String Theory on Time-Dependent Orbifolds: from breakdown to breakthrough?

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"*String Theory as a bridge between Gauge Theories and Quantum Gravity*"

❖ A.A., R.Finotello, I.Pesando - *On the Origin of the Divergences in Time-Dependent Orbifolds* - Eur.Phys.J.C 80, (2020)
❖ A.A., I.Pesando - *Light-Cone Quantization of a Scalar Field on Time-Dependent Backgrounds* - Eur.Phys.J.C 82, (2022)
❖ A.A., R.J.Szabo, I.Pesando - *The Role of Noncommutativity on the Null Boost Orbifold* - To appear, (2023)

Talk Outline

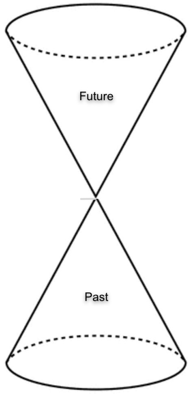
- Introduction and motivations
- The Null-Boost Orbifold (NBO)
- Scattering amplitudes divergences
- The origin of the divergences
- The role of noncommutativity

The cosmological point of view

- The **inflationary scenario** is the current paradigm of early universe cosmology, as it offers a successful solution to several problems (flatness, horizon, structure formation…), but it doesn't address the singularities matter.
- **Bouncing cosmologies**, like the Big Crunch/Big Bang scenario or the Ekpyrotic (cyclic) scenario, are more ambitious as they admit cosmological singularities.
- Besides the "origin of the universe" issue, **spacetime singularities** are anyway an open problem in General Relativity: the theory breaks down at the Planck scale and quantum gravity is needed.

How do we study the Big Bang?

- Conceptual issues:
  - Is it the beginning of time?
  - If there is a bounce, how are the information before and after the singularity related?
  - How do we define observables and the S-matrix?



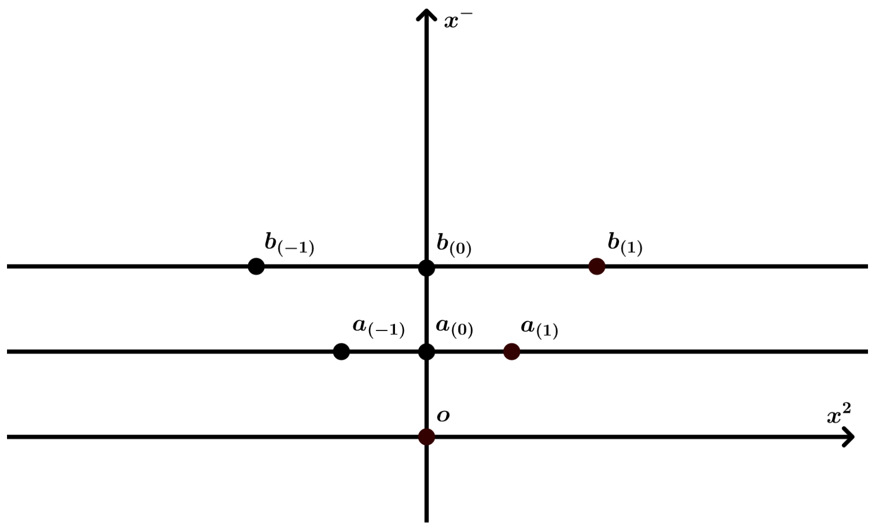
- We want to construct toy models capable of reproducing a **spacelike** singularity which appears in a certain region of space at a specific value of the time coordinate and then disappears.

- **String theory**, as a promising candidate for a theory of quantum gravity, is supposed to provide a satisfactory description of Big Bang type singularities.

The geometry of the NBO/1

The NBO is defined on $(\mathbb{M}^3/\Gamma) \otimes \mathbb{R}^{D-3}$ where Γ is the subgroup of $ISO(2,1)$ whose generator in lightcone coordinates is the Poincaré Killing vector $k = 2\pi i \Delta J_{+2}$ with $k^3 = 0$.

Spacetime points are identified as $x = \begin{pmatrix} x^- \\ x^2 \\ x^+ \\ \vec{x} \end{pmatrix} \sim e^{nk} x = \begin{pmatrix} x^- \\ x^2 + n(2\pi\Delta)x^- \\ x^+ + n(2\pi\Delta)x^2 + \frac{1}{2}n^2(2\pi\Delta)^2 x^- \\ \vec{x} \end{pmatrix}$, $n \in \mathbb{Z}$.



Scattering Amplitudes and Divergences: Closed Strings

Among the various issues which we can encounter when dealing with strings on time-dependent backgrounds, the most urgent of this model concerns **interactions**.

It was shown that the four tachyons **closed string** $2 \to 2$ tree-level scattering amplitude, the analogous of **Virasoro-Shapiro** amplitude in flat spacetime, exhibits unusual divergences in a specific kinematical regime (the Regge limit of high energy and small fixed angle):

$$A_{4T}^{closed} \sim \int^{q\sim\infty} \frac{dq}{|q|} q^{4-\alpha' \vec{p}_{\perp t}^{\,2}} ,$$

[*H.Liu, G.Moore, N.Seiberg (2002) - M.Berkooz, B.Craps, D.Kutasov, G.Rajesh (2005)*]

which is divergent when $\alpha' \vec{p}_{\perp t}^{\,2} < 4$.

Scattering Amplitudes and Divergences: Open Strings

This pathological behaviour has been interpreted in the literature as "*the result of a large* ***gravitational backreaction*** *of the incoming matter into the singularity due to the exchange of a single graviton*".

[*H.Liu, G.Moore, N.Seiberg (2002) - M.Berkooz, B.Craps, D.Kutasov, G.Rajesh (2005) - L.Cornalba, M.S.Costa (2005)*]

But if we perform an analogous computation for the four tachyons **open string "Veneziano"** amplitude we find:

$$A_{4T}^{open} \sim \int^{q\sim\infty} \frac{dq}{|q|} q^{1-\alpha' \vec{p}^{\,2}_{\perp t}} \ \mathrm{tr}\left(\{T_1, T_2\}\{T_3, T_4\}\right) \ ,$$

[*AA, R.Finotello, I.Pesando (2020)*]

which is divergent when $\alpha' \vec{p}^{\,2}_{\perp t} < 1$!

Our claim is therefore that **string perturbation theory fails**, but the nature of the divergences is not strictly related to a gravitational backreaction, since here we are dealing with open string at tree level.

The geometry of the NBO/2

- "Good" NBO coordinates

$$\begin{cases} x^- &= u \\ x^2 &= \Delta u z \\ x^+ &= v + \frac{1}{2}\Delta^2 u z^2 \end{cases}$$

- New metric

$$ds^2 = -2\,du\,dv + (\Delta u)^2 (dz)^2 + d\vec{x}^{\,2}$$

- New identifications

$$(u, v, z, \vec{x}) \sim (u, v, z + 2\pi n, \vec{x})$$

What happens if we study simple effective QFT models on this background?

Scalar QED on the NBO

$$\mathrm{S}_{sQED} = \int_\Omega \mathrm{d}^D x \sqrt{-\det g} \left(-(D^\mu \phi)^* D_\mu \phi - M^2 \phi^* \phi - \frac{1}{4} f^{\mu\nu} f_{\mu\nu} - \frac{g_4}{4} |\phi|^4 \right)$$

with $\quad D_\mu \phi = (\partial_\mu - i\, e\, a_\mu)\phi \qquad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$

First we derive the eigenfunctions of the scalar d'Alembertian, which near the singularity $u = 0$ reads:

$$\phi(u) \propto \frac{1}{\sqrt{|u|}} e^{i\frac{C}{u}}$$

with $\boldsymbol{C \sim l^2}$, where l is the discrete momentum associated to the compact direction $\boldsymbol{z}$.

Now if we compute the interaction contact term of N scalar $\boldsymbol{\phi_i}$ and consider the specific case where all $l_i = 0$, we obtain:

$$\mathcal{A}_N \propto \int_{u\sim 0} du\, |u|^{-\frac{N}{2}+1}$$

This result is technically not integrable for $N \geq 4$.
The absence of a continuous exponential term (the C_i's have isolated zeros due to the discrete momenta) does not allow for a distributional interpretation of A_N near the singularity.

The origin of the divergences/1

- We have shown that **fourth and higher order contact terms are ill defined** and we therefore claim that the root of the divergences lies in the **non existence of a well-defined underlying effective QFT**.

- If we think of String Theory as a UV completion of QFT amplitudes, we expect to find analogous behaviours in the string scattering amplitudes.

- That's exactly what happens, for example, with the open string 3-point amplitude with two tachyons and the first massive state, since it may be divergent when some physical polarizations are chosen and all discrete momenta $l_i = 0$:

$$\mathcal{A}^{open}_{TTM} \propto \int_{u\sim 0} du \, \frac{1}{|u|^{5/2}} \, \mathrm{tr}\left(\{T_{(1)}, T_{(2)}\} T_{(3)}\right)$$

[*AA, R.Finotello, I.Pesando (2020)*]

The origin of the divergences/2

- We found an analogous result on the boost orbifold (BO).
- The gravitational backreaction may however reappear at the one loop level in the open string case.
- A recent analysis of these models at the quantum mechanical level suggests that the full Hamiltonian theory is well-defined. [*I.Pesando (2022)*]
- **Is it possible to recover a meaningful perturbative formulation on this background?**

Seiberg-Witten Map

Inspired by the resolution of some static orbifold singularities, we add a Kalb-Ramond background B-field:

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left(g_{\mu\nu}\partial_a x^\mu \partial^a x^\nu - 2\pi i\alpha' \epsilon^{ab} B_{\mu\nu}\partial_a x^\mu \partial_b x^\nu \right) \quad \text{with} \quad B = B_{-2}\, dx^- \wedge dx^2 .$$

Applying the Seiberg-Witten map $\dfrac{1}{(g+2\pi\alpha' B)_{\mu\nu}} = (G + \dfrac{\theta}{2\pi\alpha'})^{\mu\nu}$ we get:

$$G^{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -(2\pi\alpha' b)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \theta^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (2\pi\alpha')^2 b \\ 0 & -(2\pi\alpha')^2 b & 0 \end{pmatrix} \quad \text{with} \quad b = B_{-2} .$$

In the decoupling zero slop limit $\alpha' \to 0$ with $b \sim (\alpha')^{-2}$:

- $\theta = (2\pi\alpha')^2 b$ stays finite
- $G^{\mu\nu} = \eta^{\mu\nu}$ (after a coordinate change)
- Boundary propagator: $\langle x^\mu(\tau) x^\nu(\tau') \rangle = -\alpha' G^{\mu\nu} \log(\tau - \tau')^2 + \frac{i}{2}\theta^{\mu\nu}\epsilon(\tau - \tau')$

Noncommutative QFT

In the ordinary QFT, the N-point scalar function reads

$$\mathcal{A}_N \propto \int_{u\sim 0} du\,|u|^{-\frac{N}{2}+1}$$

In the noncommutative QFT, the N-point scalar function reads

$$\mathcal{A}_N \propto \int_{u\sim 0} du\,|u|^{-\frac{N}{2}+1} e^{i\frac{\theta f(p_{-i})}{u}}$$

with $f(p_{-i})$ a combination of the continuous momenta p_{-i} along the u direction

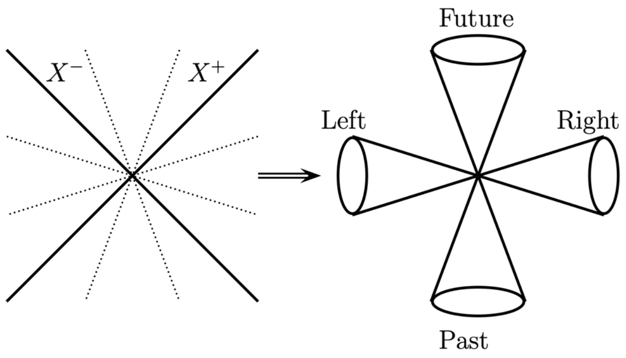
This means that we are avoiding the presence of isolated zeros and therefore **noncommutativity makes plausible the interpretation of A_N as a distribution**: in particular, this integral satisfies the properties of ***distributionally integrable functions*** defined by R.Estrada and J.Vindas in 2012.

Time-dependent orbifolds: the other models

Up to ISO(2,1) conjugations, there exist only four different $\mathbb{M}^3$ time-dependent orbifolds:

Orbifold	Generator
Boost (BO)	$2\pi i \Delta J_{+-}$
Null-Boost (NBO)	$2\pi i \Delta J_{+2}$
Shifted-Boost (SBO)	$2\pi i (\Delta J_{+-} + RP_2)$
O-Plane (OPO)	$2\pi i (\Delta J_{+2} + RP_-)$

[*L.Cornalba, M.S.Costa (2005)*]



The Boost Orbifold

To sum up

- Time-dependent orbifolds are useful toy models to reproduce and study cosmological singularities

- The origin of the unexpected divergences which appear in string scattering amplitudes can be traced back to the failure of the standard Feynman diagrammatic approach of the underlying effective QFT

- The noncommutative QFT which arises as a decoupling limit after the introduction of a B-field seems promisingly well-defined

Conclusions and Outlooks

- Understand and define better the noncommutative QFT [*AA, I.Pesando, R.J.Szabo (2023)*]
- Extend this singularity resolution to the other time-dependent orbifolds
- Discuss the relation of the noncommutative QFT with different decoupling limits which lead to Noncommutative Open String Theory (S-duality? OM theory?)
- Verify if we can recover a full string perturbation theory in presence of the B-field, even beyond tree-level

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Thank you
for your attention!