Turbulent Magnetic Field Amplification in Binary Neutron Star Mergers

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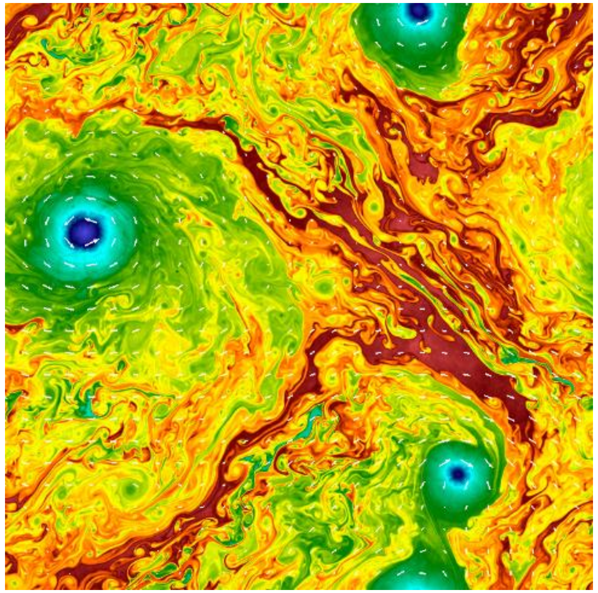


JENAS GWPFP, *12th February 2024*

Computational resources:
«LESBNS» project (20th PRACE Regular Call)
MareNostrum BSC

Long LES BNS project (21th PRACE Regular Call)
MareNostrum BSC

PRACE

IEEC
Institut d'Estudis Espacials de Catalunya

CONICET

UNC Universidad Nacional de Córdoba

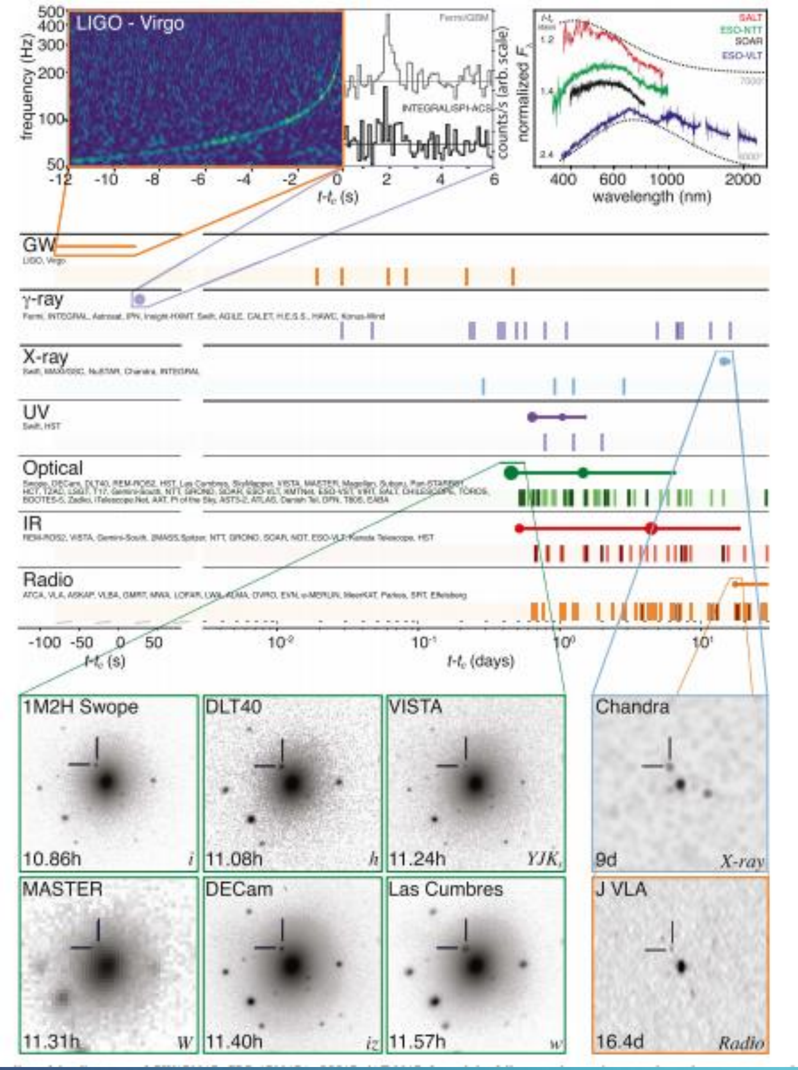
I F E G

Credit:
Maria Pereira

UIB Universitat de les Illes Balears

IAC3 Institute of Applied Computing & Community Code.

UH Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

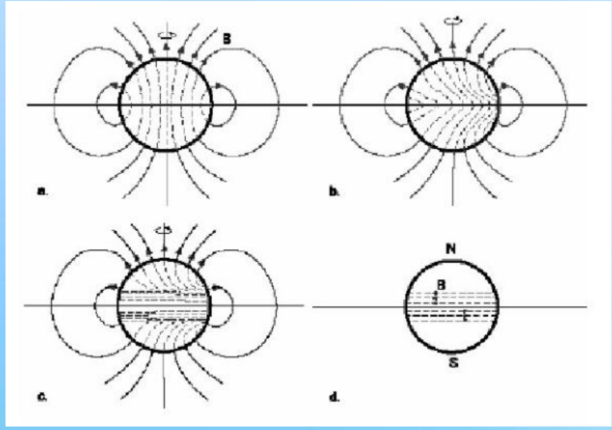


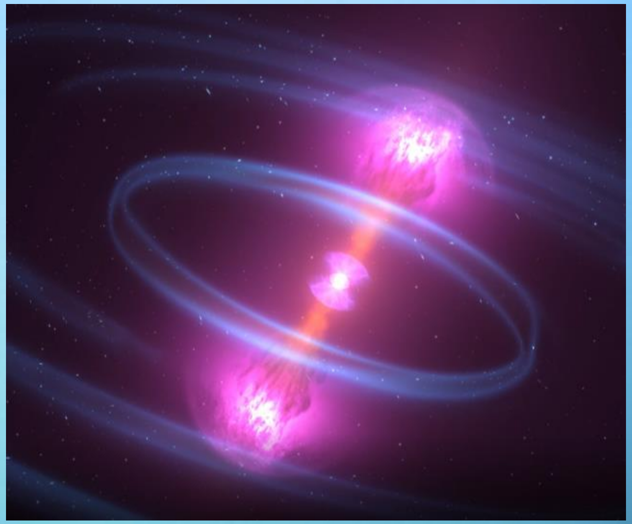
GW170817:
the beginning of the multi-messenger with GW era (first BNS merger detection)

Things that we can learn:

- Internal properties of NSs (eq. state) → *Jan-Erik's previous talk! and Bhaskar Biswas's talk (Friday)*
- **Magnetic field amplification mechanisms**
- Test General Relativity (or alternative theories to GR)
- Production of heavy elements
- EM counterpart (short GRB, kilonova)
- Formation of massive NS and/or light BH

- **PROCESSES DURING AND AFTER THE MERGER:**
  - Kelvin-Helmholtz instability (small scale)
  - Winding up (large scale)
  - Magneto-rotational instability (large scale)





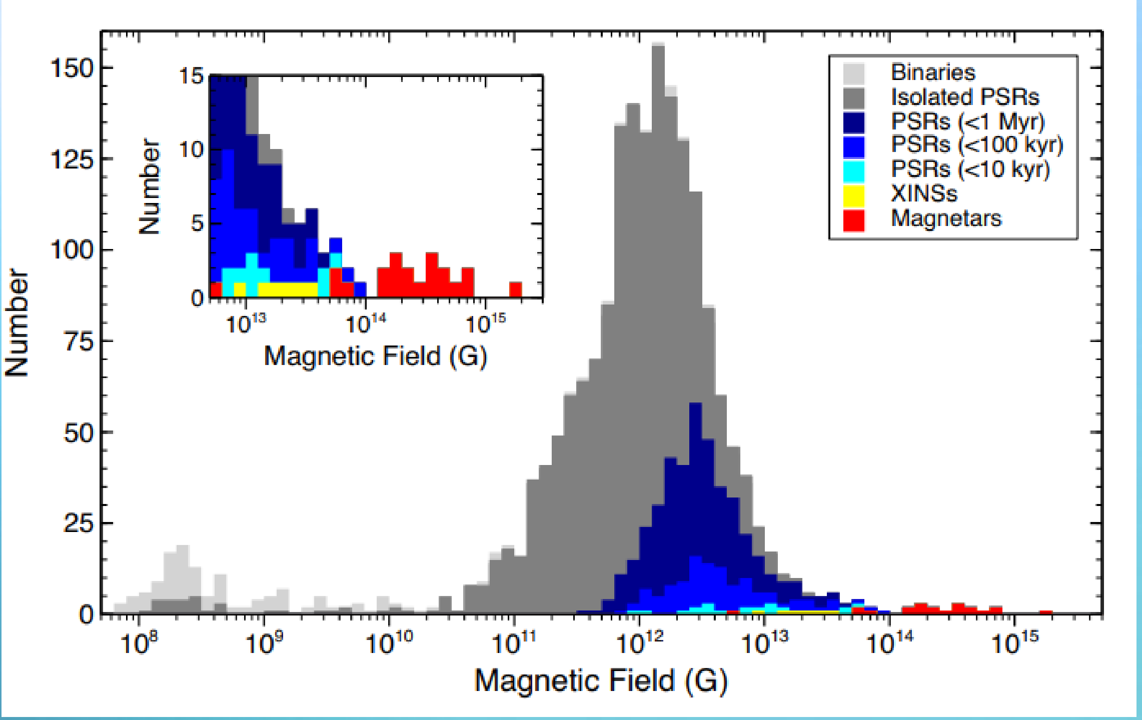
Jet appear during the merger → Most of current models need a **strong large-scale magnetic field**

sGRB

Simulate these mechanisms via BNS merger simulations

What are the typical magnetic fields expected for merging neutron stars?

Most works for simplicity (and for convenience) start with unrealistic magnetar-like values of purely dipolar fields (**10^{15} G**), either in the pre-merger or directly in the post-merger stage

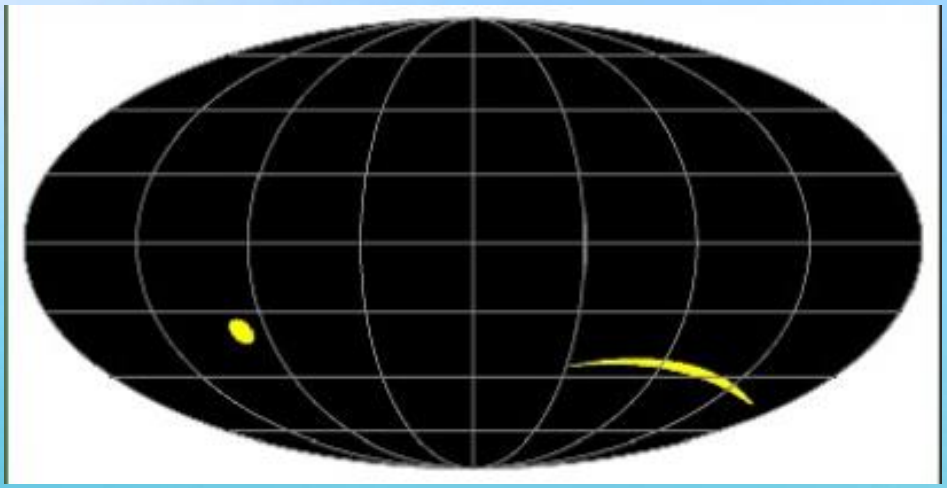


$B_{BNS} \sim 10^8 - 10^{10}$ G

[Olausen & Kaspi 2014]

What is the typical magnetic field topology of neutron stars?

Most works for simplicity (and for convenience) start with unrealistic magnetar-like values of **purely dipolar** fields (10^{15} G), either in the pre-merger or directly in the post-merger stage

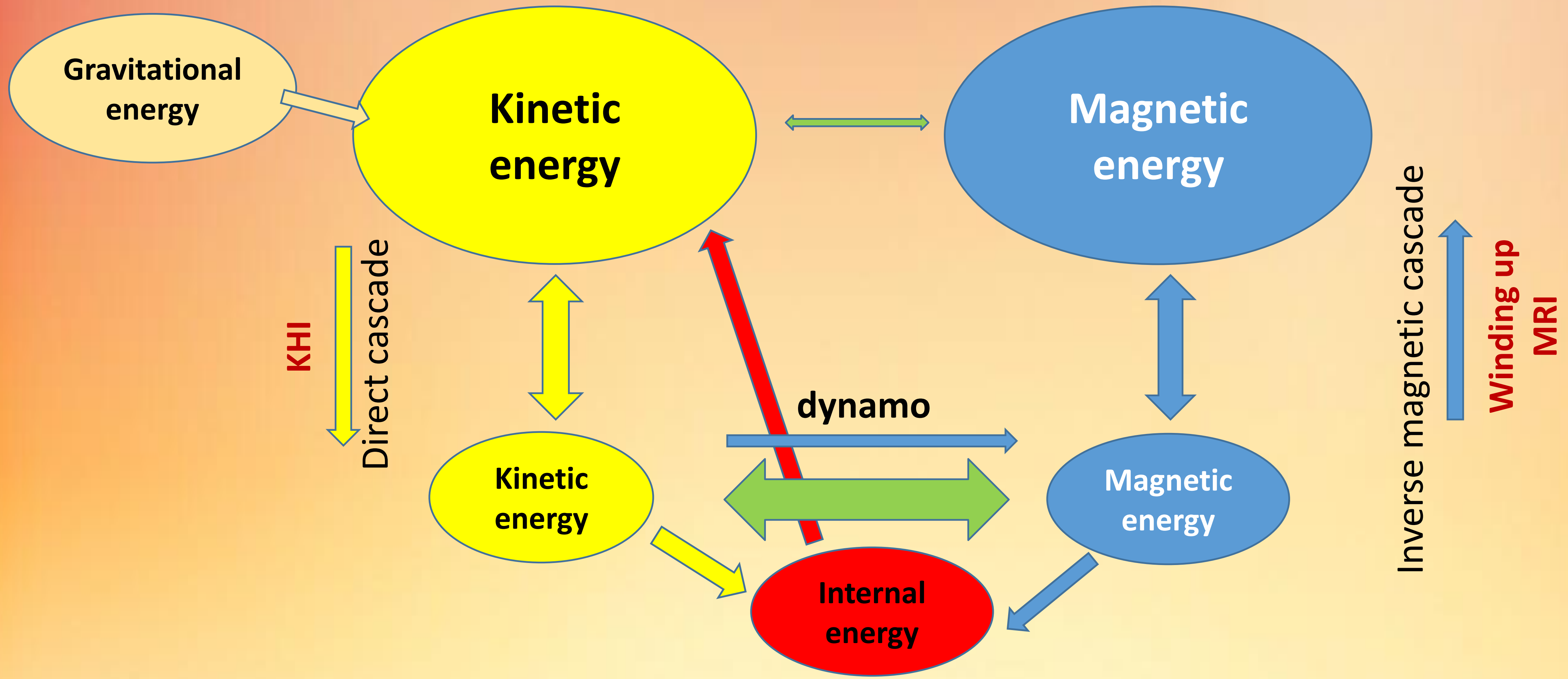


NICER results
[Riley++ 2019, Bogdanov++ 2019, Miller++ 2019]

Strong indications of a multipolar structure in NS
→
Assuming a strong dipolar magnetic field topology is unsupported by the NICER results.

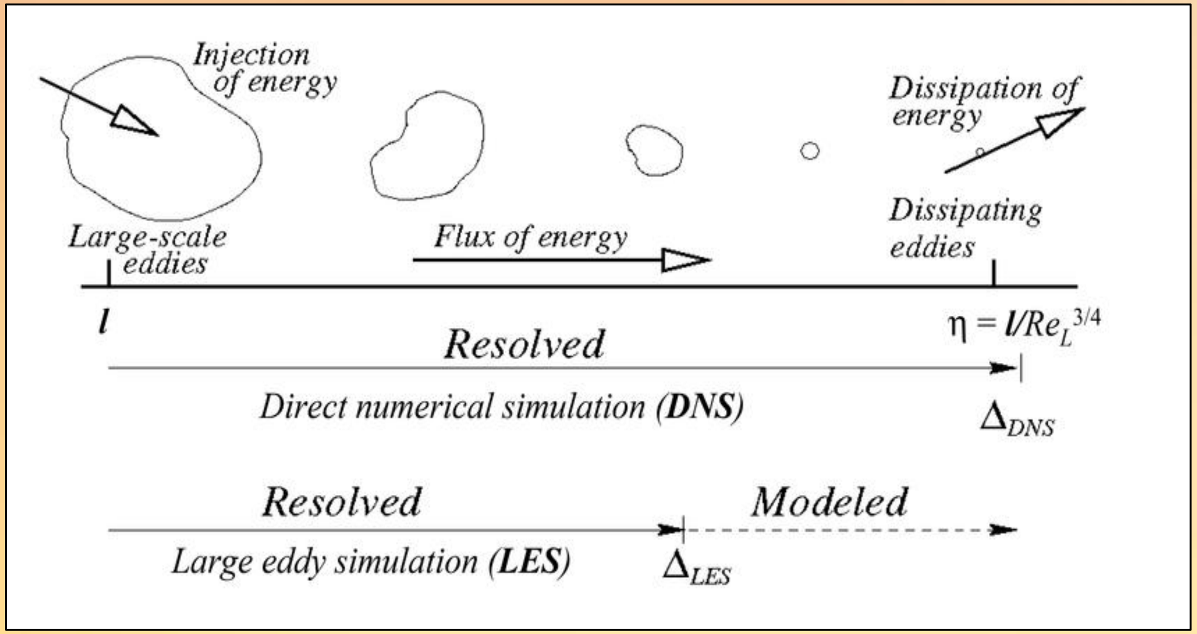
Does the initial magnetic field strength and topology matter at all in BNS mergers?

2.1. Filtering



2.1. Filtering

Resolve all the scales → Costs lots of computational resources



[Foroozani 2015]

The finite resolution of a simulation corresponds to an effective spatial filter for the fields:

$$u(\vec{x},t) = \bar{u}(\vec{x},t) + u'(\vec{x},t)$$

$$\bar{u}(\vec{x},t) = \int_{-\infty}^{\infty} G(\vec{x}-\vec{x}')\, u(\vec{x}',t)\, d^3x'$$

2.1. Filtering

Take the simplest non-linear evolution equation, Burgers:

$$\partial_t u + \frac{1}{2}\partial_x u^2 = 0 \Rightarrow \partial_t \bar{u} + \frac{1}{2}\partial_x \overline{u^2} = 0 \Rightarrow \boxed{\partial_t \bar{u} + \frac{1}{2}\partial_x \bar{u}^2 = \frac{1}{2}\partial_x \bar{\tau}}$$

$$\boxed{\bar{\tau} \equiv \bar{u}^2 - \overline{u^2}}$$

The new sub-filter-scale tensor is not known, by definition.
It needs to be modelled (or ignored)

Filtering a non-linear term: **loosing information**

2.1. Filtering

$$\bar{u}(\vec{x},t) = \int_{-\infty}^{\infty} G(\vec{x}-\vec{x}')\, u(\vec{x}',t)\, d^3x'$$

- Ideally, with infinite resolution, the kernel function is a Krönecker delta: $\delta(\vec{x}-\vec{x}')$
- **BUT:**
  - Our world is not ideal ☹. Our simulations have a finite grid cell size (Δ)
- Simplest kernel function: Step function:

$$G_i(|\vec{x}-\vec{x}'|) = \begin{cases} 1/\Delta_f & if\ |\vec{x}-\vec{x}'| \leq \Delta_f/2 \\ 0 & otherwise \end{cases}$$

- **BUT:**
  - It is not suitable for analytical calculations involving derivatives… Better to use this other smooth kernel:

$$G_i(|x_i - x_i'|) = \left(\frac{1}{4\pi\xi}\right) \exp\left(-\frac{|x_i - x_i'|^2}{4\xi}\right)$$

$\xi = \Delta^2/24$ → We obtain a Gaussian function that resembles to a step function up to the **third moment!**

2.2. The gradient model

$$G_i(|x_i - x_i'|) = \left(\frac{1}{4\pi\xi}\right)\exp\left(-\frac{|x_i - x_i'|^2}{4\xi}\right) \quad \text{with} \quad \boxed{\xi = \Delta^2/24}$$

- The gradient model assumes a Gaussian kernel. After performing a Fourier transformation of the kernel and expand it in Taylor series we can rewrite:

$$\overline{fg} \approx \bar{f}\bar{g} + 2\xi\nabla\bar{f}\cdot\nabla\bar{g}$$

$$\overline{fgh} \approx \bar{f}\bar{g}\bar{h} + 2\xi\left(\bar{h}\nabla\bar{f}\cdot\nabla\bar{g} + \bar{g}\nabla\bar{f}\cdot\nabla\bar{h} + \bar{f}\nabla\bar{g}\cdot\nabla\bar{h}\right)$$

2.3. Compressible non-relativistic MHD evolution equations

[Viganò, R. A-M.+ 2019]

$$\partial_t \bar{\rho} + \partial_k N^k(\tilde{P}) = \partial_k \bar{\tau}_N^k$$
$$\partial_t \bar{N}^i + \partial_k T^{ki}(\tilde{P}) = \partial_k \bar{\tau}_T^{ki}$$
$$\partial_t \bar{U} + \partial_k S^k(\tilde{P}) = \partial_k \bar{\tau}_S^k$$
$$\partial_t \bar{B} + \partial_k M^{ki}(\tilde{P}) = \partial_k \bar{\tau}_M^{ki}$$

2.3. Compressible non-relativistic MHD evolution equations

[Viganò, R. A-M.+ 2019]

$$
\begin{aligned}
\partial_t \bar{\rho} + \partial_k N^k(\tilde{P}) &= \partial_k \bar{\tau}_N^k \\
\partial_t \bar{N}^i + \partial_k T^{ki}(\tilde{P}) &= \partial_k \bar{\tau}_T^{ki} \\
\partial_t \bar{U} + \partial_k S^k(\tilde{P}) &= \partial_k \bar{\tau}_S^k \\
\partial_t \bar{B} + \partial_k M^{ki}(\tilde{P}) &= \partial_k \bar{\tau}_M^{ki}
\end{aligned}
$$

$$
\begin{aligned}
N^k(\tilde{P}) &= \bar{\rho}\tilde{v}^k \\
T^{ki}(\tilde{P}) &= \tilde{v}^i \tilde{v}^j \bar{\rho} - \bar{B}^i \bar{B}^j + \delta^{ij}\left(\tilde{p} + \frac{\bar{B}^2}{2}\right) \\
S^k(\tilde{P}) &= \left(\bar{U} + \tilde{p} + \frac{\bar{B}^2}{2}\right)\tilde{v}^k - \tilde{v}\cdot\bar{B}\bar{B}^k \\
M^{ki}(\tilde{P}) &= \tilde{v}^k \bar{B}^i - \tilde{v}^i \bar{B}^k
\end{aligned}
$$

UNKNOWN SFS TERMS

$$
\begin{aligned}
\bar{\tau}_N^k &= N^k(\tilde{P}) - \overline{N^k(P)} \\
\bar{\tau}_T^{ki} &= T^{ki}(\tilde{P}) - \overline{T^{ki}(P)} \\
\bar{\tau}_S^k &= S^k(\tilde{P}) - \overline{S^k(P)} \\
\bar{\tau}_M^{ki} &= M^{ki}(\tilde{P}) - \overline{M^{ki}(P)}
\end{aligned}
$$

$$
\begin{aligned}
\tau_{\mathrm{N}} &= 0 \\
\tau_T^{ki} &= \tau_{kin}^{ki} - \tau_{mag}^{ki} + \delta^{ki}\tau_{pres} \\
\tau_S^k &= \tau_{ener}^k + \tilde{v}_{pres}^{k\tau} \\
\tau_M^{ki} &= \tau_{ind}^{ki}
\end{aligned}
$$

GRADIENT SGS MODEL TERMS

$$
\begin{aligned}
\tau_{kin}^{ki} &= -2\xi\bar{\rho}\nabla\tilde{v}^k \cdot \nabla\tilde{v}^i \\
\tau_{mag}^{ki} &= -2\xi\nabla\bar{B}^k \cdot \nabla\bar{B}^i \\
\tau_{pres}^{ki} &= -\xi\left[\nabla\frac{d\tilde{p}}{d\tilde{\rho}}\cdot\nabla\bar{\rho} + \nabla\frac{d\tilde{p}}{d\tilde{\epsilon}}\cdot\nabla\bar{\epsilon} - \frac{2}{\tilde{\rho}}\frac{d\tilde{p}}{d\tilde{\epsilon}}\nabla\tilde{\rho}\cdot\nabla\bar{\epsilon} + \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \frac{1}{\tilde{\rho}}\frac{d\tilde{p}}{d\tilde{\epsilon}}\left(\bar{\rho}\nabla\tilde{v}_j\cdot\nabla\tilde{v}^j + \nabla\bar{B}_j\cdot\nabla\bar{B}^j\right)\right] \\
\tau_{ener}^k &= -2\xi\left[\nabla\tilde{\Theta}\cdot\nabla\tilde{v}^k + \left(\bar{B}^k\bar{B}_j\nabla\tilde{v}^j - \tilde{\Theta}\nabla\tilde{v}^k\right)\cdot\nabla(\ln\tilde{\rho}) - \bar{B}^k\nabla\bar{B}_j\cdot\nabla\tilde{v}^j - \nabla(\tilde{v}\cdot\bar{B})\cdot\nabla\bar{B}^k\right] \\
\boldsymbol{\tau_{ind}^{ki}} &= \boldsymbol{-4\xi\left[\nabla\tilde{v}^{[k}\cdot\nabla\bar{B}^{i]} + \bar{B}^{[i}\nabla\tilde{v}^{k]}\cdot\nabla(\ln\bar{\rho})\right]}
\end{aligned}
$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$
\begin{aligned}
&\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}\left(\widetilde{N^k} - \bar{\tau}_N^k\right)\right] = 0 \\
&\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}\left(\widetilde{T_i^k} - \gamma_{ij}\bar{\tau}_T^{jk}\right)\right] = \sqrt{\gamma}\overline{R_i^S} \\
&\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}\left(\widetilde{S^k} - \bar{\tau}_S^k\right)\right] = \sqrt{\gamma}\overline{R^U} \\
&\partial_t\left(\sqrt{\gamma}\overline{B^i}\right) + \partial_k\left[\sqrt{\gamma}\left(-\beta^k\overline{B^i} + \beta^i\overline{B^k}\right)\right. \\
&\qquad \left. + \alpha\sqrt{\gamma}\left(\gamma^{ki}\bar{\Phi} + \widetilde{M}^{ki} - \bar{\tau}_M^{ki}\right)\right] = \sqrt{\gamma}\overline{R_B^i} \\
&\partial_t(\sqrt{\gamma}\bar{\Phi}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}\right] = \sqrt{\gamma}\overline{\mathrm{R}^\Phi}
\end{aligned}
$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$\partial_t(\sqrt{\gamma}\overline{D}) + \partial_k\left[-\beta^k\sqrt{\gamma}\overline{D} + \alpha\sqrt{\gamma}\left(\widetilde{N^k} - \bar{\tau}_N^k\right)\right] = 0$$
$$\partial_t(\sqrt{\gamma}\overline{S_i}) + \partial_k\left[-\beta^k\sqrt{\gamma}\overline{S_i} + \alpha\sqrt{\gamma}\left(\widetilde{T_i^k} - \gamma_{ij}\bar{\tau}_T^{jk}\right)\right] = \sqrt{\gamma}\overline{R_i^S}$$
$$\partial_t(\sqrt{\gamma}\overline{U}) + \partial_k\left[-\beta^k\sqrt{\gamma}\overline{U} + \alpha\sqrt{\gamma}\left(\widetilde{S^k} - \bar{\tau}_S^k\right)\right] = \sqrt{\gamma}\overline{R^U}$$
$$\partial_t\left(\sqrt{\gamma}\overline{B^i}\right) + \partial_k\left[\sqrt{\gamma}\left(-\beta^k\overline{B^i} + \beta^i\overline{B^k}\right) + \alpha\sqrt{\gamma}\left(\gamma^{ki}\overline{\Phi} + \widetilde{M}^{ki} - \bar{\tau}_M^{ki}\right)\right] = \sqrt{\gamma}\overline{R_B^i}$$
$$\partial_t(\sqrt{\gamma}\overline{\Phi}) + \partial_k\left[-\beta^k\sqrt{\gamma}\overline{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}\right] = \sqrt{\gamma}\overline{\mathrm{R}^\Phi}$$

$$\bar{\tau}_N^k = -C_N\xi H_N^k, \qquad \bar{\tau}_T^{ki} = -C_T\xi H_T^{ki},$$
$$\bar{\tau}_S^k = 0, \qquad \bar{\tau}_M^{ki} = -C_M\xi H_M^{ki}$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}(\widetilde{N^k} - \bar{\tau}_N^k)] = 0$$
$$\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}\left(\widetilde{T_i^k} - \gamma_{ij}\bar{\tau}_T^{jk}\right)\right] = \sqrt{\gamma}\overline{R_i^S}$$
$$\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}(\widetilde{S^k} - \bar{\tau}_S^k)] = \sqrt{\gamma}\overline{R^U}$$
$$\partial_t(\sqrt{\gamma}\overline{B^i}) + \partial_k[\sqrt{\gamma}(-\beta^k\overline{B^i} + \beta^i\overline{B^k}) + \alpha\sqrt{\gamma}(\gamma^{ki}\bar{\Phi} + \widetilde{M}^{ki} - \bar{\tau}_M^{ki})] = \sqrt{\gamma}\overline{R_B^i}$$
$$\partial_t(\sqrt{\gamma}\bar{\Phi}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}] = \sqrt{\gamma}\overline{\mathrm{R}^\Phi}$$

$$\bar{\tau}_N^k = -C_N\xi H_N^k, \qquad \bar{\tau}_T^{ki} = -C_T\xi H_T^{ki},$$
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$$H_\varepsilon = H_p - \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \tilde{E}_k H_E^k,$$

$$H_\Theta = \tilde{\Psi}_\Theta + \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}H_p, \qquad H_v^k = \tilde{\Psi}_v^k - \left(\tilde{v}^k + \frac{\overline{\tilde{v}\cdot B}}{\tilde{\varepsilon}}\bar{B}^k\right)\frac{H_\Theta}{\tilde{\Theta}}$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}\left(\widetilde{N^k} - \bar{\tau}_N^k\right)\right] = 0$$
$$\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}\left(\widetilde{T_i^k} - \gamma_{ij}\bar{\tau}_T^{jk}\right)\right] = \sqrt{\gamma}\overline{R_i^S}$$
$$\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}\left(\widetilde{S^k} - \bar{\tau}_S^k\right)\right] = \sqrt{\gamma}\overline{R^U}$$
$$\partial_t\left(\sqrt{\gamma}\overline{B^i}\right) + \partial_k\left[\sqrt{\gamma}\left(-\beta^k\overline{B^i} + \beta^i\overline{B^k}\right)\right.$$
$$\left.+\alpha\sqrt{\gamma}\left(\gamma^{ki}\bar{\Phi} + \widetilde{M}^{ki} - \bar{\tau}_M^{ki}\right)\right] = \sqrt{\gamma}\overline{R_B^i}$$
$$\partial_t(\sqrt{\gamma}\bar{\Phi}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}\right] = \sqrt{\gamma}\overline{R^\Phi}$$

$$\bar{\tau}_N^k = -C_N\xi H_N^k, \qquad \bar{\tau}_T^{ki} = -C_T\xi H_T^{ki},$$
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$$H_\varepsilon = H_p - \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \tilde{E}_k H_E^k,$$

$$H_\Theta = \tilde{\Psi}_\Theta + \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}H_p, \qquad H_v^k = \tilde{\Psi}_v^k - \left(\tilde{v}^k + \frac{\tilde{v}\cdot B}{\tilde{\varepsilon}}\bar{B}^k\right)\frac{H_\Theta}{\tilde{\Theta}}$$

$$H_N^k = 2\nabla\bar{D}\cdot\nabla\tilde{v}^k + \bar{D}H_v^k,$$

$$H_T^{ki} = 2\left[\nabla\tilde{\varepsilon}\cdot\nabla\left(\tilde{v}^i\tilde{v}^k\right) + \tilde{\varepsilon}\left(\tilde{v}^{(i}H_v^{k)} + \nabla\tilde{v}^i\cdot\nabla\tilde{v}^k\right)\right]$$
$$+\tilde{v}^i\tilde{v}^k H_\varepsilon - 2\left[\nabla\bar{B}^i\cdot\nabla\bar{B}^k + \nabla\tilde{E}^i\cdot\nabla\tilde{E}^k + \tilde{E}^{(i}H_E^{k)}\right]$$
$$+\delta^{ki}\left[H_p + \nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j + \tilde{E}_j H_E^j\right],$$

$$\boldsymbol{H_M^{ki} = 4\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + 2\bar{B}^{[i}H_v^{k]}} \rightarrow H_E^i = \frac{1}{2}\epsilon^i_{jk}H_M^{jk}$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}(\widetilde{N^k} - \tau_N^k)] = 0$$
$$\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}\left(\widetilde{T_i^k} - \gamma_{ij}\tau_T^{jk}\right)\right] = \sqrt{\gamma}\overline{R_i^S}$$
$$\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}(\widetilde{S^k} - \tau_S^k)] = \sqrt{\gamma}\overline{R^U}$$
$$\partial_t(\sqrt{\gamma}\overline{B^i}) + \partial_k[\sqrt{\gamma}(-\beta^k\overline{B^i} + \beta^i\overline{B^k})$$
$$+\alpha\sqrt{\gamma}(\gamma^{ki}\bar{\Phi} + \widetilde{M}^{ki} - \tau_M^{ki})] = \sqrt{\gamma}\overline{R_B^i}$$
$$\partial_t(\sqrt{\gamma}\bar{\Phi}) + \partial_k[-\beta^k\sqrt{\gamma}\bar{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}] = \sqrt{\gamma}\overline{R^\Phi}$$

$$\tau_N^k = -C_N\xi H_N^k, \qquad \tau_T^{ki} = -C_T\xi H_T^{ki},$$
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$$H_\varepsilon = H_p - \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \tilde{E}_k H_E^k,$$
$$H_\Theta = \tilde{\Psi}_\Theta + \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}H_p, \qquad H_v^k = \tilde{\Psi}_v^k - \left(\tilde{v}^k + \frac{\overline{\tilde{v}\cdot B}}{\tilde{\varepsilon}}\bar{B}^k\right)\frac{H_\Theta}{\tilde{\Theta}}$$

$$H_N^k = 2\nabla\bar{D}\cdot\nabla\tilde{v}^k + \bar{D}H_v^k,$$
$$H_T^{ki} = 2\left[\nabla\tilde{\varepsilon}\cdot\nabla(\tilde{v}^i\tilde{v}^k) + \tilde{\varepsilon}\left(\tilde{v}^{(i}H_v^{k)} + \nabla\tilde{v}^i\cdot\nabla\tilde{v}^k\right)\right]$$
$$+\tilde{v}^i\tilde{v}^k H_\varepsilon - 2\left[\nabla\bar{B}^i\cdot\nabla\bar{B}^k + \nabla\tilde{E}^i\cdot\nabla\tilde{E}^k + \tilde{E}^{(i}H_E^{k)}\right]$$
$$+\delta^{ki}\left[H_p + \nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j + \tilde{E}_j H_E^j\right],$$

$$\boldsymbol{H_M^{ki} = 4\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + 2\bar{B}^{[i}H_v^{k]}} \rightarrow H_E^i = \frac{1}{2}\epsilon^i_{jk}H_M^{jk}$$

$$\tilde{\Psi}_v^k = \frac{2}{\tilde{\Theta}}\left\{\nabla(\tilde{v}\cdot\bar{B})\cdot\nabla\bar{B}^k - \nabla\tilde{\Theta}\cdot\nabla\tilde{v}^k + \frac{\bar{B}^k}{\tilde{\varepsilon}}\left[\tilde{\Theta}\nabla\bar{B}_j\cdot\nabla\tilde{v}^j + \bar{B}_j\nabla\bar{B}^j\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}_j\nabla\tilde{v}^j\cdot\nabla\tilde{\Theta}\right]\right\},$$
$$\tilde{\Psi}_M^{ki} = \frac{4}{\tilde{\Theta}}\left[\tilde{\Theta}\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + \bar{B}^{[i}\nabla\bar{B}^{k]}\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}^{[i}\nabla\tilde{v}^{k]}\cdot\nabla\tilde{\Theta}\right],$$
$$\tilde{\Psi}_\Theta = \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}\{\nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \bar{B}_{[i}\tilde{v}_{k]}\tilde{\Psi}_M^{ki}\},$$
$$\tilde{\Psi}_A = \tilde{W}^2\left(\tilde{p}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\rho}^2\frac{d\tilde{p}}{d\tilde{\rho}}\right)$$

[Carrasco+, 2020]

$$\frac{H_p}{\tilde{\Theta} - \tilde{E}^2} = \frac{\tilde{\varepsilon}\tilde{W}^2}{(\tilde{\rho}\tilde{\varepsilon} - \tilde{\Psi}_A)(\tilde{\Theta} - \tilde{E}^2)\tilde{W}^2 + \tilde{\Psi}_A\tilde{\Theta}}\left\{\tilde{\rho}\left(\nabla\frac{d\tilde{p}}{d\tilde{\rho}}\cdot\nabla\tilde{\rho} + \nabla\frac{d\tilde{p}}{d\tilde{\epsilon}}\cdot\nabla\tilde{\epsilon}\right) - 2\frac{d\tilde{p}}{d\tilde{\epsilon}}\nabla\tilde{\rho}\cdot\nabla\tilde{\epsilon}\right.$$
$$-\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} - \tilde{\Psi}_A\right)\left[\frac{\tilde{W}^2}{4}\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2} + \nabla\tilde{W}^{-2}\cdot\nabla(\ln\tilde{\rho})\right] - \frac{2}{\tilde{W}^2}\frac{d\tilde{p}}{d\tilde{\epsilon}}\left[\nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{W}^2\cdot\nabla\tilde{h}\right]$$
$$\left.-\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)\left[\tilde{v}_k\tilde{\Psi}_v^k + \nabla\tilde{v}_j\cdot\nabla\tilde{v}^j + \tilde{W}^2\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2}\right] + \frac{1}{\tilde{\varepsilon}}\left[\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)(\tilde{\Theta} - \tilde{E}^2) - \frac{\tilde{\Psi}_A\tilde{\Theta}}{\tilde{W}^2}\right]\frac{\tilde{\Psi}_\Theta}{\tilde{\Theta}}\right\}$$

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]

$$\partial_t(\sqrt{\gamma}\bar{D}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{D} + \alpha\sqrt{\gamma}\left(\widetilde{N^k} - \bar{\tau}^k_N\right)\right] = 0$$
$$\partial_t(\sqrt{\gamma}\bar{S}_i) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{S}_i + \alpha\sqrt{\gamma}\left(\widetilde{T^k_i} - \gamma_{ij}\bar{\tau}^{jk}_T\right)\right] = \sqrt{\gamma}\overline{R^S_i}$$
$$\partial_t(\sqrt{\gamma}\bar{U}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{U} + \alpha\sqrt{\gamma}\left(\widetilde{S^k} - \bar{\tau}^k_S\right)\right] = \sqrt{\gamma}\overline{R^U}$$
$$\partial_t(\sqrt{\gamma}\overline{B^i}) + \partial_k[\sqrt{\gamma}(-\beta^k\overline{B^i} + \beta^i\overline{B^k}) + \alpha\sqrt{\gamma}(\gamma^{ki}\bar{\Phi} + \widetilde{M}^{ki} - \bar{\tau}^{ki}_M)] = \sqrt{\gamma}\overline{R^i_B}$$
$$\partial_t(\sqrt{\gamma}\bar{\Phi}) + \partial_k\left[-\beta^k\sqrt{\gamma}\bar{\Phi} + \alpha c_h^2\sqrt{\gamma}\overline{B^k}\right] = \sqrt{\gamma}\overline{R^\Phi}$$

$$\bar{\tau}^k_N = -C_N\xi H^k_N, \qquad \bar{\tau}^{ki}_T = -C_T\xi H^{ki}_T,$$
$$\bar{\tau}^k_S = 0, \qquad \bar{\tau}^{ki}_M = -C_M\xi H^{ki}_M$$

$$H_\varepsilon = H_p - \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \tilde{E}_k H^k_E,$$
$$H_\Theta = \tilde{\Psi}_\Theta + \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}H_p, \qquad H^k_v = \tilde{\Psi}^k_v - \left(\tilde{v}^k + \frac{\overline{\tilde{v}\cdot B}}{\tilde{\varepsilon}}\bar{B}^k\right)\frac{H_\Theta}{\tilde{\Theta}}$$

$$H^k_N = 2\nabla\bar{D}\cdot\nabla\tilde{v}^k + \bar{D}H^k_v,$$
$$H^{ki}_T = 2\left[\nabla\tilde{\varepsilon}\cdot\nabla(\tilde{v}^i\tilde{v}^k) + \tilde{\varepsilon}\left(\tilde{v}^{(i}H^{k)}_v + \nabla\tilde{v}^i\cdot\nabla\tilde{v}^k\right)\right] + \tilde{v}^i\tilde{v}^k H_\varepsilon - 2\left[\nabla\bar{B}^i\cdot\nabla\bar{B}^k + \nabla\tilde{E}^i\cdot\nabla\tilde{E}^k + \tilde{E}^{(i}H^{k)}_E\right] + \delta^{ki}\left[H_p + \nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j + \tilde{E}_j H^j_E\right],$$
$$\mathbf{H^{ki}_M = 4\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + 2\bar{B}^{[i}H^{k]}_v} \rightarrow H^i_E = \frac{1}{2}\epsilon^i_{jk}H^{jk}_M$$

$$\tilde{\Psi}^k_v = \frac{2}{\tilde{\Theta}}\left\{\nabla(\tilde{v}\cdot\bar{B})\cdot\nabla\bar{B}^k - \nabla\tilde{\Theta}\cdot\nabla\tilde{v}^k + \frac{\bar{B}^k}{\tilde{\varepsilon}}\left[\tilde{\Theta}\nabla\bar{B}_j\cdot\nabla\tilde{v}^j + \bar{B}_j\nabla\bar{B}^j\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}_j\nabla\tilde{v}^j\cdot\nabla\tilde{\Theta}\right]\right\},$$
$$\tilde{\Psi}^{ki}_M = \frac{4}{\tilde{\Theta}}\left[\tilde{\Theta}\nabla\bar{B}^{[i}\cdot\nabla\tilde{v}^{k]} + \bar{B}^{[i}\nabla\bar{B}^{k]}\cdot\nabla(\tilde{v}\cdot\bar{B}) - \bar{B}^{[i}\nabla\tilde{v}^{k]}\cdot\nabla\tilde{\Theta}\right],$$
$$\tilde{\Psi}_\Theta = \frac{\tilde{\Theta}}{\tilde{\Theta} - \tilde{E}^2}\left\{\nabla\bar{B}_j\cdot\nabla\bar{B}^j - \nabla\tilde{E}_j\cdot\nabla\tilde{E}^j - \bar{B}_{[i}\tilde{v}_{k]}\tilde{\Psi}^{ki}_M\right\},$$
$$\tilde{\Psi}_A = \tilde{W}^2\left(\tilde{p}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\rho}^2\frac{d\tilde{p}}{d\tilde{\rho}}\right)$$

[Carrasco+, 2020]

$$\frac{H_p}{\tilde{\Theta} - \tilde{E}^2} = \frac{\tilde{\varepsilon}\tilde{W}^2}{(\tilde{\rho}\tilde{\varepsilon} - \tilde{\Psi}_A)(\tilde{\Theta} - \tilde{E}^2)\tilde{W}^2 + \tilde{\Psi}_A\tilde{\Theta}}\left\{\tilde{\rho}\left(\nabla\frac{d\tilde{p}}{d\tilde{\rho}}\cdot\nabla\tilde{\rho} + \nabla\frac{d\tilde{p}}{d\tilde{\epsilon}}\cdot\nabla\tilde{\epsilon}\right) - 2\frac{d\tilde{p}}{d\tilde{\epsilon}}\nabla\tilde{\rho}\cdot\nabla\tilde{\epsilon}\right.$$
$$-\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} - \tilde{\Psi}_A\right)\left[\frac{\tilde{W}^2}{4}\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2} + \nabla\tilde{W}^{-2}\cdot\nabla(\ln\tilde{\rho})\right] - \frac{2}{\tilde{W}^2}\frac{d\tilde{p}}{d\tilde{\epsilon}}\left[\nabla\bar{B}_j\cdot\nabla\bar{B}^j + \nabla\tilde{W}^2\cdot\nabla\tilde{h}\right]$$
$$\left.-\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)\left[\tilde{v}_k\tilde{\Psi}^k_v + \nabla\tilde{v}_j\cdot\nabla\tilde{v}^j + \tilde{W}^2\nabla\tilde{W}^{-2}\cdot\nabla\tilde{W}^{-2}\right] + \frac{1}{\tilde{\varepsilon}}\left[\left(\tilde{\varepsilon}\frac{d\tilde{p}}{d\tilde{\epsilon}} + \tilde{\Psi}_A\right)(\tilde{\Theta} - \tilde{E}^2) - \frac{\tilde{\Psi}_A\tilde{\Theta}}{\tilde{W}^2}\right]\frac{\tilde{\Psi}_\Theta}{\tilde{\Theta}}\right\}$$

2.5. Effects of LES in BNS mergers

[Palenzuela, R. A-M+ 2020]

MHDuet code generated with Simflowny software

Numerical methods:

- Einstein equation 4^{th} order accurate finite differences
- Kreiss-Oliger 6^{th} order dissipation
- Fluid MP5 reconstruction scheme + Lax-Friedrichs flux splitting formula
- **LES 4^{th} order differential operators for SGS terms**
- 4^{th} order Runge-Kutta

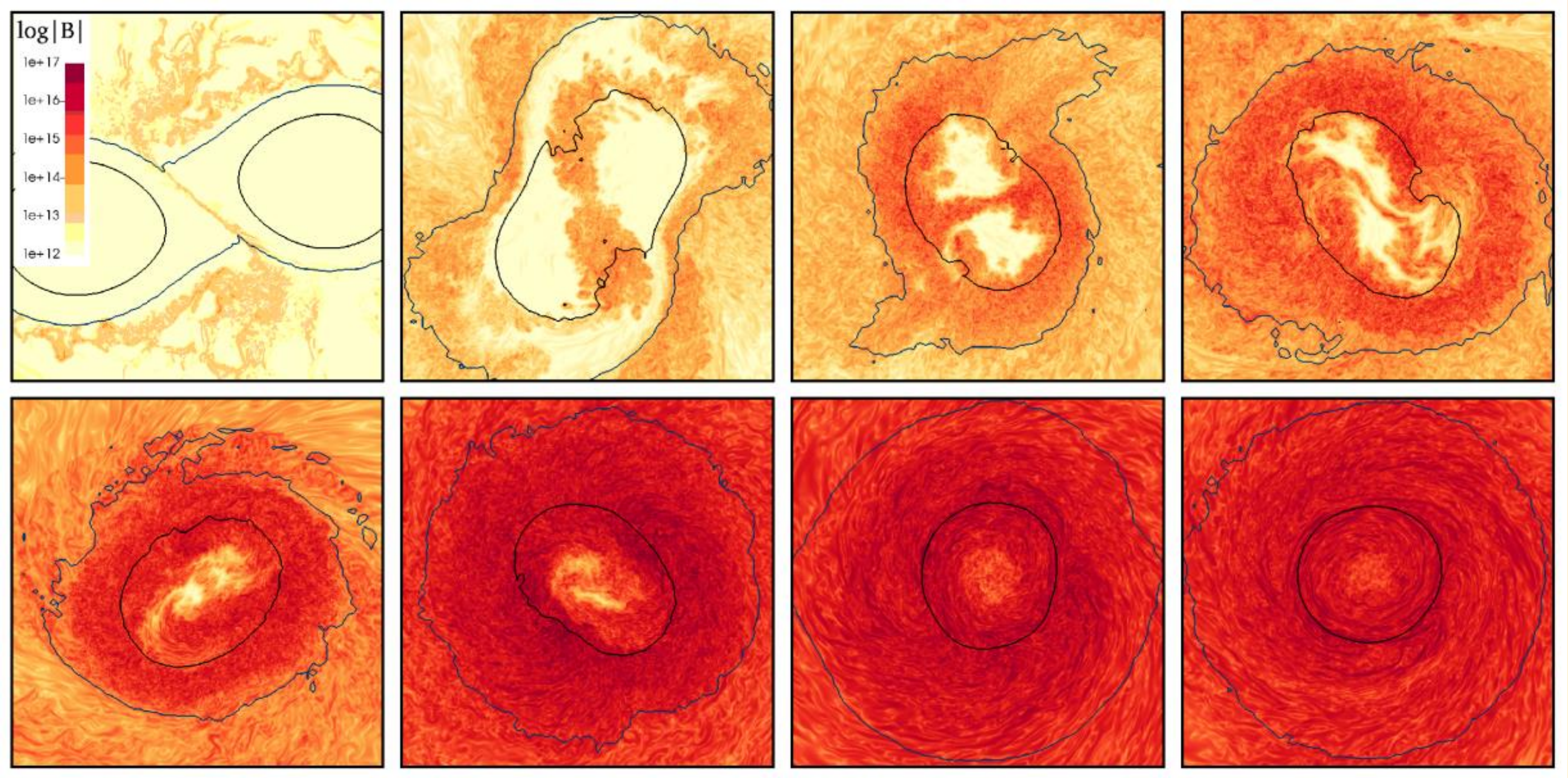
Physical setup:

- CCZ4 formulation of Einstein equations.
- Initial data by Lorene code, equal masses (1.3 Msun), quasi-circular orbits separated by 45 km
- Magnetic fields initially **10^{11} G**, confined to each star
- Hybrid EoS: piecewise APR4 + ideal

Case	$\mathcal{C}_{\mathcal{M}}$	Refinement levels	ΔL_{min} [km]	Δ_{min} [m]
LR	0	7 FMR	[-28,28]	120
MR	0	7 FMR + 1 AMR	[-13,13]	60
HR	0	7 FMR + 2 AMR	[-11,11]	30
LR LES	8	7 FMR	[-28,28]	120
MR LES	8	7 FMR + 1 AMR	[-13,13]	60
HR LES	8	7 FMR + 2 AMR	[-11,11]	30
MR BO	8	7 FMR + 1 AMR	[-13,13]	60

2.5. Effects of LES in BNS mergers

[Palenzuela, R. A-M+ 2022]

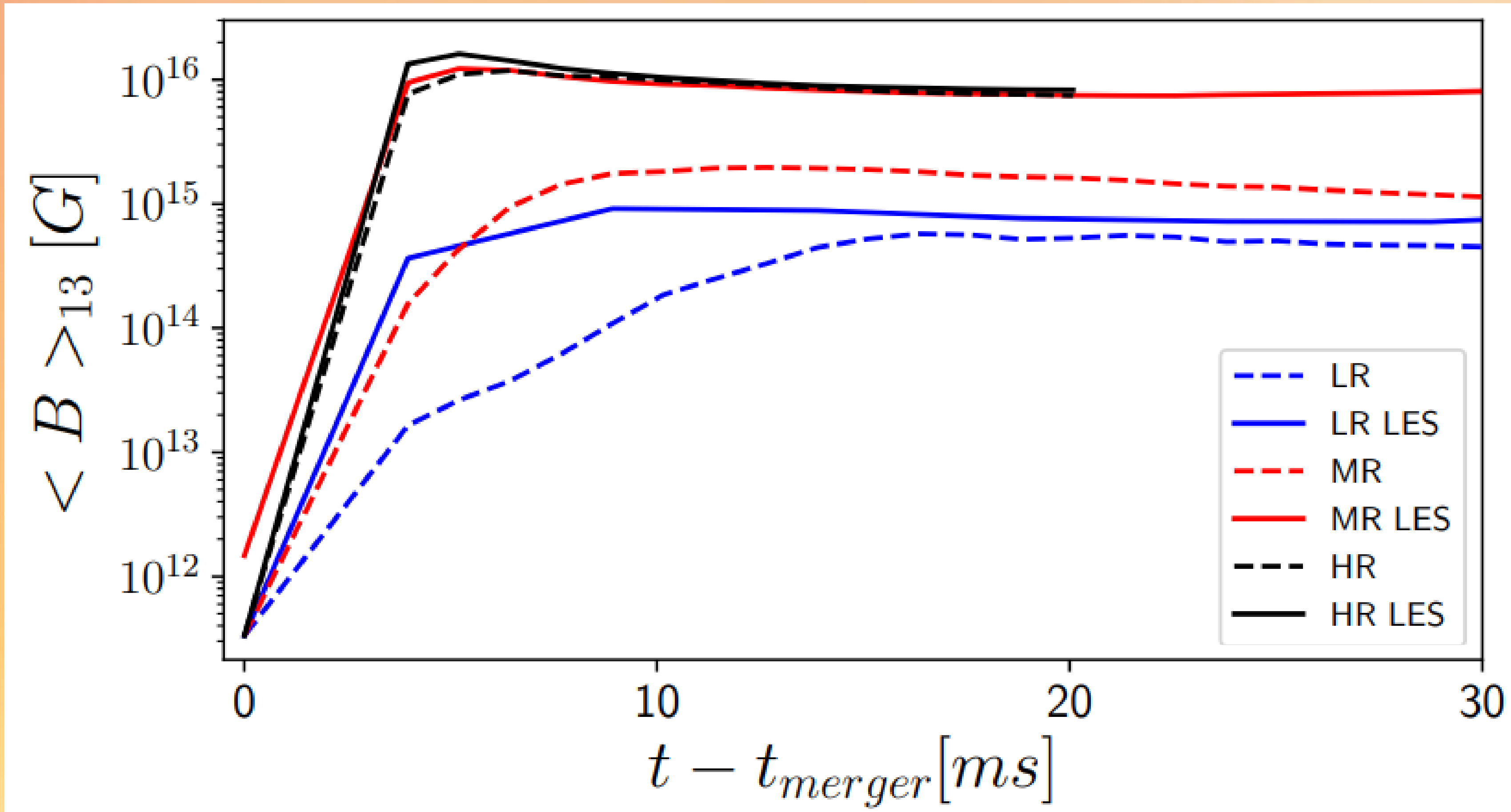


t = {0.5, 1.5, 2, 2.5, 3.5, 5, 10, 15} ms

Constant density surfaces in 10^{13} and 5×10^{14} g / cm^3

2.5. Effects of LES in BNS mergers

[Palenzuela, R. A-M+ 2022]

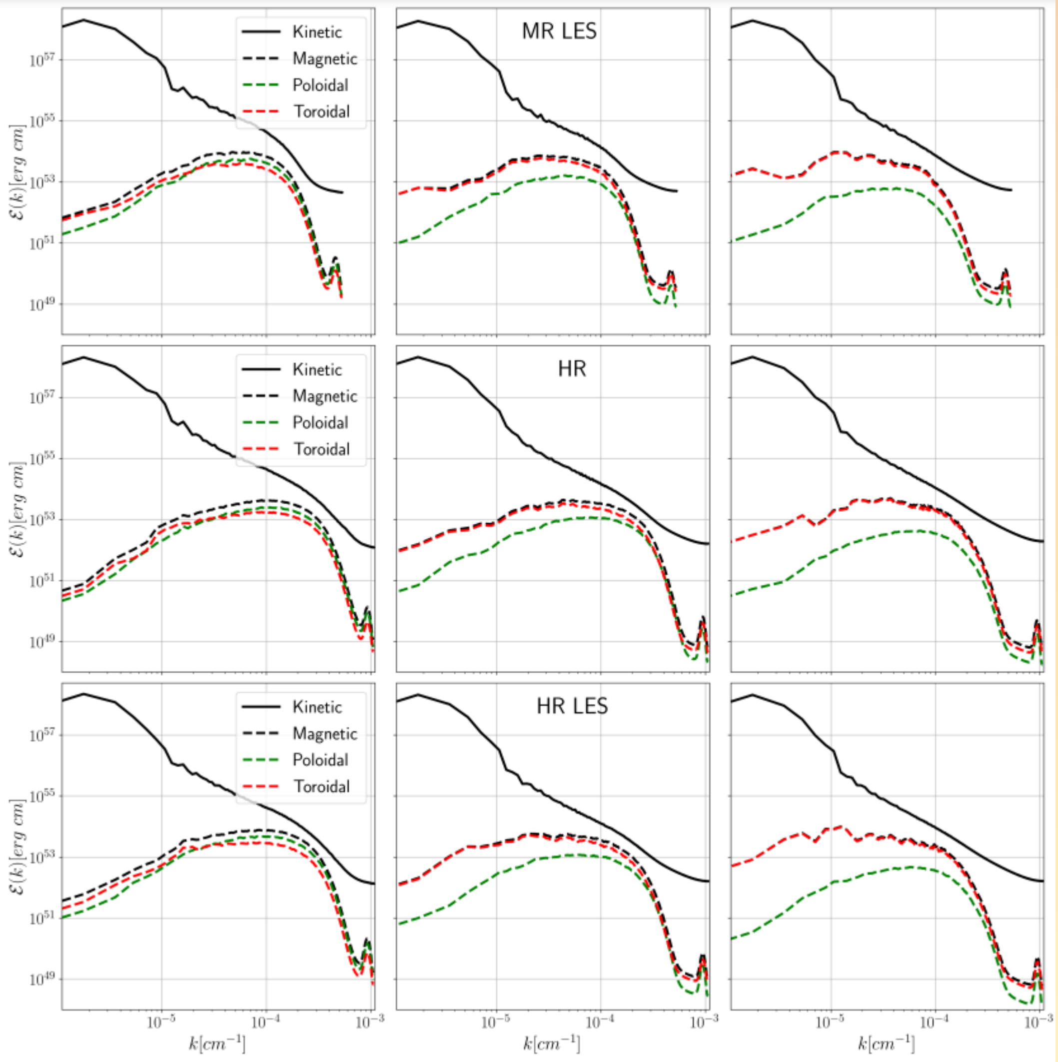


Saturation of magnetic field in t < 5 ms and convergence of averaged magnetic field strength and components!!

2.5. Effects of LES in BNS mergers

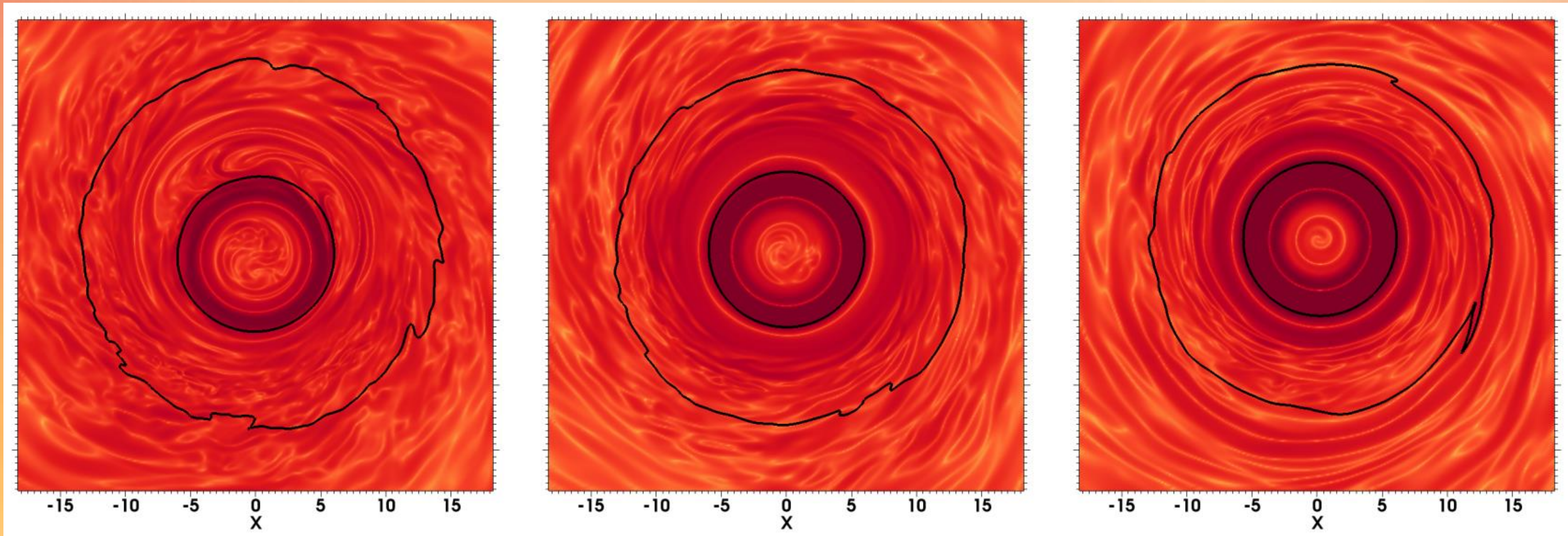
[Palenzuela , R. A-M+ 2022]

t={5, 10, 20} ms



2.6. Magnetic field evolution

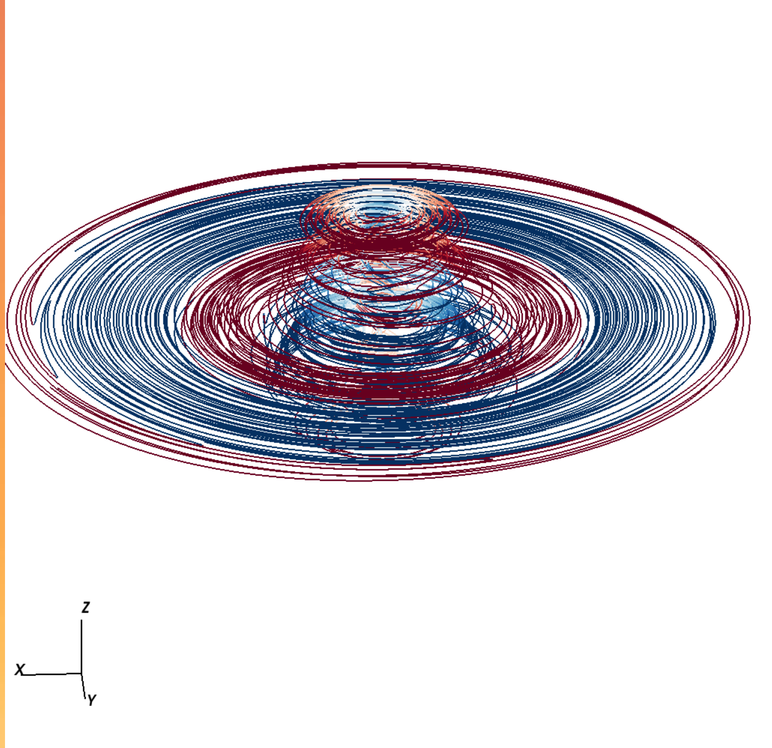
[Aguilera-Miret+ 2023]

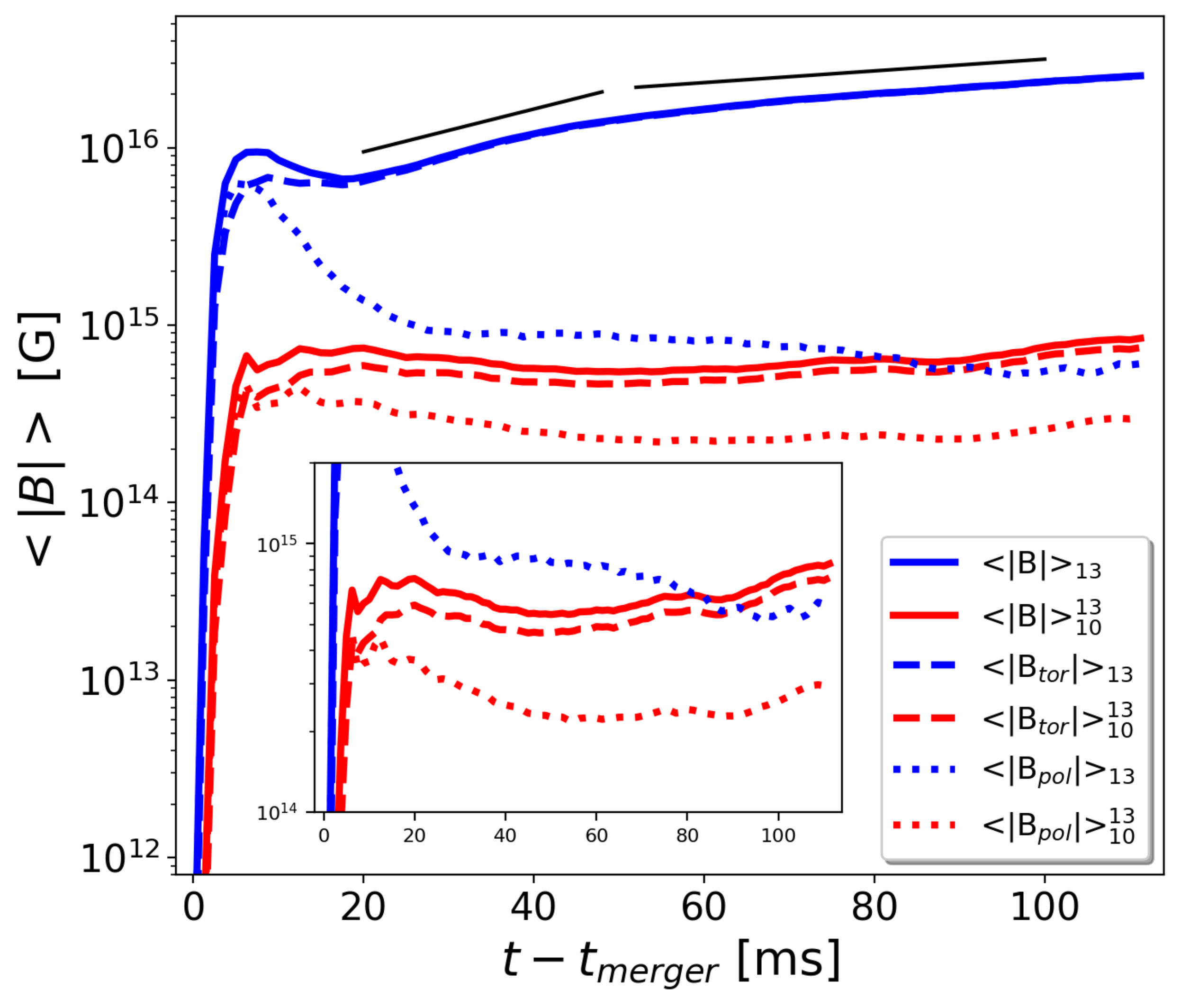


t = 25 ms **t =50 ms** **t = 110 ms**

2.6. Magnetic field evolution

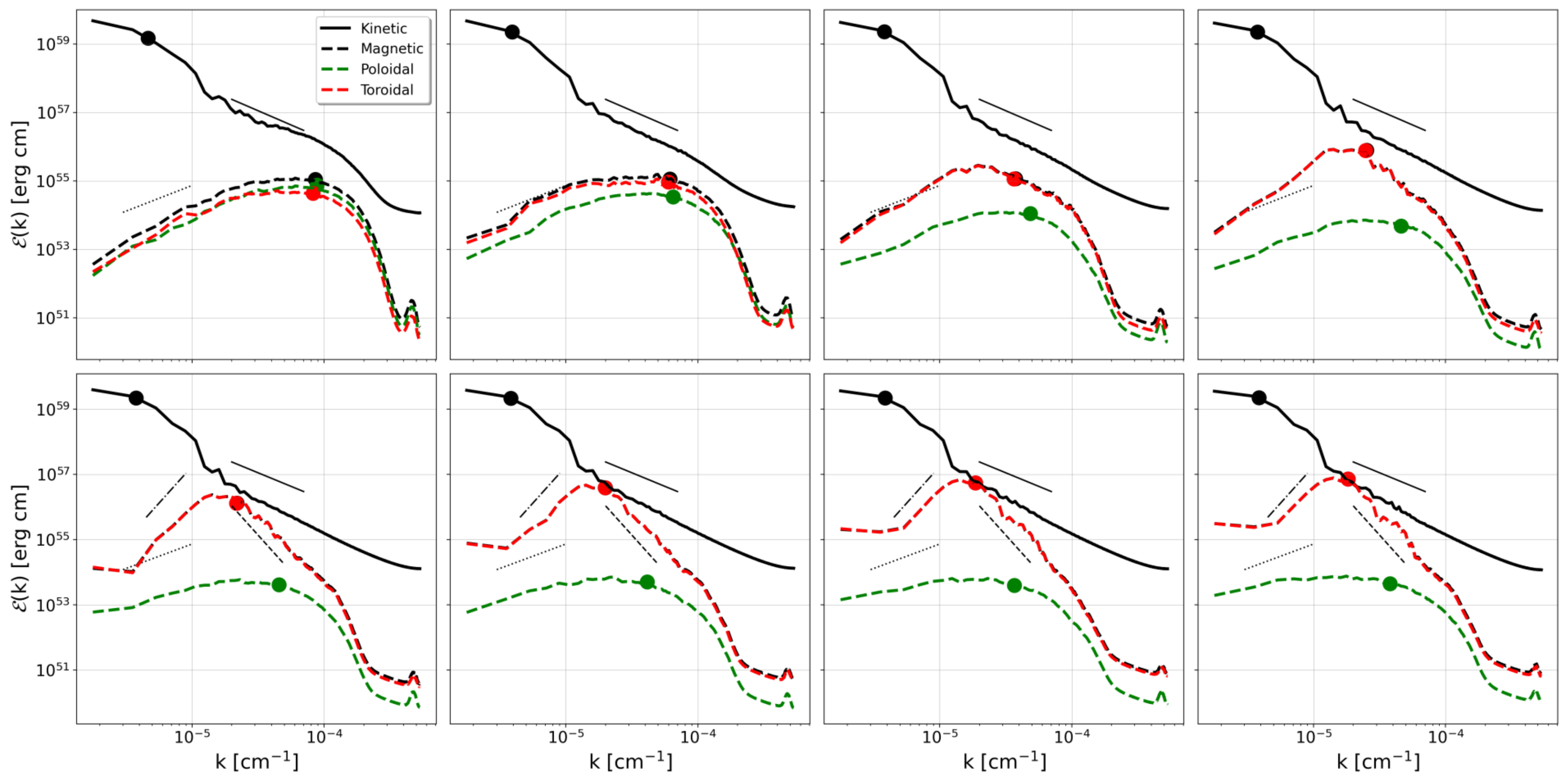
[Aguilera-Miret+ 2023]





2.6. Magnetic field evolution

[Aguilera-Miret+ 2023]



t = {5, 11, 21, 31, 50, 78, 100, 111} ms

Average magnetic structure length-scale:

t = 10 ms → 700 m
t = 100 ms → 3,5 km

$$\bar{k} \equiv \frac{\int_k k\mathcal{E}(k)dk}{\int_k \mathcal{E}(k)dk}$$

New slopes: ±9/2

2.7. Importance of the magnetic field topology

[Aguilera-Miret+ 2022]

Does the initial magnetic field strength and topology matter at all in BNS mergers?

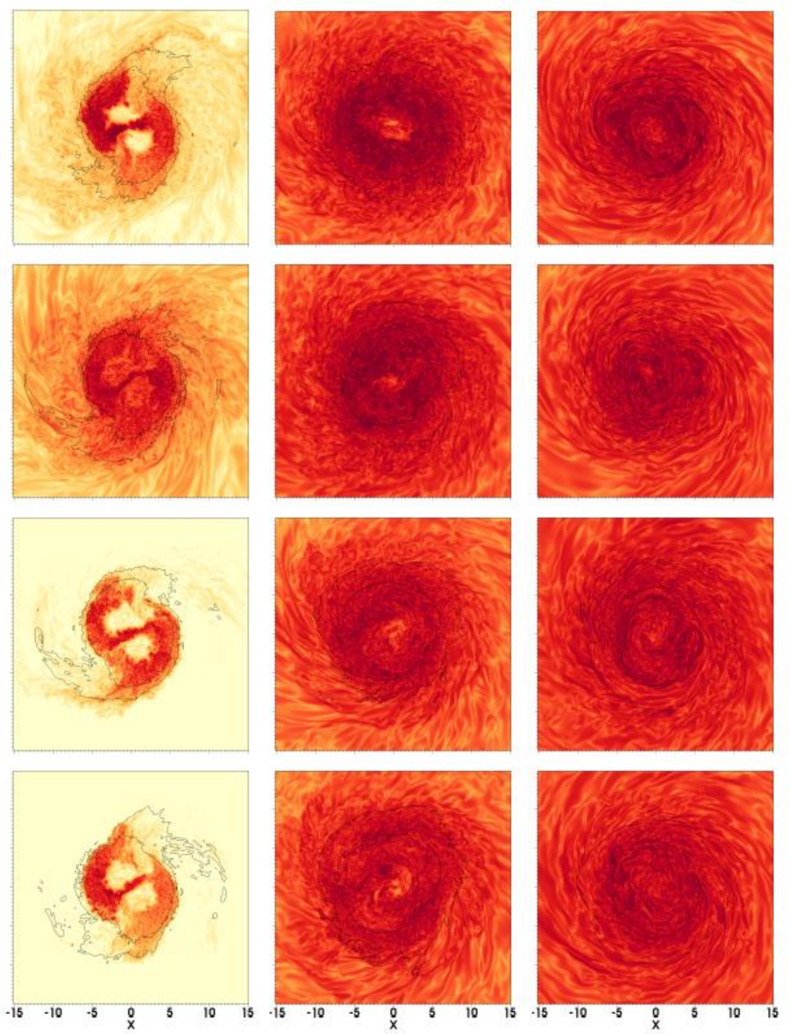
Dipolar magnetic field <B> ~ 10^{11} G
(Dip)

Dipolar magnetic field <B> ~ 10^{14} G
(Bhigh)

Dipolar with magnetic moment perpendicular to the z-axis <B> ~ 10^{11} G
(Misaligned)

Multipolar magnetic field <B> ~ 10^{11} G
(Multipolar)

$$A_\Phi \propto \sin^4\theta\,(1+\cos\theta)r^2(P-P_{cut})$$



t = 2 ms **t = 5 ms** **t = 10 ms**

2.7. Importance of the magnetic field topology

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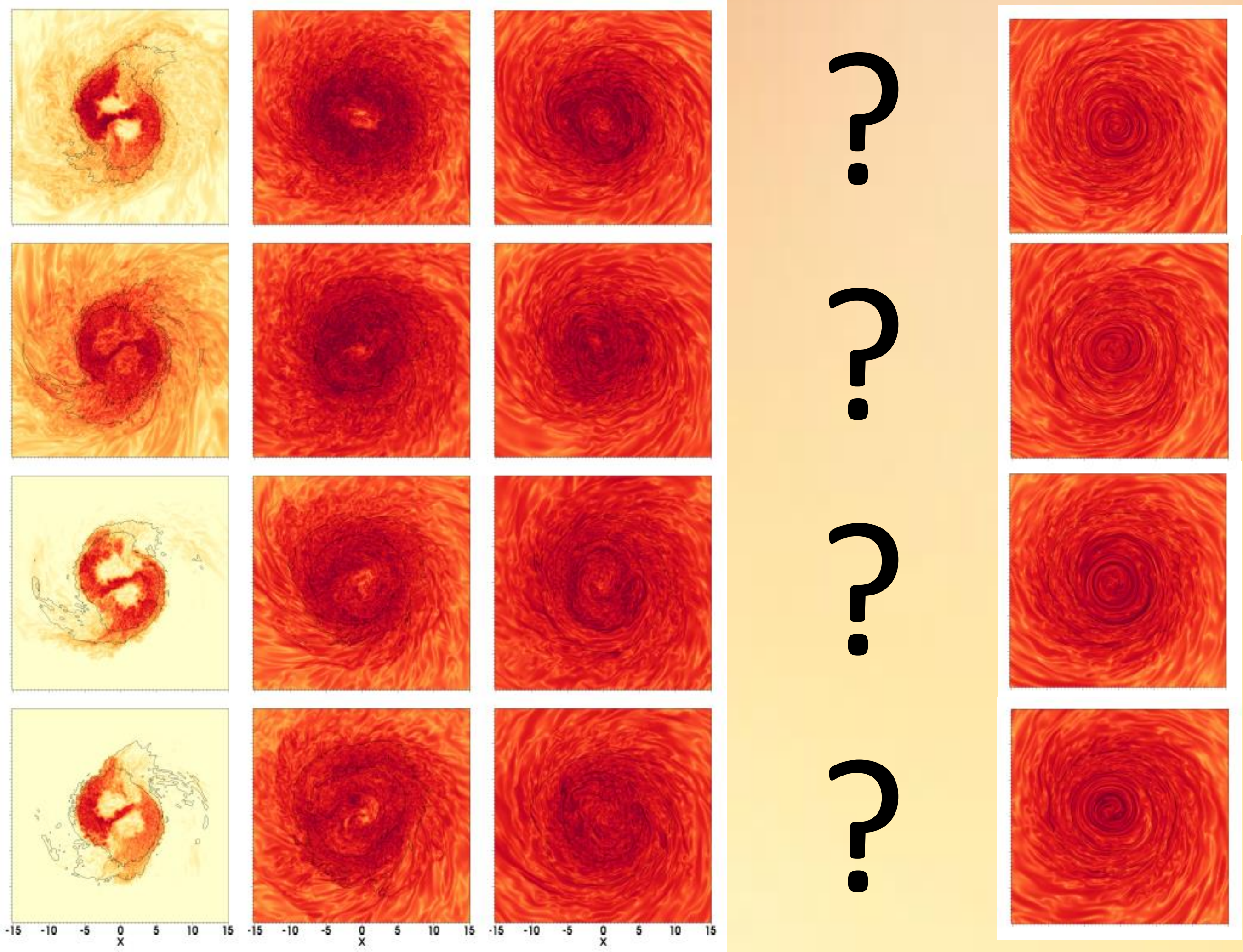

t = 2 ms **t = 5 ms** **t = 10 ms** **t = 20 ms**

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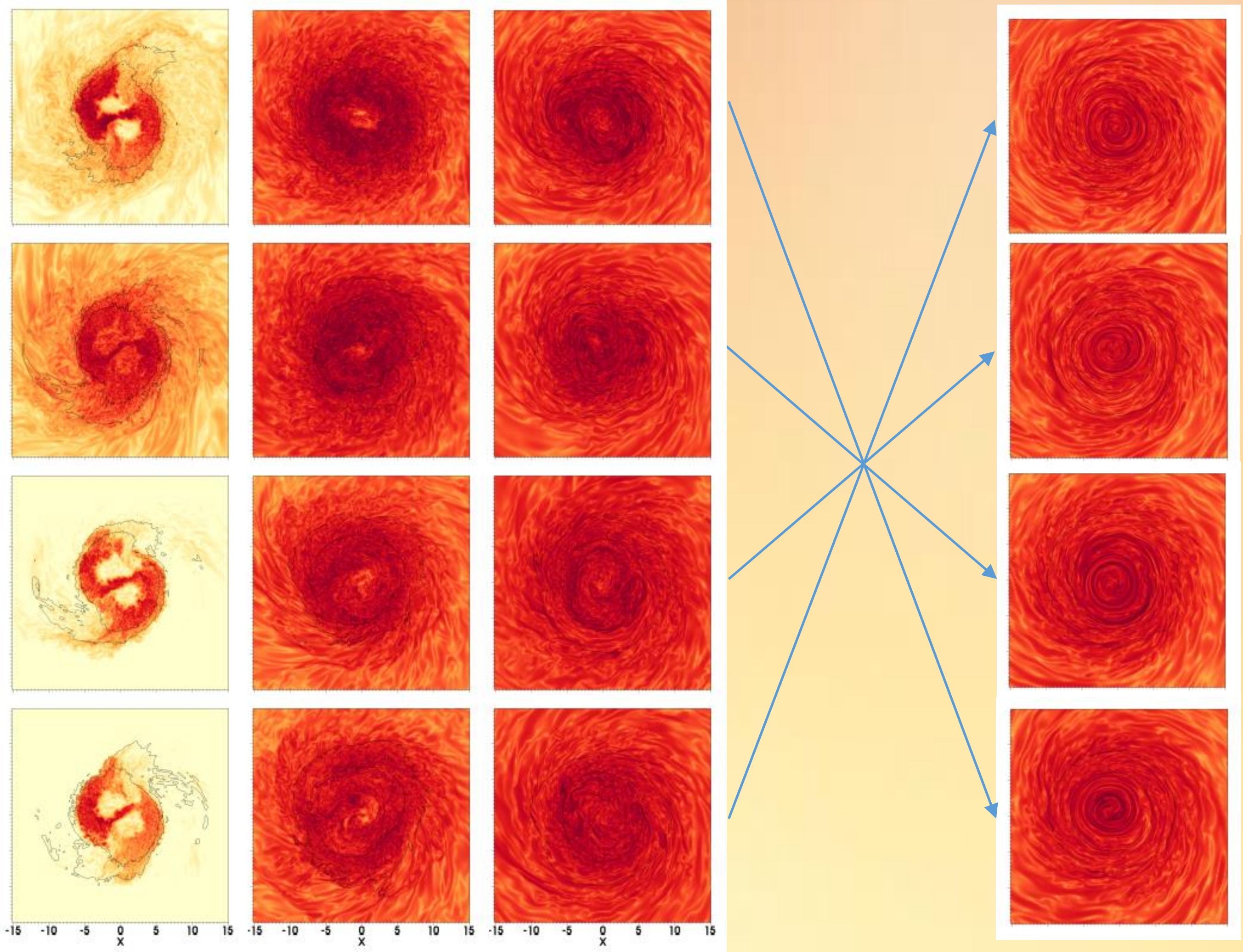

t = 2 ms **t = 5 ms** **t = 10 ms** **t = 20 ms**

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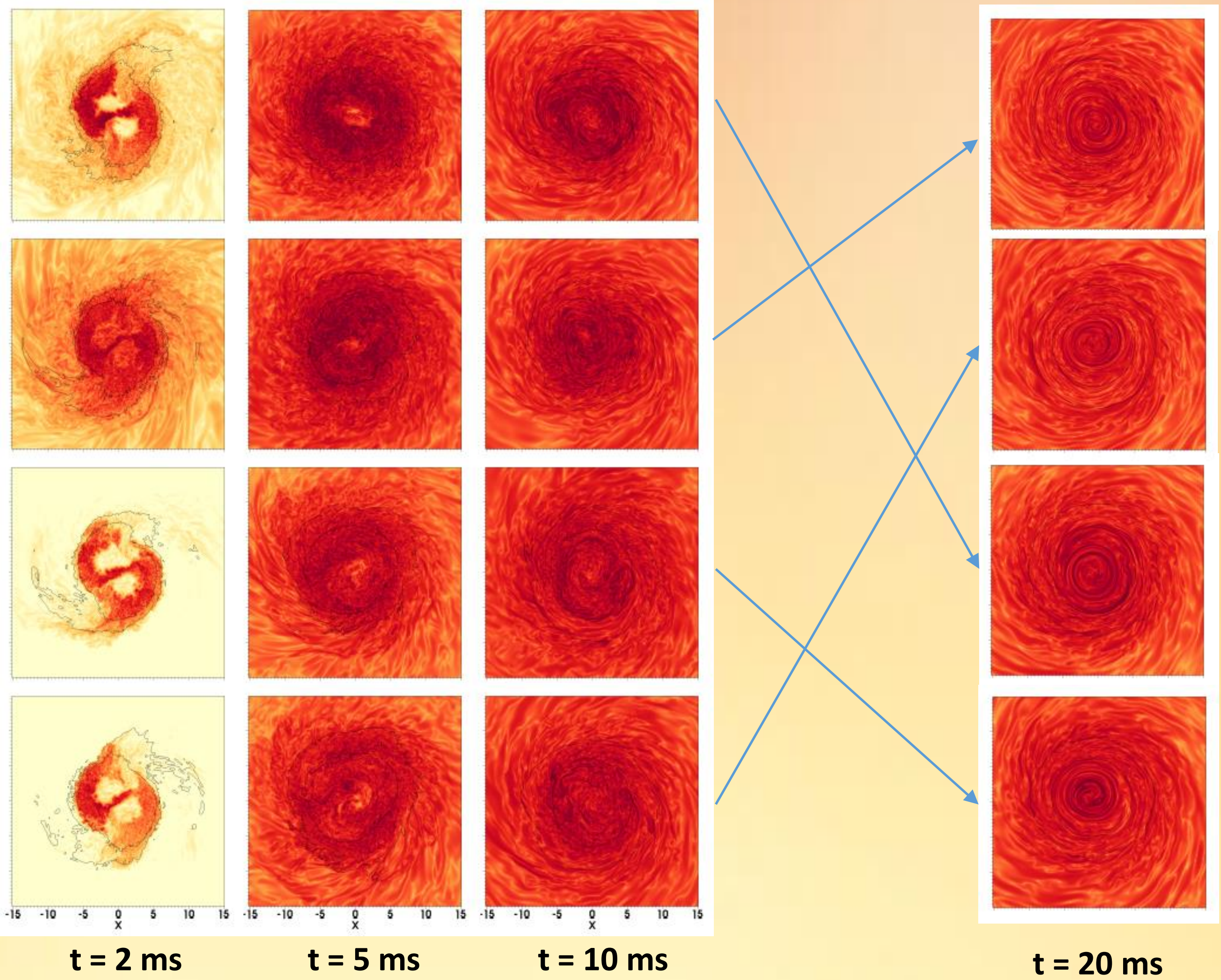
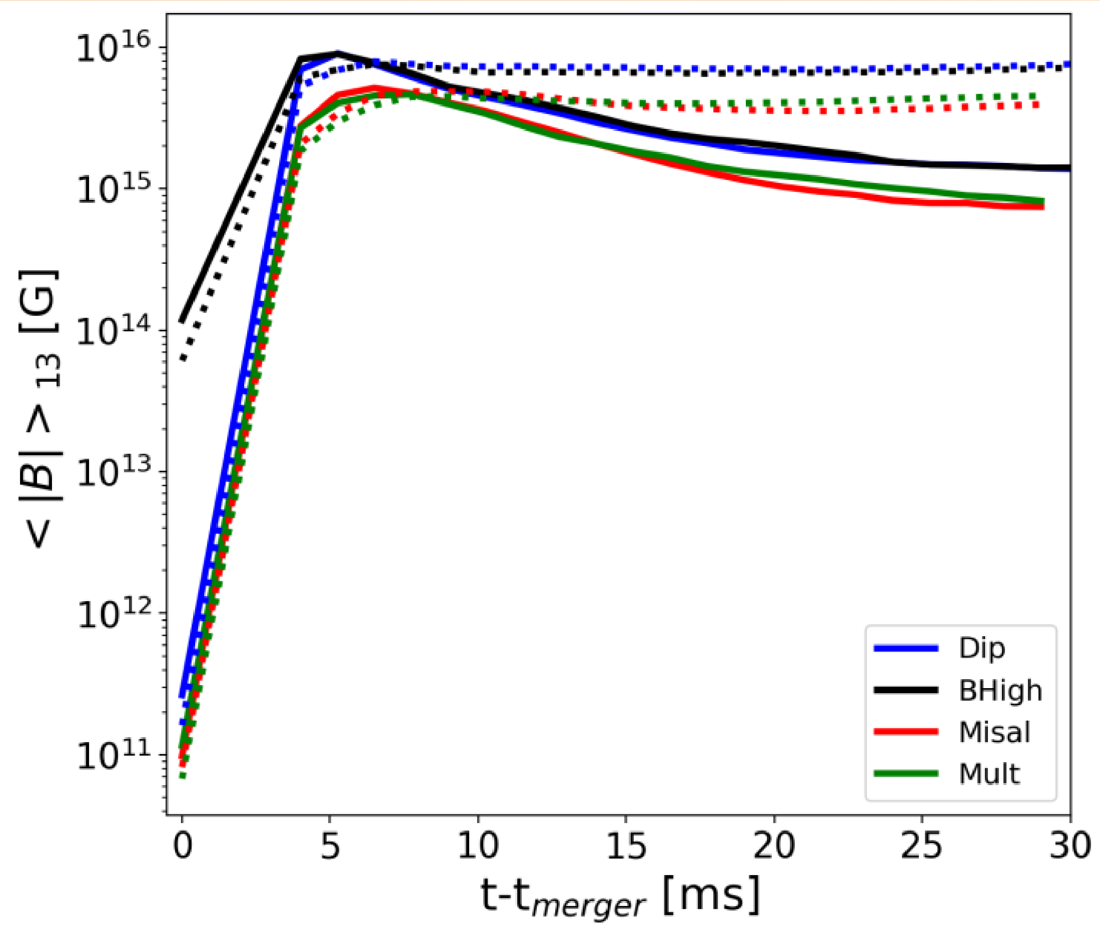


2.7. Importance of the magnetic field topology

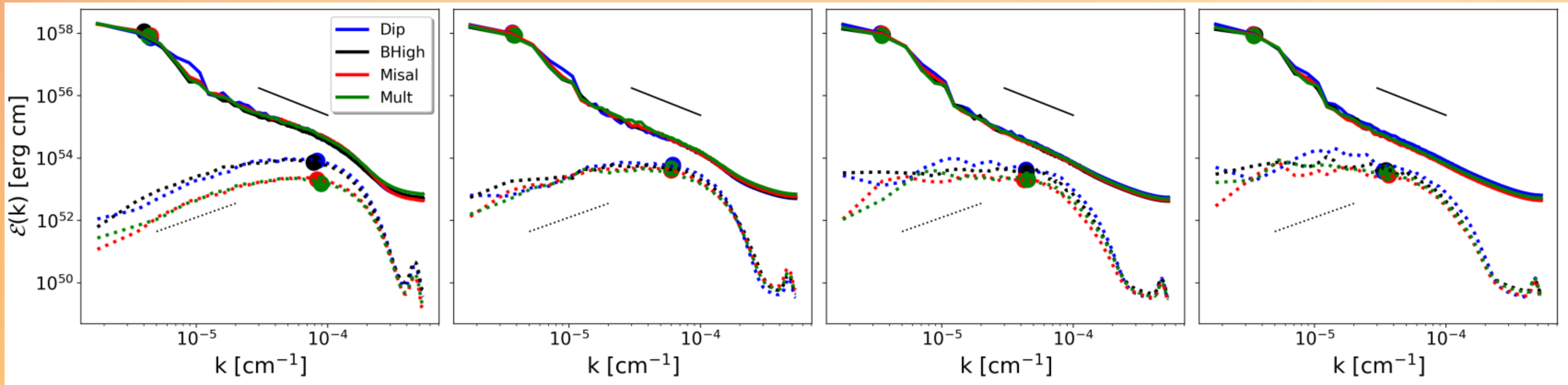
[Aguilera-Miret+ 2022]



Comparable averaged magnetic fields!!

2.7. Importance of the magnetic field topology

[Aguilera-Miret+ 2022]



t = 5 ms **t = 10 ms** **t = 20 ms** **t = 30 ms**

Comparable magnetic field spectra (both toroidal and poloidal part)!!

2.8. Conclusions

1. Average magnetic fields are amplified <B>~10^{11} G → **10^{16} G** in t < 5 ms after merger (bulk) by the KHI. The winding up effect change the magnetic spectra after the KHI from Kazantsev (3/2) to a ±9/2 power law in the equipartition point (located at ~3.5 km)

2. **Our results do not imply that one can effectively model the exponential amplification produced during the KHI by starting with a strong, large-scale, poloidal magnetic field.** In that case, the final state might be already contaminated by the large-scale magnetic field, leading to an accelerated growth due to the winding mechanism and ignoring completely the dominant, small-scale structures. The posterior evolution might be, at best, shifted in time with respect to the correct one. In the worst case, the non-linear dynamics might produce unrealistic results (i.e., like the early production of jets when there should be none, since they are facilitated by large-scale magnetic fields). **In the absence of enough numerical resolution, the use of strong magnetic fields could be physically acceptable IF AND ONLY IF their topology is dominated by an axisymmetric toroidal component with highly turbulent homogeneous perturbations, as seen for the saturation state after the KHI phase.**

3. **The initial magnetic field strength and topology DOES NOT MATTER at all…** as long as you can resolve the KHI that causes a turbulent amplification of the magnetic field. The turbulent magnetic field is isotropic and **erases any dependence on the initial magnetic field topology and strength.**

4. The formulation is general and **can be applied to BNS post-merger or any scenario where the small scales are important**

2.8. Conclusions

1. Average magnetic fields are amplified <B>~10^{11} G → **10^{16} G** in t < 5 ms after merger (bulk) by the KHI. The winding up effect change the magnetic spectra after the KHI from Kazantsev (3/2) to a ±9/2 power law in the equipartition point (located at ~3.5 km)

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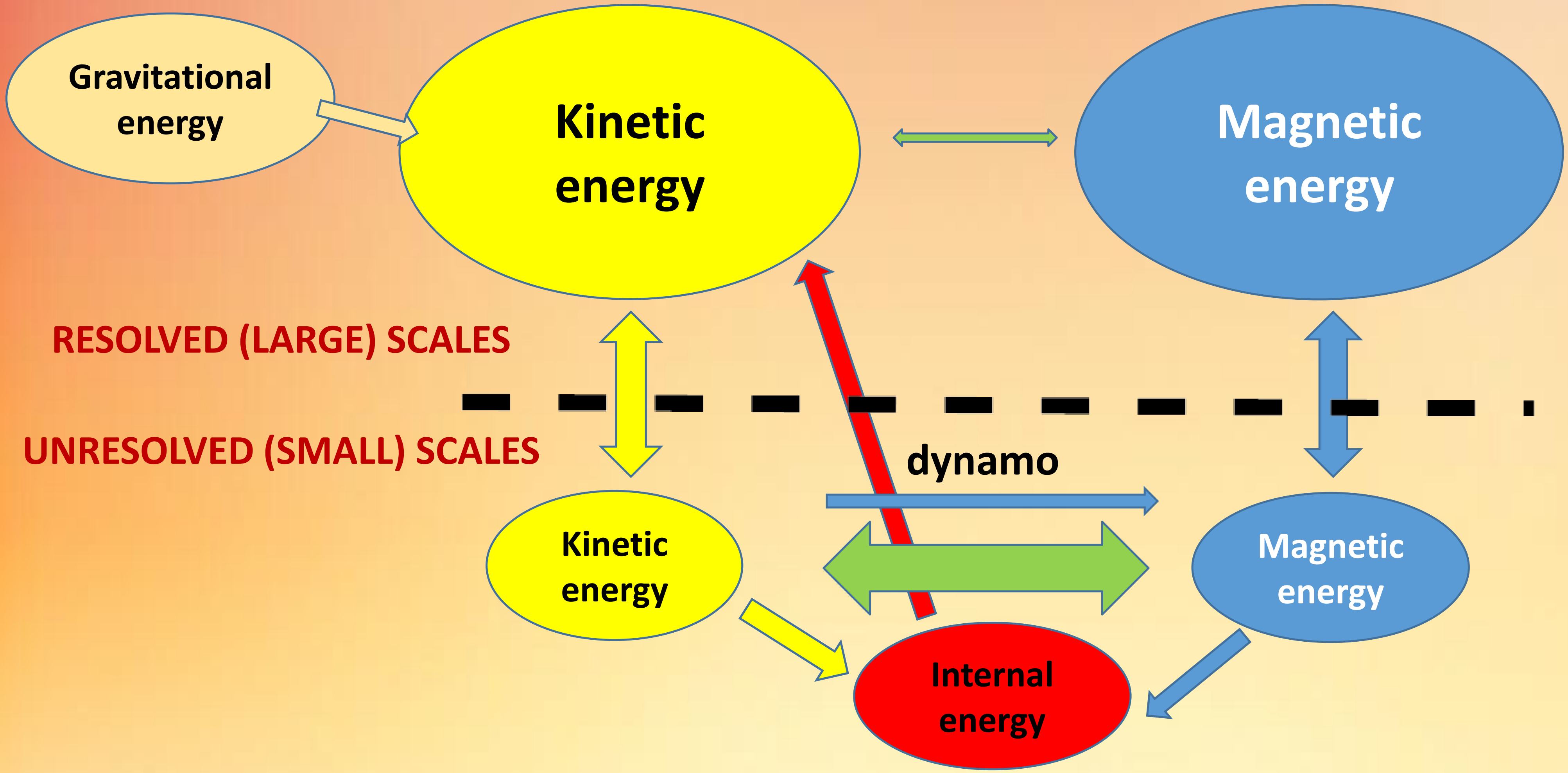
2.1. Filtering



2.3. Compressible non-relativistic MHD evolution equations

[Viganò, R. A-M.+ 2019]

$$\partial_t\bar{\rho} + \partial_k N^k(\tilde{P}) = \partial_k \bar{\tau}_N^k$$
$$\partial_t\bar{N}^i + \partial_k T^{ki}(\tilde{P}) = \partial_k \bar{\tau}_T^{ki}$$
$$\partial_t\bar{U} + \partial_k S^k(\tilde{P}) = \partial_k \bar{\tau}_S^k$$
$$\partial_t\bar{B} + \partial_k M^{ki}(\tilde{P}) = \partial_k \bar{\tau}_M^{ki}$$

$$N^k(\tilde{P}) = \bar{\rho}\tilde{v}^k$$
$$T^{ki}(\tilde{P}) = \tilde{v}^i\tilde{v}^j\bar{\rho} - \bar{B}^i\bar{B}^j + \delta^{ij}\left(\tilde{p} + \frac{\bar{B}^2}{2}\right)$$
$$S^k(\tilde{P}) = \left(\bar{U} + \tilde{p} + \frac{\bar{B}^2}{2}\right)\tilde{v}^k - \tilde{v}\cdot\bar{B}\bar{B}^k$$
$$M^{ki}(\tilde{P}) = \tilde{v}^k\bar{B}^i - \tilde{v}^i\bar{B}^k$$

UNKNOWN SFS TERMS

$$\bar{\tau}_N^k = N^k(\tilde{P}) - \overline{N^k(P)}$$
$$\bar{\tau}_T^{ki} = T^{ki}(\tilde{P}) - \overline{T^{ki}(P)}$$
$$\bar{\tau}_S^k = S^k(\tilde{P}) - \overline{S^k(P)}$$
$$\bar{\tau}_M^{ki} = M^{ki}(\tilde{P}) - \overline{M^{ki}(P)}$$

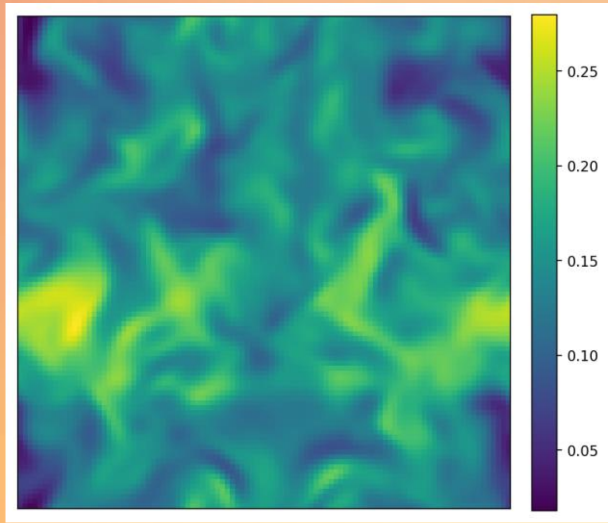
$$\tau_{\mathrm{N}} = 0$$
$$\tau_T^{ki} = \tau_{kin}^{ki} - \tau_{mag}^{ki} + \delta^{ki}\tau_{pres}$$
$$\tau_S^k = \tau_{ener}^k + \tilde{v}_{pres}^{k\tau}$$
$$\tau_M^{ki} = \tau_{ind}^{ki}$$

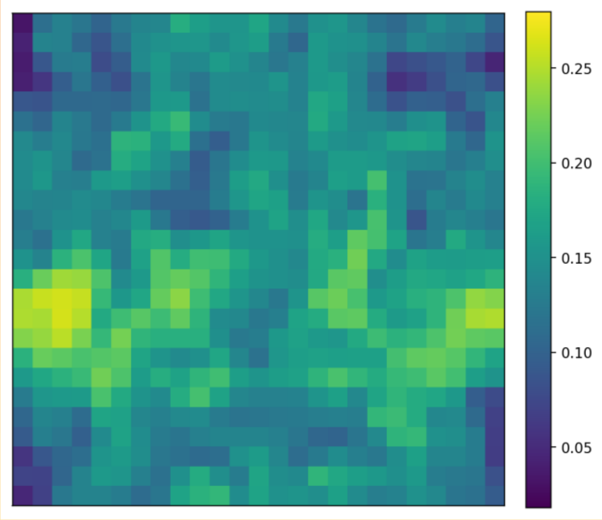
GRADIENT SGS MODEL TERMS

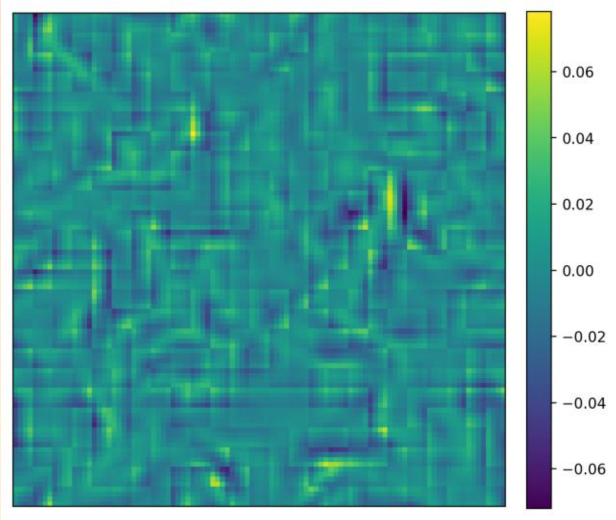
$$\tau_{kin}^{ki} = -2\xi\bar{\rho}\nabla\tilde{v}^k\cdot\nabla\tilde{v}^i$$
$$\tau_{mag}^{ki} = -2\xi\nabla\bar{B}^k\cdot\nabla\bar{B}^i$$
$$\tau_{pres}^{ki} = -\xi\left[\nabla\frac{d\tilde{p}}{d\tilde{\rho}}\cdot\nabla\bar{\rho} + \nabla\frac{d\tilde{p}}{d\tilde{\epsilon}}\cdot\nabla\bar{\epsilon} - \frac{2}{\tilde{\rho}}\frac{d\tilde{p}}{d\tilde{\epsilon}}\nabla\tilde{\rho}\cdot\nabla\tilde{\epsilon} + \nabla\bar{B}_j\cdot\nabla\bar{B}^j - \frac{1}{\tilde{\rho}}\frac{d\tilde{p}}{d\tilde{\epsilon}}\left(\bar{\rho}\nabla\tilde{v}_j\cdot\nabla\tilde{v}^j + \nabla\bar{B}_j\cdot\nabla\bar{B}^j\right)\right]$$
$$\tau_{ener}^k = -2\xi\left[\nabla\tilde{\Theta}\cdot\nabla\tilde{v}^k + \left(\bar{B}^k\bar{B}_j\nabla\tilde{v}^j - \tilde{\Theta}\nabla\tilde{v}^k\right)\cdot\nabla(\ln\tilde{\rho}) - \bar{B}^k\nabla\bar{B}_j\cdot\nabla\tilde{v}^j - \nabla(\tilde{v}\cdot\bar{B})\cdot\nabla\bar{B}^k\right]$$
$$\boldsymbol{\tau_{ind}^{ki} = -4\xi\left[\nabla\tilde{v}^{[k}\cdot\nabla\bar{B}^{i]} + \bar{B}^{[i}\nabla\tilde{v}^{k]}\cdot\nabla(\ln\bar{\rho})\right]}$$

2.3. Compressible non-relativistic MHD evolution equations

[Viganò, R. A-M.+ 2019]







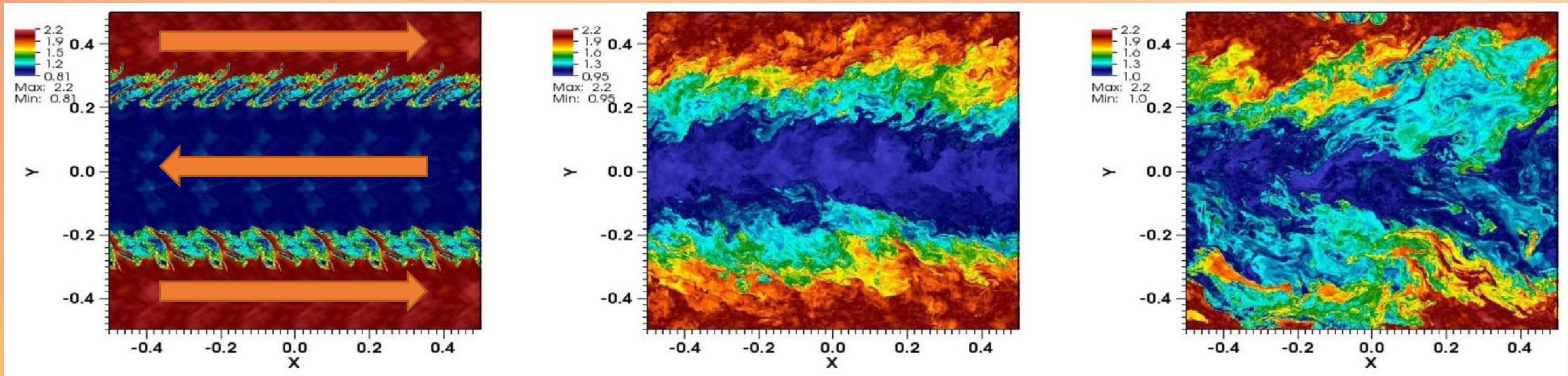
$$u(\vec{x},t) = \bar{u}(\vec{x},t) + u'(\vec{x},t)$$

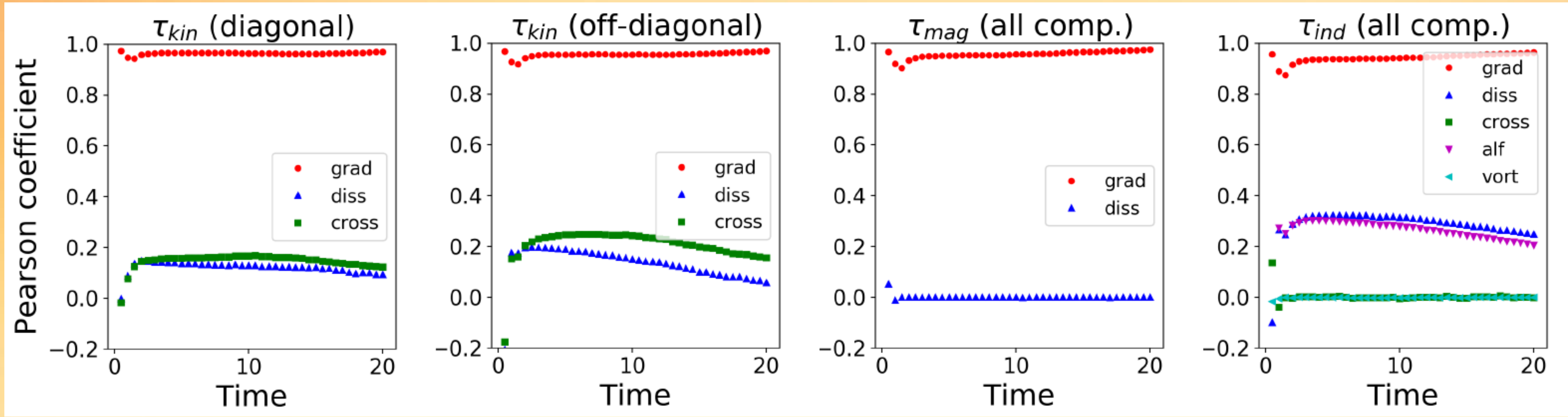
$$\bar{\tau} = \bar{u}\bar{u} - \overline{uu}$$

$$\mathcal{P} = Corr\{\bar{\tau}^{ki}, \tau^{ki}\} = [-1,1], \quad \mathrm{C}^{\mathrm{ki}}_{\mathrm{best}} = \frac{\Sigma \bar{\tau}^{ki} \tau^{ki}}{\Sigma \left(\tau^{ki}\right)^2}$$

2.3. Compressible non-relativistic MHD evolution equations

[Viganò, R. A-M.+ 2019]

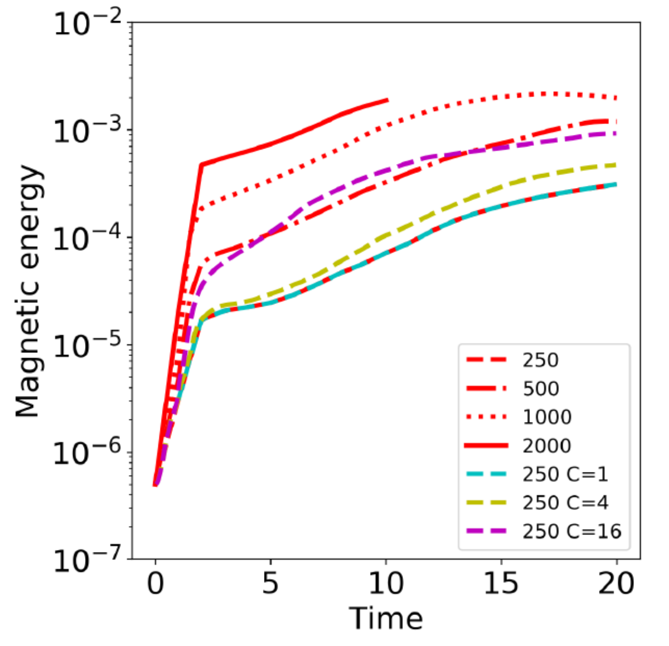


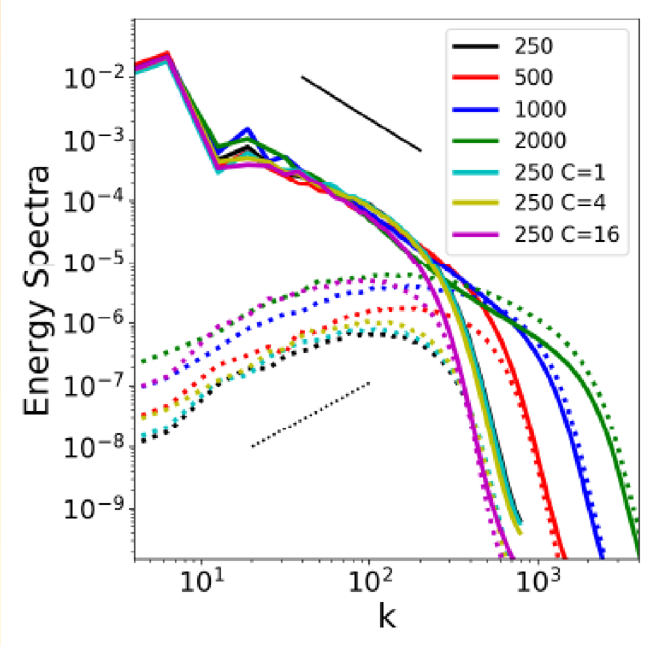


2.3. Compressible non-relativistic MHD evolution equations

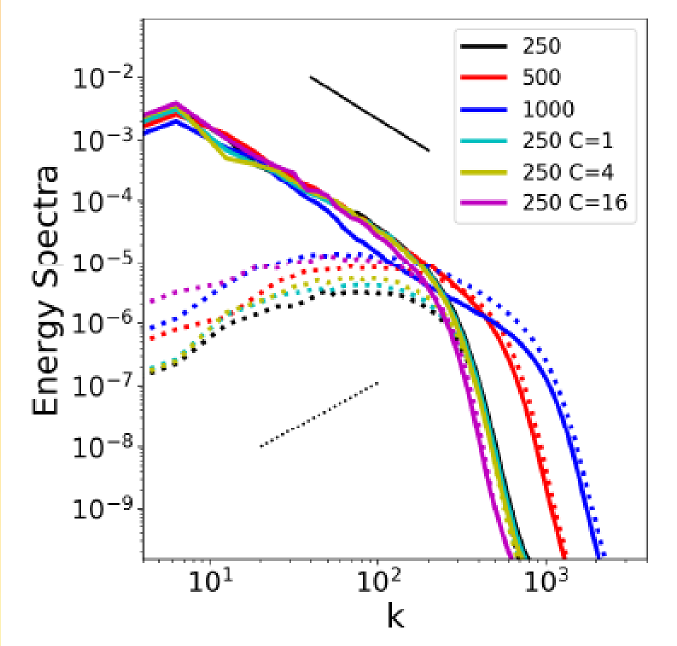
[Viganò, R. A-M.+ 2019]

- Amplification of the magnetic energy with SGS
- Effective resolution x2 in the magnetic energy and x8 in the spectra at low *k*!





t = 10



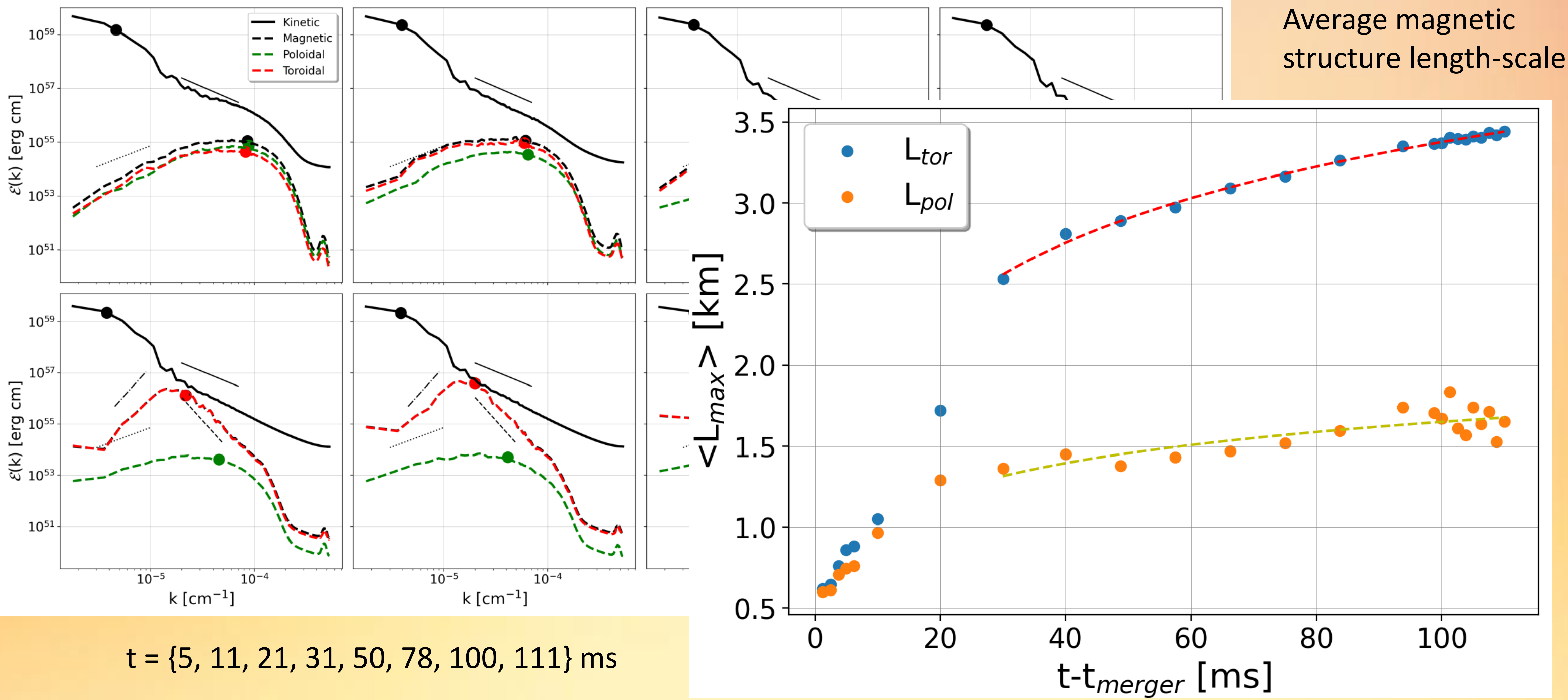
t = 20

Assumptions & Caveats

- **The space-time metric is not "turbulent"**, i.e., the gradient terms arising from metric components in the fluid equations are neglected (verified by a-priori tests under typical conditions)
- Similarly, **the SGS terms arising in the Einstein equations are not included**, i.e., the steepness (derivatives) of MHD fields are dominating the non-linearity of the turbulence.
- The **gradient SGS model can be thought as a reconstruction scheme because mimics the dynamics down to finite "depths"** inside the cell **without assuming any physical dynamics**: if physical dynamics qualitatively differ at much smaller scales, there is nothing one can do.

2.6. Magnetic field evolution

[Aguilera-Miret+ 2023]

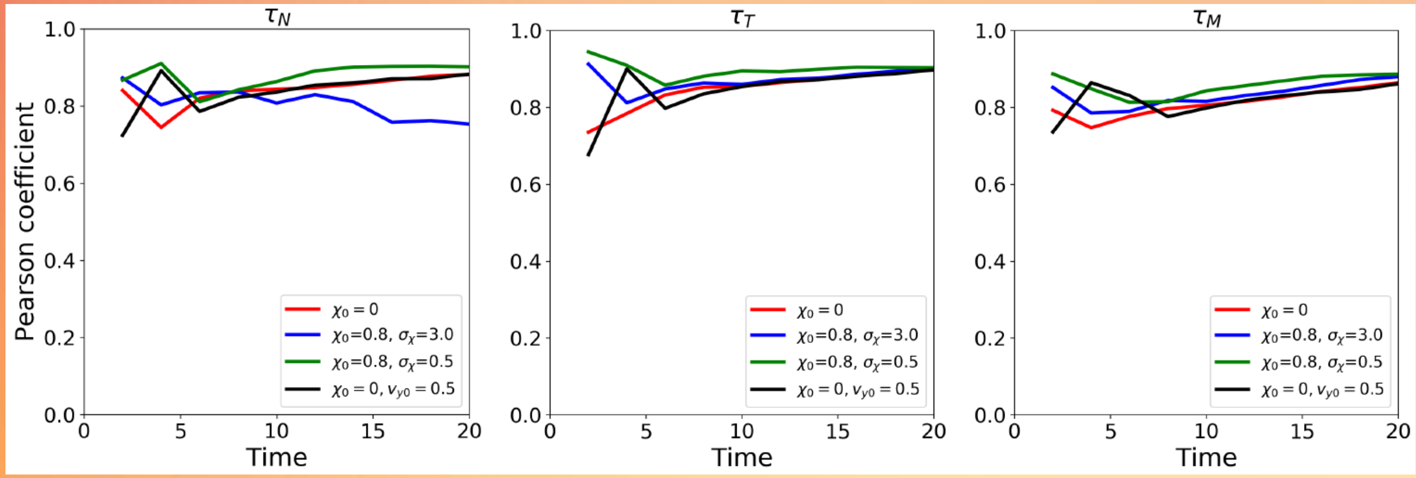


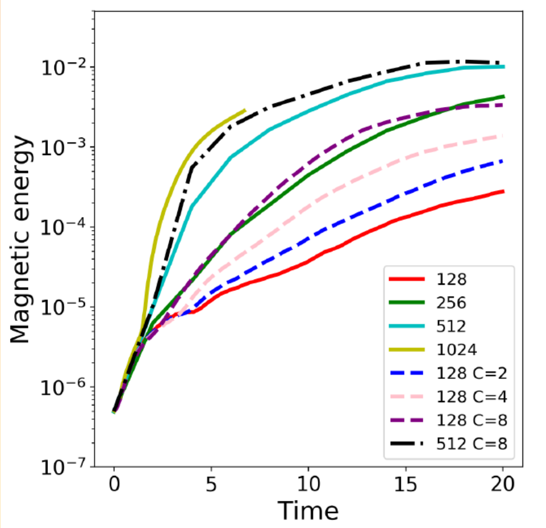
Average magnetic structure length-scale:

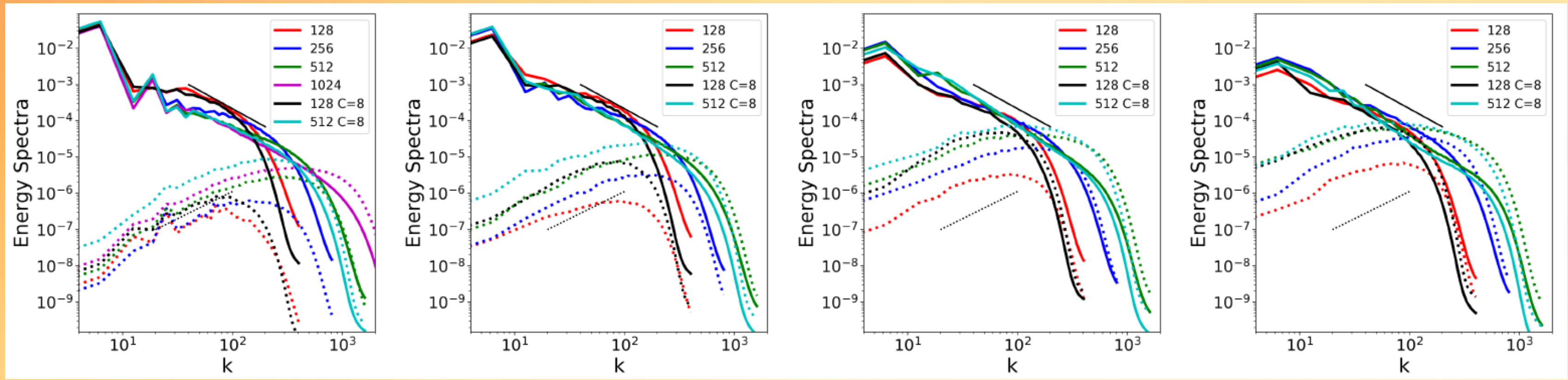
t = {5, 11, 21, 31, 50, 78, 100, 111} ms

2.4. GRMHD evolution equations

[Viganò, R. A-M.+ 2020]





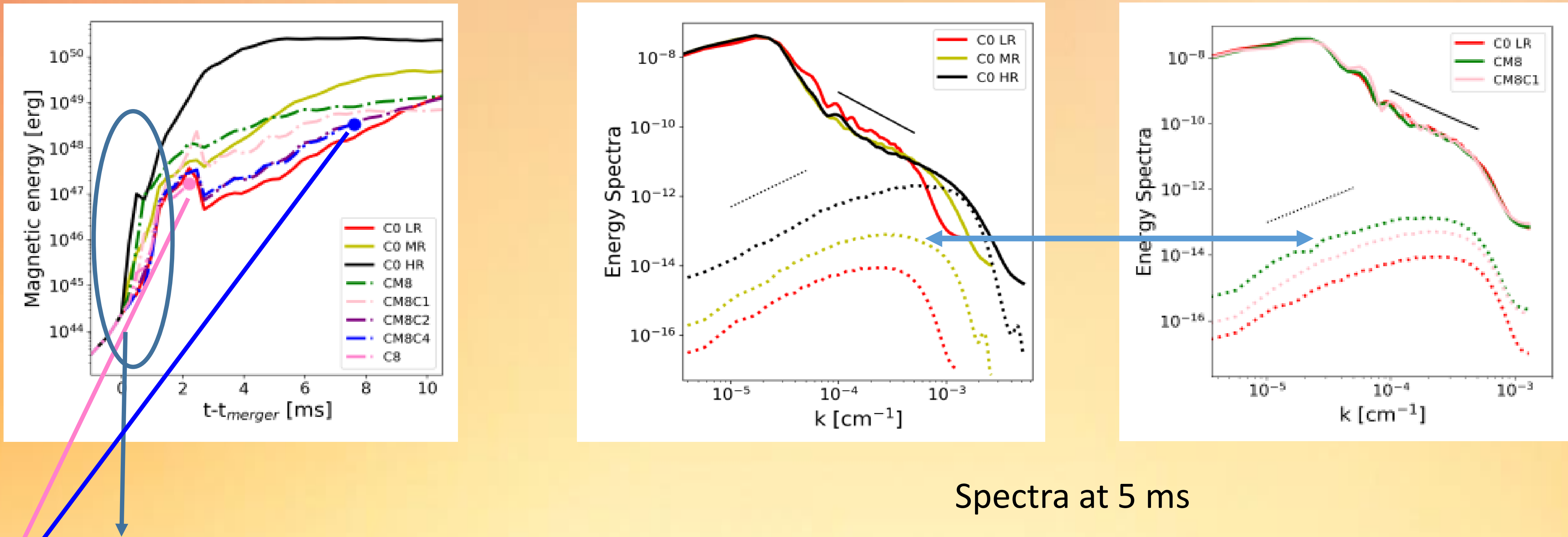


t = 6 t = 10 t = 16 t = 20

Similar results for different resolutions and background metrics

2.5. Effects of LES in BNS mergers

[Aguilera-Miret+ 2020]



Spectra at 5 ms

Effectiveness of SGS terms is evident mostly in the fast amplification phase.

Non-linearity causes resolution/SGS-dependent collapse (adds an extra dissipation).

2.8. Importance of the initial configuration of the NSs?

[Aguilera-Miret+ in prep, 2024]

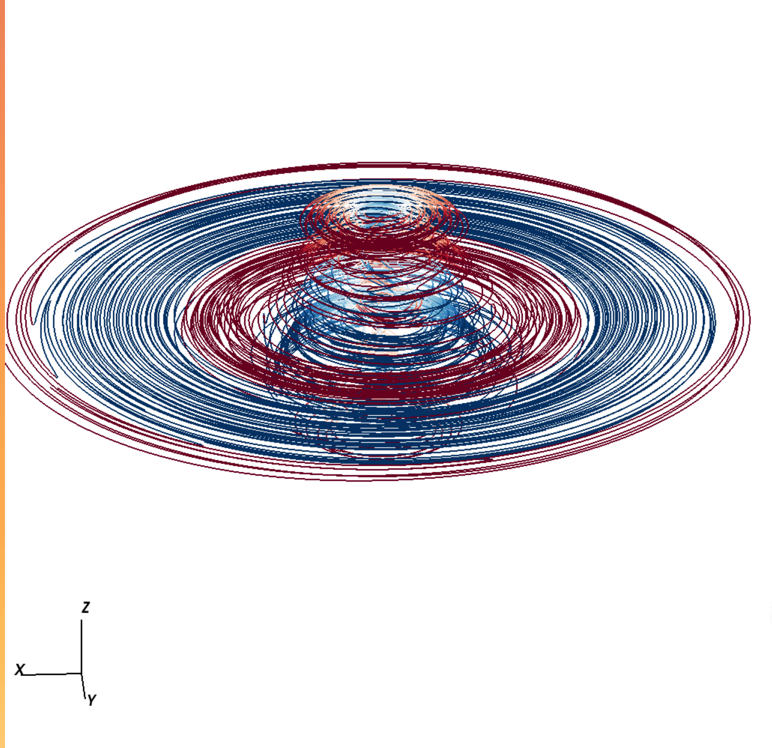
Ongoing work…

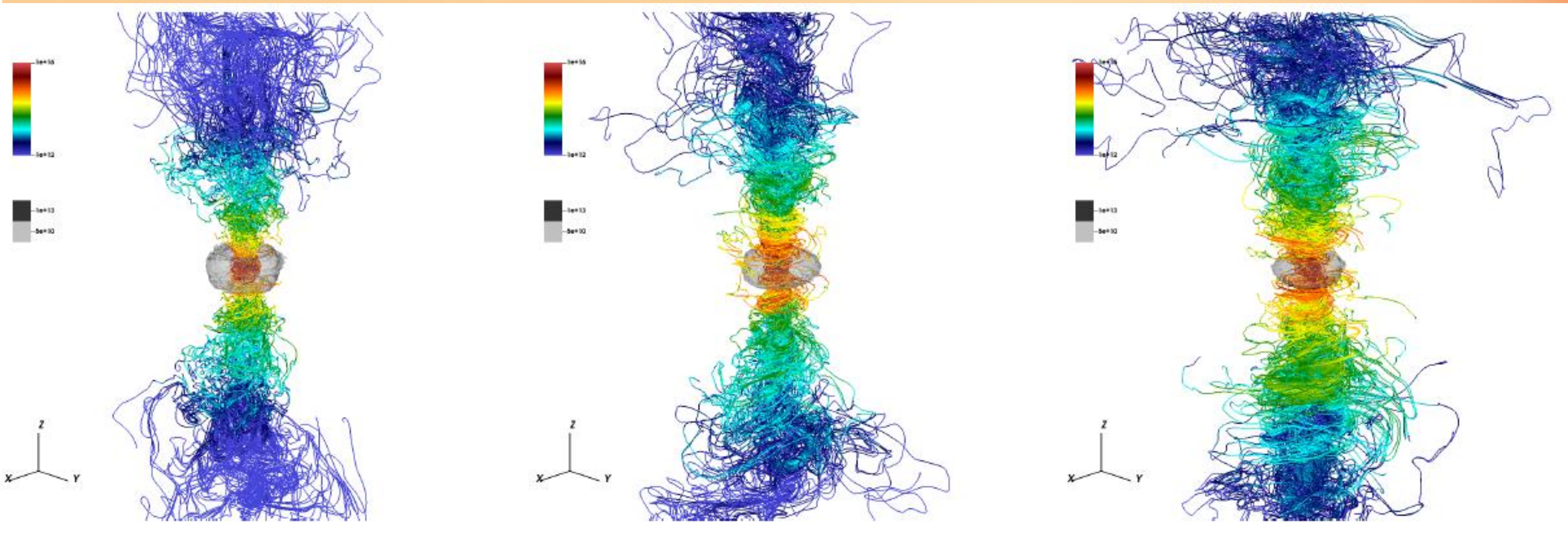
- Is this amplification **independent from the initial data of the neutron stars**?
    - q = 1, total mass = {1.8 - 3.0} Msun ---› **Firsts results seem not to change at all!** ☺
    - q ≠ 1 ---› **Firsts results seem not to change at all!** ☺
    - …

Large Eddy Simulations of BNS mergers

2.6. Magnetic field evolution

[Aguilera-Miret+ 2023]





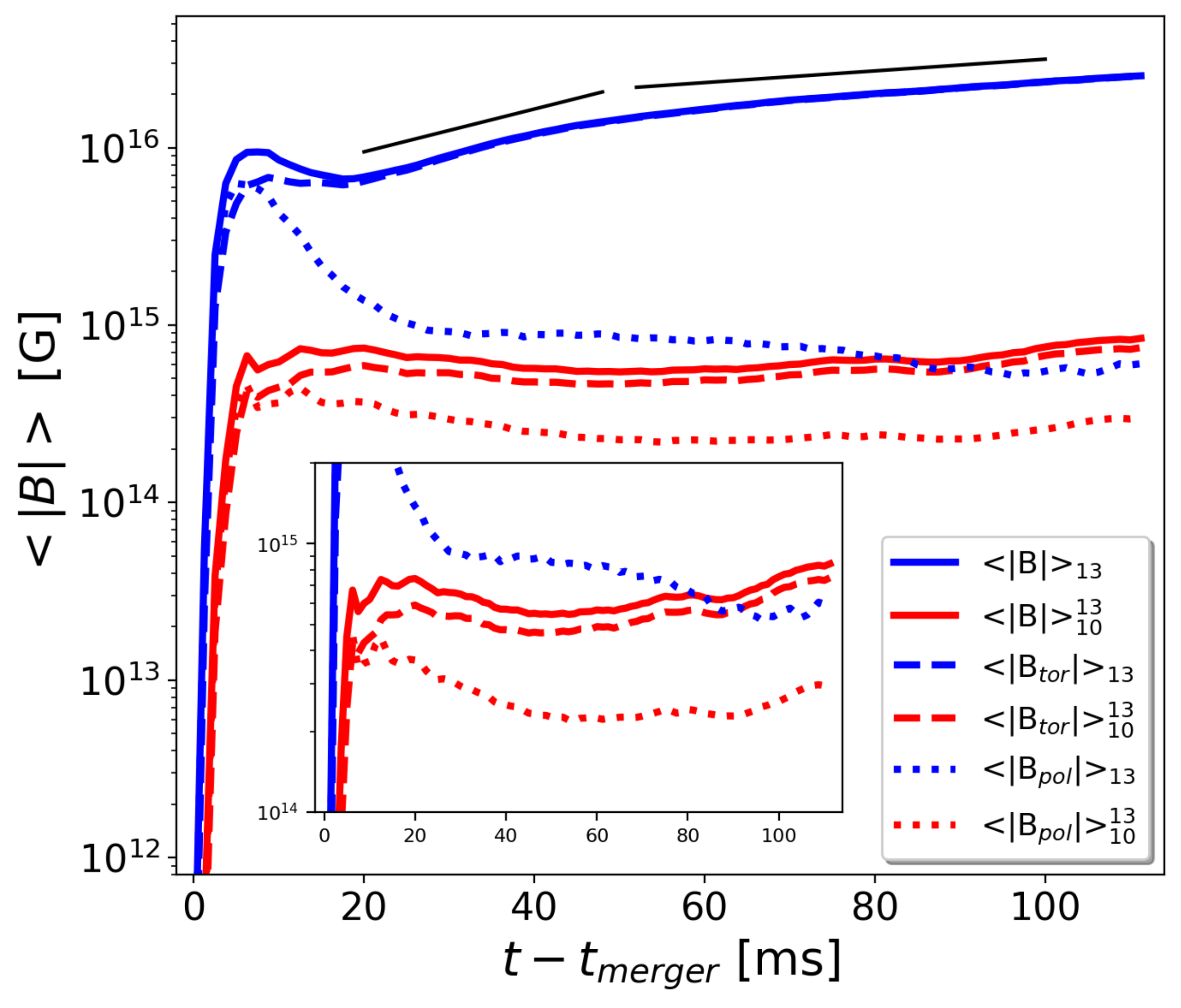
15 ms

50 ms

100 ms

2.6. Magnetic field evolution

[Aguilera-Miret+ 2023]



2.8. Importance of the neutron star's configurations?

[Aguilera-Miret+, in prep. 2024]

Is this universality of the amplification of the magnetic field THAT universal?

2.8. Importance of the neutron star's configurations?

[Aguilera-Miret+, in prep. 2024]

Is this universality of the amplification of the magnetic field THAT universal?

Varying the total mass (keeping q = 1 and q != 1) →