Dynamical friction on compact binary systems

Vincent Desjacques

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[VD, Nusser, Buehler ApJ, arXiv: 2111.07366]

[Buehler & VD PRD, arXiv: 2207.13740]

[Buehler, Kolyada & VD, arXiv: 2310.05244]

GWFPF workshop, Rome, 15/2/24

Outline

Dynamical Friction (DF) in structure formation

Solving circular DF with a multipole expansion

Perfect fluid vs. ULA dark matter

Including eccentricity and density profiles

Dynamical Friction (DF)

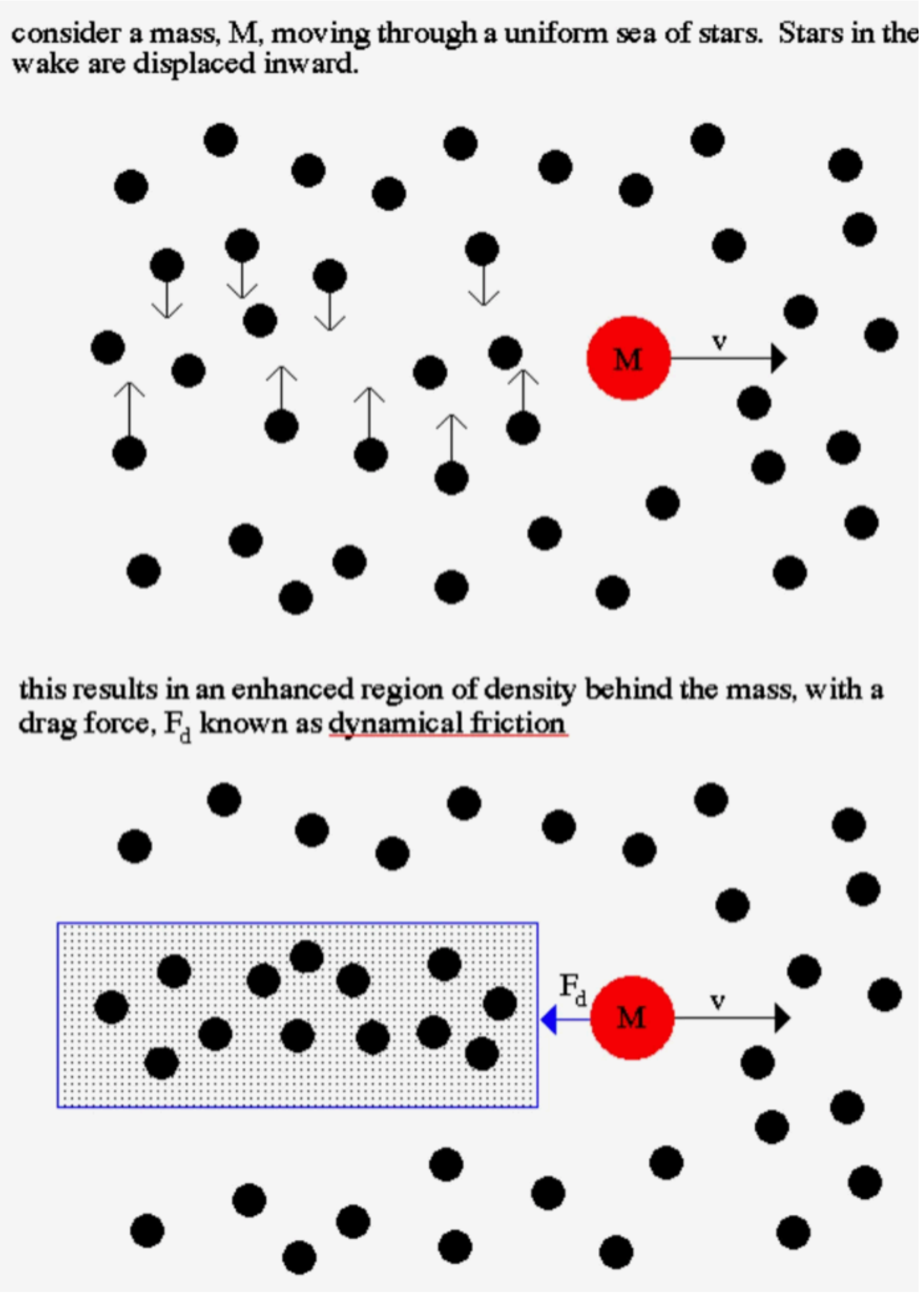


Figure @ Luke Leisman

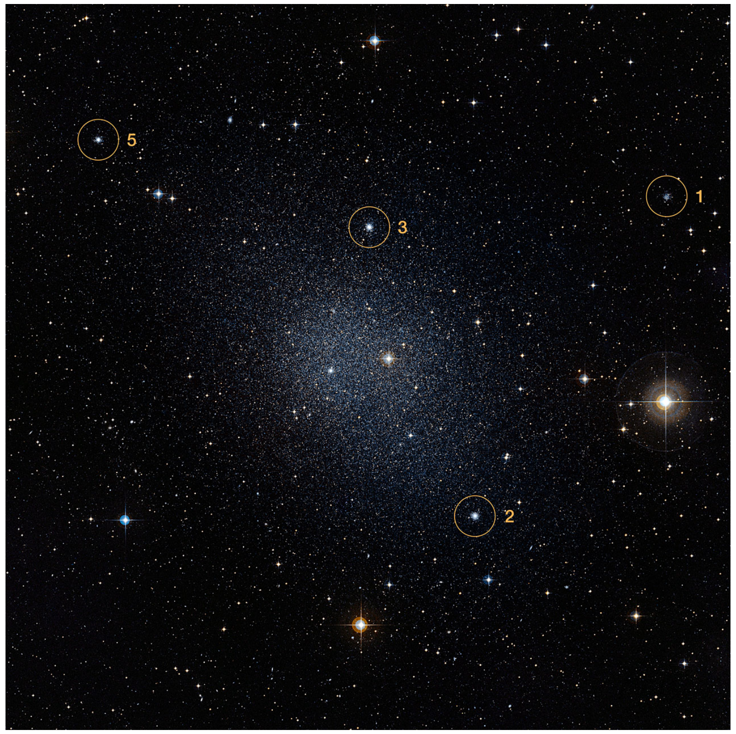
- *Pioneering work by Chandrasekhar (1943)*
- *The density wake is global (far-field matters)*
- *The medium can be anything (stars, DM, gas…)*

DF in structure formation

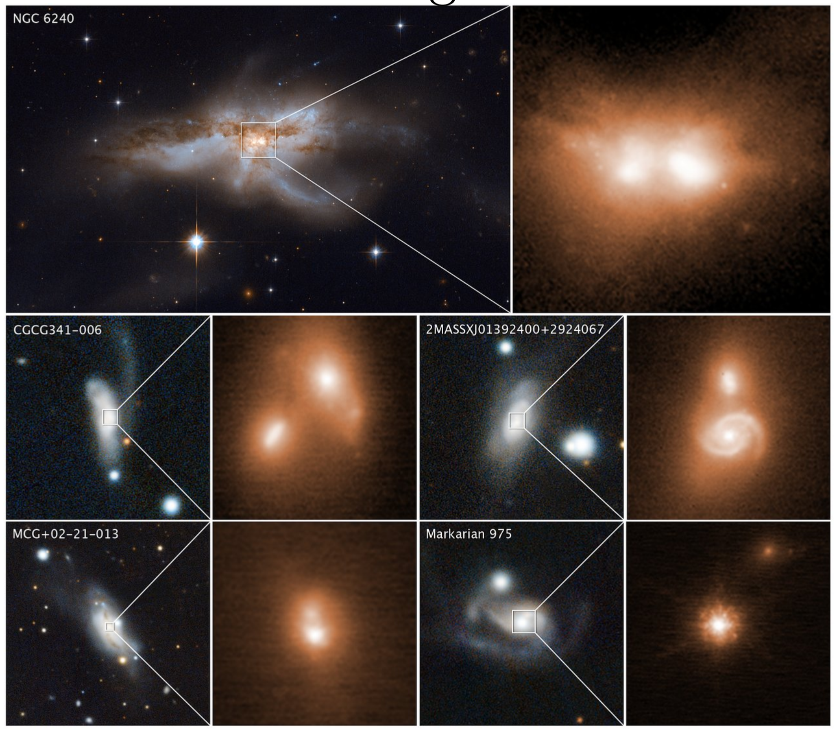
DF is a key mechanism which controls

- *The merging of DM halos, galaxies, Globular clusters*
- *The slow-down of spinning galactic bars*
- *The coalescence of compact objects*

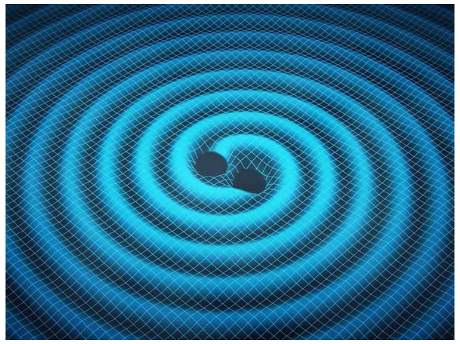
Fornax Globular clusters



coalescing AGNs



Signature of DF in GW data



Keplerian binary:

$$t_{\rm GW} \simeq 5.8 \times 10^{17}\ {\rm yr} \left(\frac{\mu}{\rm M_\odot}\right)^{-1} \left(\frac{M}{M_\odot}\right)^{-2} \left(\frac{r_0}{\rm AU}\right)^{4}$$

$$t_{\rm DF} \simeq 1.3 \cdot 10^{17}\ {\rm yr} \left(\frac{\mu}{\rm M_\odot}\right) \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}} \left(\frac{r_0}{\rm AU}\right)^{-\frac{3}{2}} \left(\frac{\rho}{0.01\ {\rm M_\odot pc^{-3}}}\right)^{-1} I^{-1}$$

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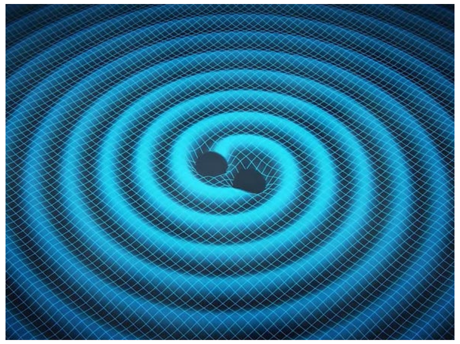

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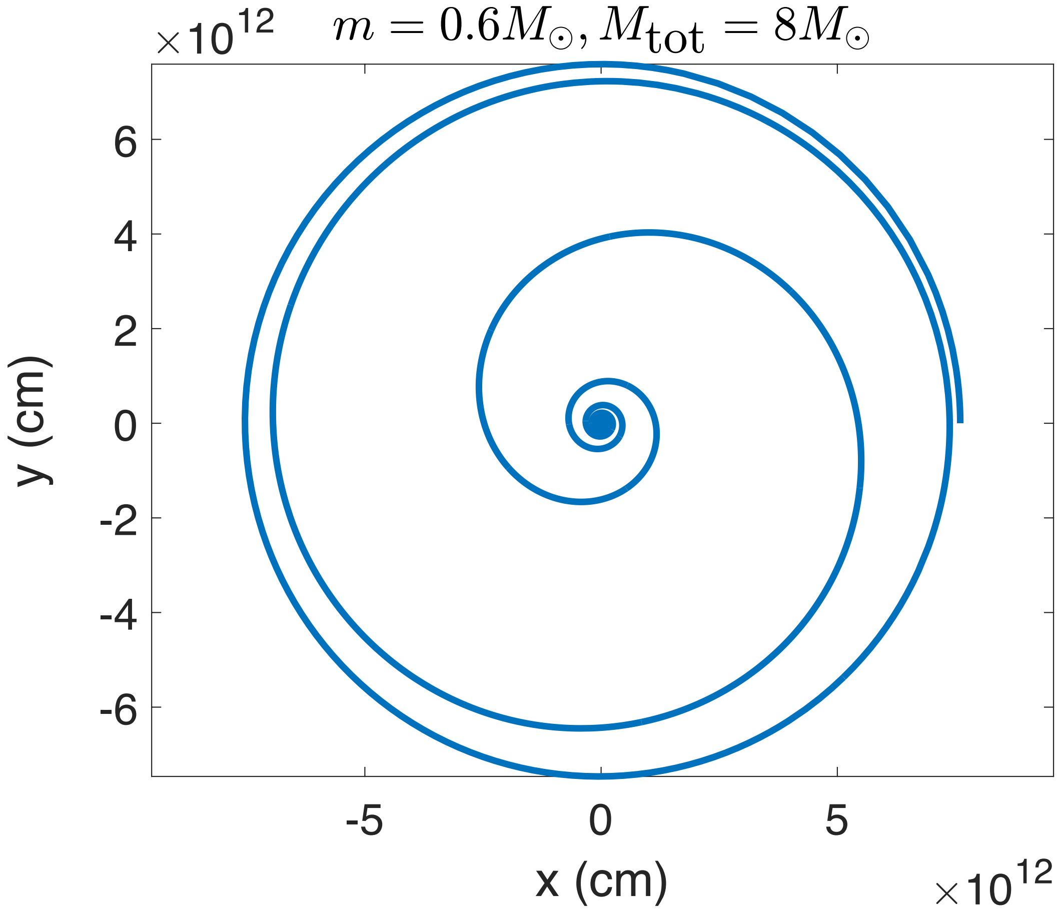

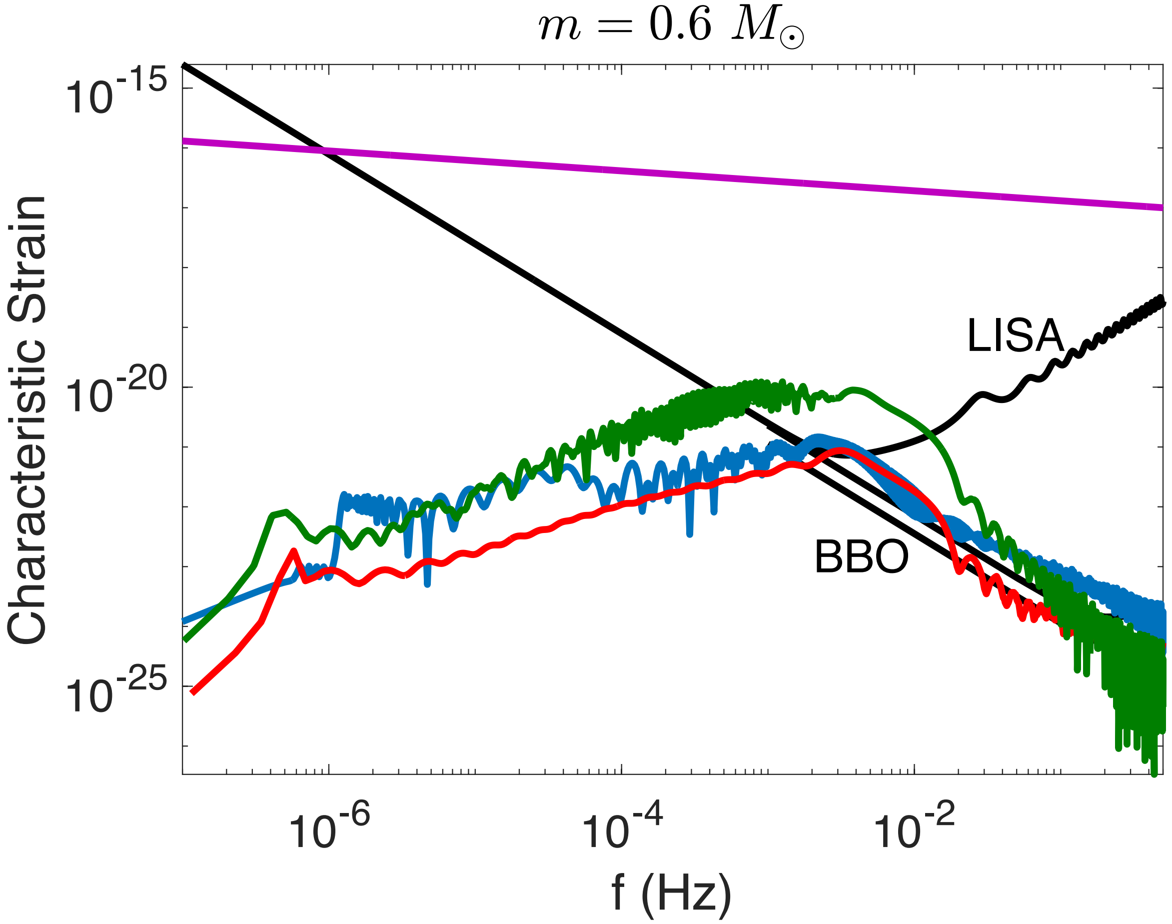

[Ginat, Glanz, Perets, Grishin, VD, MNRAS, arXiv: 1903.11072]

Analytical approaches to DF

Standard assumptions:

- *Linear response theory*
- *The perturber is a point mass in straight-line motion at constant velocity*
- *Infinite, homogeneous and isotropic medium*
- *Self-gravity of the medium is ignored*

[Chandrasekhar 43; Dokuchaev 64; Ruderman+ 71; Rephaeli+ 80; Just+ 91; Ostriker 99, …]

DF for perturbers in straight-line motion

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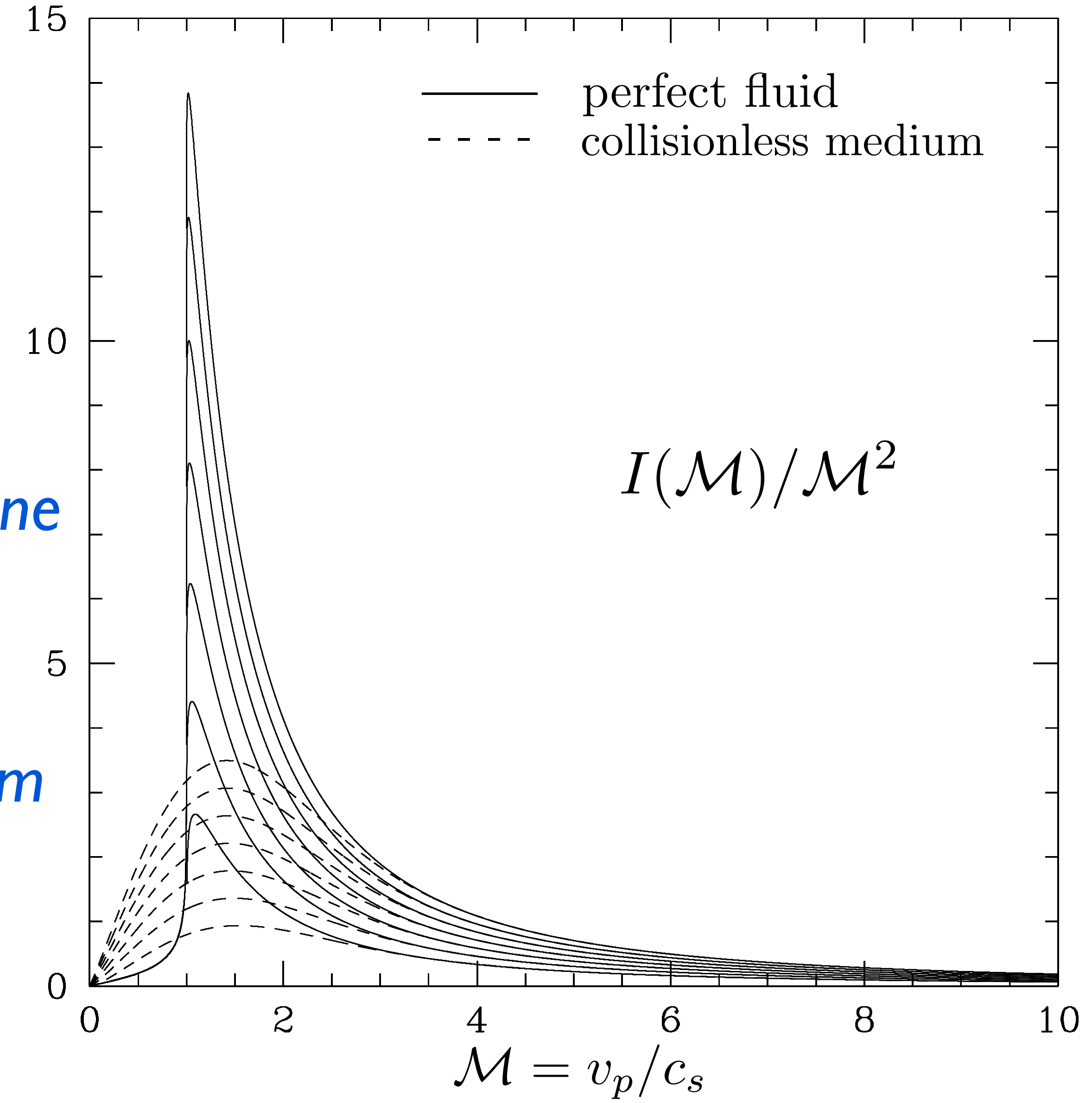


$$\vec{F}_{\rm DF} = -4\pi \left(\frac{GM}{v_p}\right)^2 \bar{\rho}\, I(\mathcal{M})\, \frac{\vec{v}_p}{v_p} \quad \propto \begin{cases} v_p & (\mathcal{M} \ll 1) \\ \frac{\ln\Lambda}{v_p^2} & (\mathcal{M} \gg 1) \end{cases}$$

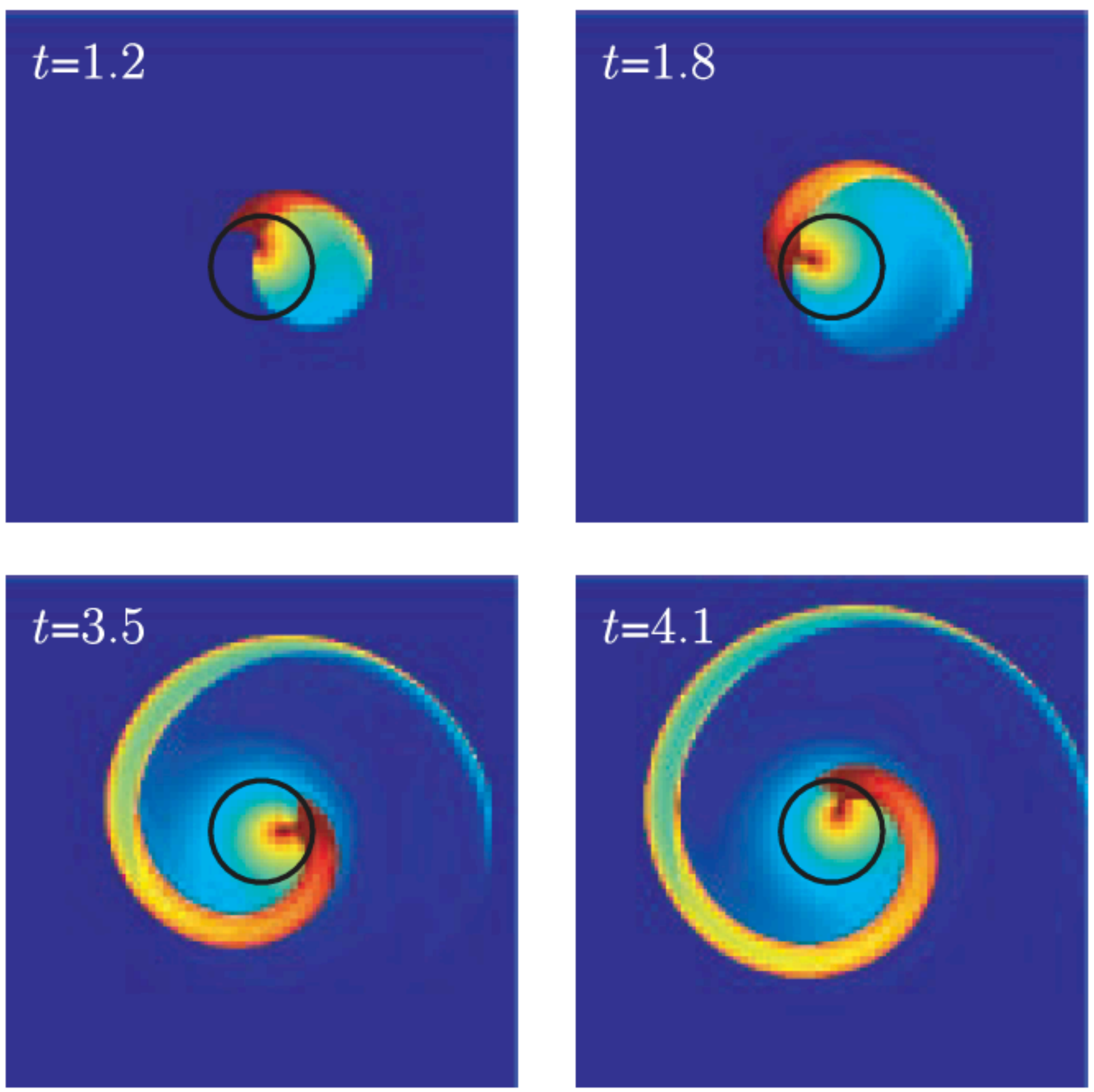
[Chandrasekhar 43; Dokuchaev 64; Ruderman+ 71; Rephaeli+ 80; Just+ 91; Ostriker 99, …]

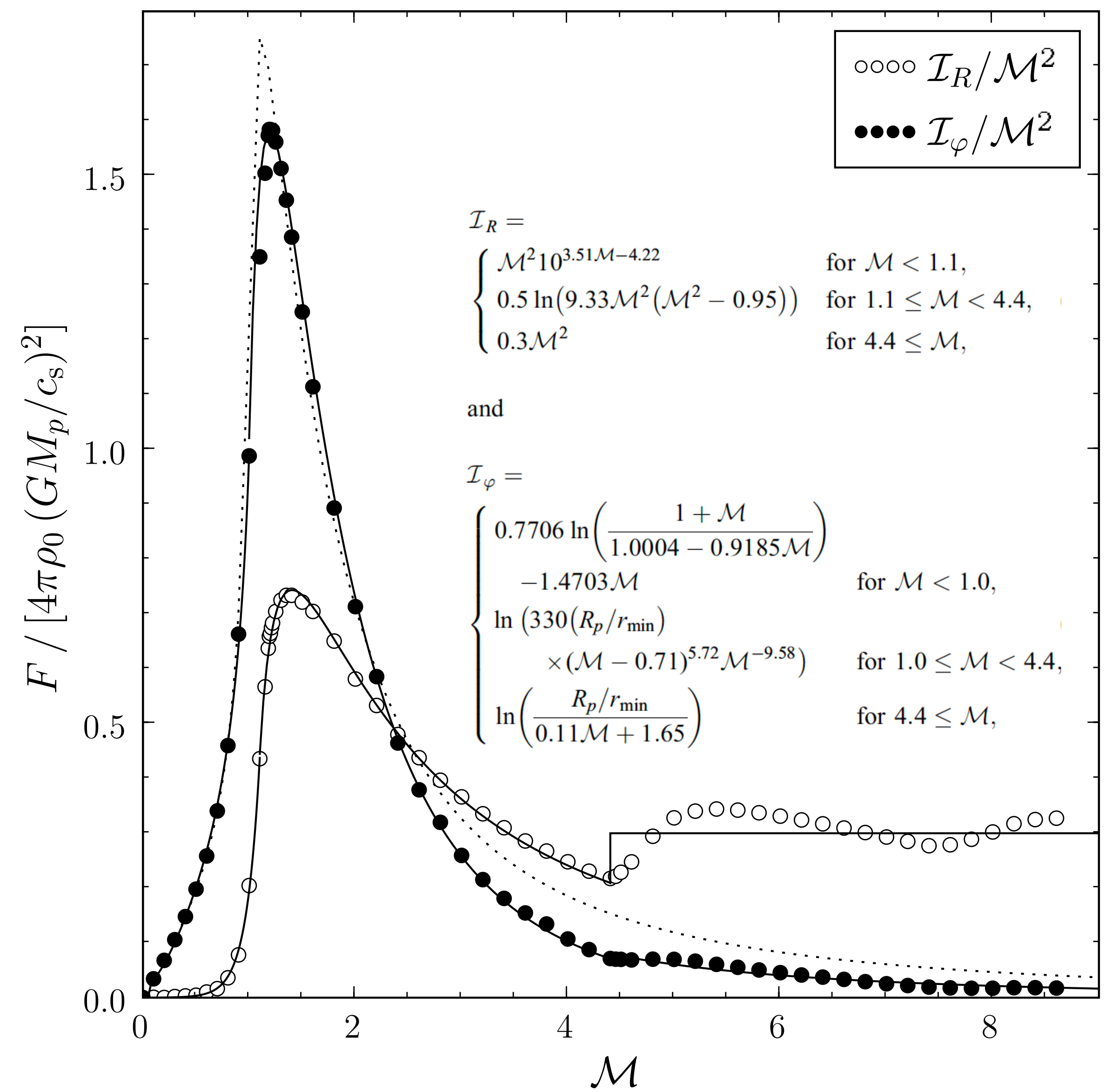
Compact binary coalescence: DF on bound eccentric orbits

DF for perturbers on circular orbits

- *Few analytical results* [Tremaine+ 84]
- *Several semi-analytical studies* [Kim+ 07; Kim +08, …]

Perfect fluid (Kim+ 07)





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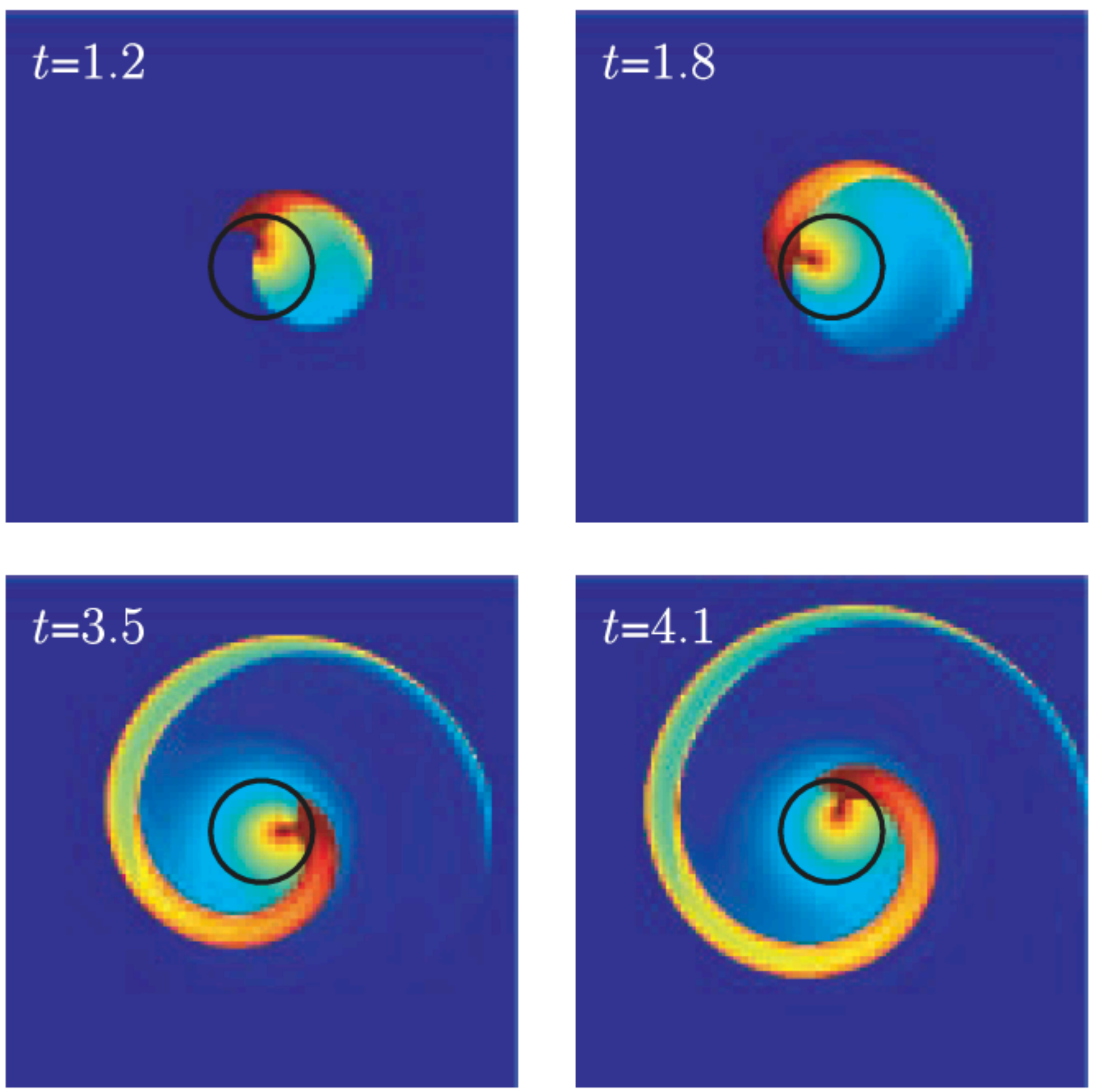

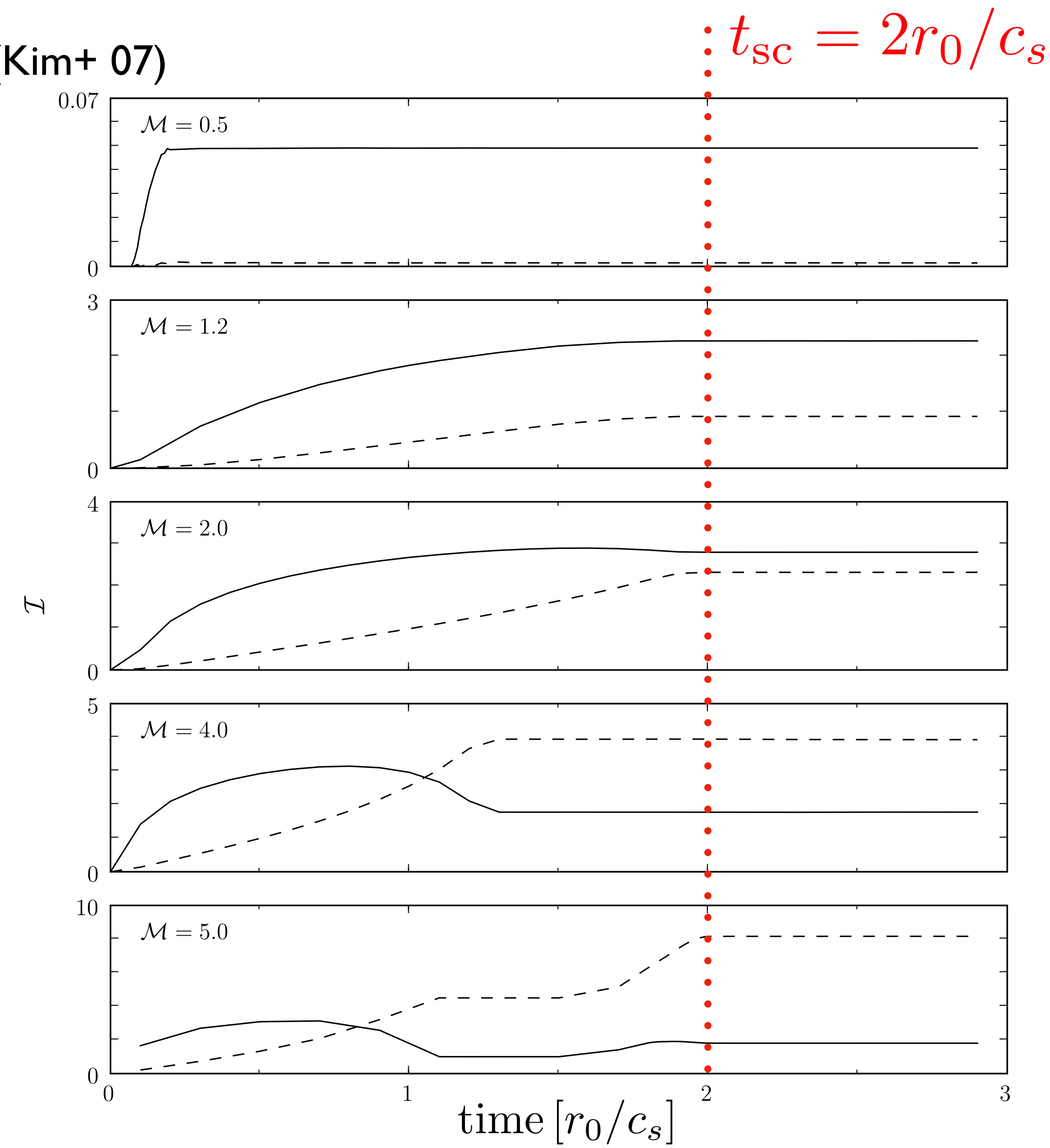

Circular DF in a perfect fluid

[VD, Nusser, Buehler ApJ, arXiv: 2111.07366]

- *Linear response theory:*

$$\ddot{\delta}\rho - c_s^2 \nabla^2 \delta\rho = 4\pi G \bar{\rho}\, \rho_p$$

$$G_{\rm ret}(\vec{r}, \tau) = \frac{1}{r}\, \delta_D\left(\tau - \frac{r}{c_s}\right)$$

$$\delta\rho \sim \frac{GM}{c_s^2 r} \bar{\rho}$$

- *Multipole expansion:*

$$I(\mathcal{M}) = \sum_{\ell m} I_{\ell m}(\mathcal{M})$$

$$I_{\ell m}(\mathcal{M}) \sim j_\ell(m\mathcal{M})\, h_\ell^{(1)}(m\mathcal{M})$$

$$\vec{F}_{\rm DF}(t) = -4\pi \left(\frac{GM}{v_p}\right)^2 \bar{\rho} \left(\Re(I)\hat{r}(t) + \Im(I)\hat{\varphi}(t)\right) \frac{\vec{v}_p}{v_p} \qquad (v_p = \Omega r_0)$$

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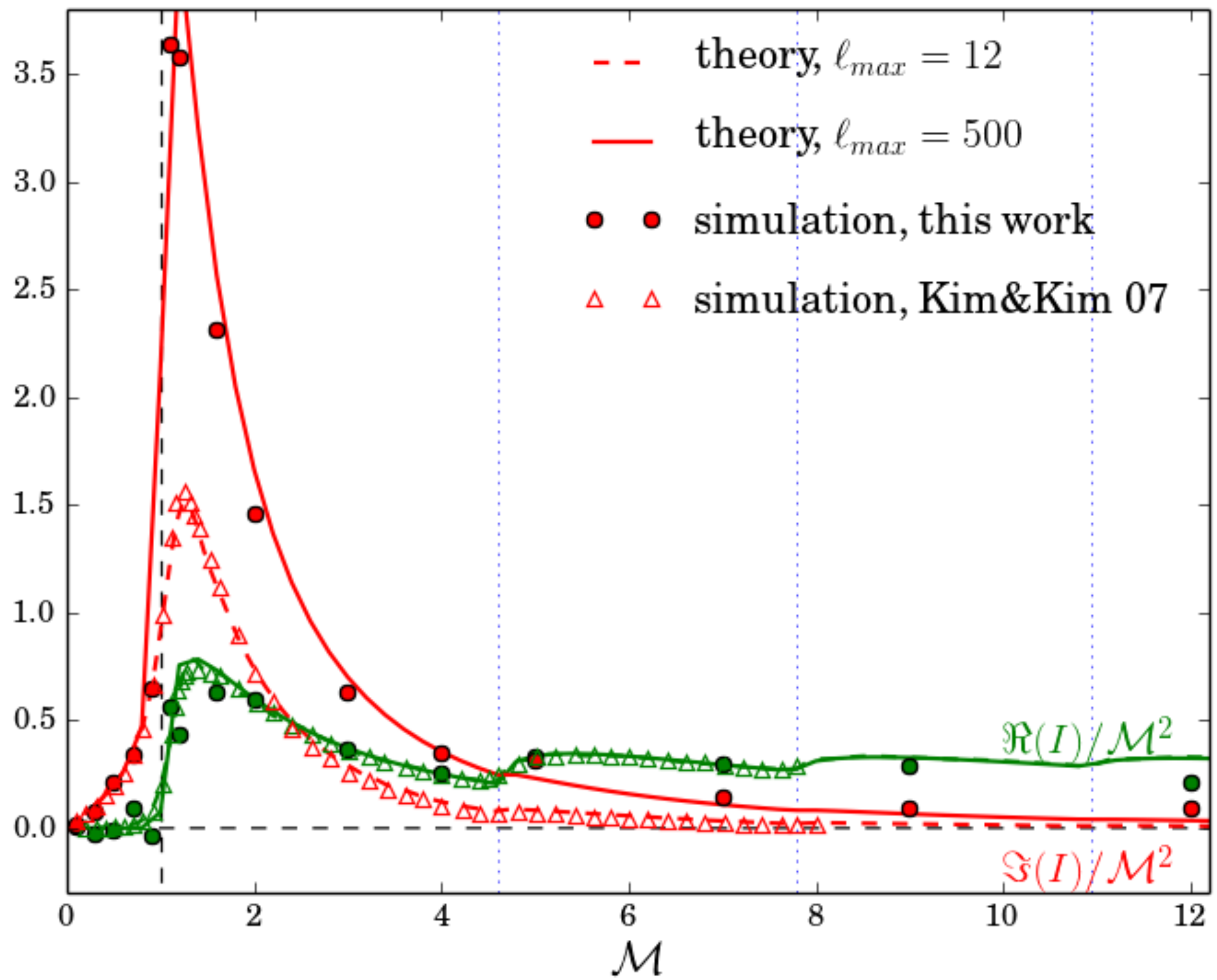

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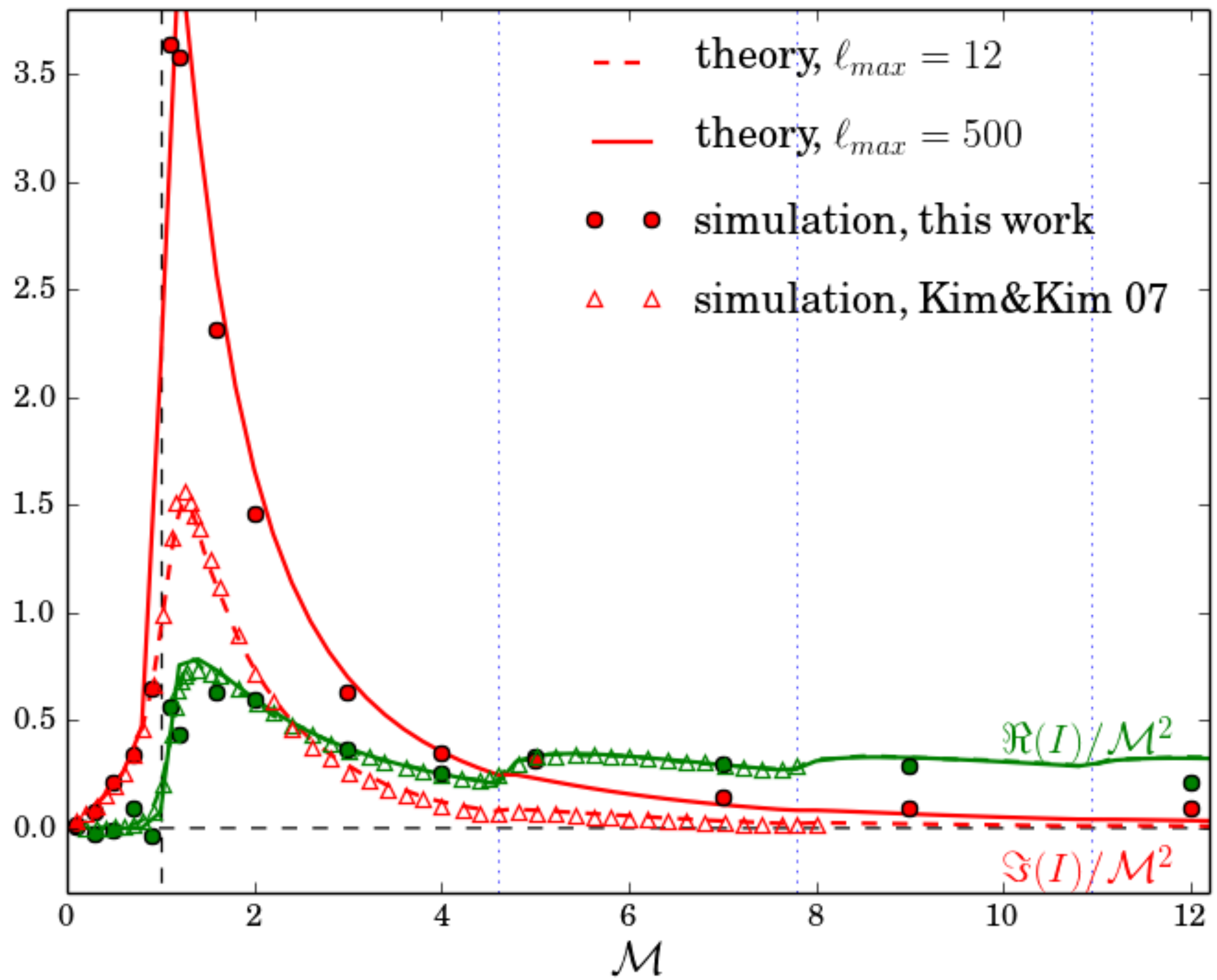
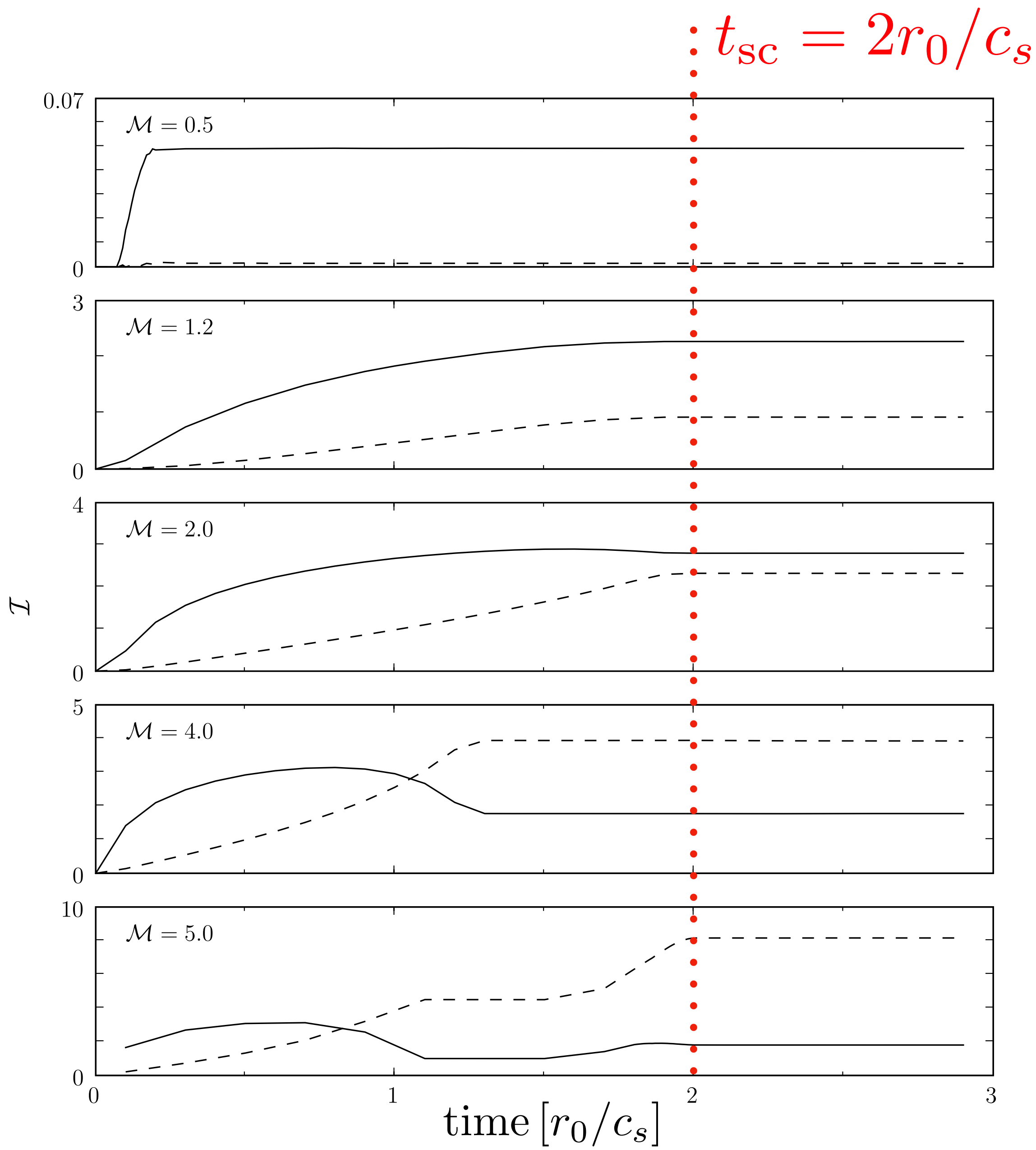


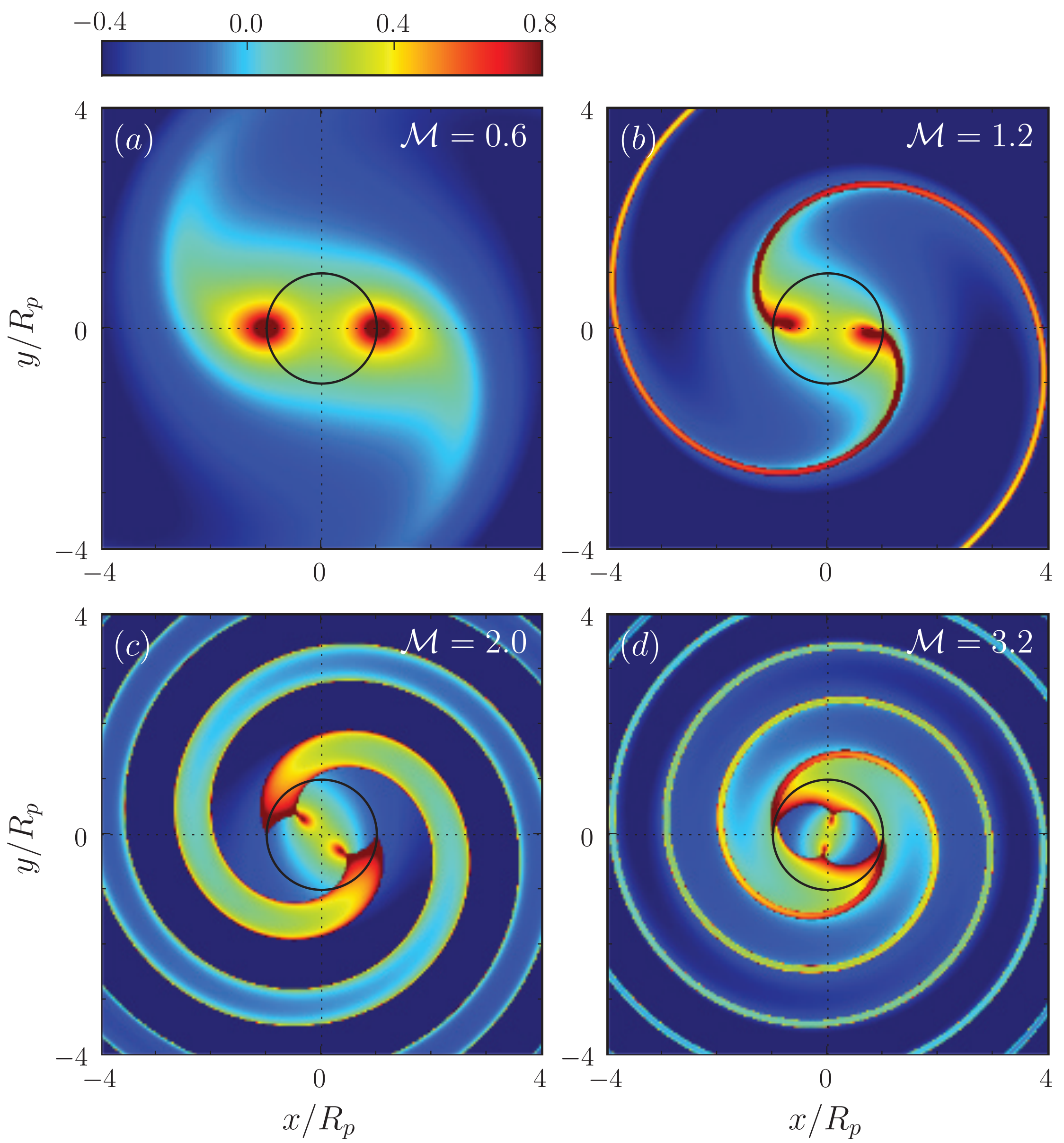


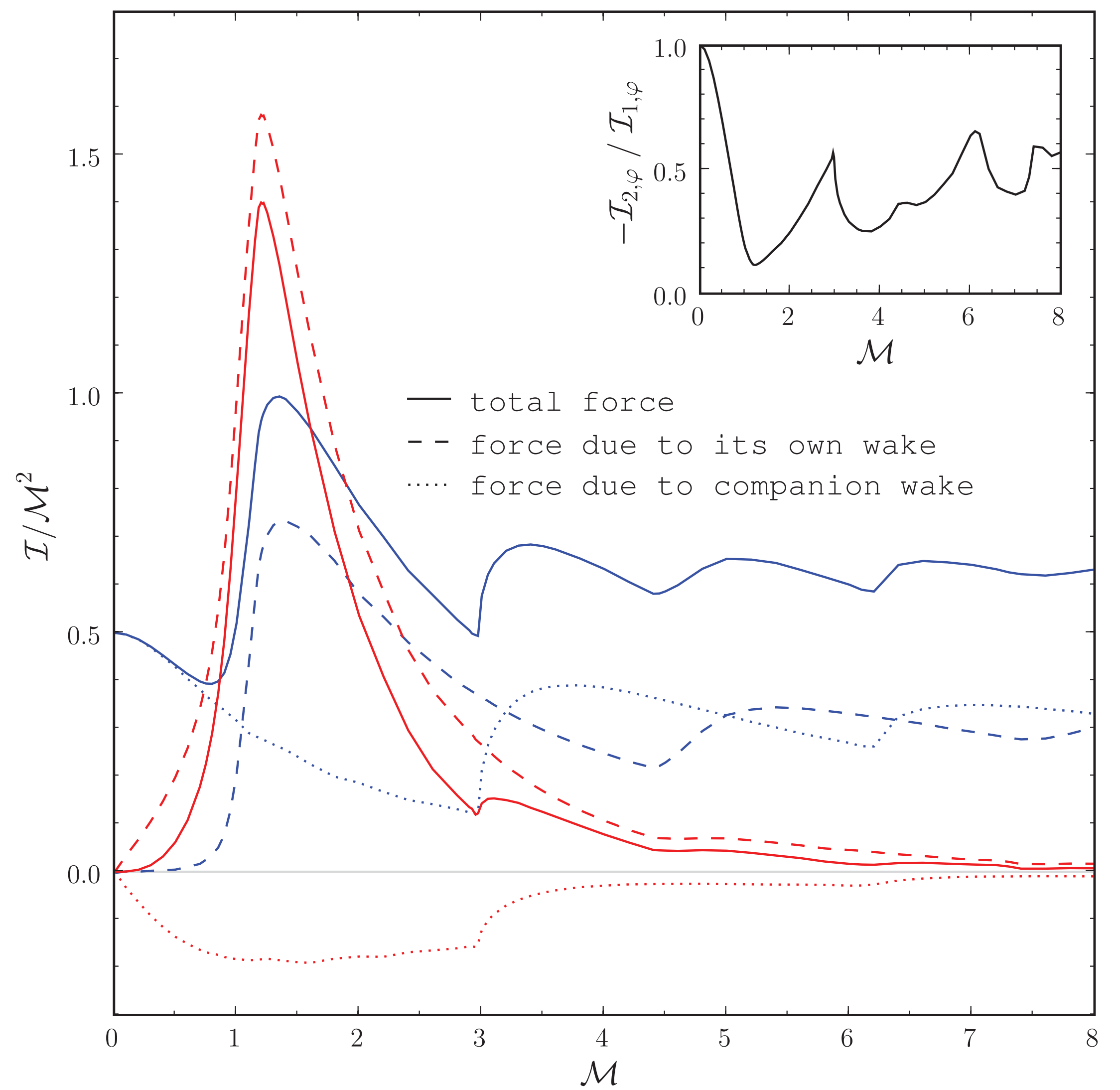
$$I_{\ell m} \supset -\frac{e^{2im\mathcal{M}\tilde{t}}}{2}\int_{-\infty}^{+\infty} dz\, e^{2iz\tilde{t}}\,\frac{j_\ell(z)j_{\ell-1}(z)}{z+m\mathcal{M}+i\epsilon}$$

$$(\tilde{t} = t/t_{\rm sc})$$

Compact circular binary

$$\text{Equal-mass binary: } q_1 = q_2 = \frac{1}{2}$$



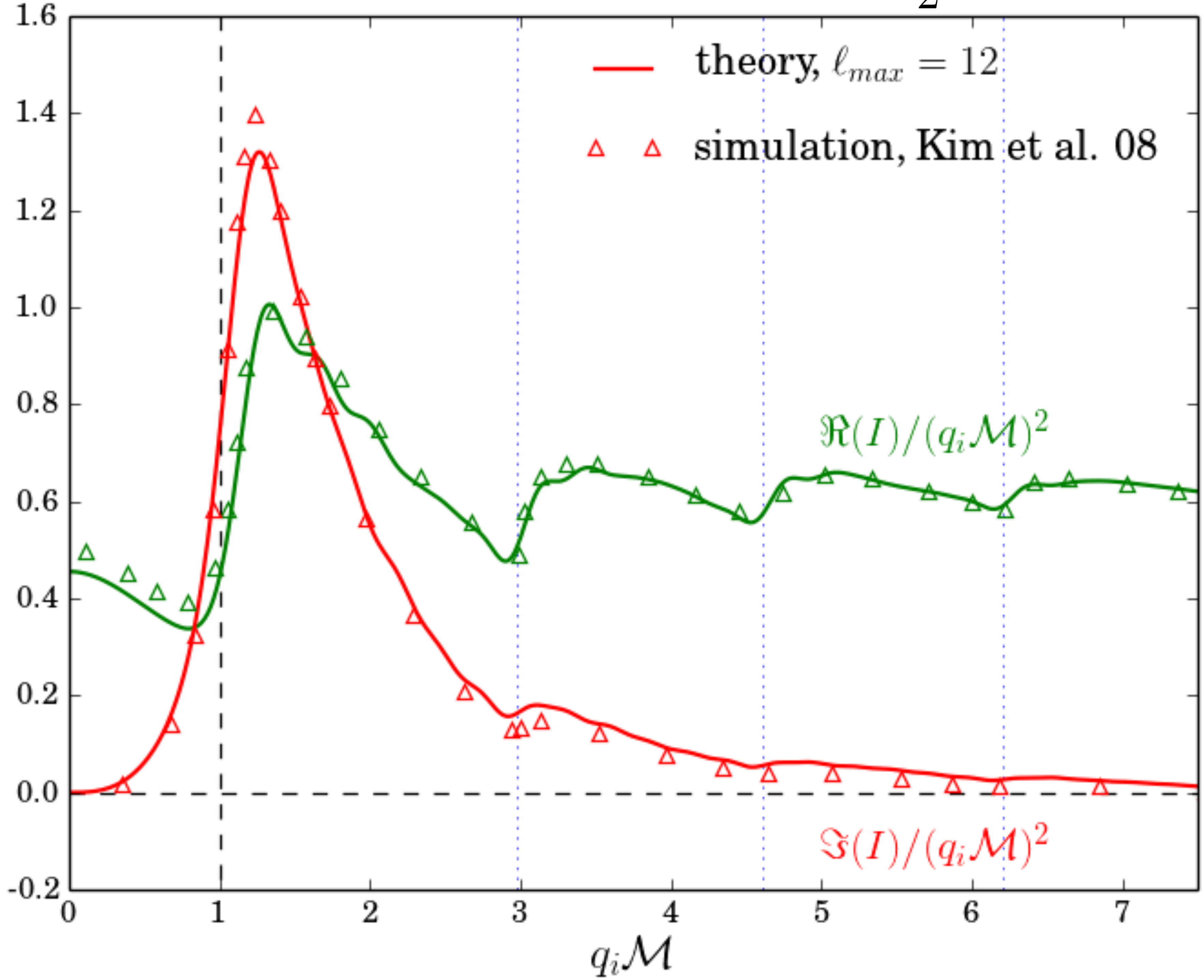


[Kim+ 08]

Compact circular binary: friction coefficient

[VD, Nusser, Buehler ApJ, arXiv: 2111.07366]

Equal-mass binary: $q_1 = q_2 = \frac{1}{2}$



Ultra-light dark matter (ULDM)

ULDM is a Bose-Einstein condensate characterized by a wave function

- *Wave-like description (Gross-Pitaevskii-Poisson):*

$$i\partial_t\psi = -\frac{\hbar}{2m_a}\nabla^2_{\mathbf{r}}\psi + \frac{m_a}{\hbar}\Phi\,\psi$$
$$\nabla^2_{\mathbf{r}}\Phi = 4\pi G\rho \qquad (\rho \equiv |\psi|^2)$$

- *Hydrodynamic description (Madelung):*

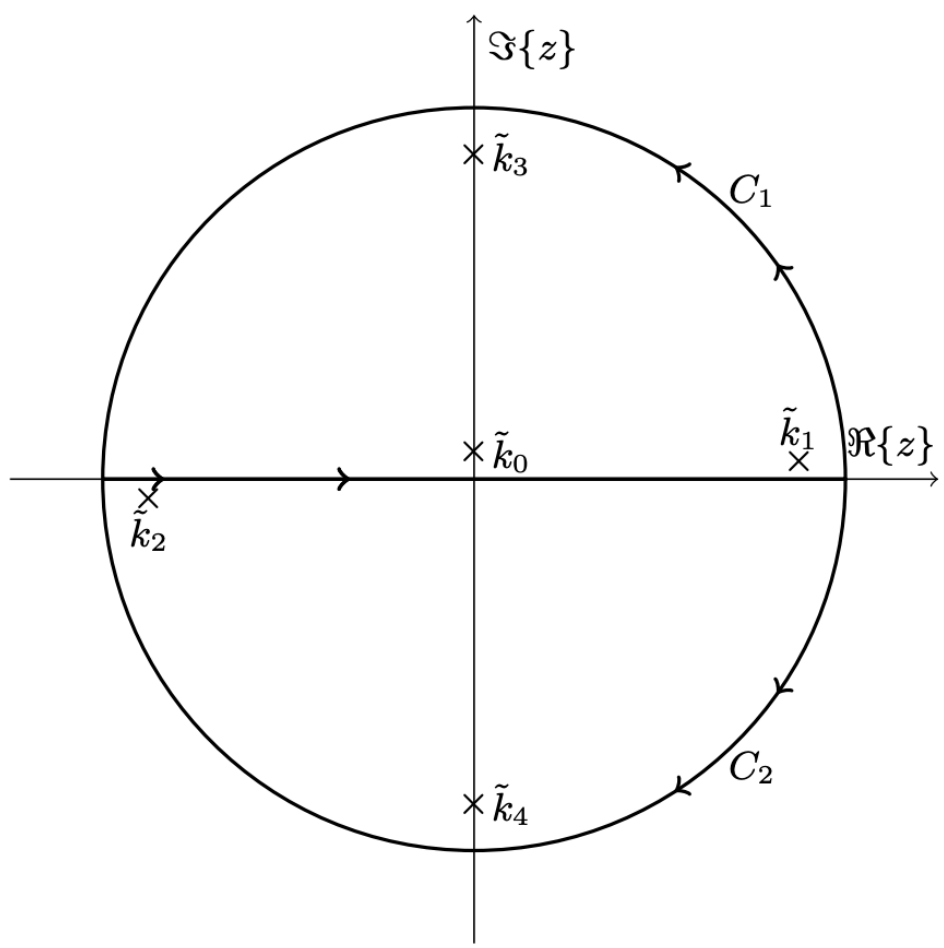
$$\psi = \sqrt{\rho}\,\mathrm{e}^{i\theta} \qquad \mathbf{v} = \frac{\hbar}{m_a}\nabla_{\mathbf{r}}\theta \quad \longrightarrow \quad \partial_t\rho + \nabla_{\mathbf{r}}(\rho\mathbf{v}) = 0 \qquad \partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla_{\mathbf{r}})\mathbf{v} = -\nabla_{\mathbf{r}}(Q+\Phi)$$

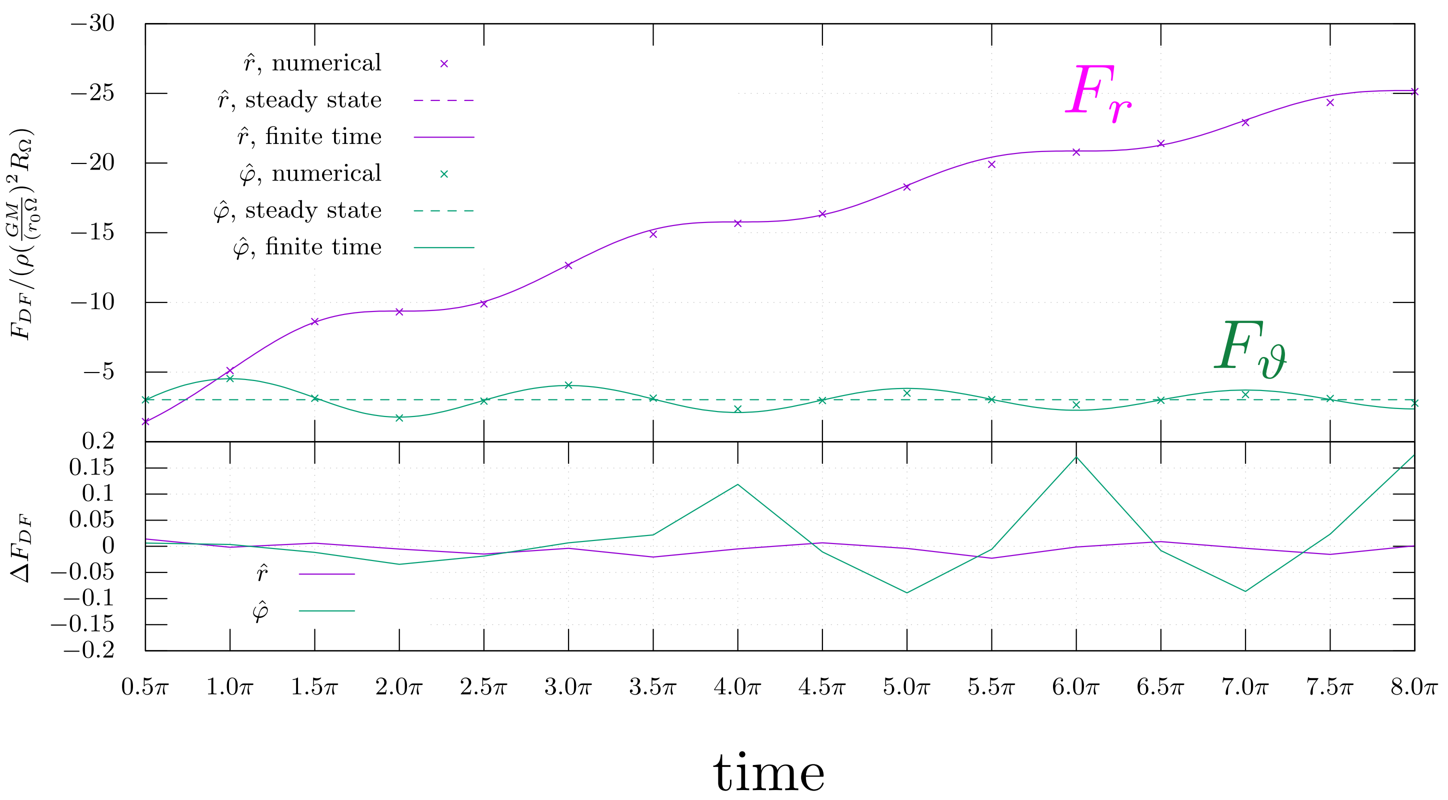
[Ruffini+ 68; Baldeschi+83; Sin 92; Hu+ 99; Chavanis 11; Hui+ 17]

Circular DF in ULDM

[Buehler & VD PRD, arXiv: 2207.13740]

$$I_{\ell m} \propto \int_0^\infty dz\, z \frac{j_\ell(z) j_{\ell-1}(z)}{z^4/R_\Omega^2 - (m+i\epsilon)^2}$$
$$- \frac{R_\Omega}{2} e^{imt} \int_0^\infty \frac{dz}{z} j_\ell(z) j_{\ell-1}(z) \left(\frac{e^{-i(z^2/R_\Omega - i\epsilon)t}}{z^2/R_\Omega - m - i\epsilon} + \frac{e^{i(z^2/R_\Omega + i\epsilon)t}}{z^2/R_\Omega + m + i\epsilon} \right)$$



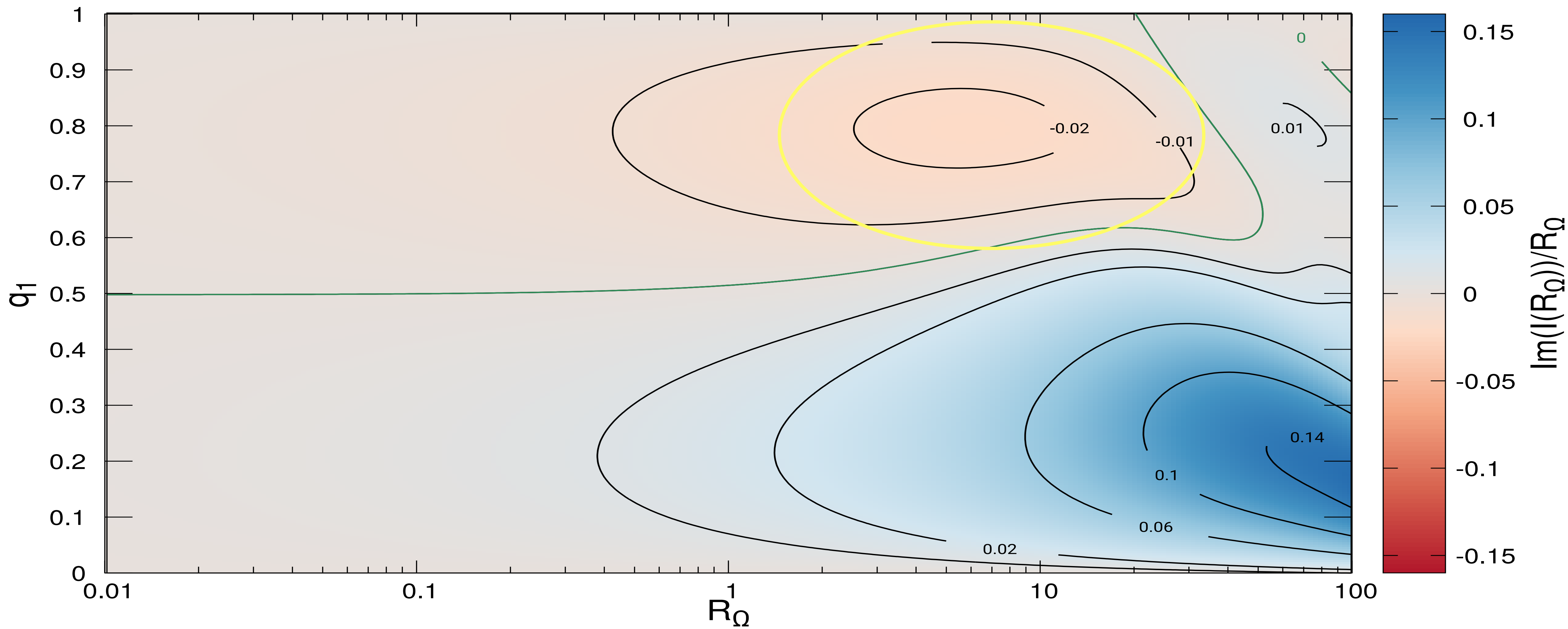


Circular DF in ULDM

[Buehler & VD PRD, arXiv: 2207.13740]

$$G_{\rm ret}(\vec{r},\tau) \propto \Im\left[\mathrm{erf}\left(\frac{1+i}{2\sqrt{2}}\sqrt{R_\Omega}\frac{r}{\sqrt{\tau}}\right)\right] \qquad (R_\Omega = 2m_a\Omega r_0^2/\hbar)$$

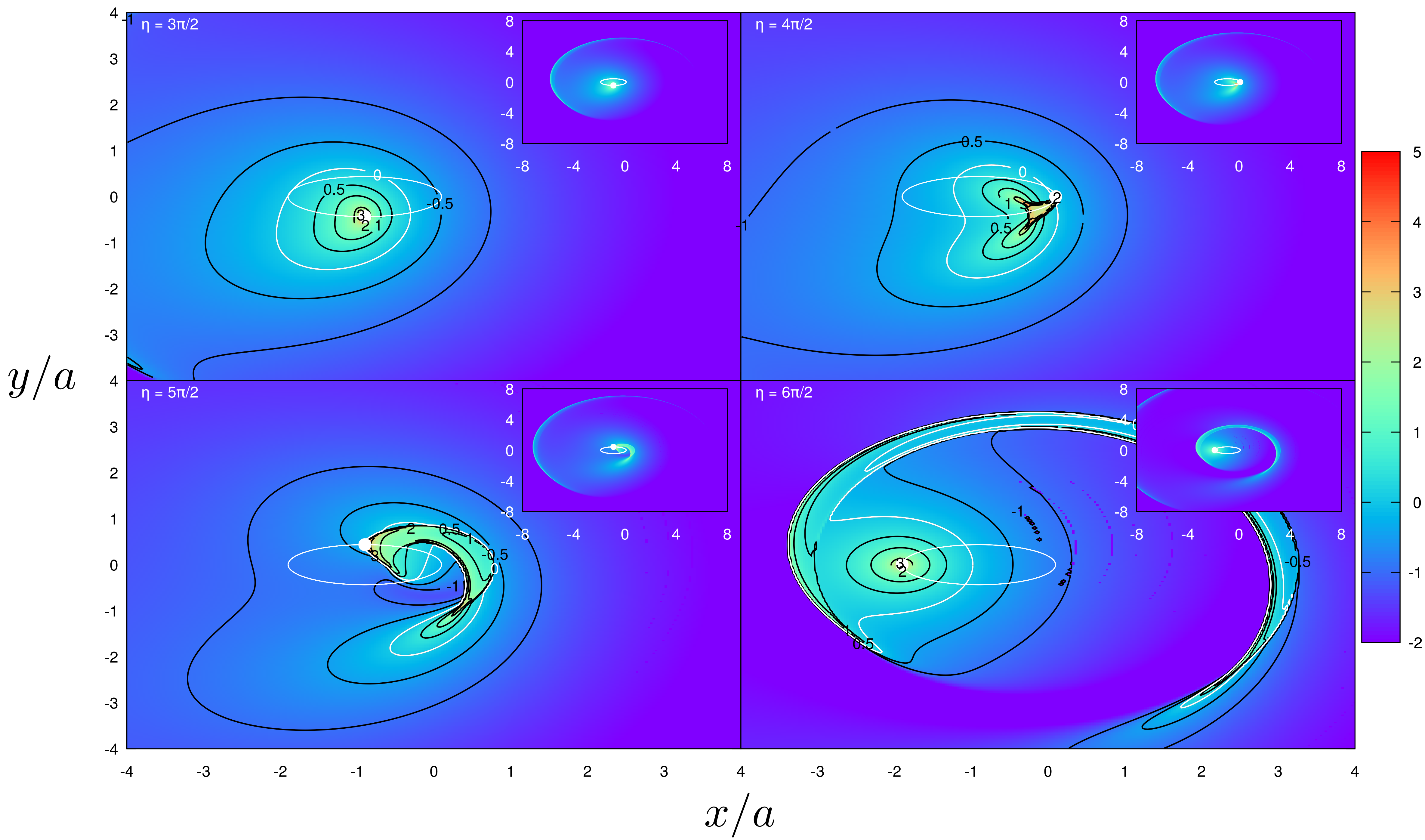
Friction coefficient for a compact circular binary



Eccentric evolution in perfect fluid

[Buehler, Kolyada & VD, arXiv: 2310.05244]

$\mathcal{M}_a = 0.9, \quad e = 0.9 \qquad (\mathcal{M}_a = \Omega a / c_s)$



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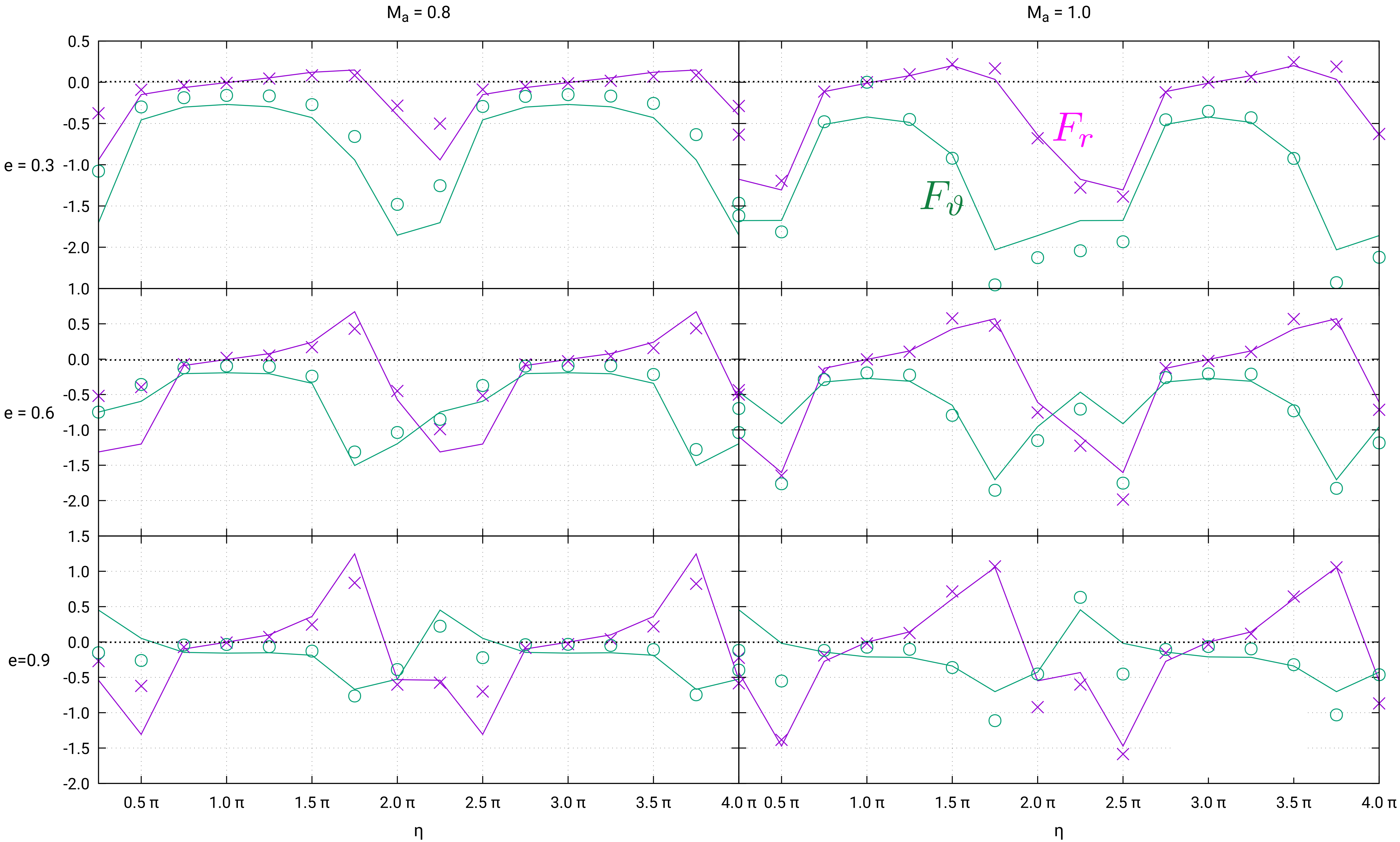

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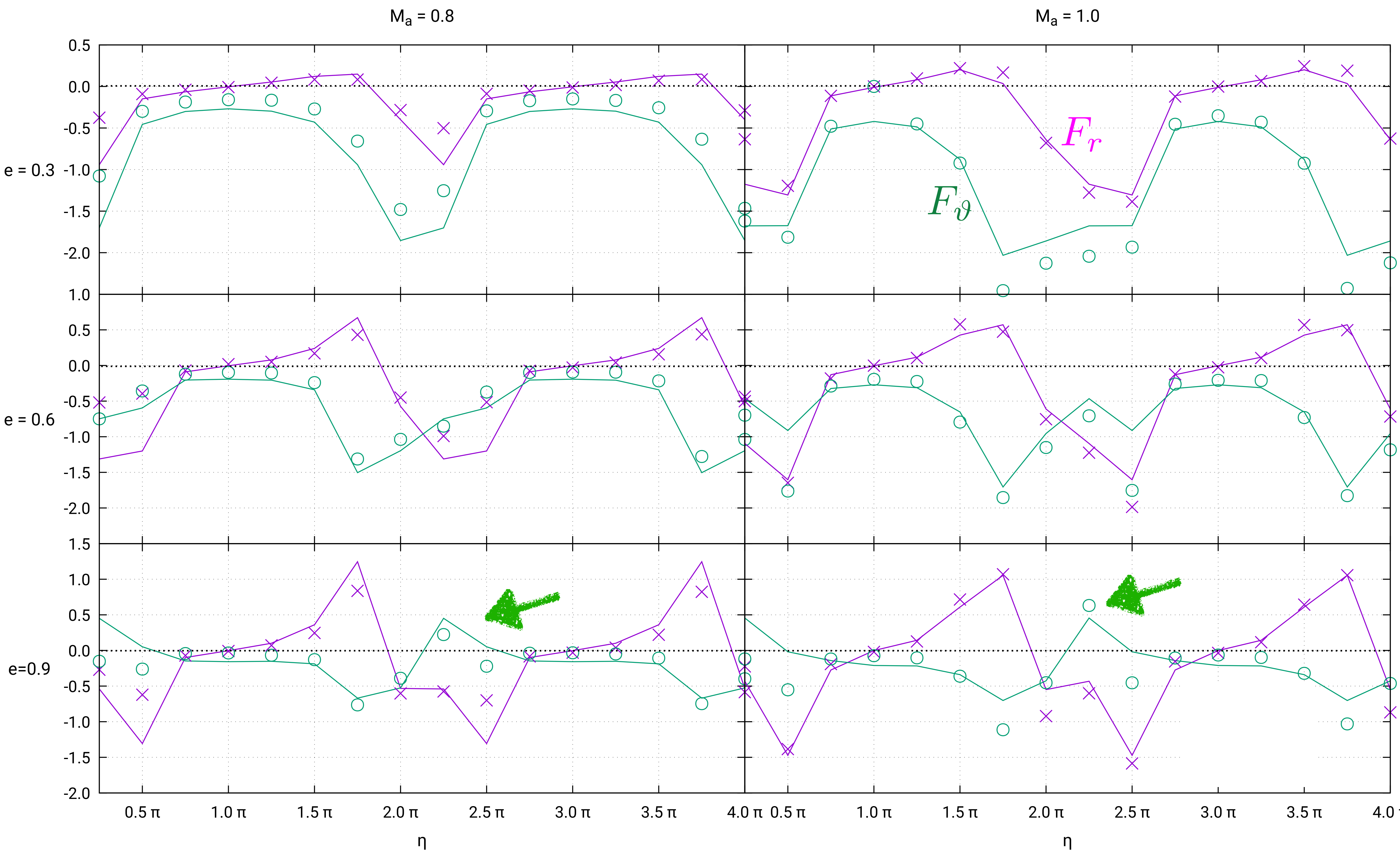

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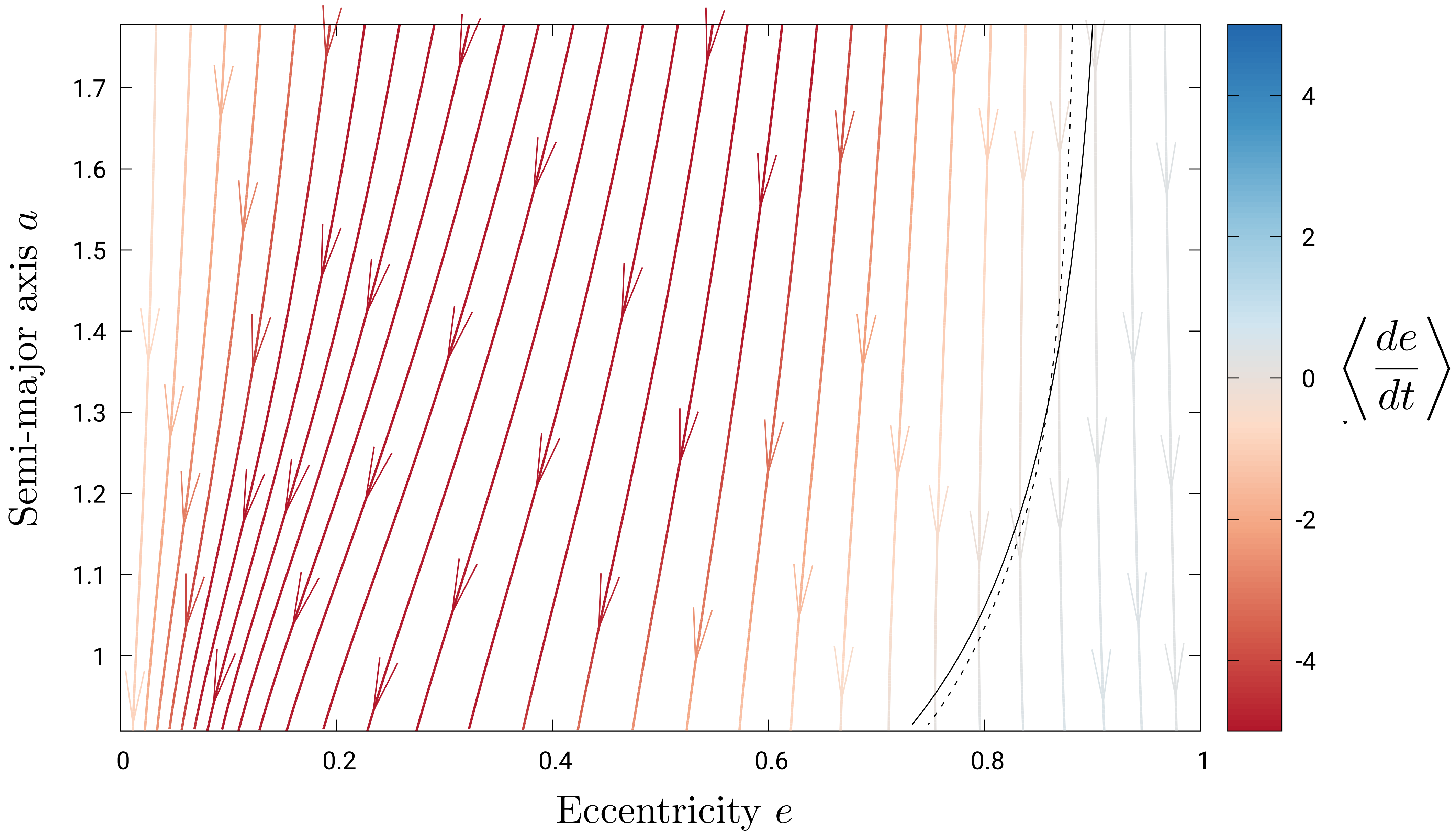


Circular DF in isothermal density profiles

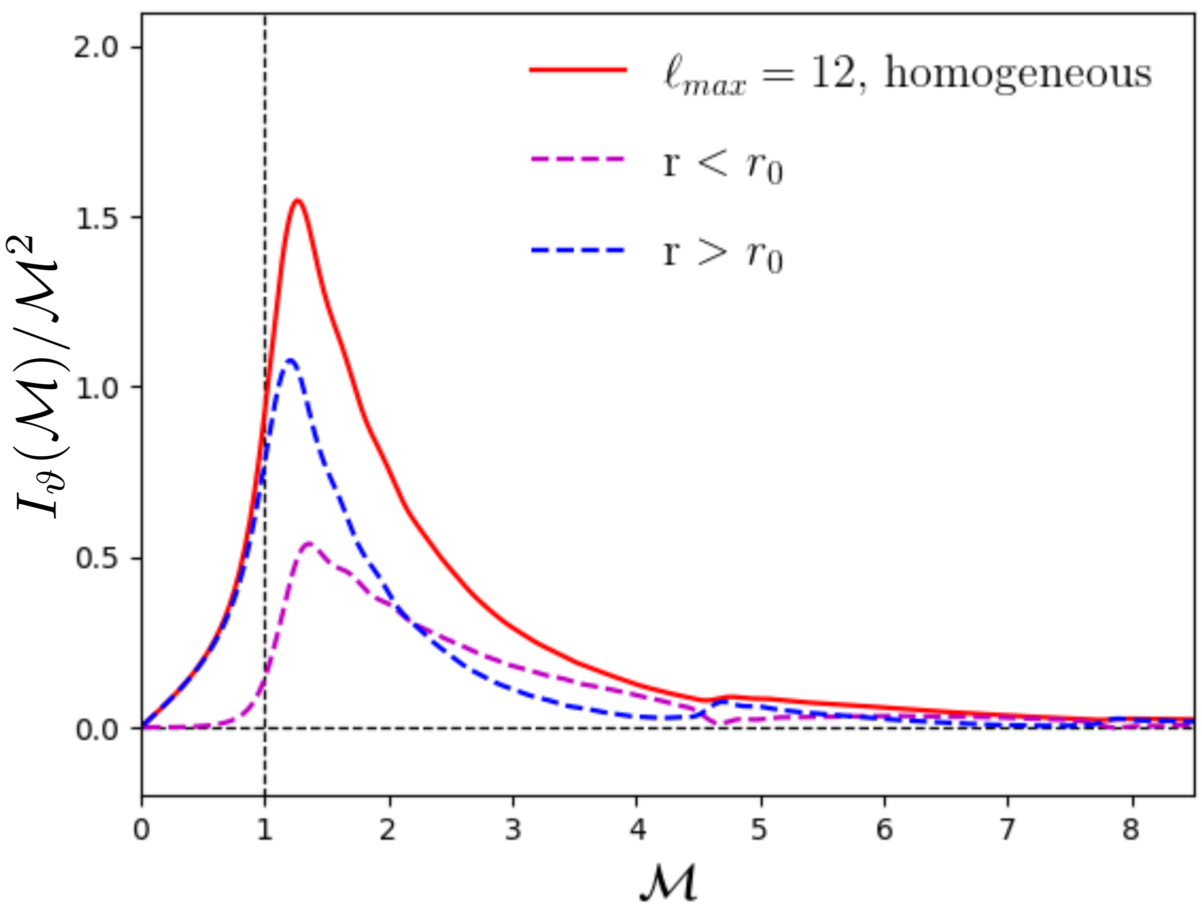
w/ Gali Eytan and Robin Buehler

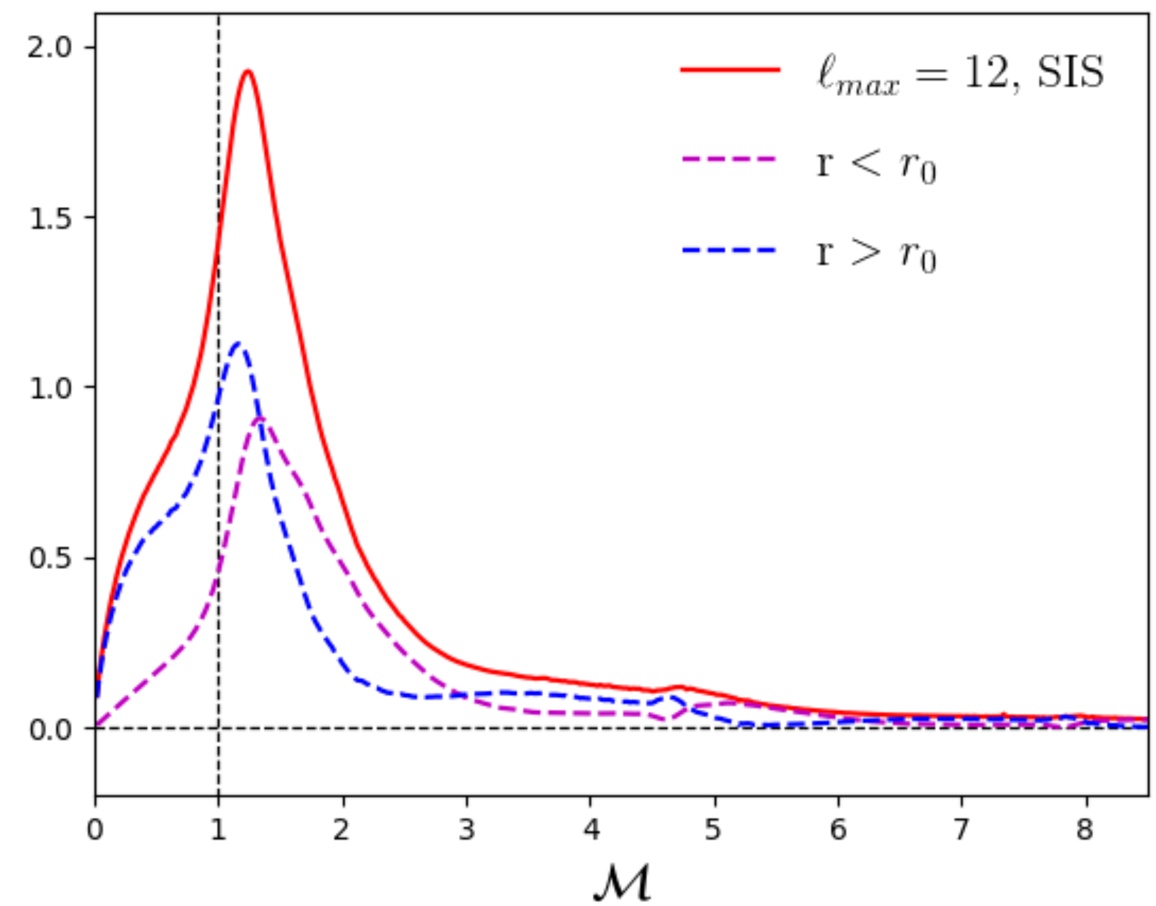
$$P = c_s^2 \rho$$

Singular Isothermal Sphere (SIS):

$$\rho_0(r) = \frac{c_s^2}{2\pi G} r^{-2}$$

$$\phi_0(r) = 2c_s^2 \ln(r) + \text{const}$$





Outlook

- *Linear response theory DF can be solved analytically in the circular case*
- *Include eccentric orbits, inhomogeneous backgrounds, other matter "excitations"*
- *Provide (i) useful physical insights and (ii) a versatile tool to explore DF for a wide range of media and dynamical systems (single perturber, binaries …)*