + b_1 \frac{GM}{r} + \dots \right) \right]$$

$$\boxed{\lambda_\ell^{(s)} \propto k_\ell^{(s)\text{Love}} (\omega = 0)\ R^{2\ell+1}}$$

- $s = 0$: Scalar response
- $s = 1$: Electric/Magnetic polarization
- $s = 2$: Electric/Magnetic tidal response

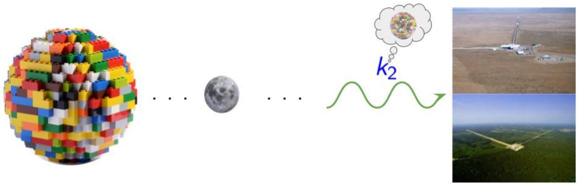
# Measuring Love Flanagan & Hinderer (2007), Cardoso et al. (2014), Chatziioannou (2020)

## Leading order tidal effects enter at 5PN Flanagan & Hinderer (2007)

$$\Psi_{\rm LO}^{\rm tidal}(f) = -\frac{9}{16}\frac{v^5}{\mu\,(GM)^4}\left[(m_1+12m_2)\,\frac{\lambda_1}{m_1} + 1 \leftrightarrow 2\right],$$

$$v = (GM\pi f)^{1/3}\,, \qquad \lambda \equiv \lambda_{\ell=2} = \tfrac{1}{3}k_{\ell=2}\frac{R^5}{G}$$

*Love numbers probe the internal structure of the compact body*



# Static scalar Love numbers of Schwarzschild black hole

- Massless scalar field in Schwarzschild black hole background ($r_{\text{h}} = r_s = 2GM$):

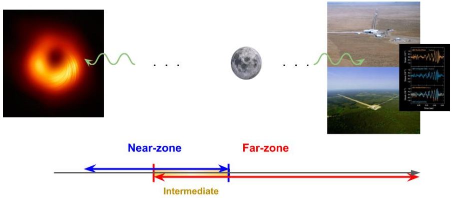
$$ds^2 = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2\,, \quad \Delta(r) = r(r - r_s)$$

- Full e.o.m.: $\mathbb{O}_{\text{full}}\,\Phi_{\omega\ell m} = \left[\partial_r \Delta\, \partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$

## Response as low-energy scattering problem

Starobinsky (1965), Chia (2020), CDI (2021a)

- Near-zone region: $\omega(r - r_{\text{h}}) \ll 1$
- Far-zone region: $\omega r \gg 1$
- Intermediate region: $r_{\text{h}} \ll r \ll \frac{1}{\omega}$



# Static scalar Love numbers of Schwarzschild black hole

- Full e.o.m.: $\mathbb{O}_{\text{full}}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell\,(\ell+1)\,\Phi_{\omega\ell m}$

## Scalar Love numbers of $d=4$ Schwarzschild black hole

- Near-zone approximation: $\frac{\omega^2 r^4}{\Delta} \simeq \frac{\omega^2 r_s^4}{\Delta} \Rightarrow \mathbb{O}_{\text{NZ}} = \partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2$
- *Not* unique but *exact* for $\omega = 0$.
- $\ell \in \mathbb{N} \to \ell \in \mathbb{R}$ to perform source/response split ($A_\ell(\omega) \in \mathbb{R}_+^*$):

$$k_{\ell m}^{(0)}(\omega) = A_\ell(\omega)\left(\tan\pi\ell\cosh 2\pi\omega r_s + i\sinh 2\pi\omega r_s\right) \xrightarrow{\ell\in\mathbb{N}} \in i\mathbb{R}$$

$$\Rightarrow \boxed{\boxed{k_{\ell m}^{\text{Love},(0)}(\omega) = 0}} + \mathcal{O}\left(\omega^2 r_s^2\right)$$

# The tune of Love

- Totalitarian principle: "*Everything not forbidden is compulsory!*"
- 't Hooft naturalness (1980): "*At any energy scale, a physical parameter is allowed to be small if setting it to zero enhances the symmetry of the system. Otherwise, its natural value is an* $\mathcal{O}(1)$ *number*".

## Magic zeroes in the black hole response problem

- For *all* isolated asymptotically flat GR (Kerr-Newman) black holes:

Fang & Lovelace (2005), Damour & Nagar (2009), Binnington & Poisson (2009), Hinderer (2009), Poisson (2015), Le Tiec & Casals (2020), Chia (2020), Le Tiec et. al (2020), CDI (2021a)

$$\boxed{k_{\ell m}^{\text{Love},(s)}(\omega=0)=0} \Rightarrow \boxed{\boxed{\text{Fine-tuning or enhanced symmetry?}}}$$ Porto (2016)

# Enhanced symmetry in near-zone

Recall near-zone e.o.m. for massless scalar field in Schwarzschild background:

$$\mathbb{O}_{\text{NZ}}\Phi_{\omega\ell m} = \ell\left(\ell+1\right)\Phi_{\omega\ell m}\,, \quad \mathbb{O}_{\text{NZ}} = \partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2\,, \quad \Delta = r\left(r-r_s\right)$$

## $SL\left(2,\mathbb{R}\right)$ symmetry of $\mathbb{O}_{\text{NZ}}$: Bertini et al. (2012)

$$L_0 = -\beta\,\partial_t \;\;,\;\; L_{\pm1} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\,\partial_t\right] \;\;,\;\; \beta = 2r_s$$

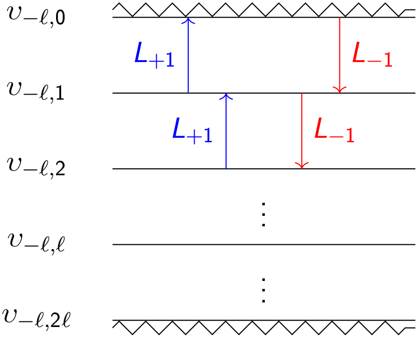
$$\left[L_m, L_n\right] = \left(m-n\right)L_{m+n} \;\;,\;\; \mathcal{C}_2 = L_0^2 - \frac{1}{2}\left(L_{+1}L_{-1} + L_{-1}L_{+1}\right) = \mathbb{O}_{\text{NZ}}$$

- Globally defined and regular at both future and past e.h.'s

$$\boxed{\begin{gathered} L_0\Phi_{\omega\ell m} = i\beta\omega\Phi_{\omega\ell m} \\ \mathcal{C}_2\Phi_{\omega\ell m} = \ell\left(\ell+1\right)\Phi_{\omega\ell m} \end{gathered}}$$

# Highest-weight banishes Love CDI (2021b, 2022)

## Massless scalar field in Schwarzschild background




- Highest-weight vector with fixed Casimir at orbital number $\ell \in \mathbb{N}$:

$$L_0 \upsilon_{-\ell,0} = -\ell \upsilon_{-\ell,0} \quad , \quad L_{+1} \upsilon_{-\ell,0} = 0$$

$$\Rightarrow \upsilon_{-\ell,0} = e^{\ell t/\beta} \Delta^{\ell/2}$$

- $\upsilon_{-\ell,0}$: *regular* at e.h., $\omega_{-\ell,0} = i\ell/\beta$.
  $\Rightarrow \upsilon_{-\ell,n} = (L_{-1})^n \upsilon_{-\ell,0}$: *regular* solutions, $\omega_{-\ell,n} = i(\ell - n)/\beta$.
- $\Phi_{\omega=0,\ell m} \propto \upsilon_{-\ell,\ell}$

$$(L_{+1})^{\ell+1} \upsilon_{-\ell,\ell} = 0 \Rightarrow \boxed{\partial_r^{\ell+1} \Phi_{\omega=0,\ell m} = 0 \Rightarrow \text{No } r^{-\ell-1} \text{ term!}}$$

# Love symmetry for Kerr-Newman black holes CDI (2021b, 2022)

$$L_0^{(s)} = -\beta\,\partial_t + s$$

$$L_{\pm1}^{(s)} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\,\partial_t + \frac{a}{\sqrt{\Delta}}\,\partial_\phi - s\,(1\pm1)\left(\sqrt{\Delta}\right)'\right]$$

*Teukolsky (1973)*

$$\phi_0 = F_{\mu\nu}l^\mu m^\nu\,, \quad \phi_1 = \tfrac{1}{2}F_{\mu\nu}(l^\mu n^\nu + m^{*\mu}m^\nu)\,, \quad \phi_2 = F_{\mu\nu}m^{*\mu}n^\nu\,, \tag{1.1}$$

$$\psi_0 = -C_{\alpha\beta\gamma\delta}l^\alpha m^\beta l^\gamma m^\delta\,, \quad \psi_4 = -C_{\alpha\beta\gamma\delta}n^\alpha m^{*\beta}n^\gamma m^{*\delta}\,, \tag{1.3}$$

$$ds^2 = (1 - 2Mr/\Sigma)dt^2 + (4Mar\sin^2(\theta)/\Sigma)dtd\varphi - (\Sigma/\Delta)dr^2 - \Sigma d\theta^2 - \sin^2(\theta)(r^2 + a^2 + 2Ma^2r\sin^2(\theta)/\Sigma)d\varphi^2\,. \tag{4.1}$$

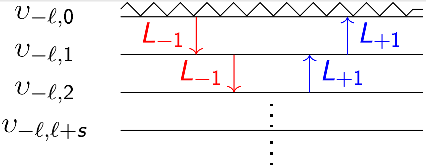
Here $M$ is the mass of the black hole, $aM$ its angular momentum, $\Sigma = r^2 + a^2\cos^2\theta$, and[1] $\Delta = r^2 - 2Mr + a^2$. When $a = 0$, the metric reduces to the Schwarzschild metric, a nonrotating black hole.

TABLE 1

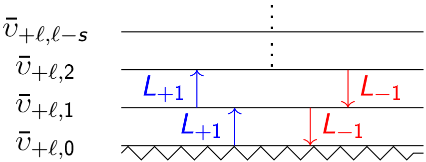
FIELD QUANTITIES $\psi$, SPIN-WEIGHT $s$, AND SOURCE TERMS $T$ FOR EQUATION (4.7)

| $\psi$ | $s$ | $T$ |
|---|---|---|
| $\Phi$ | 0 | $\Box\Phi = 4\pi T$ |
| $\chi_0$<br>$\rho^{-1}\chi_1$ | $\frac{1}{2}$<br>$-\frac{1}{2}$ | See references in Appendix B |
| $\phi_0$<br>$\rho^{-2}\phi_2$ | 1<br>−1 | $J_0$ (eq. [3.6])<br>$\rho^{-2}J_2$ (eq. [3.8]) |
| $\psi_0{}^B$<br>$\rho^{-4}\psi_4{}^B$ | 2<br>−2 | $2T_0$ (eq. [2.13])<br>$2\rho^{-4}T_4$ (eq. [2.15]) |

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2} - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi} - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,. \tag{4.7}$$




(a) Highest-weight repr. $= \{\upsilon_{-\ell,n}\}$:
Regular (Singular) at future (past) e.h.




(b) Lowest-weight repr. $= \{\bar\upsilon_{+\ell,n}\}$:
Regular (Singular) at past (future) e.h.

$$R_{s,\omega=0,\ell m}(r) = \left(\frac{r-r_+}{r-r_-}\right)^{im\Omega\beta/2}\Delta^{-s}\sum_{n=0}^{\ell+s}c_{n,\ell m}r^n \Rightarrow$$ No induced multipole moment term

# Infinite extension CDI (2021b, 2022)

$$\mathrm{SL}\left(2,\mathbb{R}\right)_{\text{Love}} \to \mathrm{SL}\left(2,\mathbb{R}\right)_{\text{Love}} \ltimes \hat{U}\left(1\right) \supset \mathrm{SL}\left(2,\mathbb{R}\right)_{(\alpha)}$$

$$L_m\left(\alpha\right) = L_m^{\text{Love}} + \alpha\, \upsilon_m\, \beta\Omega\, \partial_\phi \quad , \quad \upsilon_m \in \mathcal{V}$$

## Starobinsky near-zone $\mathrm{SL}\left(2,\mathbb{R}\right) \times SO\left(3\right)$

Starobinsky (1973) $\alpha = 1$

$$L_0 = -\beta\left(\partial_t + \Omega\,\partial_\phi\right)$$

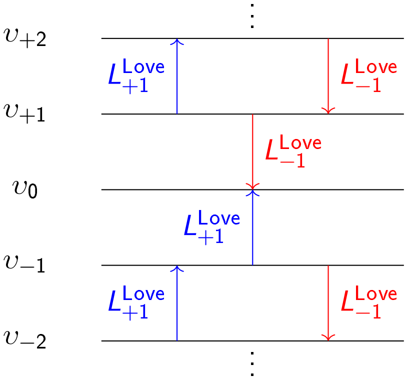
$$L_{\pm1} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\left(\partial_t + \Omega\,\partial_\phi\right)\right]$$

$$L_0\Phi_{\omega\ell m} = i\beta\left(\omega - m\Omega\right)\Phi_{\omega\ell m}$$

$$\mathcal{C}_2\Phi_{\omega\ell m} = \ell\left(\ell+1\right)\Phi_{\omega\ell m}$$

*Geometrically more interesting!*

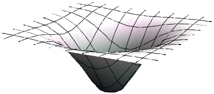
Hui et al. (2021, 2022)



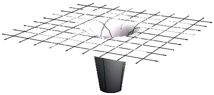
# Relation to enhanced NHE isometry CDI (2022)

- Near-horizon geometry of extremal black holes develops an infinite throat and *decouples* from far-horizon geometry.
- Decoupled throat has enhanced isometry from acquired AdS factors.
- RN: $\text{SL}(2,\mathbb{R})_{\text{NHE}} \times SO(3)$
- Kerr: $\text{SL}(2,\mathbb{R})_{\text{NHE}} \times U(1)$

Bardeen & Horowitz (1999)



$Q^2 < M^2$



$Q^2 = M^2$

$$\text{RN:}\quad \lim_{Q^2\to M^2} \text{SL}(2,\mathbb{R})_{\text{Love}} = \text{SL}(2,\mathbb{R})_{\text{NHE}}$$

$$\text{Kerr:}\quad \lim_{a^2\to M^2} \left(\text{SL}(2,\mathbb{R})_{\text{BH}} \subset \text{SL}(2,\mathbb{R})_{\text{Love}} \ltimes \hat{U}(1)\right) = \text{SL}(2,\mathbb{R})_{\text{NHE}}$$

# Near-zone symmetries as isometries of subtracted geometries CDI (2022)

- 4-d Kerr-Newman black hole as a fibration over a 3-d base space:

$$ds^2 = -\Delta_0^{-1/2} G\,(dt + \mathcal{A})^2 + \Delta_0^{1/2}\left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta}{G}\sin^2\theta\, d\phi^2\right)\,,$$

$$G = \Delta - a^2\sin^2\theta\,,\quad \mathcal{A} = \frac{a\sin^2\theta}{G}\left(2Mr - Q^2\right)\,\mathrm{d}\phi\,.$$

## Black hole thermodynamics Cvetič & Larsen (2012a, 2012b)

- Ergosurface bound at locus $G = 0$.
- Position of event horizon, entropy, Hawking temperature and angular velocity are independent of the warp factor and only depend on the near-horizon behavior of the angular potential.
- $\Delta_0$ contains information about environment of black hole, and not its internal structure.

## Subtracted geometry Cvetič & Larsen (2012a, 2012b)

- Subtract information about environment

$$ds^2_{\text{sub}} = -\Delta_{0,\text{sub}}^{-1/2} G \left(dt + \mathcal{A}_{\text{sub}}\right)^2 + \Delta_{0,\text{sub}}^{1/2} \left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta}{G} \sin^2\theta \, d\phi^2\right)$$

such that $\lim_{r\to r_+} \mathcal{A}_{\text{sub}} = \lim_{r\to r_+} \mathcal{A} = -\left(r_+^2 + a^2\right) \mathrm{d}\phi$

## Love symmetry

$$\Delta_{0,\text{sub}} = \left(r_+^2 + a^2\right)^2 \left(1 + \beta^2\Omega^2 \sin^2\theta\right)$$

$$\mathcal{A}_{\text{sub}} = \frac{a\sin^2\theta}{G} \left(r_+^2 + a^2 + \beta\left(r - r_+\right)\right) \mathrm{d}\phi$$

$$\boxed{\boxed{\mathscr{L}_{L_m^{\text{Love}}} g_{\mu\nu}^{\text{sub}} = 0}}$$

## Starobinsky near-zone $SL\left(2,\mathbb{R}\right)$

$$\Delta_{0,\text{sub}} = \left(r_+^2 + a^2\right)^2$$

$$\mathcal{A}_{\text{sub}} = \frac{a\sin^2\theta}{G} \left(r_+^2 + a^2\right) \mathrm{d}\phi$$

$$\boxed{\boxed{\mathscr{L}_{L_m^{\text{Star}}} g_{\mu\nu}^{\text{sub}} = 0}}$$

# Love symmetry in higher dimensions CDI (2021b, 2022), CI(2023), in prep.

## Scalar susceptibility in $d \geq 4$ Schwarzschild Kol & Smolkin (2011), Hui et al. (2021)

$$\hat{\ell} \equiv \frac{\ell}{d-3}$$

$$k_\ell^{(0)} = \begin{cases} c_\ell \neq 0 & \text{if } 2\hat{\ell} \notin \mathbb{N} \\ c_\ell + \beta_\ell \ln \frac{r-r_s}{L} & \text{if } \hat{\ell} \in \mathbb{N} + \frac{1}{2} \\ c_\ell = 0 & \text{if } \hat{\ell} \in \mathbb{N} \end{cases}$$

- Love symmetry still exists in near-zone: Bertini et al. (2012)

$$L_0 \Phi_{\omega\ell m} = i\beta\omega\Phi_{\omega\ell m}$$
$$\mathcal{C}_2 \Phi_{\omega\ell m} = \hat{\ell}(\hat{\ell}+1)\Phi_{\omega\ell m}$$

- Regular static solution $\in$ highest-weight repr. of $\mathrm{SL}(2,\mathbb{R})$ iff $\hat{\ell} \in \mathbb{N}$ ✓

EM susceptibilities: $\begin{cases} \text{S-modes} & \checkmark \\ \text{V-modes} & \checkmark \end{cases}$

TLNs: $\begin{cases} \text{T-modes} & \checkmark \\ \text{V-modes (Regge-Wheeler)} & \checkmark \\ \text{S-modes (Zerilli)} & ? \end{cases}$

- $p$-form perturbations of Schwarzschild-Tangherlini black holes ✓
- Scalar susceptibilities for 5-d rotating (Myers-Perry) black holes ✓

# Beyond Einsteinian Love CDI (2022), in prep.

- Love symmetry might not be unique to GR, but it certainly does not exist for a generic theory of gravity.

## Generic spherically symmetric, non-extremal black hole

$$ds^2 = -f_t(r)\,dt^2 + \frac{dr^2}{f_r(r)} + r^2 d\Omega_2^2$$

- Geometric condition for existence of near-zone $SL(2,\mathbb{R})$ Love symmetry:

$$\frac{f_r(r)}{f_t(r)} = \frac{4r^2 f_t(r) + \left(\frac{\beta_s}{\beta} r_h\right)^2}{\left(r^2 f_t(r)\right)'^2}\;, \quad \beta = \frac{2}{\sqrt{f_t'(r_h)\, f_r'(r_h)}}\;, \quad \beta_s = 2r_h$$

- If such geometries are supported, the static Love numbers will vanish due to the highest-weight property.
- Counterexample: $R^3_{\mu\nu\rho\sigma}$ gravity $\rightarrow$ Condition *not* satisfied!
  $\Rightarrow$ No Love symmetry $\rightarrow$ Consistent with $k_\ell^{(0)} = c_\ell + \beta_\ell \ln\frac{r-r_h}{L}$ results

# Summary

- Love numbers capture the linear conservative "tidal" response of a compact body to external "tidal" fields and can be probed in radiation waveform signals.
- Black holes in $d = 4$ GR have vanishing static Love numbers
  $\rightarrow$ ('t Hooft-)naturalness concerns
- Enhanced $\mathrm{SL}(2,\mathbb{R})$ Love symmetry explains seemingly unnatural properties of black holes Love numbers
  $\rightarrow$ "Highest-weight banishes Love"
- Approximate near-zone symmetries are isometries of subtracted geometries.
- Closely related to enhanced NHE $\mathrm{SL}(2,\mathbb{R})$ isometry.
- In higher dimensions, Love symmetry still exists regardless of details of perturbation and is in accordance with the more intricate vanishing of the black hole static Love numbers.
- For modified GR, Love symmetry in general does not exist and Love numbers have their natural non-zero and RG-flowing values.

## What else to do with Love symmetry?

- Re-organization of perturbation theory and QNMs?
- Black $p$-branes and AdS/CFT? (in progress)
- Full symmetry structure and Kerr/CFT? (in progress)
- "Accidental" symmetry of extremal GR black holes?

  Porfyriadis & Remmen (2021)
- Near-horizon BMS-like algebra and Celestial holography?

  Donnay, Giribet, González, Pino (2016a,2016b)

*"Black holes are the hydrogen atom of the 21st century"*

*'t Hooft (2016), EHT (April 10, 2019)*

*Thank you*