New probe of non-Gaussianities with primordial black hole induced gravitational waves

Based on [arXiv:2402.XXXXX, T. Papanikolaou, X. C. He, X.-Ha. Ma, Y. F. Cai., E. N. Saridakis, M. Sasaki]

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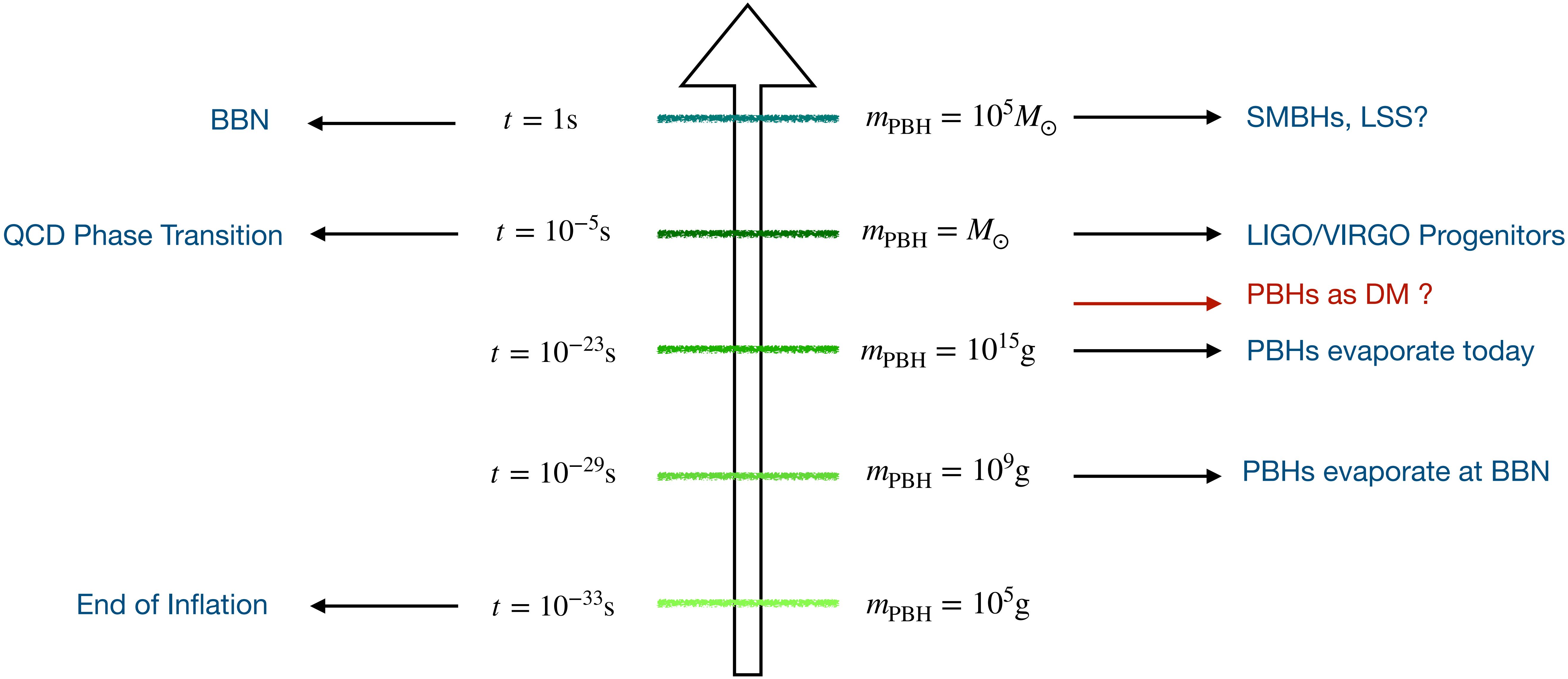
Introduction

- Primordial Black Holes (PBHs) form in the early universe out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta \equiv \dfrac{\delta\rho}{\rho_{\rm b}} > \delta_{\rm c}(w \equiv p/\rho)$ [Carr - 1975].

$$m_{\rm PBH} = \gamma M_{\rm H} \propto H^{-1} \text{ where } \gamma \sim O(1)$$



See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

PBHs and GWs

- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

  GWs   PBHs

- 2) **Relic Hawking radiated gravitons** from PBH evaporation [Anantua et al. - 2008, Dong et al. - 2015].

- 3) **GWs** emitted **by PBH mergers** [Eroshenko - 2016, Raidal et al. - 2017].

- 4) **GWs induced** at second order **by PBH energy density fluctuations** [Papanikolaou et al. - 2020].

PBH-eMD era phenomenology

- PBHs can dominate in the early Universe since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$.

PBHs with $m_{\rm PBH} < 10^9 \mathrm{g}$ (They evaporate before BBN)

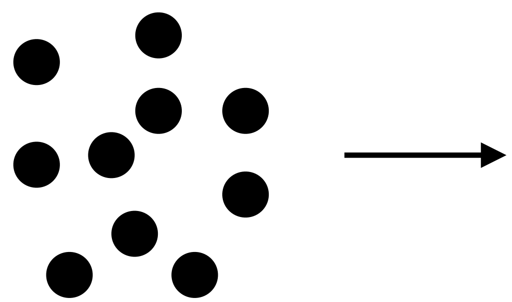
- These ultralight PBHs can **drive the reheating process** through their evaporation [Zagorac et al. - 2019, Martin et al. - 2019, Inomata et al. - 2020] during which all the SM particles can be produced.

- Hawking evaporation of ultralight PBHs can **alleviate as well the Hubble tension** [Hooper et al. - 2019, Nesseris et al. - 2019, Lunardini et al. - 2020] by injecting to the primordial plasma dark radiation degrees of freedom which can increase $N_{\rm eff}$.

- Evaporation of light PBHs can also **produce naturally the baryon asymmetry** through CP violating out-of-equilibrium decays of Hawking evaporation products [J. D. Barrow et al. - 1991, T. C. Gehrman et al. - 2022, N. Bhaumik et al. - 2022].

- **GWs induced by PBH energy density fluctuations can interpret** in a very good agreement **the recently released PTA GW data** [Lewicki et al. - 2023, Basilakos et al. - 2023]

The PBH Matter Field



Poisson Statistics [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

Same mass [Dizgah, Franciolini & Riotto - 2019]

$$\mathscr{P}_{\delta_{\rm PBH},{\rm Poisson}}(k) \equiv \frac{k^3\langle|\delta_k^{\rm PBH}|^2\rangle}{2\pi^2} = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3, \text{ where } k < k_{\rm UV} = \frac{a}{\bar{r}}$$

$\rho_{\rm PBH}$ **is inhomogeneous**

$\rho_{\rm tot}$ is **homogeneous**

} $\delta_{\rm PBH}$ can be seen as an **isocurvature perturbation.**

This **isocurvature perturbation,** $\delta_{\rm PBH}$ generated during the RD era **will convert** during the PBHD era **to a curvature perturbation** $\zeta_{\rm PBH}$, associated to a PBH gravitational potential Φ.

$$\mathscr{P}_{\Phi}(k) = S_{\Phi}^2(k)\left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}\mathscr{P}_{\delta_{\rm PBH},{\rm Poisson}}(k), \text{ where } S_{\Phi}(k) \equiv \left(\frac{k}{k_{\rm evap}}\right)^{-1/3}$$

Scalar Induced Gravitational Waves

- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as:

$$h^{s,''}_{\vec{k}} + 2\mathscr{H} h^{s,'}_{\vec{k}} + k^2 h^{s}_{\vec{k}} = 4S^{s}_{\vec{k}}\,.$$

- The source term, $S_{\vec{k}}$ can be recast as:

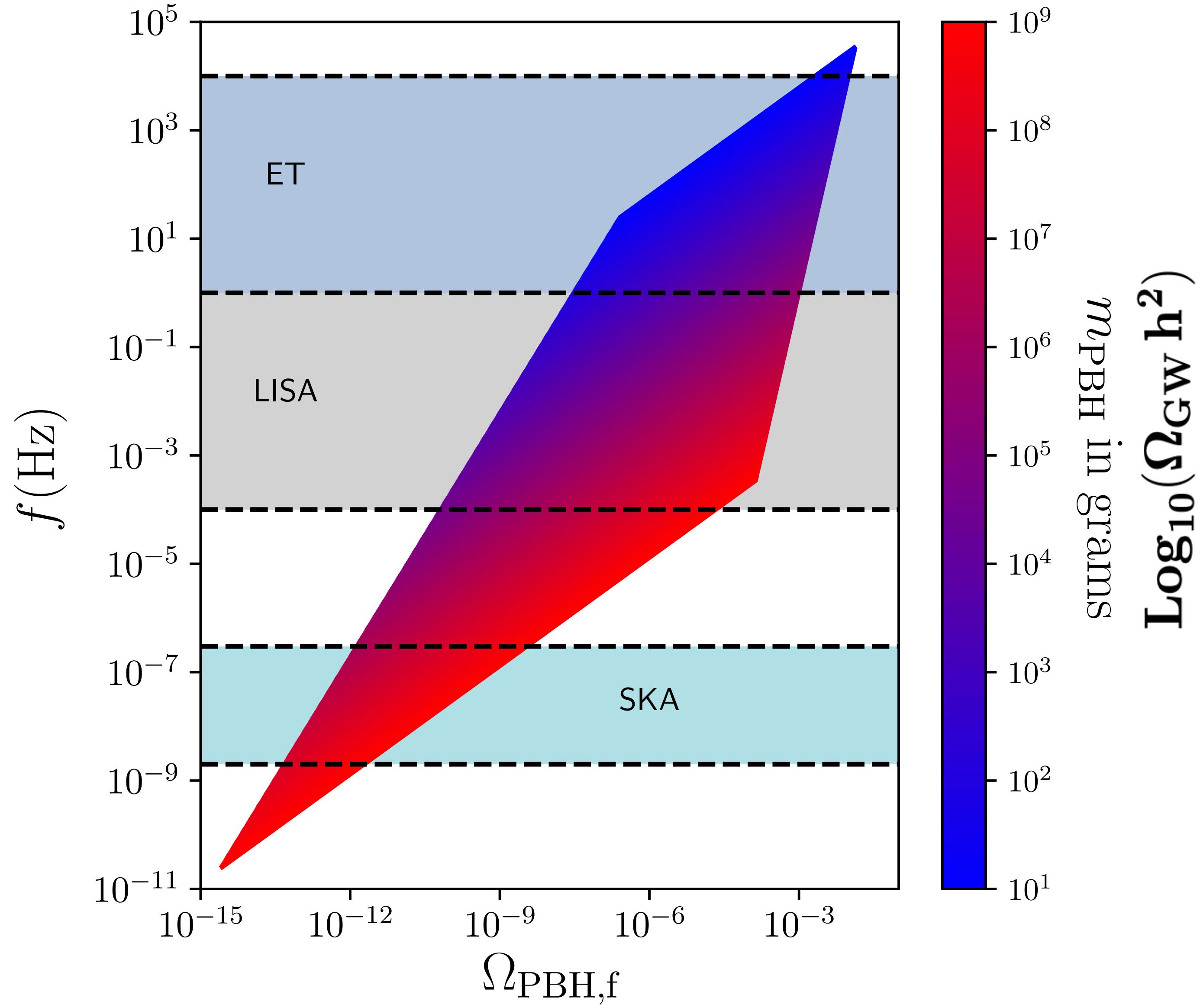
$$S^{s}_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^{s}_{ij}(\vec{k}) q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathscr{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right].$$

$$\Downarrow$$

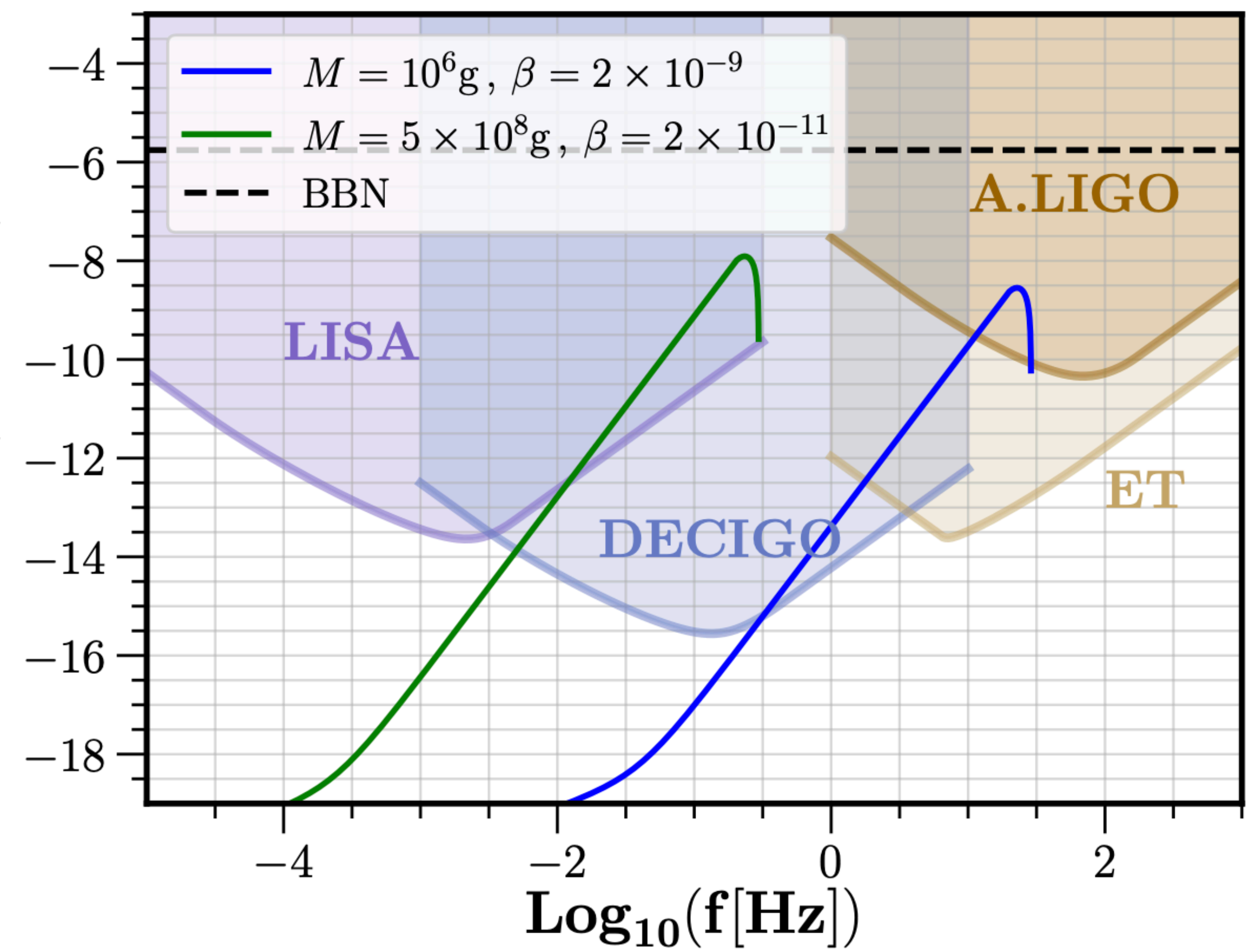
$$\Omega_{\mathrm{GW}}(\eta, k) \equiv \frac{1}{\rho_{\mathrm{tot}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \mathscr{P}_h(\eta, k),$$

with $\mathscr{P}_h(\eta, k) \equiv \frac{k^3 |h_k|^2}{2\pi^2} \propto \int \mathrm{d}v \int \mathrm{d}u \left(\int f(v, u, k, \eta)\mathrm{d}\eta\right)^2 \mathscr{P}_\Phi(kv)\mathscr{P}_\Phi(ku)\,.$

GW Detectability



[Papanikolaou et al. - 2020]



[Domenech et al. - 2020]

- By accounting on BBN bounds on the GW amplitude at $k \sim k_{\mathrm{UV}}$, one can set upper bound constraints on the $\Omega_{\mathrm{PBH,f}}$ readings as

$$\Omega_{\mathrm{PBH,f}} < 10^{-6}\left(\frac{M_{\mathrm{PBH}}}{10^4\mathrm{g}}\right)^{-17/24}.$$

The effect of local-type non-Gaussianities

Primordial non-Gaussianities of local type

$$\begin{aligned}
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k) \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}) \\
&\quad\times \frac{6}{5} f_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + 2 \text{ perms}\right] \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\mathcal{R}(\boldsymbol{k_4})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4}) \\
&\quad\times \left\{ \frac{54}{25} g_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3) + 3 \text{ perms}\right] \right. \\
&\qquad \left. + \tau_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(|\boldsymbol{k_1}+\boldsymbol{k_3}|) + 11 \text{ perms}\right] \right\}
\end{aligned}$$

Primordial non-Gaussianities of local type

$$\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k)$$

$$\begin{aligned}\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\rangle \equiv\ & (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}) \\ & \times \frac{6}{5}f_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)+2\text{ perms}\right]\end{aligned}$$

$$\begin{aligned}\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\mathcal{R}(\boldsymbol{k_4})\rangle \equiv\ & (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4}) \\ & \times \left\{\frac{54}{25}g_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3)+3\text{ perms}\right]\right. \\ & \left. + \tau_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(|\boldsymbol{k_1}+\boldsymbol{k_3}|)+11\text{ perms}\right]\right\}\end{aligned}$$

$kR \ll 1$ ↓ $R \sim 1/k_{\rm f}$

$$\mathscr{P}_{\delta_{\rm PBH}}(k) \simeq \mathscr{P}_{\mathscr{R}}(k)\nu^4\left(\frac{4}{9\sigma_R}\right)^4\int\frac{{\rm d}^3p_1{\rm d}^3p_2}{(2\pi)^6}\tau_{\rm NL}(p_1,p_2,p_1,p_2)W^2_{\rm local}(p_1)W^2_{\rm local}(p_2)P_{\mathscr{R}}(p_1)P_{\mathscr{R}}(p_2)$$

$$+\frac{k^3}{2\pi^2}(k-\text{independent terms})$$

Primordial non-Gaussianities of local type

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$\bar{\tau}_{\rm NL}$

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$$+\frac{k^3}{2\pi^2}(k-\text{independent terms}) \equiv \nu^4\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}(k)+\mathcal{P}_{\delta_{\rm PBH},\rm Poisson}(k),\ \text{where}\ \nu=\delta_c/\sigma_R\sim 8$$

Primordial non-Gaussianities of local type

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↓

$$\mathscr{P}_{\Phi}(k) = S^2_{\Phi}(k)\left(5+\frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}\left[\left(\frac{4\nu}{9\sigma_R}\right)^4\bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k)+\mathscr{P}_{\delta_{\rm PBH},\rm Poisson}(k)\right]$$

The non-Gaussian PBH matter power spectrum

$$\text{Ansatz}: \mathscr{P}_{\mathscr{R}}(k_{\rm evap} < k < k_{\rm UV}) \simeq 2\times 10^{-9}, \quad \tau_{\rm NL}(k_1,k_2,k_3,k_4) = \tau_{\rm NL}(k_{\rm f})e^{-\frac{1}{2\sigma_\tau^2}\left(\sum_{i=1,2,3,4}\ln^2\frac{k_i}{k_{\rm f}}\right)}$$

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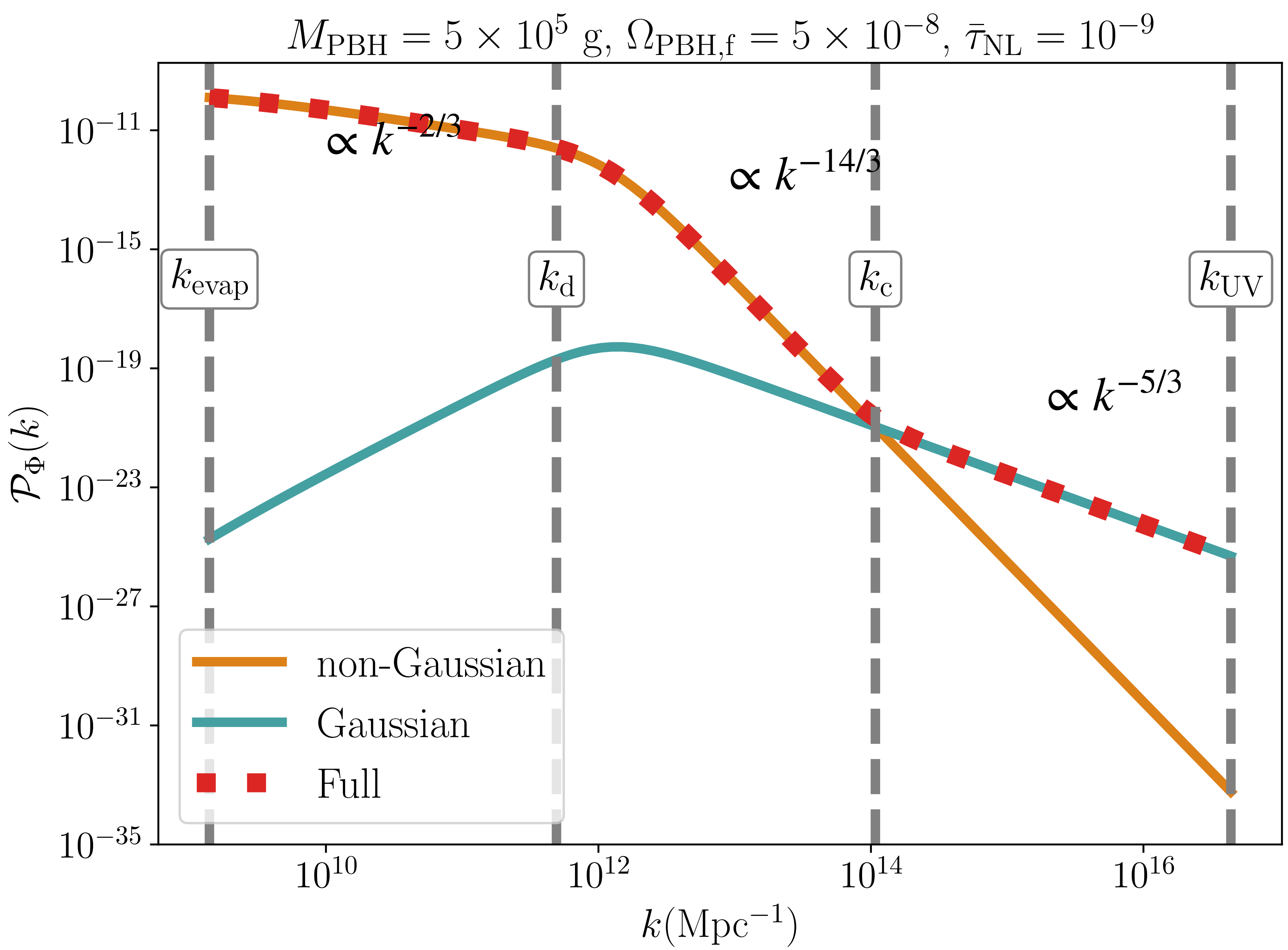

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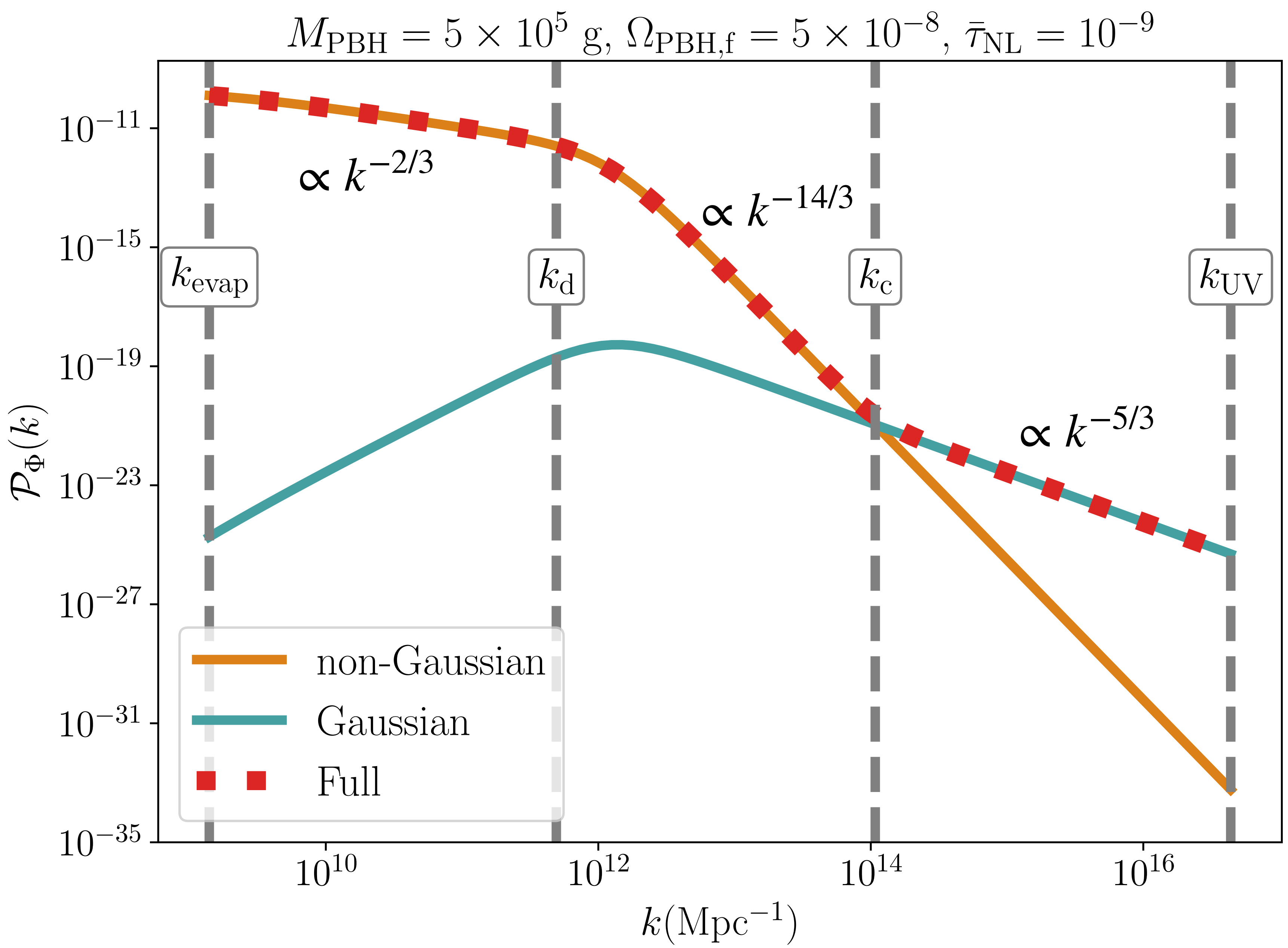
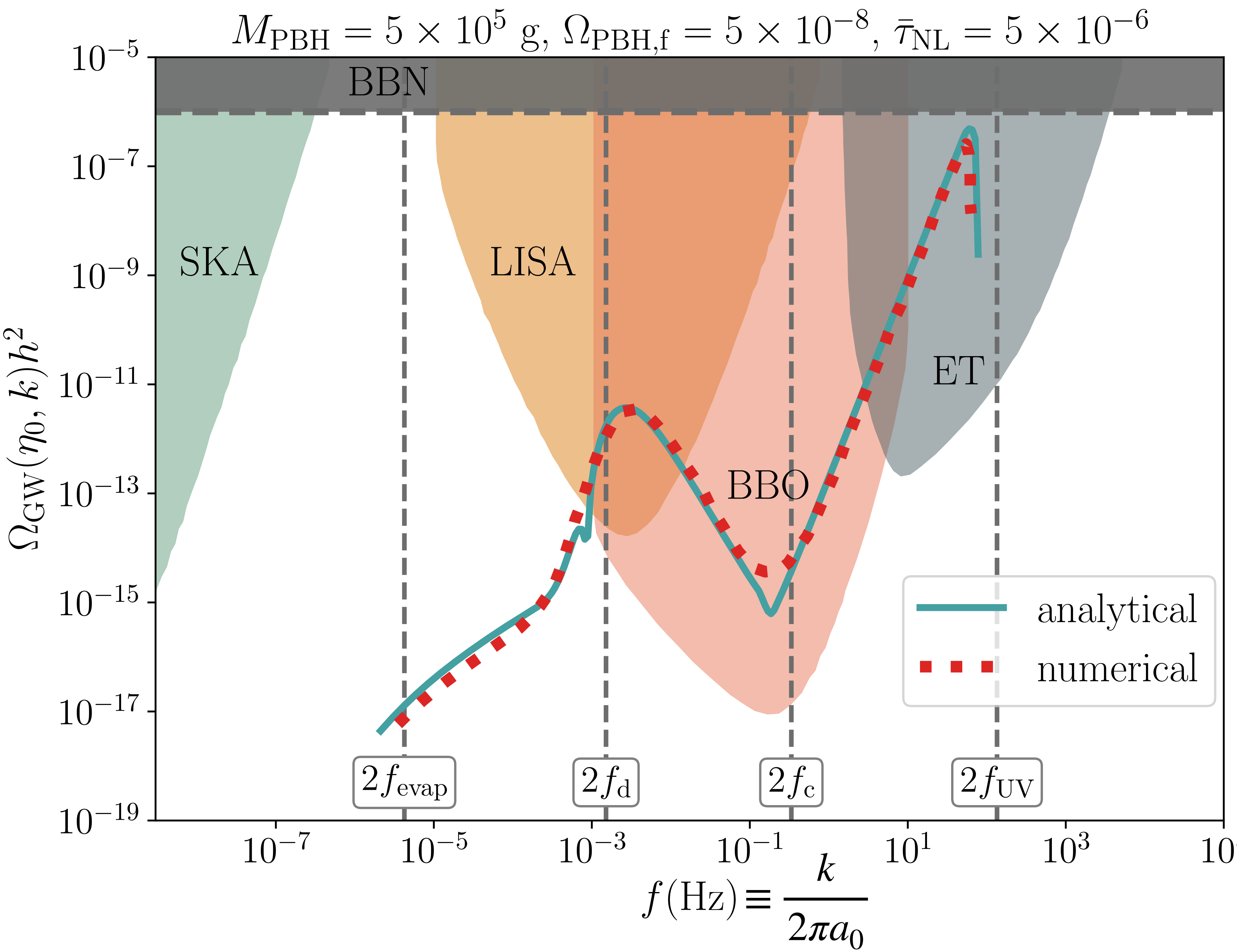
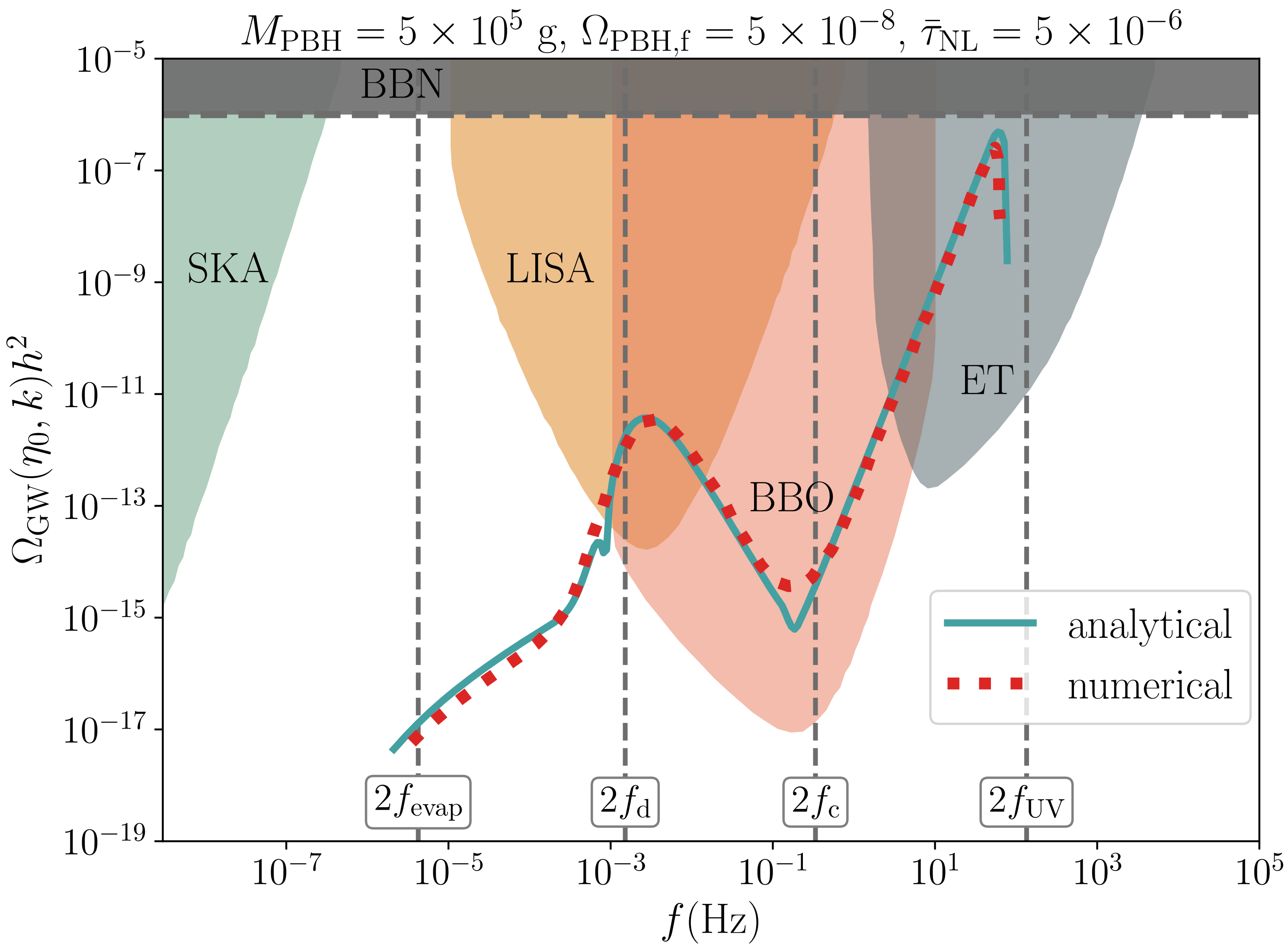


$$\text{Scale Hierarchy}: 10^5\mathrm{Mpc}^{-1} < k_{\rm evap} < k_{\rm d} < k_{\rm c} < k_{\rm UV} \ll k_{\rm f} \sim 1/R$$

Non-Gaussian Induced GWs



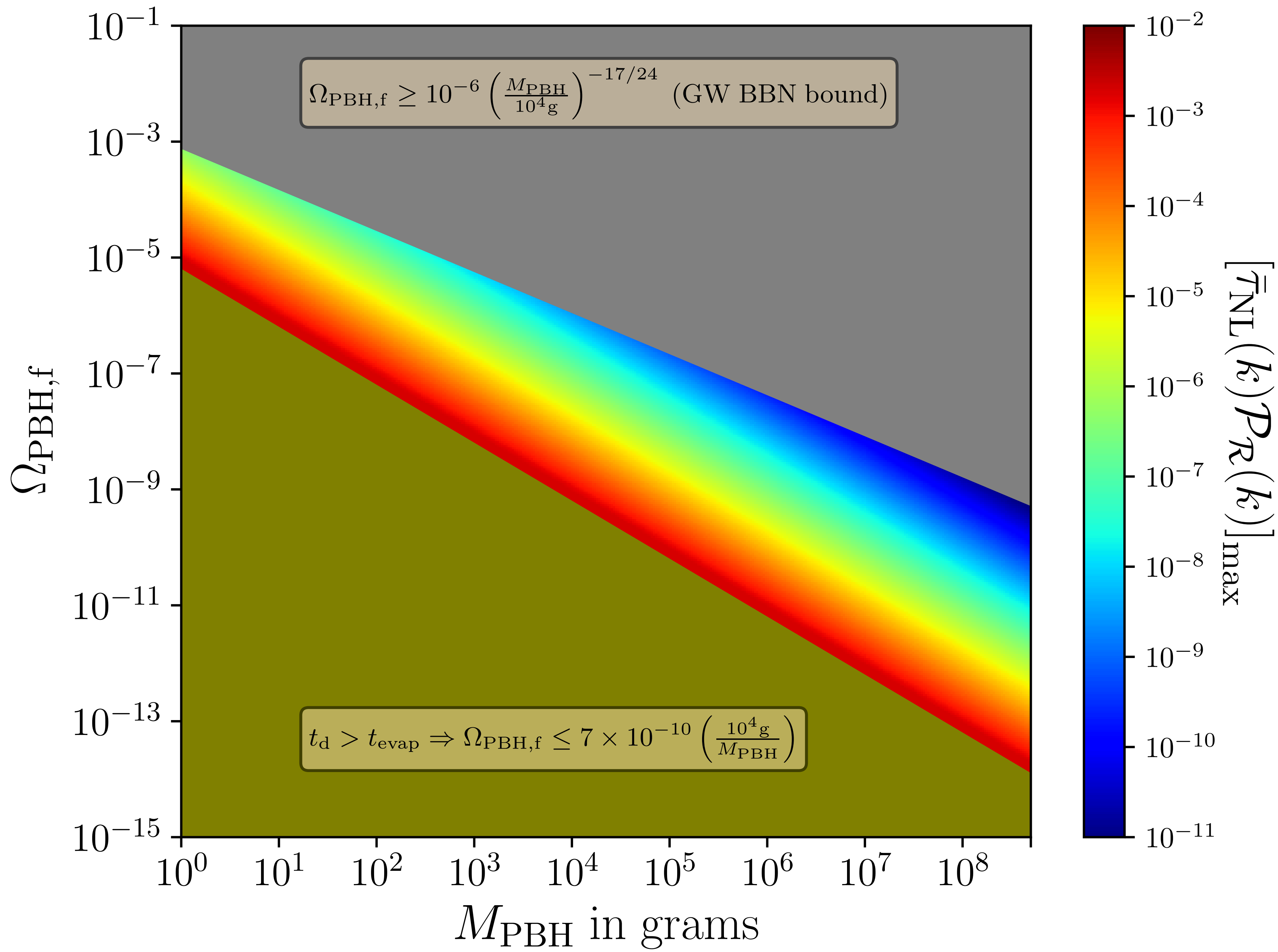
Non-Gaussian Induced GWs



$$
\Omega_{\rm GW}(\eta_0,k)h^2 \simeq \begin{cases}
3\times10^{-80}\left(\dfrac{k}{10^4{\rm Mpc}^{-1}}\right)^{11/3}\left(\dfrac{M_{\rm PBH}}{10^4{\rm g}}\right)^{41/6}\left(\dfrac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3} & 2k_{\rm c}<k<2k_{\rm UV}\\
5\times10^{-13}\left(\dfrac{k}{10^4{\rm Mpc}^{-1}}\right)^{-7/3}\left(\dfrac{M_{\rm PBH}}{10^4{\rm g}}\right)^{11/6}\left(\dfrac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3}\left(\dfrac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm d}<k<2k_{\rm c}\\
4\times10^{-75}\left(\dfrac{k}{10^4{\rm Mpc}^{-1}}\right)^{17/3}\left(\dfrac{M_{\rm PBH}}{10^4{\rm g}}\right)^{17/2}\left(\dfrac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & \\
\quad+10^{-39}\left(\dfrac{k}{10^4{\rm Mpc}^{-1}}\right)\left(\dfrac{M_{\rm PBH}}{10^4{\rm g}}\right)^{4}\left(\dfrac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{22/9}\left(\dfrac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm evap}<k<2k_{\rm d}
\end{cases}
$$

Constraining non-Gausianities

$$\Omega_{\rm GW}(2k_{\rm d},\eta_0) \leq 7\times 10^{-6} \Rightarrow \bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k) \leq 2\times 10^{-20}\Omega_{\rm PBH,f}^{-17/9}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{-17/9}$$



Conclusions

- **GWs** induced by **PBH isocurvature perturbations** can be abundantly produced in **eMD eras before BBN** driven by PBHs and give us access to the early Universe given their **potential detectability by GW experiments.**

- In particular, by requiring not to have GW overproduction at the end of BBN one can set **constraints on the abundances of ultralight PBHs** with $m_{\rm PBH} < 10^9 {\rm g}$ which are **otherwise unconstrained** by other observational probes.

- Incorporating in the analysis **the effect of local-type primordial non-Gaussianities on PBH clustering** we found a **bi-peaked structure of the induced GW signal** with the l**ow frequency peak being related to the** $\tau_{\rm NL}$ **parameter.**

- Accounting finally for BBN bounds on the GW amplitude we set **constraints on primordial non-Gaussianities on very small scales** $k > 10^5 {\rm Mpc}^{-1}$, otherwise unconstrained by current CMB and LSS probes.

- The portal of **PBH induced GWs induced** can serve as a **new messenger from the early Universe.**

Thanks for your attention!