Open-Closed Duality
in
String Field Theory

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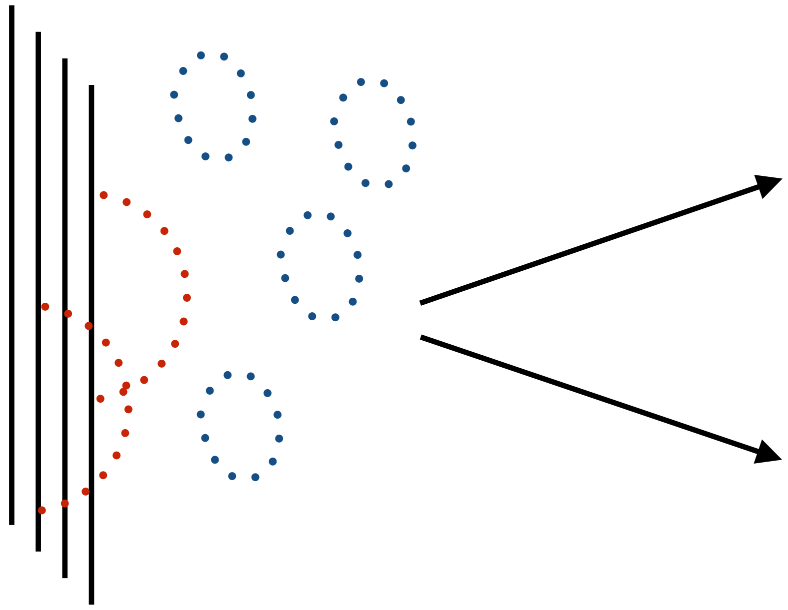
JHEP **09 (2023) 119**
+ work in progress

4th meeting of the PRIN Network

"String Theory as a bridge between Gauge Theories and Quantum Gravity"

Roma, La Sapienza, 20 October 2023

- String Theory gives important tools to better understand QFT and Gravity (***This is the goal of our PRIN!***)



Open strings: Gauge Theory

$\alpha' \to 0$ ⇕

Closed strings: Gravity

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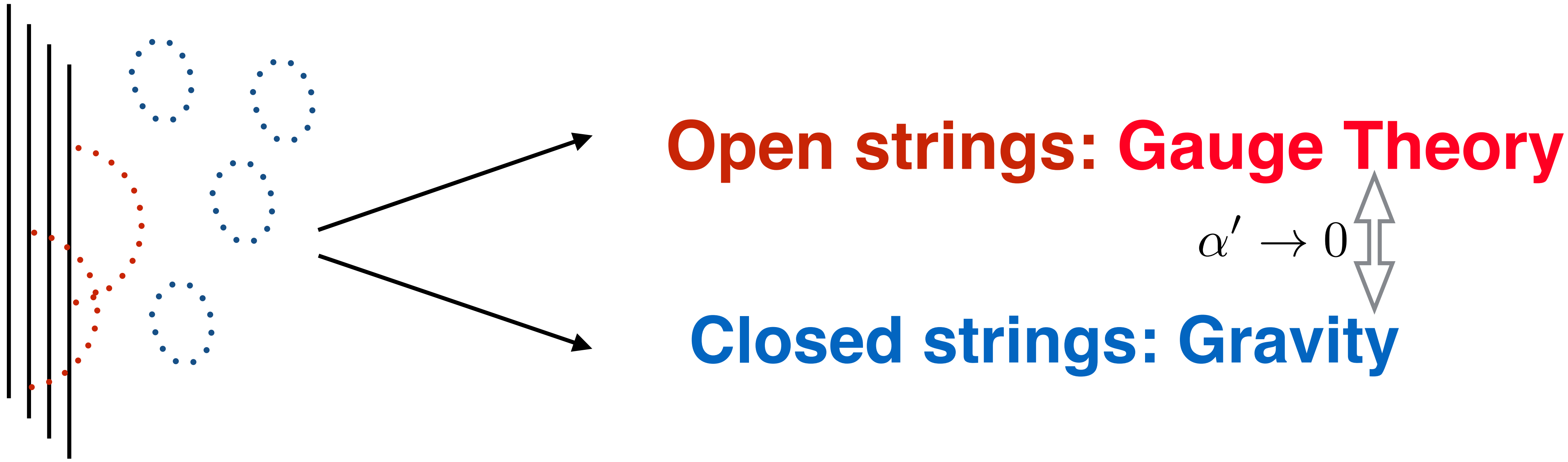

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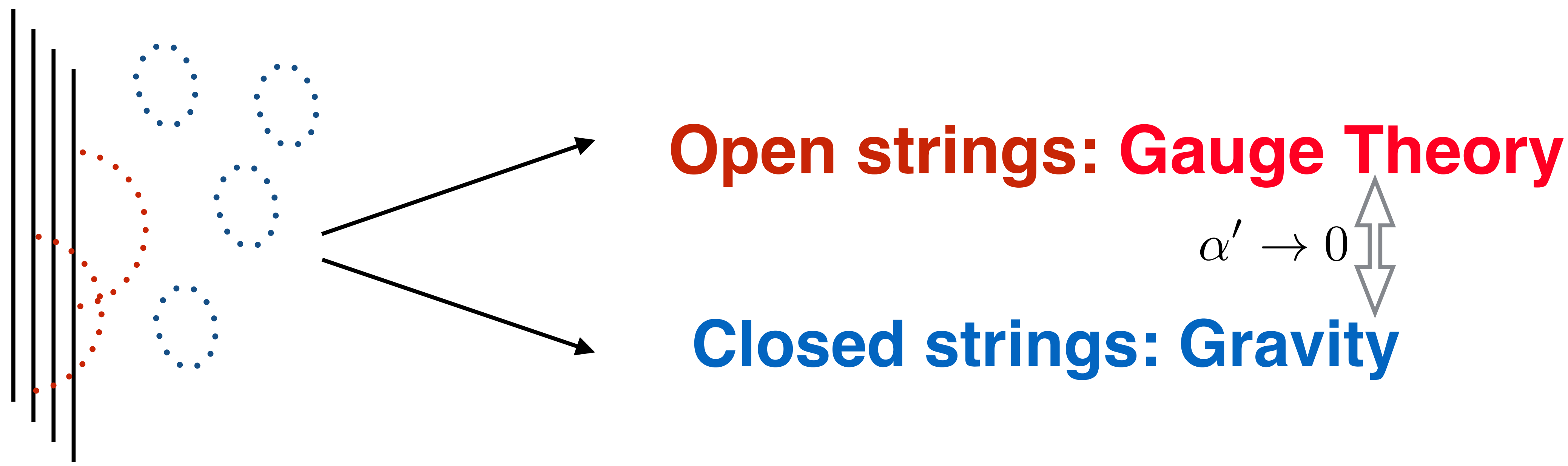

- ***Open-Closed duality is roughly the statement that the same physical process can be described from an open or a closed string perspective***

- I have always found this statement a bit confusing and perhaps imprecise, so I tried to understand it in a different way… **Use String Field Theory!**

Skeleton Summary

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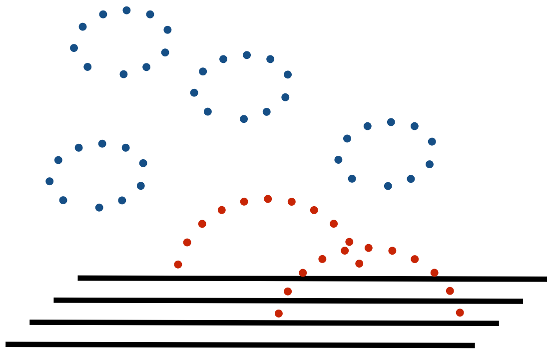
- Start with a **'Master Theory'** containing both open and closed strings. **Open-Closed SFT**



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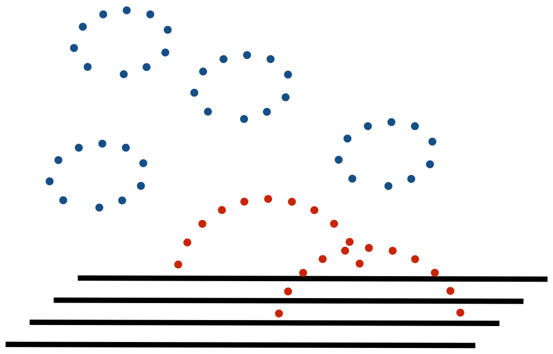
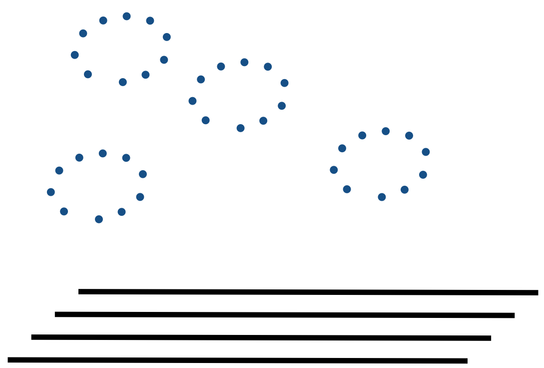


- Integrate out the open string degrees of freedom. D-branes remain as sources. **Unstable Closed SFT**.



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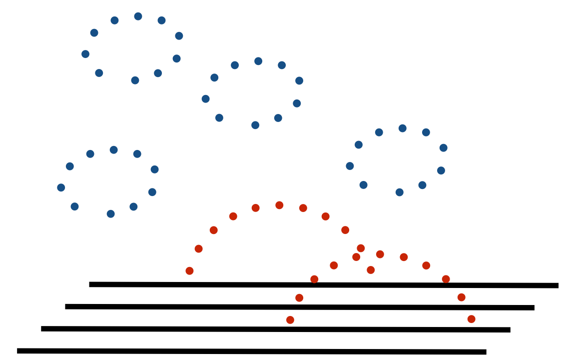

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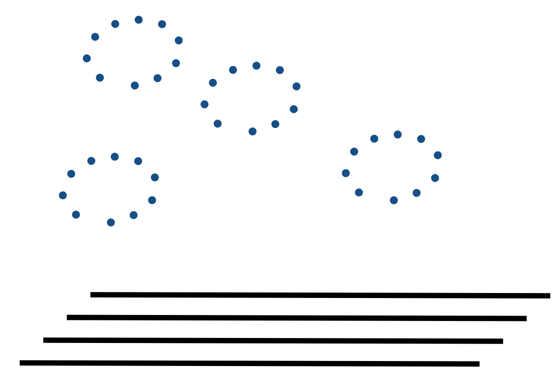
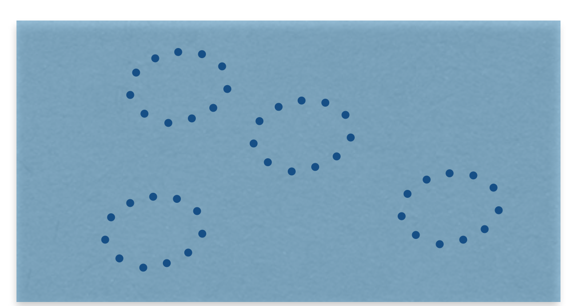


- Absorb the sources with a vacuum shift in the closed strings: closed strings in a deformed background. **Stable Closed SFT.**



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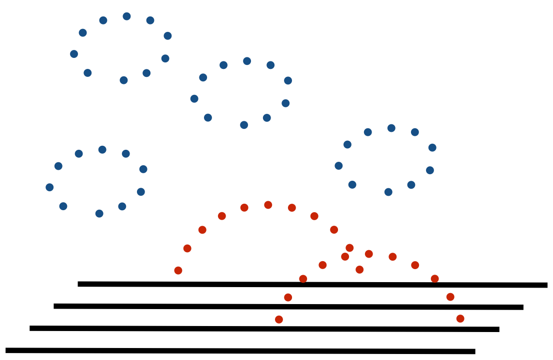

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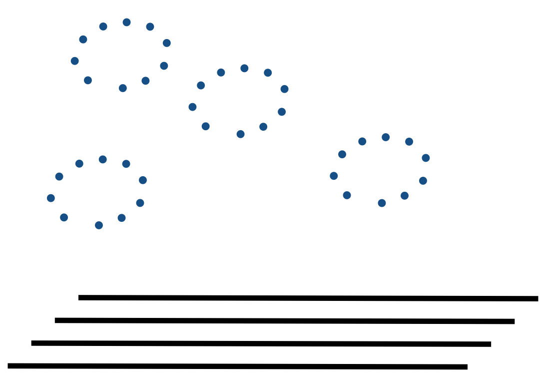

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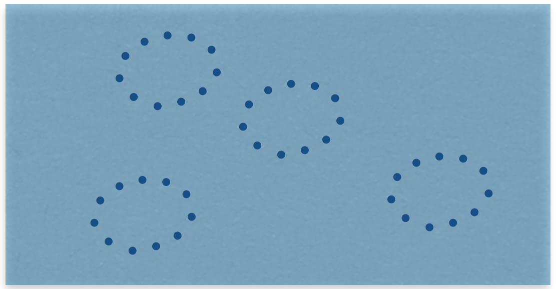

New closed string theory without D-branes!

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- Can we provide **examples** where this program is succesful? (**no obstructions**)

What is Open-Closed SFT?

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- The full interacting action has a topological decomposition in genus + boundaries

$$S(\Phi,\Psi) = \sum_{g=0}^{\infty}\sum_{b=0}^{\infty} S_{g,b}(\Phi,\Psi) \qquad S_{g,b}(\Phi,\Psi) = -g_s^{2g-2+b}\sum_{k,l_i}\mathcal{V}_{g,b}^{k,\{l_i\}}(\Phi^{\wedge k};\Psi^{\otimes l_1},\cdots,\Psi^{\otimes l_b})$$

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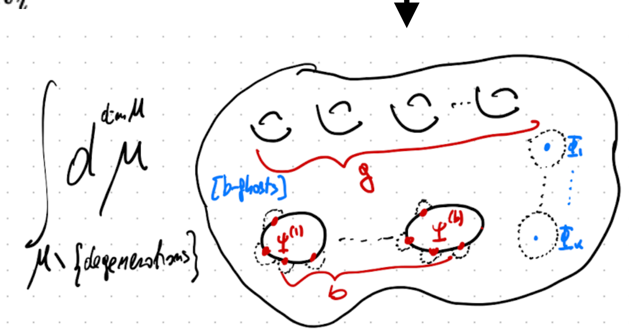
Off-shell amplitude with integration near degenerations removed ⟵



- The consistency of the construction is encoded in the quantum BV master equation (path integral is well defined, good QFT)

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- Geometrically

$$(S_{g_1,b_1},S_{g_2,b_2})_{\rm c}\in\Sigma_{g_1+g_2,b_1+b_2}$$

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$$\Delta_{\rm c}S_{g,b}\in\Sigma_{g+1,b}\,,$$

$$\Delta_{\rm o}^{(1)}S_{g,b}\in\Sigma_{g,b+1}\,,$$

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- Notice that $S_{0,0}(\Phi)$ and $S_{0,1}(0,\Psi)$ satisfy classical master equations. They are **classical closed SFT** and **classical open SFT** respectively. ***Other classical limits??***

- We can isolate the **genus zero** sector (classical closed strings) which obeys the master equation

$$(S_0, S_0)_{\rm c} + (S_0, S_0)_{\rm o} + 2\Delta_{\rm o}^{(1)} S_0 = 0 \,. \qquad S_0 \equiv \sum_{b=0}^{\infty} S_{0,b}$$

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- But it is not possible to also constrain the open strings to be classical (b=1)

$$\begin{aligned}
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- How to focus on this simplified sector? Take a ***Large N*** number of initial D-branes!

Open-Closed SFT in the Large N limit

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- Open string fields are N x N matrices and the inner product has an understood trace. Use a normalized trace (finite in the large N limit)

$$\Psi = \psi^a_{ij}\, t^{ij} o_a\,, \qquad\qquad (t^{ij})_{pq} = \delta^i_p\, \delta^j_q$$

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- The O-C SFT action rearranges in a double expansion

$$S_{\rm oc}[\Phi,\Psi] = \sum_{g=0}^{\infty}\sum_{b=0}^{\infty}\kappa^{2g+b-2}\,N^b\sum_{k=0}^{\infty}\sum_{\{l_1,\ldots,l_b\}=0}^{\infty}\frac{1}{b!k!(l_1)\cdots(l_b)}\mathcal{A}'^{g,b}_{k;\{l_1,\ldots,l_b\}}\left(\Phi^{\wedge k}\otimes'\Psi^{\odot l_1}\wedge'\ldots\wedge'\Psi^{\odot l_b}\right)$$
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Open-Closed SFT in the Large N limit

- Open string fields are N x N matrices and the inner product has an understood trace. Use a normalized trace (finite in the large N limit)

$$\Psi = \psi^a_{ij}\, t^{ij} o_a , \qquad (t^{ij})_{pq} = \delta^i_p\, \delta^j_q$$

$$\mathrm{Tr}'[M] := \frac{1}{N}\ \mathrm{Tr}[M]$$

- As well as normalized amplitudes

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- ’t Hooft coupling obviously emerges

$$\lambda := \kappa N , \qquad \kappa^{2g-2} \lambda^b = \frac{1}{N^{2g-2}} \lambda^{2g+b-2}$$

Open-Closed SFT in the Large N limit

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- Classical closed strings + quantum (but planar) open strings.

What does it mean to INTEGRATE OUT open strings?

Integrating out open strings

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- Perform the perturbative (BV) path integral on open strings with a gauge fixing

$$\int \mathcal{D}\Psi \, e^{-S_{\rm pl}(\Phi,\Psi)}\Big|_{h_{\rm o}\Psi=0} = e^{-S_{\rm eff}(\Phi)} \qquad [Q_o, h_o] = 1_o$$

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$$\mathcal{G} := e^{\wedge\Phi} \otimes' e^{\wedge'\mathcal{C}(\Psi)}$$

$$= \left[\sum_{k\geq 0}\frac{1}{k!}\Phi^{\wedge k}\right] \otimes' \left[\sum_{b\geq 0}\frac{1}{b!}\left(\sum_{l_1\geq 0}\frac{1}{(l_1)}\Psi^{\odot l_1}\right)\wedge'\cdots\wedge'\left(\sum_{l_b\geq 0}\frac{1}{(l_b)}\Psi^{\odot l_b}\right)\right]$$

GROUP ELEMENT (encode the fields)

- The open-closed action can be nicely packaged

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- The full BV master equation reflects on the (full) coderivation $\mathbf{n}$

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This only creates planar open string loops!

- With this preparation, the path integral gives

$$S_{\text{eff}}(\Phi) = \int_0^1 dt\, \frac{\omega_{\text{c}}}{\lambda^2} \left(\dot{\Phi}\,,\, \pi_1 \tilde{\boldsymbol{l}}\, e^{\wedge\Phi}\right) + \Lambda_{\text{open}}\,.$$

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- Closed string off-shell amplitudes on g=0 Riemann surfaces with boundaries, with moduli-space integration carried over all the way down to open string degeneration, but still cut-off at closed string degeneration.

Obstructions to Integrating out open strings

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- To perform the full integration-out we had to assume that the open string propagator fully inverts the BRST operator

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- Way out? ASSUME that the open string cohomology does not propagate. Strong constraint on the chosen string background!
- Amazingly this precisely happens in ***minimal string theory*** and in the ***topological string***, perhaps in other scenarios as well.

What is an UNSTABLE Closed SFT?

The unstable closed string theory

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- Assuming we safely survived the open string integration-out, let's then have a closer look at the obtained closed SFT

$$S_{\text{eff}}(\Phi) = \sum_{b=0}^{\infty} \lambda^{b-2} \sum_{k=0}^{\infty} \frac{1}{b!k!} \mathcal{A}''^{g=0,b}_{k} \left(\Phi^{\wedge k}\right),$$

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- Closed string emission from surfaces with boundaries (disk, annulus etc…) controlled by the 't Hooft coupling!

What does it mean to absorb the sources to end up with a STABLE Closed SFT?

Canceling the tadpole

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- As usual in (quantum) field theory we have to search for a new vacuum with stable fluctuations. The tadpole is a source term, solve the sourced equation of motion!

$$\sum_{k=1}^{\infty} \frac{1}{k!} \tilde{l}_k(\Phi^{\wedge k}) = -\tilde{l}_0 .$$

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- Notice in particular that $\tilde{l}_0^{(1)}$ is essentially the **boundary state**.

$$\Phi_1 = -\frac{b_0^+}{L_0^+}\, \tilde{l}_0^{(1)}$$

Cfr Di Vecchia et al '97, Sen '04

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- At higher order I don't know precisely, but it is clear that this has to do with large distance effects (IR structure).

Example: FZZT branes in the (2,1) Minimal String Theory

MINIMAL STRING THEORY

Di Francesco, Ginsparg, Zinn-Justin '95
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- This is the closed string side of the story.

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- Instead of deforming the bulk with the $\{O_{2k+1}\}$, we can add special D-branes: FZZT branes

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- So this looks like a perfect playground for testing our picture. Are the open and closed obstructions avoided??

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- Indeed we find (work in progress!) that the vacuum shift solution gives rise to the same partition function that the marginal solution

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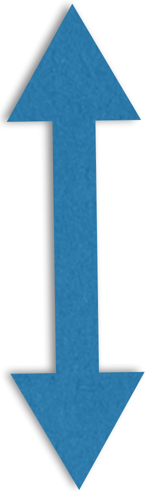

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SPHERE PARTITION FUNCTION
(GAIOTTO RASTELLI)

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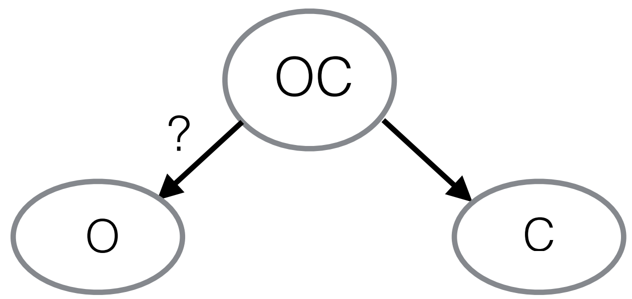

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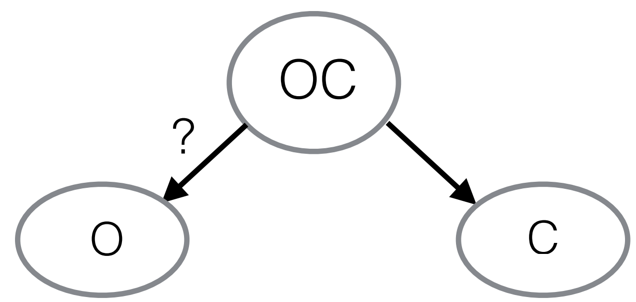

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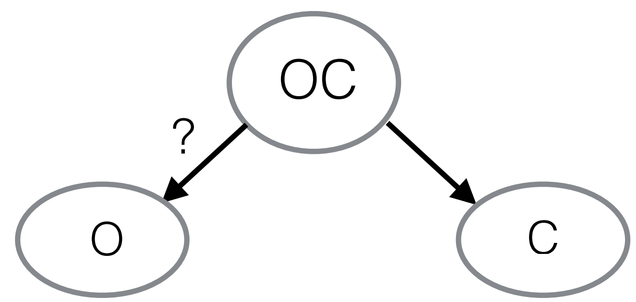

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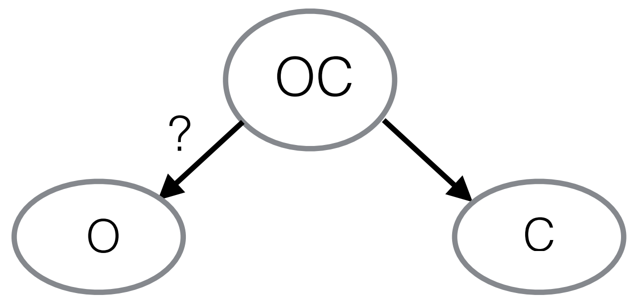

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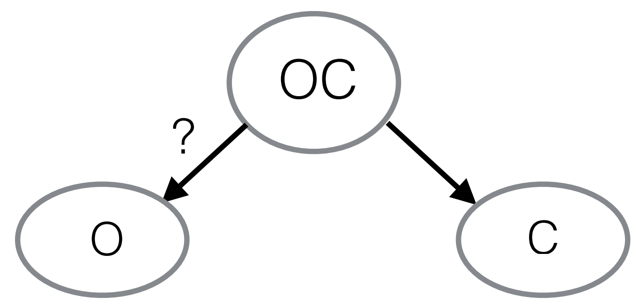

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