

Nonequilibrium evolution of quarkonium in the QGP

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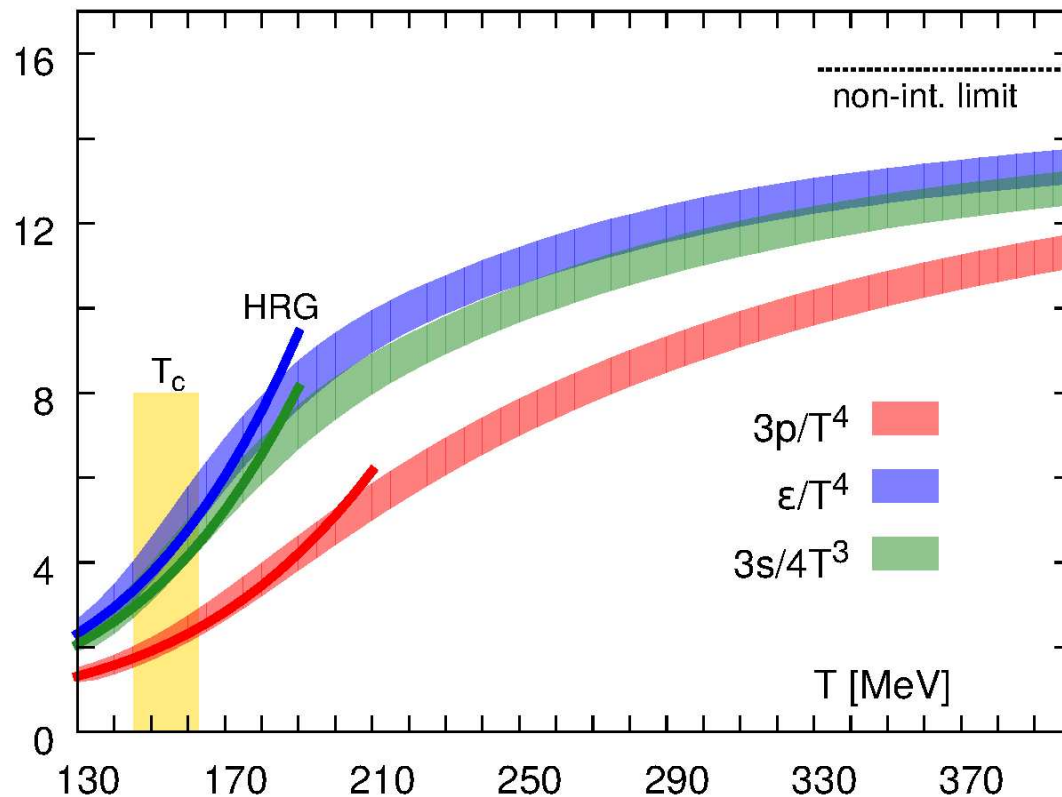


Based on

- (1) N. Brambilla, M.A. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo and P. Vander Griend
Phys. Rev. D 108 (2023) 1 [arXiv:2302.11826](#)
- (2) N. Brambilla, M.A. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo and P. Vander Griend
JHEP 08 (2022) 303 [arXiv:2205.10289](#)
- (3) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
Phys. Rev. D 104 (2021) 094049 [arXiv:2107.06222](#)
- (4) N. Brambilla, M.A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber
JHEP 05 (2021) 136 [arXiv:2012.01240](#)
- (5) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Phys. Rev. D 97 (2018) 074009 [arXiv:1711.04515](#)
- (6) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Phys. Rev. D 96 (2017) 034021 [arXiv:1612.07248](#)

Quarkonium suppression
as a Quark Gluon Plasma probe

Color deconfinement

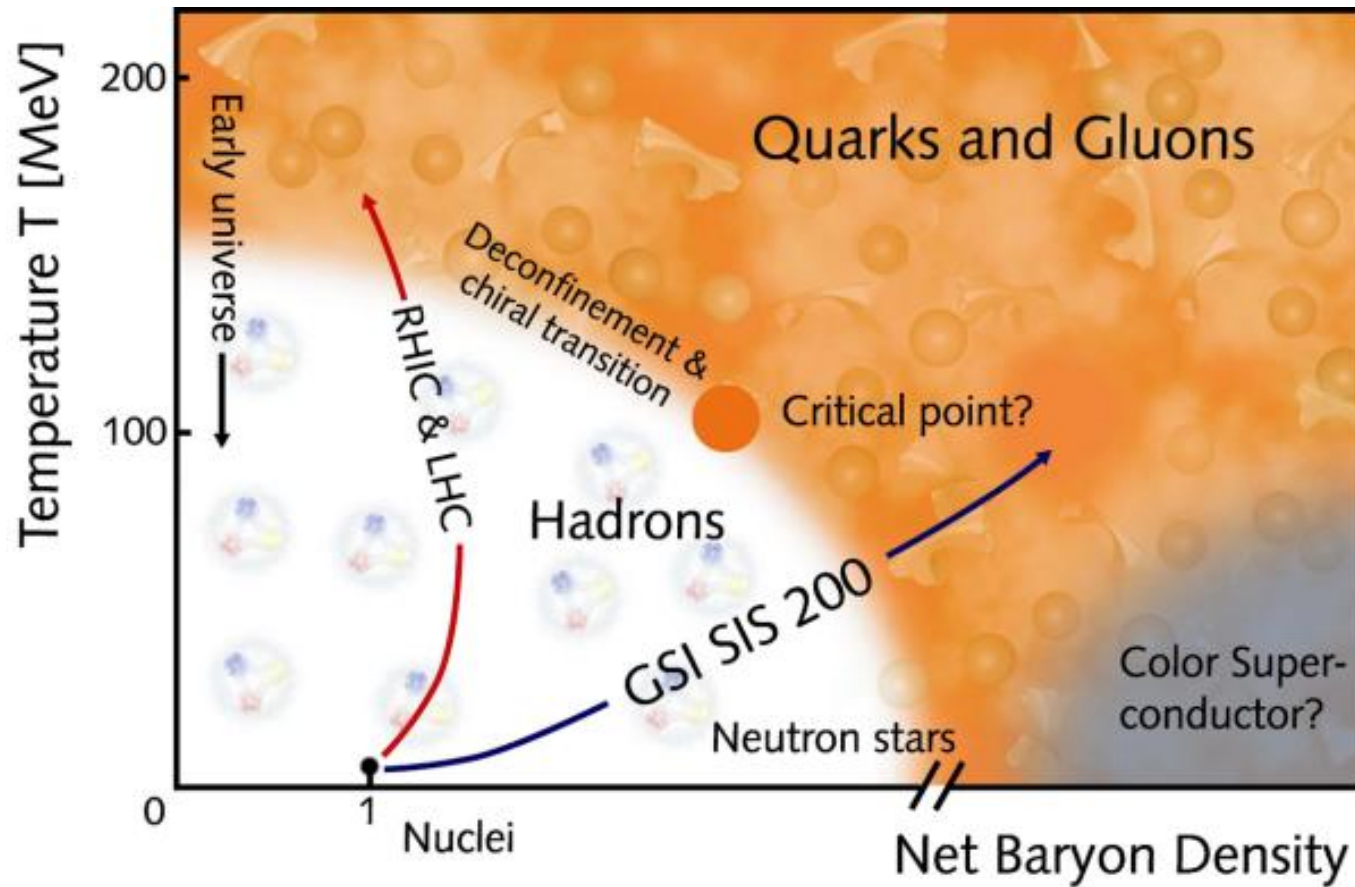


Transition from hadronic matter to a **plasma of deconfined quarks and gluons** as studied in finite temperature lattice QCD: $T_c = 154 \pm 9$ MeV.

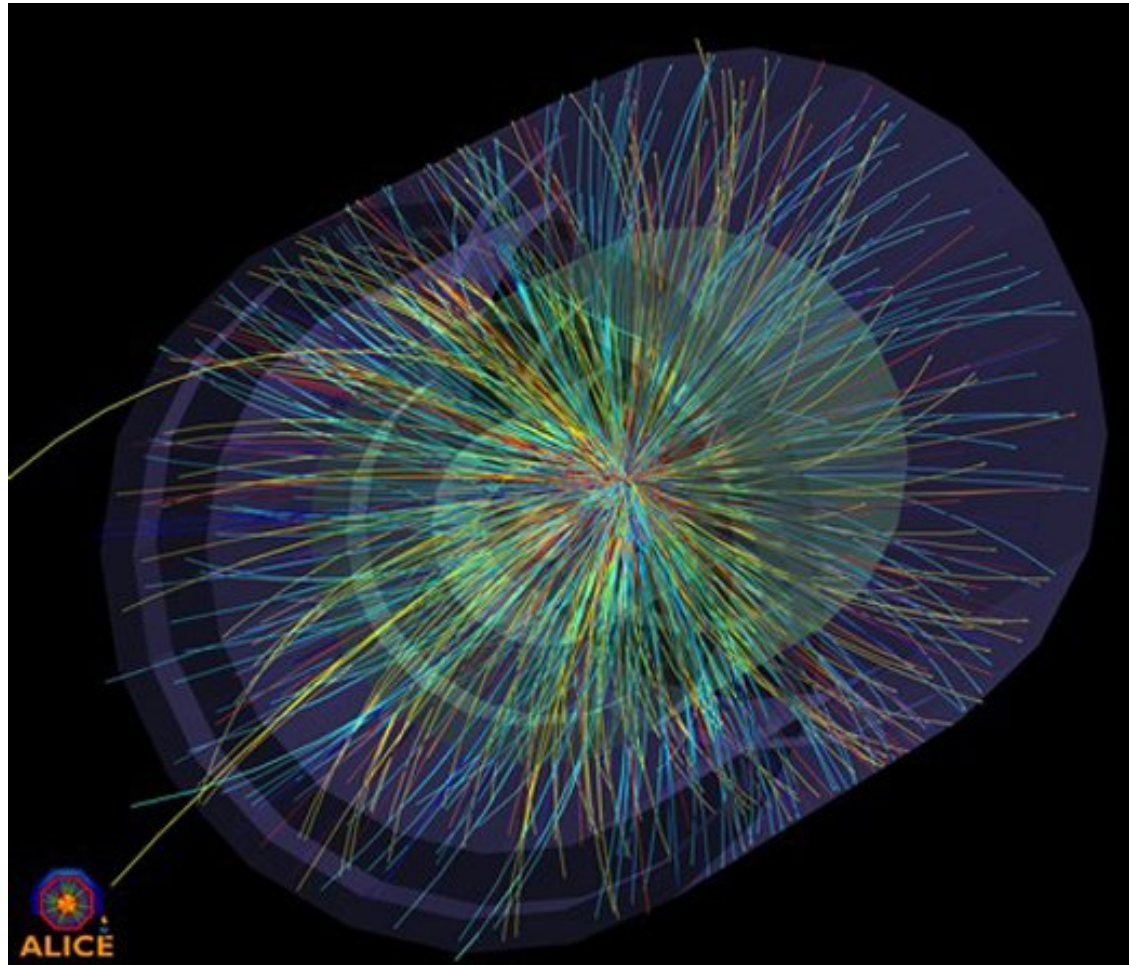
○ Wuppertal-Budapest JHEP 1009 (2010) 073

HotQCD PRD 90 (2014) 094503, TUMQCD PRD 93 (2016) 114502

QCD phase diagram



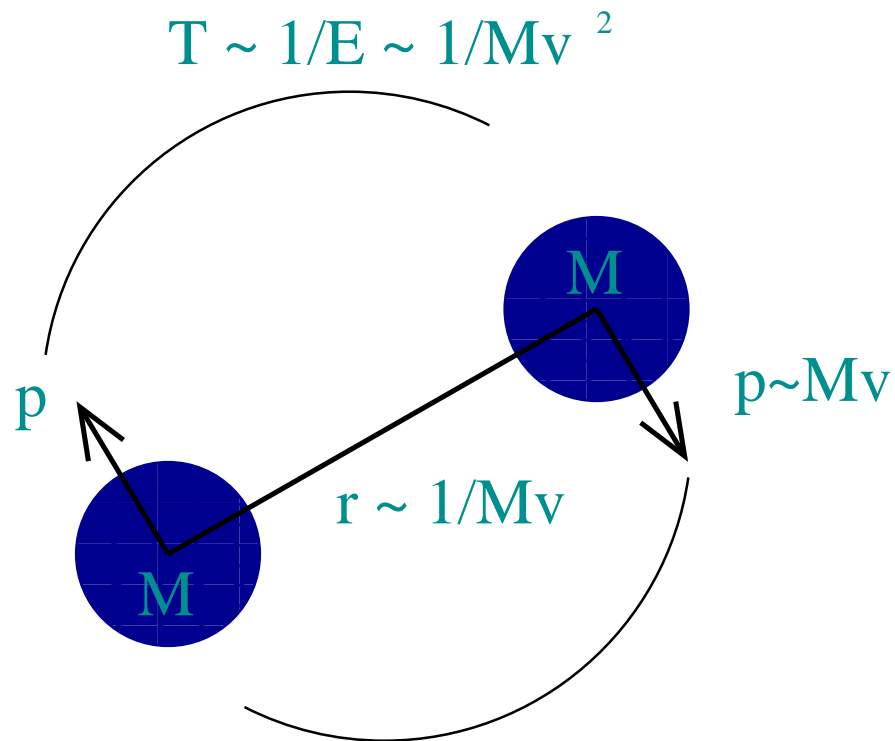
Heavy-ion experiments



We need probes to study the state of matter which is formed.

Quarkonium

Quarkonium is a hadron made of a heavy quark (c, b) and a heavy antiquark.



It is a **non-relativistic bound state** whose relative velocity is $v \ll 1$.

Hence characterized by the typical hierarchy of scales of a non-relativistic bound state.

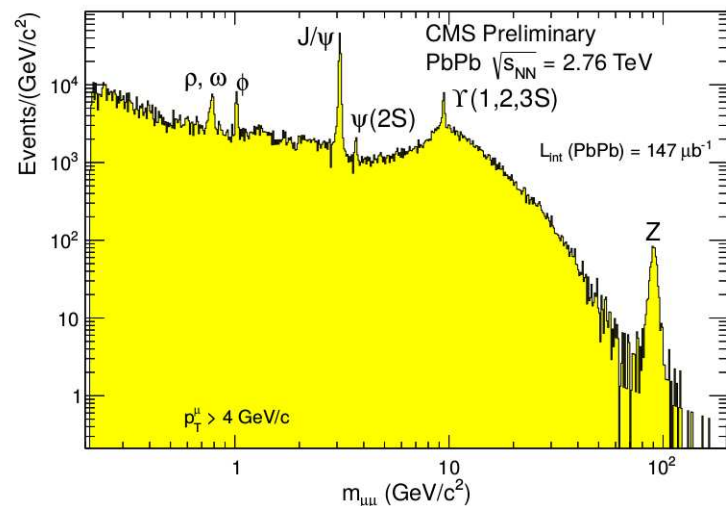
Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe.

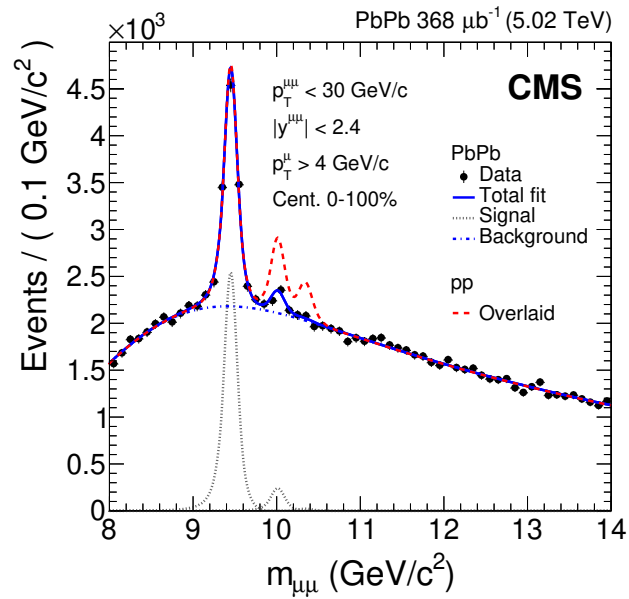
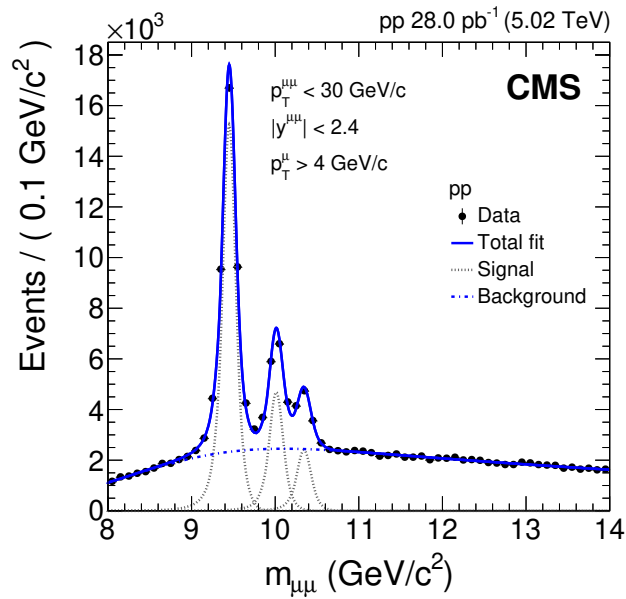
○ Matsui Satz PLB 178 (1986) 416

Experimentally

- Heavy quarks are formed early in heavy-ion collisions: $1/M \sim 0.1 \text{ fm} < 0.6 \text{ fm}$.
- Heavy quarkonium formation is sensitive to the medium.
- The dilepton signal (happening after the quark-gluon plasma has faded away) makes the quarkonium a clean experimental probe.

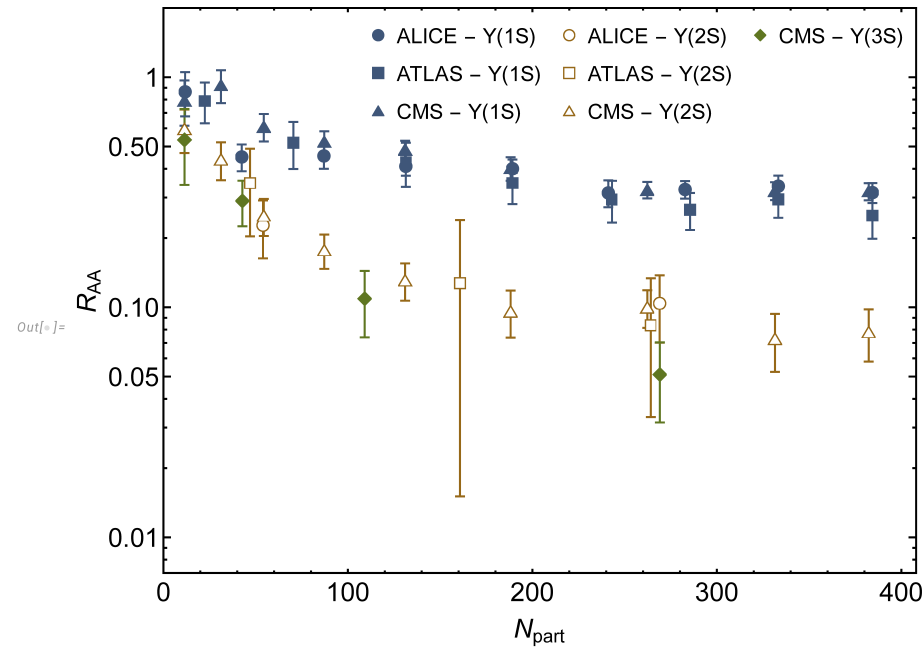


Υ suppression @ CMS



○ CMS PRL 120 (2018) 142301

Υ suppression @ LHC



R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.

- CMS PLB 790 (2019) 270
- ALICE PLB 822 (2021) 136579
- ATLAS PRC 107 (2023) 054912

Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe.

Theoretically

- The heavy-quark mass introduces one or more large scales, whose contributions may be factorized and computed in perturbation theory ($\alpha_s(M) \ll 1$).
- Low-energy scales are sensitive to the temperature.
Low-energy contributions may be accessible via lattice calculations.
- If the heavy quark is heavy enough (**bottomonium**) regeneration from the medium is negligible and only suppression due to screening and **thermal width** is relevant.

Effective field theories

Energy scales

Quarkonium in a medium is characterized by several energy scales:

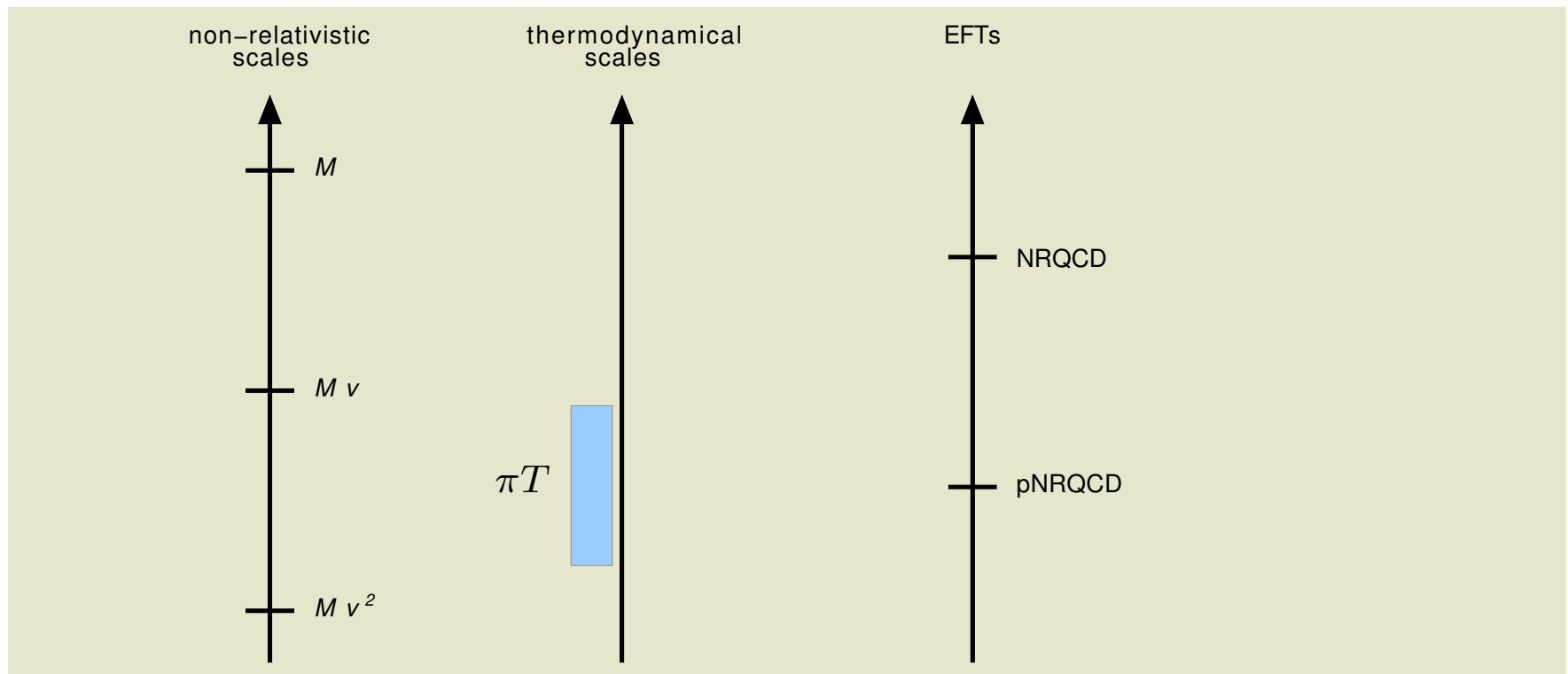
- the scales of a **non-relativistic** bound state
(v is the relative heavy-quark velocity; $v \sim \alpha_s$ for a Coulombic bound state):
 M (mass),
 Mv (momentum transfer, inverse distance),
 Mv^2 (kinetic energy, binding energy, potential V), ...
- the **thermodynamical** scales:
 T (temperature), ...

T stands for a generic inverse correlation length characterizing the medium. For definiteness we assume that the system is locally in thermal equilibrium so that a **slowly varying time-dependent temperature** can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system in terms of a hierarchy of EFTs. We assume T (~ 400 MeV) $<$ Mv (~ 1.5 GeV, for Υ).



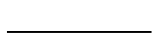
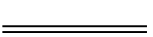
- Brambilla Pineda Soto Vairo RMP 77 (2005) 1423
Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

pNRQCD

Fields:

- S^\dagger creates a quark-antiquark pair in a **color singlet** configuration.
- O^\dagger creates a (unbound) quark-antiquark pair in a **color octet** configuration.
- **gluons** and **light quarks**.

Propagators:

- singlet  and octet  governed (in a **Coulombic** system) by the Hamiltonians $h_s = \frac{\mathbf{p}^2}{M} - \frac{4}{3} \frac{\alpha_s}{r} + \dots$ and $h_o = \frac{\mathbf{p}^2}{M} + \frac{\alpha_s}{6r} + \dots$, respectively.

Electric-dipole interactions:

-  = $O^\dagger \mathbf{r} \cdot g\mathbf{E} S$  = $O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \}$

Equation of motion:

$$i\partial_t S = h_s S + \mathbf{r} \cdot g\mathbf{E}^a \frac{O^a}{\sqrt{6}} + \dots$$

Open quantum systems

Density matrix

An arbitrary statistical ensemble of quantum states can be represented by a **density matrix** ρ , which is

- **Hermitian**: $\rho^\dagger = \rho$;
- **positive**: $\langle \psi | \rho | \psi \rangle \geq 0$ for all states $|\psi\rangle$;
- and can be **normalized** to have unit trace: $\text{Tr}\{\rho\} = 1$.

The time evolution of the density matrix is described by the **von Neumann equation**:

$$i \frac{d\rho}{dt} = [H, \rho]$$

which follows from the Schrödinger equation for $|\psi\rangle$. The evolution equation

- is **linear** in ρ ;
- **preserves the trace** of ρ ;
- is **Markovian**.

Open quantum system

In quantum information theory, one separates the full system into a **subsystem** of interest and its **environment**. A density matrix ρ for the subsystem can be obtained from the density matrix ρ_{full} for the full system by the partial trace over the environment states:

$$\rho = \text{Tr}_{\text{environment}} \{ \rho_{\text{full}} \}$$

In general the evolution of ρ is non-Markovian.

The evolution is Markovian if the time during which the subsystem is observed is much larger than the time scale for correlations between the subsystem and the environment. We must also restrict to the low-frequency behavior of the subsystem, which can be accomplished by smoothing out over times larger than the correlation time scale.

Lindblad equation

The density matrix ρ for the subsystem necessarily satisfies three basic properties: it is **Hermitian**, **positive**, and it can be **normalized**.

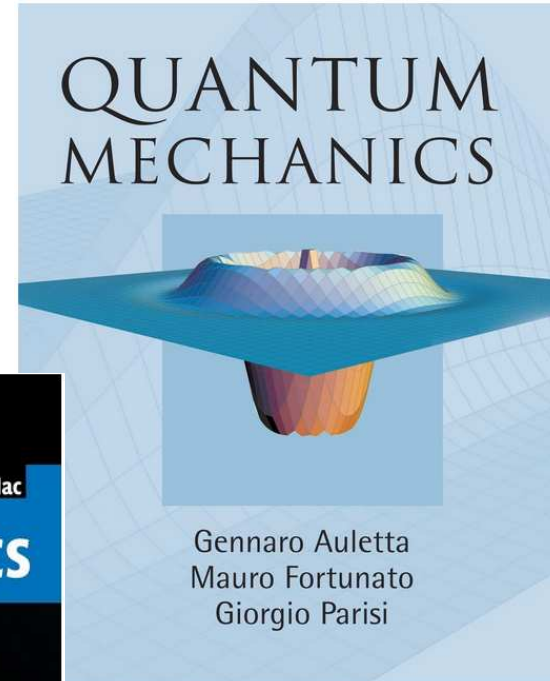
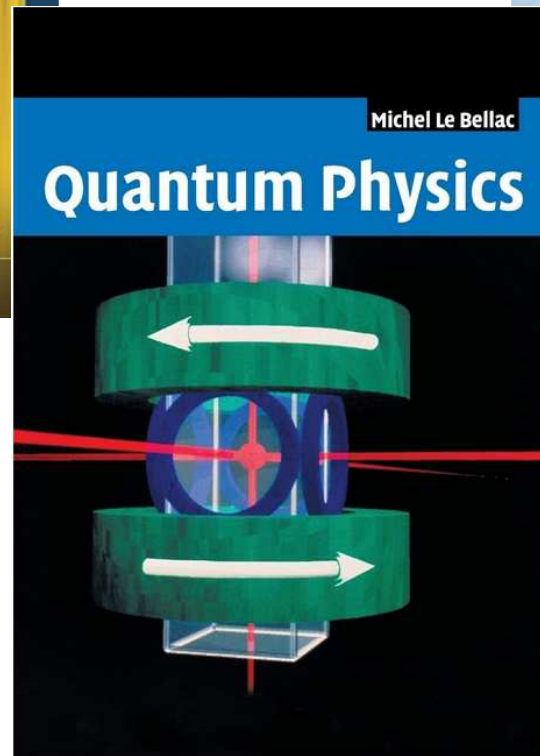
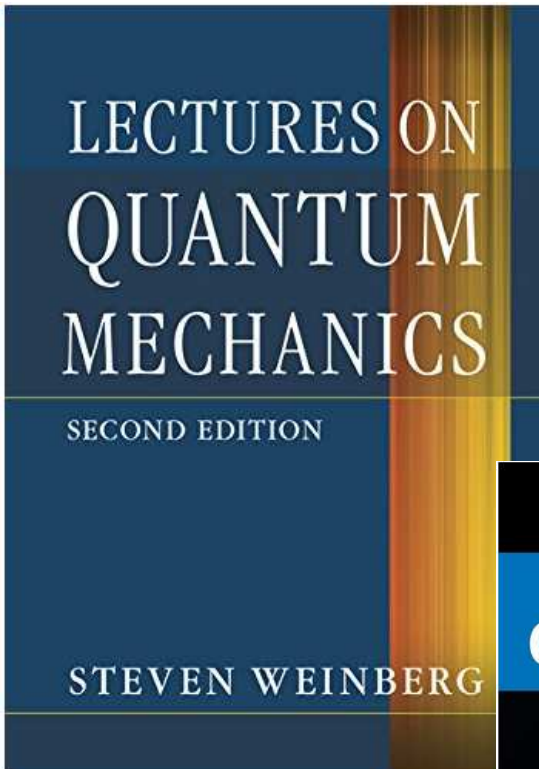
If further the time evolution is **linear** in ρ , **preserves the trace** of ρ , is **Markovian** and the linear operator that determines the time evolution of ρ is **completely positive**
 \Rightarrow then this requires the time evolution equation to have the **Lindblad form**

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$

where H is a Hermitian operator and the C_n 's are an additional set of operators called **collapse operators**.

- Lindblad CMP 48 (1976) 119
Gorini Kossakowski Sudarshan JMP 17 (1976) 821

Lindblad equation in textbooks



Quarkonium as an open quantum system

- **System:** heavy quarks/quarkonium
- **Environment:** quark gluon plasma

We may define a (reduced) **density matrix** for the heavy quark-antiquark pair in a color singlet and octet configuration:

$$\begin{aligned}\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \text{Tr}\{\rho_{\text{full}}(\tau_{\text{med}}) S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\} \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \text{Tr}\{\rho_{\text{full}}(\tau_{\text{med}}) O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\}\end{aligned}$$

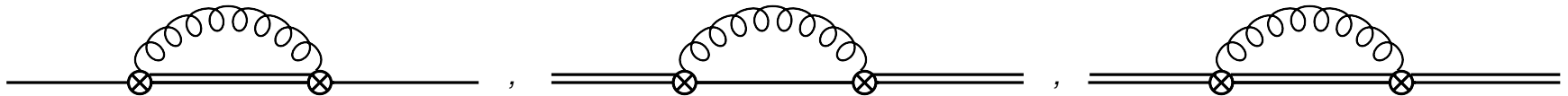
τ_{med} fm is the time formation of the plasma.

The system is in **non-equilibrium** because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although **the number of heavy quarks is conserved**: $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\} = 1$.

Expansions

- The density of heavy quarks is much smaller than the one of the light d.o.f.: we expand at **first order in the heavy quark-antiquark density**.
- We consider T **much smaller than the inverse Bohr radius** of the quarkonium: we expand up to **order r^2 in the multipole expansion**.
- We follow the evolution for **large times**.

Relevant diagram topologies are



For each topology one quark-antiquark propagator represents a density matrix insertion.

Time scales

Environment correlation time: $\tau_E \sim \frac{1}{T}$

System intrinsic time scale: $\tau_S \sim \frac{1}{E}$

System relaxation time: $\tau_R \sim \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$ $a_0 = \text{Bohr radius}, \Lambda = T, E$

- Because we have assumed $1/a_0 \gg \Lambda$, it follows $\tau_R \gg \tau_S, \tau_E$
which, for large times $t - \tau_{\text{med}} \gg \tau_R$, qualifies the system as **Markovian**.
- If $T \gg E$ then $\tau_S \gg \tau_E$
which qualifies the motion of the system as **quantum Brownian**.

From the evolution equations to the Lindblad equation

Under the Markovian

$$\tau_R \gg \tau_S, \tau_E \quad \text{or} \quad \frac{1}{a_0} \gg E, T$$

and quantum Brownian motion condition

$$\tau_S \gg \tau_E \quad \text{or} \quad T \gg E$$

at (N)LO in E/T , the evolution equations can be written in the **Lindblad form**.

Lindblad equation for a strongly coupled plasma

@ LO in E/T the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{8}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \quad C_i^1 = \sqrt{\frac{5\kappa}{16}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

@ NLO in E/T , H and the collaps operators get some higher order corrections.

Thermal width and mass shift

The quantity κ is related to the thermal decay width of the heavy quarkonium.
In particular for $1S$ states, we have (Σ_s = self-energy)

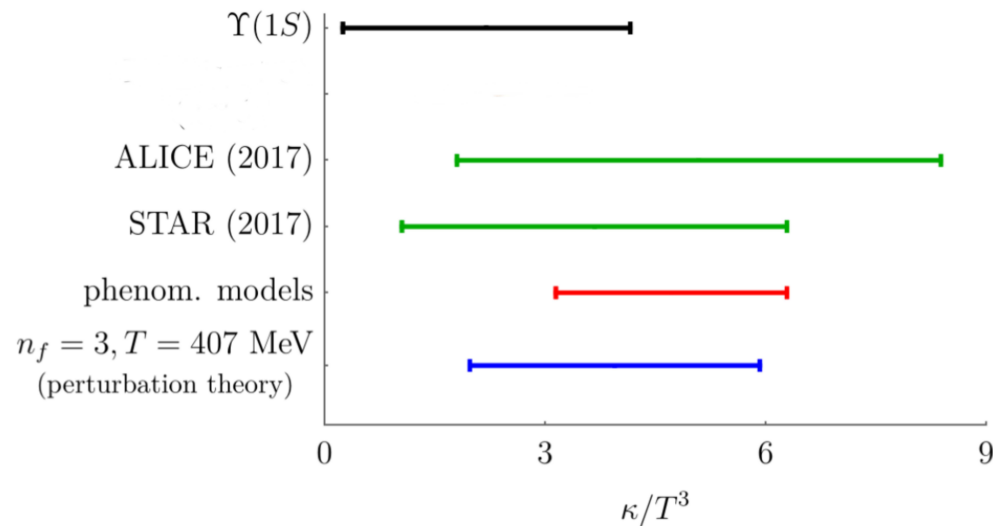
$$\Gamma(1S) = -2\langle \text{Im}(-i\Sigma_s) \rangle = 3a_0^2 \kappa$$

The quantity γ is related to the thermal mass shift of the heavy quarkonium.
In particular for $1S$ states, we have

$$\delta M(1S) = \langle \text{Re}(-i\Sigma_s) \rangle = \frac{3}{2}a_0^2 \gamma$$

κ

$$\kappa = \frac{g^2}{18} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

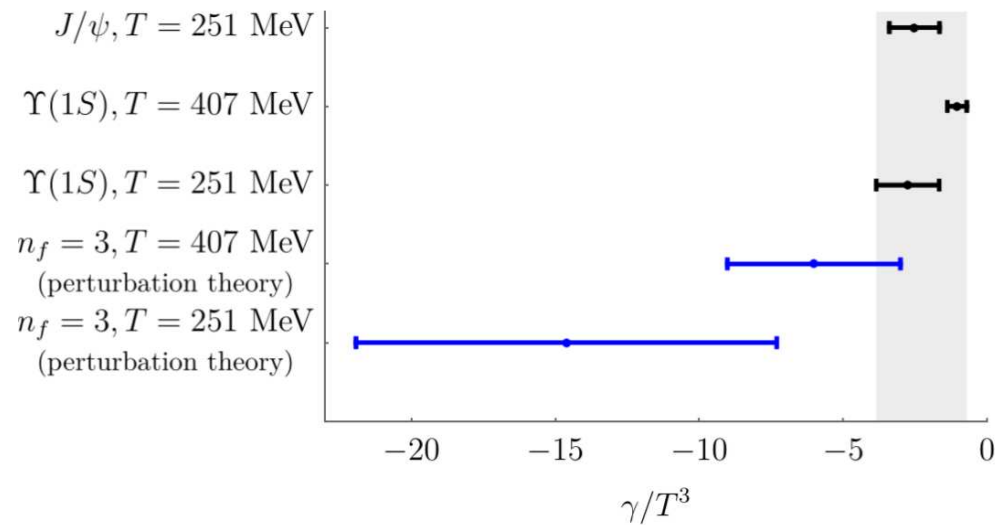


$[\hat{\kappa} \equiv \kappa/T^3]$

- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
from the lattice data of Aarts et al JHEP 11 (2011) 103
and Kim Petreczky Rothkopf JHEP 11 (2018) 088

γ

$$\gamma = \frac{g^2}{18} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$



$[\hat{\gamma} \equiv \gamma/T^3]$

- Brambilla Escobedo Vairo Vander Griend PRD 100 (2019) 054025
from the lattice data of Kim Petreczky Rothkopf JHEP 11 (2018) 088

Evolution set up

- After heavy-ion collision, quark-antiquarks propagate freely up to $\tau_{\text{med}} = 0.6$ fm.
- From τ_{med} fm to the freeze-out time t_F they propagate in medium.
- We assume the medium to be locally in thermal equilibrium.
- We use a 3+1D dissipative relativistic hydrodynamics code that makes use of the quasiparticle anisotropic hydrodynamics (aHydroQP) framework. The code uses a realistic equation of state fit to lattice QCD measurements and is tuned to soft hadronic data collected in 5.02 TeV collisions using smooth optical Glauber initial conditions.
- Alqahtani Nopoush Strickland PRC 92 (2015) 054910, 95 (2017) 034906
Alqahtani Nopoush Strickland PPNP 101 (2018) 204

Quantum trajectories algorithm

The **QTraj** code implements the **quantum trajectories algorithm** and the **waiting time approach** as follows.

- 1 Initialize a wave function $|\psi(t_0)\rangle$ at initial time t_0 , which corresponds to the initial quantum state of the particle given by $\rho(t_0) = |\psi(t_0)\rangle\langle\psi(t_0)|$.
- 2 Generate a random number $0 < r_1 < 1$ and evolve the wave function forward in time with H_{eff} until $\|e^{-i\int_{t_0}^t dt' H_{\text{eff}}(t')}|\psi(t_0)\rangle\|^2 \leq r_1$ where $H_{\text{eff}} = H - i\Gamma/2$, $\Gamma = \sum \Gamma_n$ and $\Gamma_n = C_n^\dagger C_n$. Denote the first time step fulfilling the inequality as the jump time t_j . If the jump time is greater than the simulation run time t_F , end the simulation at time t_F ; otherwise, proceed to step 3.
- 3 At time t_j , initiate a quantum jump:
 - (a) If the system is in a singlet configuration, jump to octet. If the system is in an octet configuration, generate a random number $0 < r_2 < 1$ and jump to singlet if $r_2 < 2/7$; otherwise, remain in the octet configuration.
 - (b) Generate a random number $0 < r_3 < 1$; if $r_3 < l/(2l + 1)$, take $l \rightarrow l - 1$; otherwise, take $l \rightarrow l + 1$.
 - (c) Apply the appropriate collapse operator C_i^j to the wavefunction, normalize.
- 4 Continue from step 2.

Jumps and probabilities

The probabilities in step 3 correspond to the branching fractions into a state of different color and/or angular momentum:

$$p_n = \frac{\langle \psi(t) | \Gamma_n | \psi(t) \rangle}{\langle \psi(t) | \Gamma | \psi(t) \rangle}$$

Each evolution of the wave function from time t_0 to t_F is called a **quantum trajectory**.

In practice, a large number of quantum trajectories must be generated.

As the number of trajectories considered increases, the average converges to the solution of the Lindblad equation.

- Dalibard Castin Molmer PRL 68 (1992) 580
Daley AP 63 (2014) 77

Simulation set up

We employ a radial lattice of $\text{NUM} = 4096$ lattice sites and a radial length of $L = 80 \text{ GeV}^{-1}$, corresponding to a radial lattice spacing of $a \approx 0.0195 \text{ GeV}^{-1}$. The real time integration is discretized with a time step of $\text{dt} = 0.001 \text{ GeV}^{-1}$.

@ LO in E/T , we sample approximately $7 - 9 \times 10^5$ independent physical trajectories for each choice of κ/T^3 and γ/T^3 , with approximately 50-100 quantum trajectories per physical trajectory. To generate each physical trajectory, we sample the bottomonium production point in the transverse plane using the nuclear binary collision overlap profile $N_{AA}^{\text{bin}}(x, y, b)$, the initial transverse momentum of the state p_T from an E_T^{-4} spectrum, and the initial azimuthal angle ϕ of the state's momentum uniformly in $[0, 2\pi)$. We bin the results for the survival probability as a function of centrality, p_T , and ϕ . This allows us to make predictions for differential observables such as R_{AA} as a function of p_T and elliptic flow. To ensure that the hierarchy of energy scales of the EFT is fulfilled, we evolve the state in the vacuum when the temperature falls below $T_F = 250 \text{ MeV}$.

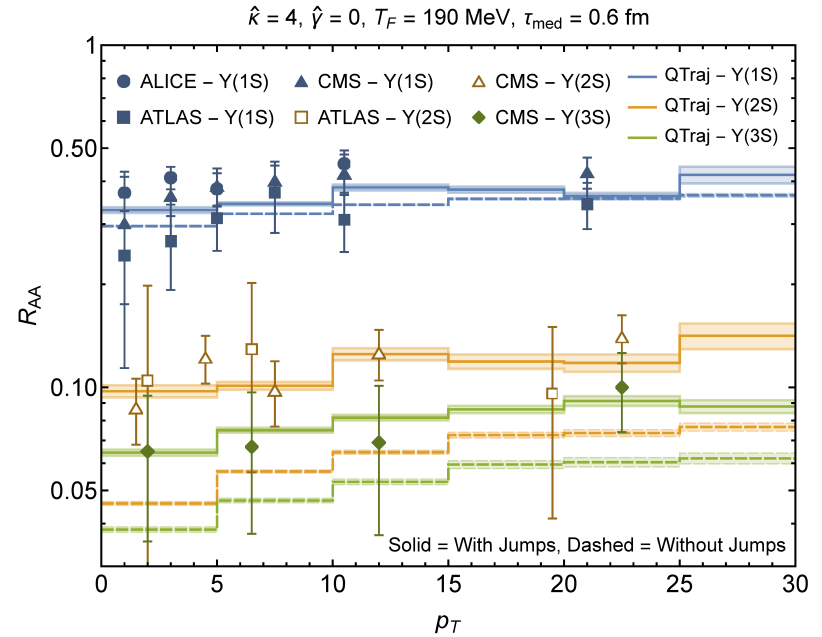
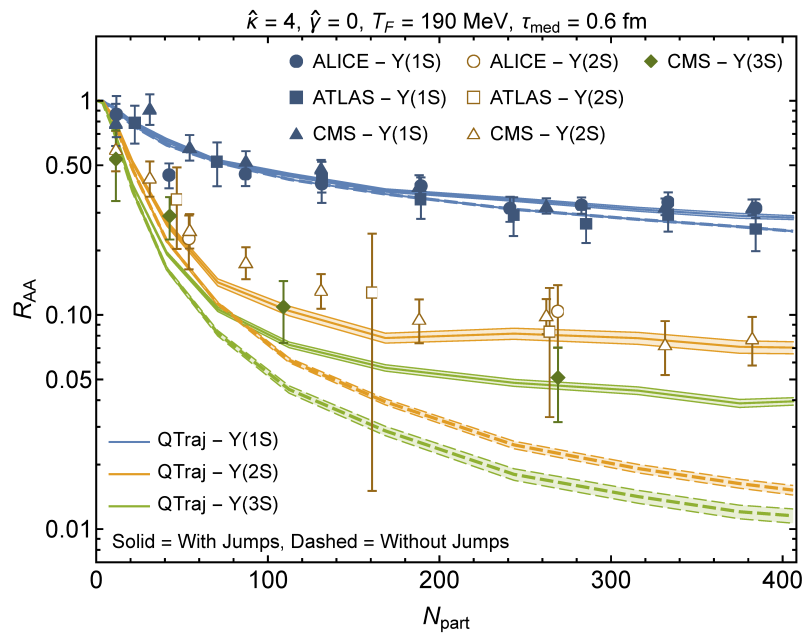
@NLO in E/T , we evaluate the evolution using an ensemble of $1-2 \times 10^5$ physical trajectories with 30 quantum trajectories per physical trajectory. To ensure that the hierarchy of energy scales of the EFT is fulfilled, we evolve the state in the vacuum when the temperature falls below $T_F = 190 \text{ MeV}$.

Results

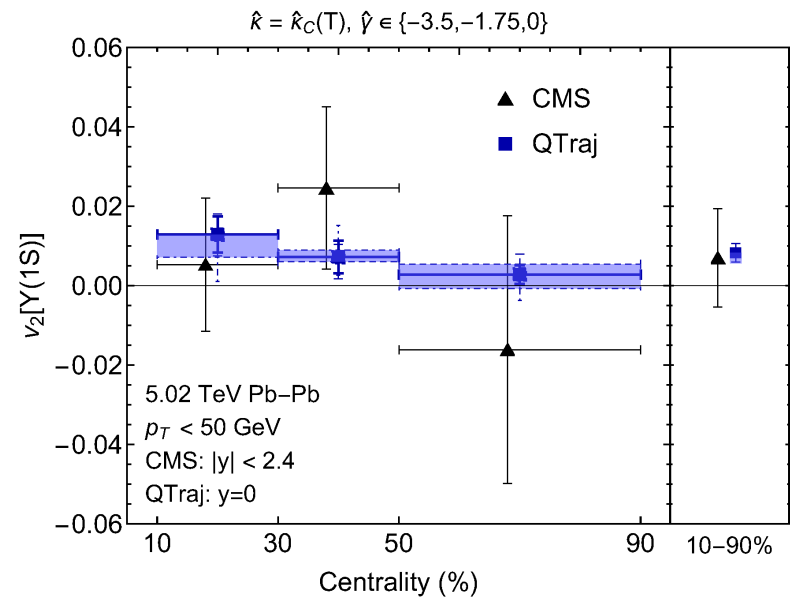
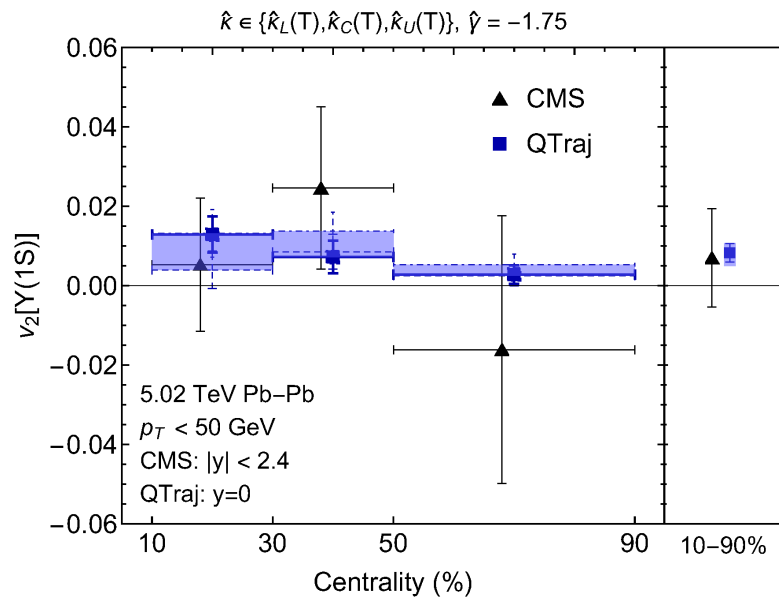
Bottomonium nuclear modification factor @ NLO in E/T

We compute the nuclear modification factor R_{AA} from

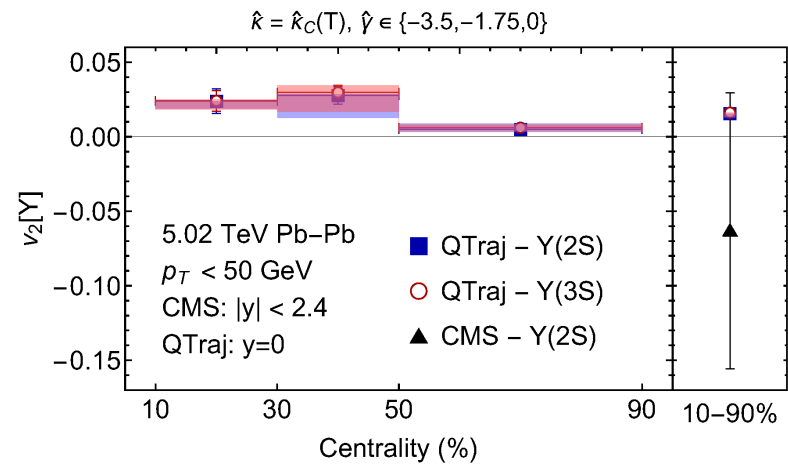
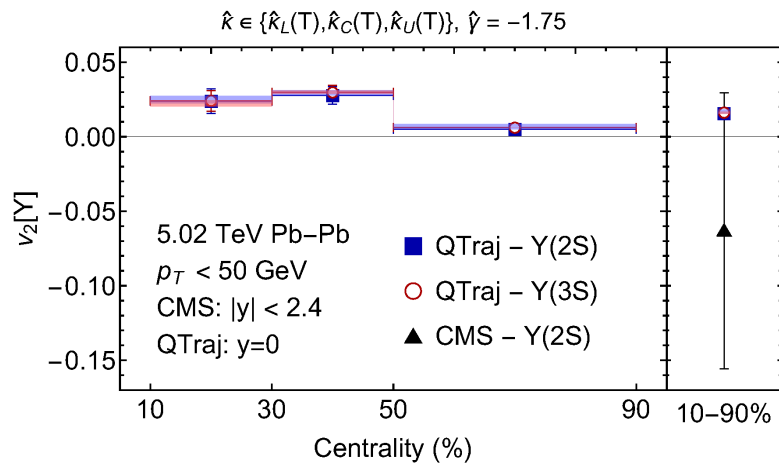
$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$



Elliptic flow v_2 of the $\Upsilon(1S)$ at LO in E/T



Elliptic flow v_2 of the $\Upsilon(2S)$ and $\Upsilon(3S)$ at LO in E/T



Conclusions & outlook

Conclusions

We have shown how the heavy quark-antiquark pair **out-of-equilibrium evolution** can be treated in the framework of QCD non relativistic EFTs. Main features are:

- the medium may be a **strongly-coupled plasma** (not necessarily a quark-gluon plasma) whose characteristics are determined by lattice calculations;
- the **total number of heavy quarks**, i.e., $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\}$, **is preserved** by the evolution equations;
- the **non-abelian** nature of QCD is fully accounted for;
- the treatment is **quantum**. And the data are sensitive to it!

The evolution equations follow from assuming the **inverse size of the quark-antiquark system to be larger than any other scale** of the medium and from being accurate at **first non-trivial order in the multipole expansion** and at **first order in the heavy-quark density**.

Under some conditions (**large time, quasistatic evolution, quantum Brownian motion**) the evolution equations are of **Lindblad form**. Their numerical solution provides a status of the art determination of $R_{AA}[\Upsilon(nS)]$ and differential observables in agreement with LHC data.

Outlook

Similar frameworks and methods may be applied to the evolution of many different systems out of equilibrium in thermal but also in non-thermal environments:

- cold nuclear matter
- matter at finite density
- dark matter in the early universe



- leptogenesis
- ...