

# Primordial Black Holes: formation and cosmological impact in the current Universe

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Gravitational Wave Probes of Fundamental Physics

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# Equation of State of the Early Universe

The radiation dominated Universe goes through 3 main transitions:

- Electroweak phase-transition
- **QCD phase-transition** (crossover)

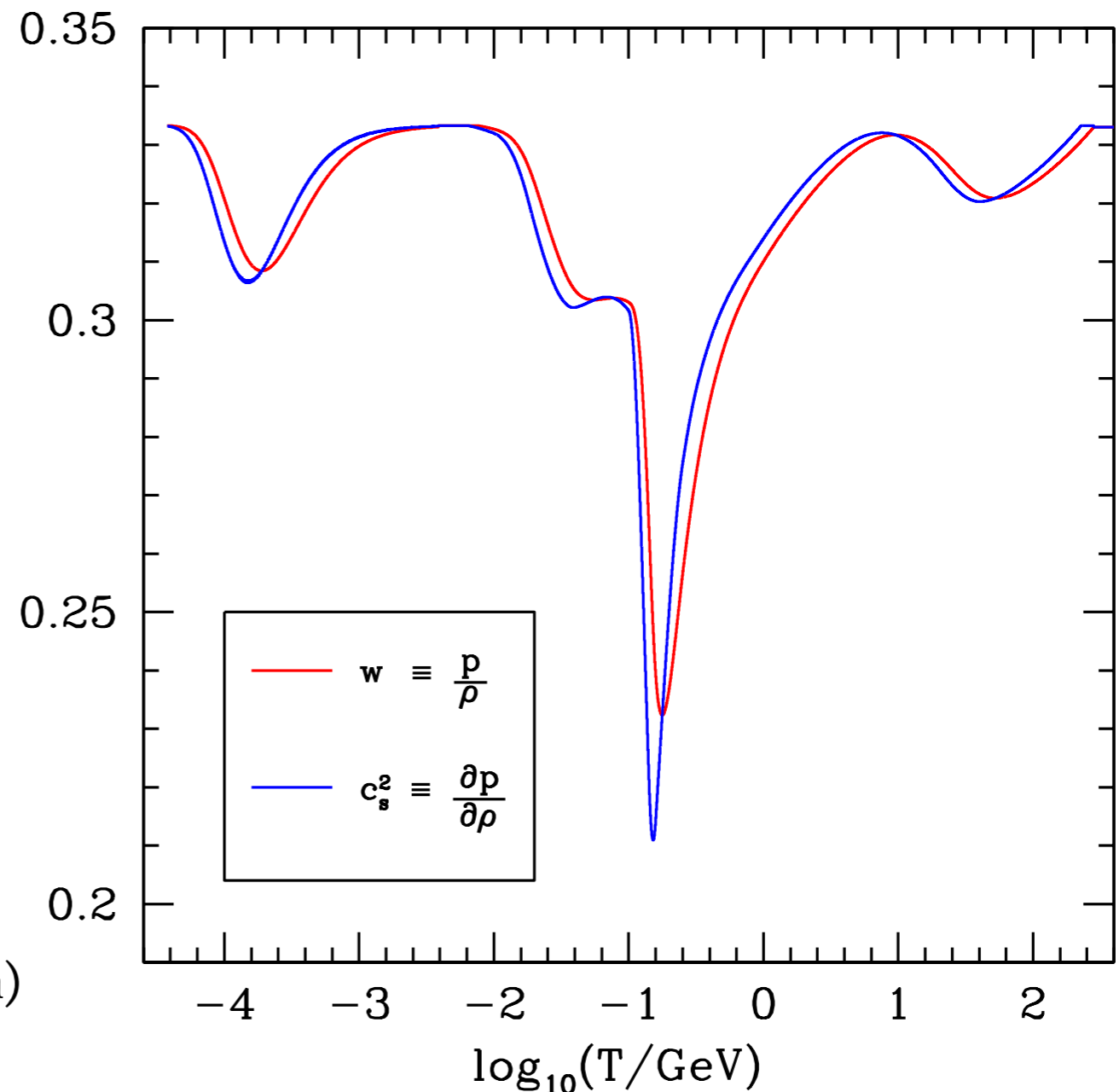
$$\rho = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4 \quad s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3$$

$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\text{eff}}(T)}{3g_{\text{eff}}(T)} - 1$$

$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\text{eff}} + Th'_{\text{eff}})}{3(4g_{\text{eff}} + Tg'_{\text{eff}})} - 1$$

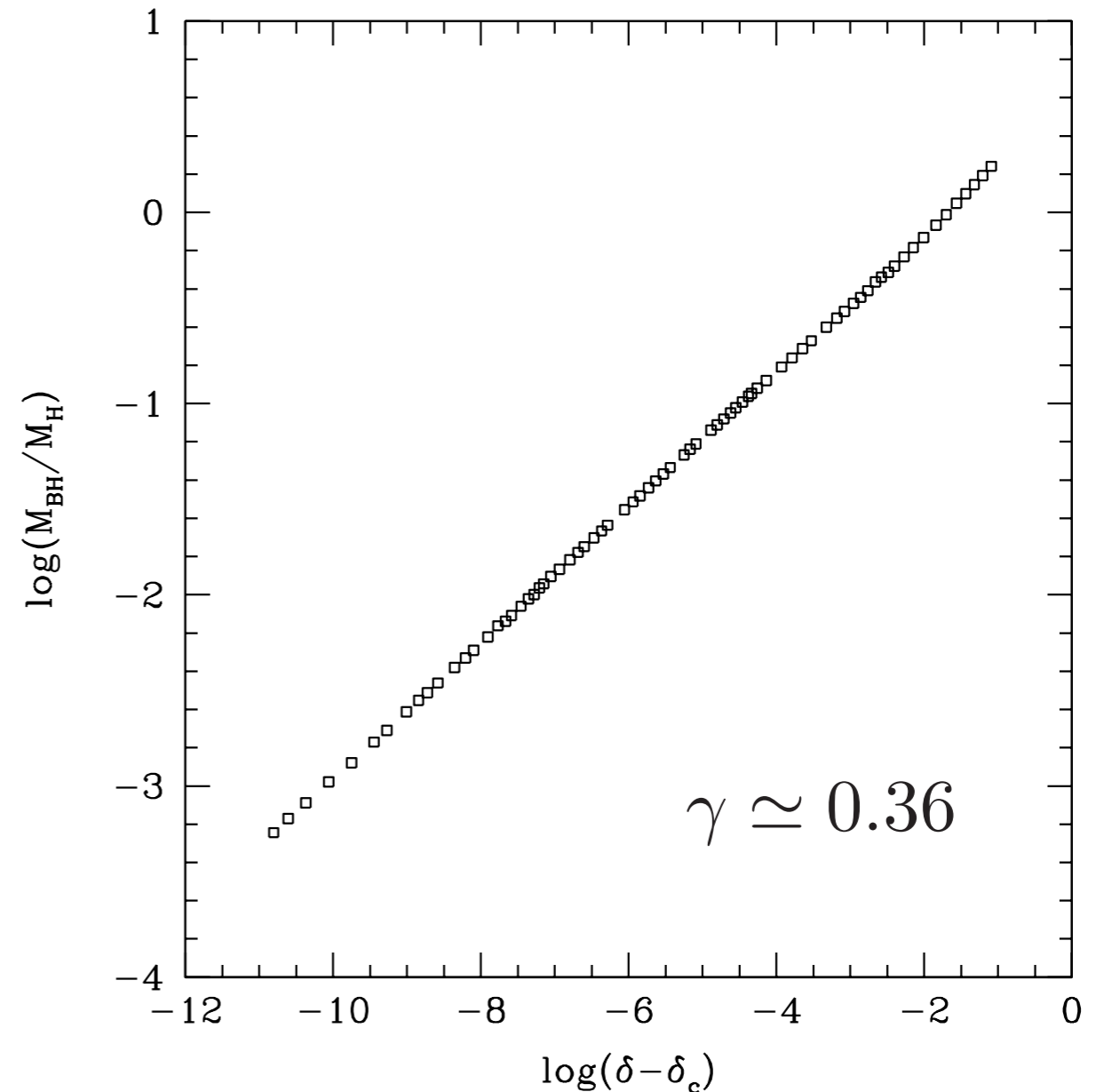
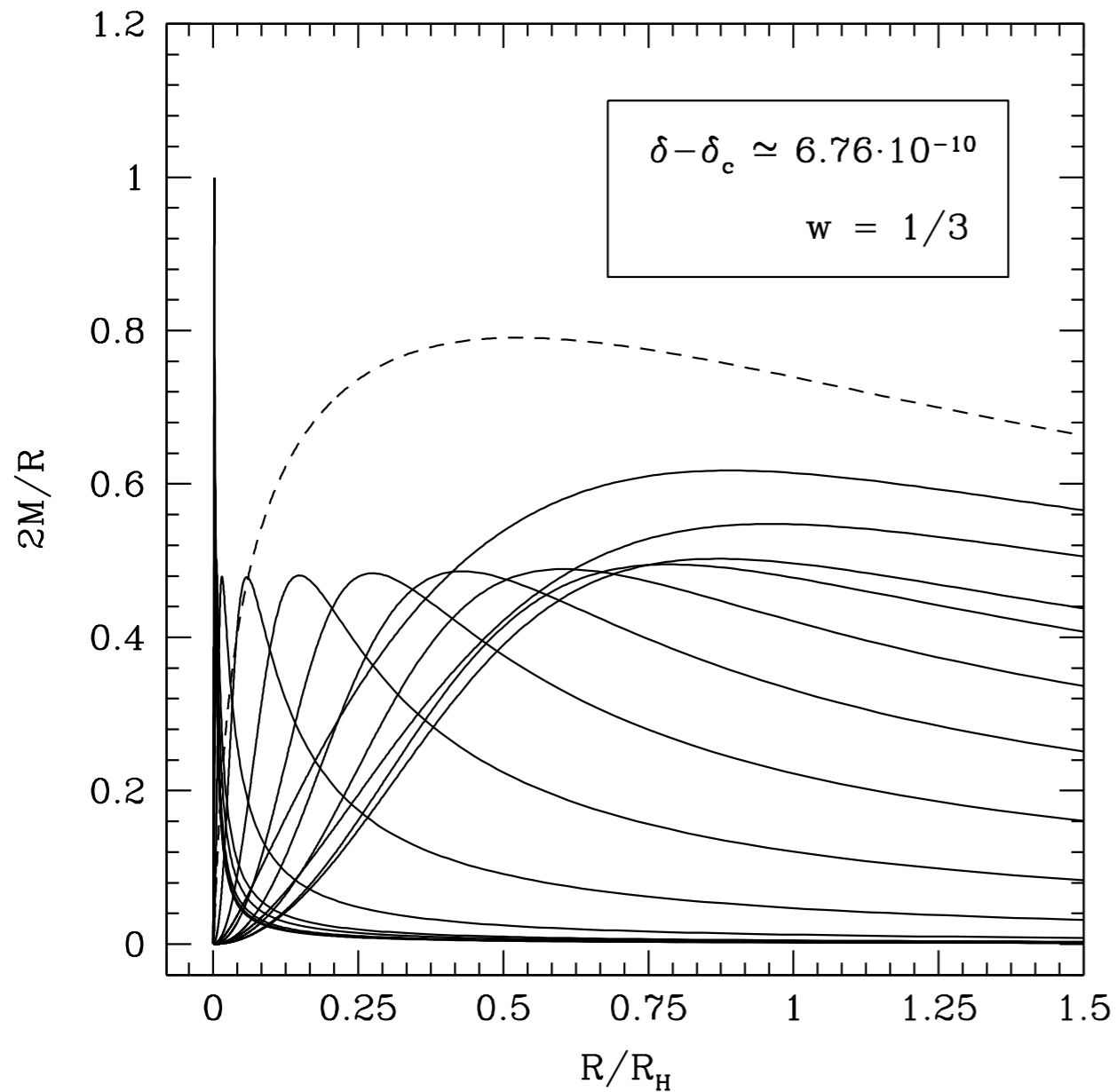
- Nucleosynthesis (e+e- annihilation)



# PBH formation in radiation dominated Universe

$$R(r, t) = 2M(r, t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$



## Initial conditions: curvature profile

- The asymptotic metric ( $t \rightarrow 0$ ), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)} [dr^2 + r^2 d\Omega^2]$$

- In the “linear regime” of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the quasi-homogeneous / gradient expansion approach.

$$\frac{\delta\rho}{\rho_b} = - \left( \frac{1}{aH} \right)^2 \frac{4}{9} \left[ \nabla^2 \zeta(r) + \frac{1}{2} (\nabla \zeta(r))^2 \right] e^{-2\zeta(r)}$$

- The perturbation amplitude  $\delta$  is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

$$\mathcal{C}(r) := \frac{2[M(r, t) - M_b(r, t)]}{R(r, t)} = -\frac{4}{3} \tilde{r} \zeta'(r) \left[ 1 + \frac{1}{2} \tilde{r} \zeta'(r) \right] \Rightarrow \delta = \delta_G \left[ 1 - \frac{3}{8} \delta_G \right]$$

# PBH Abundance (Peak Theory)

*C.Germani, IM - PRL (2019)*

*C. Yoo, T. Harada, J. Garriga, K. Kohri (2019)*

• PDF of  $\delta$  follows a Gaussian distribution: 
$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (kr_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left( \frac{k_*}{a_m H_m} \right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \quad \nu_c \equiv \frac{\delta_c}{\sigma}$$

• **If**  $M_{PBH} \sim 10^{16} g$  are Dark Matter  $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

• **Narrow peak:**  $\frac{k_*}{\sigma} \gg 1 \Rightarrow \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \Rightarrow \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$

• **Broad peak:**  $\frac{k_*}{\sigma} \ll 1 \Rightarrow \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \Rightarrow \mathcal{P}_0 \sim 3 \times 10^{-3}$

• **Non linear effects:**  $\delta = \delta_G \left[ 1 - \frac{3}{8} \delta_G \right] \Rightarrow 1.5 \lesssim \frac{\mathcal{P}_{0NL}}{\mathcal{P}_{0L}} = \frac{16 \left( 1 - \sqrt{1 - \frac{3}{2} \delta_c} \right)^2}{9\delta_c^2} \lesssim 4$

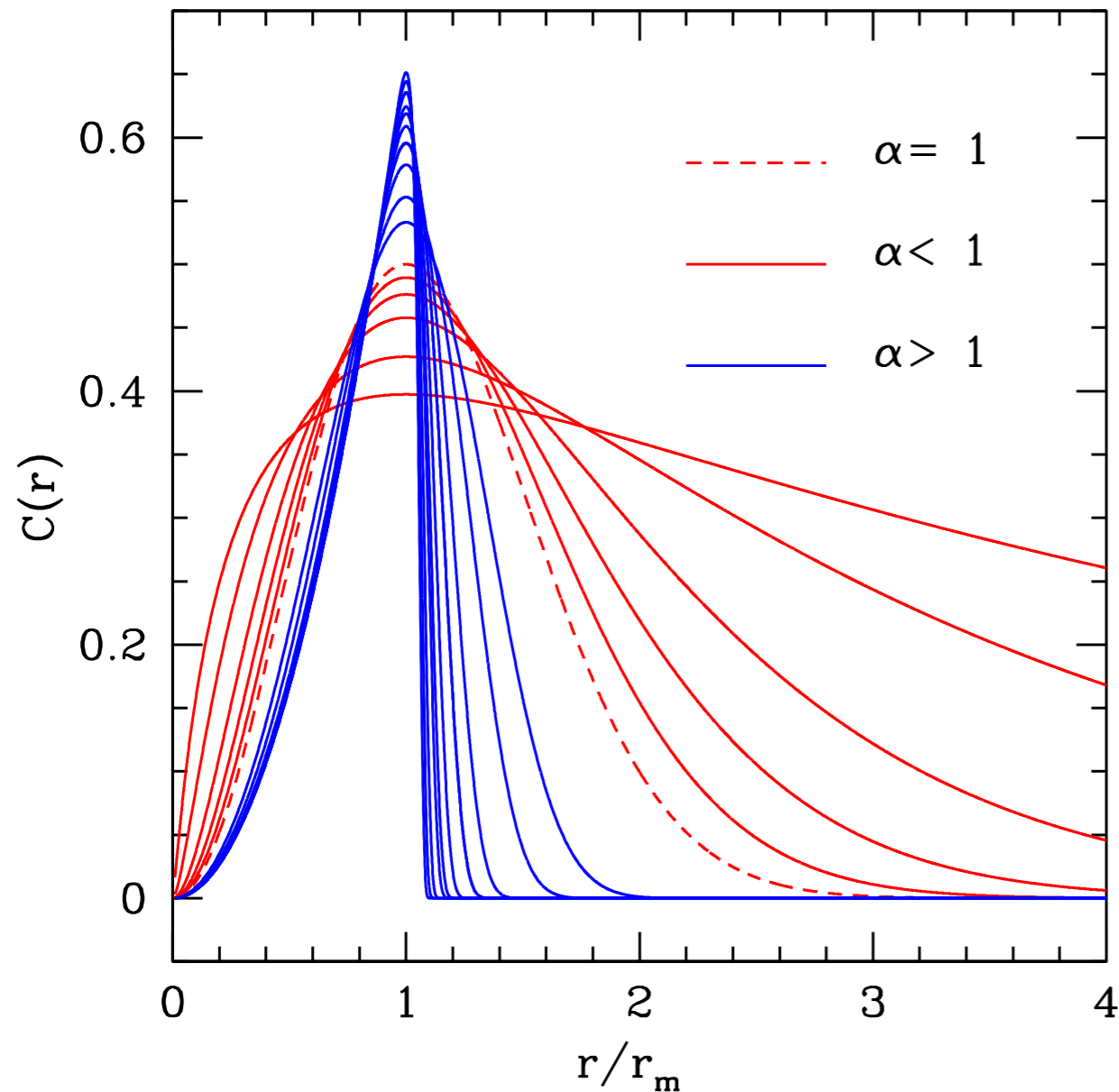
*S.Young, IM, C.Byrnes JCAP (2019)*

*De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)*

# PBH threshold

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\alpha \equiv -\frac{\mathcal{C}''(\tilde{r}_m) \tilde{r}_m^2}{4\mathcal{C}(\tilde{r}_m)} = \frac{\alpha_G}{(1 - \frac{1}{2}\Phi_m)(1 - \Phi_m)}$$



$$\tilde{r} = r e^{\zeta(r)}$$

$$0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}$$

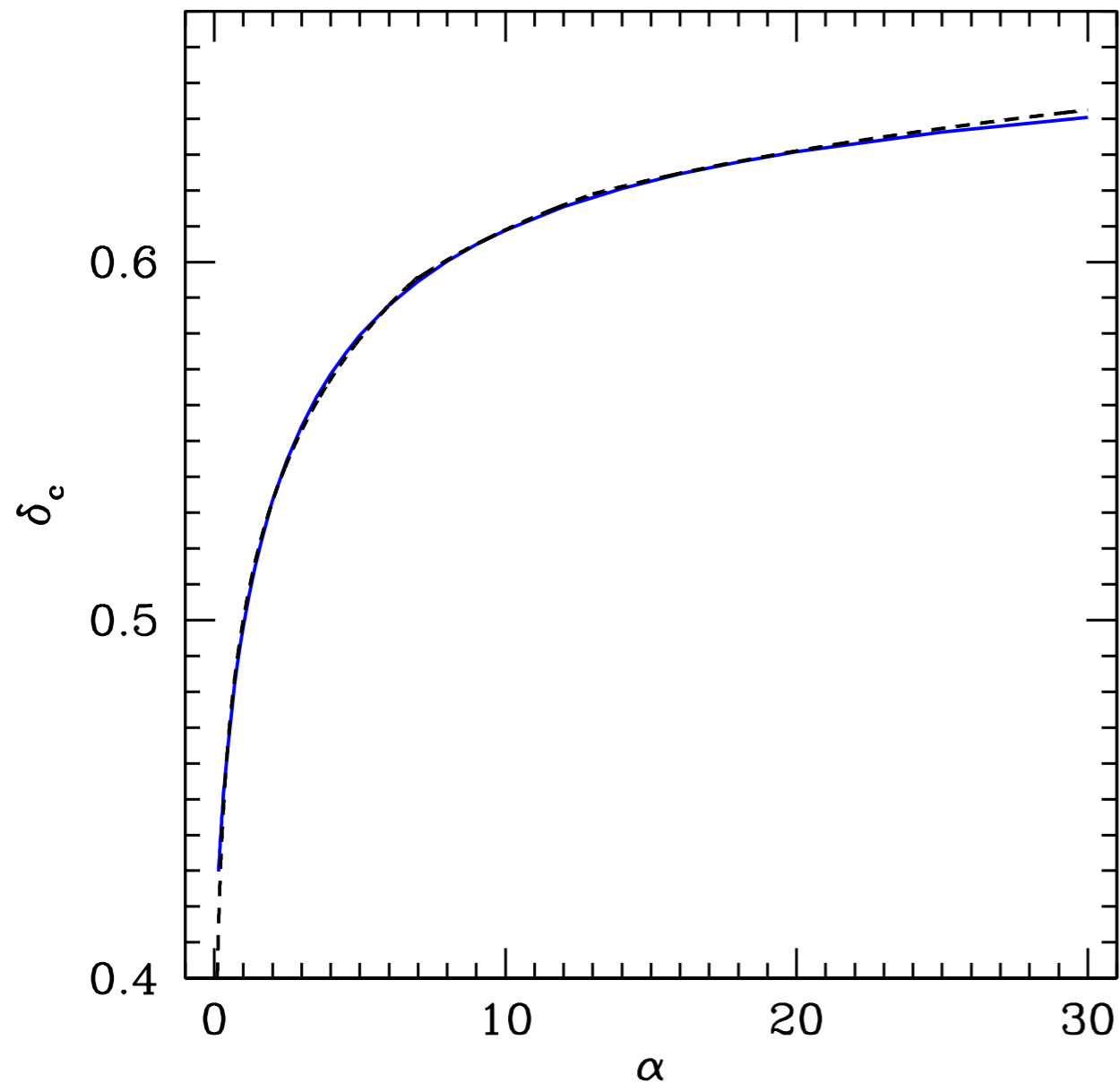
$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

*I. Musco - PRD (2019)*

# PBH threshold

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*I. Musco - PRD (2019)*

# PBH threshold prescription

Curvature power spectrum  $\mathcal{P}_\zeta$



Characteristic overdensity scale  $k_* \hat{r}_m$



Characteristic shape parameter  $\alpha$



Threshold  $\delta_c$

*IM, De Luca, Franciolini, Riotto - PRD (2021)*

1. **The power spectrum of the curvature perturbation:** take the primordial power spectrum  $\mathcal{P}_\zeta$  of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function  $T(k, \eta)$

$$P_\zeta(k, \eta) = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) T^2(k, \eta).$$

2. **The comoving length scale  $\hat{r}_m$**  of the perturbation is related to the characteristic scale  $k_*$  of the power spectrum  $P_\zeta$ . Compute the value of  $k_* \hat{r}_m$  by solving the following integral equation

$$\int dk k^2 \left[ (k^2 \hat{r}_m^2 - 1) \frac{\sin(k \hat{r}_m)}{k \hat{r}_m} + \cos(k \hat{r}_m) \right] P_\zeta(k, \eta) = 0.$$

3. **The shape parameter:** compute the corresponding shape parameter  $\alpha$  of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha) [1 + F(\alpha)] \alpha = -\frac{1}{2} \left[ 1 + \hat{r}_m \frac{\int dk k^4 \cos(k \hat{r}_m) P_\zeta(k, \eta)}{\int dk k^3 \sin(k \hat{r}_m) P_\zeta(k, \eta)} \right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5} e^{-1/\alpha} \frac{\alpha^{1-5/2\alpha}}{\Gamma(\frac{5}{2\alpha}) - \Gamma(\frac{5}{2\alpha}, \frac{1}{\alpha})}}.$$

4. **The threshold  $\delta_c$ :** compute the threshold as function of  $\alpha$ , fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ( $aHr_m = 1$ ).

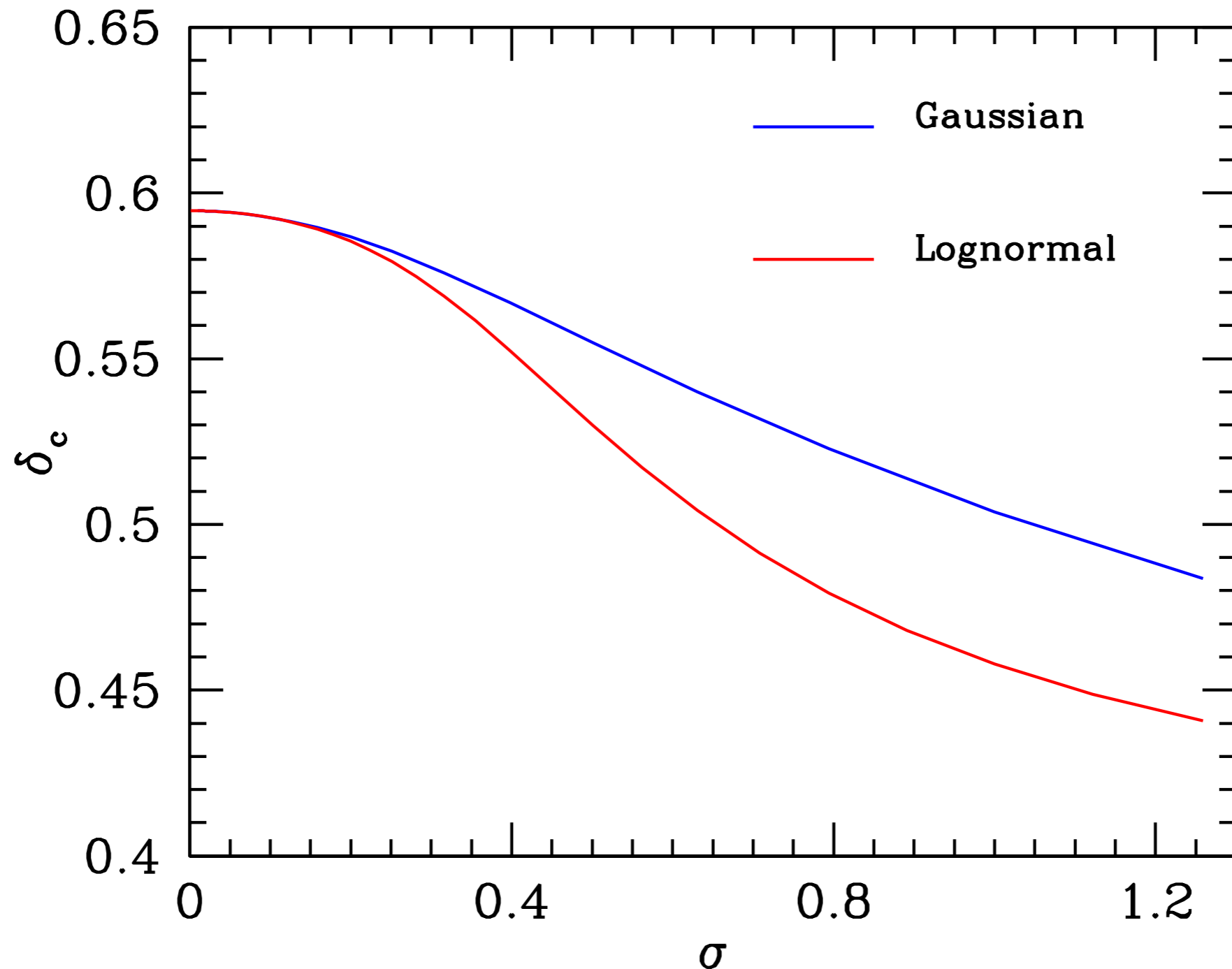
$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$



## Power Spectrum:

Gaussian:  $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp \left[ -(k - k_*)^2 / 2\sigma^2 \right]$

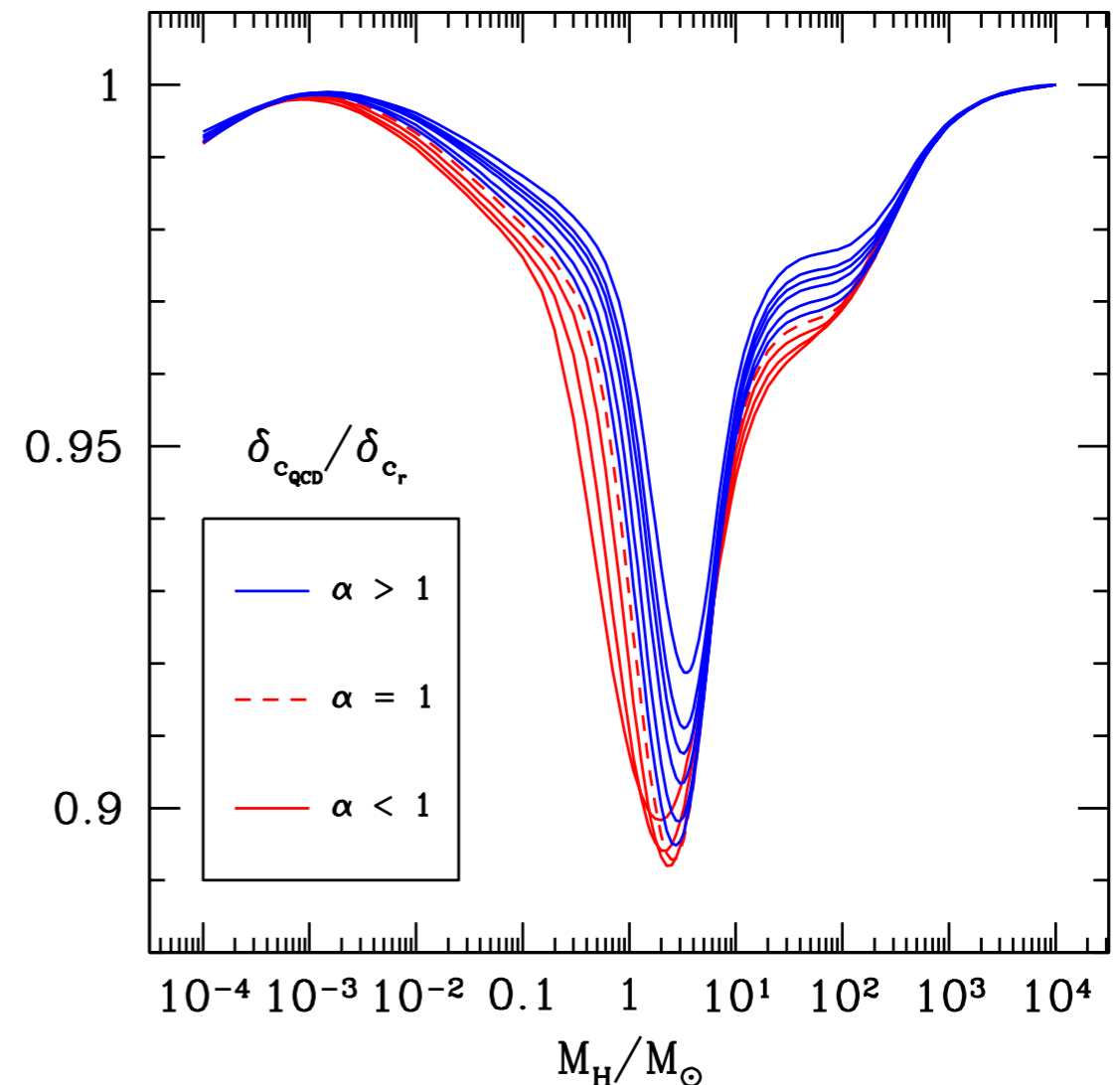
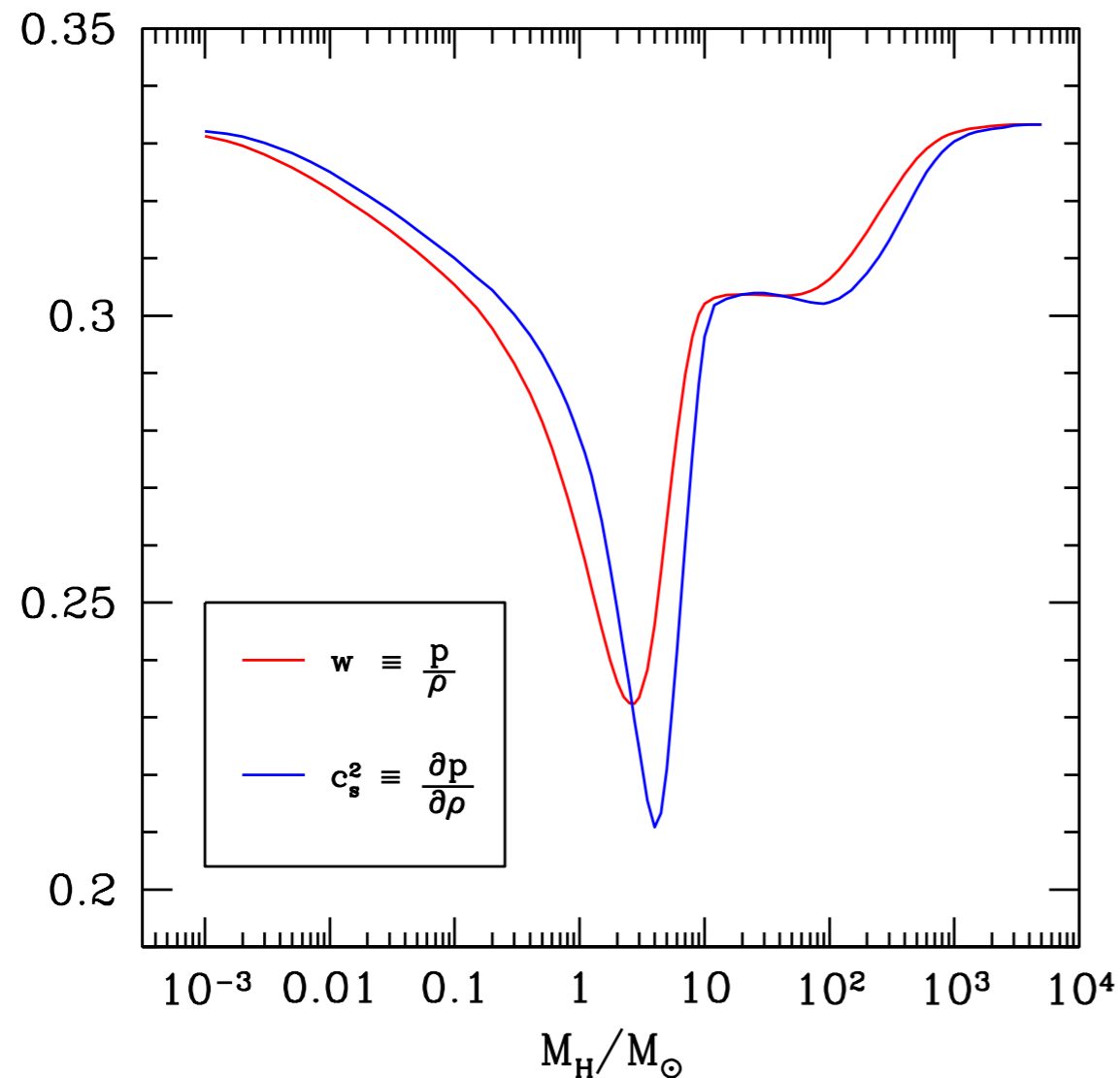
Lognormal:  $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp \left[ -\ln^2 (k/k_*) / 2\sigma^2 \right]$



# PBH Threshold during the QCD

*IM, K. Jedamzik, Sam Young - arXiv:2303.07980*

*Escriva, Bagui, Clesse (2023)*



Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of  $w(T)$ .

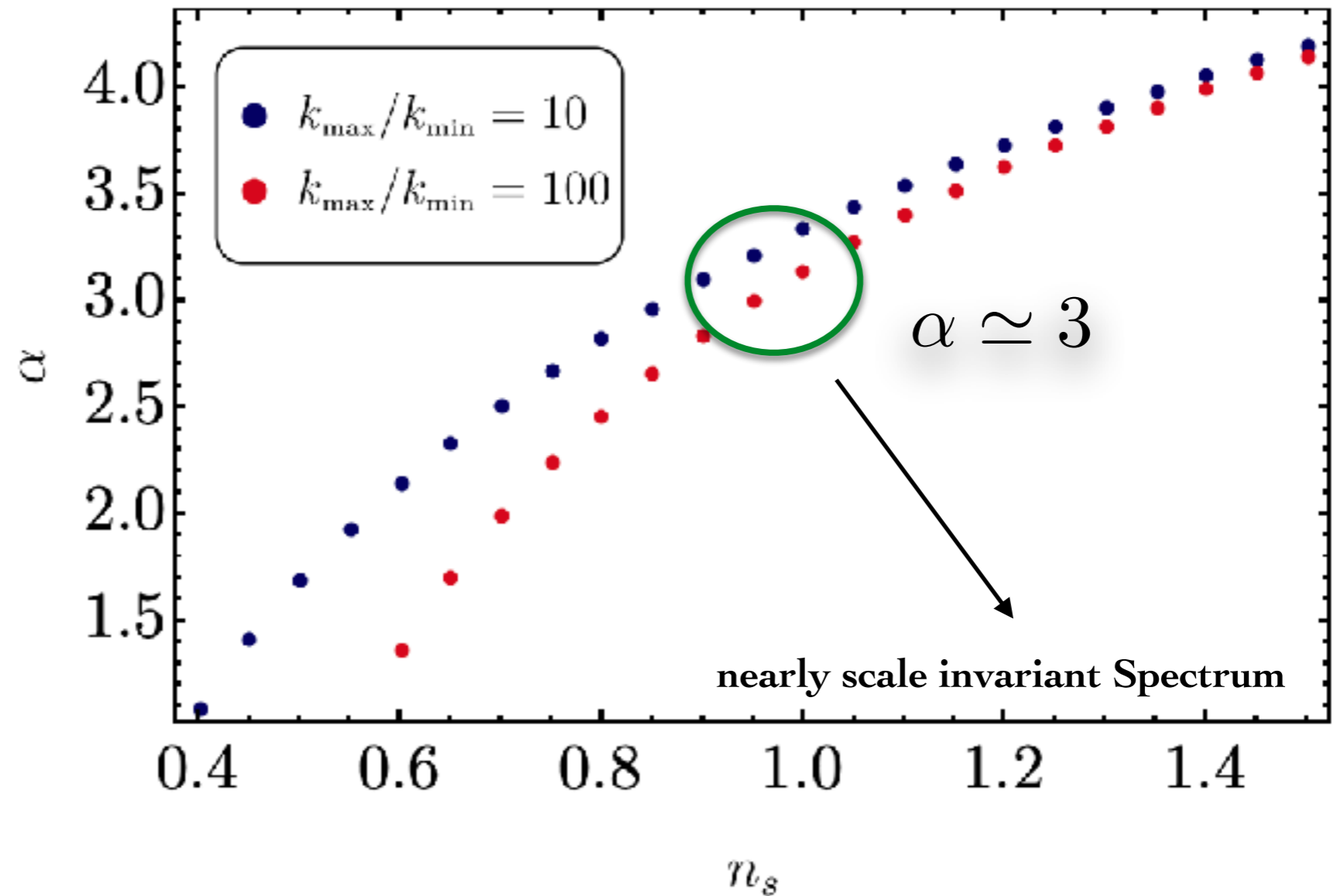
Significant enhancement of PBH formation around the solar mass scale: abundance increased of about  $O(3)$  with respect radiation!

## Scale invariant Power Spectrum

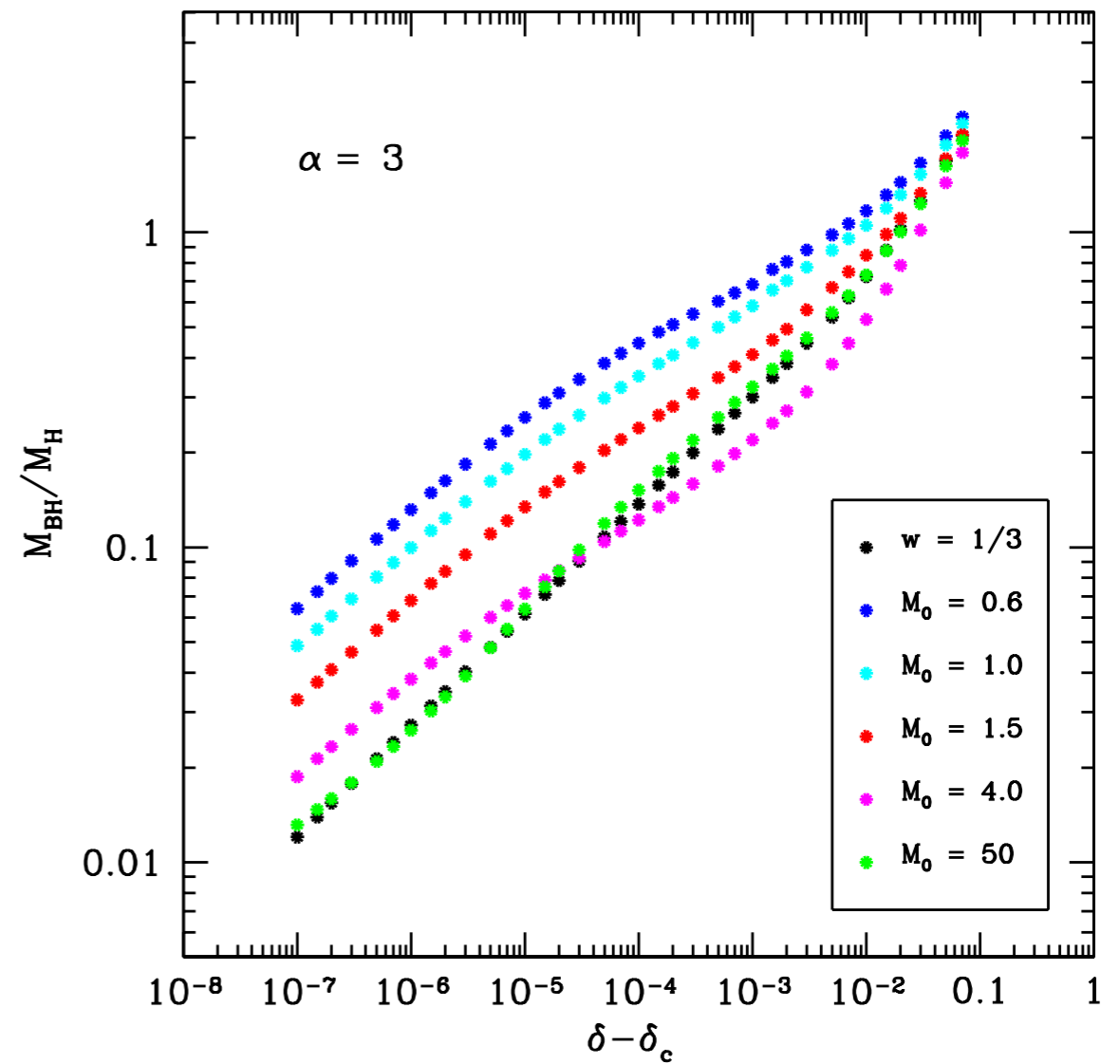
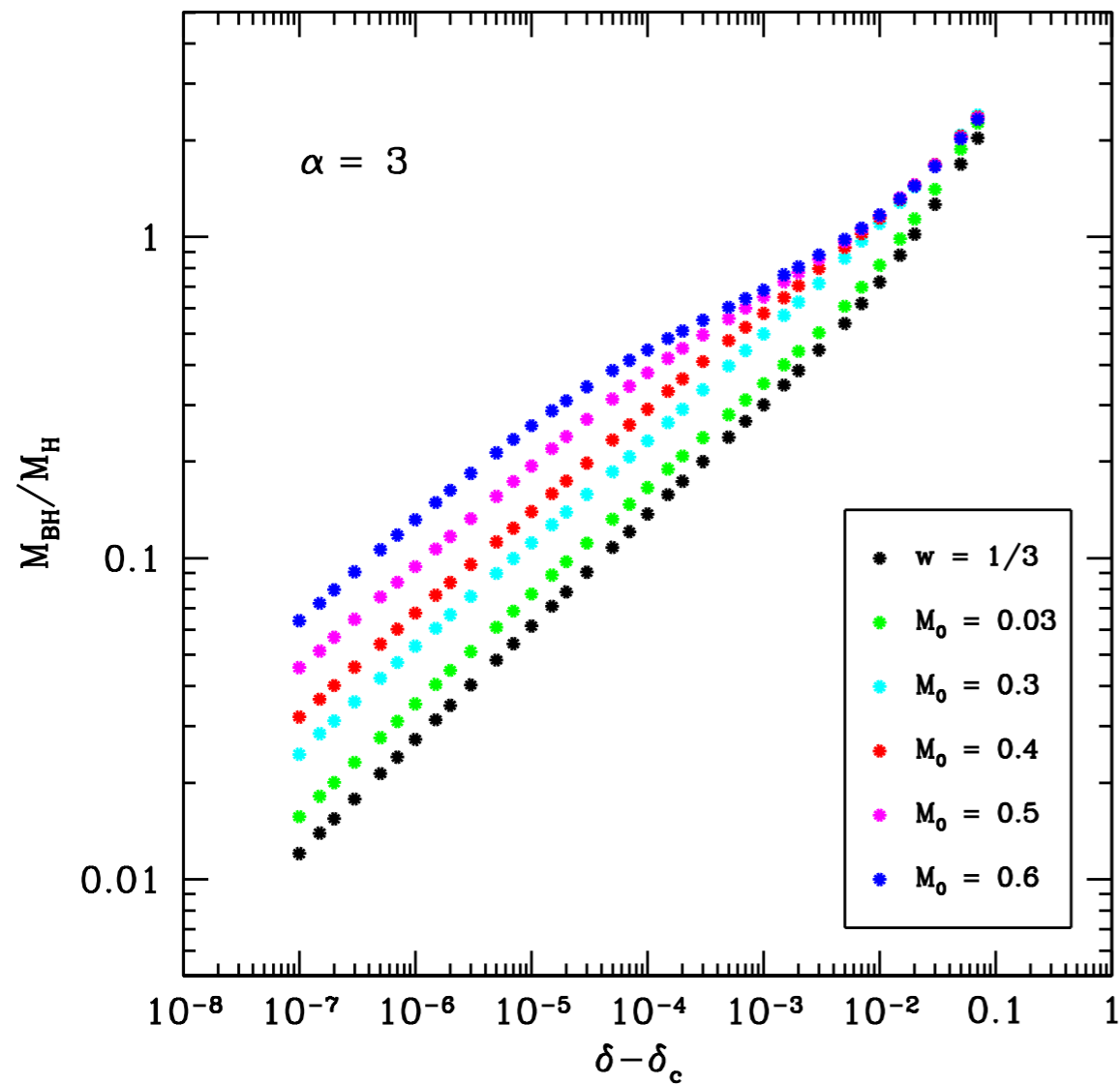
$$P_{\zeta}(k) = A (k/k_{\min})^{n_s - 1} \Theta(k - k_{\min}) \Theta(k_{\max} - k)$$

$n_s$  — spectrum tilt

$k_{\max}/k_{\min}$  — cut-off scale



# PBH scaling law during the QCD



$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

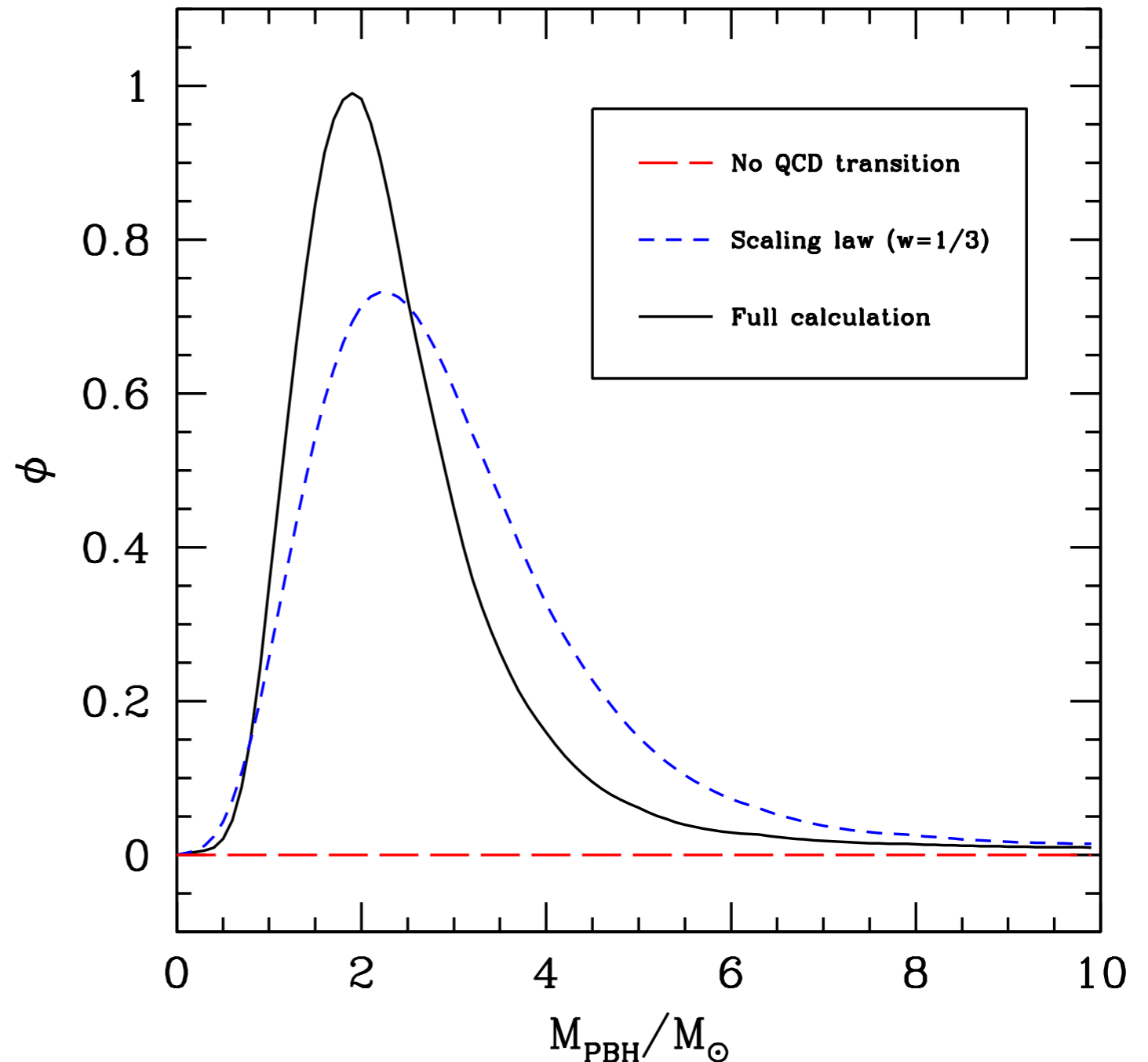
# PBH mass function during the QCD

Mass Function  $\psi(m_{\text{PBH}})$ : fraction of PBHs with mass in the infinitesimal interval of  $M_{\text{PBH}}$

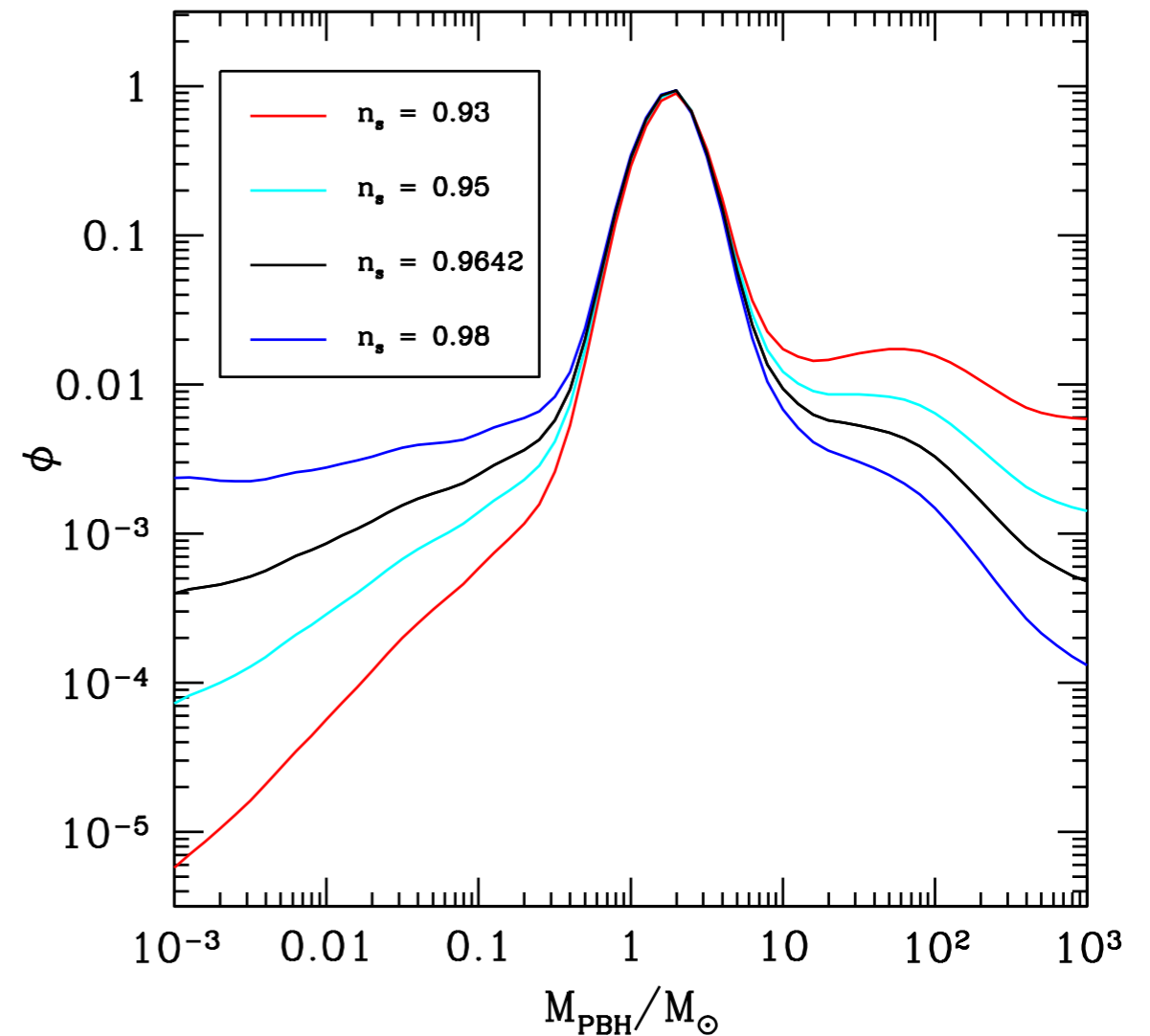
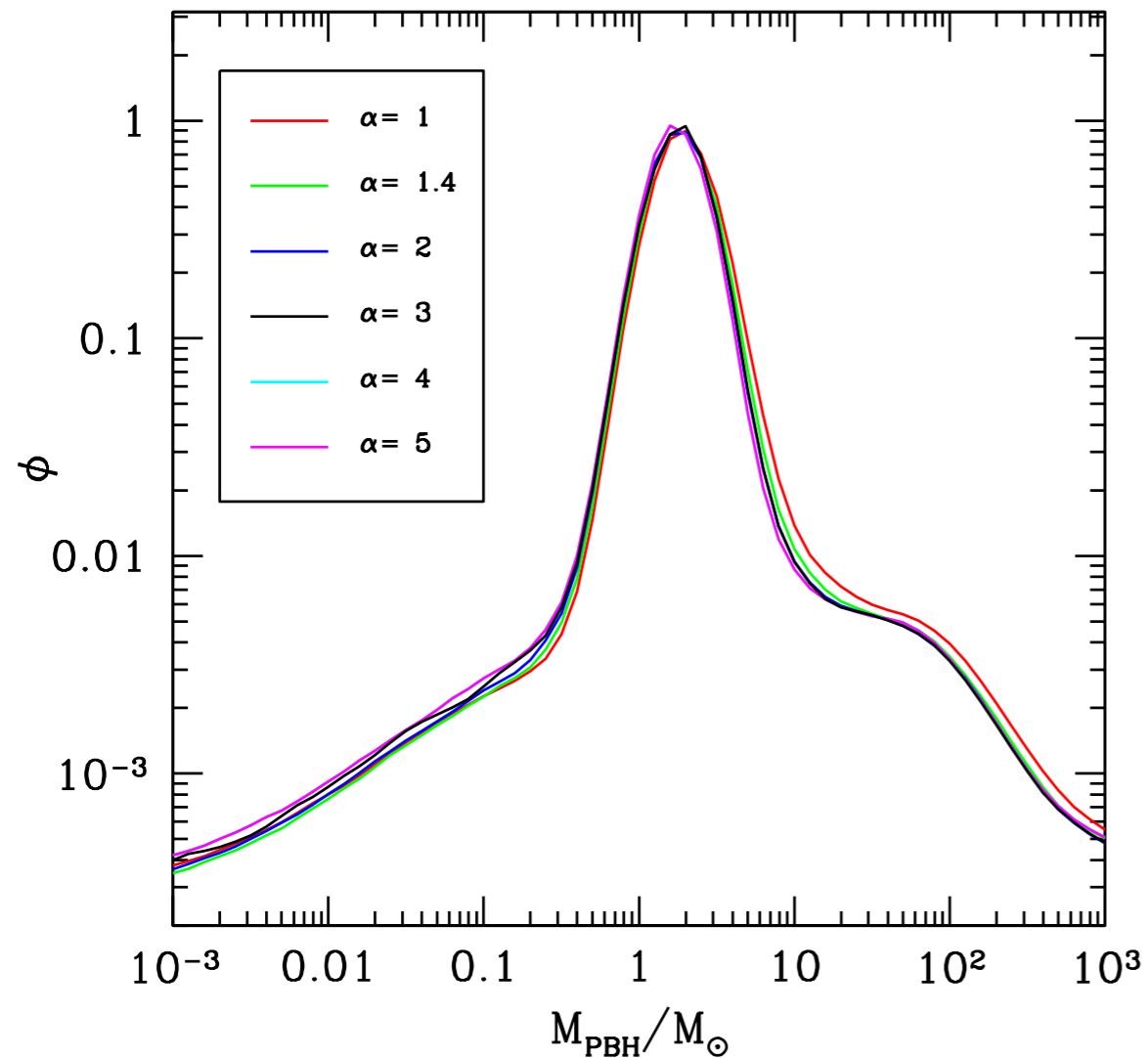
$$\phi(M_{\text{PBH}}) = \frac{1}{\Omega_{\text{PBH}}} \frac{d\Omega_{\text{PBH}}}{dM_{\text{PBH}}}$$

$$\int dm_{\text{PBH}} \phi(m_{\text{PBH}}) = 1$$

- The main effect is given by the modification of the threshold.
- The modified scaling law gives a pile up of PBHs on smaller masses.

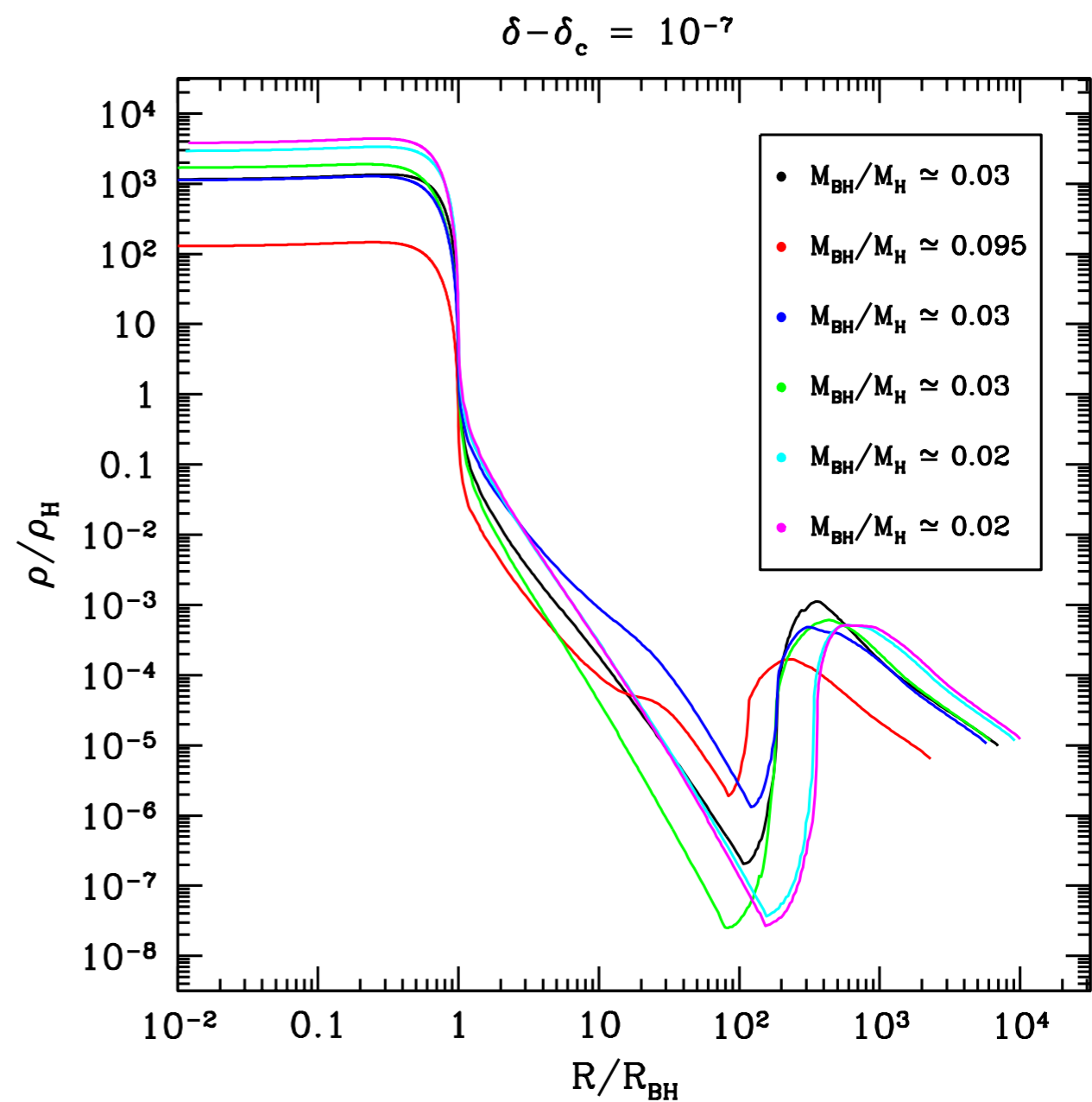
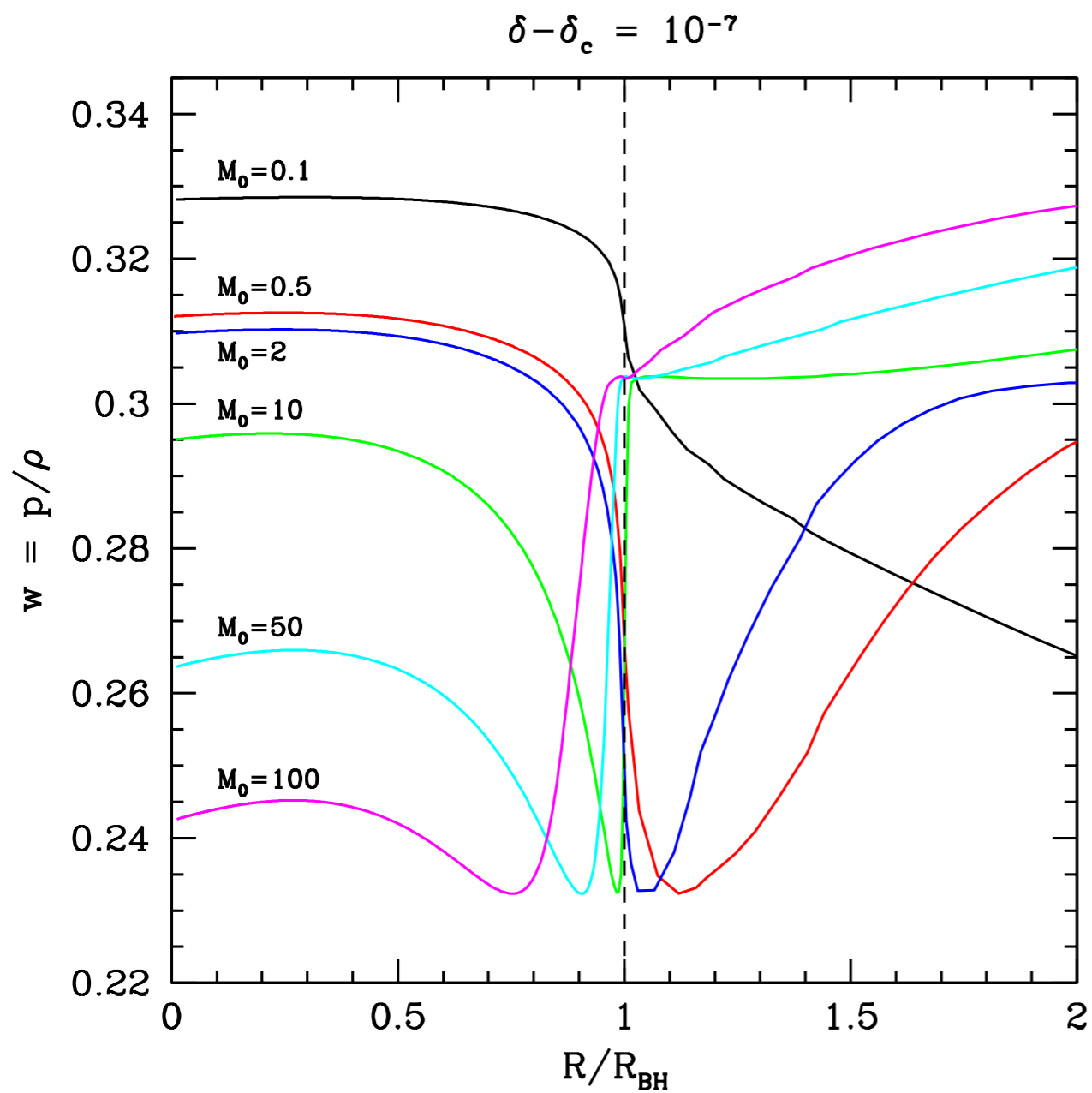


# PBH mass function during the QCD: shape/tilt dependence

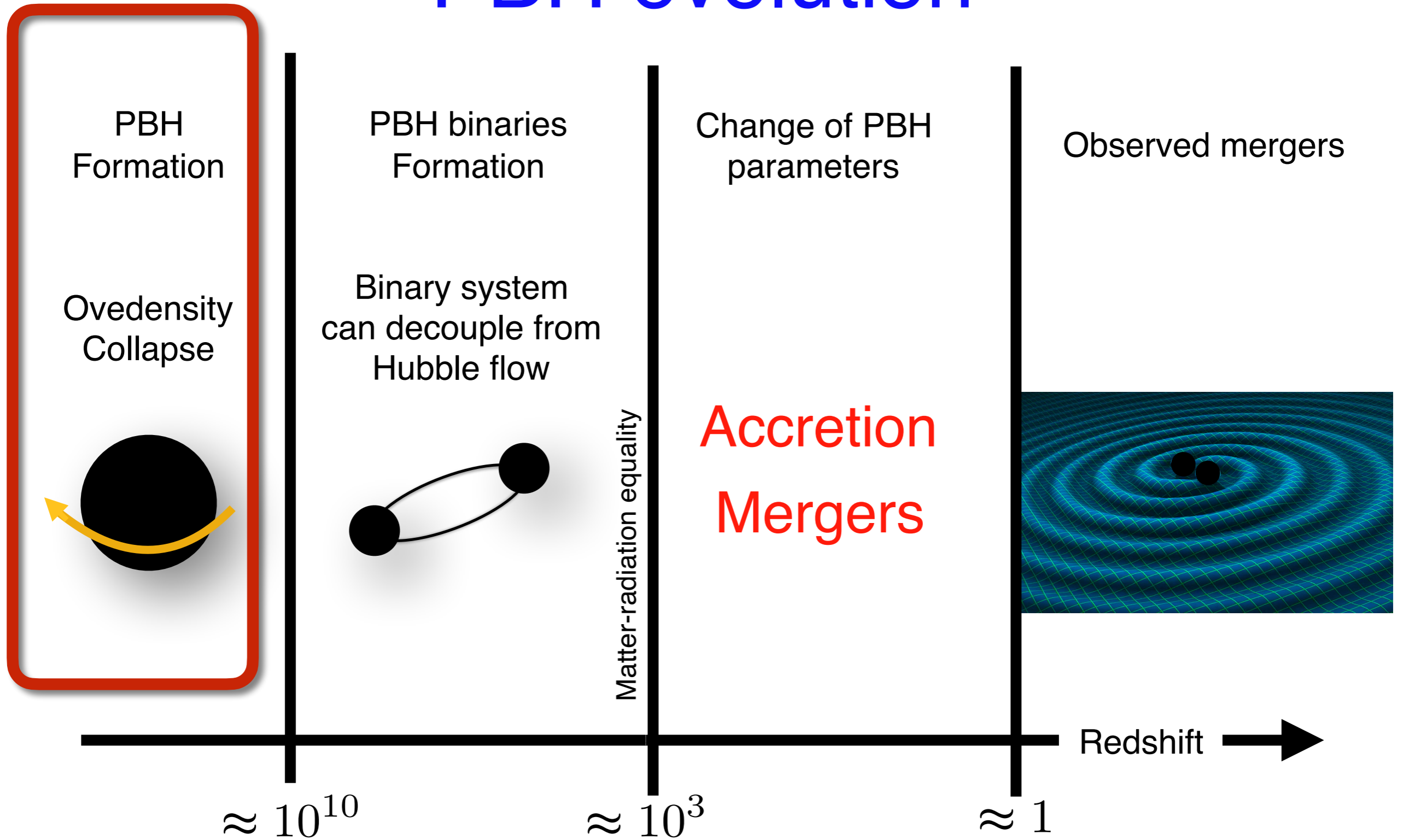


- Given the PBH abundance, the shape does not play a significant role on the mass function (attractor solution)!
- The tilt of the power spectrum does not affect the peak of the mass function.

# PBH formation during the QCD



# PBH evolution

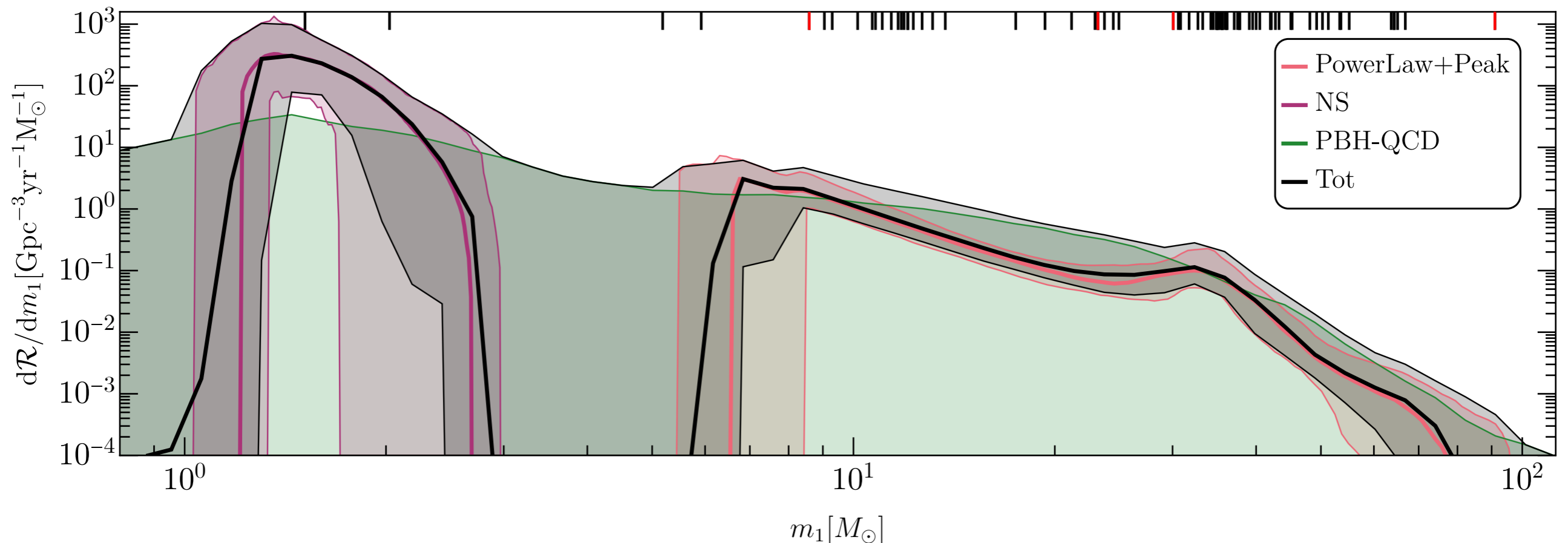




# GWs from PBH mergers

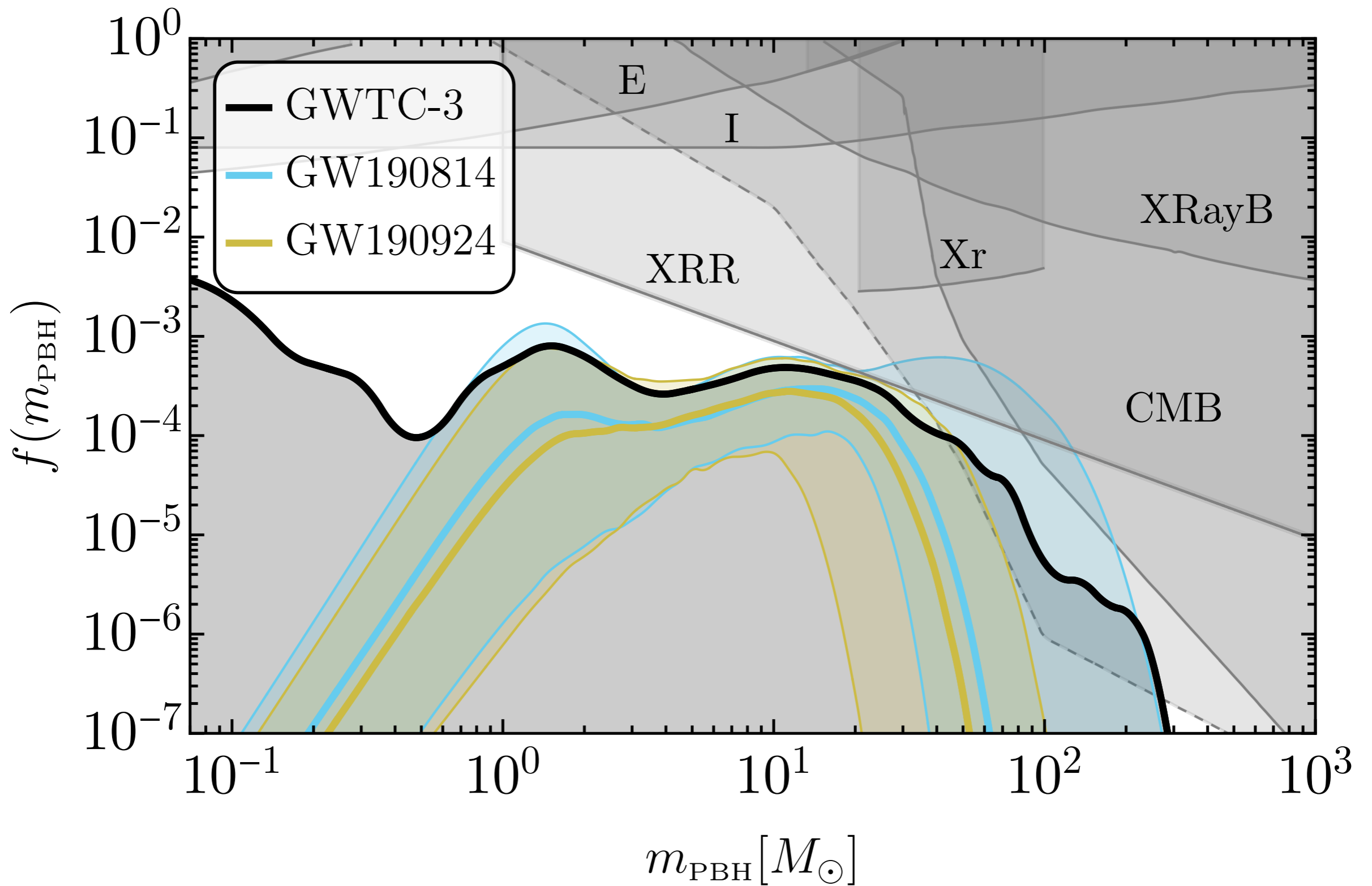
*G. Franciolini, IM, P.Pani, A urbano - PRD (2022)*

- Making **Bayesian inference analysis** we found that a sub-population of PBHs is compatible with the LVK catalog.
- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).

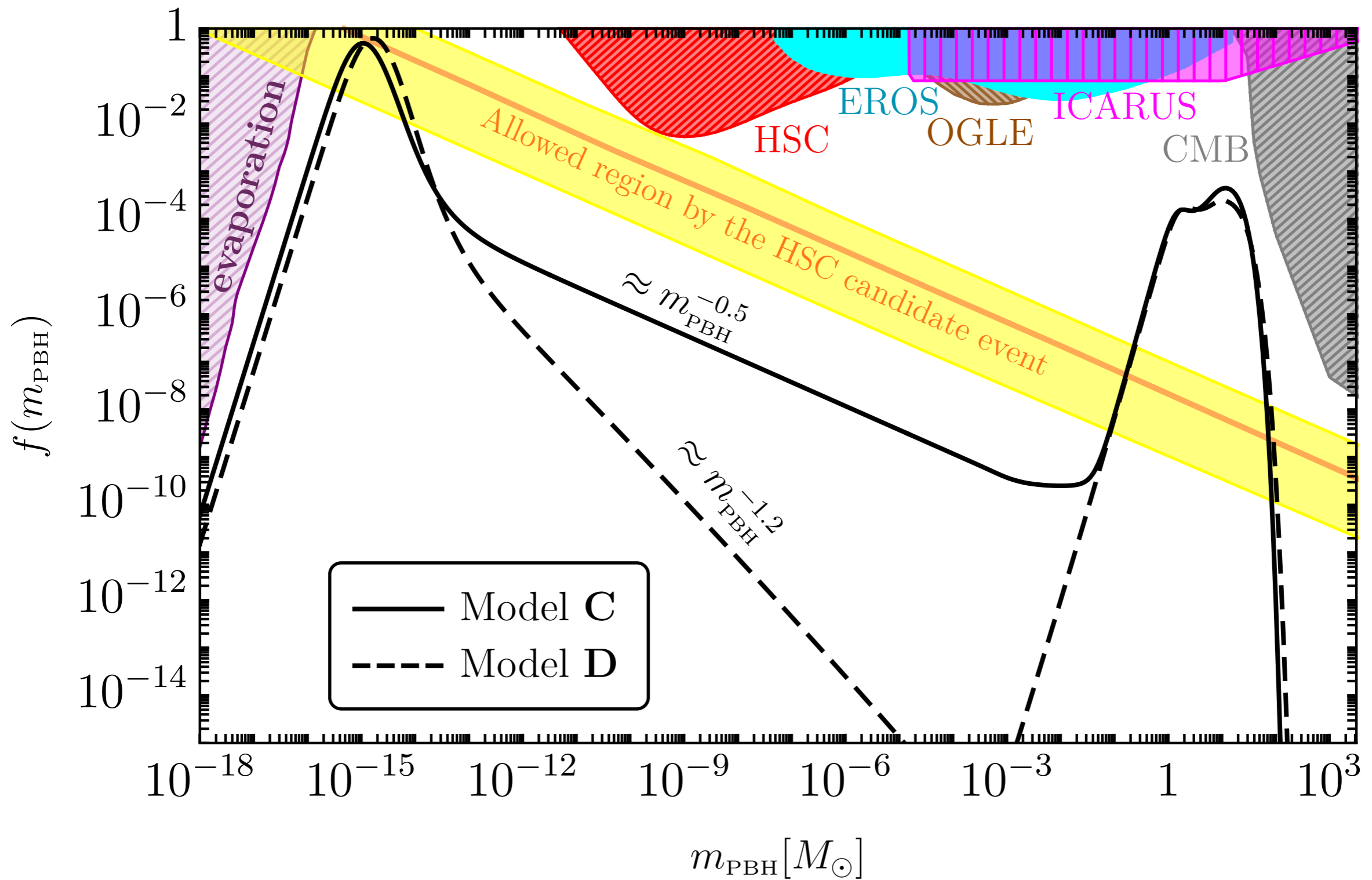


GW event	PBH prob. [%]	$m_1[M_\odot]$	$m_2[M_\odot]$
GW151012	1.2	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$
GW190412	25.4	$30.1^{+4.7}_{-5.1}$	$8.3^{+1.6}_{-0.9}$
GW190512_180714	1.6	$23.3^{+5.3}_{-5.8}$	$12.6^{+3.6}_{-2.5}$
GW190519_153544	1.5	$66.0^{+10.7}_{-12.0}$	$40.5^{+11.0}_{-11.1}$
GW190521	7.2	$95.3^{+28.7}_{-18.9}$	$69.0^{+22.7}_{-23.1}$
GW190602_175927	2.7	$69.1^{+15.7}_{-13.0}$	$47.8^{+14.3}_{-17.4}$
GW190701_203306	1.4	$53.9^{+11.8}_{-8.0}$	$40.8^{+8.7}_{-12.0}$
GW190706_222641	1.3	$67.0^{+14.6}_{-16.2}$	$38.2^{+14.6}_{-13.3}$
GW190828_065509	2.8	$24.1^{+7.0}_{-7.2}$	$10.2^{+3.6}_{-2.1}$
GW190924_021846	40.3	$8.9^{+7.0}_{-2.0}$	$5.0^{+1.4}_{-1.9}$
GW191109_010717	2.9	$65^{+11}_{-11}$	$47^{+15}_{-13}$
GW191129_134029	1.2	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$
GW190425	2.8	$2.0^{+0.6}_{-0.3}$	$1.4^{+0.3}_{-0.3}$
GW190426_152155	1.2	$5.7^{+3.9}_{-2.3}$	$1.5^{+0.8}_{-0.5}$
GW190814	29.1	$23.2^{+1.1}_{-1.0}$	$2.59^{+0.08}_{-0.09}$
GW190917_114630	3.0	$9.3^{+3.4}_{-4.4}$	$2.1^{+1.5}_{-0.5}$
GW200105_162426	3.6	$8.9^{+1.2}_{-1.5}$	$1.9^{+0.3}_{-0.2}$
GW200115_042309	1.2	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$

# PBH - DM constraints



# PBHs and Dark Matter (asteroidal mass)

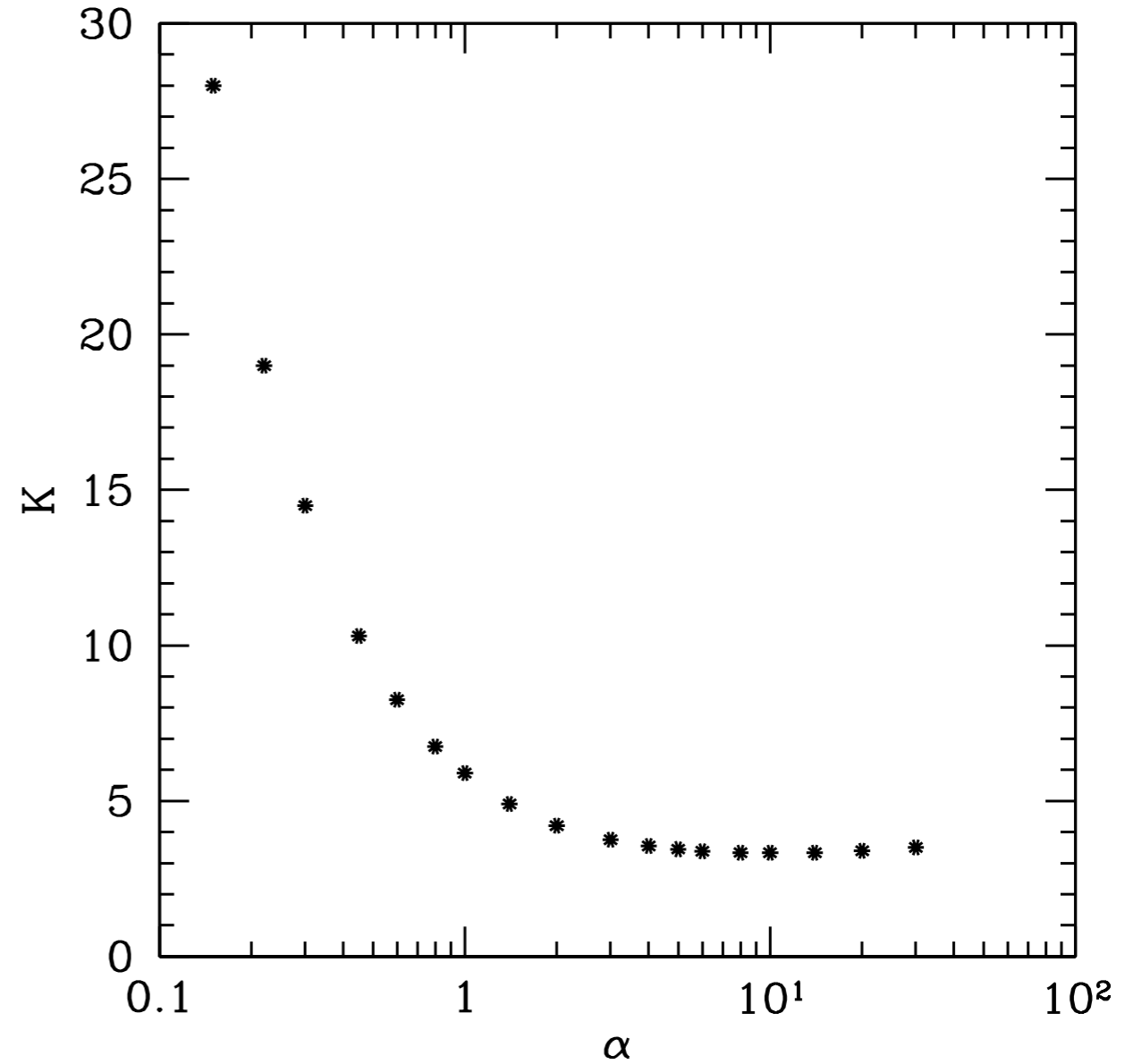
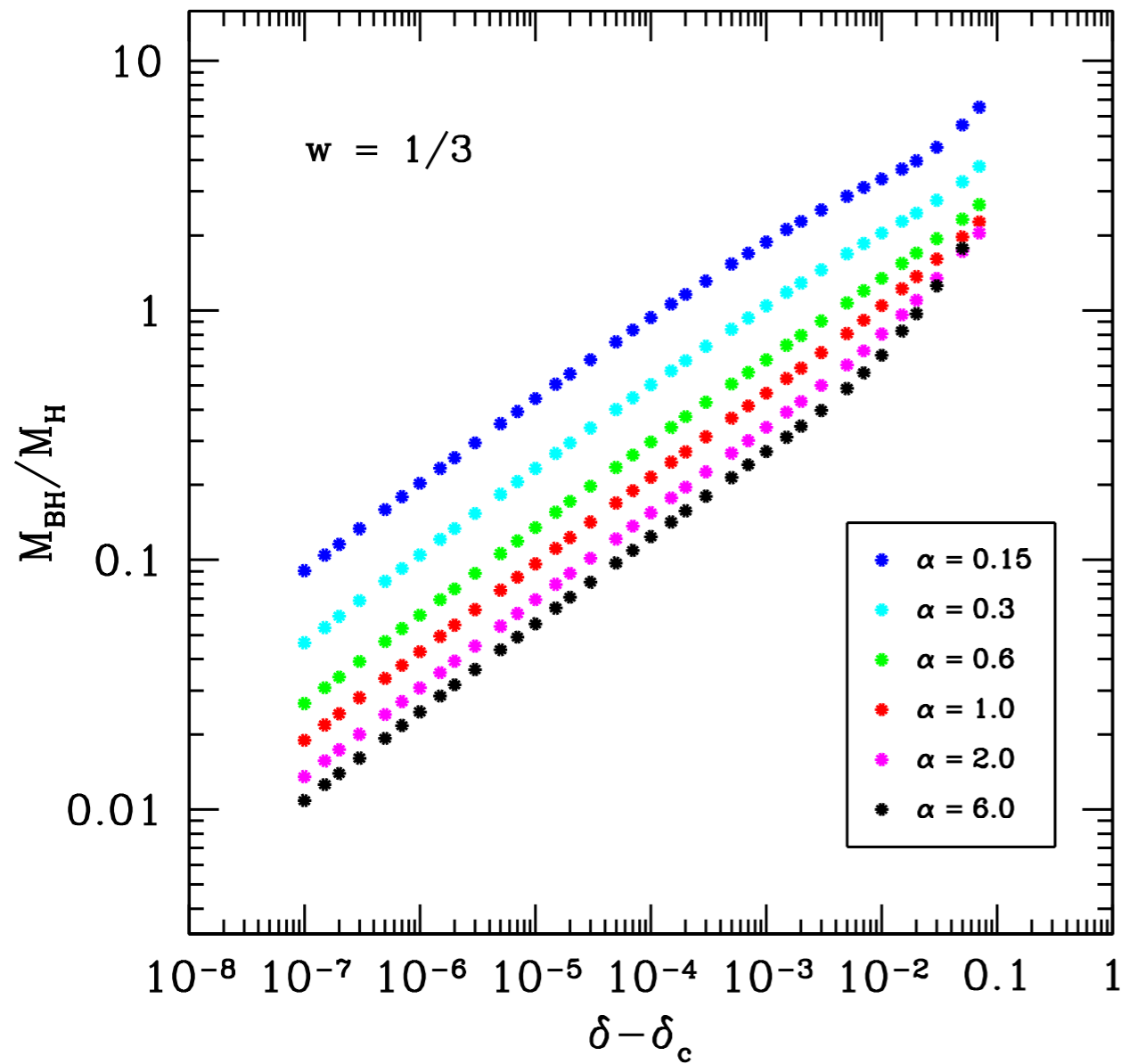


# Conclusions

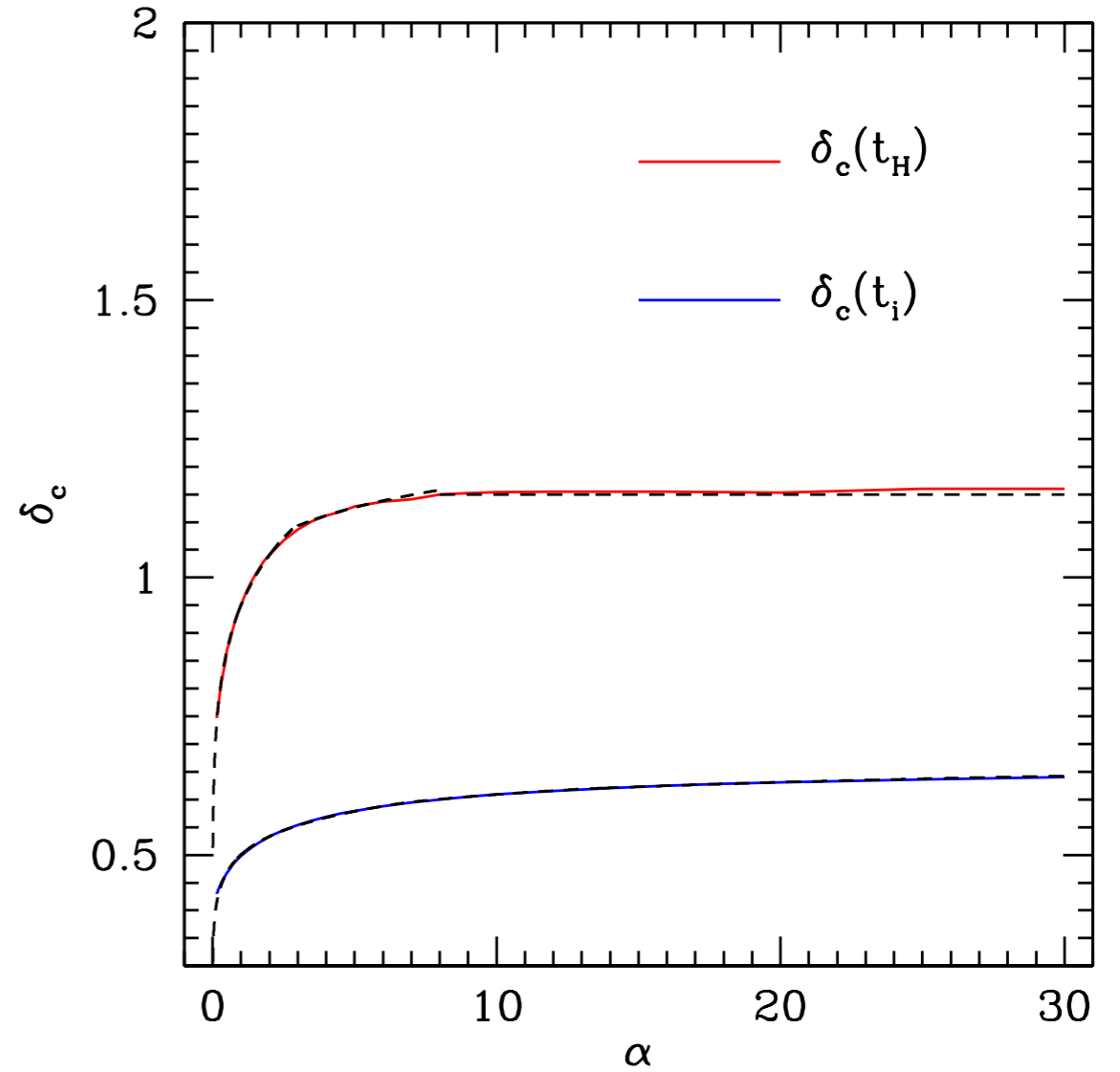
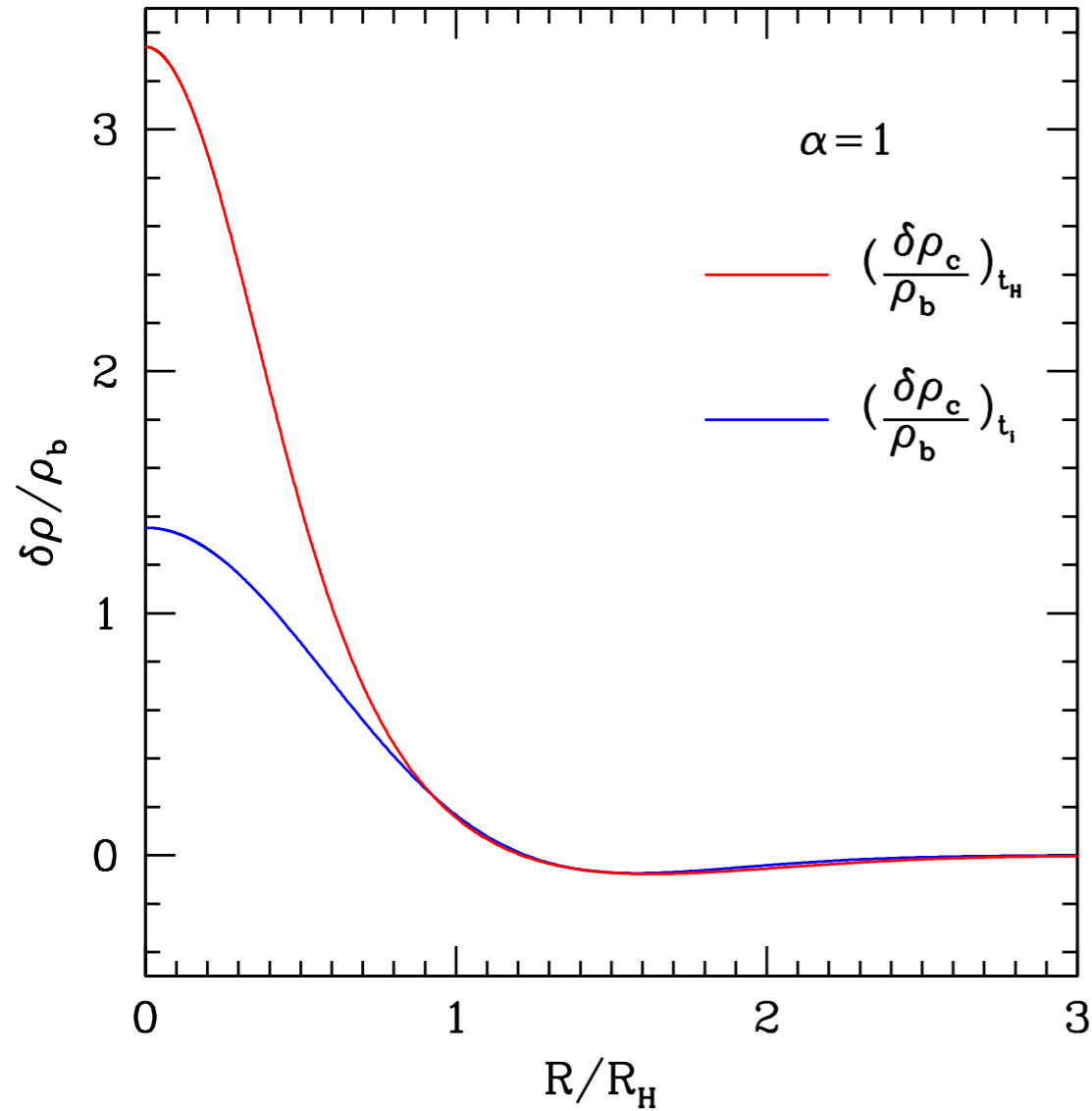
- The non linear threshold for PBH and the mass function could be fully computed from the shape of the power spectrum of cosmological perturbations, making relativistic numerical simulations.
- A softening of the equation of state (QCD) significantly enhances the formation of PBHs, with a mass distribution peaked between 1 and 2 solar masses (the range of heavy NSs and light BHs).
- This could give a sub-population of BH mergers compatible with the LVK catalog, explaining mass gap events as GW190814.
- Our analysis predicts a constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%), compatible with the current observational constraints.
- A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range (USR inflation models).

*Additional slides*

# Scaling law (critical collapse)



# Non linear horizon crossing

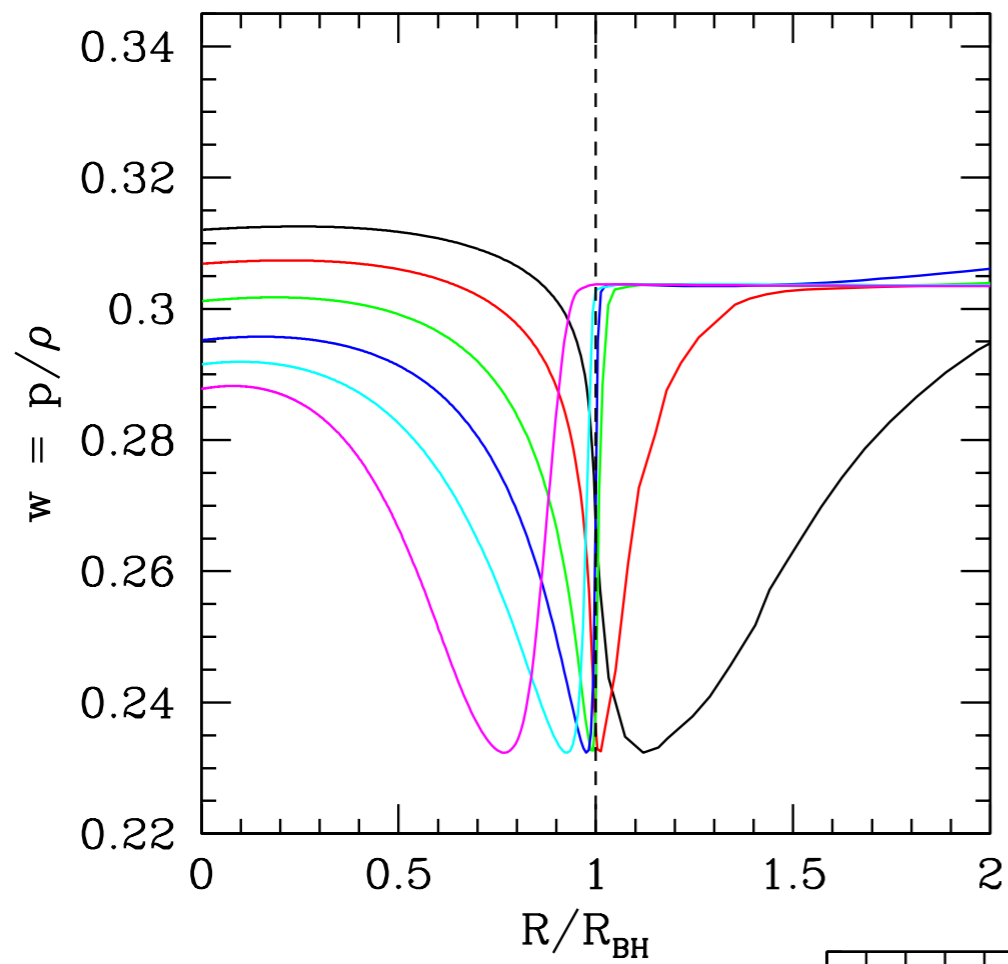
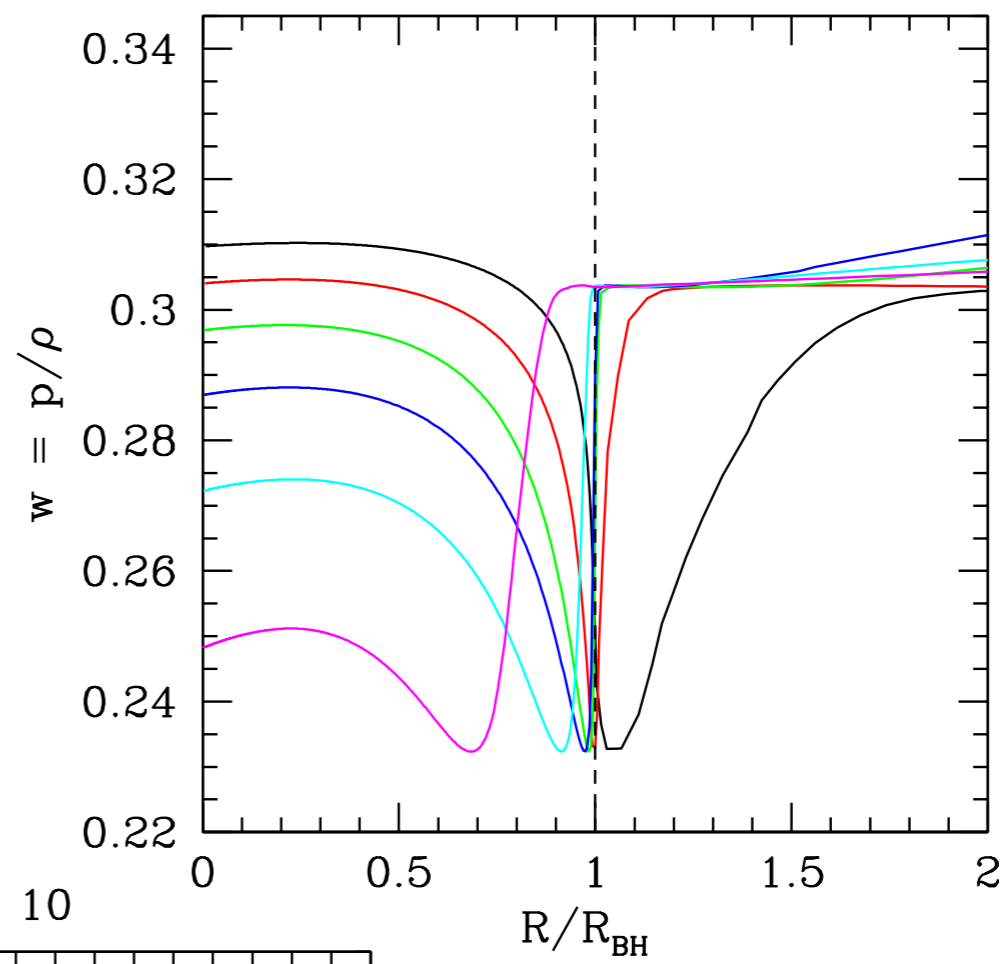
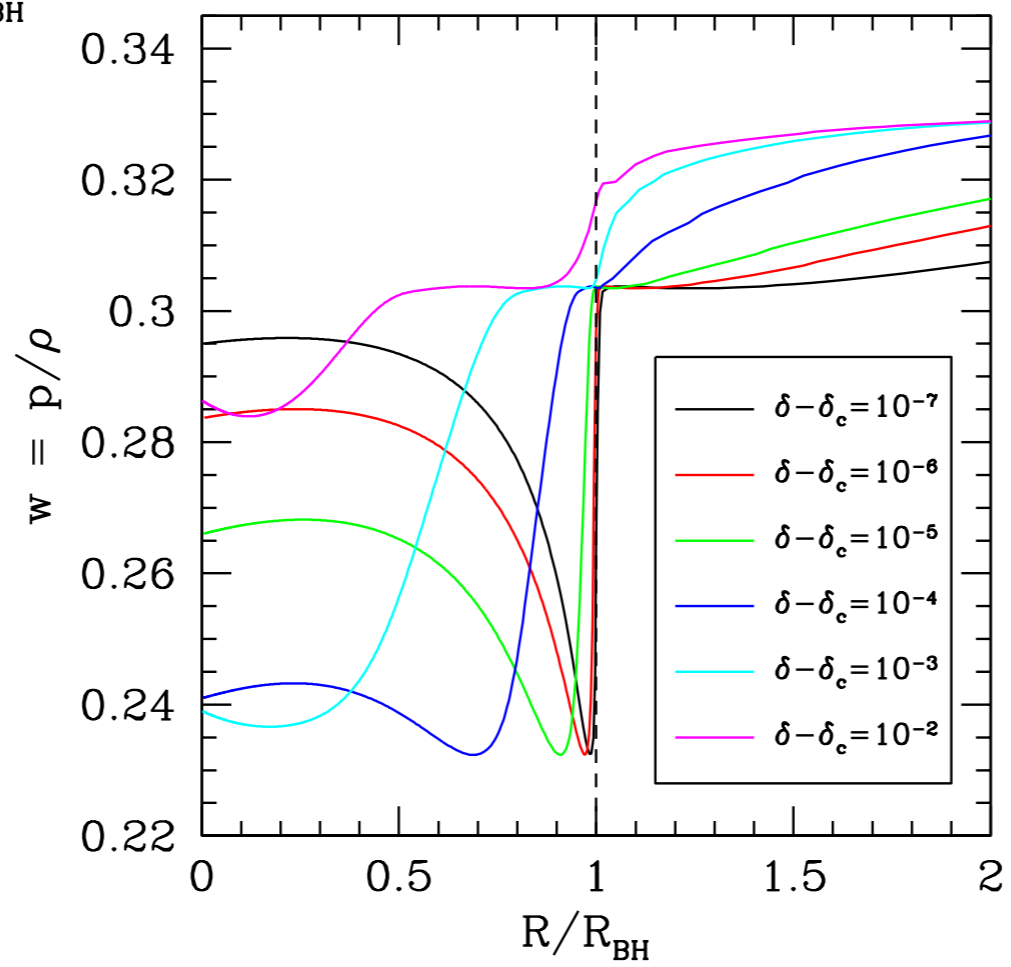


$$\delta_c(t_i) \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

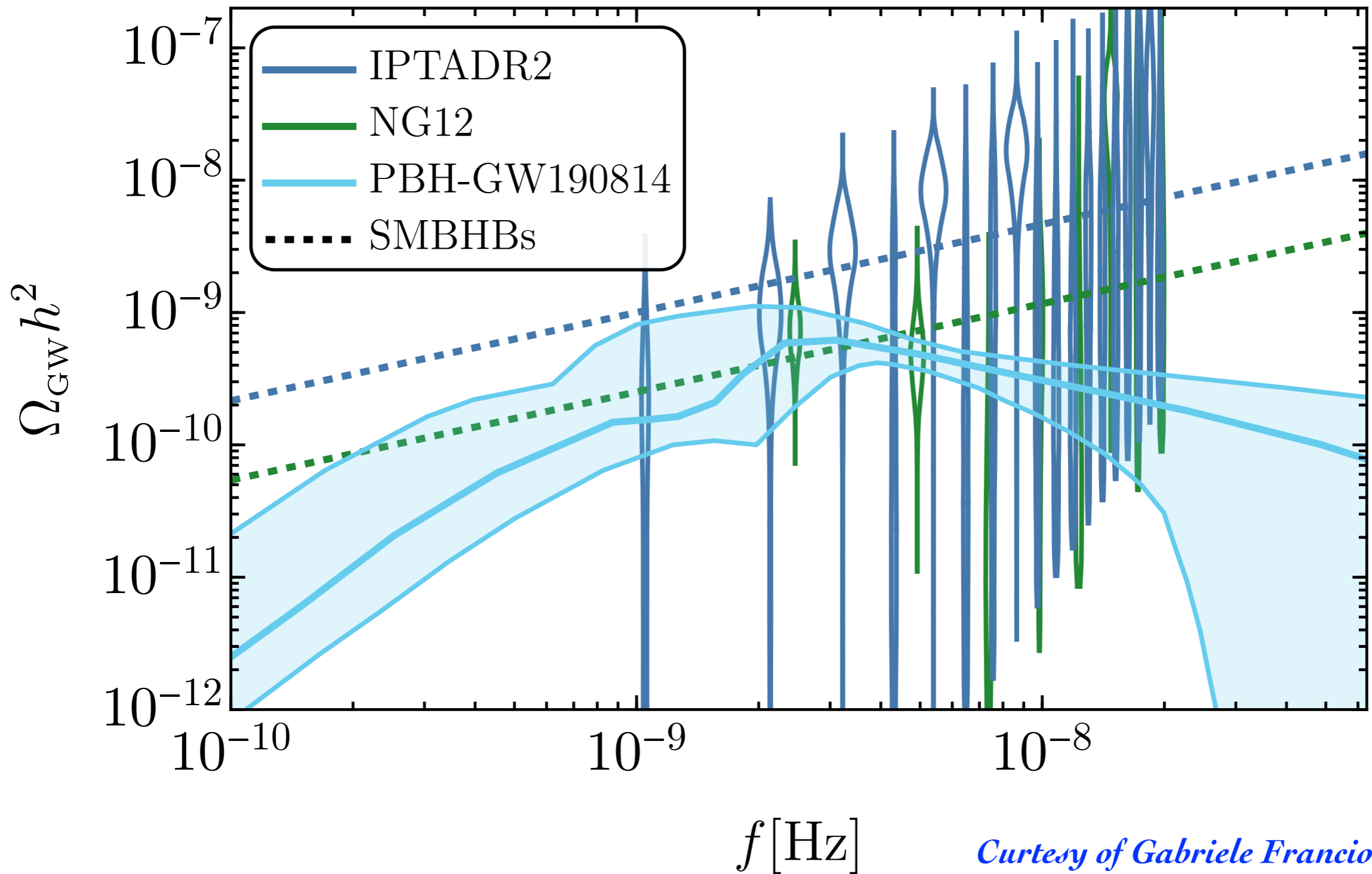
$$\delta_c(t_H) \simeq \begin{cases} \alpha^{0.125} - 0.05 & 0.1 \lesssim \alpha \lesssim 3 \\ \alpha^{0.06} + 0.025 & 3 \lesssim \alpha \lesssim 8 \\ 1.15 & \alpha \gtrsim 8 \end{cases}$$

$0.4 \leq \delta_c(t_i) < 0.6$	$0.7 \lesssim \delta_c(t_H) \lesssim 1.15$
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$M_0 = 0.5$  $M_0 = 2$  $M_0 = 10$ 

# PBH - DM constraints (PTA / NANOGrav)



*Courtesy of Gabriele Franciolini*