NEW YORK TIMES BESTSELLING AUTHOR

ADRIEN KUNTZ

THE THREE-BODY PROBLEM IN GR

WINNER OF THE HUGO AWARD

“Extraordinary.”
—THE NEW YORKER

In collaboration with :
Enrico Trincherini
Francesco Serra
Konstantin Leyde

SISSA
INFN Istituto Nazionale di Fisica Nucleare
ifpU

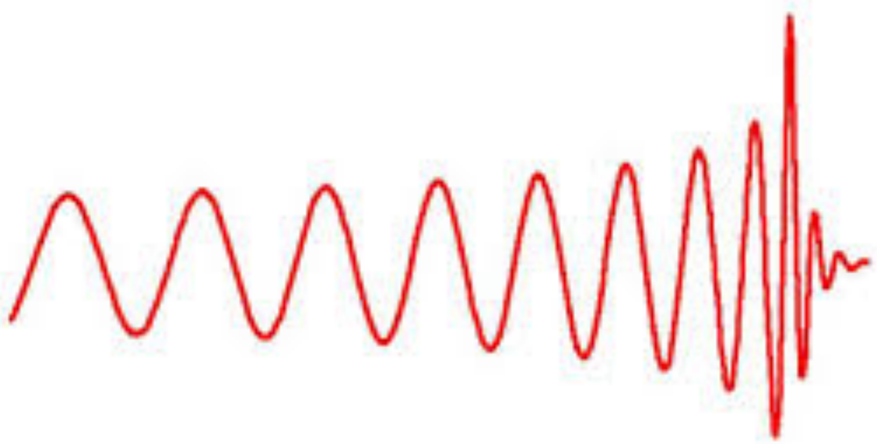
MARIE CURIE ACTIONS
H2020-MSCA-RISE-2020
GRU 101007855

MUR PRIN

European Research Council
Established by the European Commission
ERC-2018-COG GRAMS 815673

GRAVITATIONAL WAVES

Detection of GW so far beautifully corresponds to two-body systems

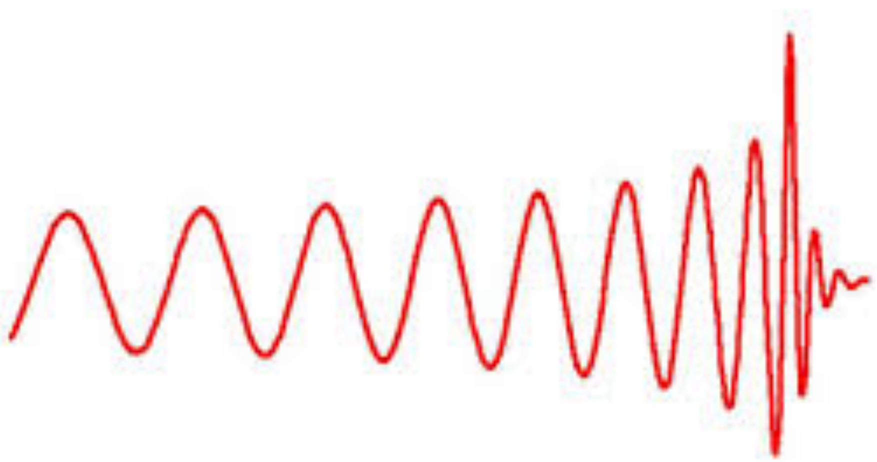


$$\Phi(f) = \phi_0 + 2\pi f t_0 + \sum_{k=0}^{7} \alpha_k f^{(k-5)/3}$$

m_1, m_2, χ_1, χ_2

GRAVITATIONAL WAVES

Detection of GW so far beautifully corresponds to two-body systems



$$\Phi(f) = \phi_0 + 2\pi f t_0 + \sum_{k=0}^{7} \alpha_k f^{(k-5)/3}$$

m_1, m_2, χ_1, χ_2

If we ever detect a new feature in data, we have (as 19th century astronomers) two possible explanations:

- Modification of GR
- Perturbation by an « environment » (this talk)

GRAVITATIONAL WAVES

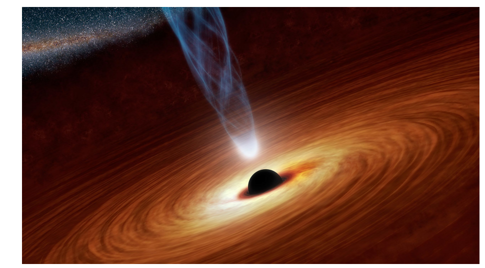
THIS QUESTION IS NOT PURELY ACADEMIC !

- 90% of low-mass binaries are expect to belong to a 'hierarchical' triple system

Tokovinin et al. 2006

- 'Migration traps' around SMBH at $R \sim 20 - 600 R_{\rm sch}$

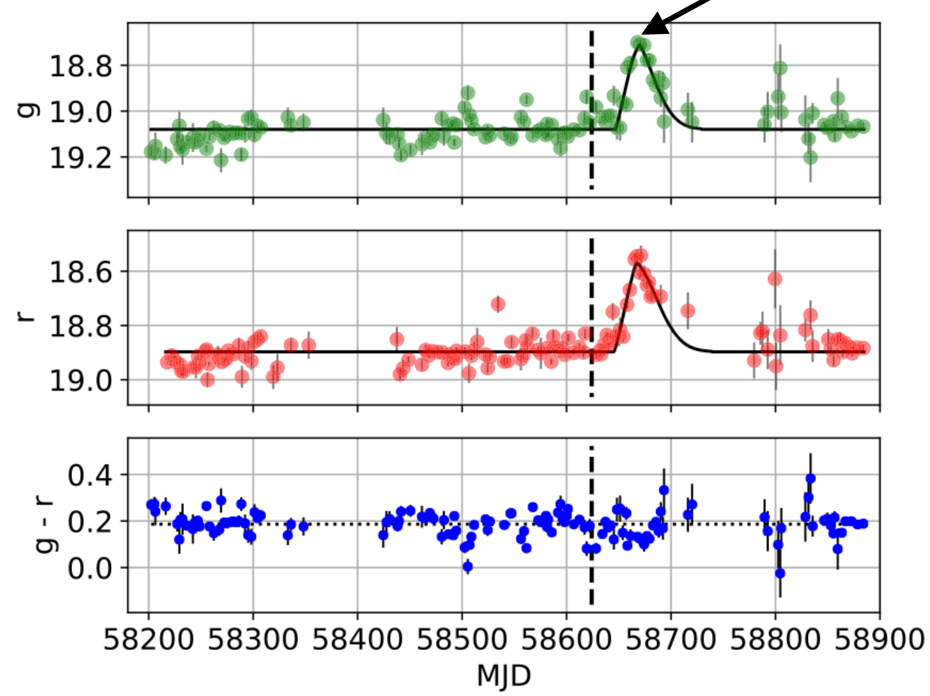
Bellovary et al. 2015



GW190521 close to a SMBH!

Graham et al. (2020)

GW190521



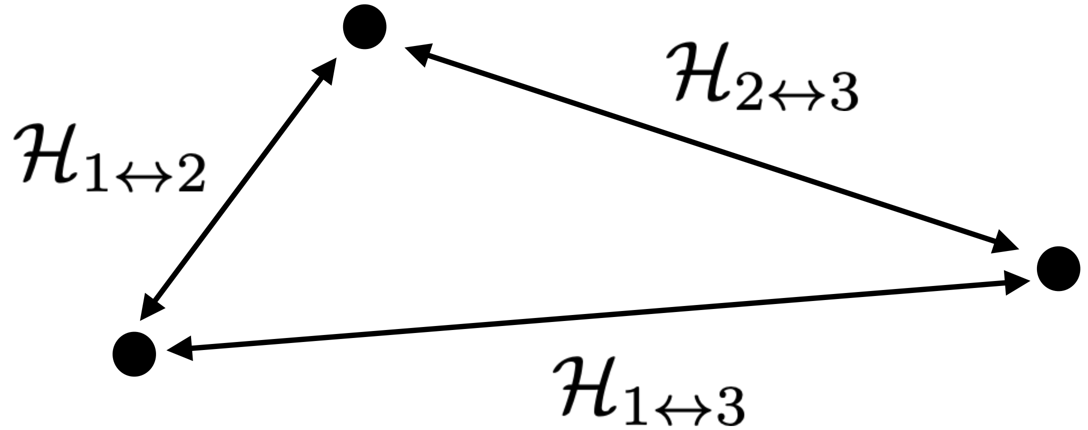
(ZTF data)

Can we see the imprint of third body in waveform ?

Three-body problem in GR: Theory

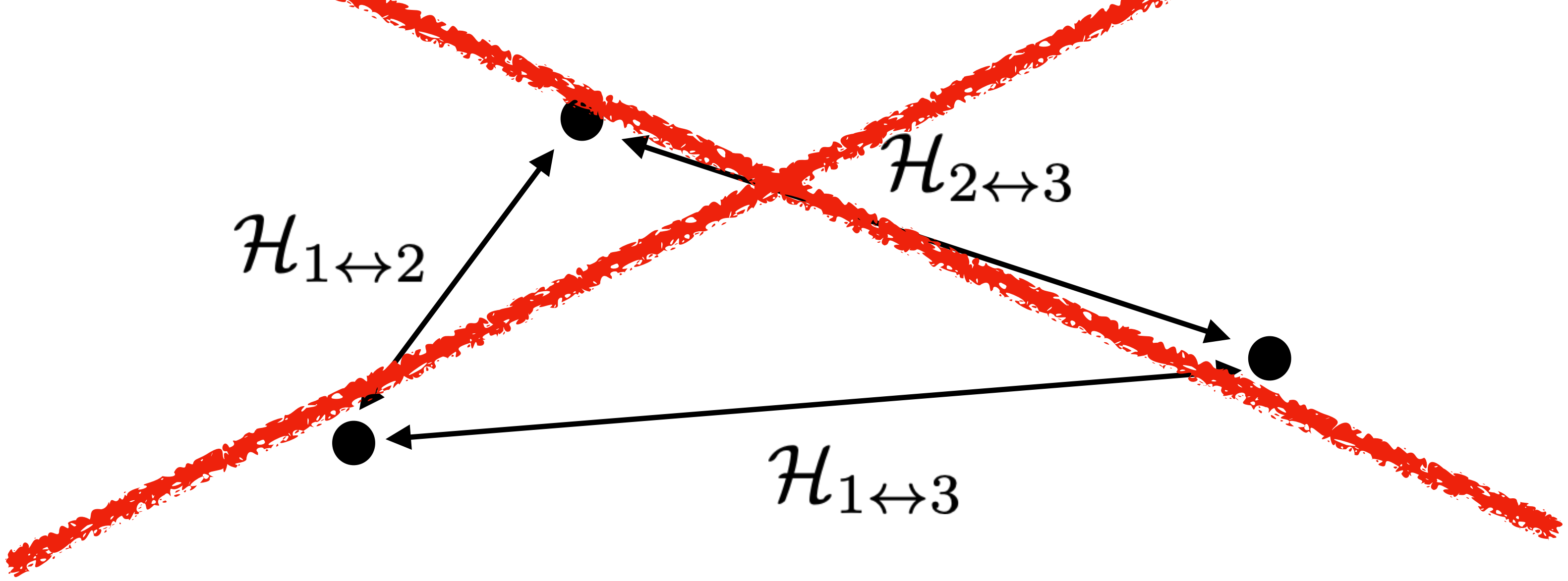
A COMMON MISCONCEPTION

$$\mathcal{H}_{3-\mathrm{body}} = \mathcal{H}_{1\leftrightarrow 2} + \mathcal{H}_{1\leftrightarrow 3} + \mathcal{H}_{2\leftrightarrow 3}$$



A COMMON MISCONCEPTION

$$\mathcal{H}_{3-\mathrm{body}} = \mathcal{H}_{1\leftrightarrow 2} + \mathcal{H}_{1\leftrightarrow 3} + \mathcal{H}_{2\leftrightarrow 3}$$

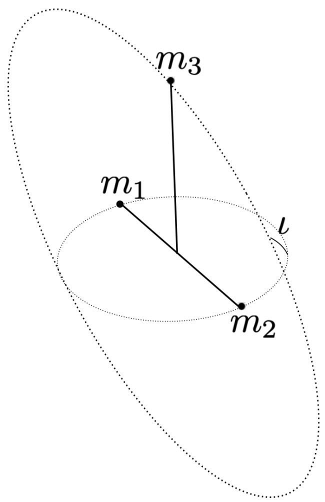


GR IS A NONLINEAR THEORY !

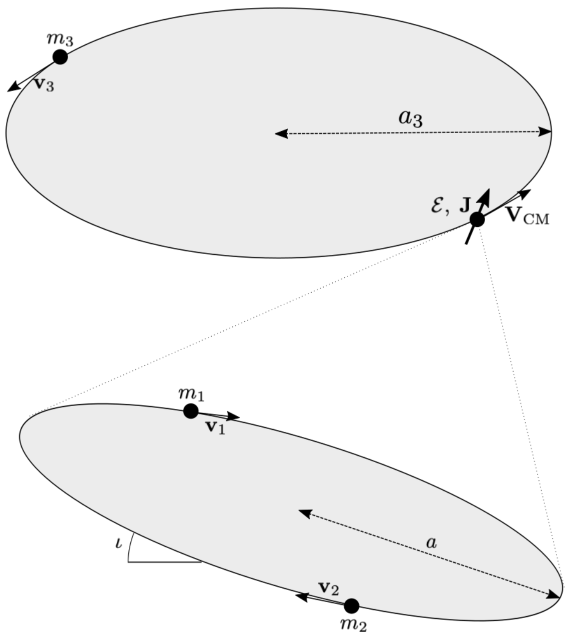
$$g_{\mu\nu} \neq g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu} \qquad \text{(would not solve } R_{\mu\nu} = 0 \text{)}$$

EFFECTIVE TWO-BODY

AK, F. Serra, E. Trincherini 2021

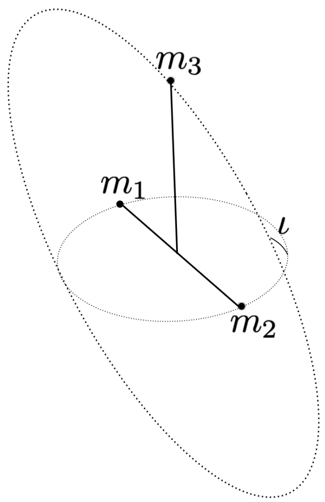


3-body motion = 2-body with spin!

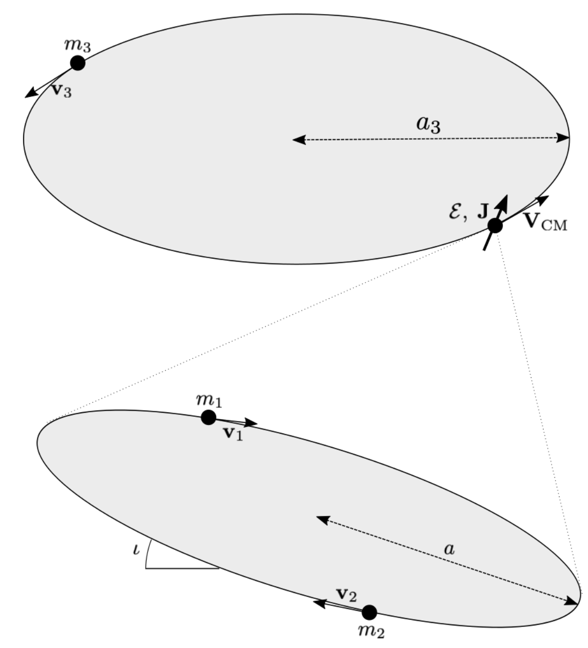


EFFECTIVE TWO-BODY

AK, F. Serra, E. Trincherini 2021



3-body motion = 2-body with spin!

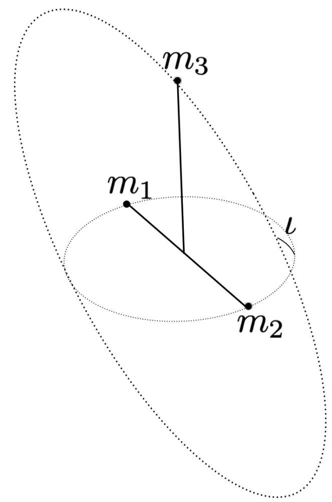


$$\mathscr{L}_{\rm full} = \sum_{A=1}^{3} - m_A \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu} \quad \Leftrightarrow \quad \mathscr{L}_{\rm EFT} = -\,\mathscr{E}\sqrt{-g_{\mu\nu}V^\mu_{\rm CM}V^\nu_{\rm CM}} + \frac{1}{2}J_{\mu\nu}\Omega^{\mu\nu} - m_3\sqrt{-g_{\mu\nu}v_3^\mu v_3^\nu}$$

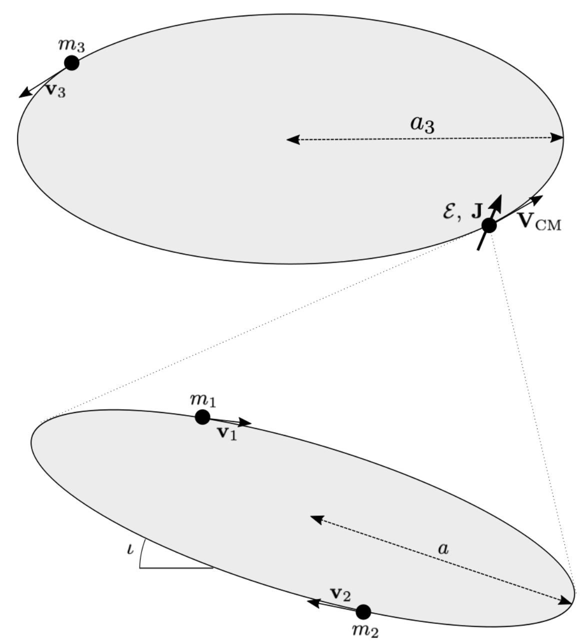
PROPER TIME

EFFECTIVE TWO-BODY

AK, F. Serra, E. Trincherini 2021



3-body motion = 2-body with spin!



$$\mathscr{L}_{\rm full} = \sum_{A=1}^{3} - m_A \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu} \quad \Leftrightarrow \quad \mathscr{L}_{\rm EFT} = -\mathscr{E}\sqrt{-g_{\mu\nu} V_{\rm CM}^\mu V_{\rm CM}^\nu} + \frac{1}{2} J_{\mu\nu}\Omega^{\mu\nu} - m_3\sqrt{-g_{\mu\nu} v_3^\mu v_3^\nu}$$

PROPER TIME

The equivalence principle fixes nearly everything!

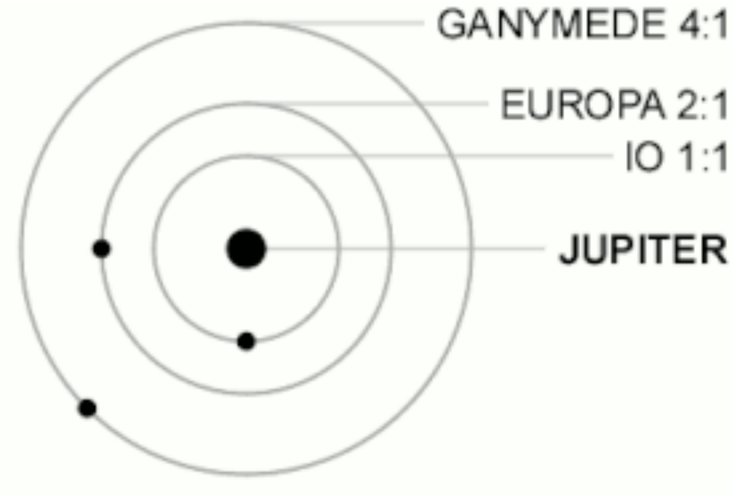
$$\mathscr{E} = m - \frac{G_N m \mu}{2a}, \qquad J_{ij} = \epsilon_{ijk} J^k, \quad \Omega_{ij} = \epsilon_{ijk}\Omega^k, \qquad \mathbf{J} = \sqrt{G_N m a (1-e^2)}\; \hat{\mathbf{j}}, \qquad \mathbf{\Omega} = \hat{\mathbf{e}} \times \dot{\hat{\mathbf{e}}}$$

$\hat{\mathbf{e}} \equiv$ UNIT RUNGE-LENZ VECTOR

PRECESSION RESONANCES

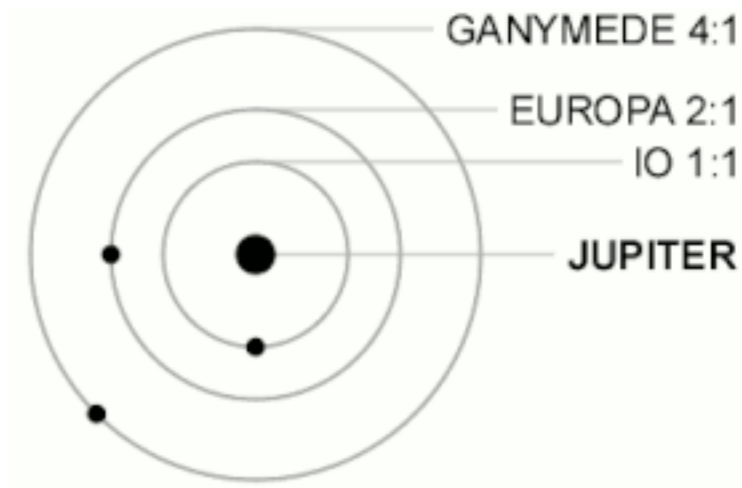
PRECESSION RESONANCES

- Resonances are a fascinating phenomenon of the 3-body problem

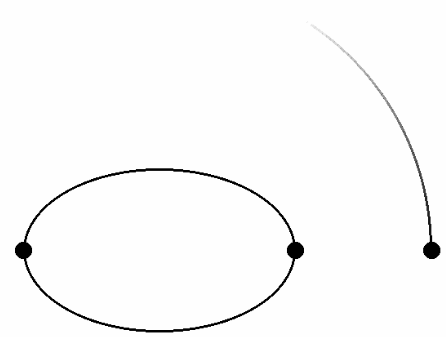


PRECESSION RESONANCES

- Resonances are a fascinating phenomenon of the 3-body problem



- When relativistic effects are included, there are other kinds of resonances



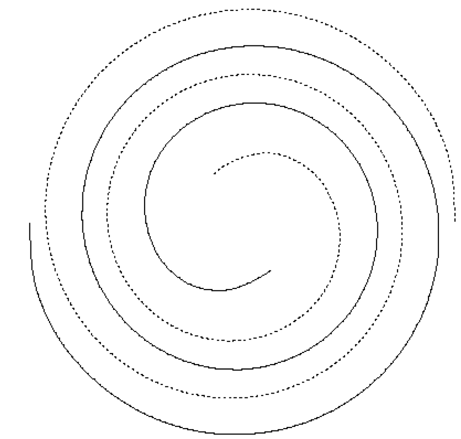
Perihelion angle

Outer orbit frequency

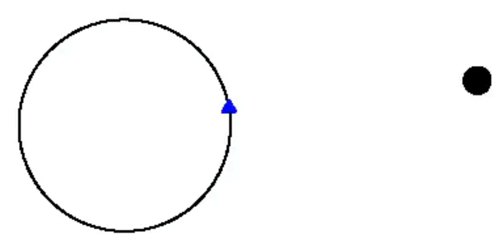
$$p\,\dot{\omega} + q\sqrt{\frac{GM}{a_3^3}} = 0$$

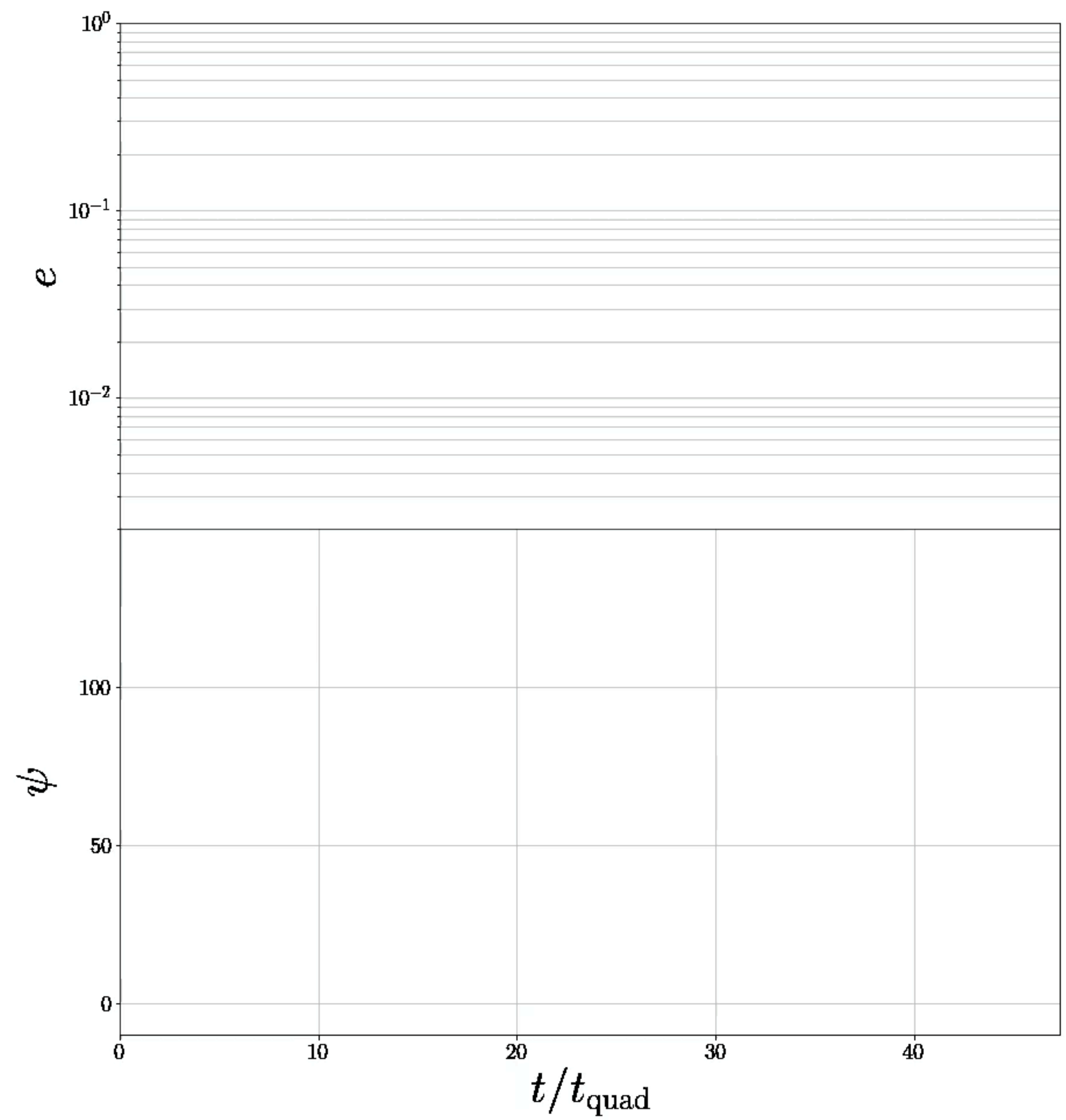
$$p, q \in \mathbb{Z}$$

PRECESSION RESONANCES



$$a(t) = a_0\left(1 - \frac{t}{t_{\rm RR}}\right)^{1/4}$$





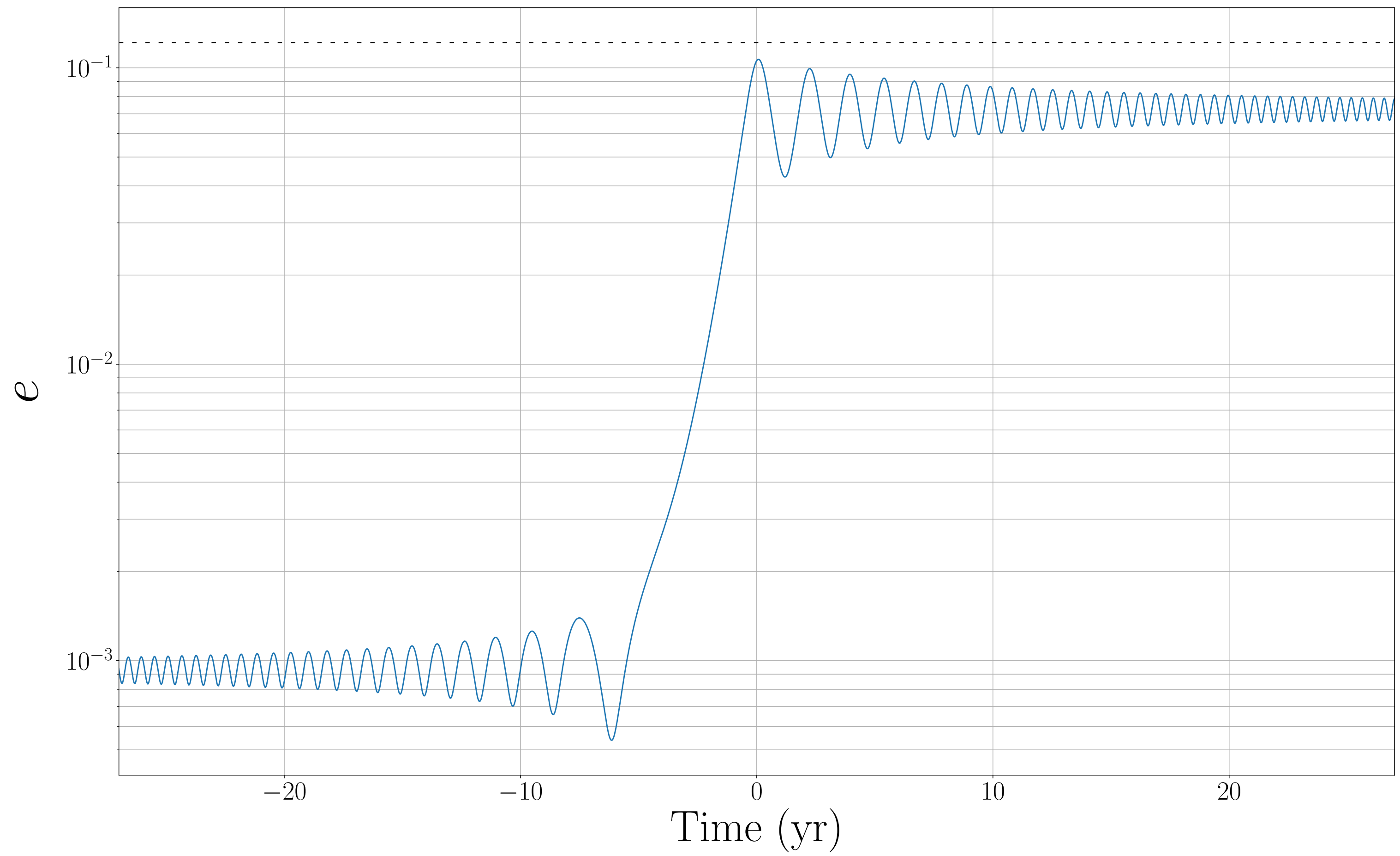
$$\psi \equiv \omega - \sqrt{G_N M/a_3^3}\; t$$

AK 2021

Precession resonances

Features of the resonance

- Happen in the very relativistic regime where usually $e \simeq 0$
- Exchange of (Newtonian) energy of outer orbit and (PN) energy of inner orbit
- At the resonance, exponential eccentricity growth
- Could happen in or close to LISA bandwidth!



$$\dot{e} = \frac{e \sin \psi}{t_{\rm KL}}$$

$m_3 = m = 50\ M_\odot$

$a_3 = 0.5$ AU

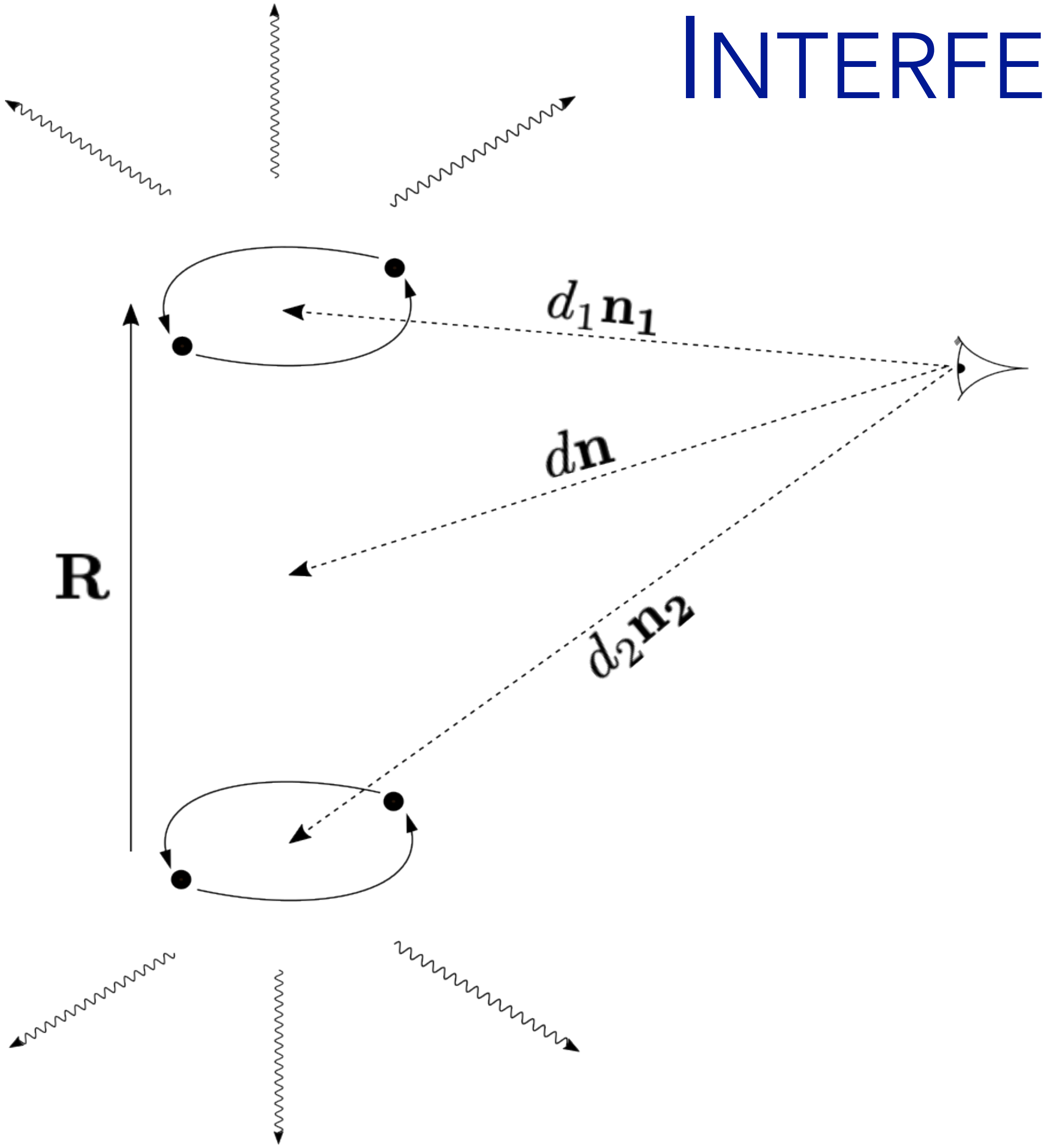
$a = 4 \times 10^{-3}$ AU

$e_3 = 0.7$

AK 2021

INTERFERENCE OF GW

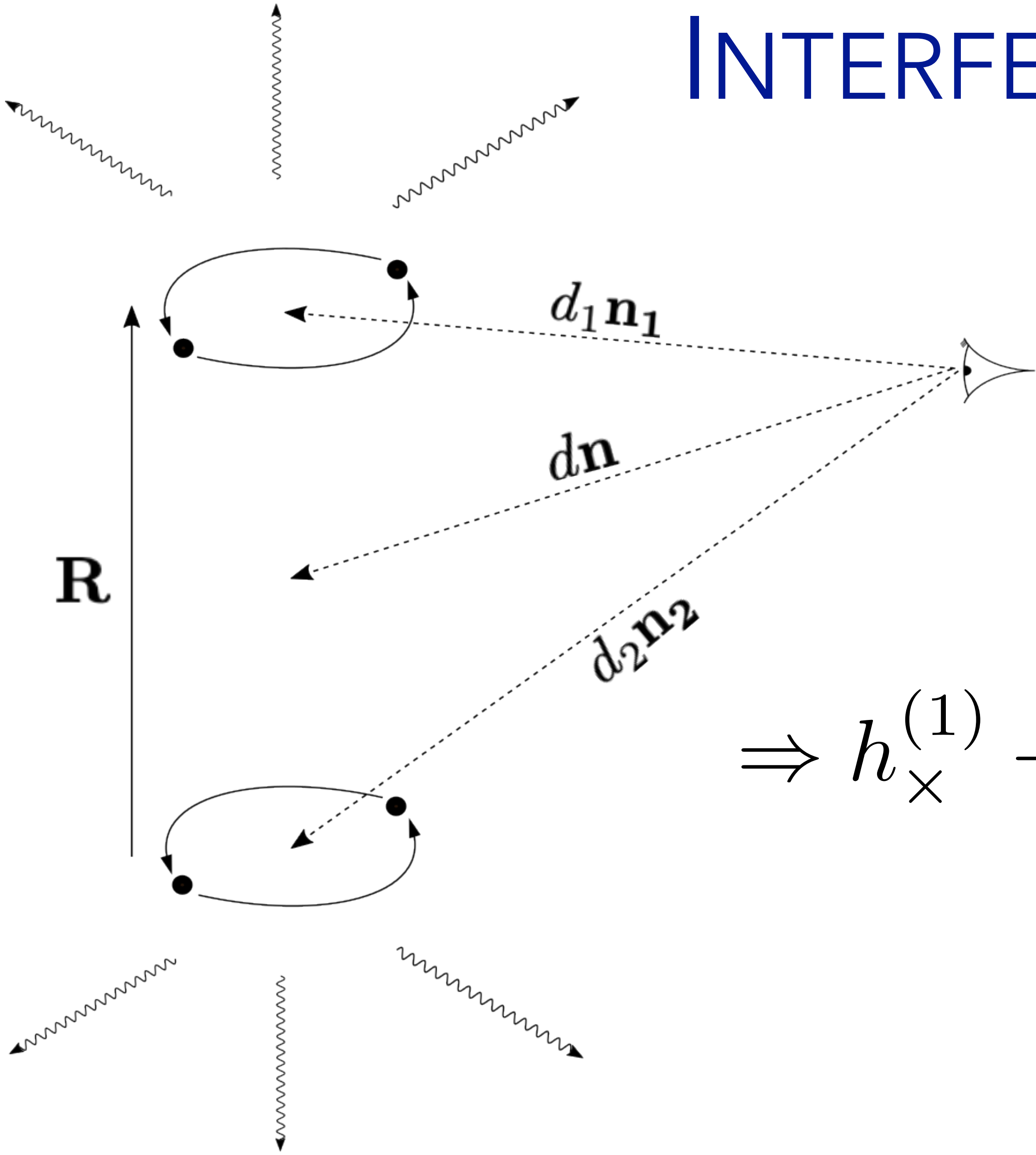
INTERFERENCE OF GW



$$h_\times^{(1)} = \mathcal{A}\sin\big(2\omega(t-d_1)+2\phi_1\big)$$

$$h_\times^{(2)} = \mathcal{A}\sin\big(2\omega(t-d_2)+2\phi_2\big)$$

Interference of GW



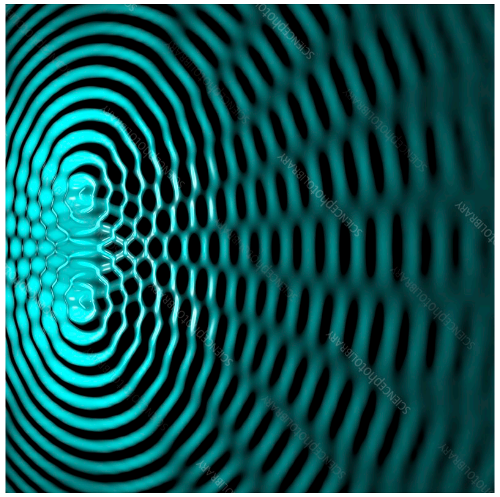
$$h_\times^{(1)} = \mathcal{A}\sin\left(2\omega(t-d_1)+2\phi_1\right)$$

$$h_\times^{(2)} = \mathcal{A}\sin\left(2\omega(t-d_2)+2\phi_2\right)$$

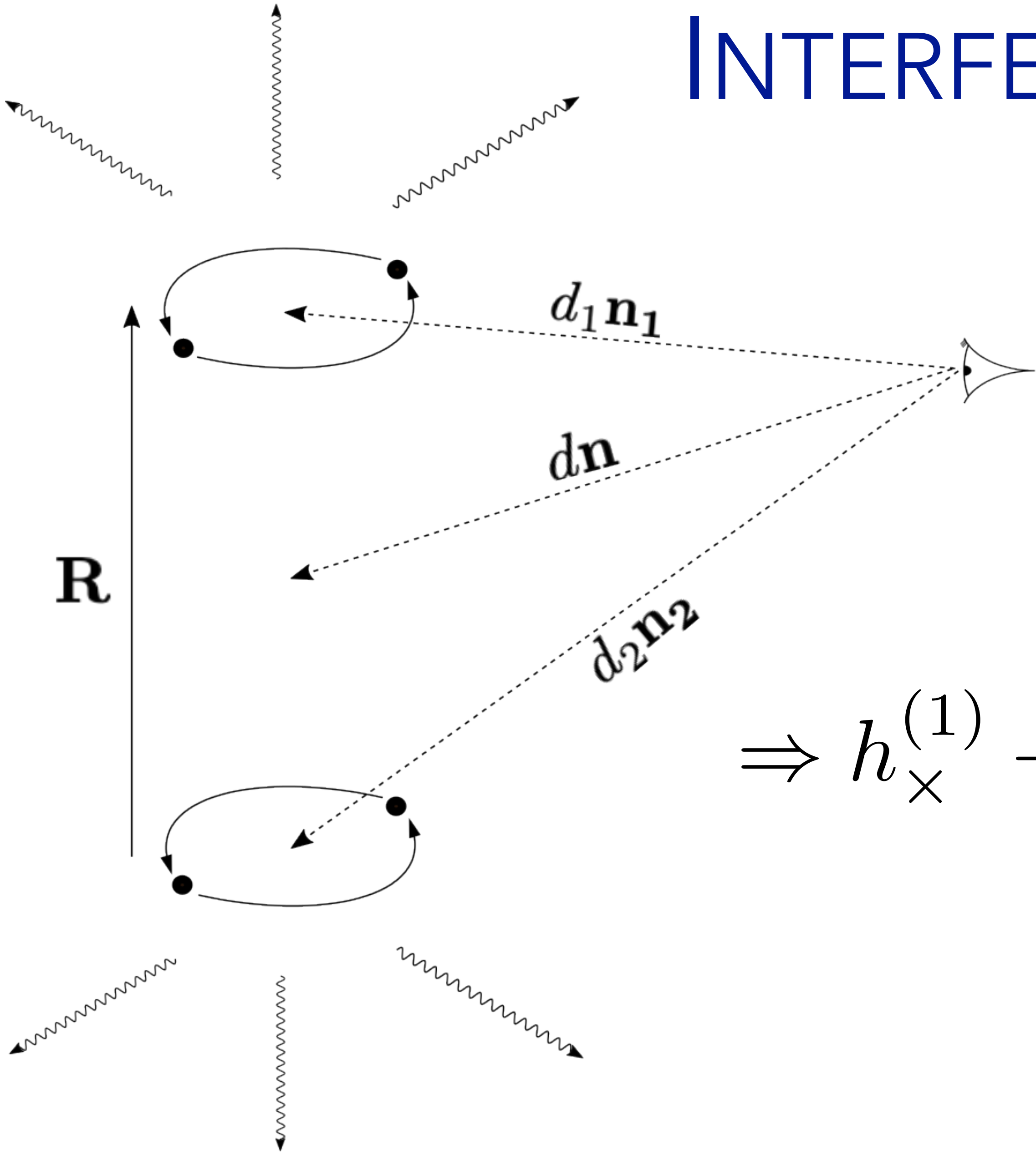
$$\Rightarrow h_\times^{(1)} + h_\times^{(2)} = 2\mathcal{A}\sin\left[2\omega(t-d)+\phi_1+\phi_2\right]$$

$$\times\cos\left[\phi_2-\phi_1+\omega\mathbf{n}\cdot\mathbf{R}\right]$$

Interference pattern!



Interference of GW

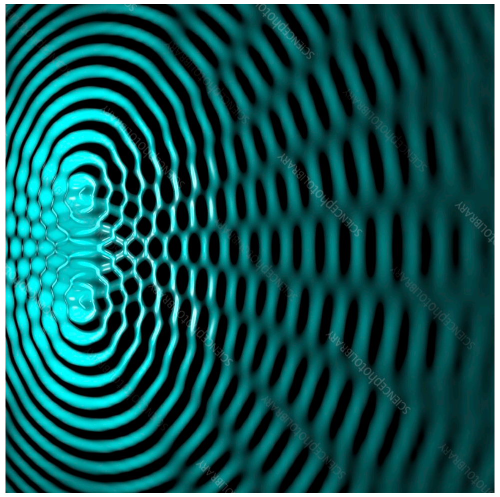


$$h_\times^{(1)} = \mathcal{A}\sin\left(2\omega(t-d_1)+2\phi_1\right)$$

$$h_\times^{(2)} = \mathcal{A}\sin\left(2\omega(t-d_2)+2\phi_2\right)$$

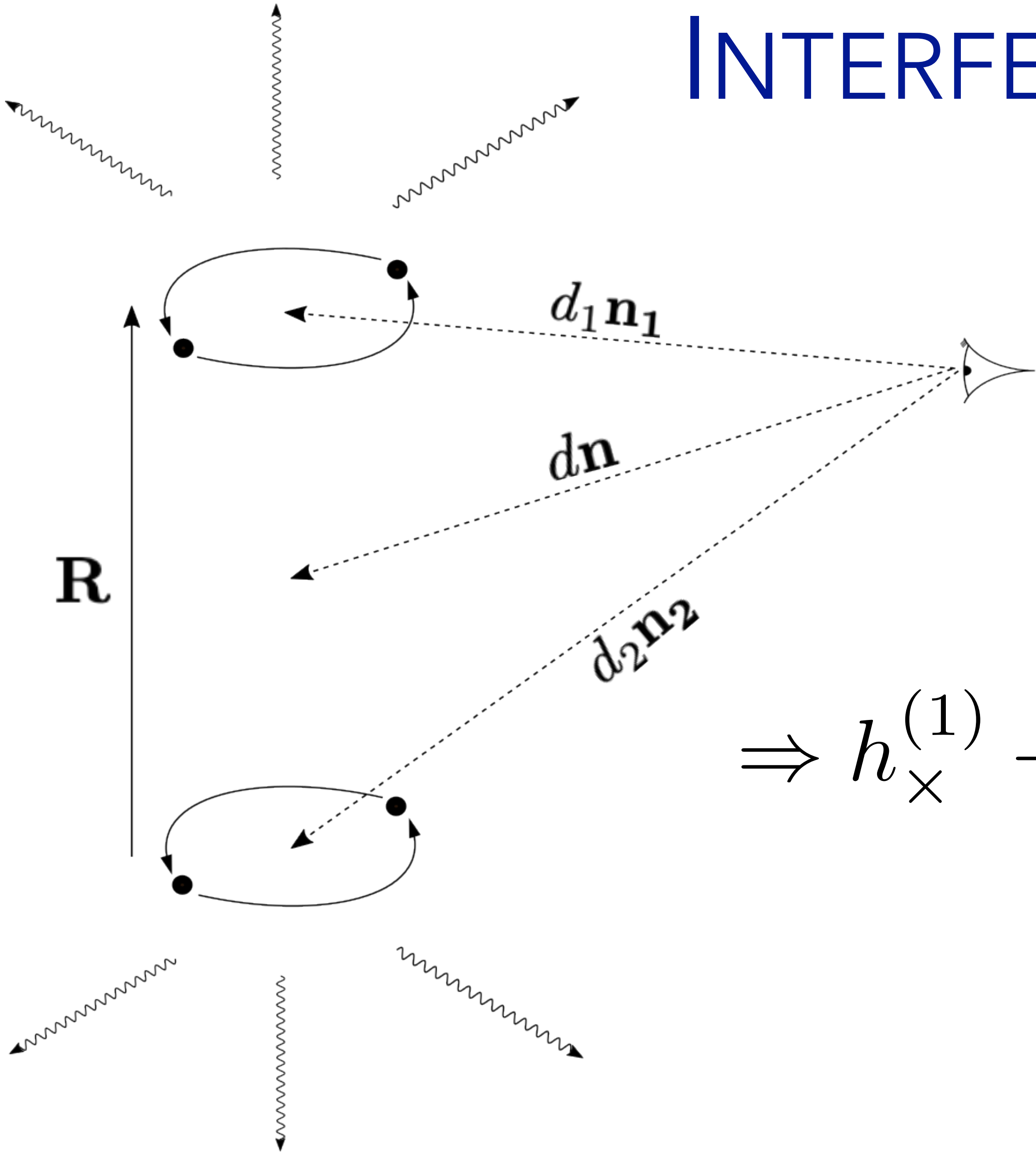
$$\Rightarrow h_\times^{(1)} + h_\times^{(2)} = 2\mathcal{A}\sin\left[2\omega(t-d)+\phi_1+\phi_2\right]$$
$$\times\cos\left[\phi_2-\phi_1+\omega\mathbf{n}\cdot\mathbf{R}\right]$$

Interference pattern!



If $\phi_2 = \phi_1 + \frac{\pi}{2}$ and $\omega R \ll 1 \Leftrightarrow R \ll \lambda_{\mathrm{GW}}$:

INTERFERENCE OF GW

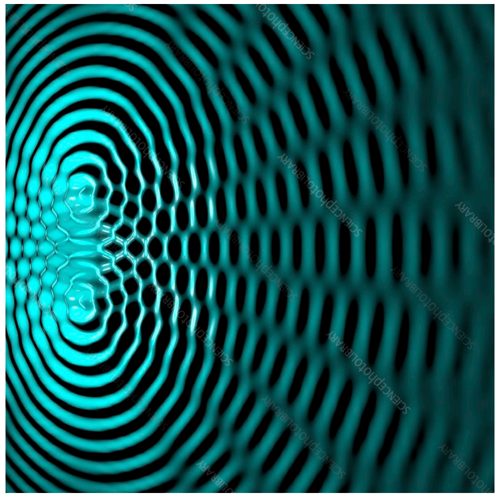


$$h_\times^{(1)} = \mathcal{A}\sin\left(2\omega(t-d_1)+2\phi_1\right)$$

$$h_\times^{(2)} = \mathcal{A}\sin\left(2\omega(t-d_2)+2\phi_2\right)$$

$$\Rightarrow h_\times^{(1)} + h_\times^{(2)} = 2\mathcal{A}\sin\left[2\omega(t-d)+\phi_1+\phi_2\right]$$
$$\times\cos\left[\phi_2-\phi_1+\omega\mathbf{n}\cdot\mathbf{R}\right]$$

Interference pattern!



If $\phi_2 = \phi_1 + \frac{\pi}{2}$ and $\omega R \ll 1 \Leftrightarrow R \ll \lambda_{\rm GW}$:

Destructive interference everywhere!

Radiation-reaction forces

In the case of coherent emission $\omega R \ll 1$:

Energy loss is modified!

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = \left.\frac{\mathrm{d}a_1}{\mathrm{d}t}\right|_{\mathrm{Peters}} \big(1 + \cos(\phi_1 - \phi_2)\big)$$

RADIATION-REACTION FORCES

In the case of coherent emission $\omega R \ll 1$:

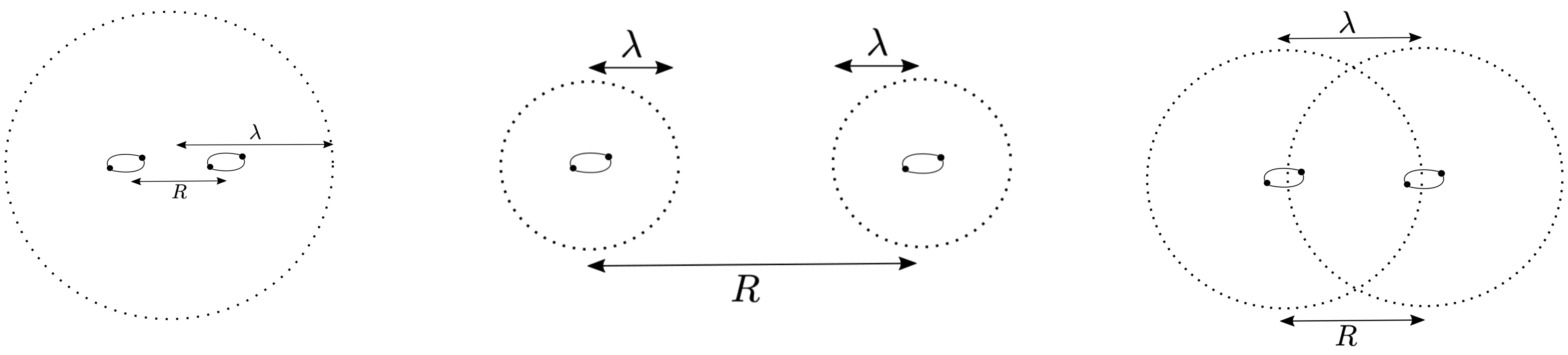
Energy loss is modified!

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = \left.\frac{\mathrm{d}a_1}{\mathrm{d}t}\right|_{\mathrm{Peters}} \left(1 + \cos(\phi_1 - \phi_2)\right)$$

Remarks:

- When there is no coherent emission the radiation-reaction force is also modified

AK 2023



- Even if the quadrupole emission is suppressed, there will be octupolar waves

Conclusions

- Very rich phenomenology in the Newtonian 3-body problem, even more in the relativistic one…
- Eccentricity can be non-negligible even for very relativistic systems due to precession resonance
- Gravitational waves can interfere!
- Future work: more precise waveforms for 3-body problem