Properties of quark matter in extreme conditions

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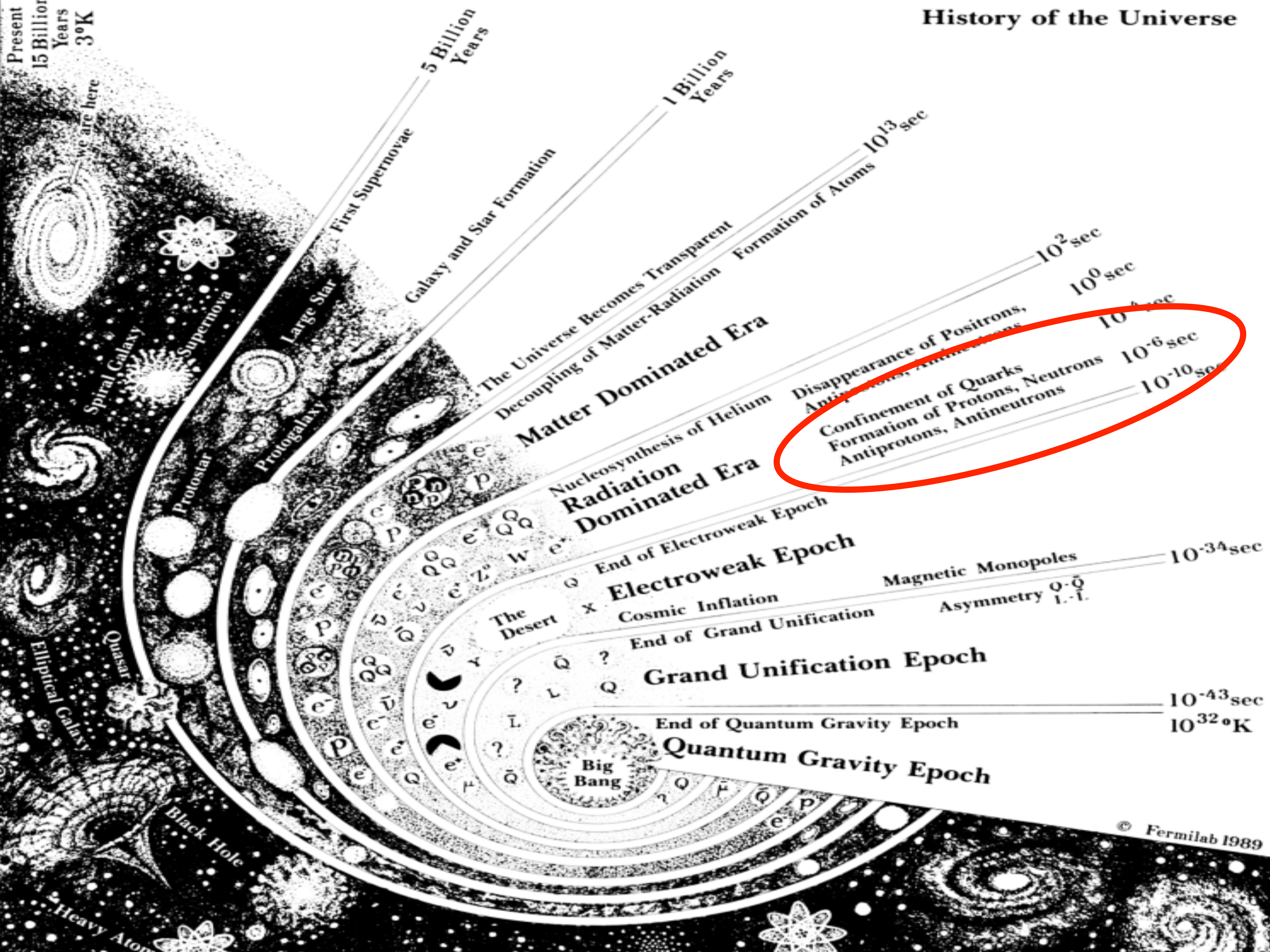
It is always with the best intentions
that the worst work is done

O. Wilde

Outline

- Preliminary remarks
- Phases of quark matter
- Heating, compressing or altering matter
- Conclusions

History of the Universe



$t \sim 10^{-6}s$

Quark hadron transition

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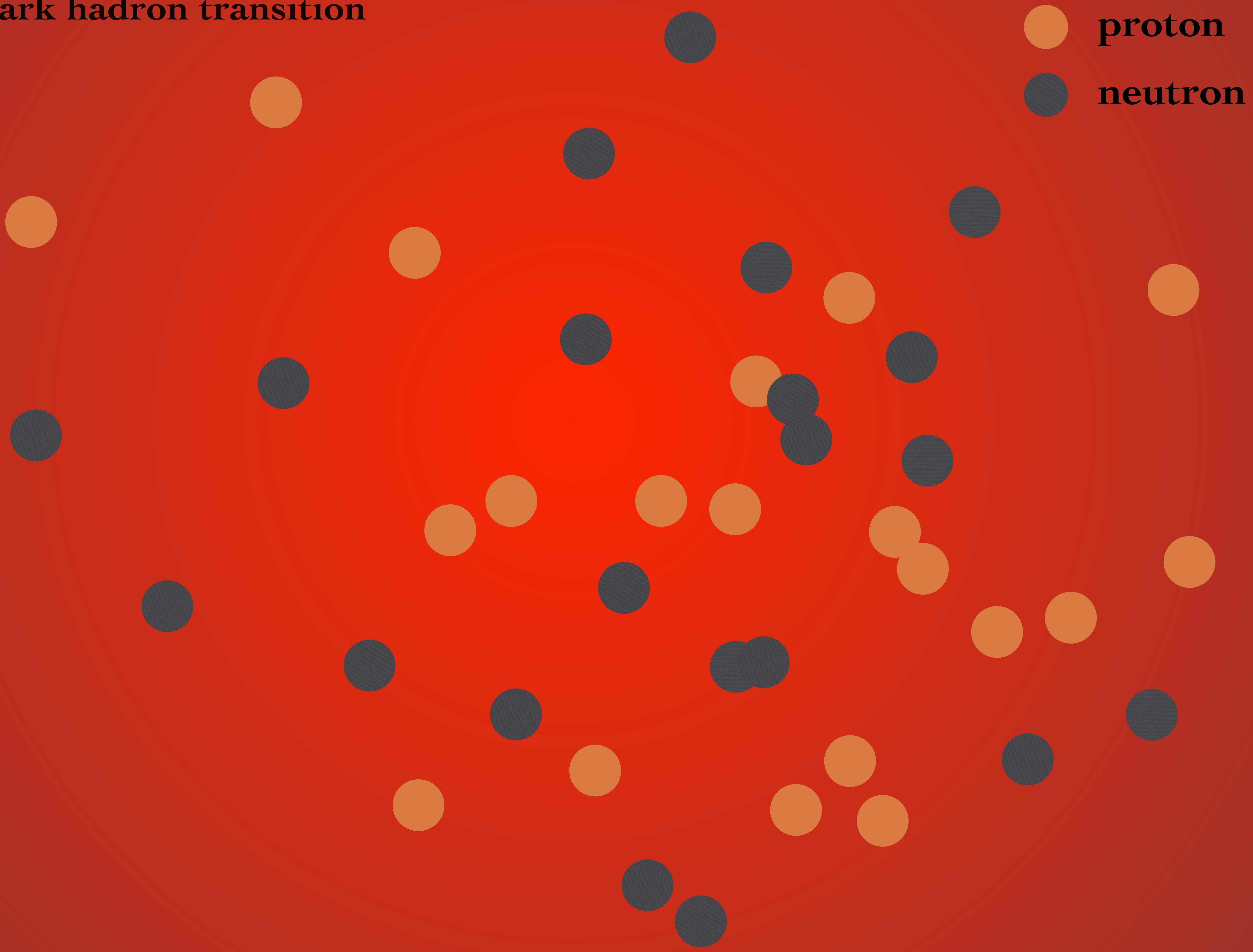

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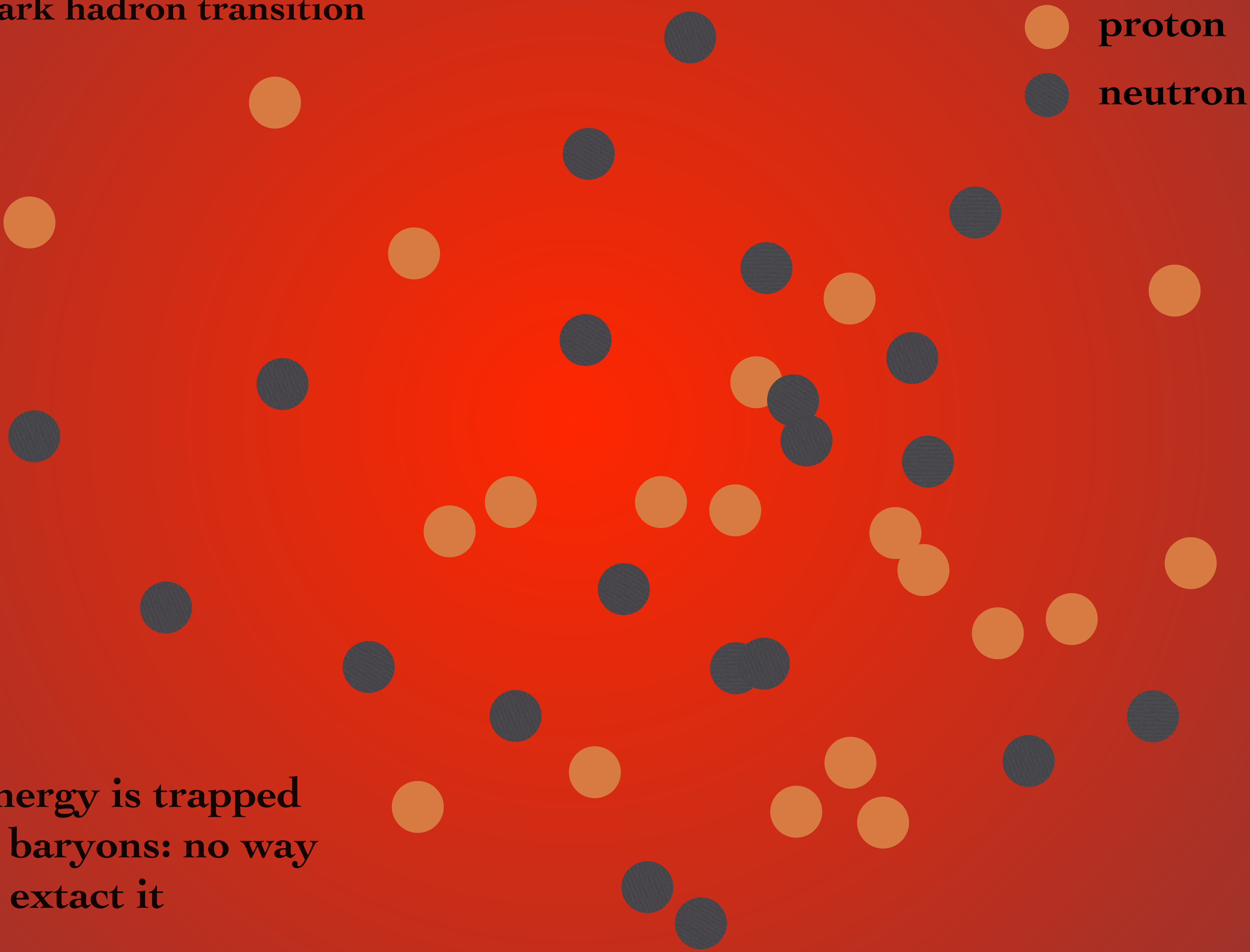


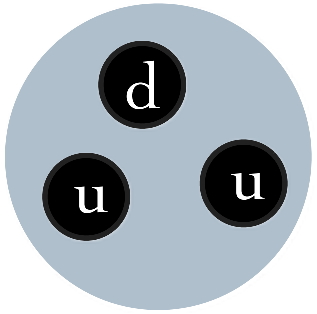
Energy is trapped
in baryons: no way
to extact it

Can we understand this process?

CONFINED HADRONS

BARYONS (fermions)

proton



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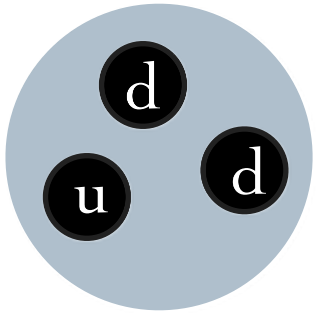

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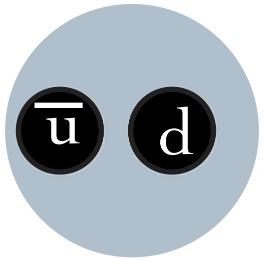
MESONS (bosons)

pions…



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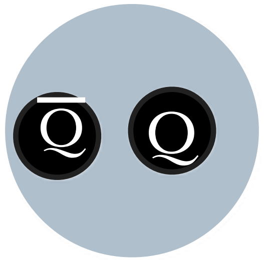

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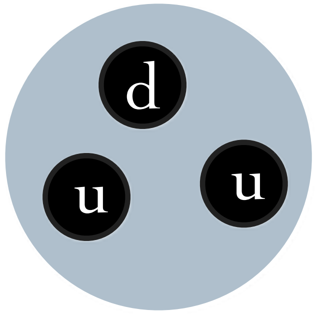

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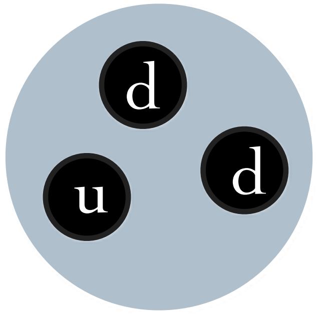

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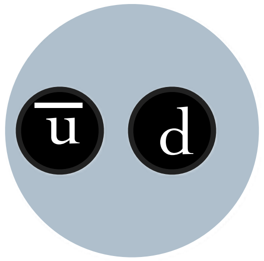

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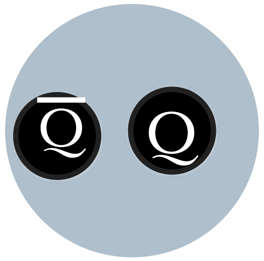

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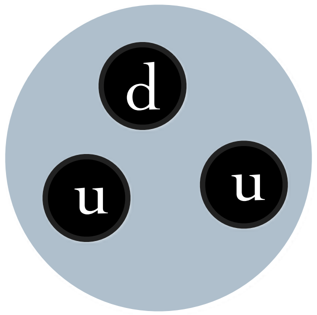

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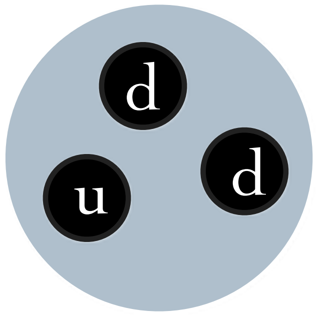

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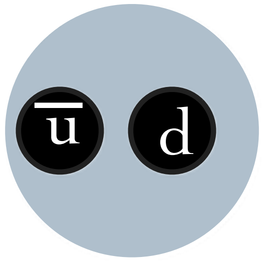

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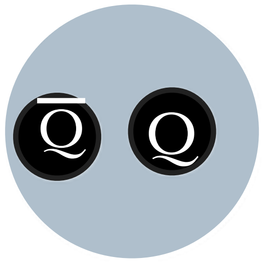

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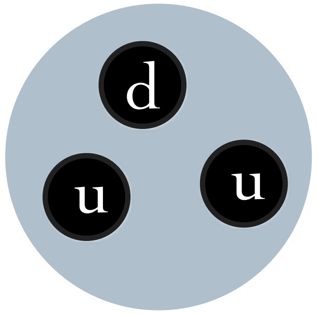

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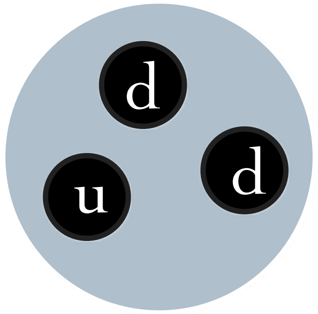

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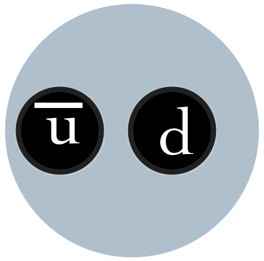

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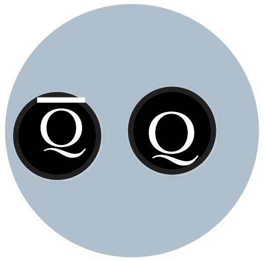

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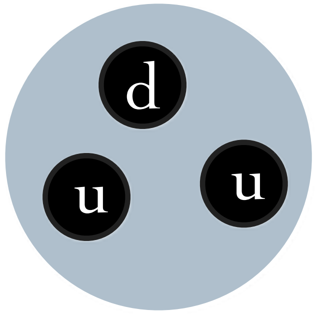

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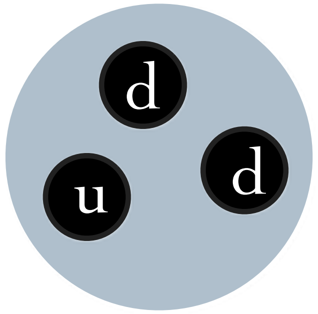

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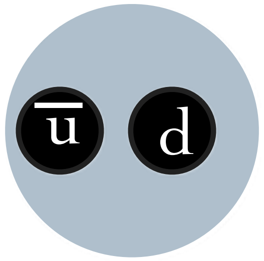

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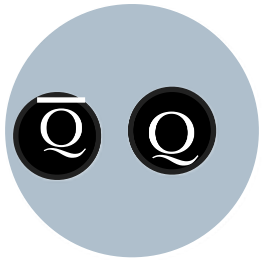

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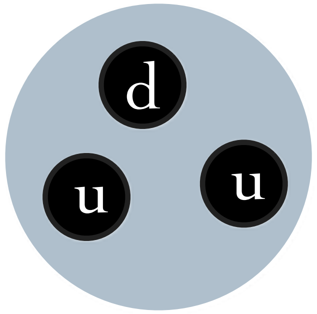

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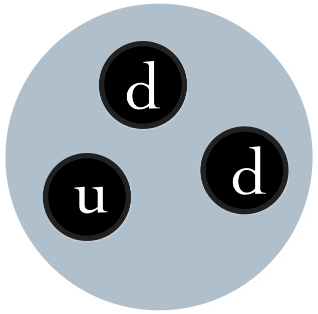

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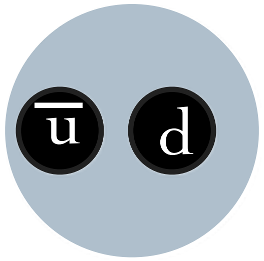

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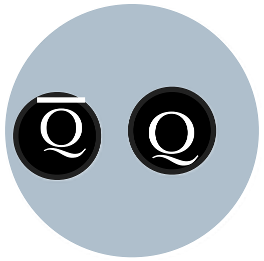

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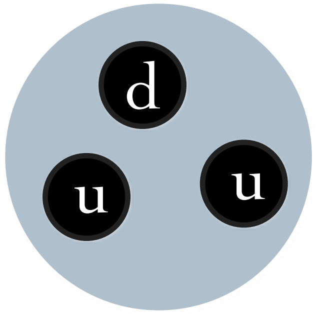

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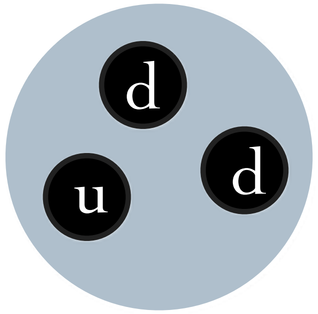

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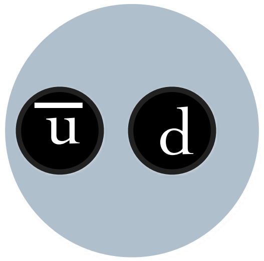

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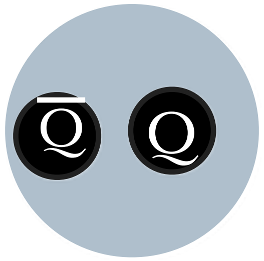

Nonrelativistic object

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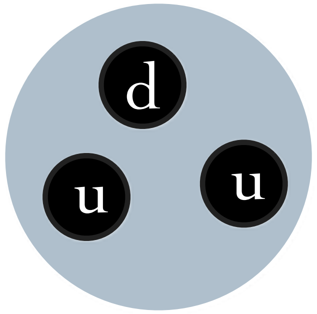

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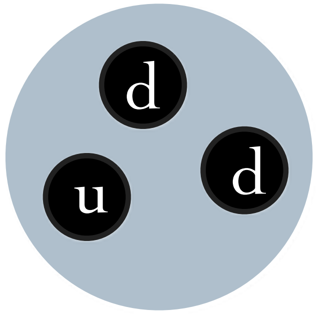

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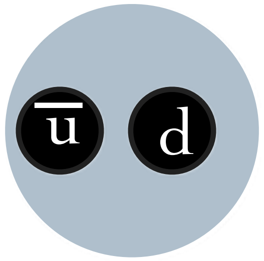

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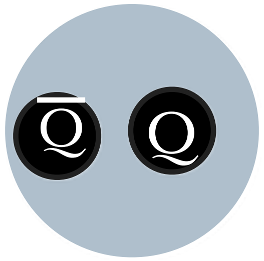
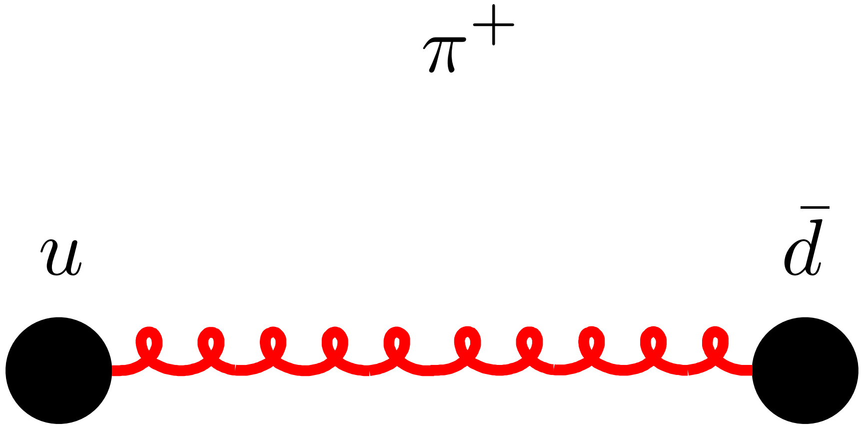


Nonrelativistic object

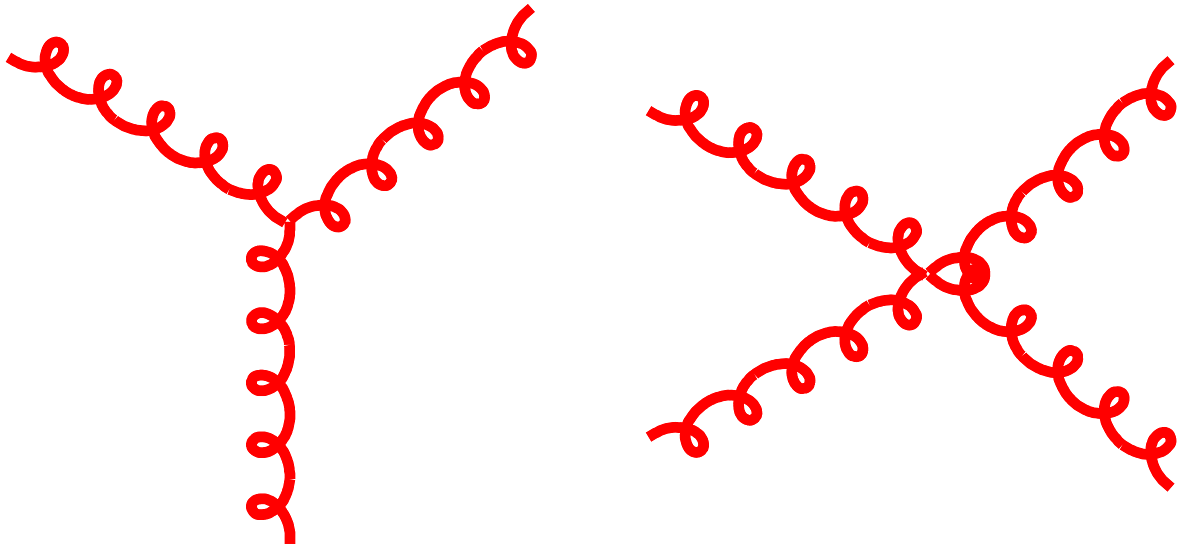
Talk tomorrow by A.Vairo

Quantum Chromodynamics

Interaction between quarks



Interaction between gluons



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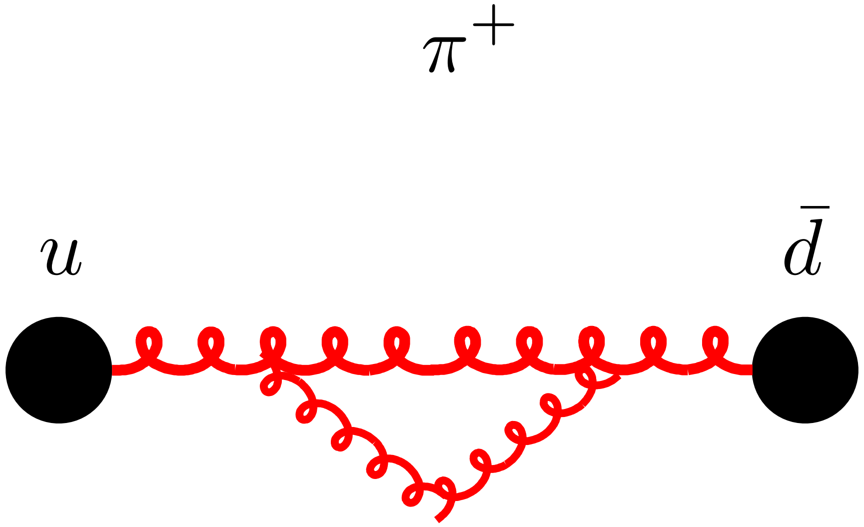

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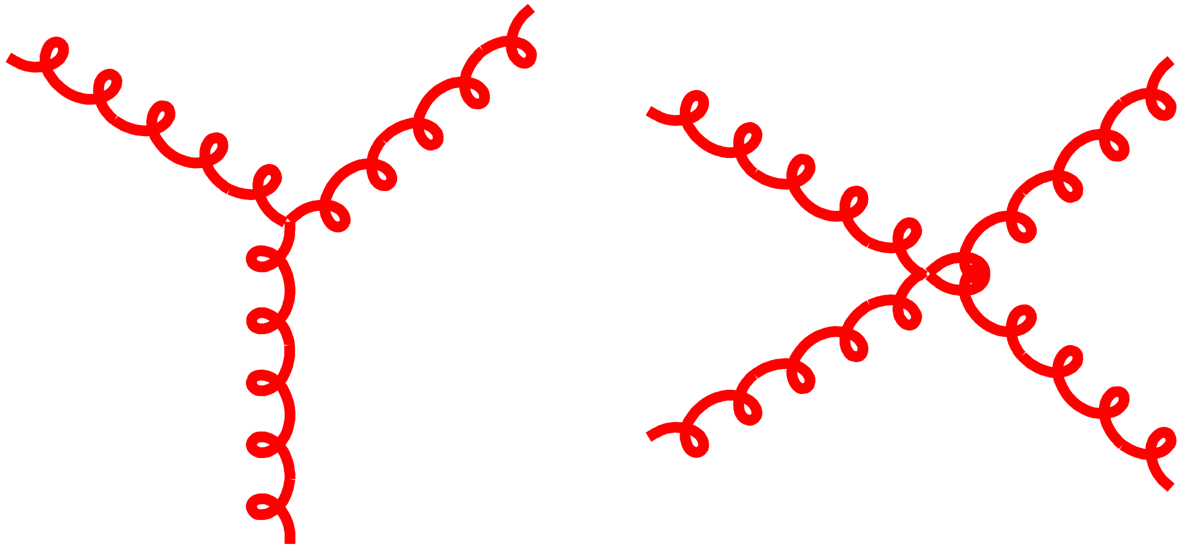

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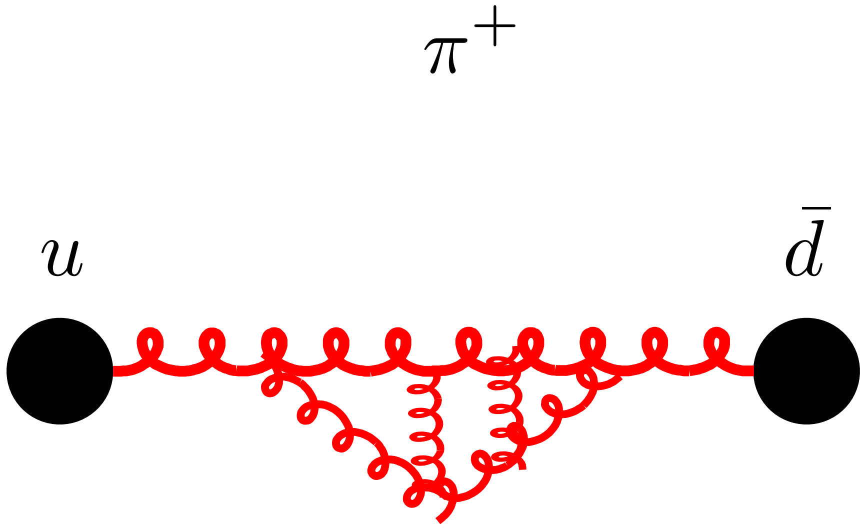

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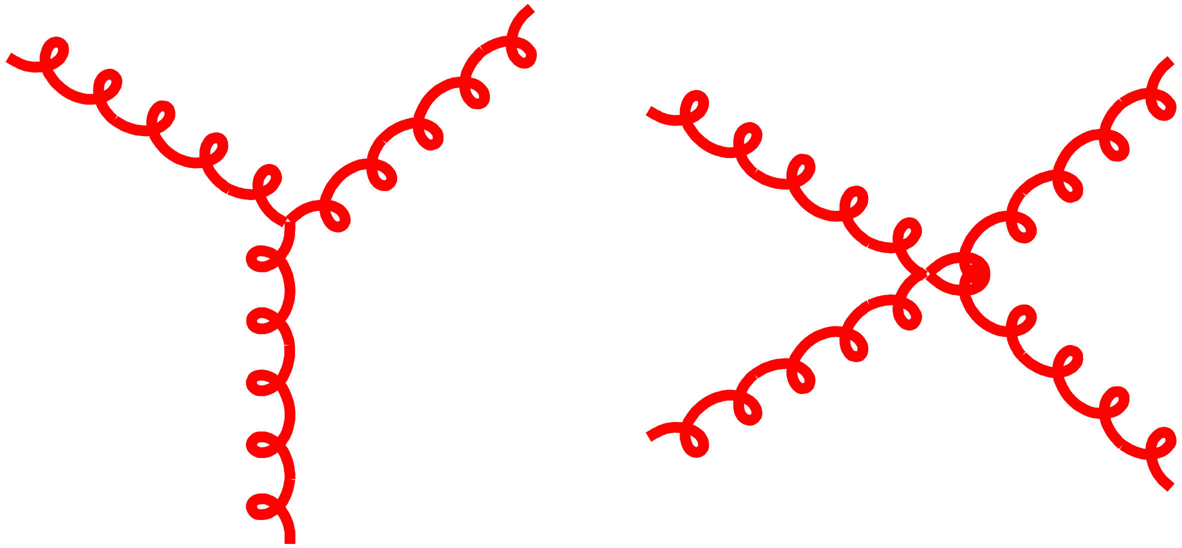

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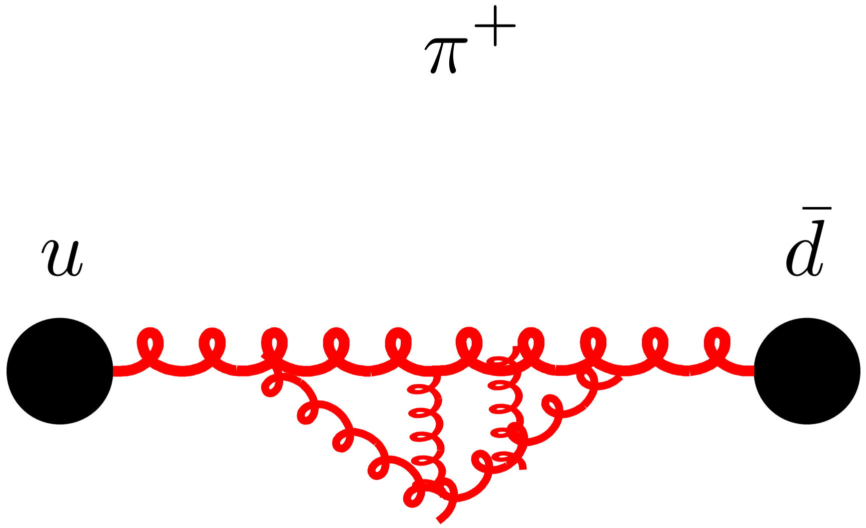

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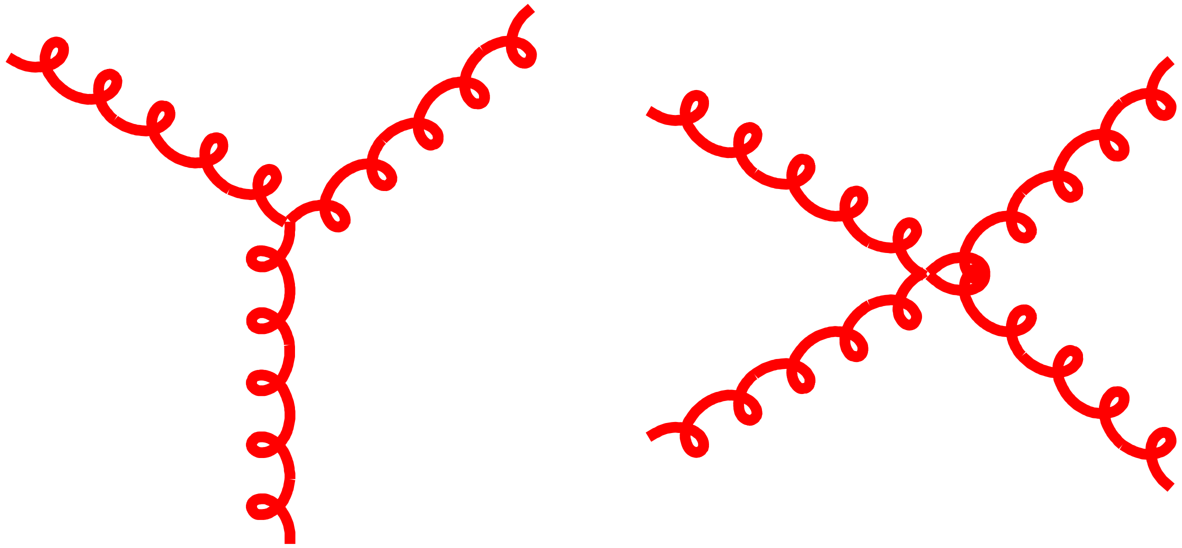
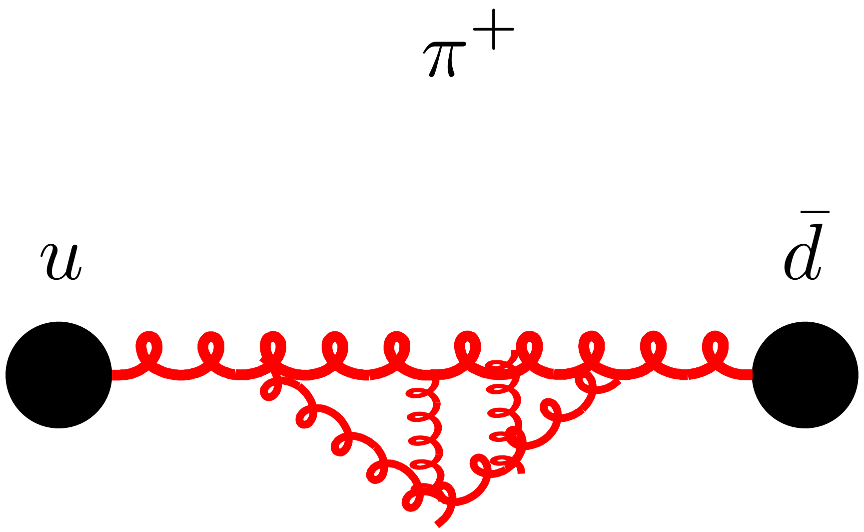


Non-Abelian gauge theory

Quantum Chromodynamics

Interaction between quarks



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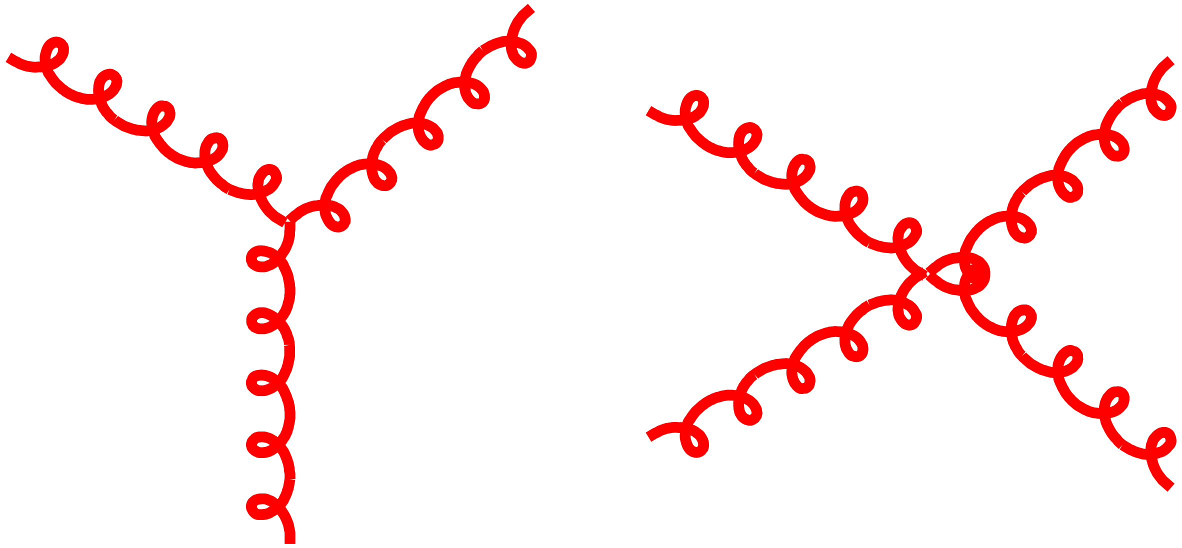
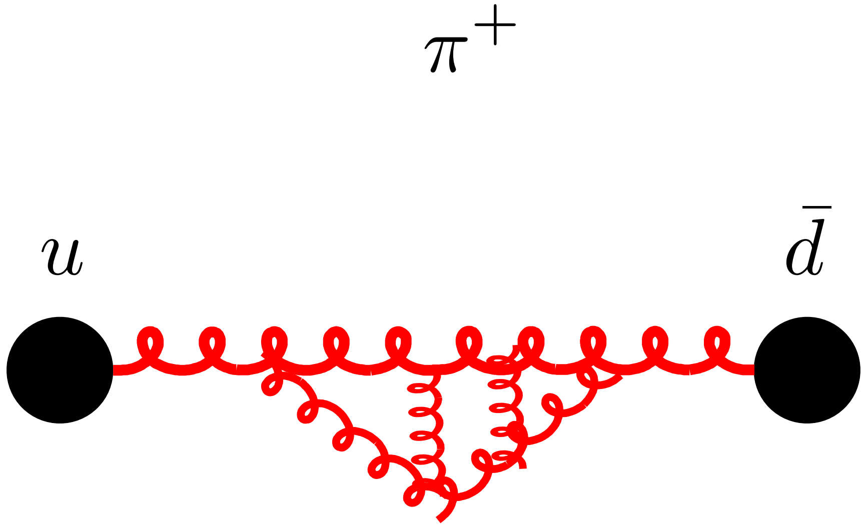


Non-Abelian gauge theory

interactions mediated by gauge bosons (gluons)

Quantum Chromodynamics

Interaction between quarks



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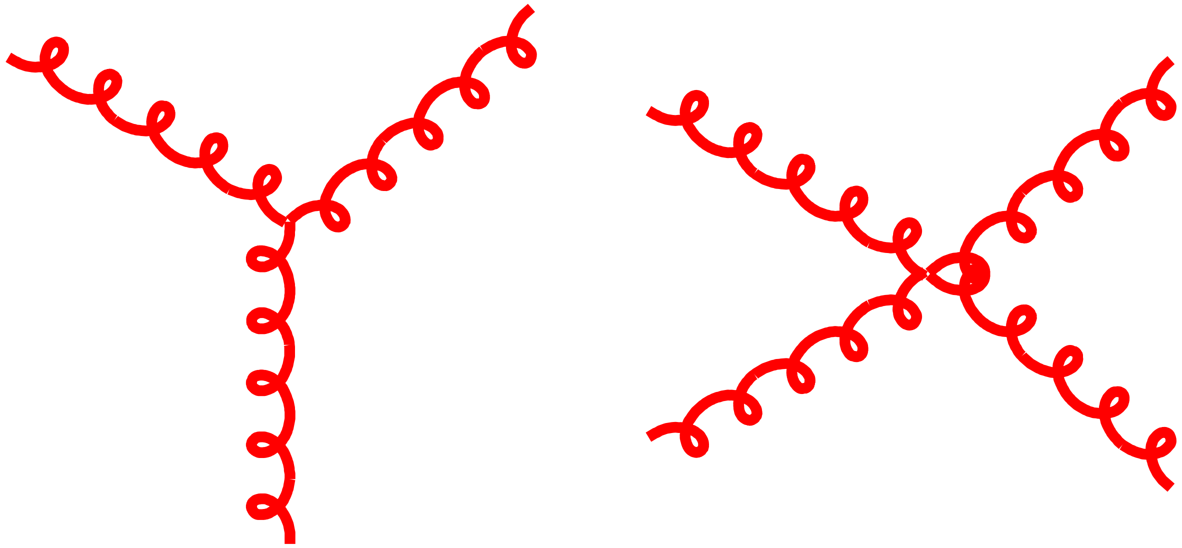


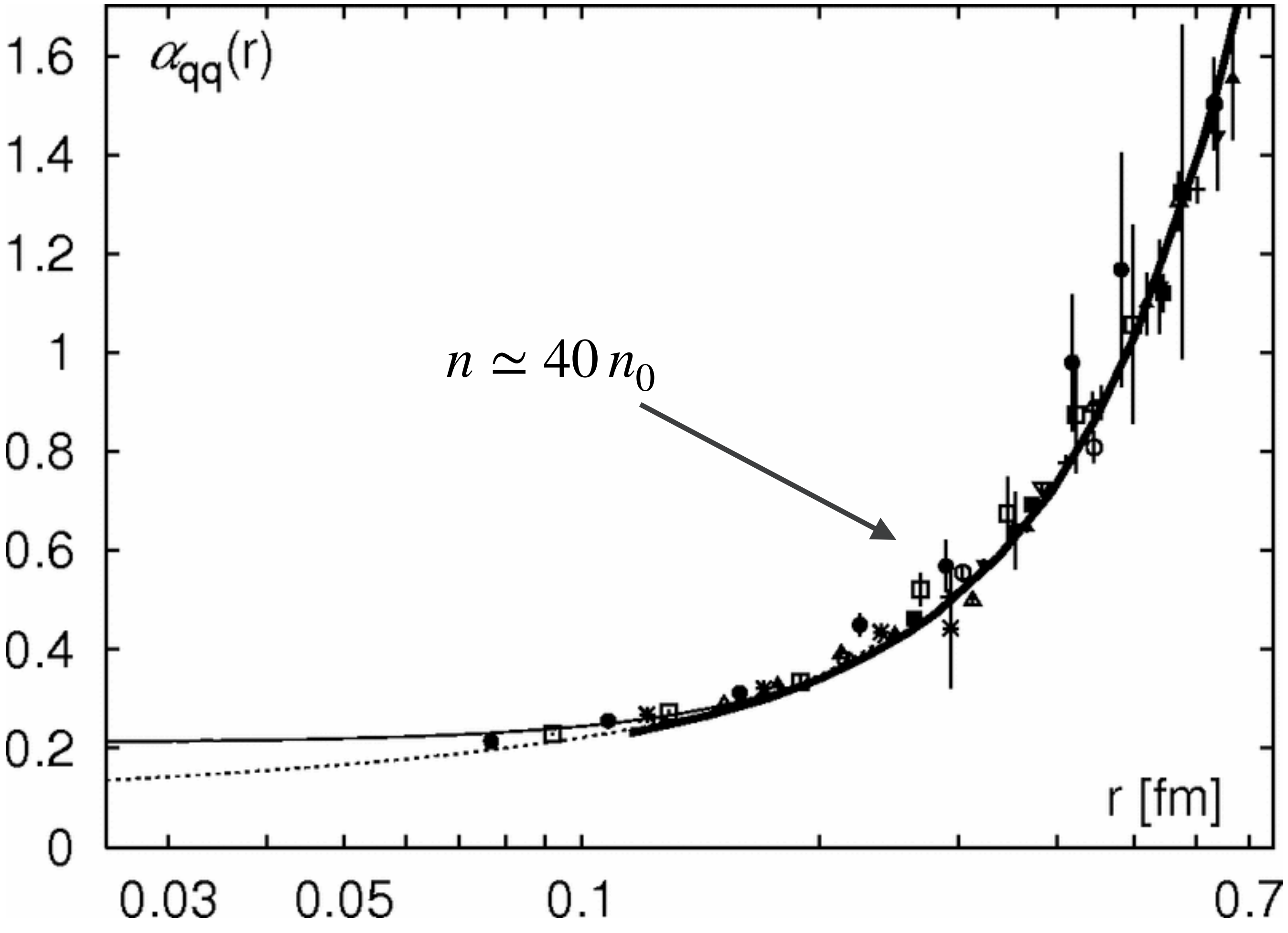
Non-Abelian gauge theory

gluons interact with gluons

interactions mediated
by gauge bosons (gluons)

Modeling the strong interaction

Any description of QCD should have confinement and asymptotic freedom



Kaczmarek and Zantow
Physical Review D 71(11):114510

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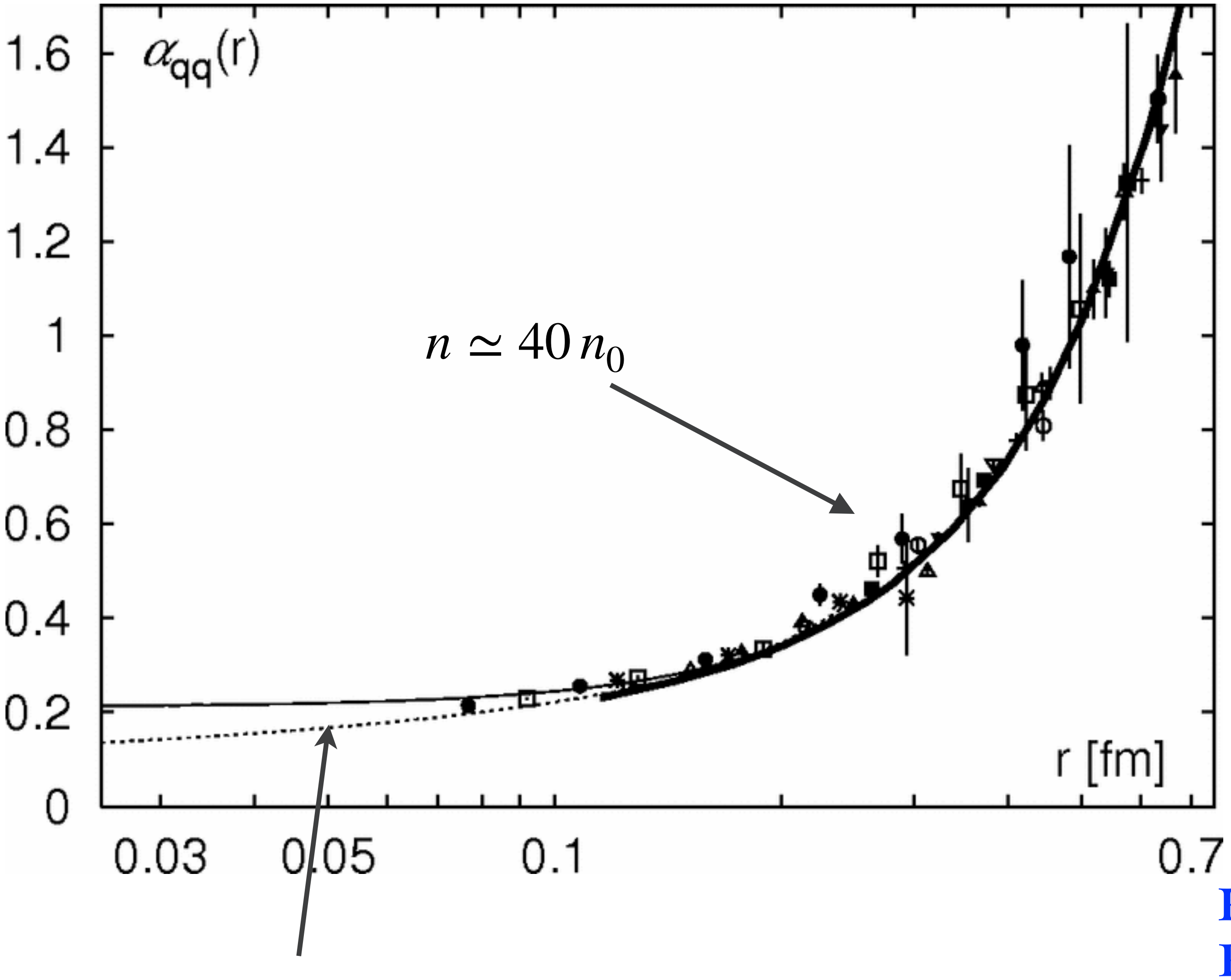

$n \simeq 40\, n_0$

Asymptotic freedom (perturbation theory)

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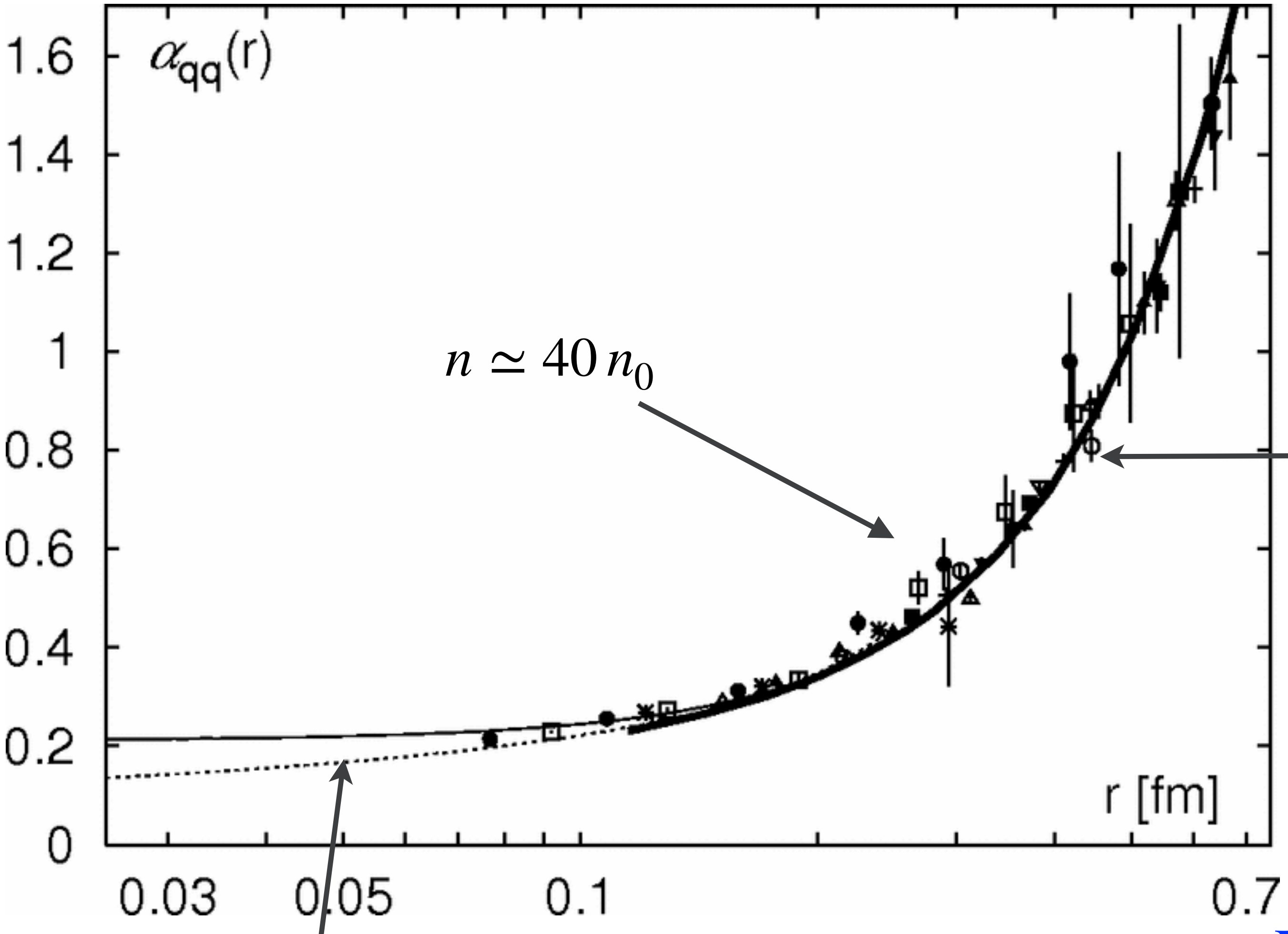

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Strongly interacting

Asymptotic freedom (perturbation theory)

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Modeling the strong interaction

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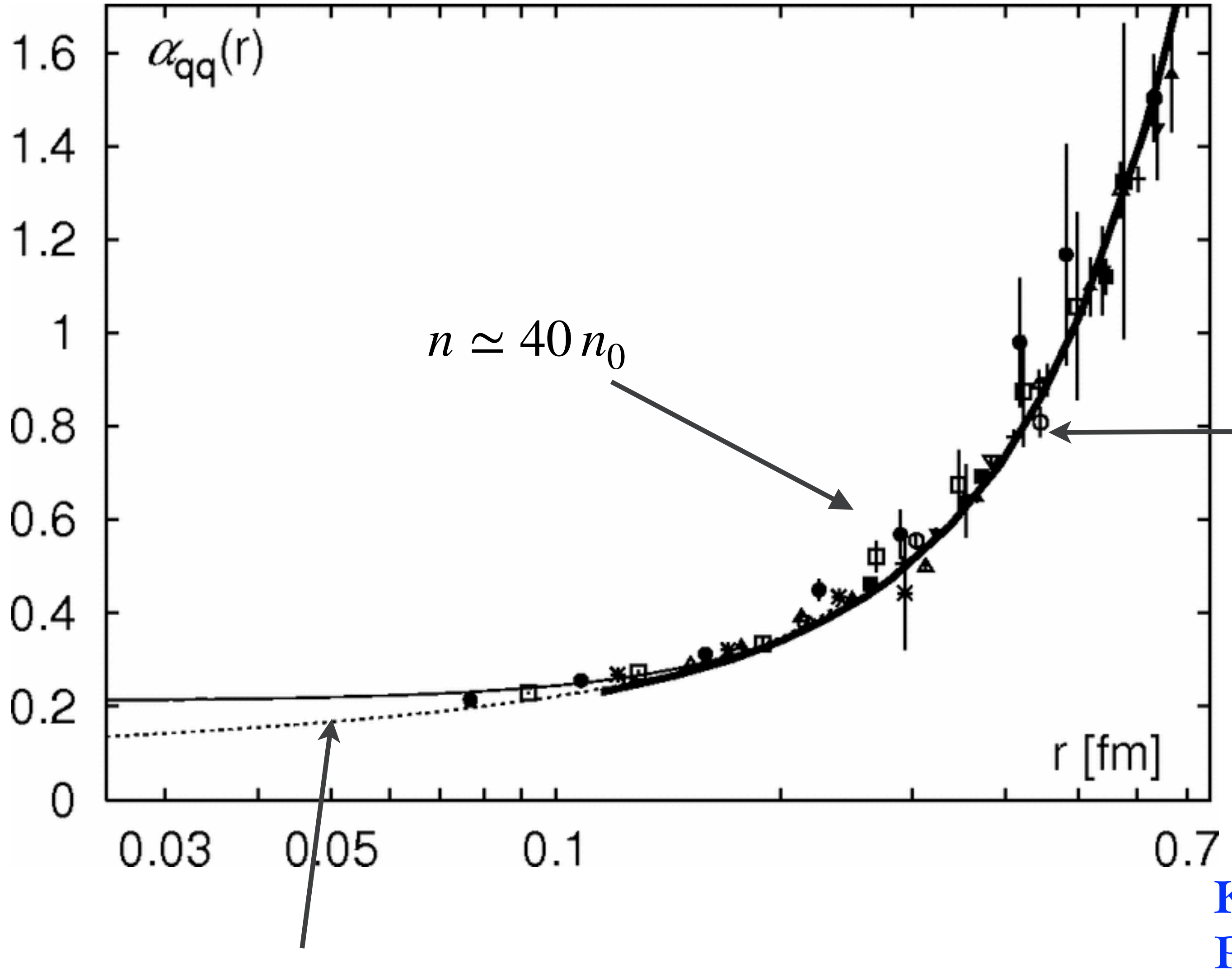
Confinement



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Nuclear interaction as Van der Waals forces

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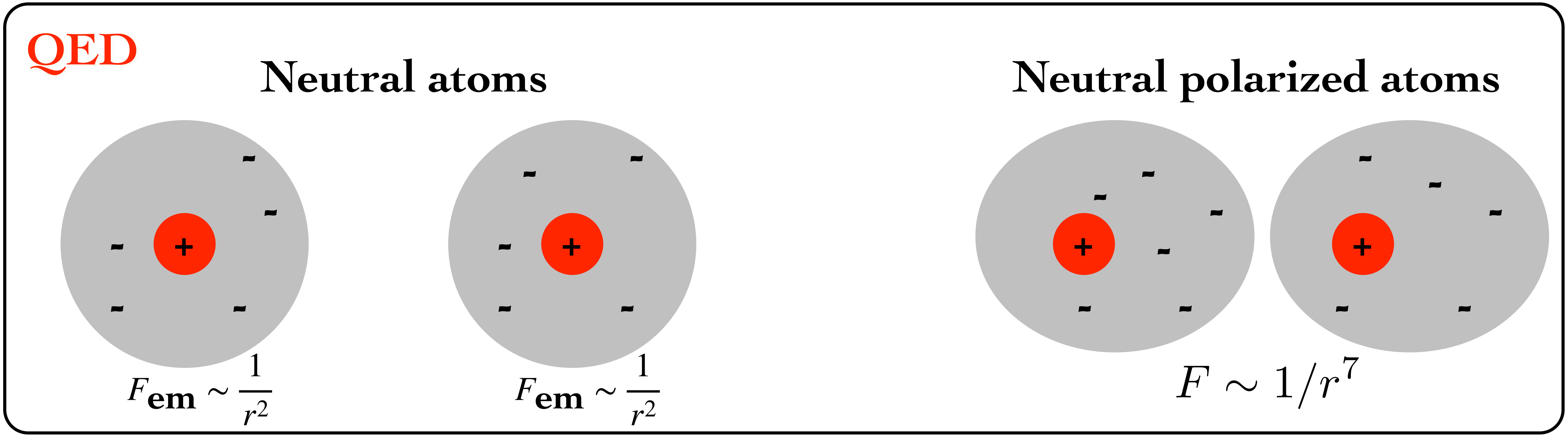

QED

Neutral atoms

Neutral polarized atoms

$F_{\mathbf{em}} \sim \frac{1}{r^2}$

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$F \sim 1/r^7$

Nuclear interaction as Van der Waals forces

QED

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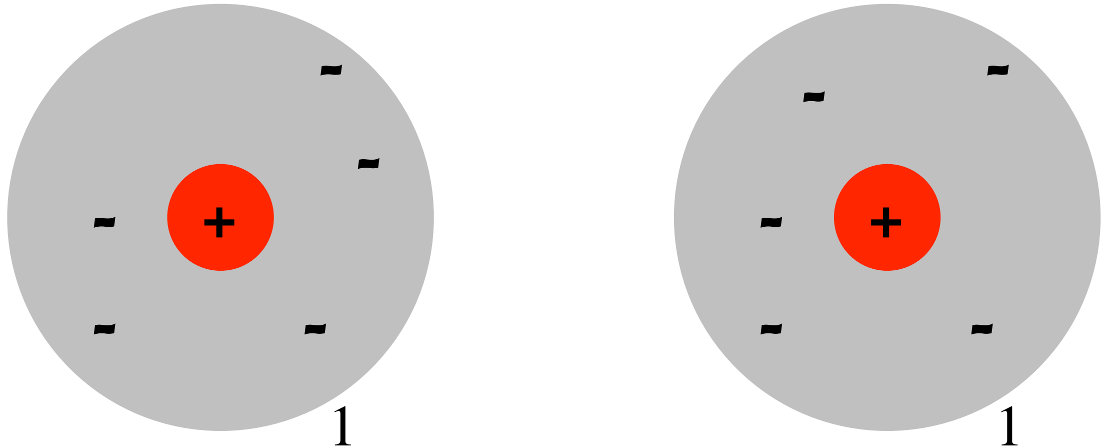

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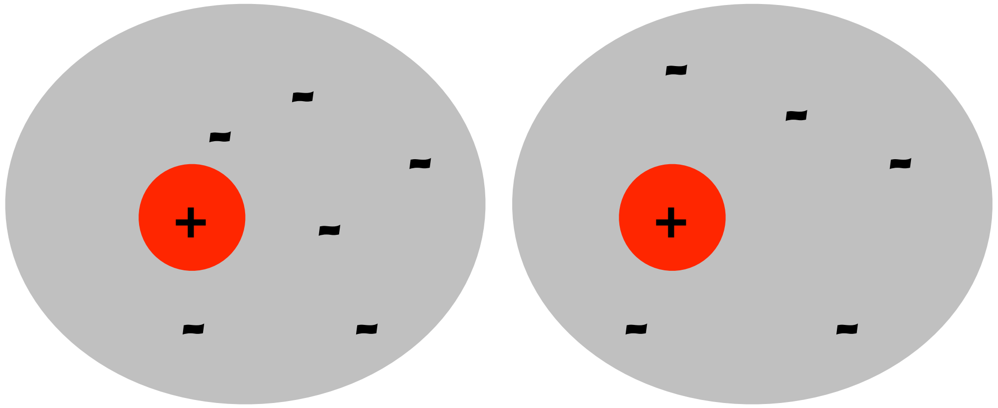

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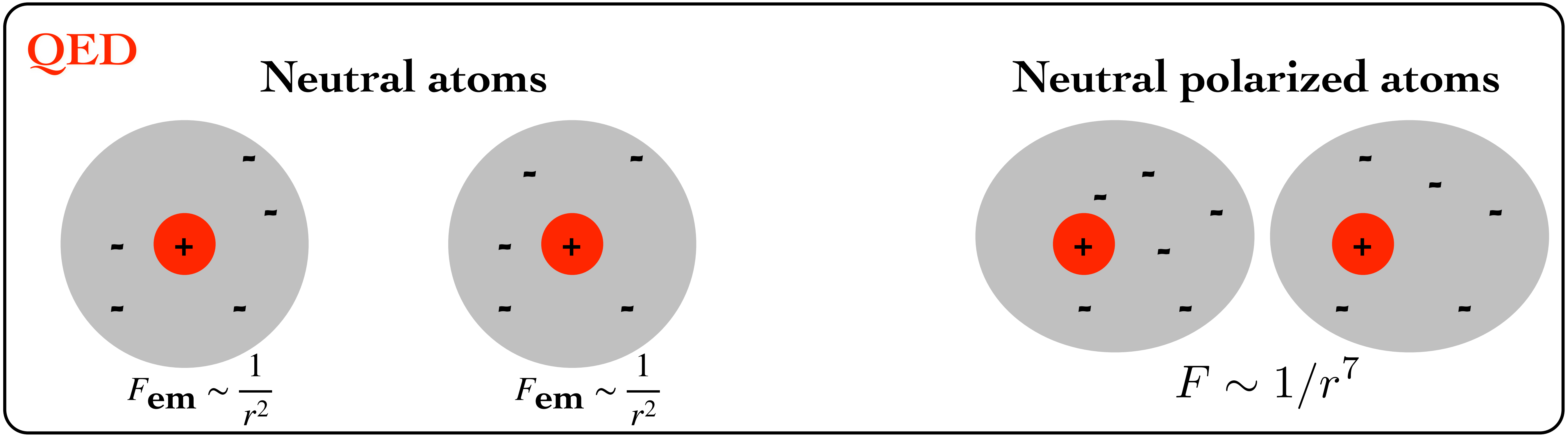
QCD

Neutral nucleons

“Polarized” nucelons

Nuclear interaction as Van der Waals forces



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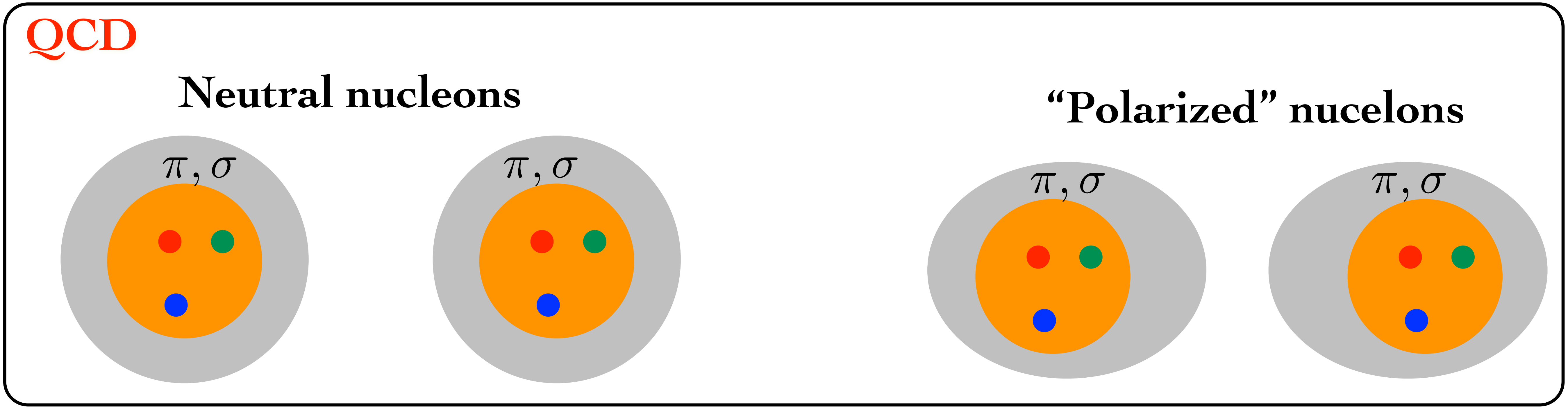

QCD

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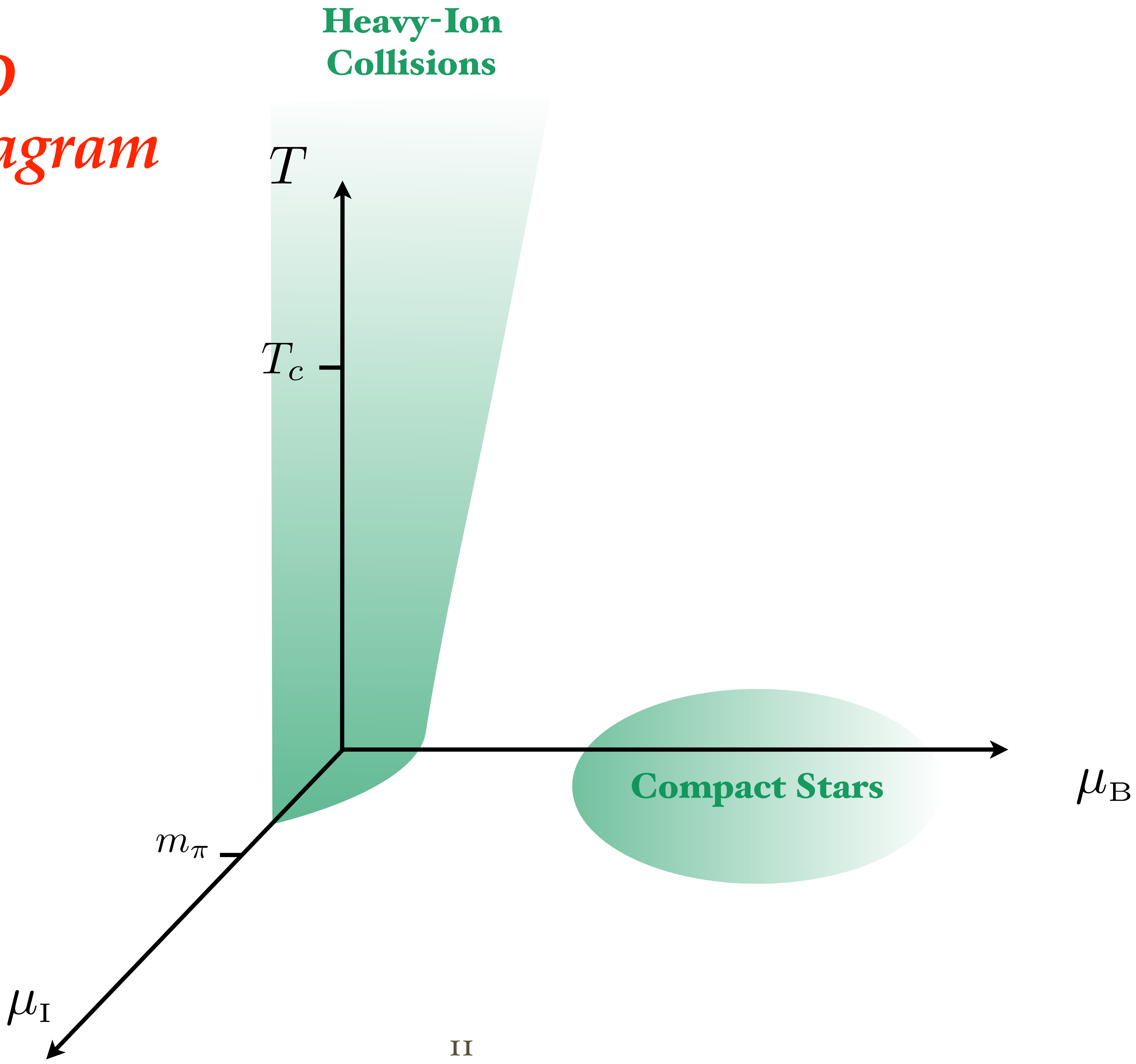
“Polarized” nucelons

The fundamental interaction is “stronger” than the derived interaction

The QCD phase diagram



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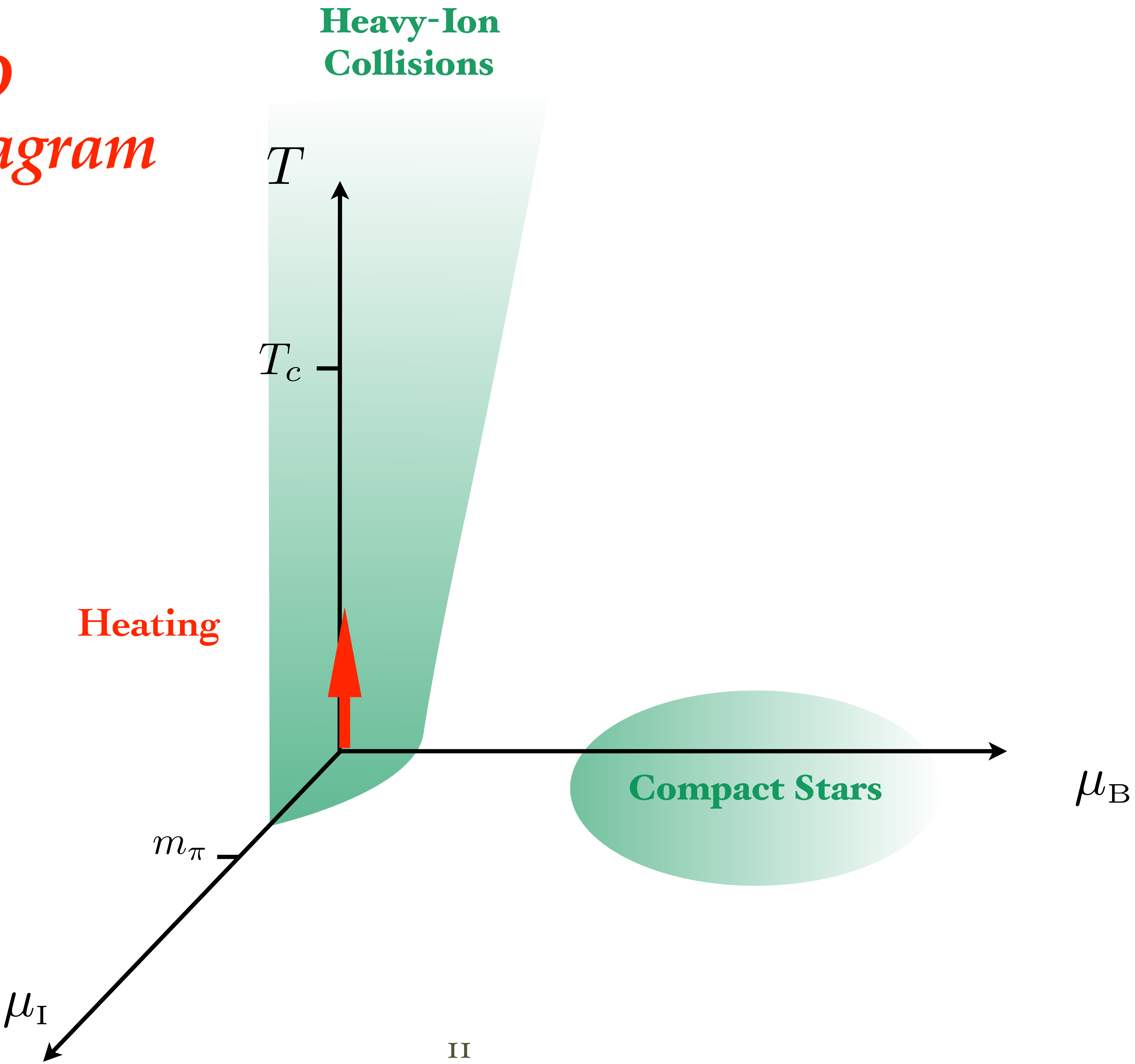

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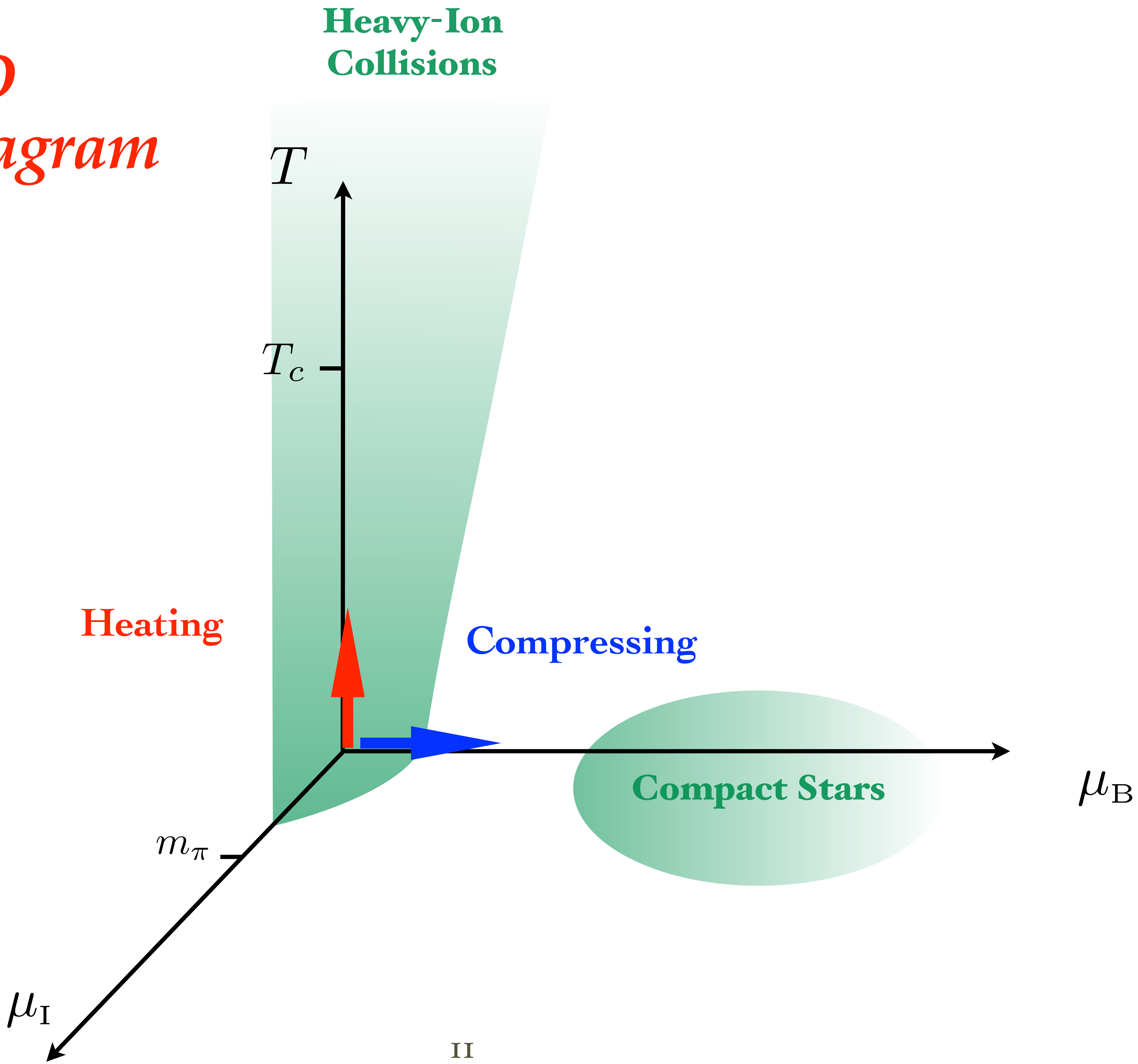

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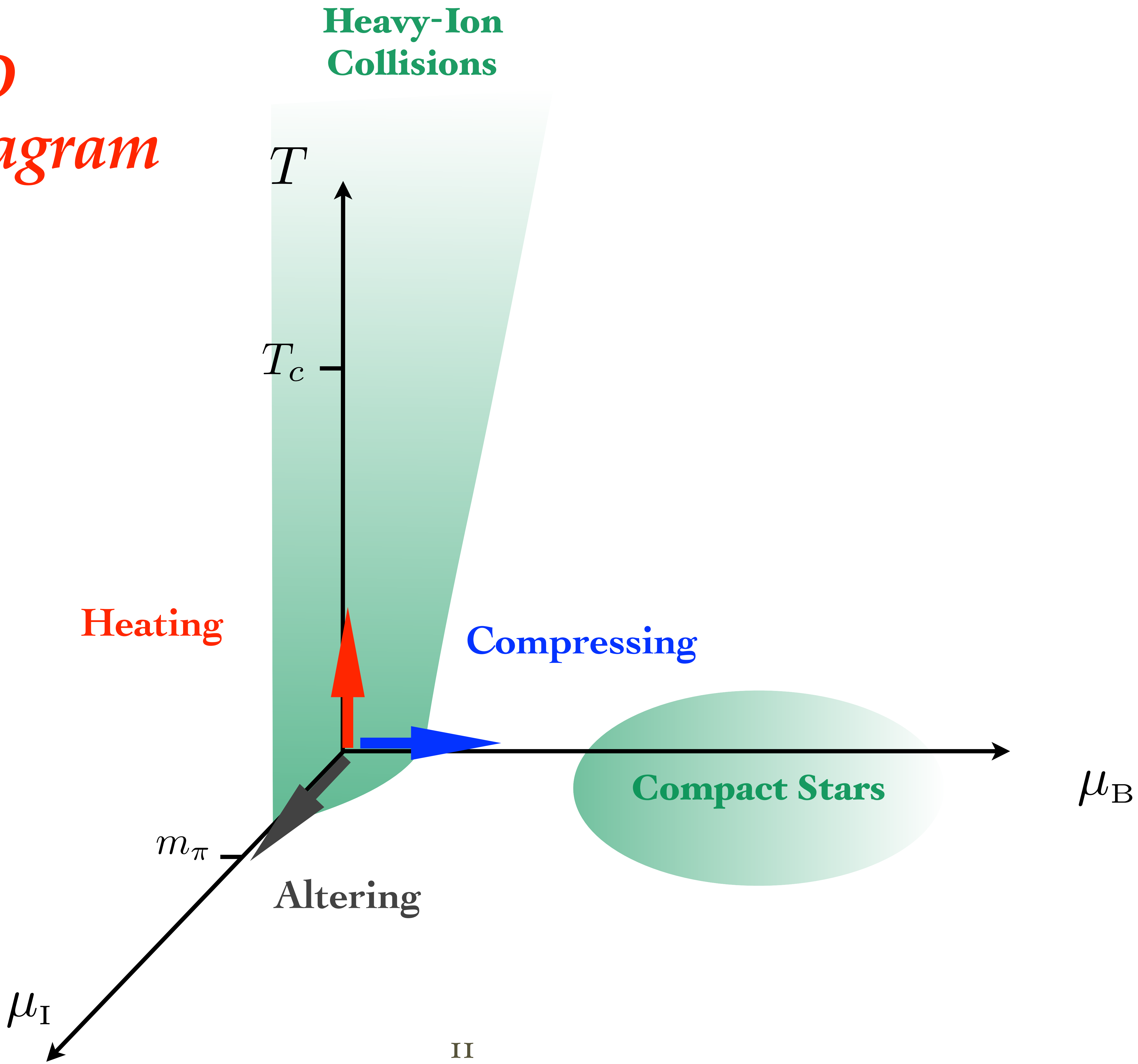
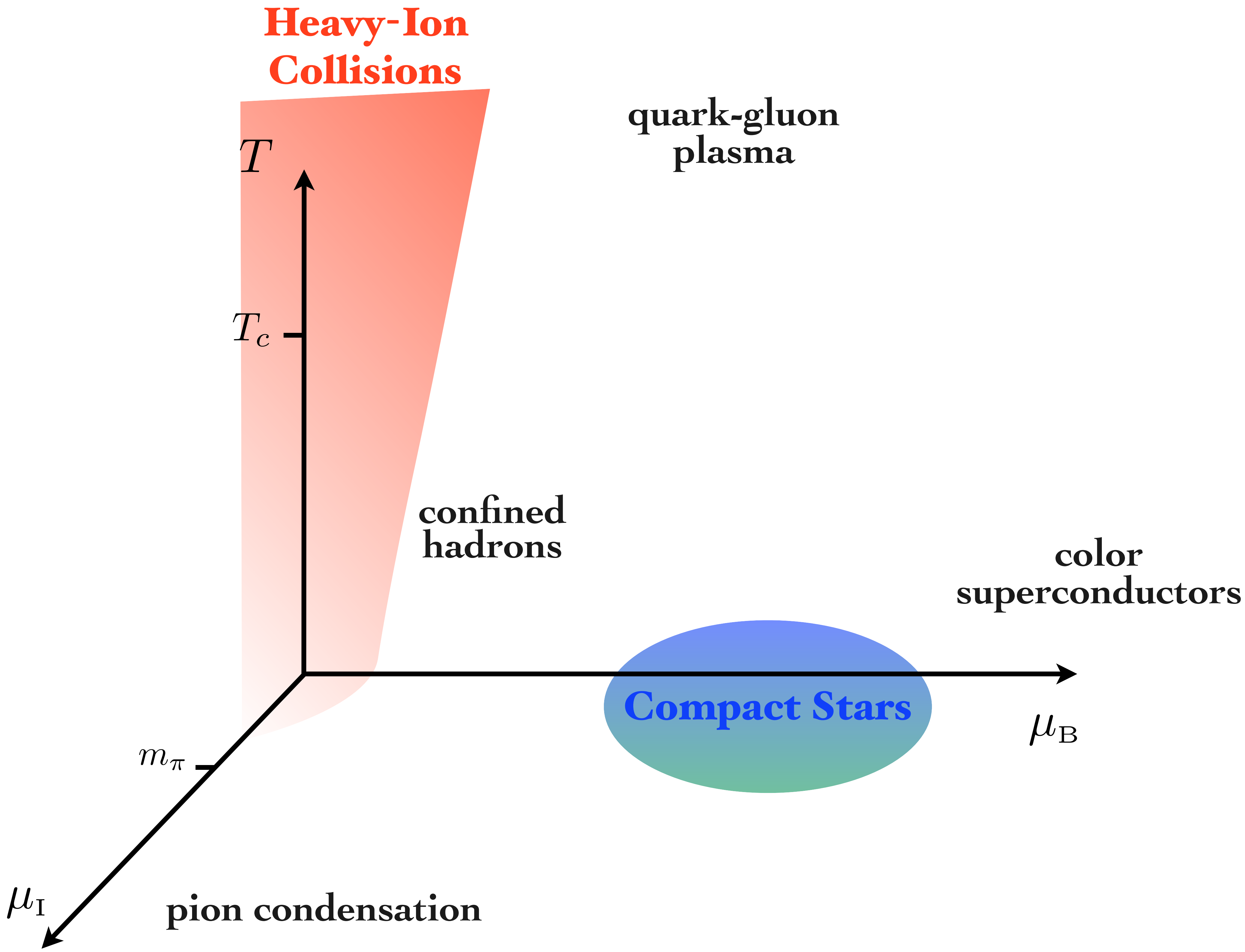


Heating


Heavy-Ion Collisions
quark-gluon plasma
T
T_c
confined hadrons
color superconductors
Compact Stars
μ_{B}
m_π
μ_{I}
pion condensation

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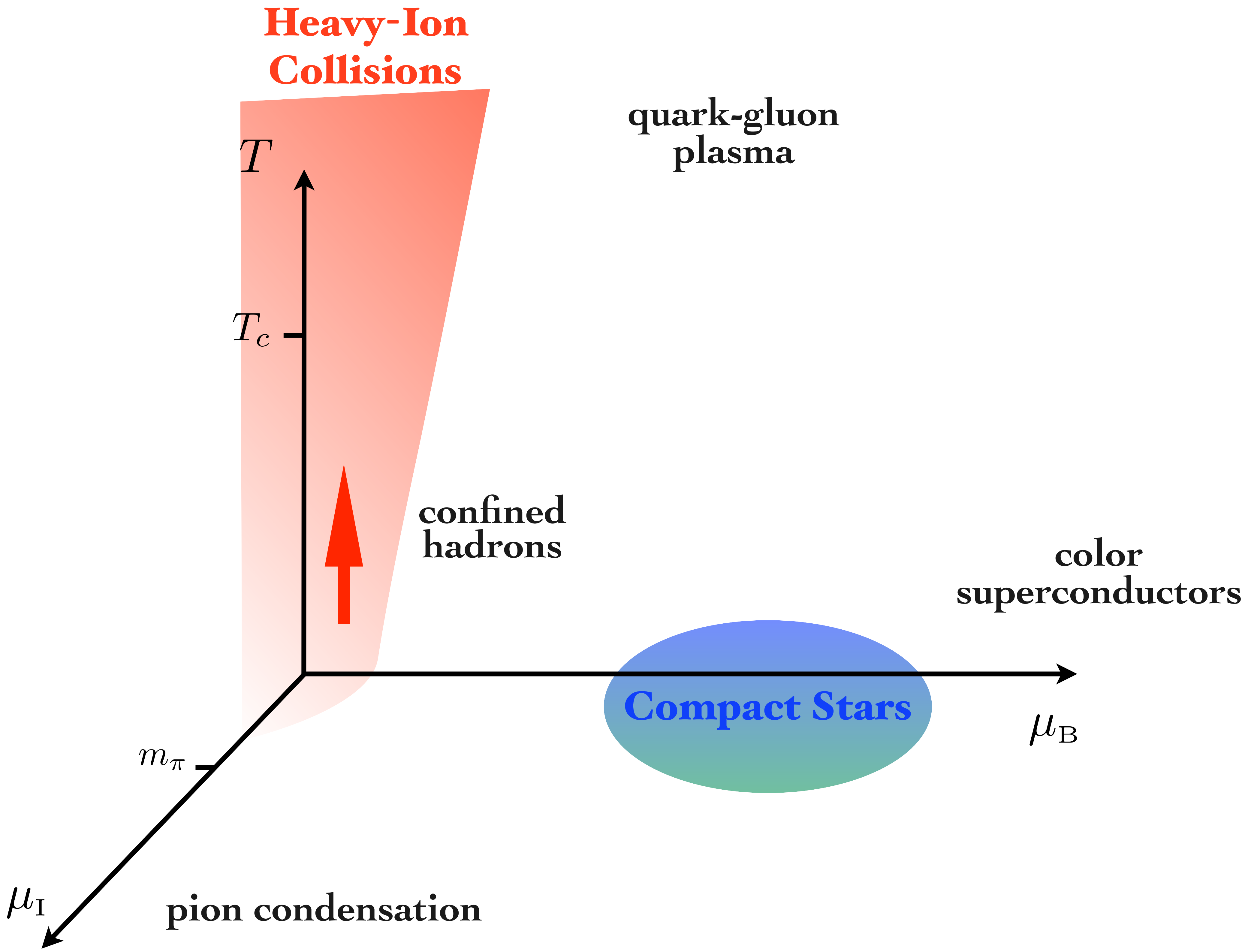
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Thermodynamics of hadrons

R. Hagedorn (1964/65) “statistical bootstrap”: Exponential growth of states with temperature.

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A limiting temperature, T_c, for hadronic matter

Thermodynamics of hadrons

R. Hagedorn (1964/65) "statistical bootstrap": Exponential growth of states with temperature.

A limiting temperature, T_c, for hadronic matter

Roughly: close to T_c, putting energy into the system increases the number of particles, not the temperature

Quark liberation

N. Cabibbo and G.Parisi PLB 59, Issue 1, 13 October 1975

"We suggest that the “observed” exponential spectrum is connected to the existence of a different phase of the vacuum in which quarks are not confined."

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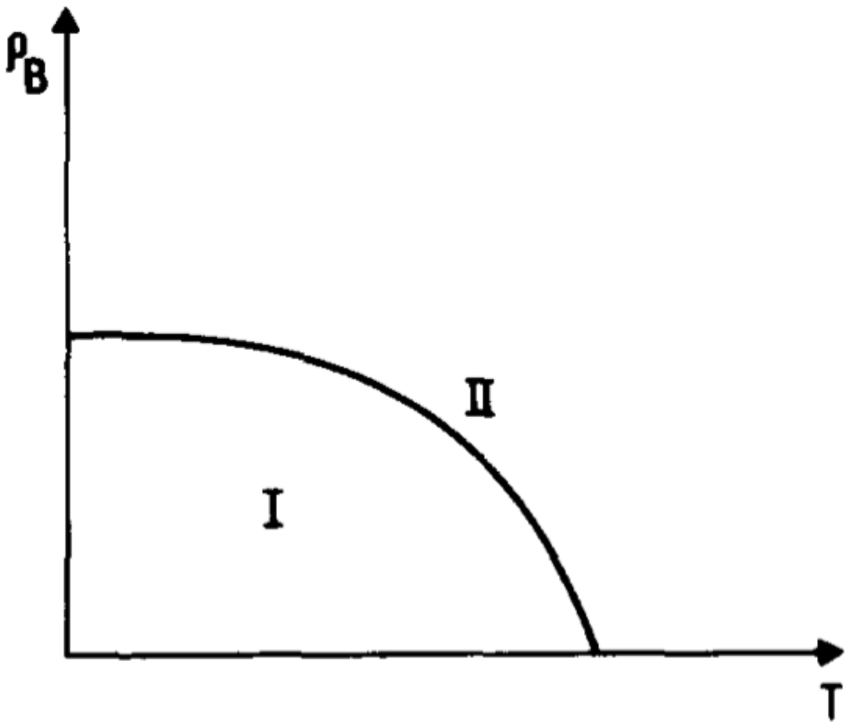

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

In the $V \to \infty$ limit the exponential spectrum is typical of second order phase transitions

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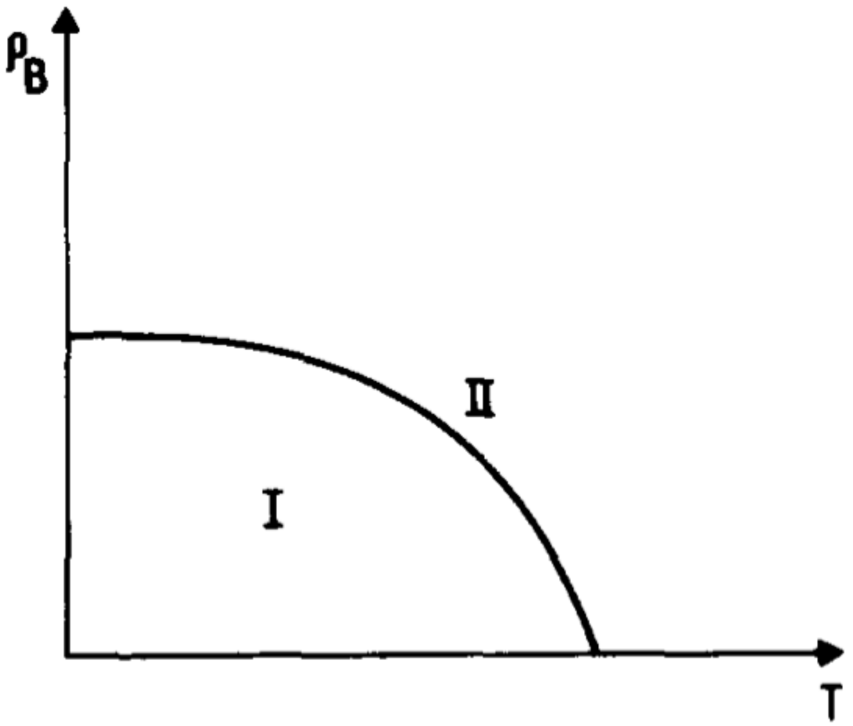

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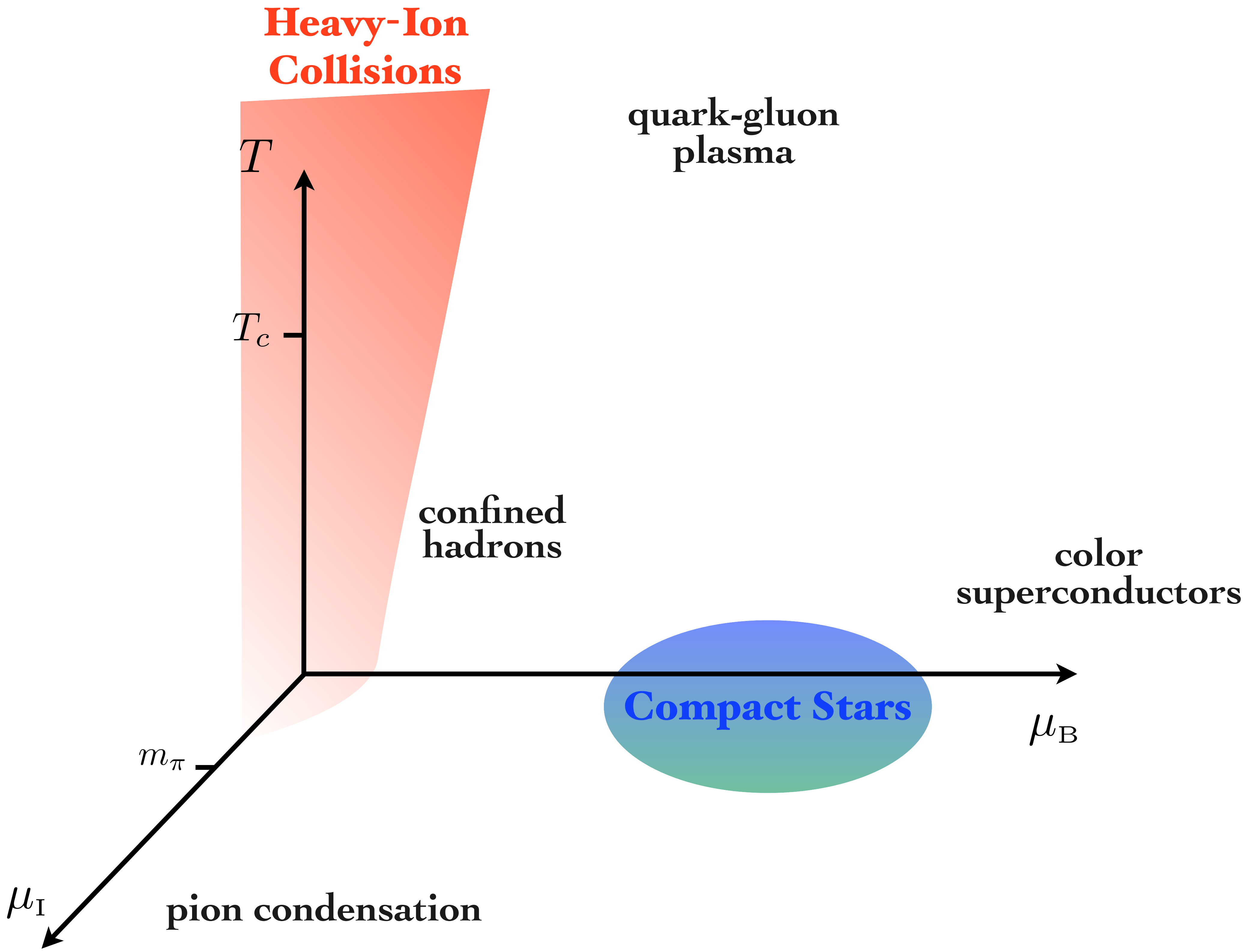
In the $V \to \infty$ limit the exponential spectrum is typical of second order phase transitions

Close to the phase transition there is a kind of “critical opalescence” of hadrons

Squeezing hadronic matter



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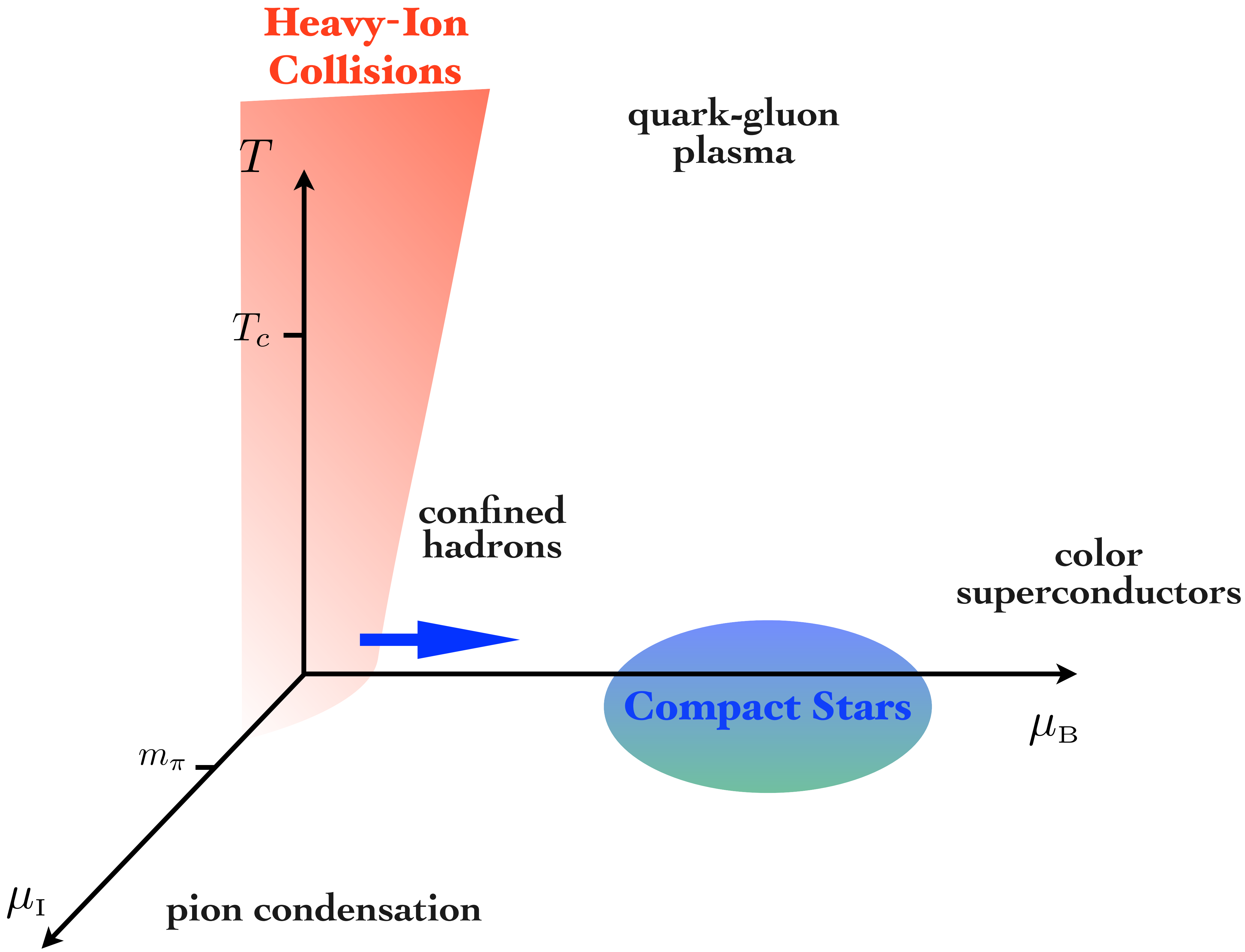
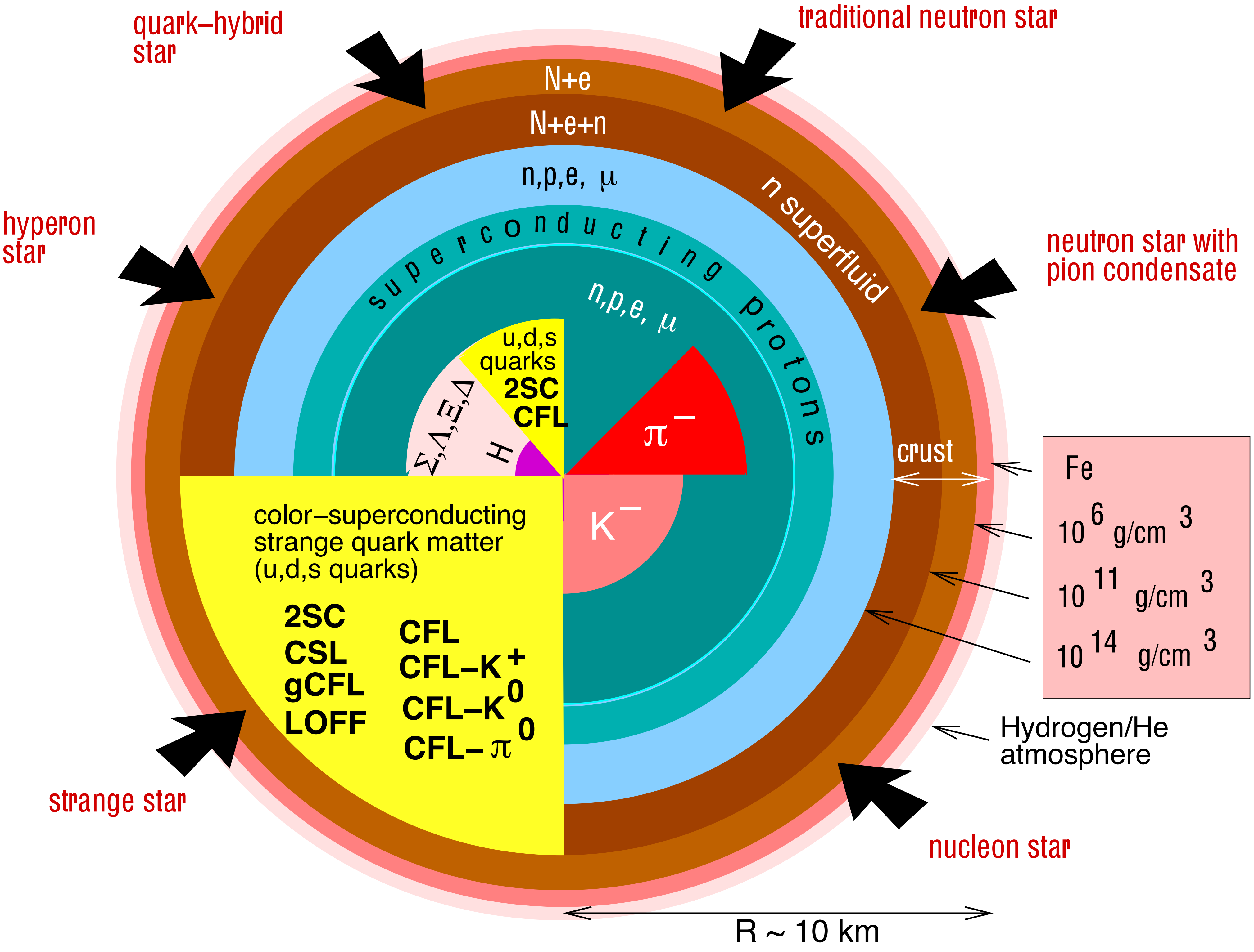


The squeezer: Compact Star



$$M_\odot \lesssim M \lesssim 2M_\odot$$

$$R \sim 10 \text{ km}$$

$$T \lesssim 10^6 \text{ K}$$

F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193

Compressed nuclear matter

The weak equilibrium in Neutron Stars has all the time to work.

Neutron rich matter

Isotope	Z/A	$\rho_t(\mathrm{g/cm^3})$	μ_e (MeV)
^{56}Fe	0.464	7.96×10^6	0.95
^{62}Ni	0.452	2.71×10^8	2.61
^{64}Ni	0.437	1.3×10^9	4.31
^{66}Ni	0.424	1.48×10^9	4.45
^{86}Kr	0.419	3.12×10^9	5.66
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^{126}Ru	0.350	1.29×10^{11}	18.3
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Haensel and Pichon
Astron.Astrophys. 283 (1994) 313

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← neutron drip

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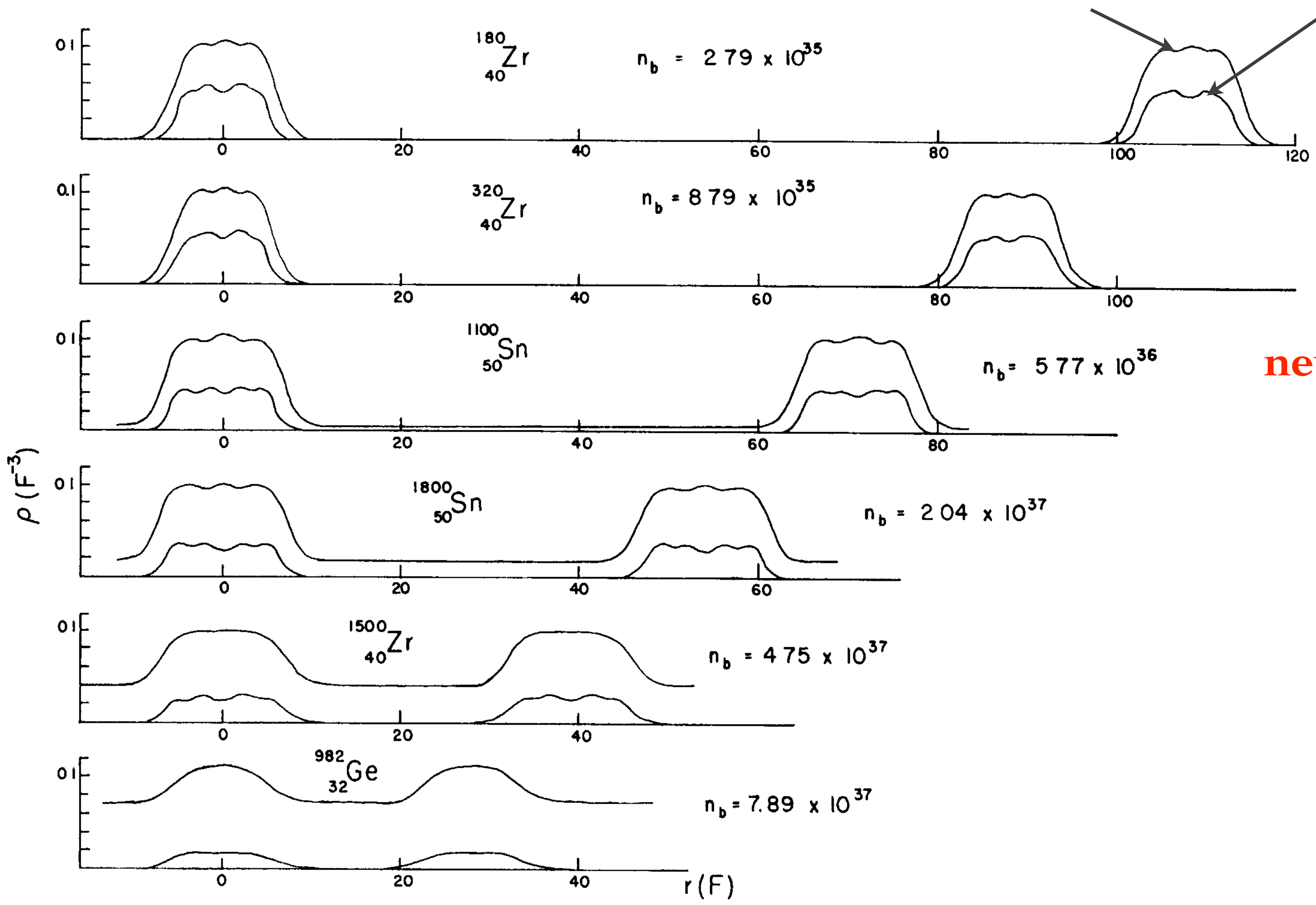
Neutron rich matter

many electrons

neutron drip

Haensel and Pichon
Astron.Astrophys. 283 (1994) 313

NS crust



neutrons

protons

neutron drip

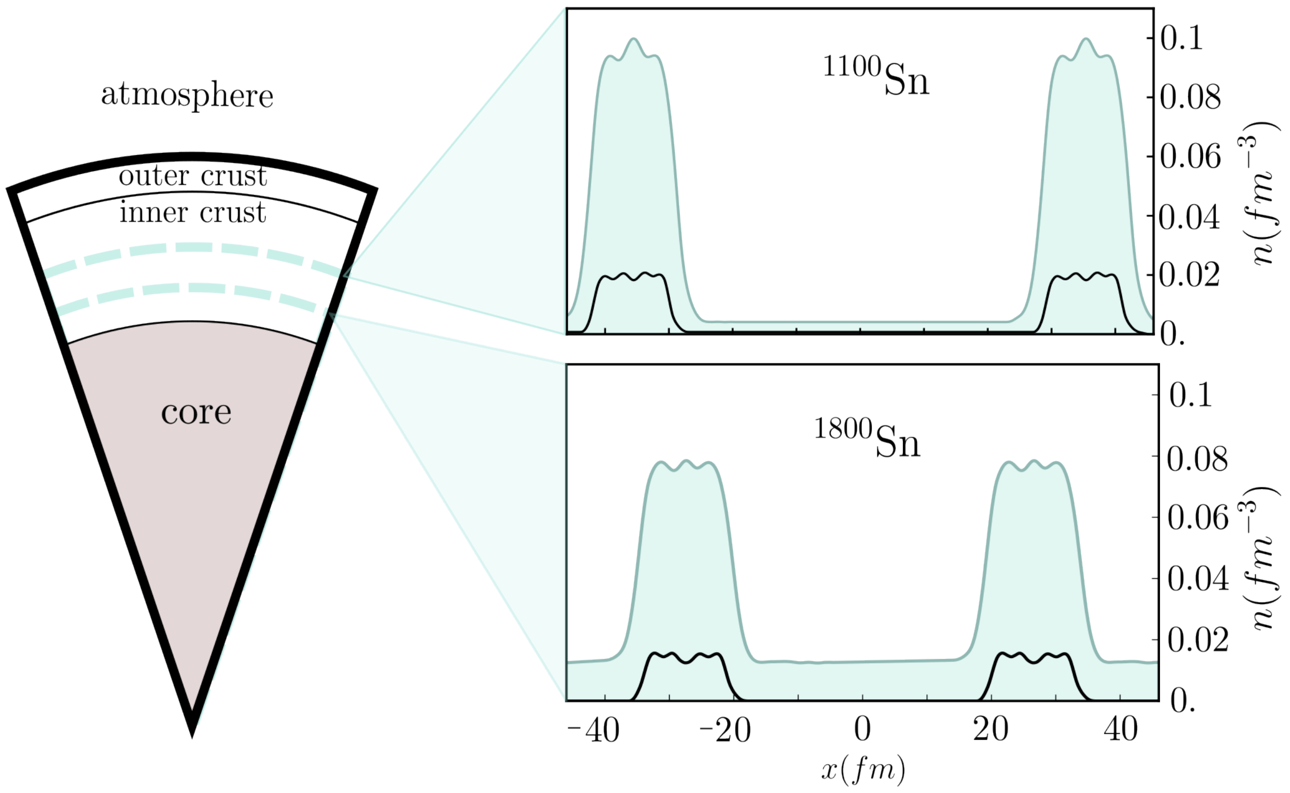
Proton and neutron density profiles along the lines joining two nuclei

J. W. Negele and D. Vautherin, Nucl. Phys. A207, 298 (1973)

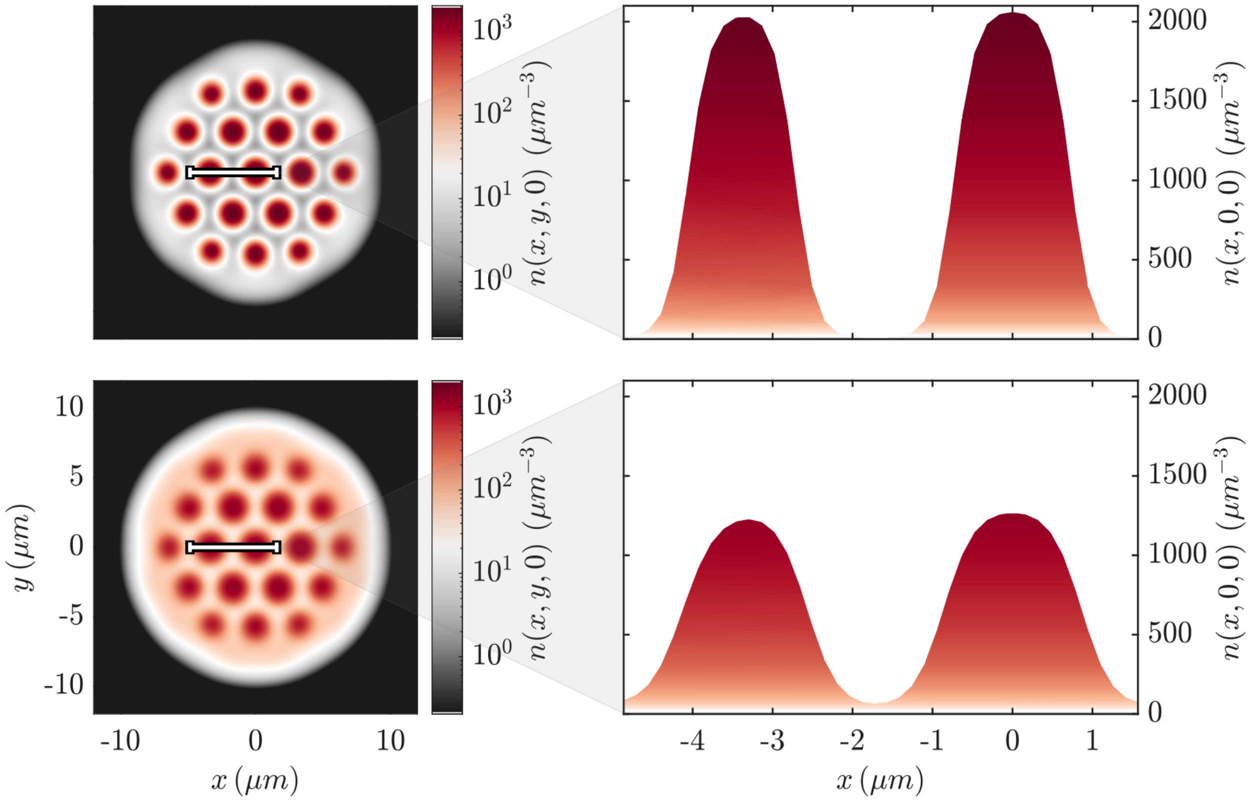
Quantum simulations: Supersolid inner crust

Ultracold atom systems emulate some properties of Neutron Stars

Neutron Star

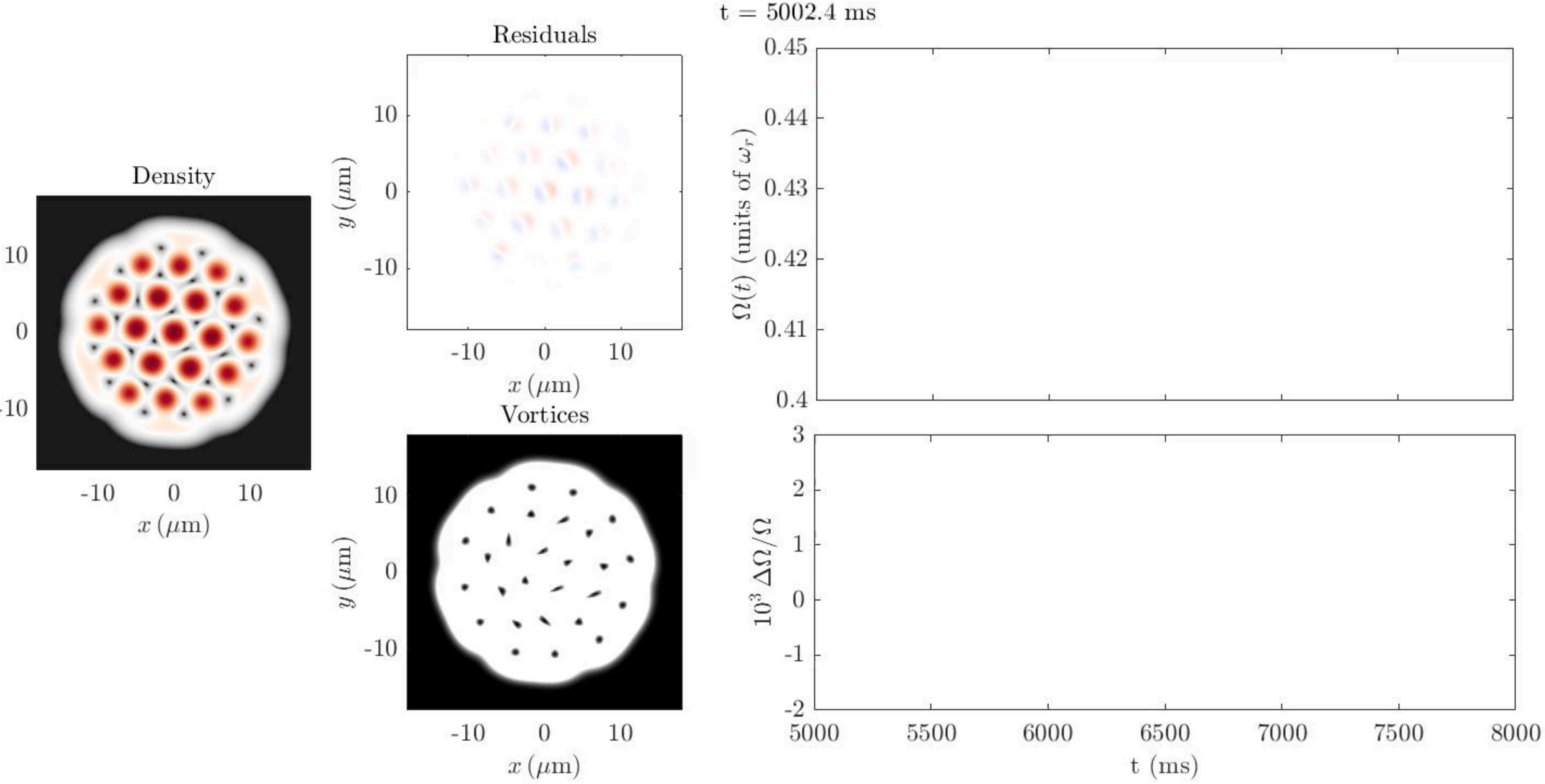


Atomic Supersolid



Poli, Bland, White, Mark, Ferlaino, Trabucco, Mannarelli
Phys.Rev.Lett. 131 (2023) 22, 223401

Glitches in supersolids



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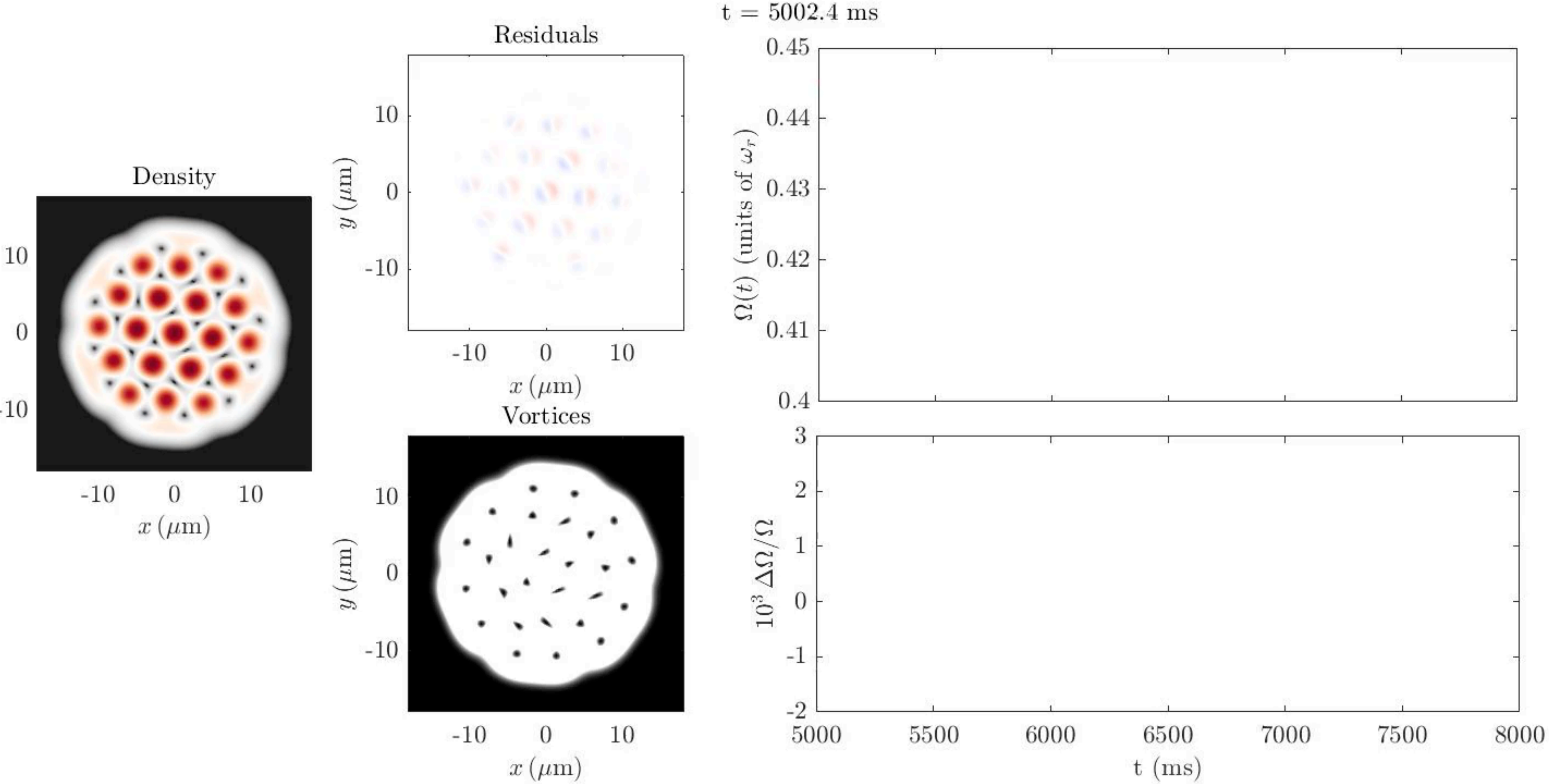

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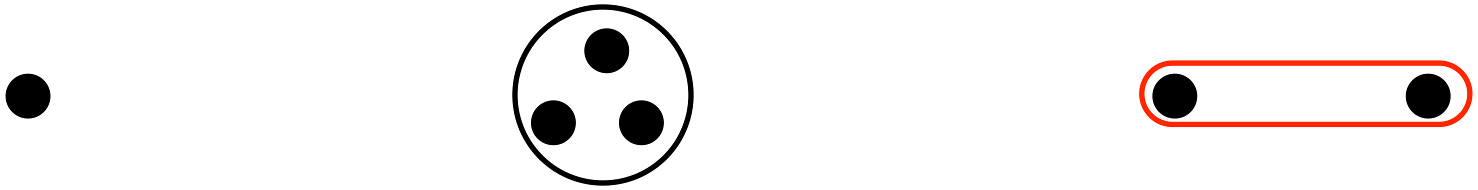
Increasing the density: deconfined quark pairs



quark	baryon	diquark
point-like	~1 fm	~10 fm

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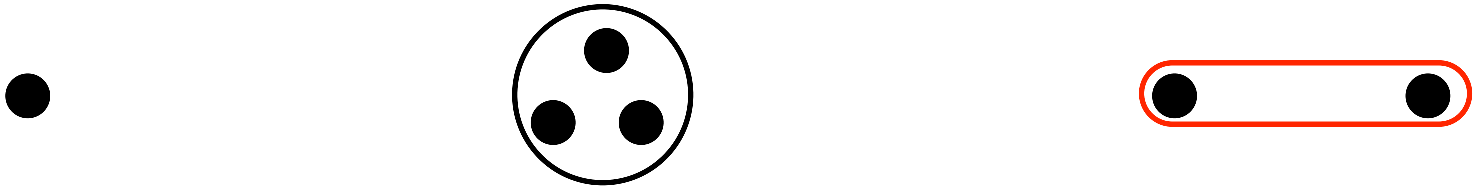
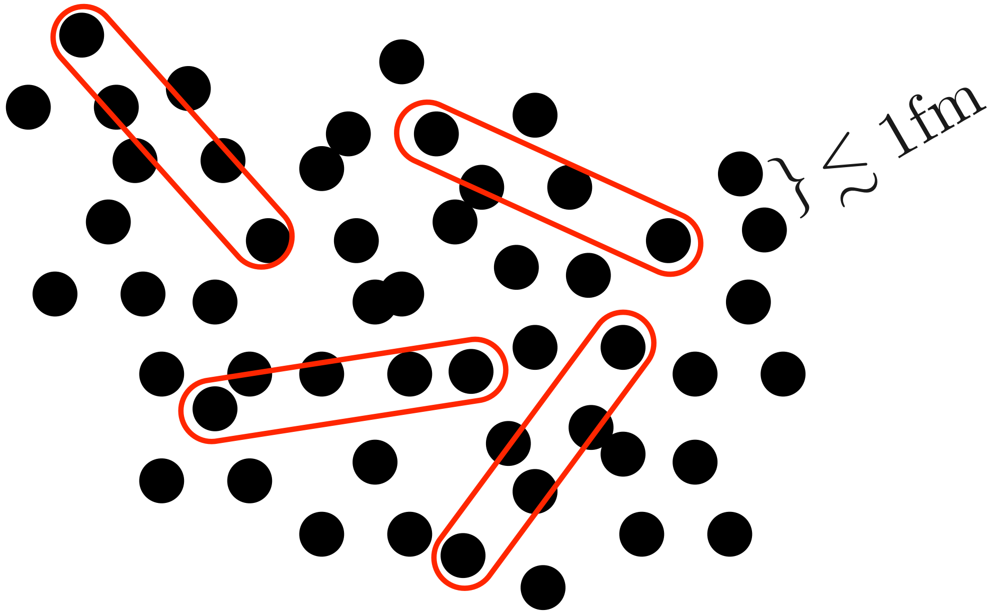


quark — point-like

baryon — ~1 fm

diquark — ~10 fm

Very high density (Compact Star inner core)



Liquid of quarks with
correlated diquarks

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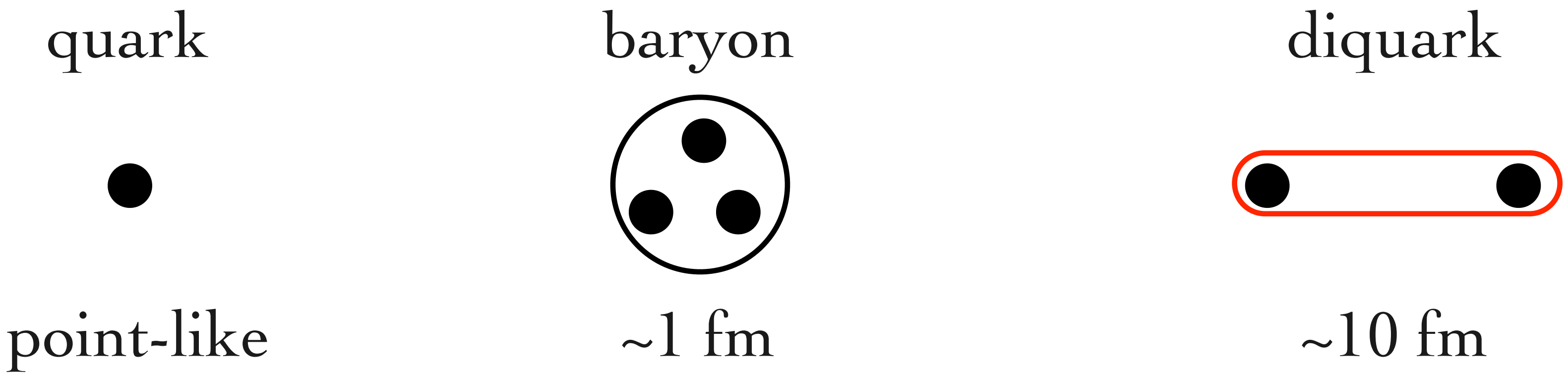

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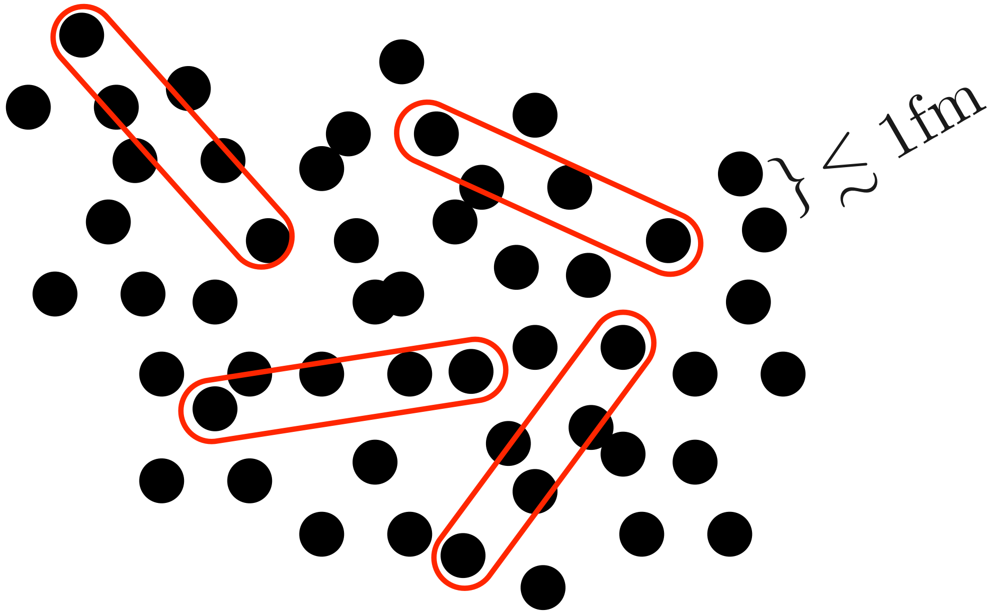


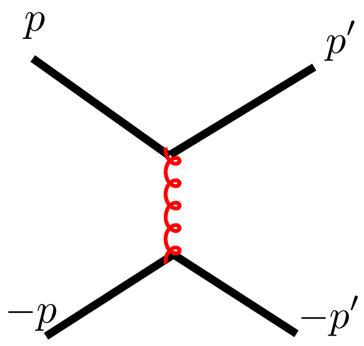
Liquid of quarks with correlated diquarks

Attractive interaction (perturbative)

$$3 \times 3 = \bar{3}_A + 6_S$$

$\uparrow$

attractive channel

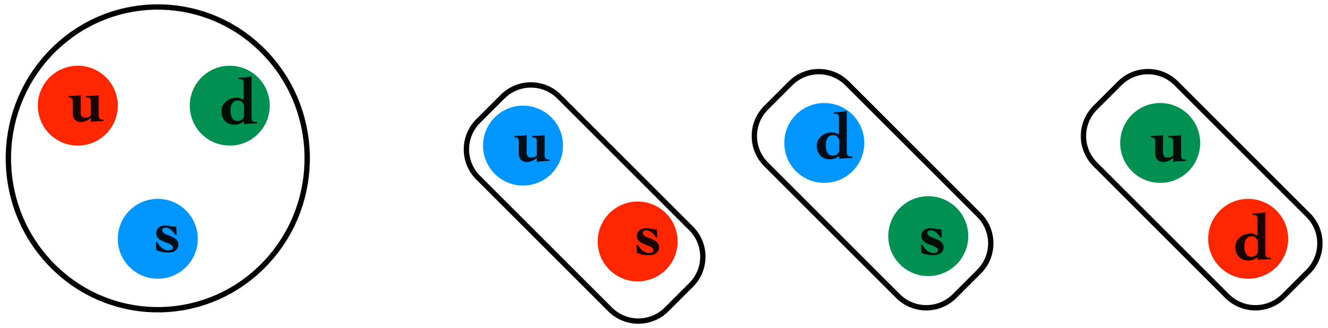


$p, p' \simeq p_{\rm F}$

Color-flavor locking

CFL pairing

(Alford, Rajagopal, Wilczek hep-ph/9804403)



quarks of all flavors and colors form Cooper pairs

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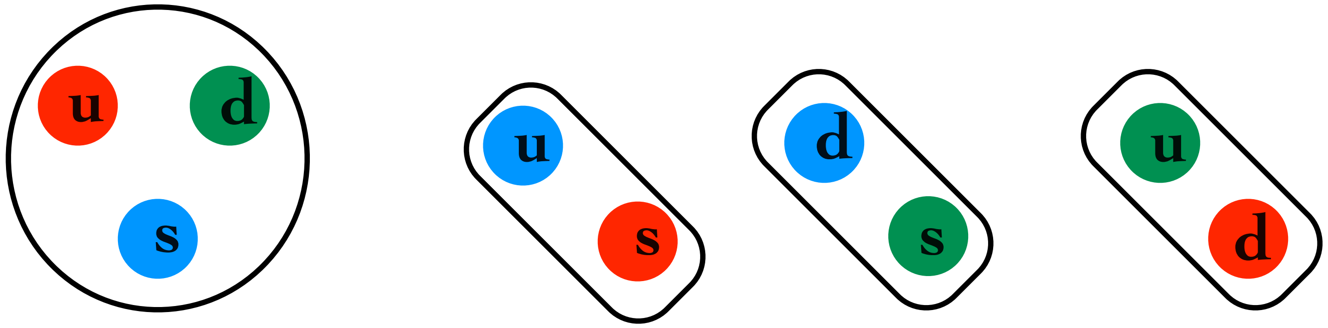

quarks of all flavors and colors form Cooper pairs

- Anderson-Higgs mechanism (gluons acquire mass) COLOR SUPERCONDUCTOR
- $U(1)_B$ breaking **SUPERFLUID**
- χSB: 8 (pseudo) Nambu-Goldstone bosons… as in the confined phase

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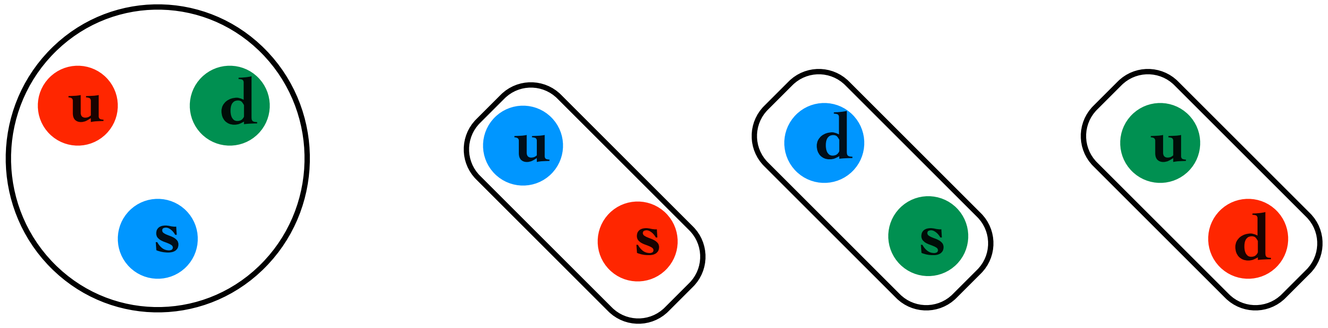

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Symmetry breaking

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times Z_2$$

Supersolid quark matter

R. Anglani, MM et al. "Crystalline color superconductors"
Review of Modern Physics 86, 509 (2014)

Bulk quark matter in compact stars

sizable strange quark mass

\+

weak equilibrium

\+

electric neutrality

→

mismatched
Fermi momenta

Bulk quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

→

mismatched
Fermi momenta

No pairing case

weak interactions

$$u \rightarrow d + \bar{e} + \nu_e$$
$$u \rightarrow s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

→

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality →

$$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$

Bull quark matter in compact stars

sizable strange quark mass
+
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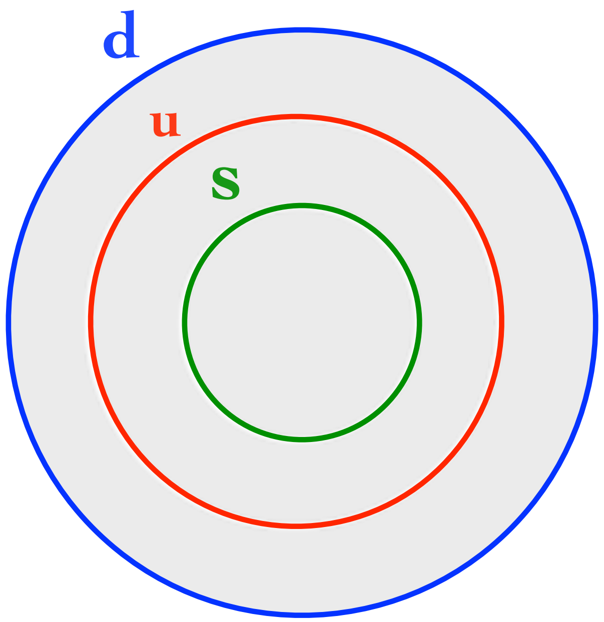
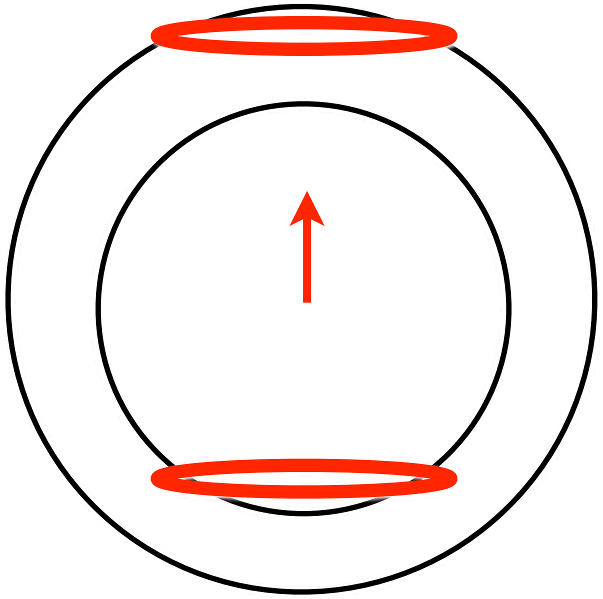


Fermi spheres of
u,d, s quarks

Crystalline structures



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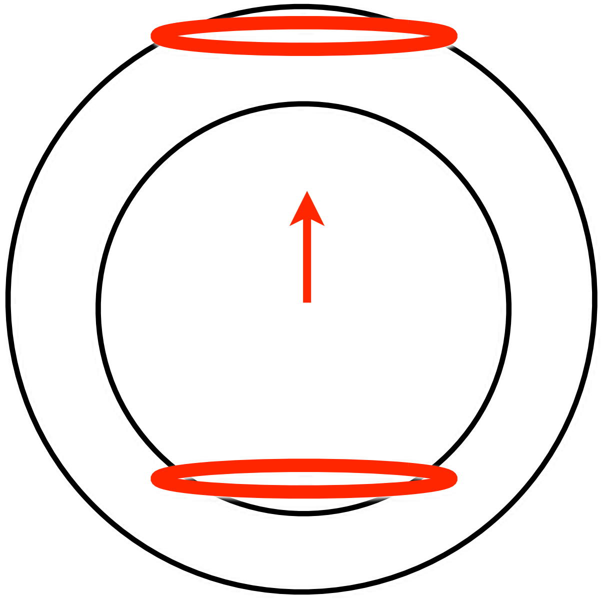
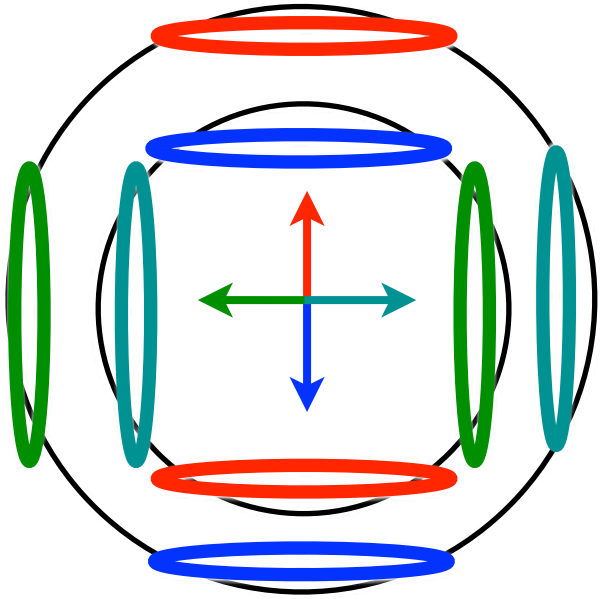


- Complicated structures can be obtained combining plane waves

Crystalline structures



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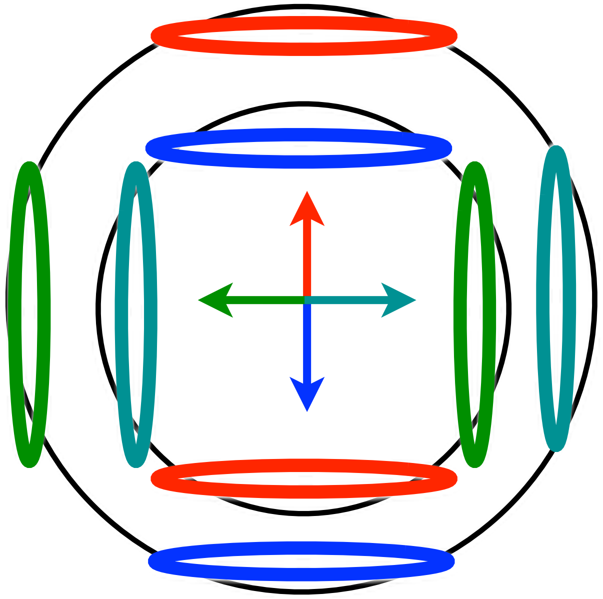

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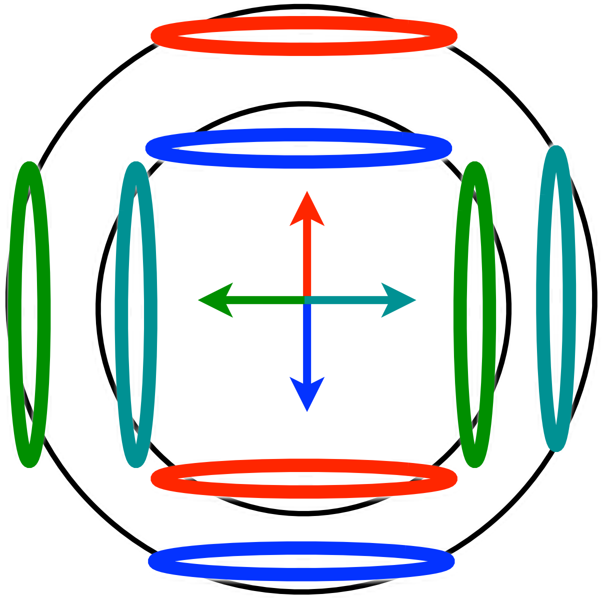

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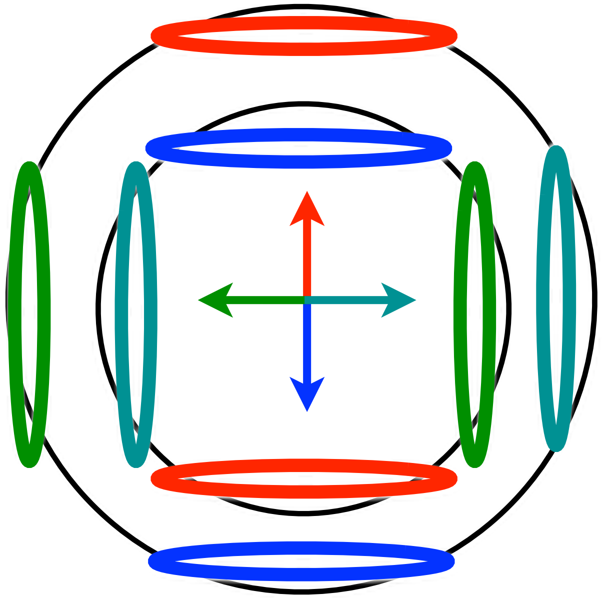
Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Crystalline structures



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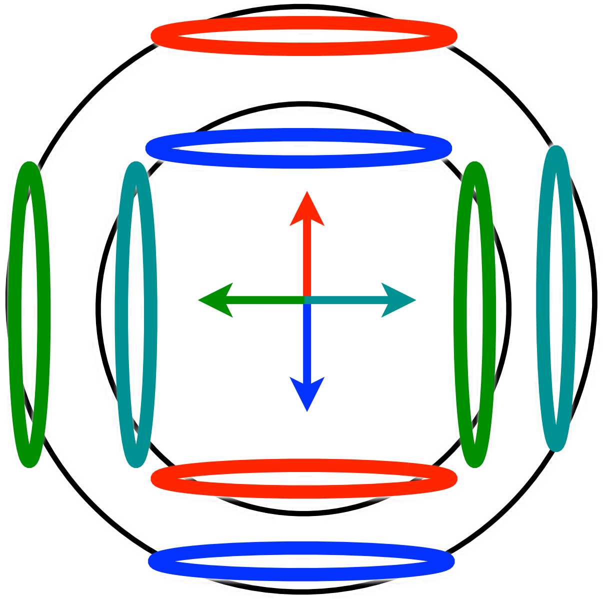
Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Rigidity

Crystalline structures



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Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Rigidity

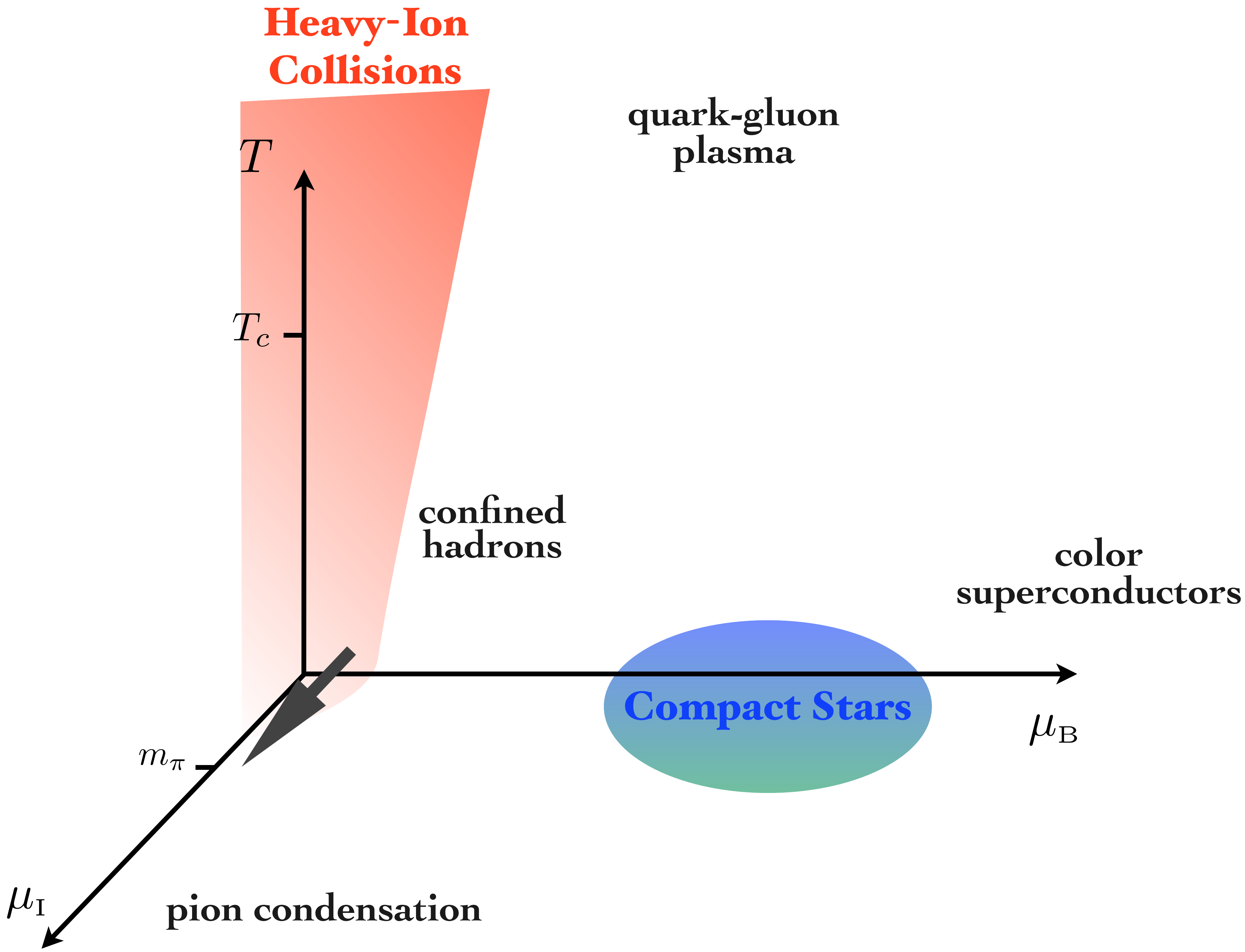
$\nu_{\mathrm{CCSC}} \sim 2.47\,\mathrm{MeV/fm}^3$

20 to 1000 times more rigid than the crust of neutron stars

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Meson condensation

Altering matter composition

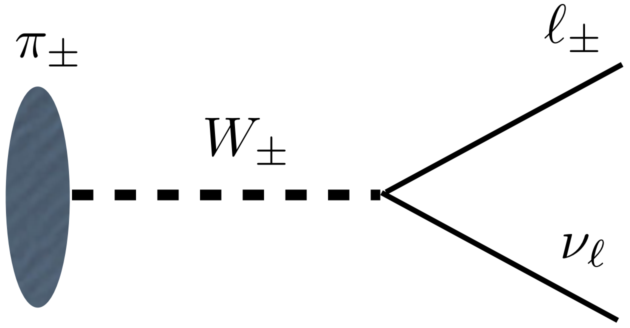


Pion condensation

Stabilization

The pion decay can be Pauli blocked by a large lepton chemical potential

pion decay in vacuum



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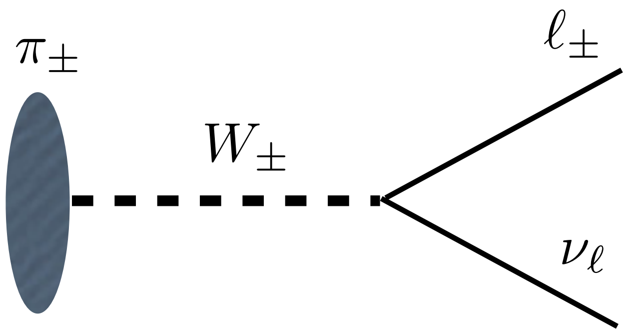

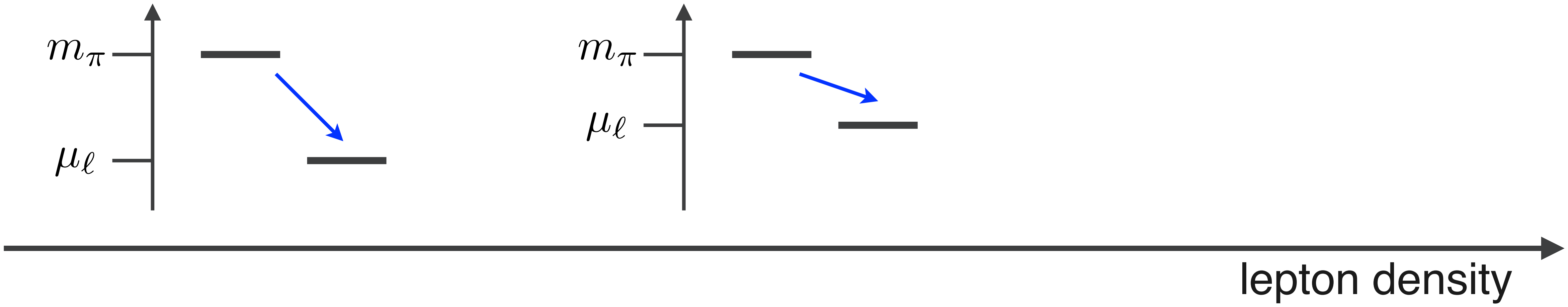

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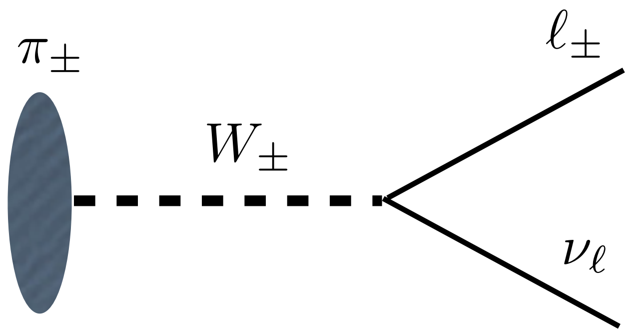

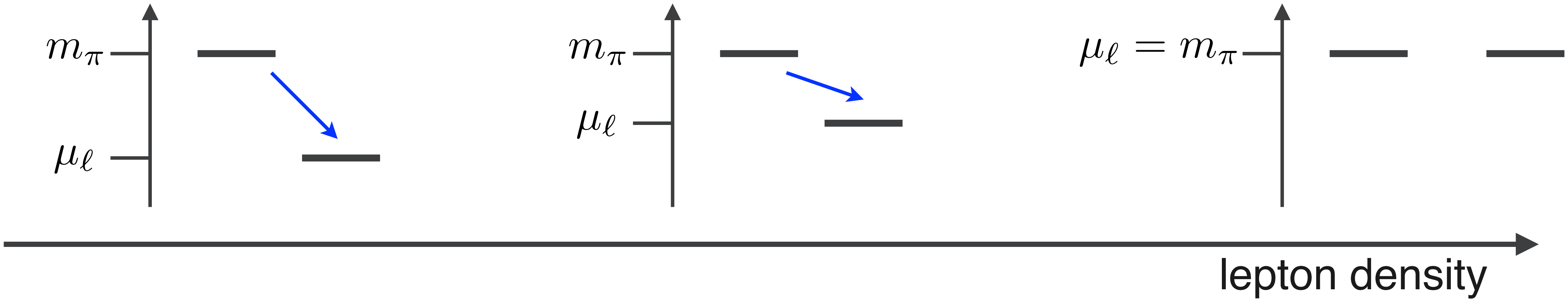

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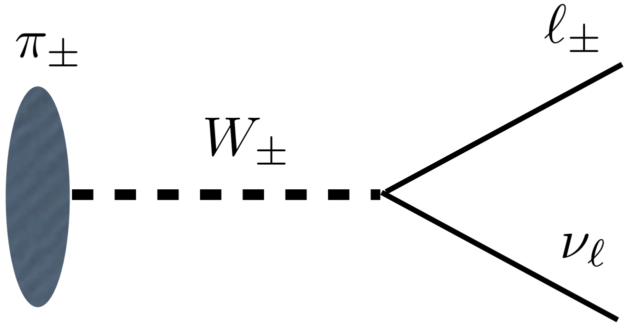
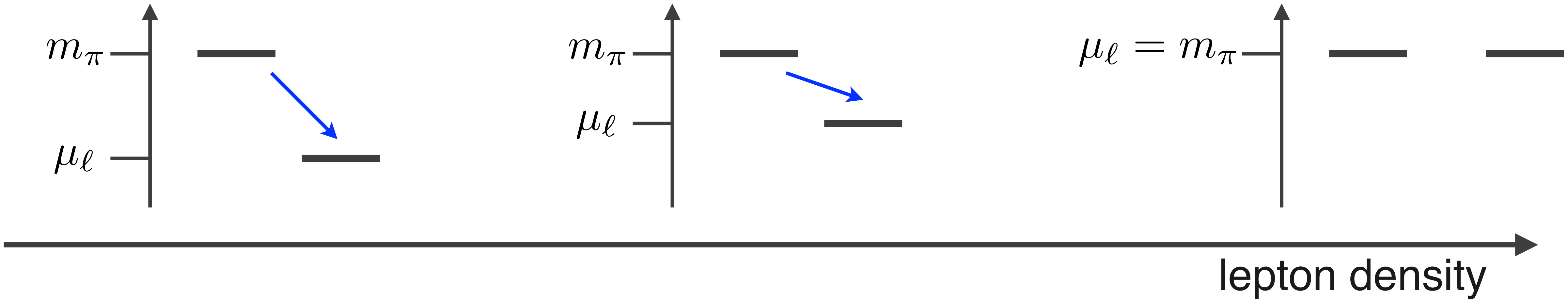




Energy spectrum splitting
Stark-like effect

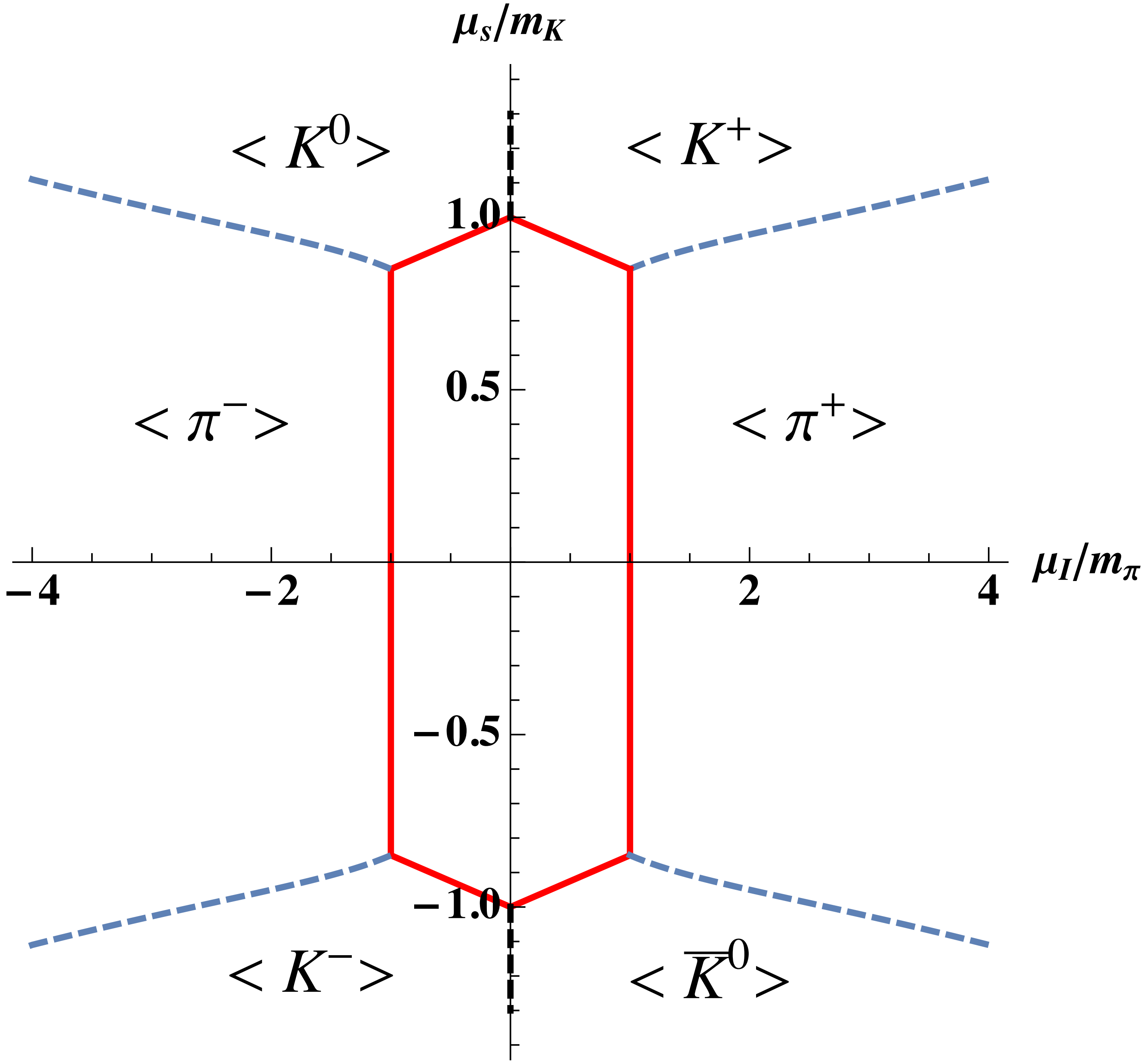
$$E_{\pi^0} = \sqrt{m_\pi^2 + p^2}$$

$$E_{\pi^-} = +\mu_I + \sqrt{m_\pi^2 + p^2}$$

$$E_{\pi^+} = -\mu_I + \sqrt{m_\pi^2 + p^2}$$

$\boxed{m_{\pi^+}^{\text{eff}} = m_\pi - \mu_I}$ → At $\mu_I = m_\pi$ a massless mode appears: pion condensation $\langle \bar{\psi} \sigma_2 \gamma_5 \psi \rangle$

Phases of meson condensates



Dashed: first order
Solid: second order

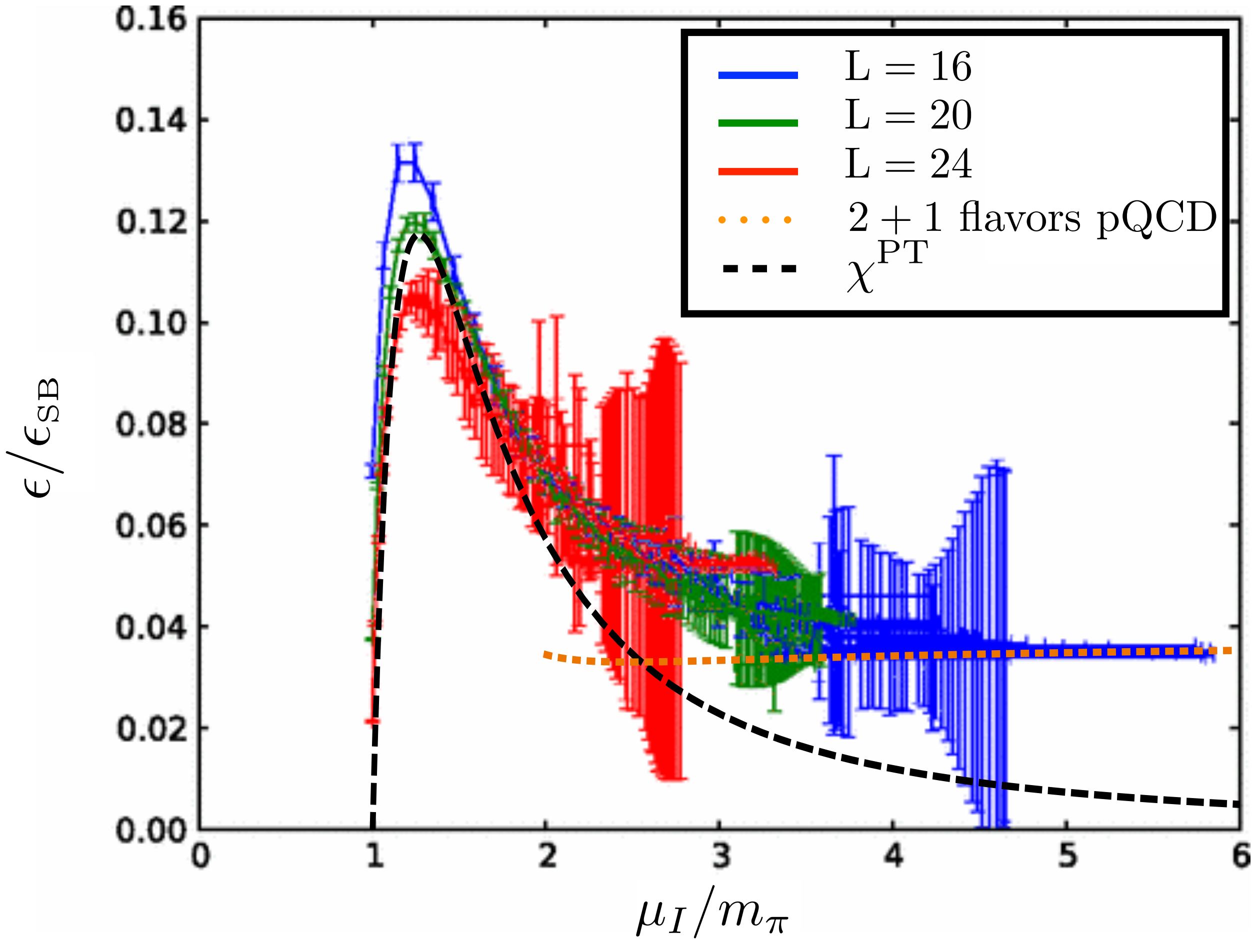
Kogut and Toublan PhysRevD.64.034007

MM, Particles 2, no.3, 411-443 (2019)

At asymptotic μ_I and/or μ_S matter should be deconfined in a rather unusual way

Pion condensation

Energy density



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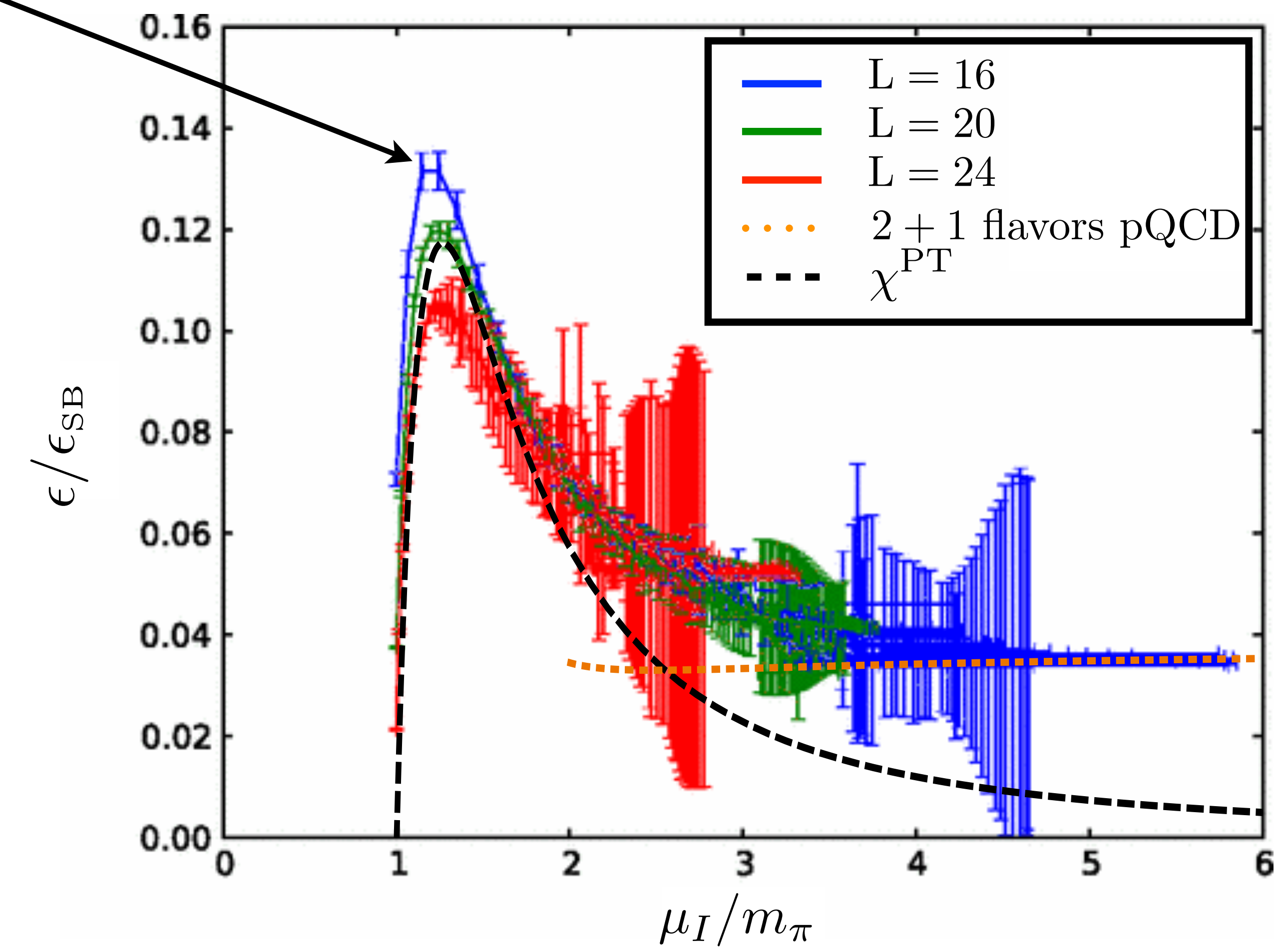

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NPLQCD Collaboration
Phys.Rev.D 108 (2023) 11, 114506

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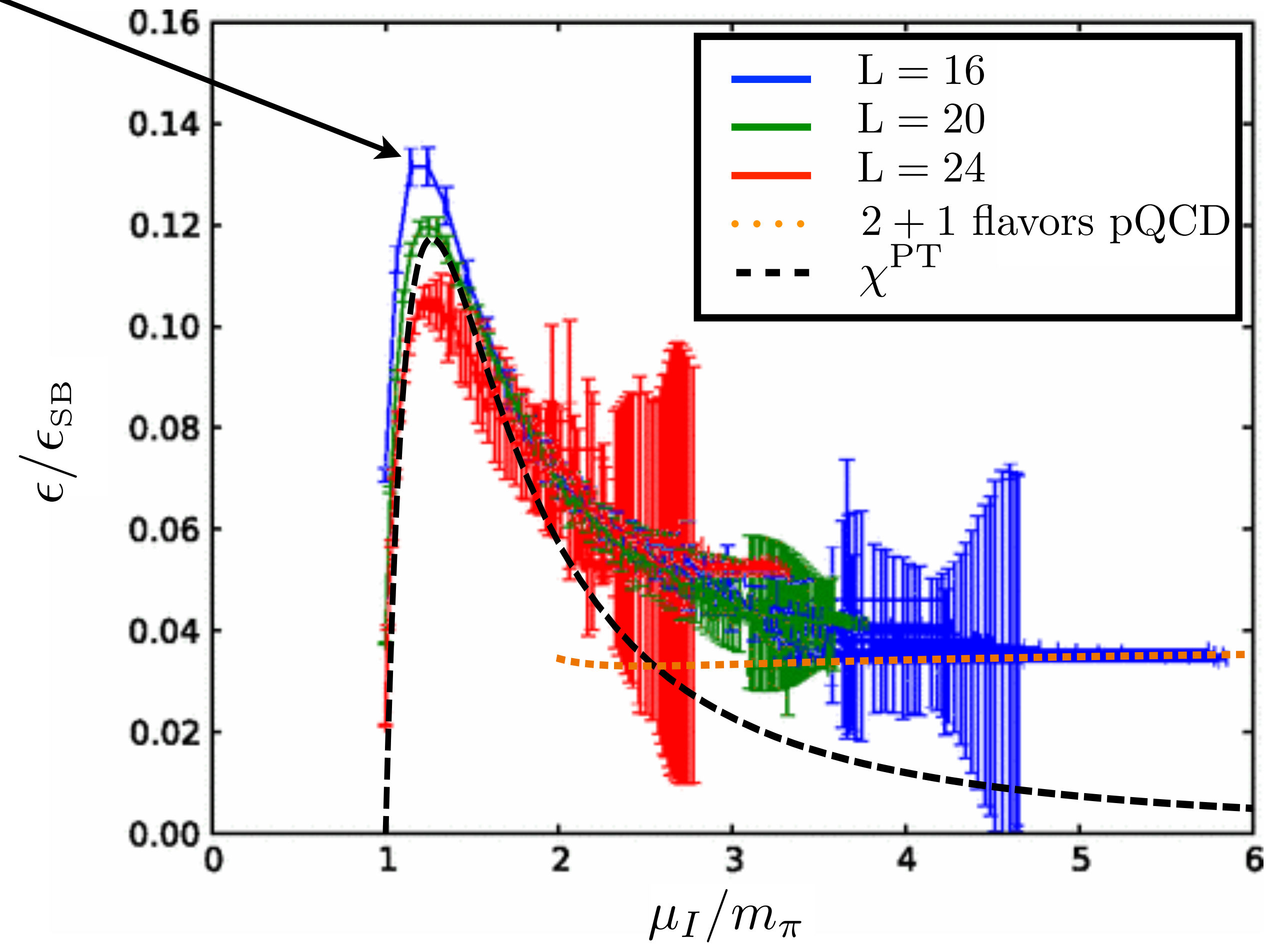

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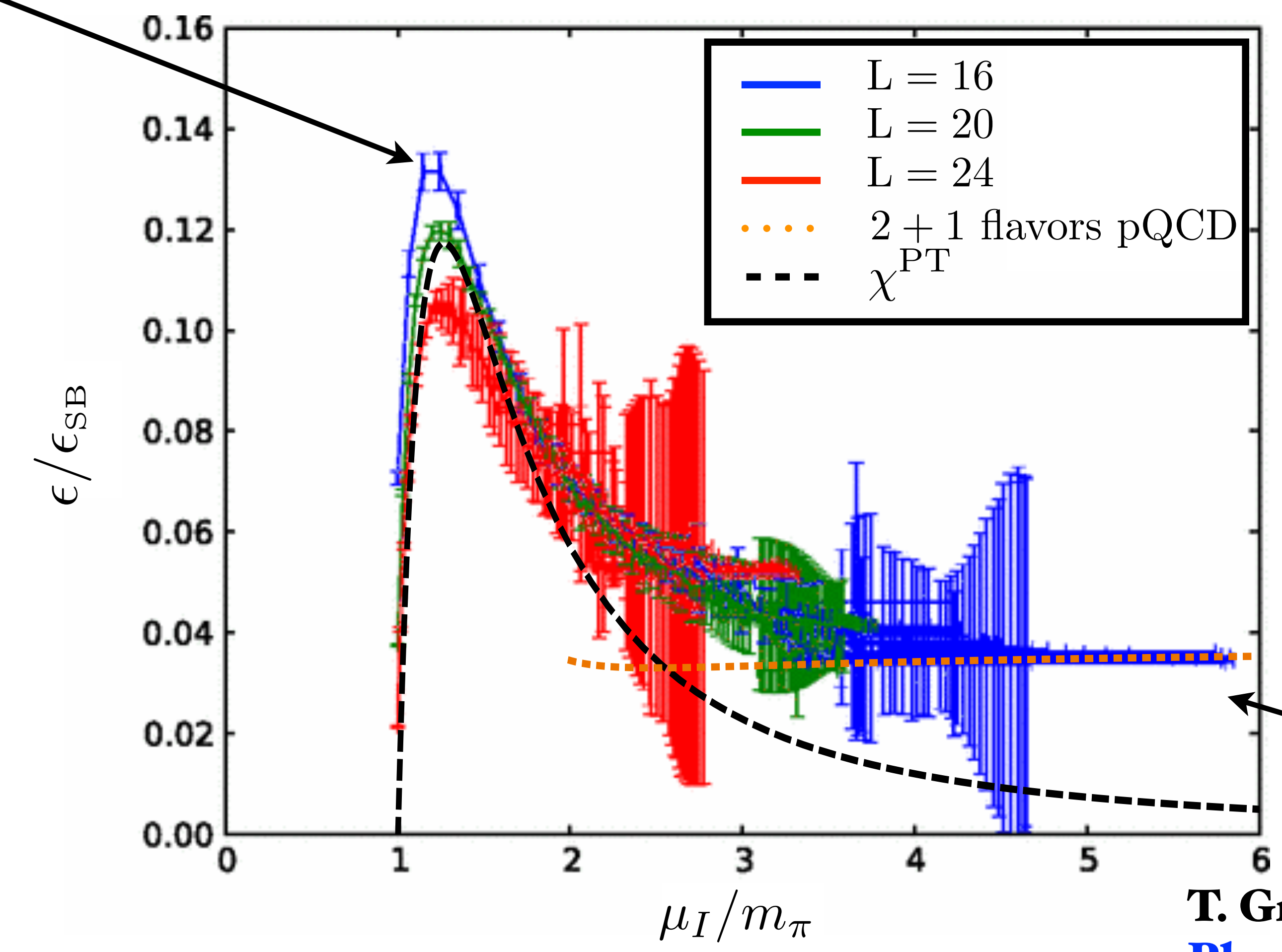

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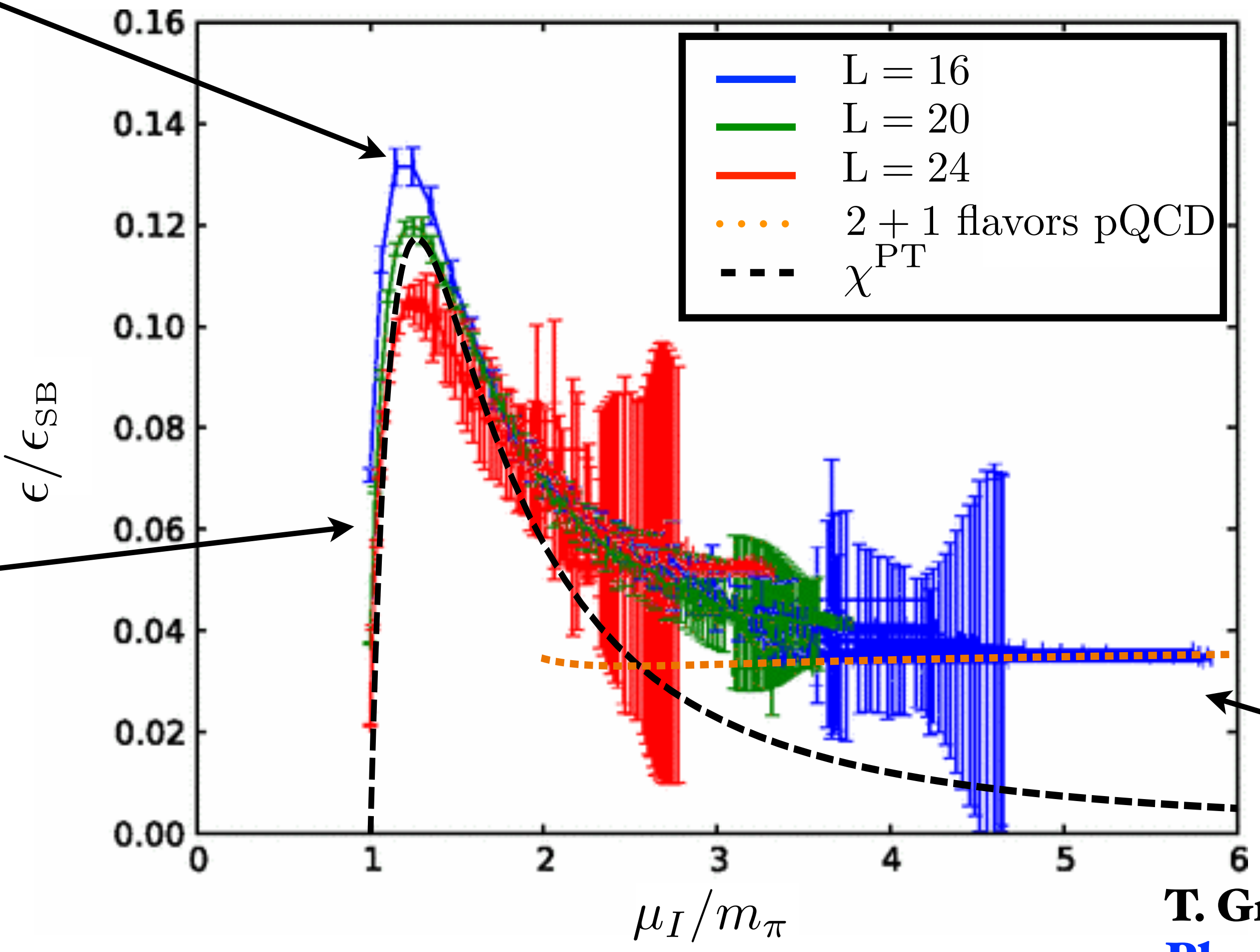

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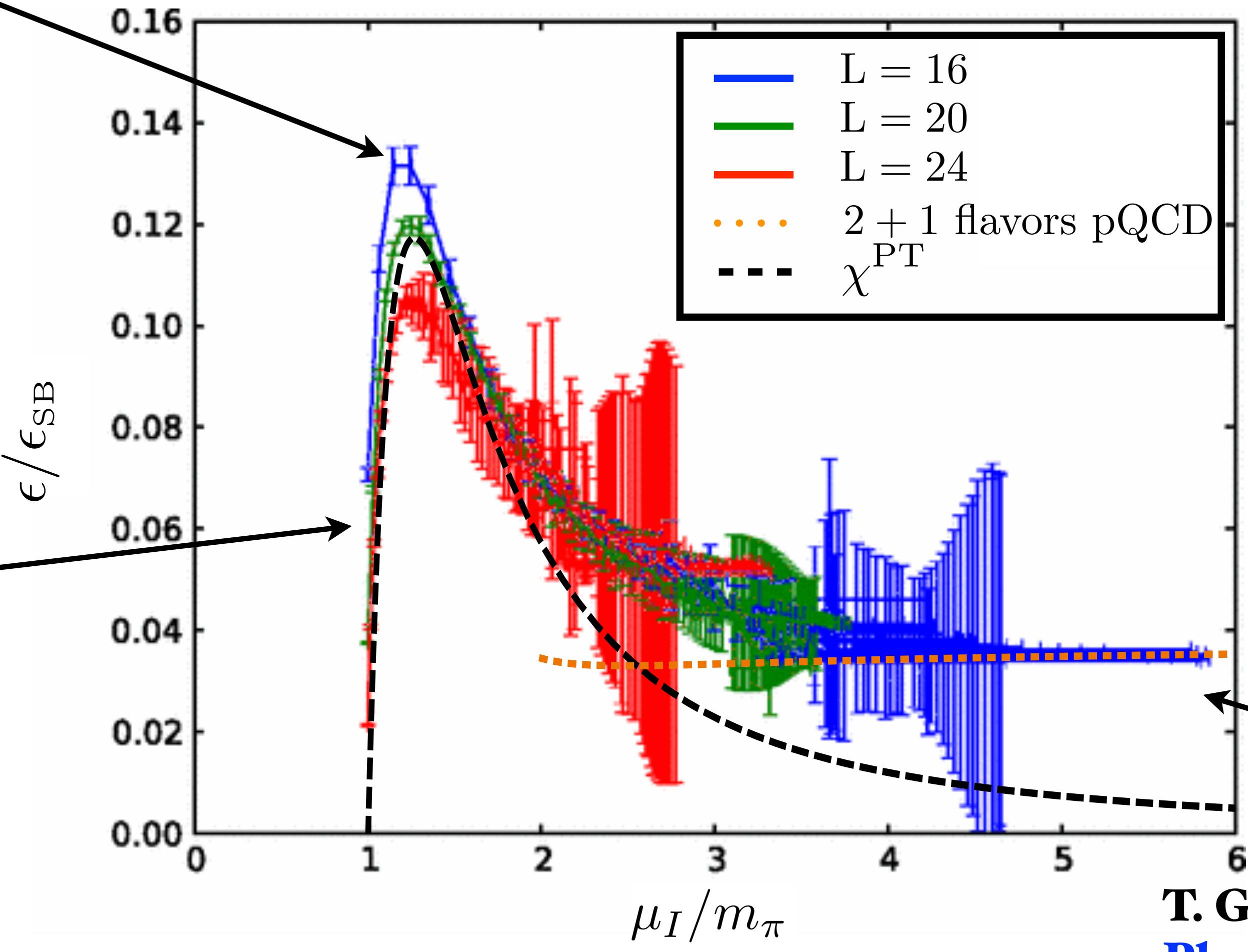

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Pions in a box

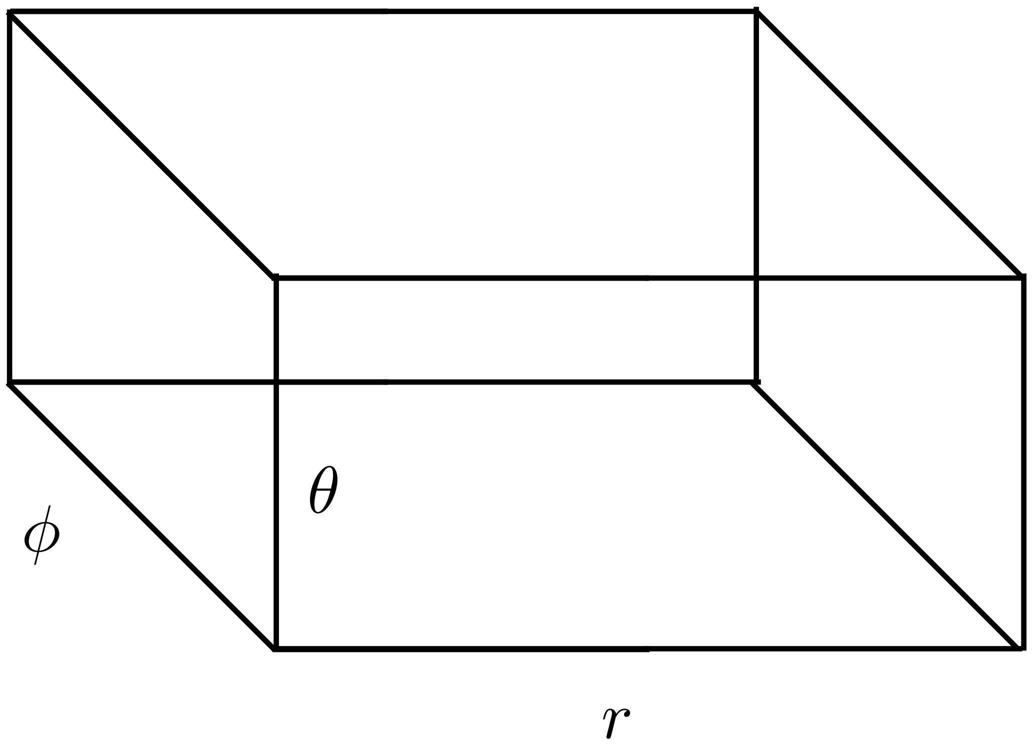
Metric

$$ds^2 = dt^2 - \ell^2 \left(dr^2 + d\theta^2 + d\phi^2\right)$$

$$\ell = \frac{b}{m_\pi}$$

$$0 \leq r \leq 2\pi \ , \quad 0 \leq \theta \leq \pi \ , \quad 0 \leq \phi \leq 2\pi$$

$$V = 4\pi^3 \ell^3$$



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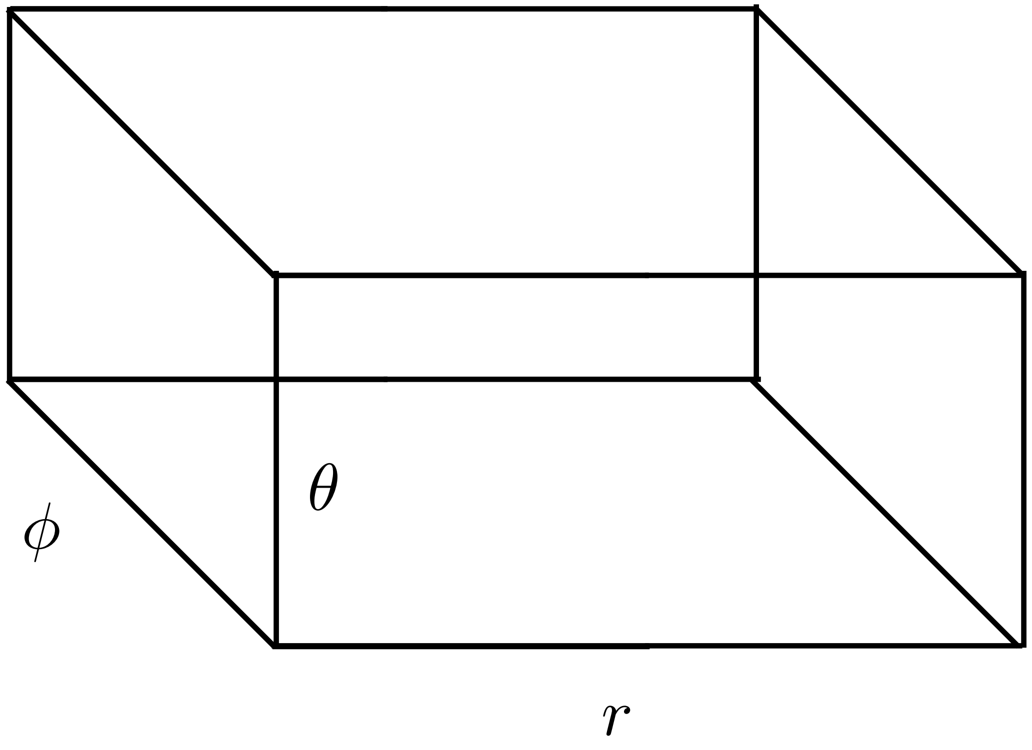


Boundary conditions

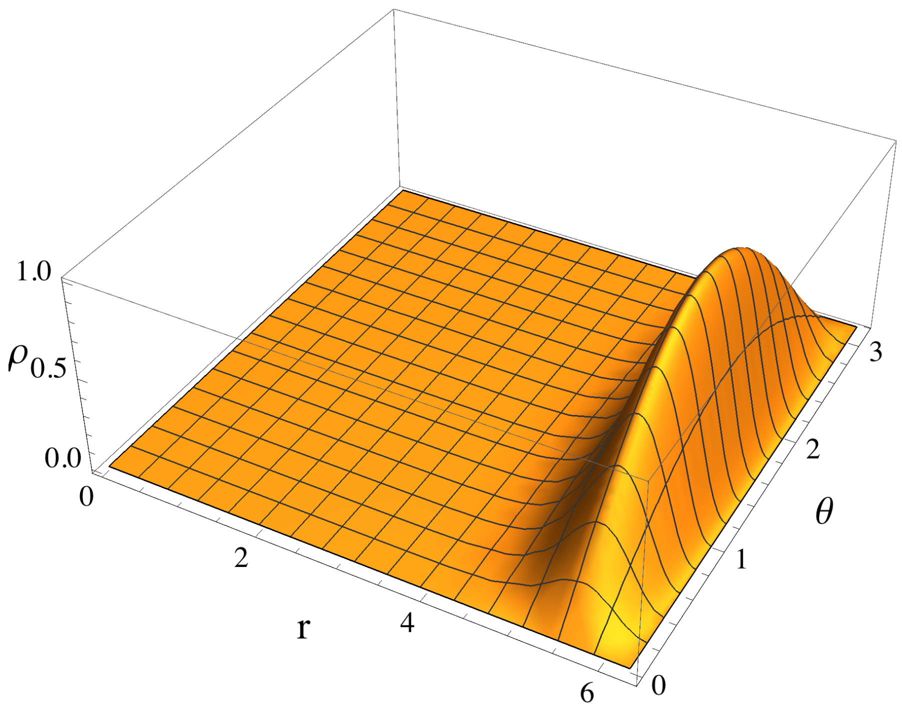
$$\Sigma(0,\theta,\phi) = \Sigma(2\pi,\theta,\phi)$$
$$n(r,0,\phi) = -n(r,\pi,\phi)$$
$$n(r,\theta,0) = n(r,\theta,2\pi)$$

different BCs can be easily implemented

Supersolid of pions

We can define a topological charge, B_m, which depends on the boundary conditions.

$B_m = 1$



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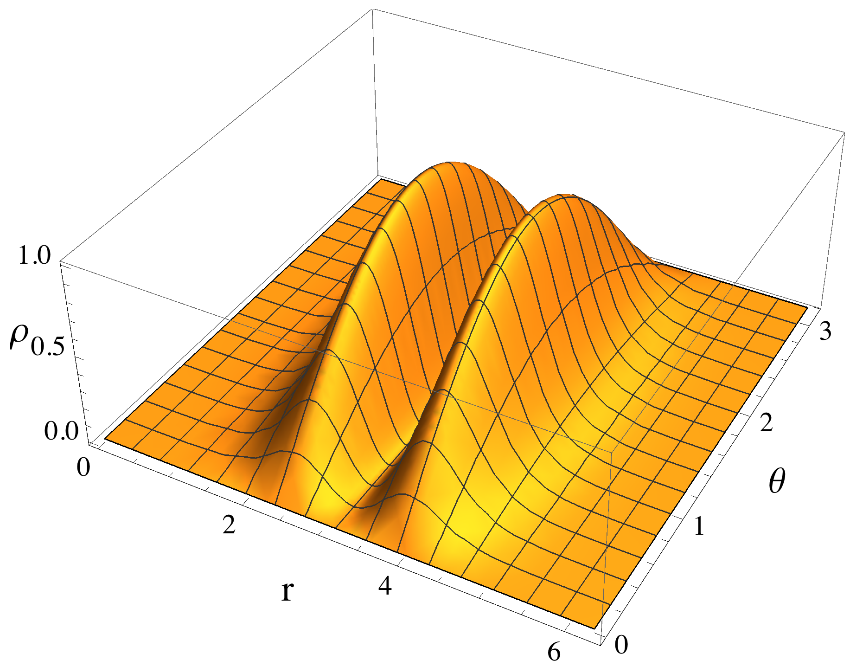

F. Canfora, S. Carignano, M. Lagos, MM, A. Vera
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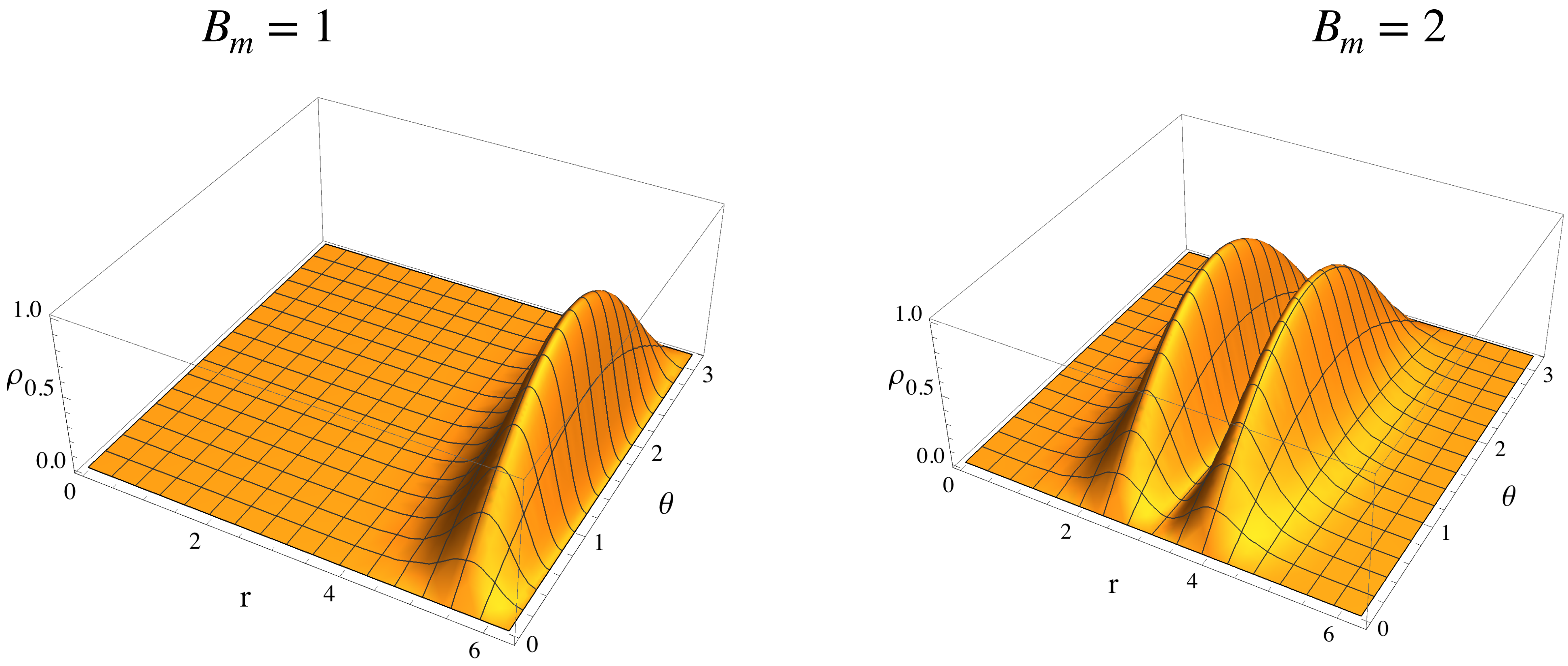

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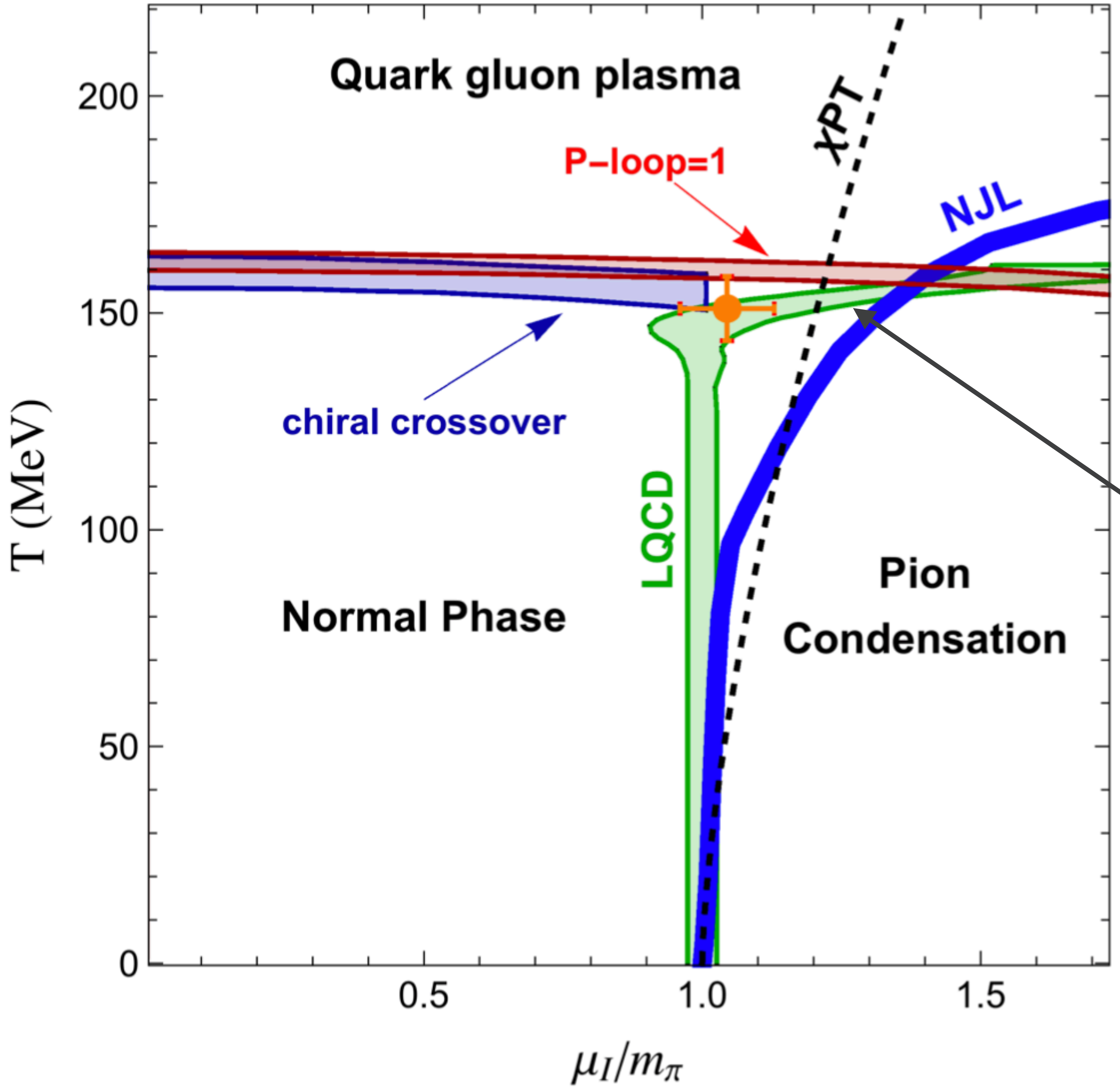
Phys.Rev.D 103 (2021) 7, 076003

T-μ_I phase diagram

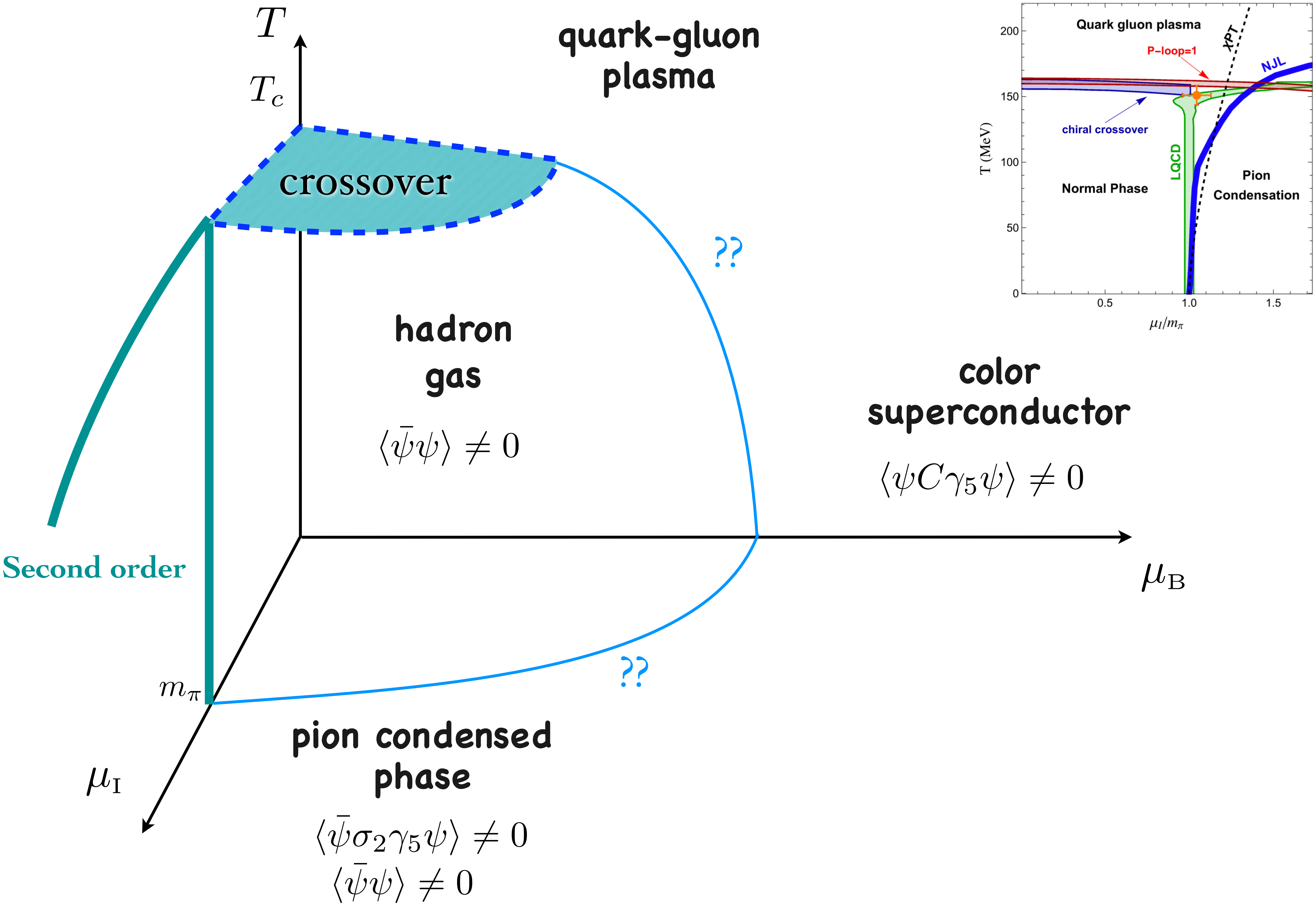


Second order

Combination of LQCD by Brandt et al, PRD 97, 054514 (2018) with effective field methods.

M.M. Particles 2 (2019) no.3, 411-443

Revisiting the QCD phase diagram



T

T_c

quark-gluon plasma

crossover

hadron gas

$\langle \bar{\psi}\psi \rangle \neq 0$

??

color superconductor

$\langle \psi C \gamma_5 \psi \rangle \neq 0$

μ_{B}

Second order

m_π

??

μ_{I}

pion condensed phase

$\langle \bar{\psi}\sigma_2\gamma_5\psi \rangle \neq 0$

$\langle \bar{\psi}\psi \rangle \neq 0$

M.M. Particles 2 (2019) no.3, 411-443

Conclusions

- **Nucleons can be thought as bubbles of trapped (confined) energy**

- **Extreme conditions help to understand quark matter**

- **There is a richness of phases**

- **We expect critical temperatures and chemical potentials for confined hadronic matter**

Thanks for your attention!

Back up

Identify the hadronic phases by quark condensates

In each phase different quark condensates are realized

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$$\langle\bar{\psi}\psi\rangle$$

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no condensate

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Each quark condensate **breaks** or **locks** the symmetries of QCD in a different way

Effective field theories

Effective field theories

If we do not use QCD we want theories that preserve (part of) its symmetries
and that are capable of describing the symmetry breaking patterns

Lattice QCD

Discretization on a lattice.
Does not work at large baryonic densities

Effective field theories

Describe global symmetries of QCD
Lack the gauge field dynamics

Qualitative picture

Any effective theory can be characterized by

1) separation scale
2) particle content
3) matching condition
4) method of regularization/cancelation of divergencies

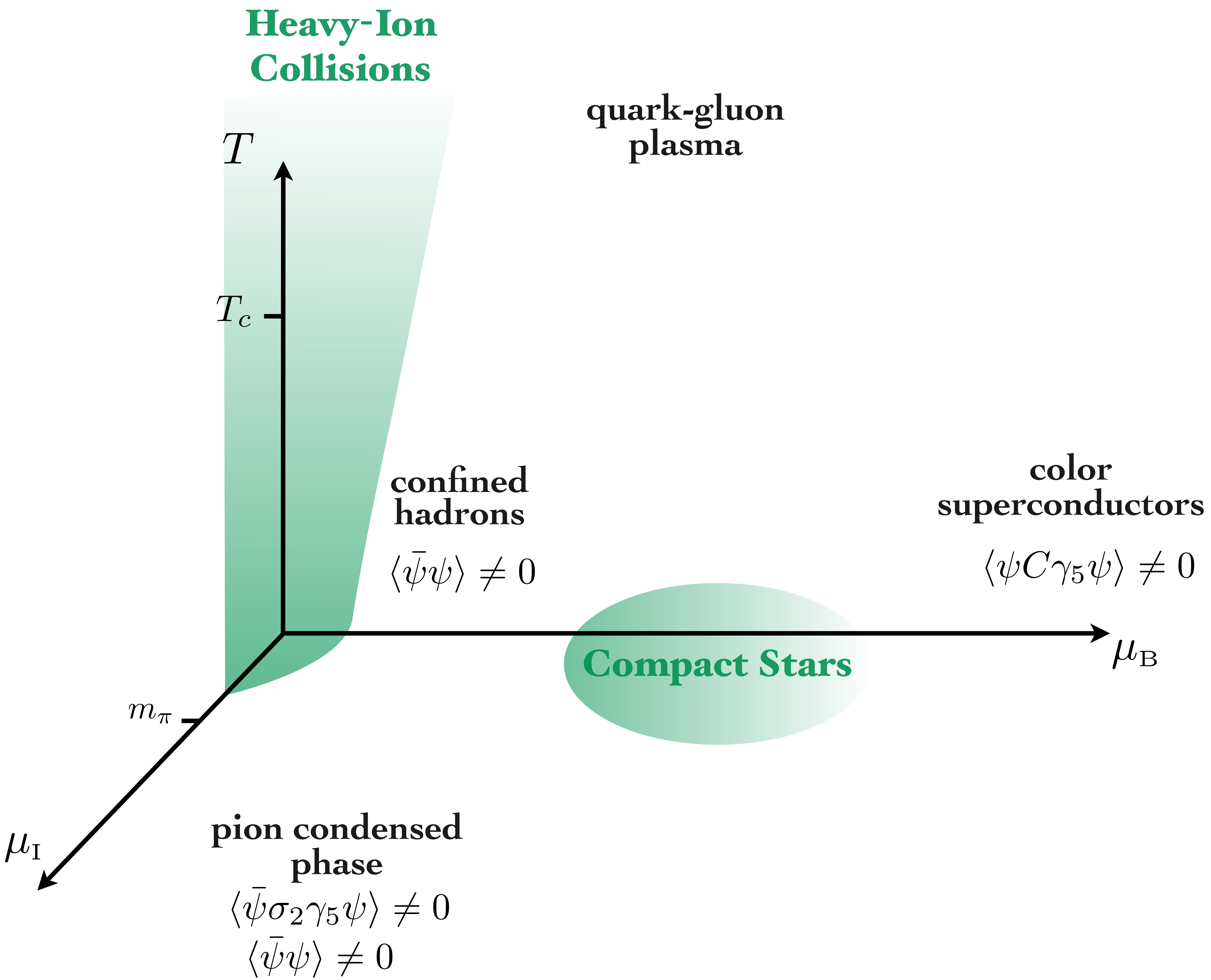
QCD is a renormalizable theory: any divergency can be removed.

This results in a theory which has been very successfully compared to experiments. No UV scale has appeared so far. In other words, if QCD is the low energy EFT of a more fundamental one, we still have not found the breaking scale.

When dealing with EFT of QCD, we always have to keep in mind that there exists a breaking scale. The scale is associated to a change of degrees of freedom or to an internal inconsistency of the EFT.
Example: chiral perturbation theory is a low-energy theory with breaking scale $\Lambda_\chi \sim 1\ \mathrm{GeV}$

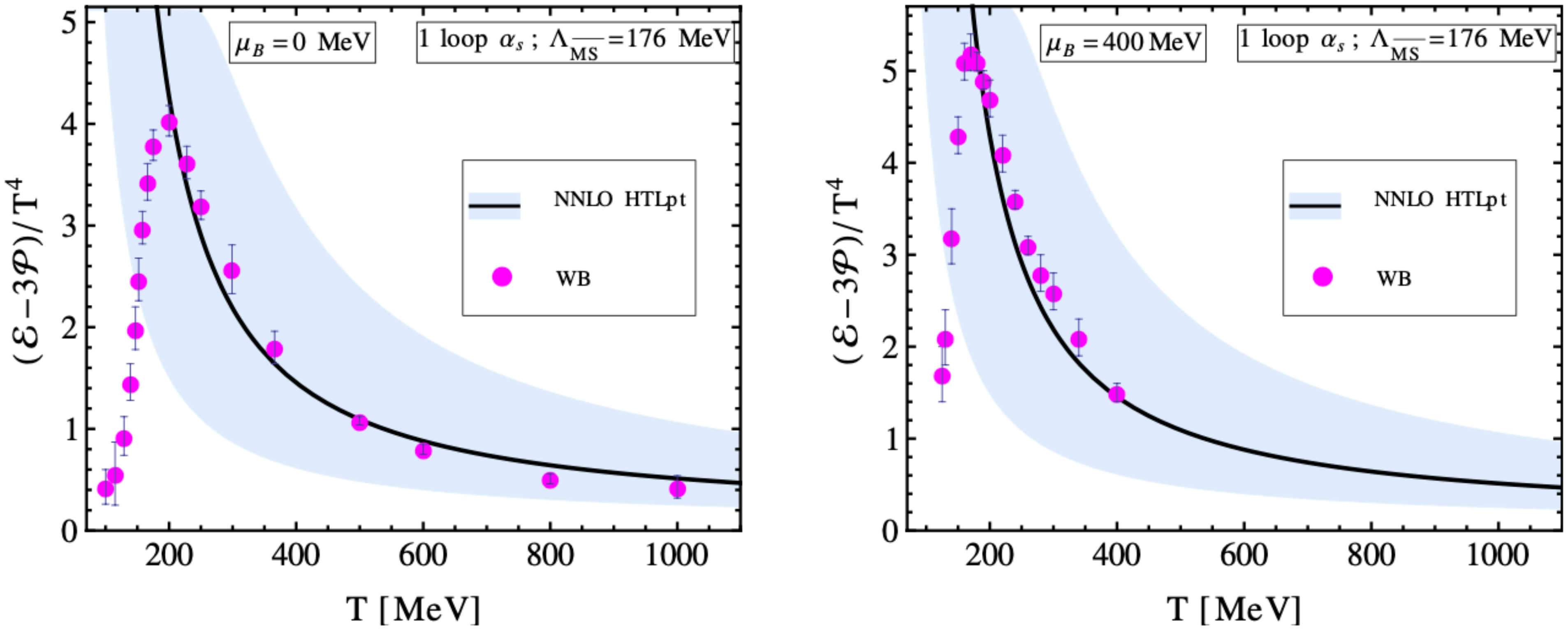
Beyond this point one has to consider the mesonic resonances, baryons and then quarks and gluons. Which means changing the degrees of freedom, of interaction etc. Thi is not impossible, it is only extremely hard and does not seem to be simpler than solving QCD itself.

The QCD phase diagram



Hard thermal loop (HTL)

Resummed perturbation theory



Pion condensation

More on the method

- **The $\mathcal{O}(p^2)$ Lorentz invariant chiral Lagrangian density for pions**

$$\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{F_0^2 m_\pi^2}{2}\mathrm{Tr}(\Sigma)$$

- $\mathrm{SU}(2)$ **variational vev**

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

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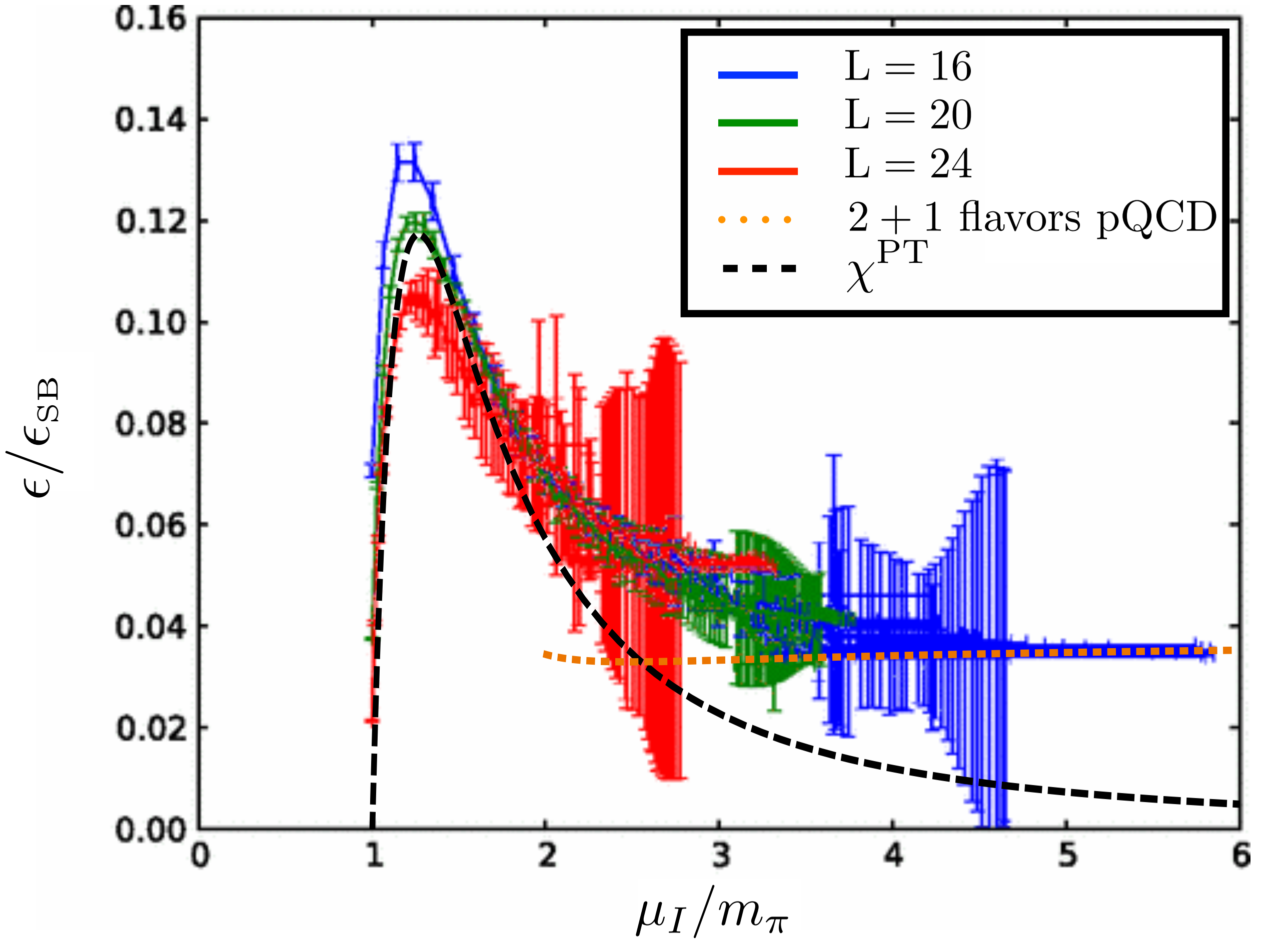

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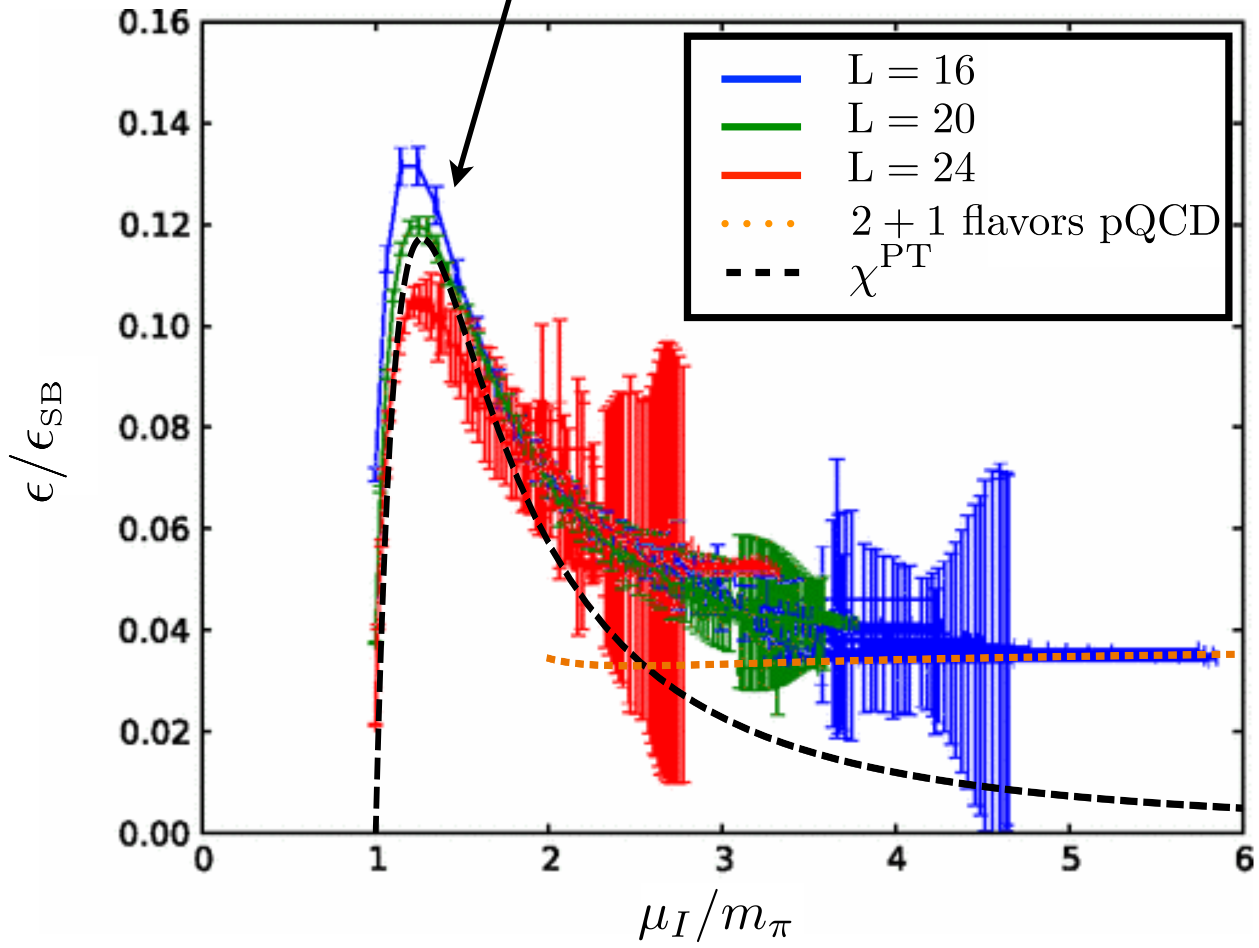

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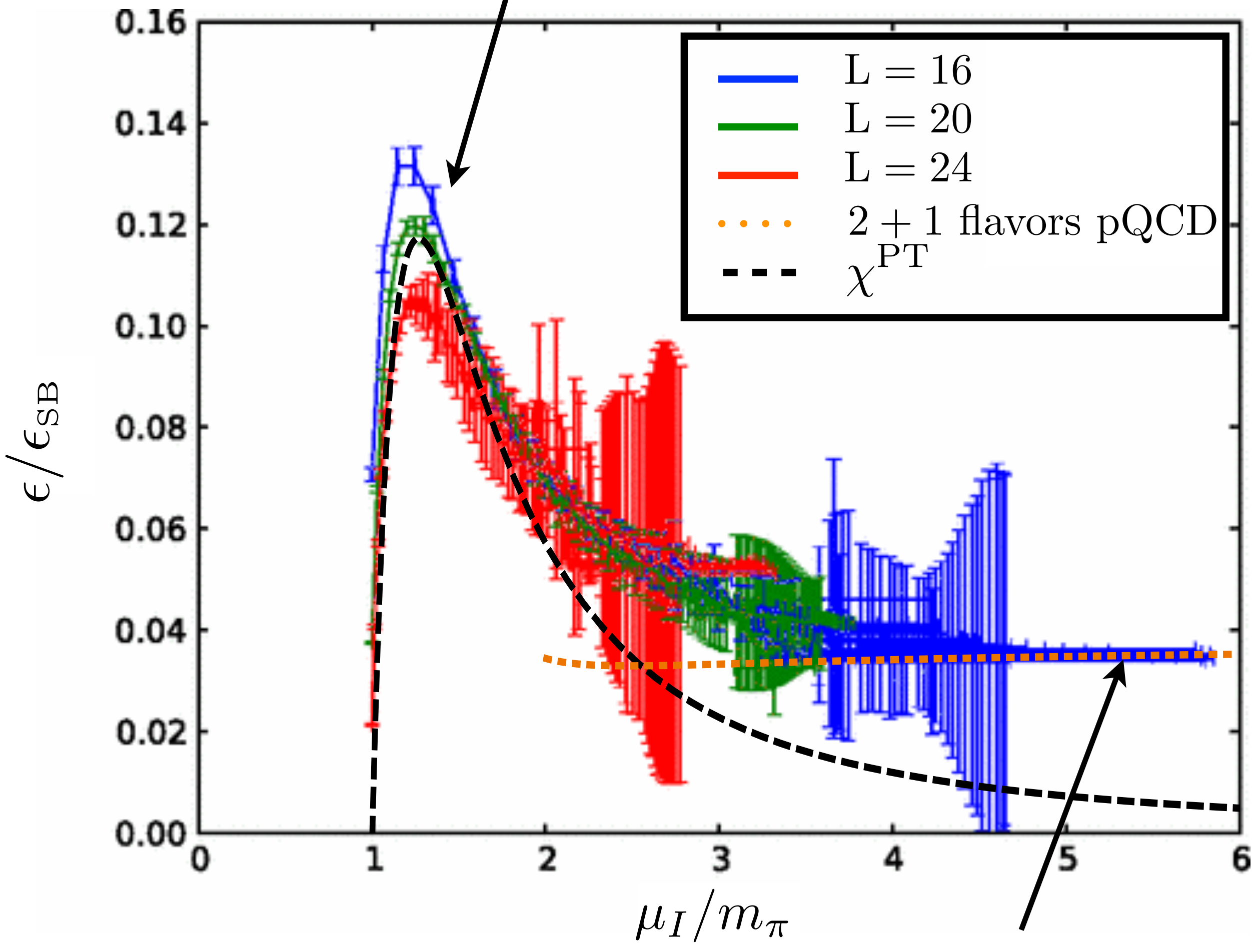

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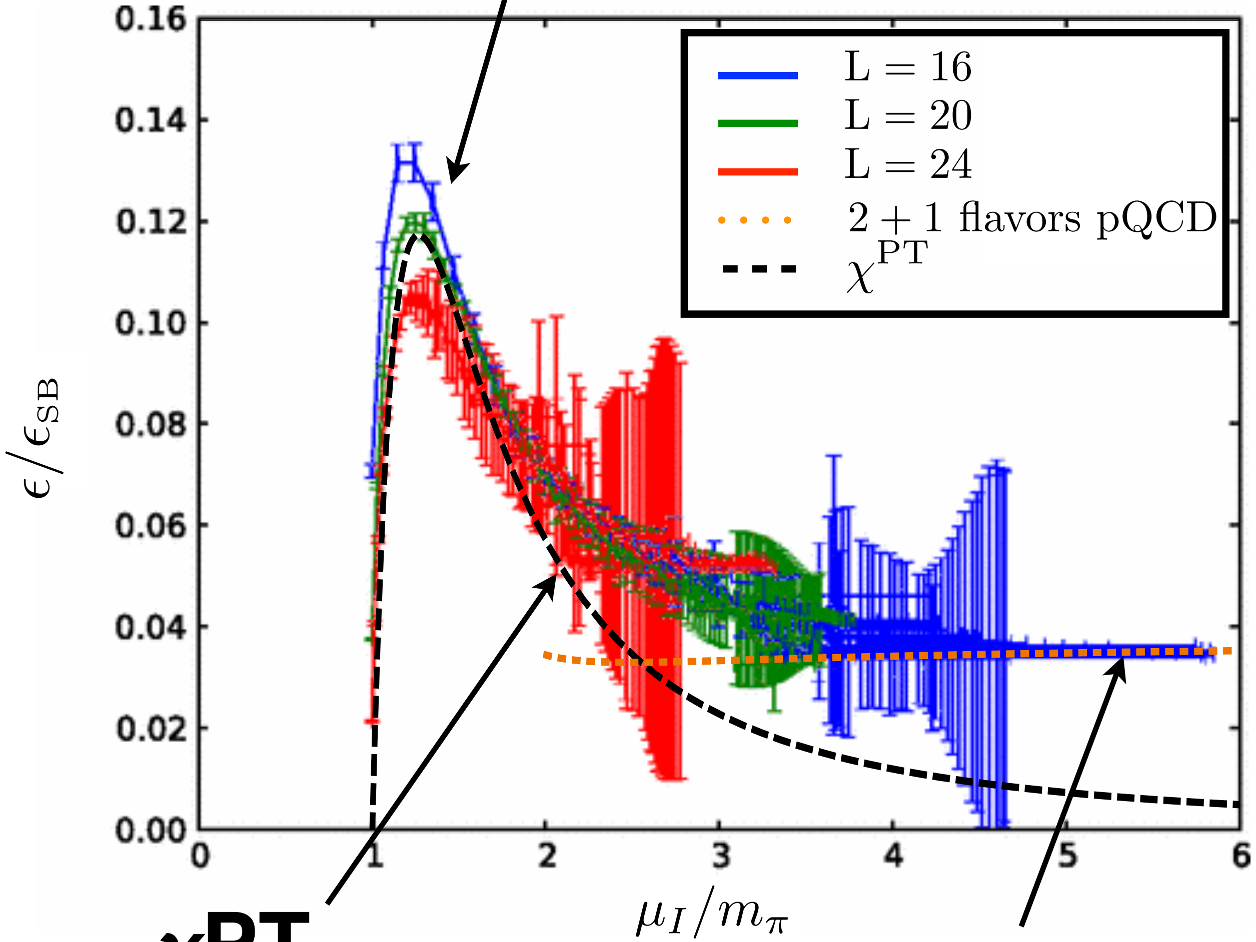

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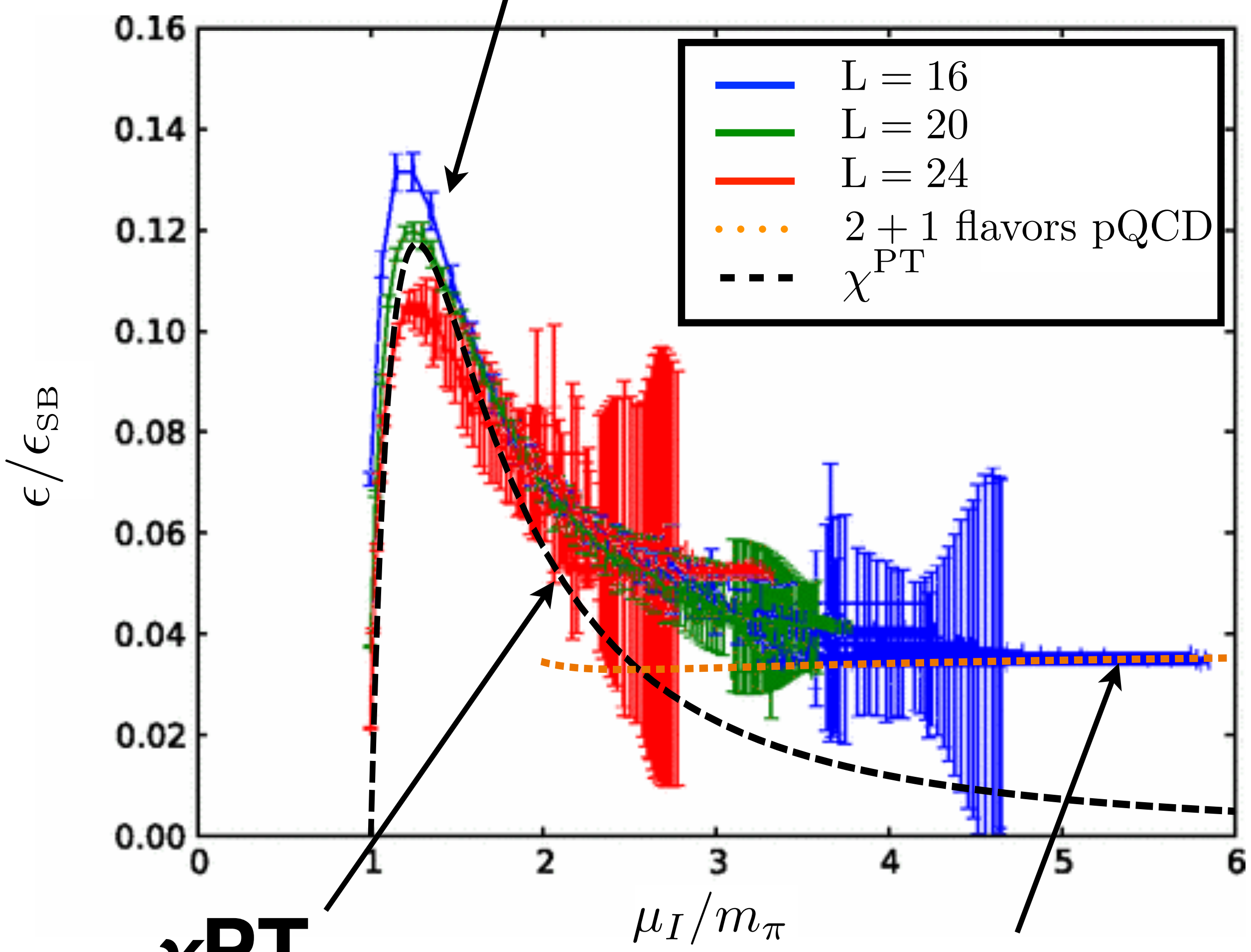

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Static Lagrangian

$$\mathcal{L}_0(\alpha, \mu_I, n_3) = F_0^2 m_\pi^2 \cos\alpha + \frac{F_0^2}{2}\mu_I^2 \sin^2\alpha(1 - n_3^2)$$

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Maximising the Lagrangian

for $\mu_I < m_\pi$	$\cos\alpha = 1$	$\mathcal{L}_0$ independent of $\boldsymbol{n}$
for $\mu_I > m_\pi$	$\cos\alpha_\pi = m_\pi^2/\mu_I^2$	$n_3 = 0$ residual $O(2)$ symmetry

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We now look for solutions in which the rotation is local

More about the leading order Lagrangian

The $\mathcal{O}(p^2)$Lorentz invariant Lagrangian density for pions

$$\boxed{\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{F_0^2 m_\pi^2}{2}\mathrm{Tr}(\Sigma)}$$

Trick for introducing the isospin. We define the covariant derivative

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2}[v_\mu, \Sigma]$$

**Gasser and Leutwyler,
Annals Phys. 158, 142 (1984)**

Formally preserving the Lorentz invariance

Then we take $v^\mu = \mu_I \, \sigma_3 \, \delta^{\mu 0}$

BEC of pions!

Rotated condensates

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \propto \cos\alpha$$
$$\langle \bar{d}\gamma_5 u + \text{h.c.} \rangle \propto \sin\alpha$$

Control parameter $$\gamma = \frac{\mu_I}{m_\pi}$$

Pressure

$$P = \frac{f_\pi^2 m_\pi^2}{2}\gamma^2\left(1 - \frac{1}{\gamma^2}\right)^2$$

Ground state occupation number

$$n_I = f_\pi^2 m_\pi \gamma \left(1 - \frac{1}{\gamma^4}\right)$$

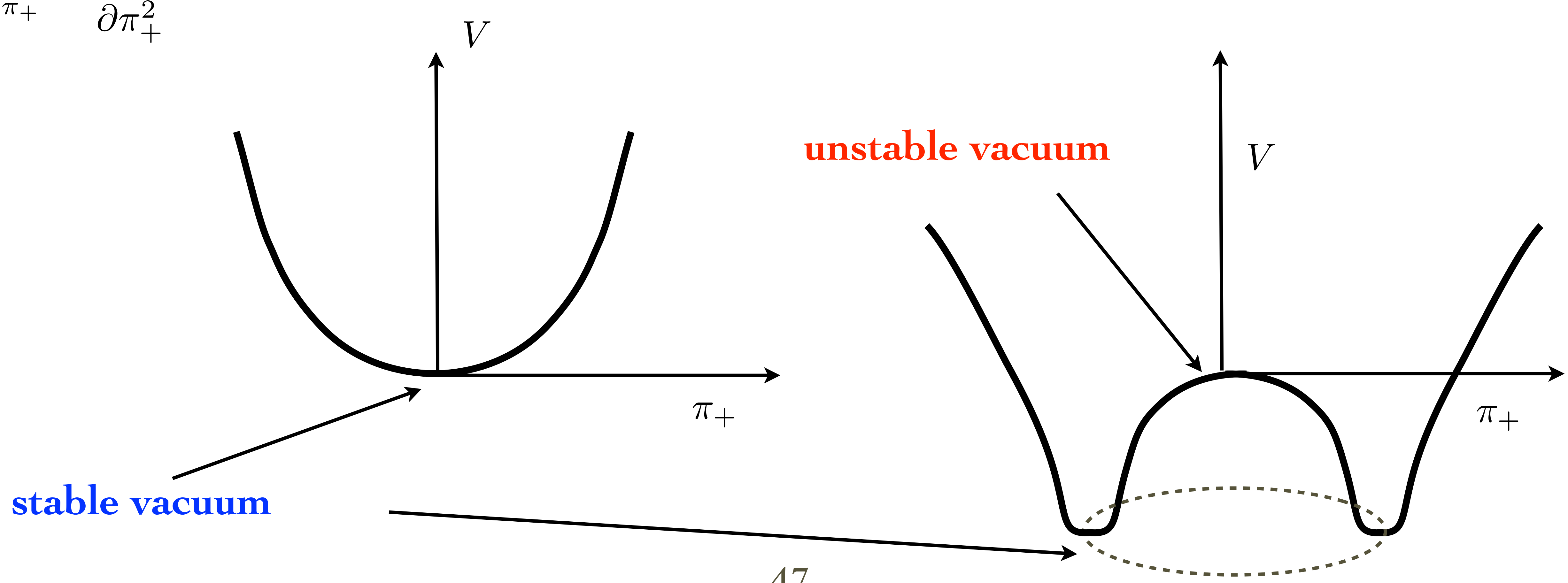
Pion fluctuations

Mass splitting
proportional to the isospin charge

$$m_{\pi^0} = m_\pi$$
$$m_{\pi^-} = m_\pi + \mu_I$$
$$m_{\pi^+} = m_\pi - \mu_I$$

The meson mass vanishes at the phase transition

$$m^2_{\pi_+} \sim \frac{\partial^2 V}{\partial \pi_+^2}$$

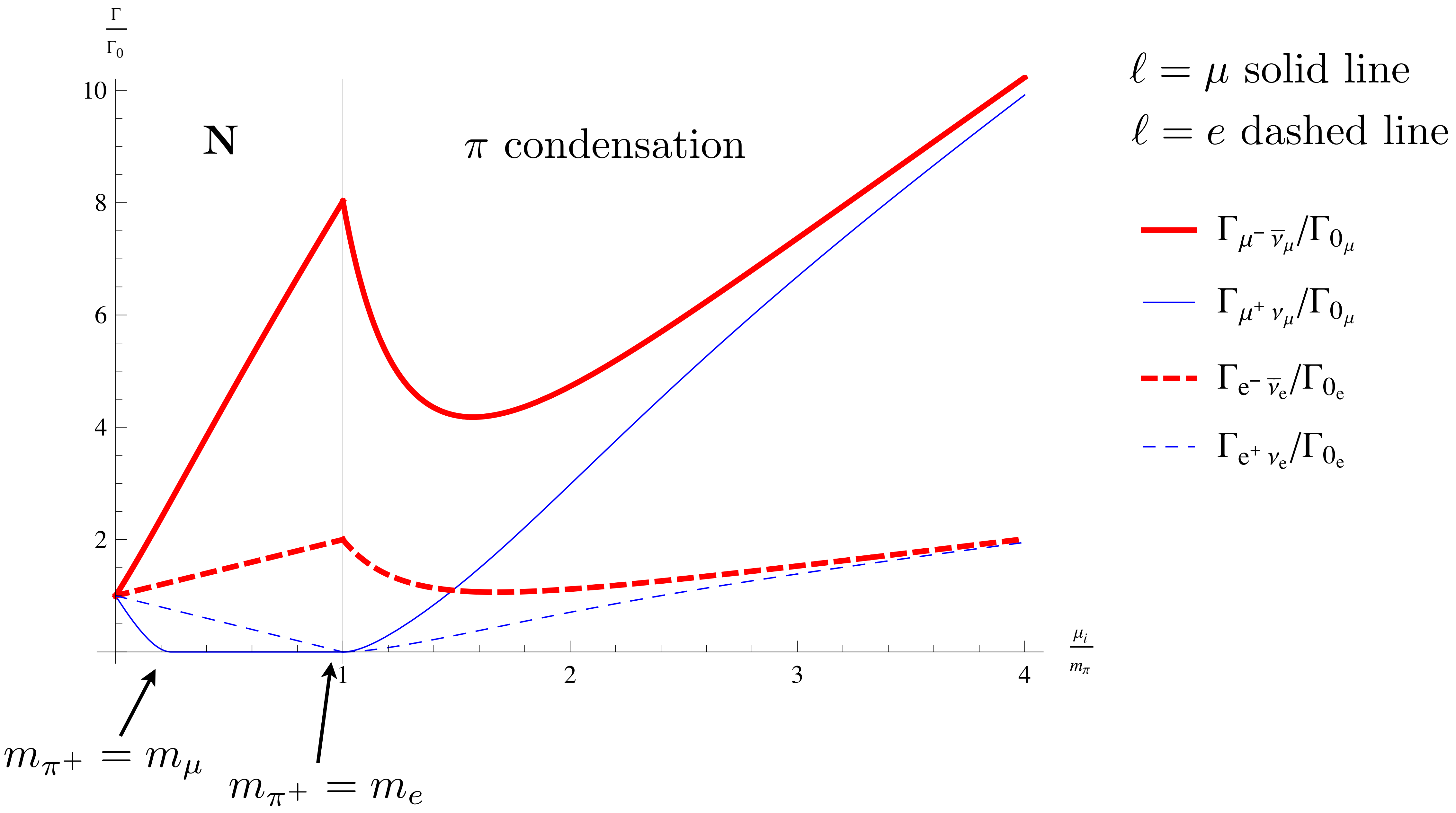


unstable vacuum

stable vacuum

Leptonic decays

Processes $\tilde{\pi}_- \to \ell^\pm \nu_\ell$**and** $\tilde{\pi}_+ \to \ell^\pm \nu_\ell$



$\ell = \mu$ solid line

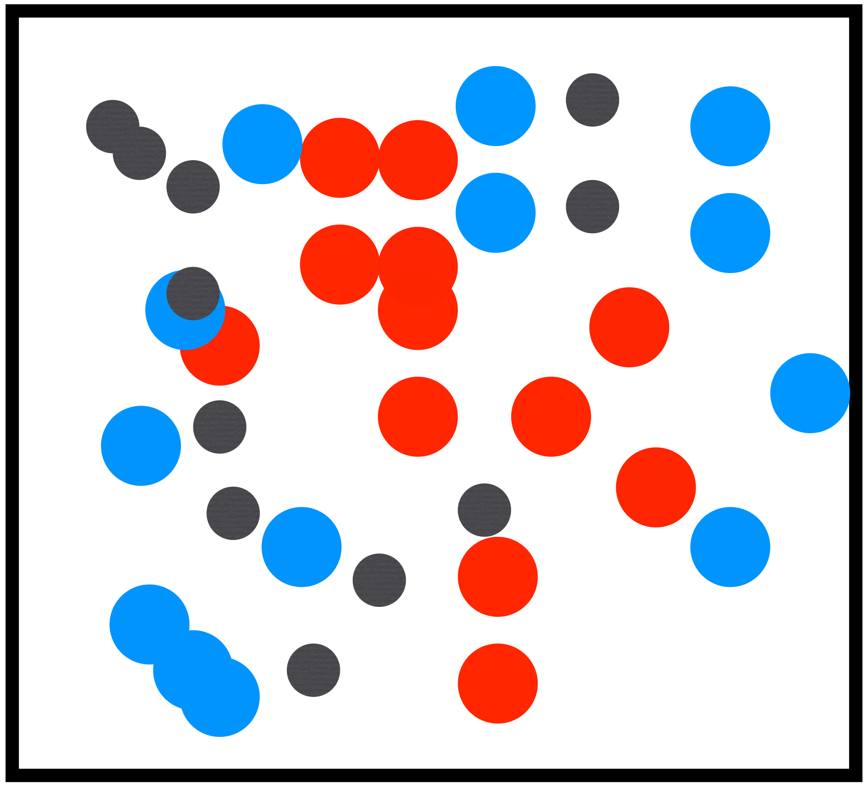
$\ell = e$ dashed line

A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025

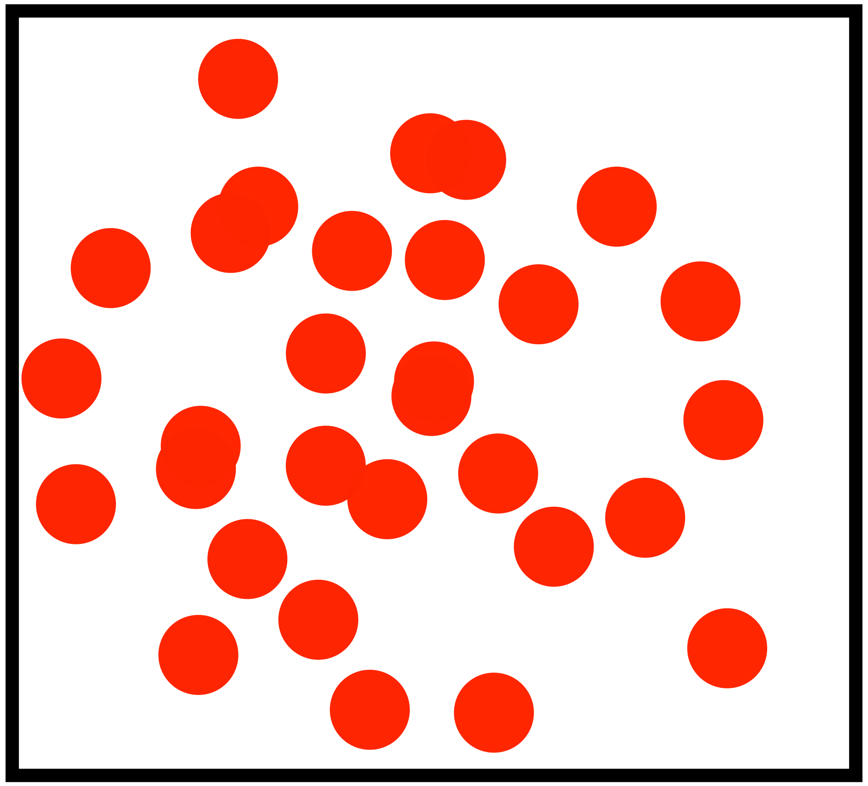
Deconfinement by increasing temperature

Pions ●　　Baryons ●　　Anti Baryons ●

Increasing T
Fixed low μ_B



Fixed Low T
Increasing μ_B



Deconfinement by increasing temperature

Pions ● Baryons ● Anti Baryons ●

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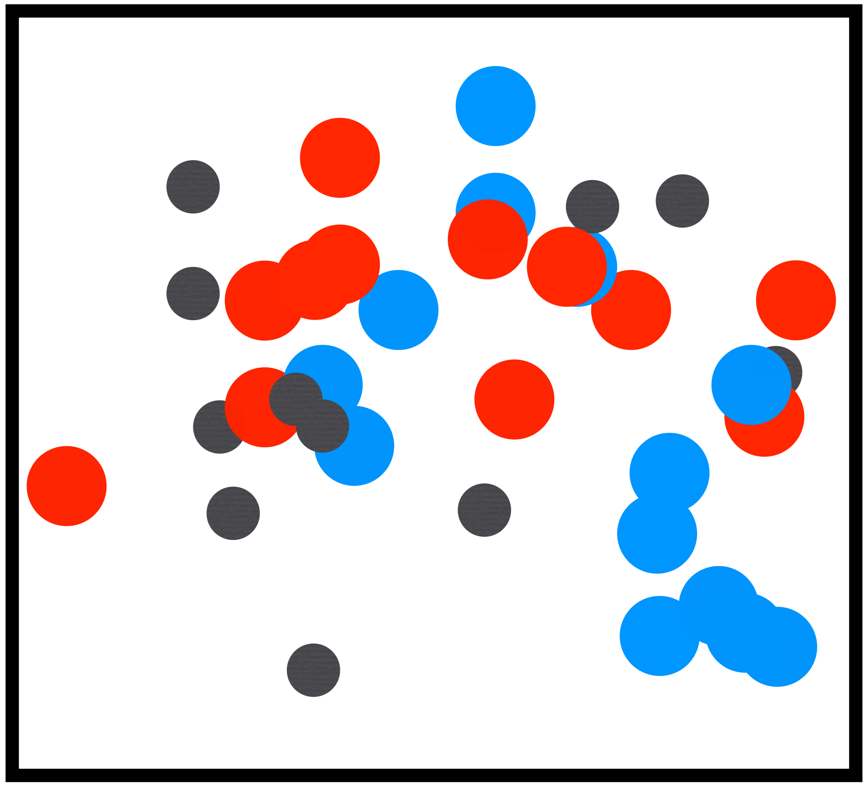
Fixed low μ_B



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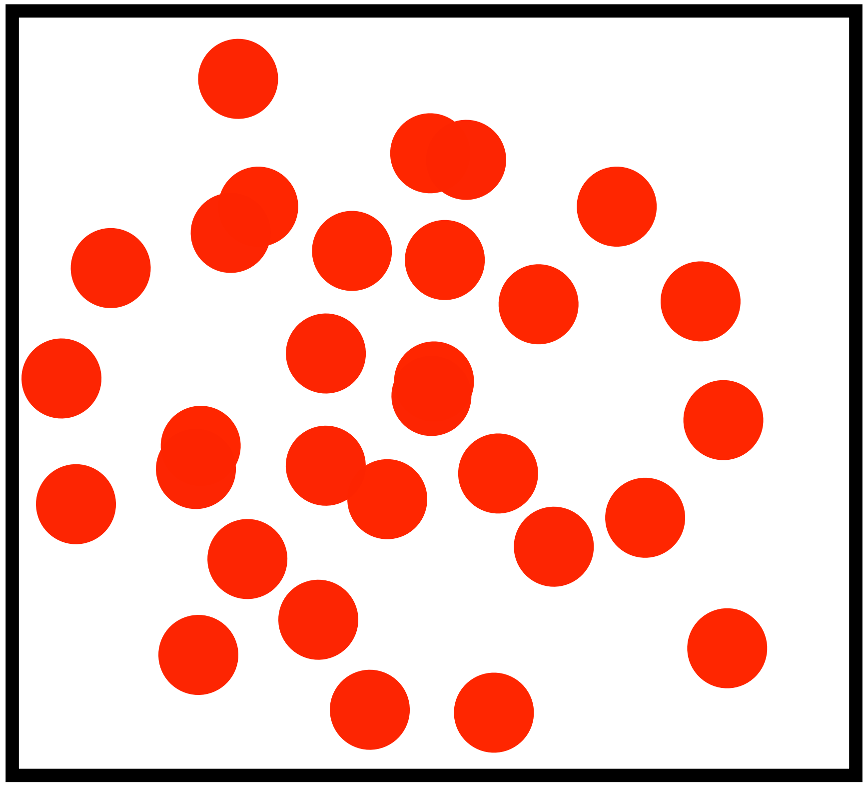

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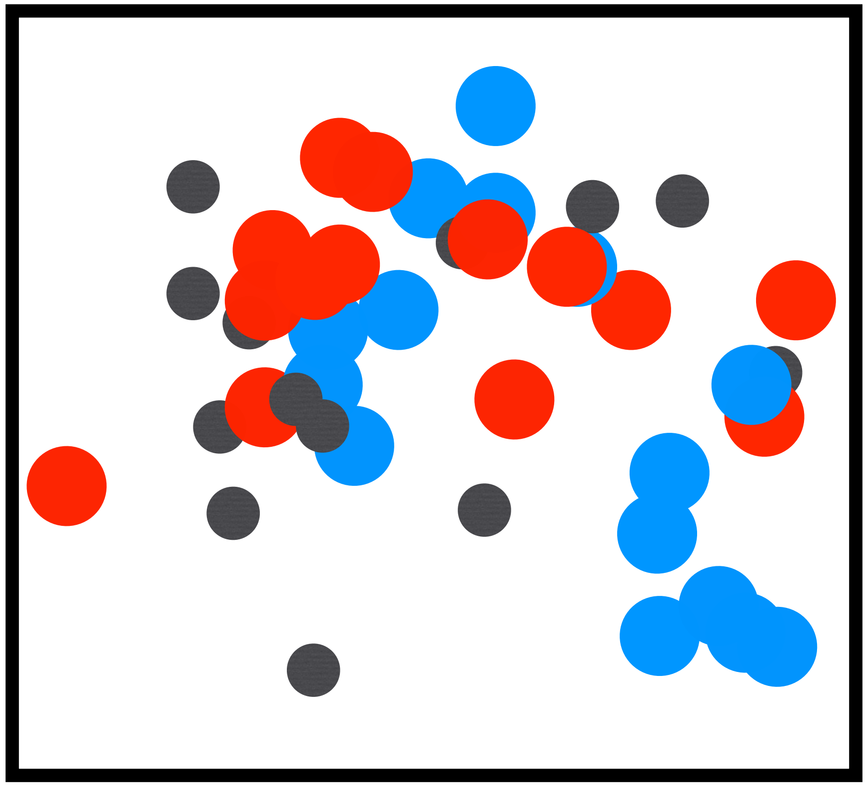

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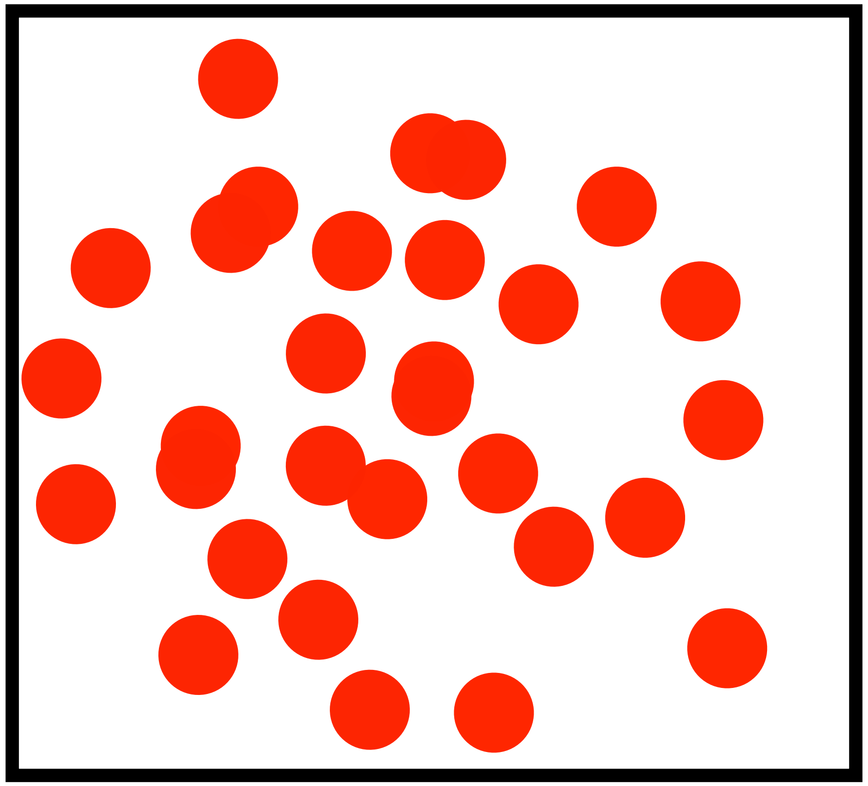

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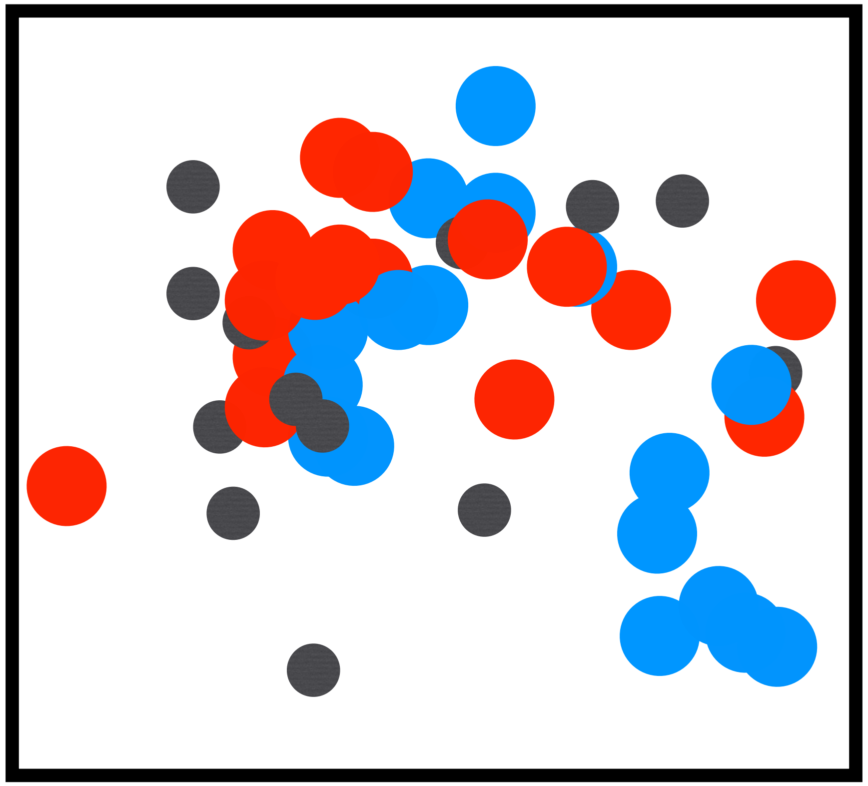

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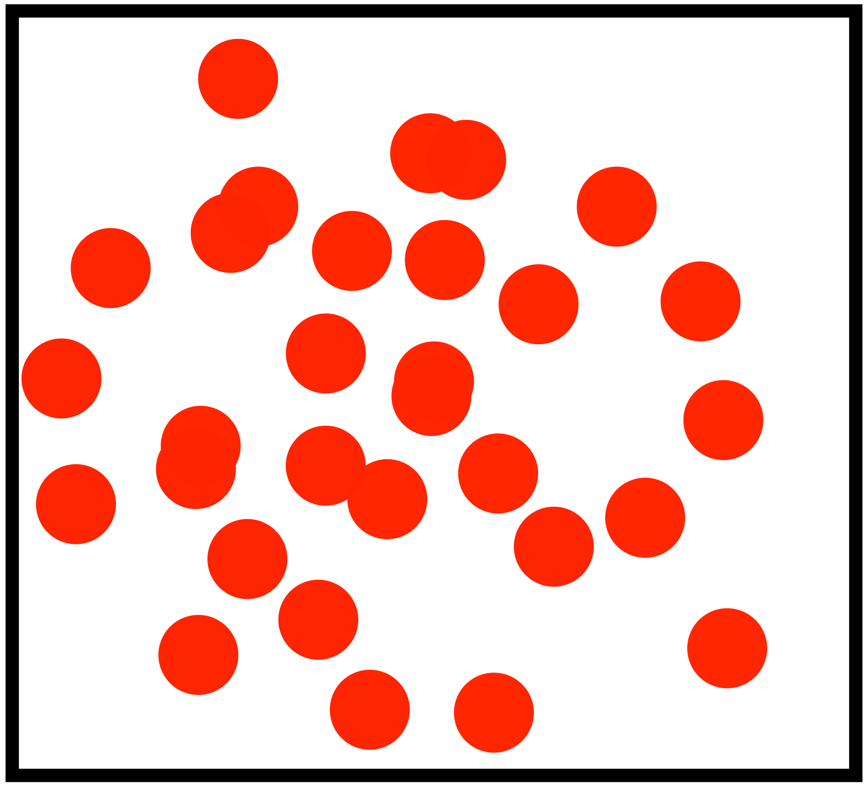

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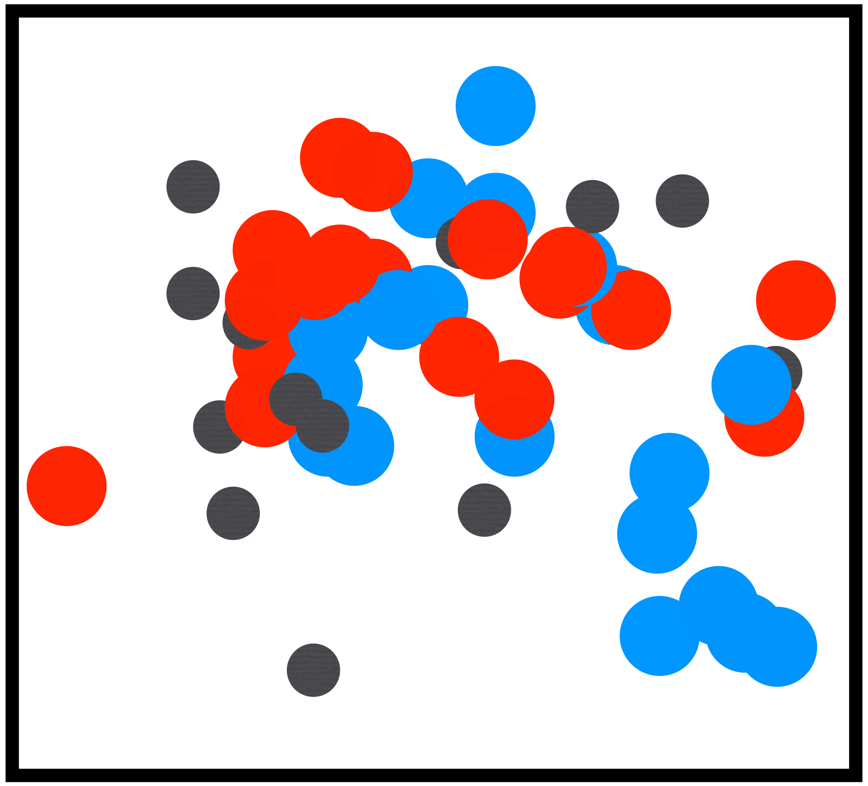

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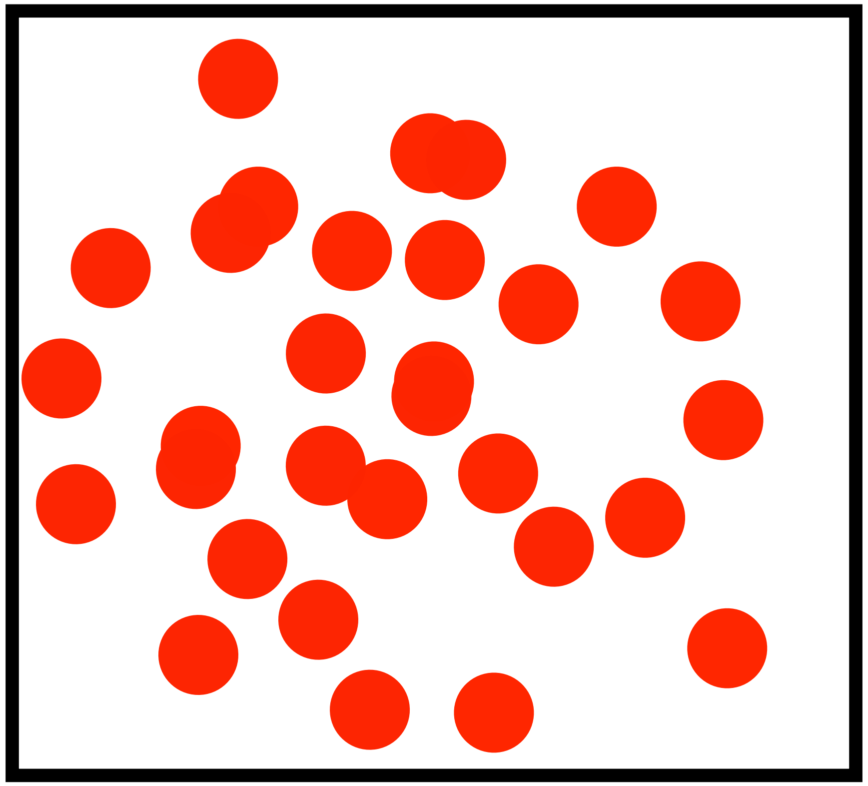

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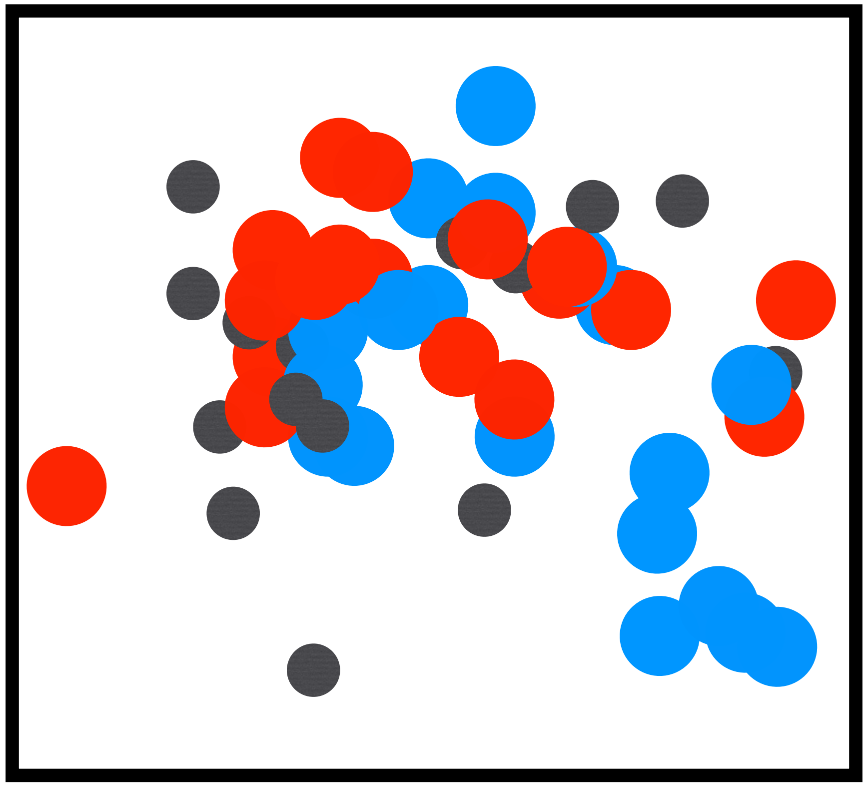

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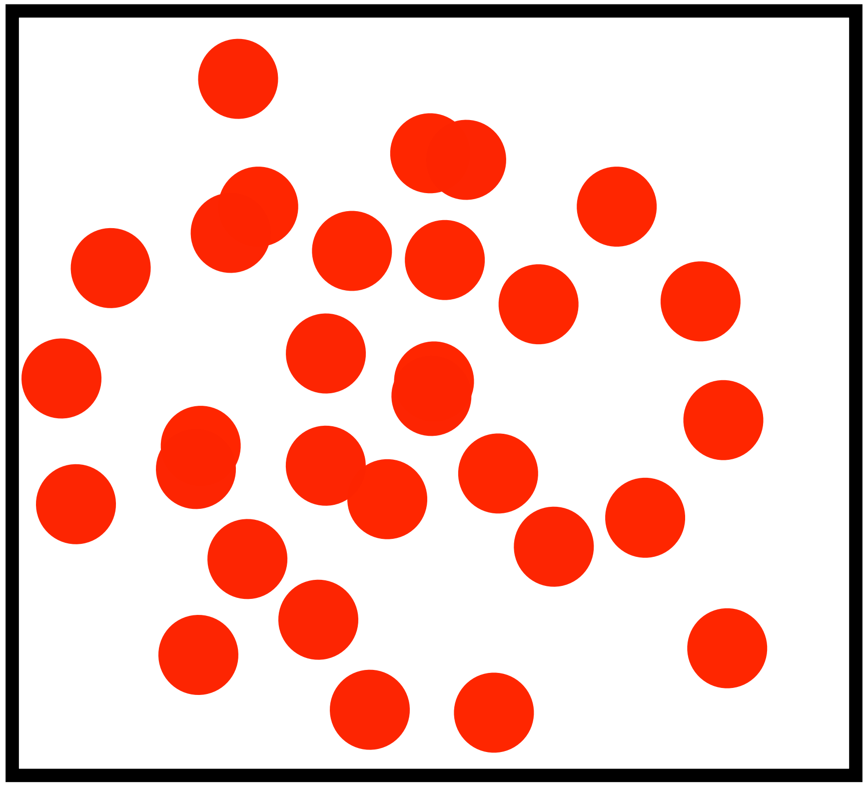

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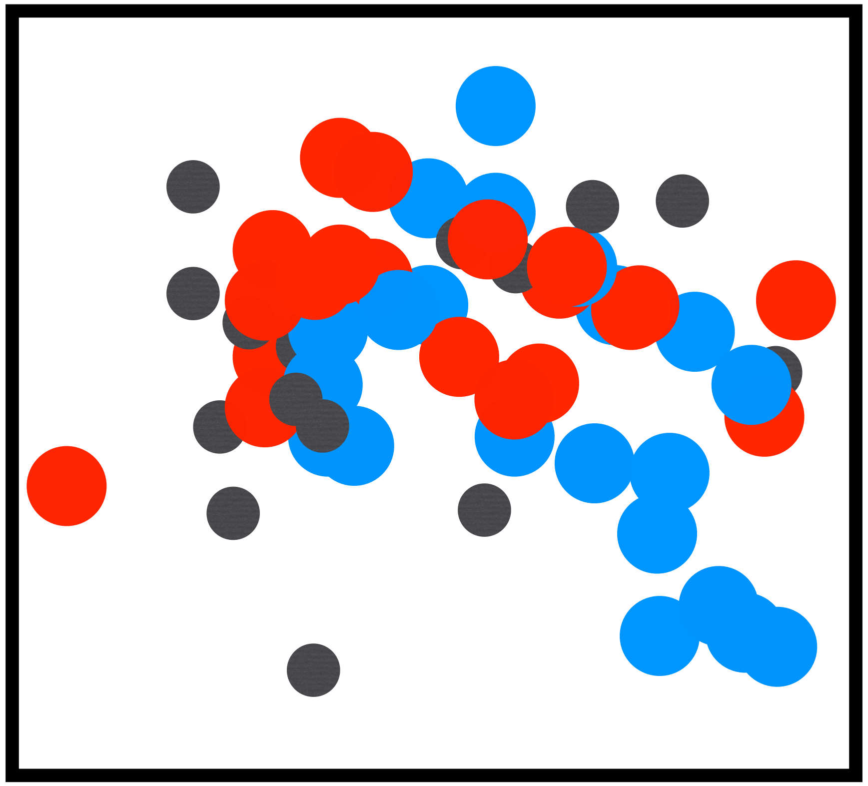

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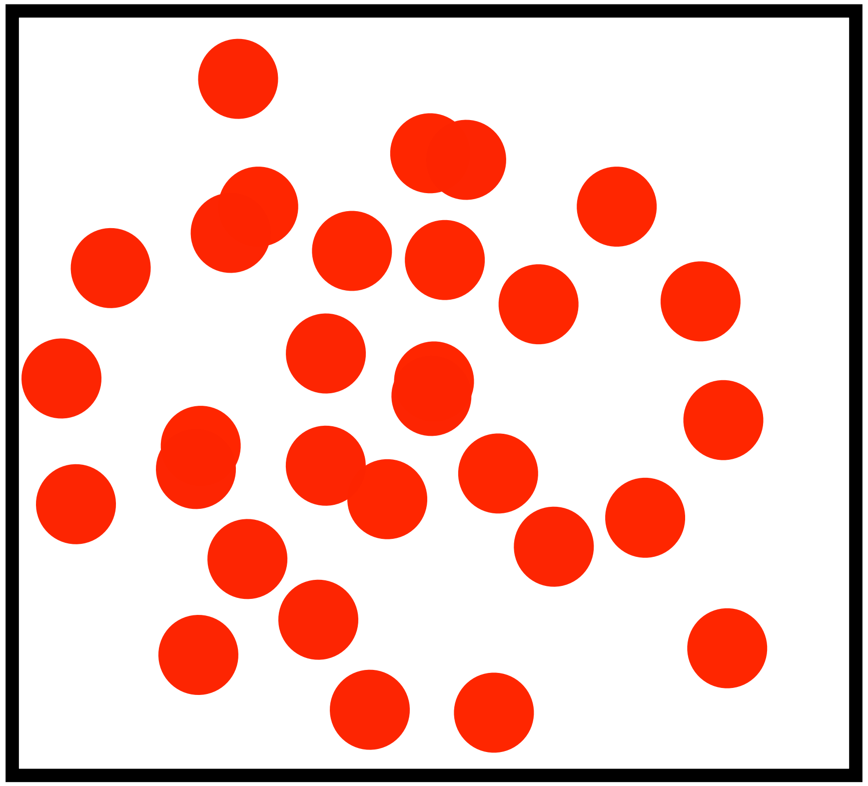

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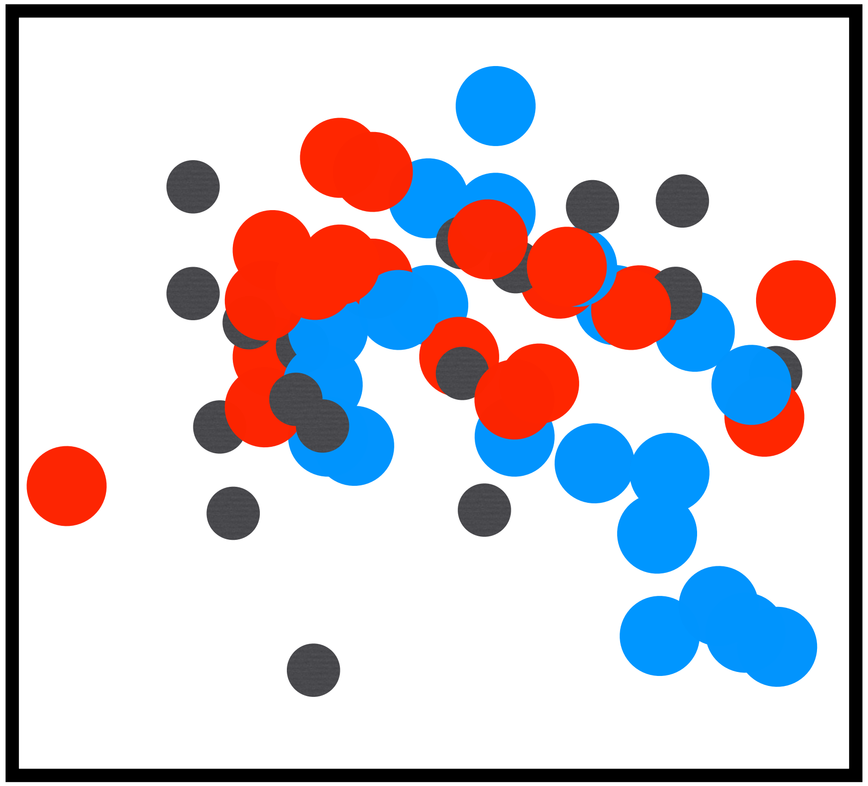

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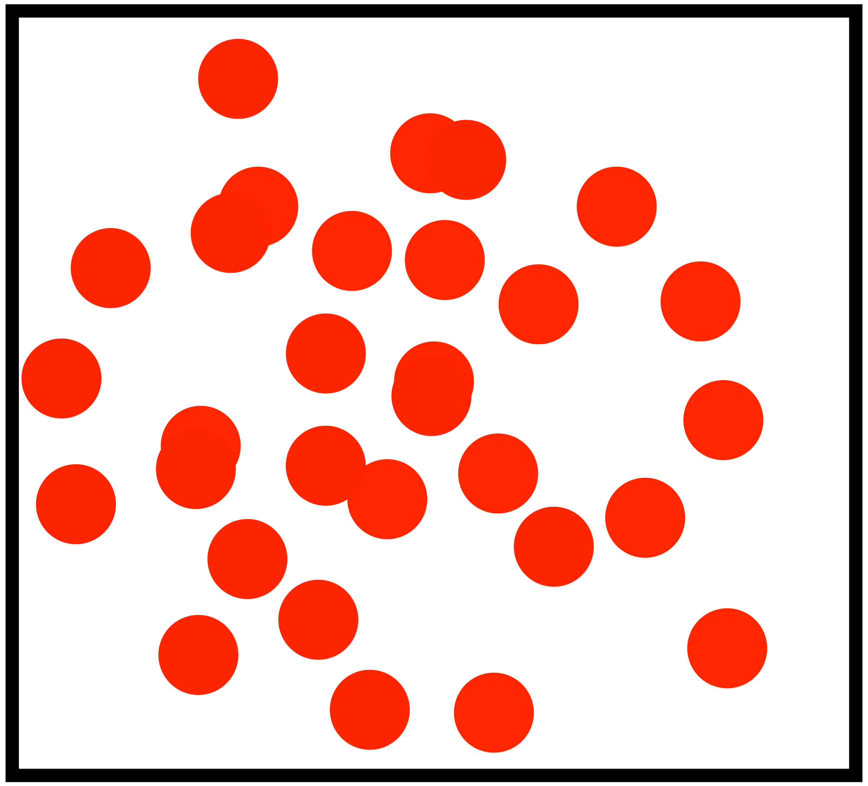

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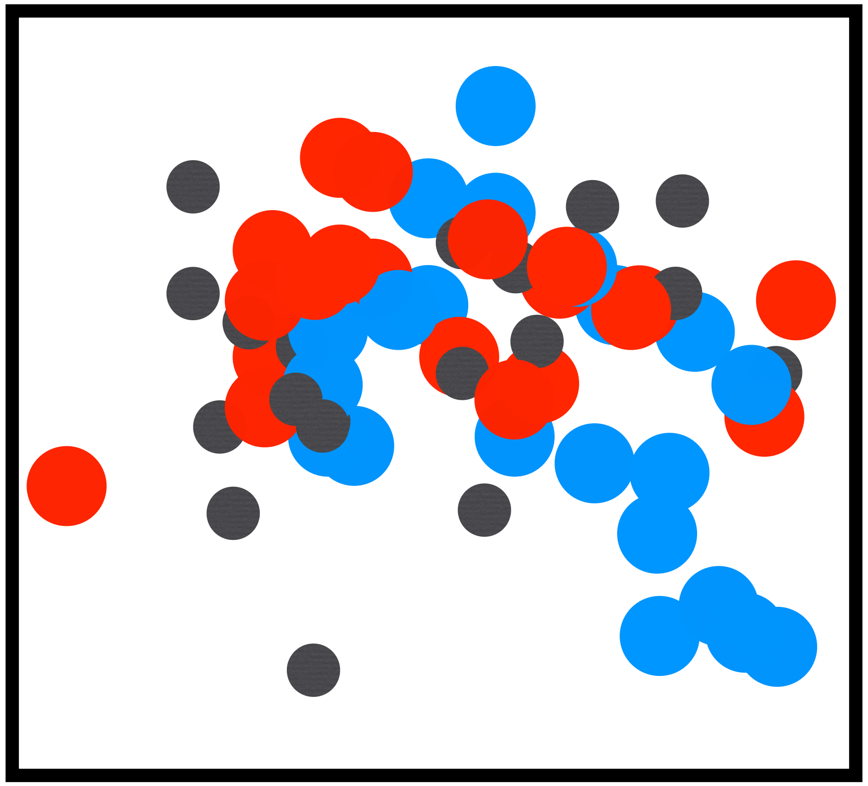

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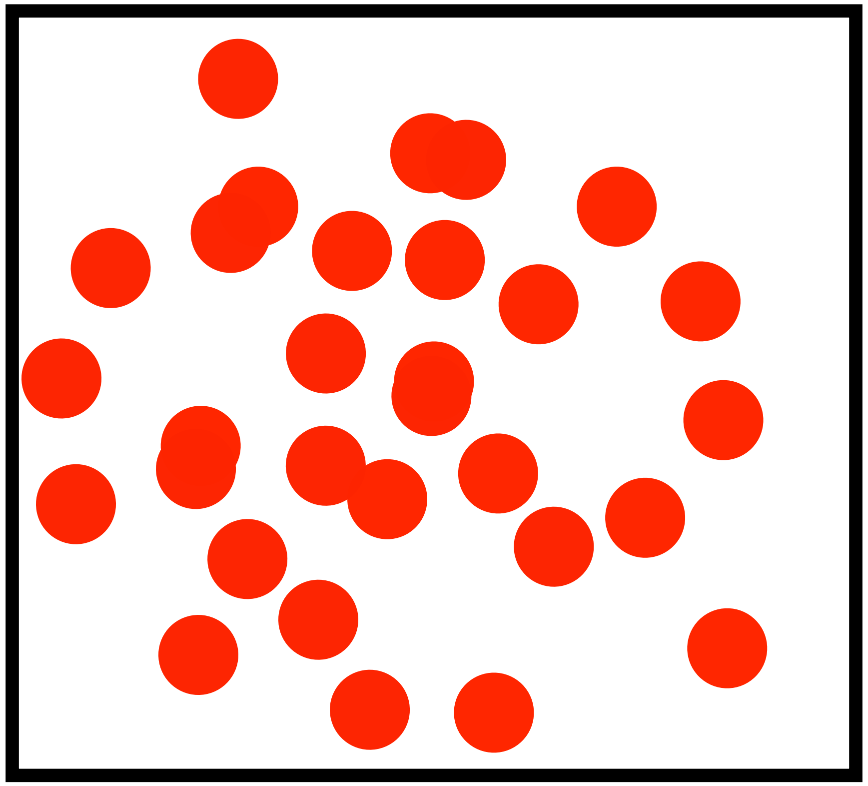

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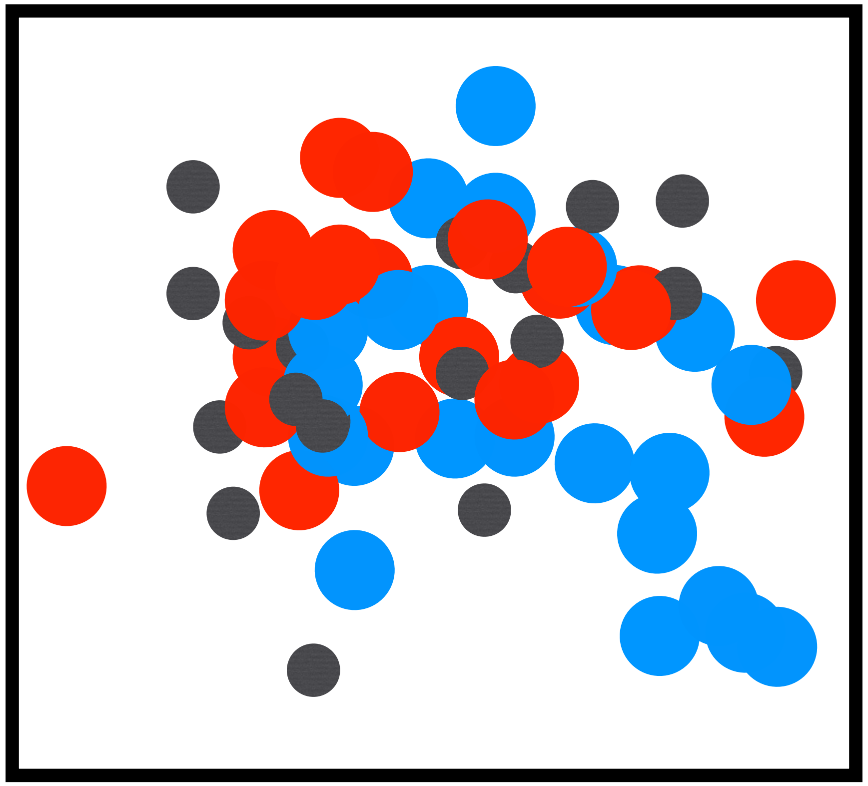

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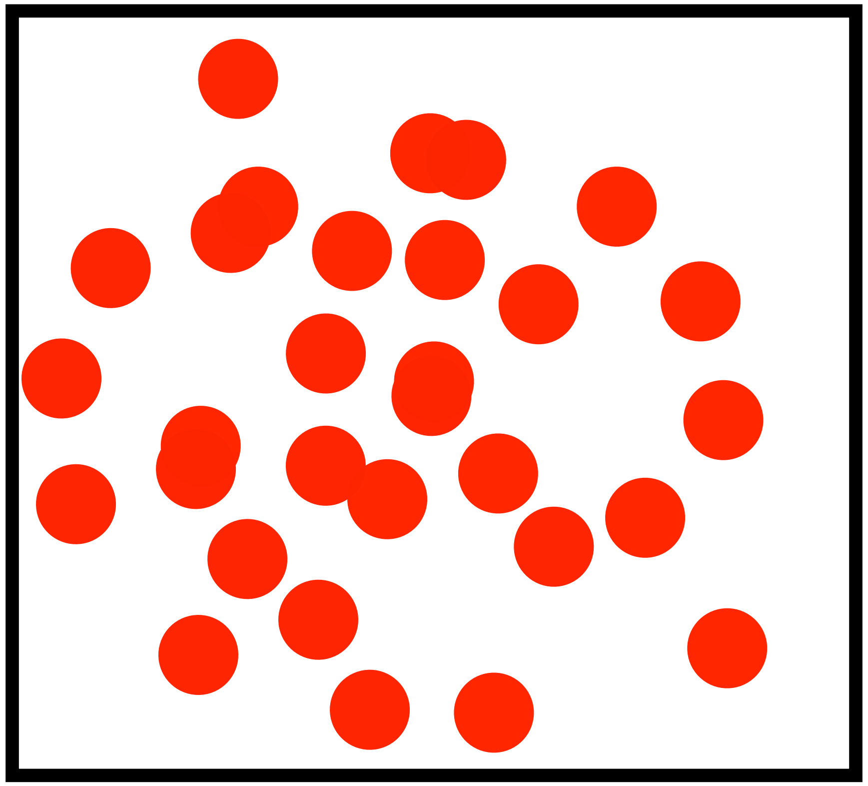

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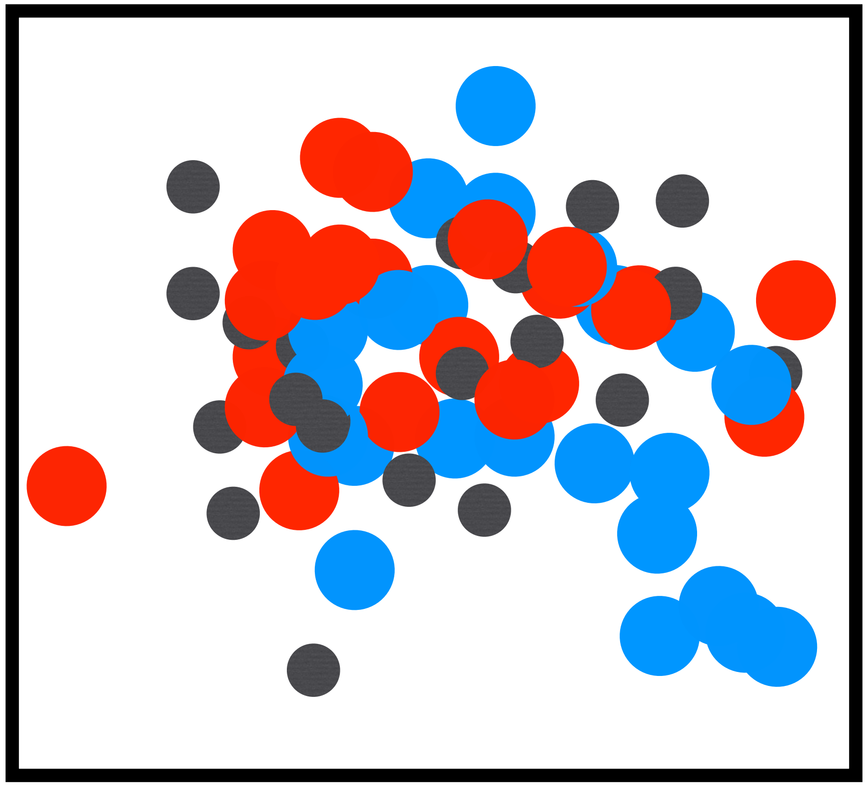

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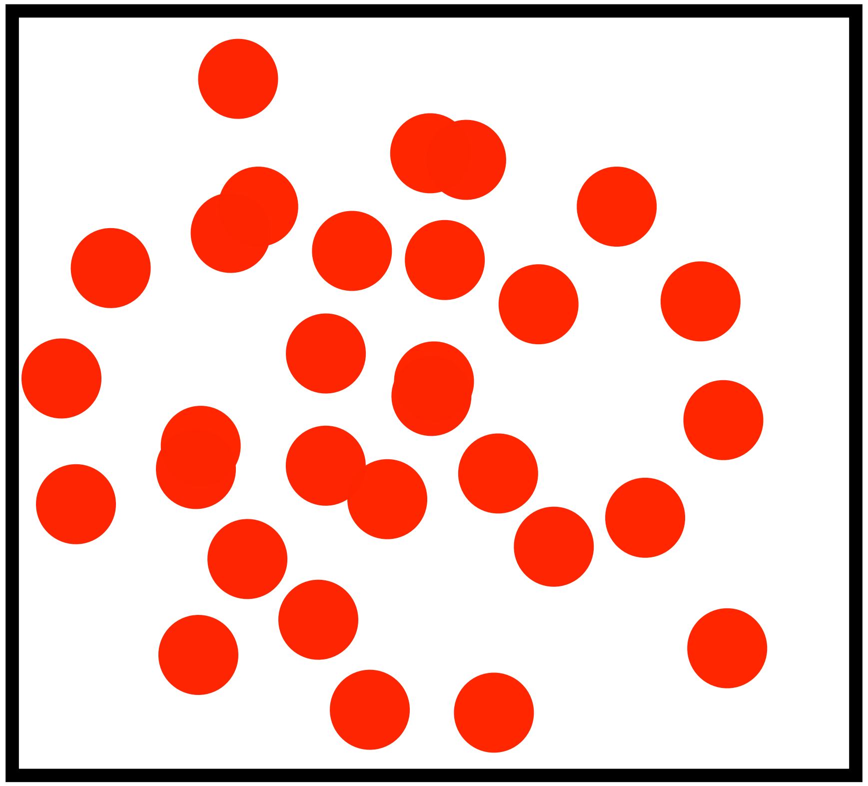

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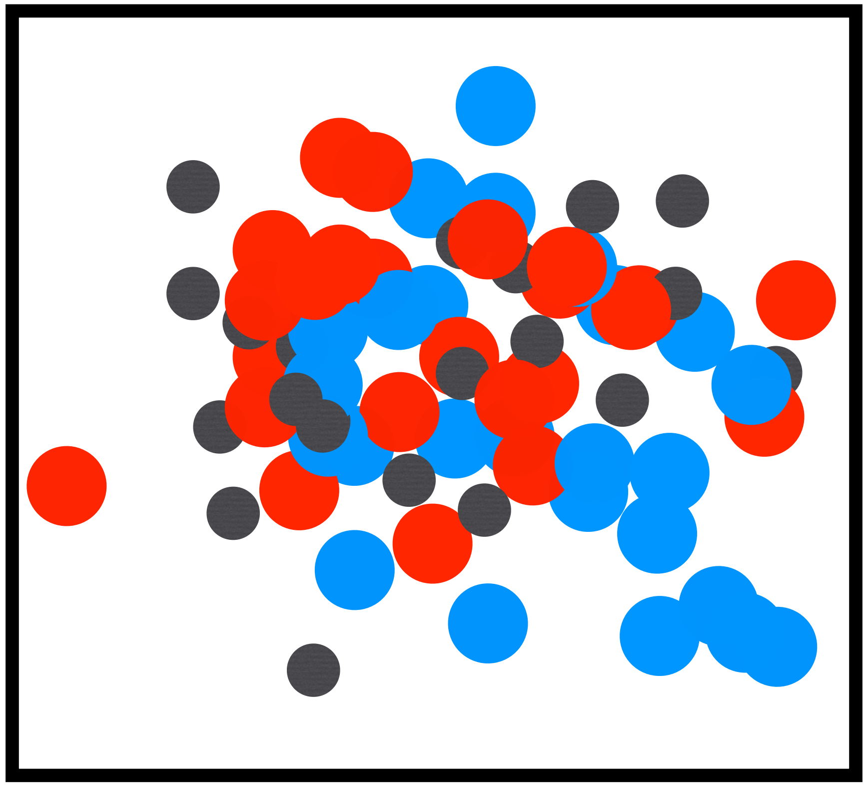

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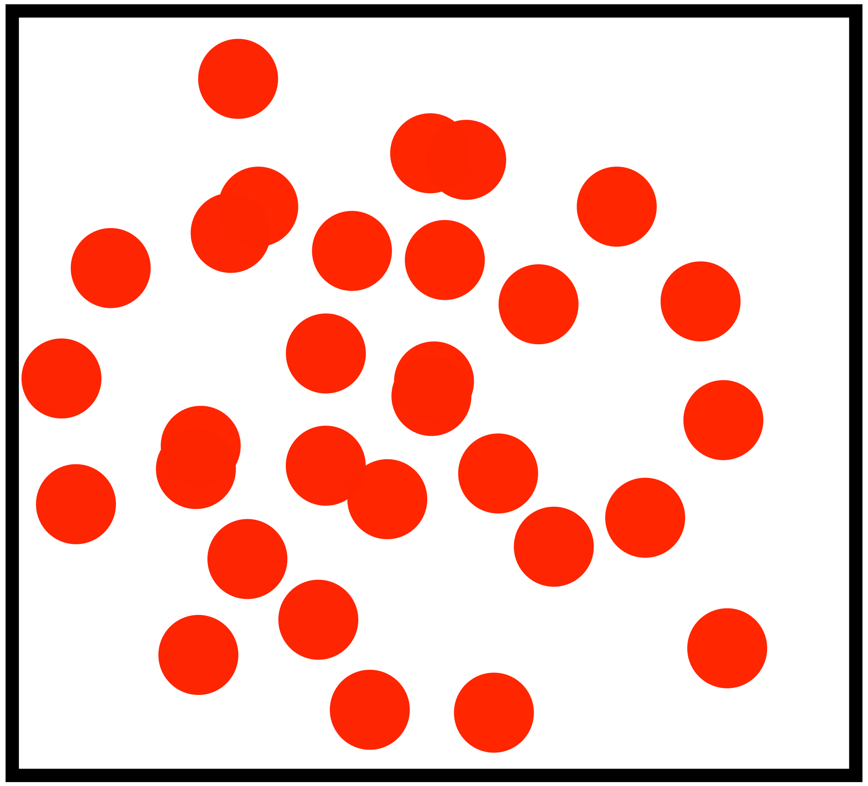

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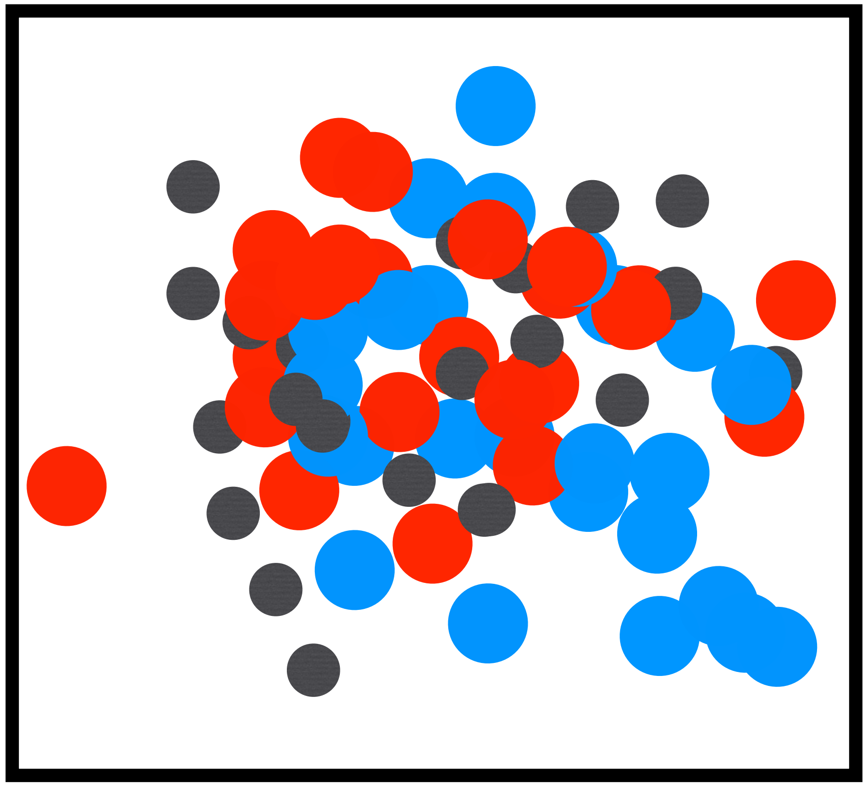

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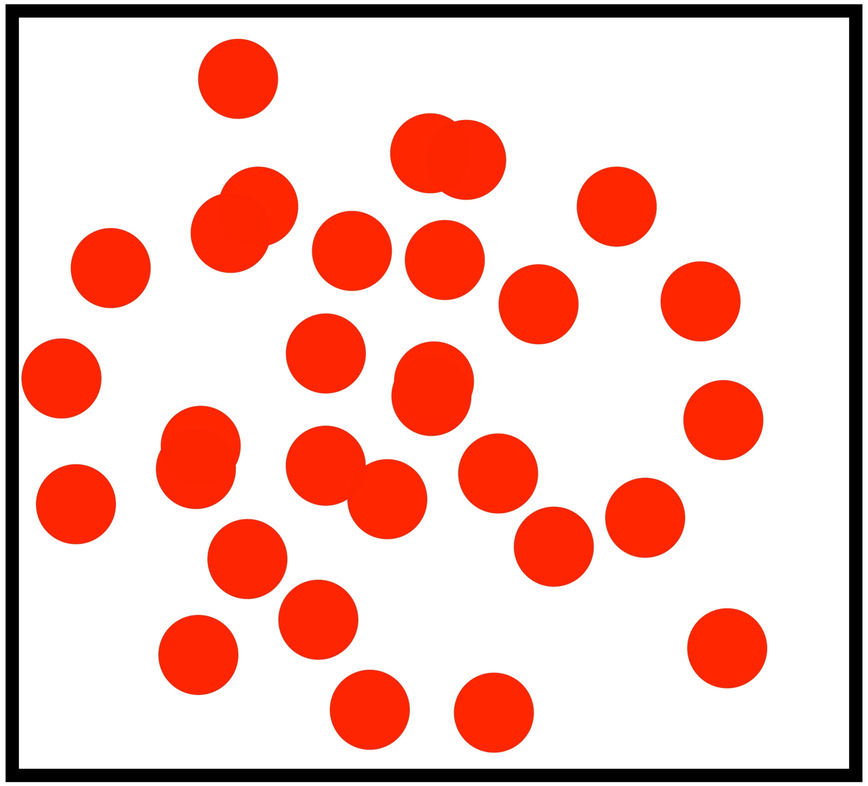

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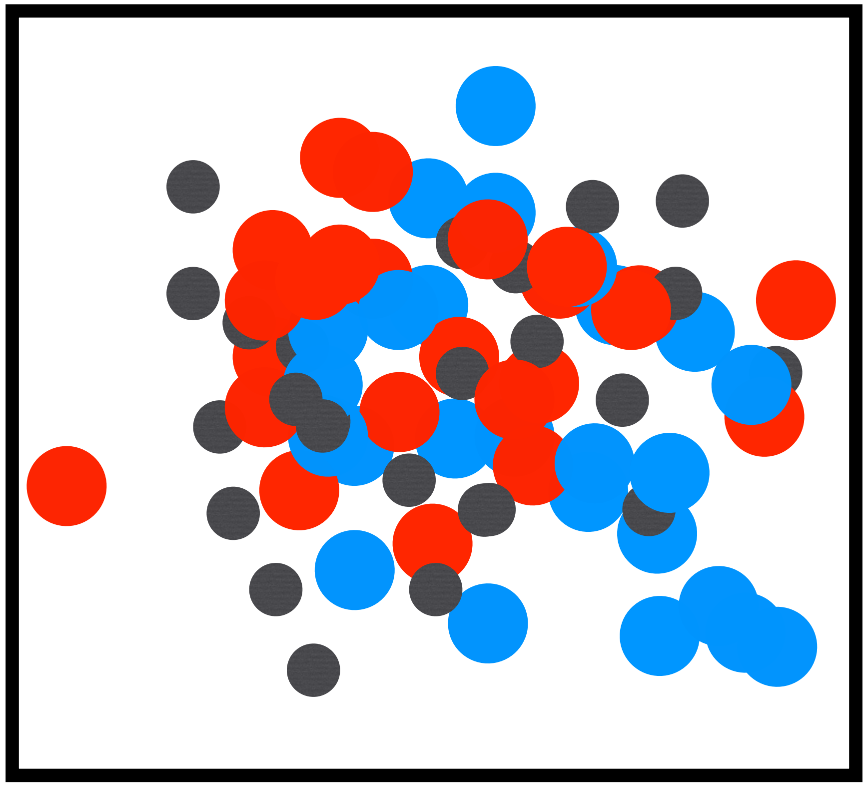

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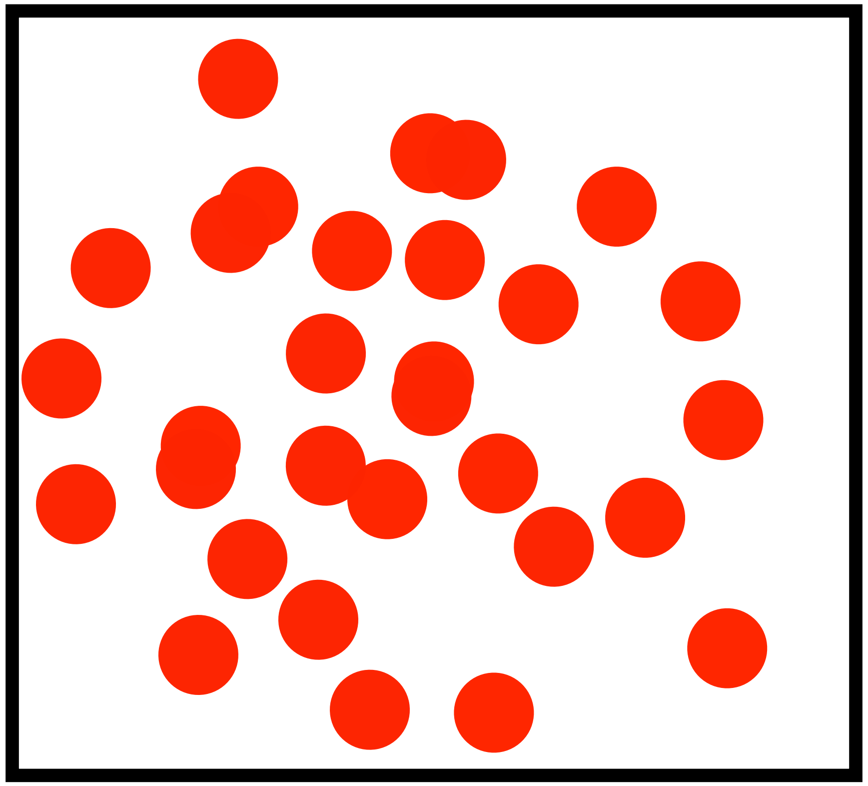

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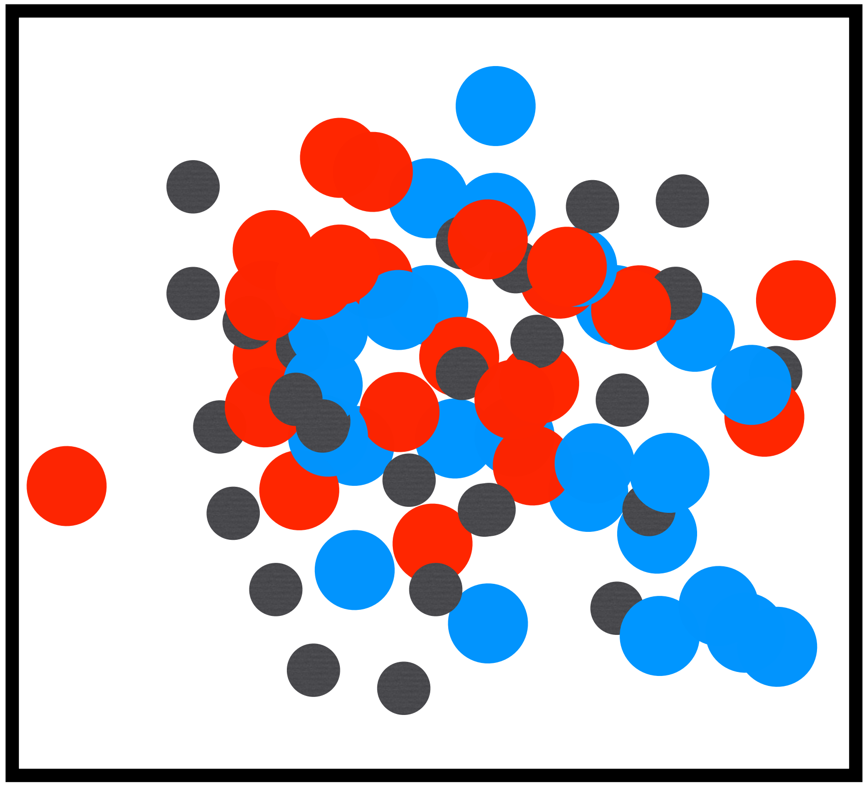

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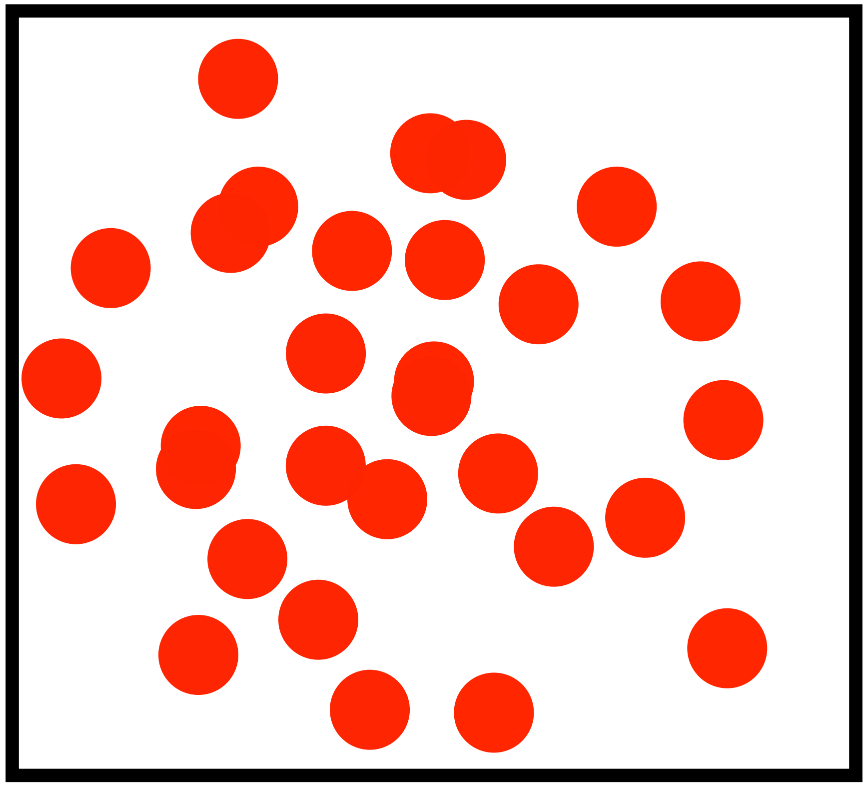

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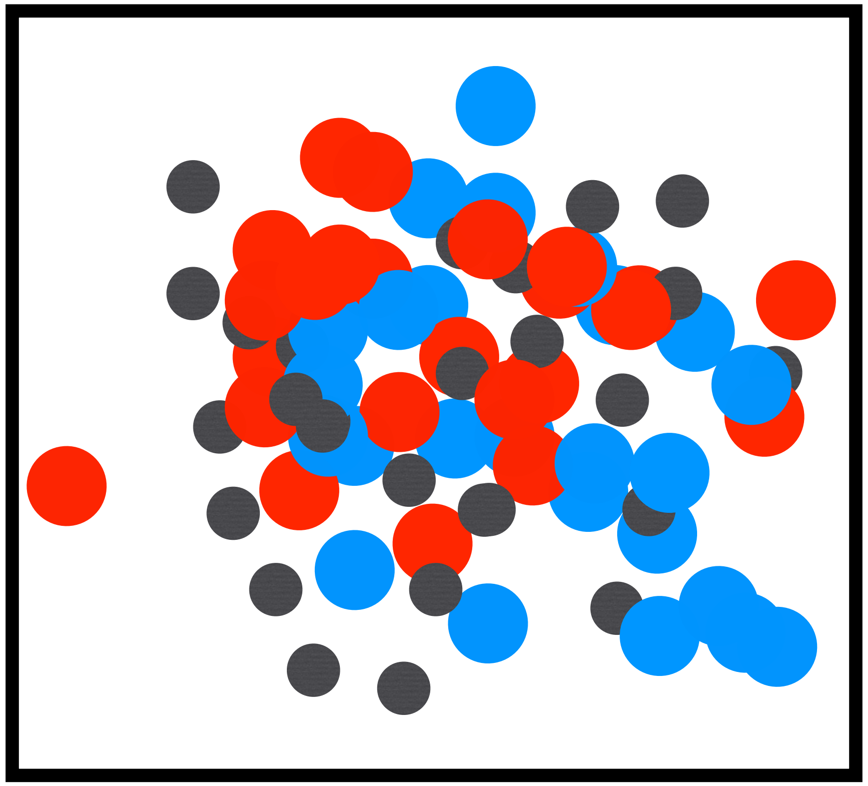

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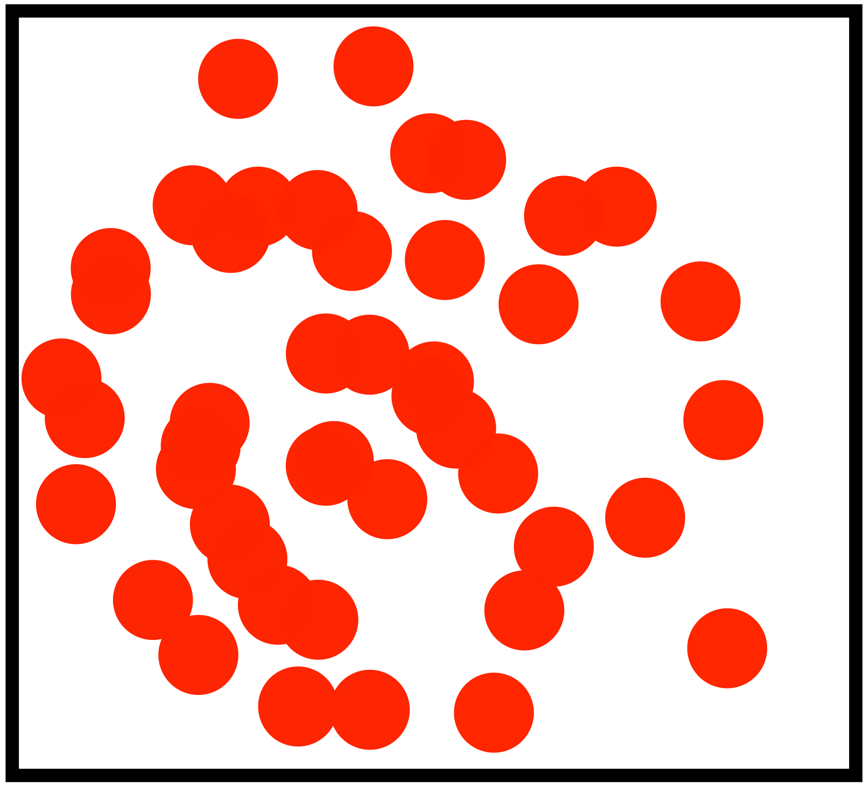

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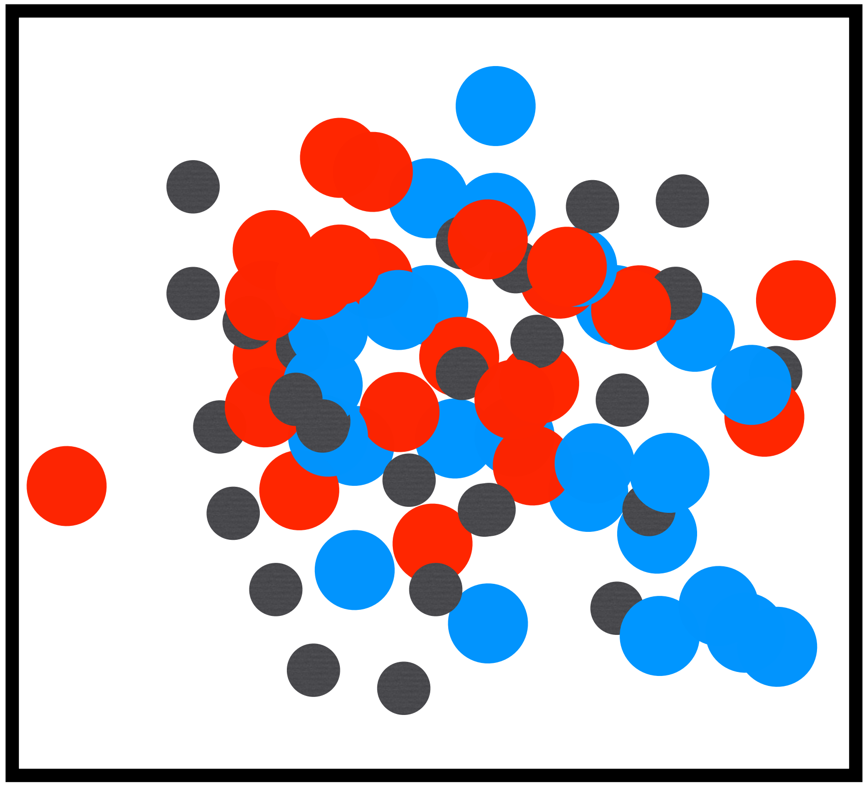

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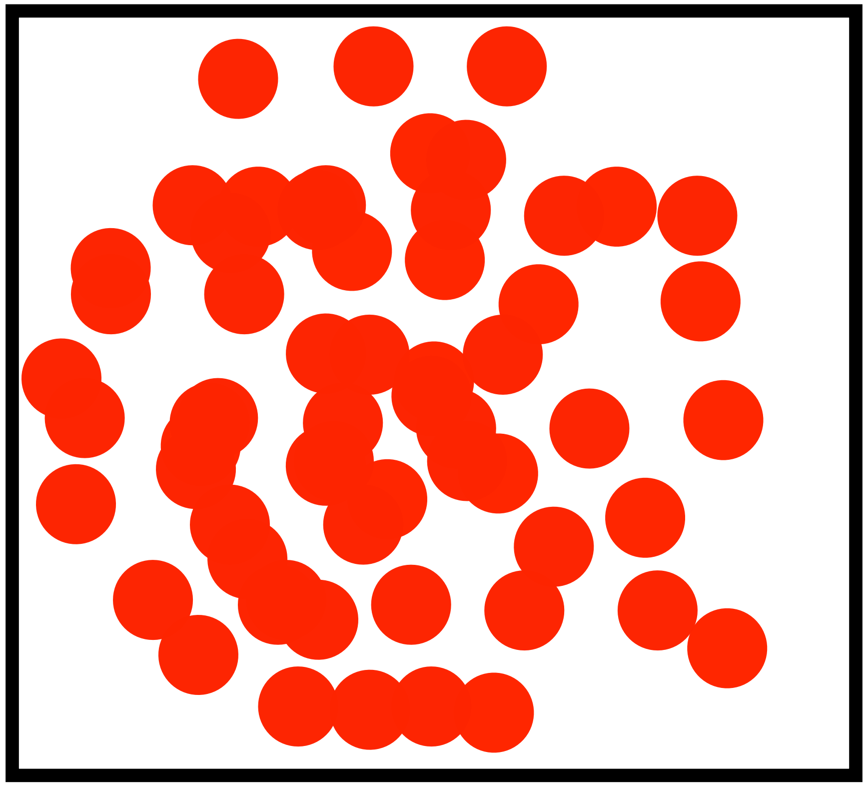

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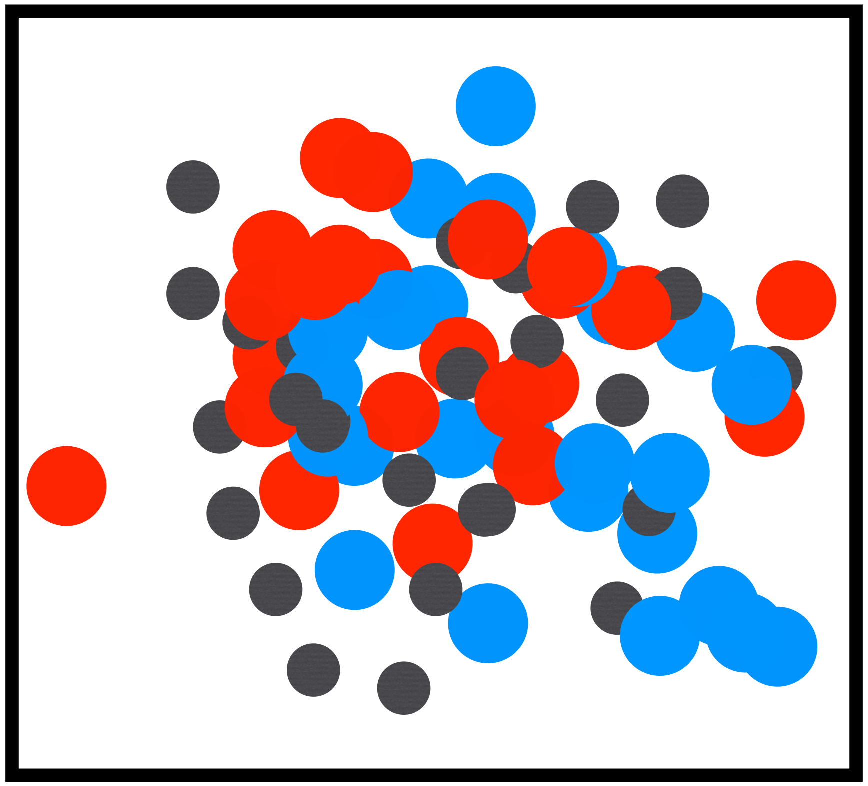

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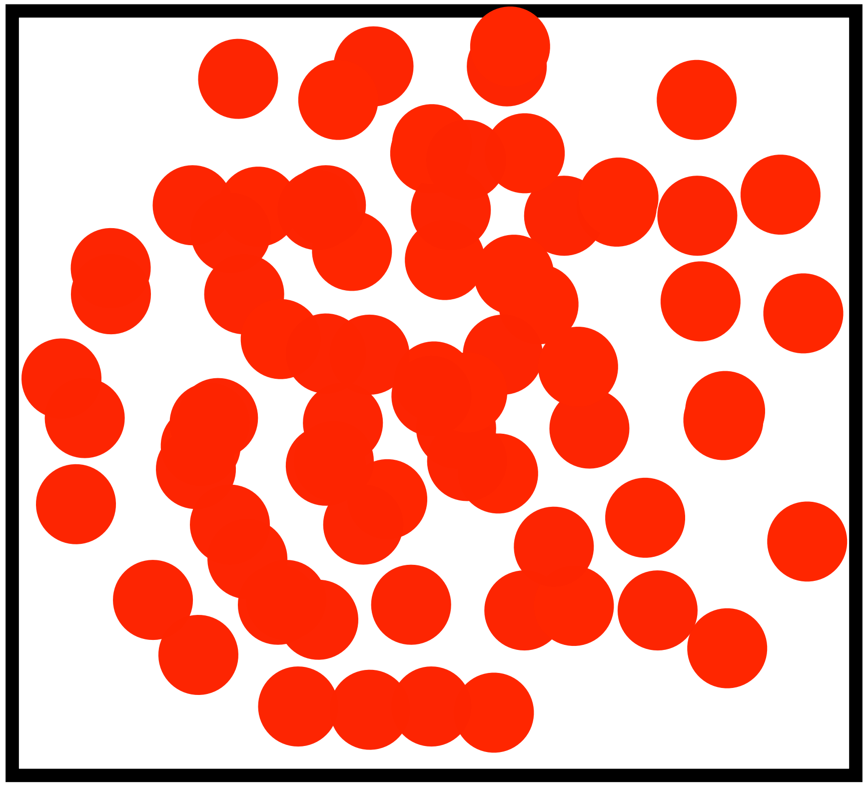

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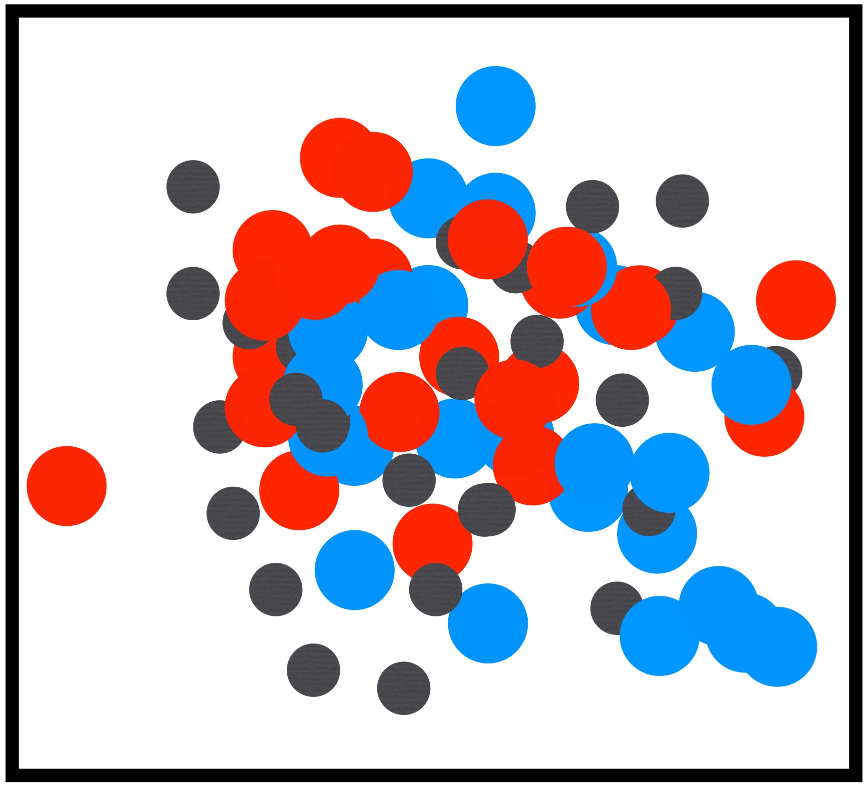

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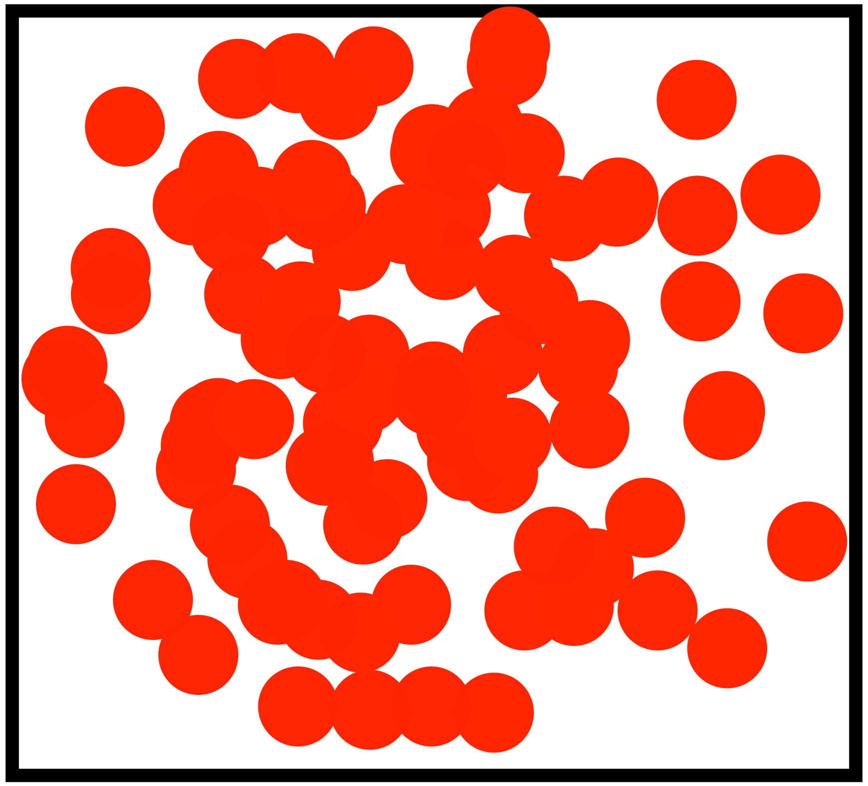
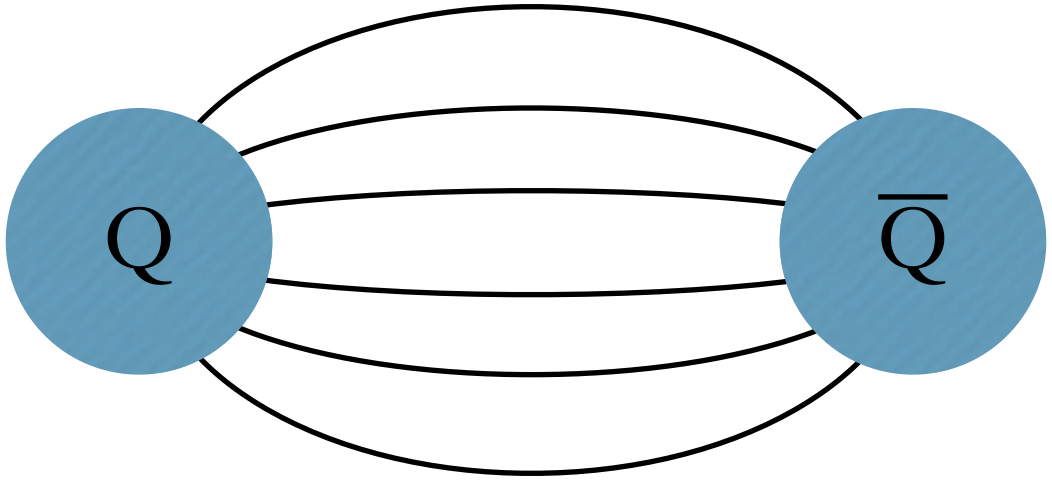


Color screening

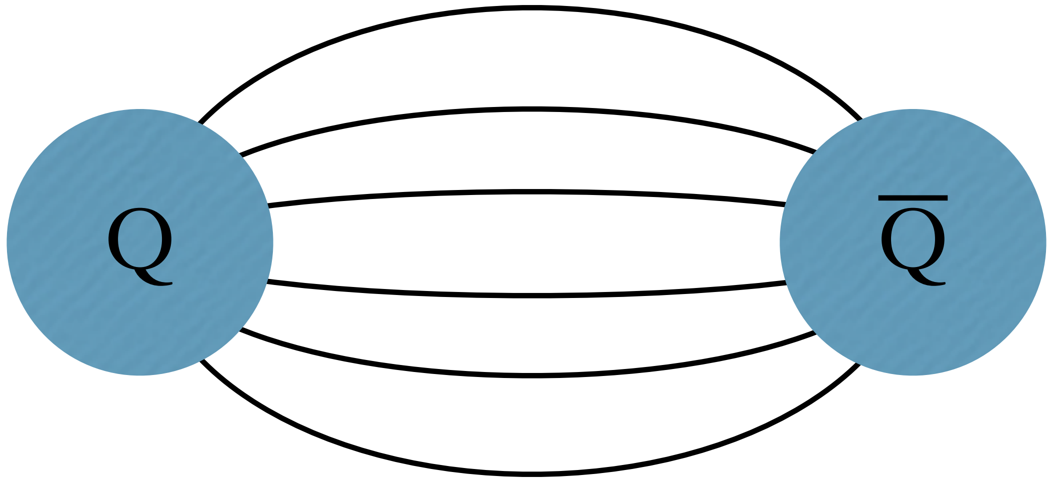


VACUUM
Anti-screening

M. Laine et al. JHEP 0703, 054 (2007)

See Thursday talks

Color screening



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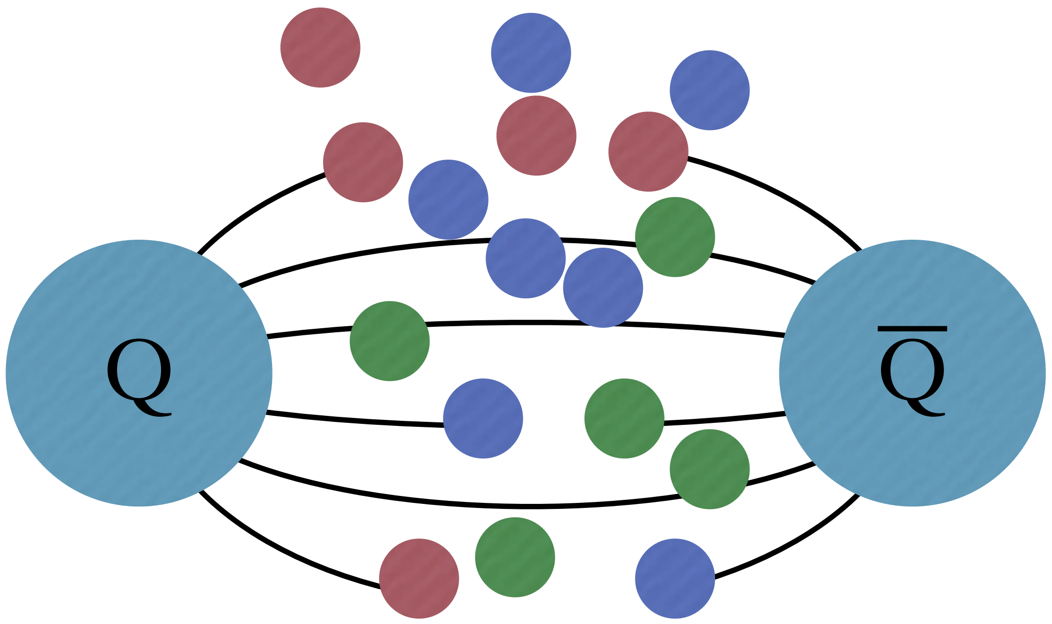

QGP
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See Thursday talks

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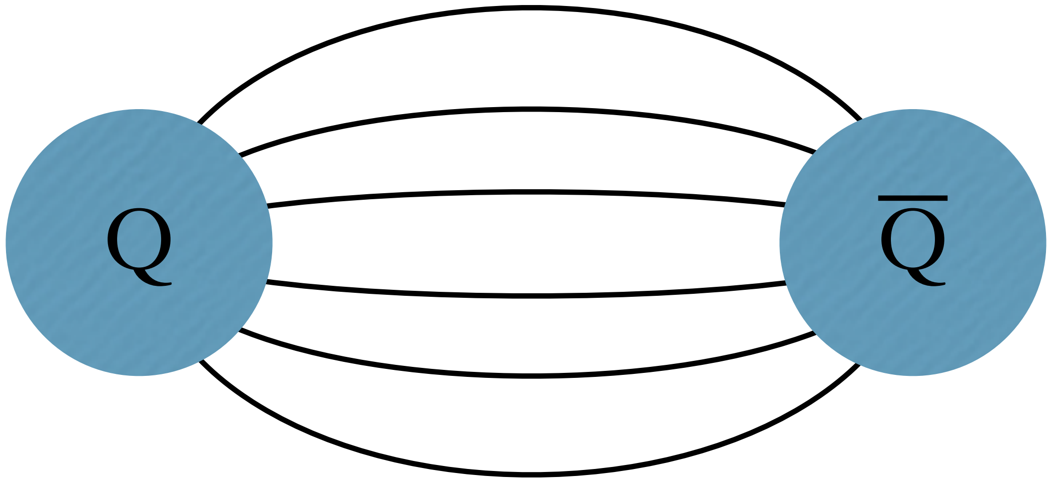

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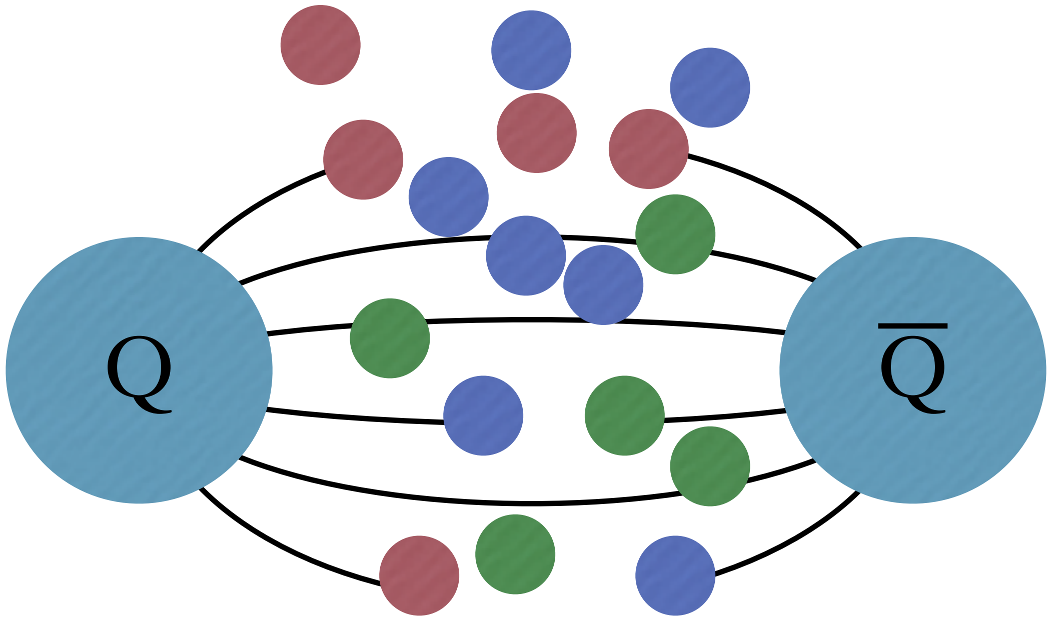

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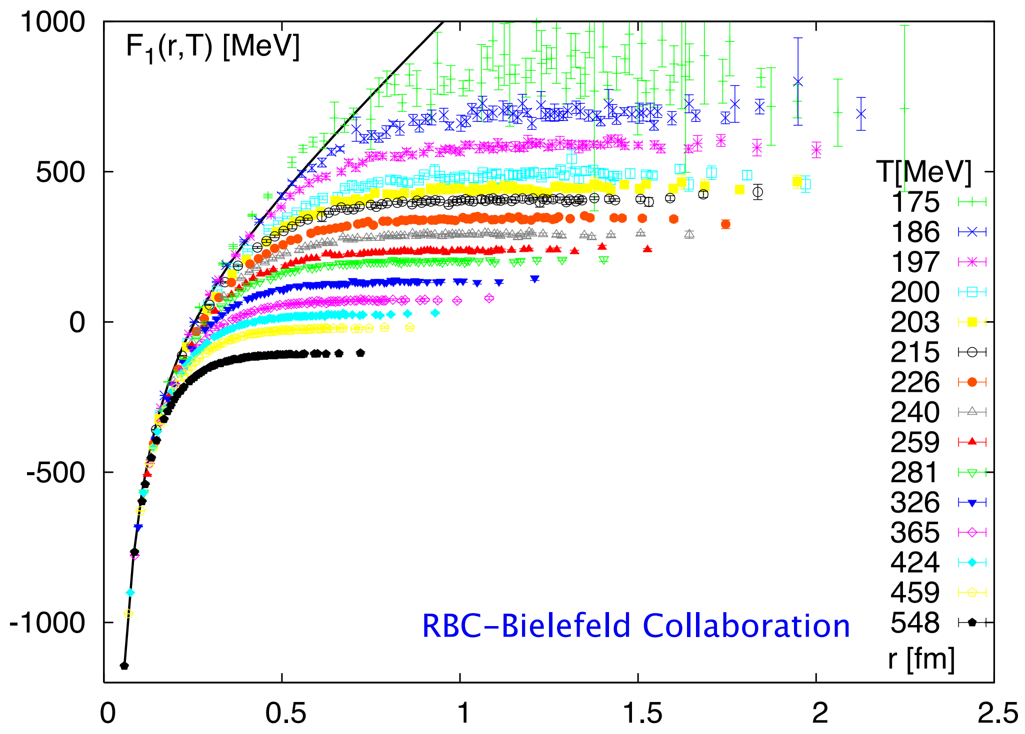


Pioneering work by Matsui and Satz
Charmonia melting by Debye Screening
Phys.Lett. B178 (1986) 416-422

One can use quarkonia melting as a "thermometer" of the QGP temperature

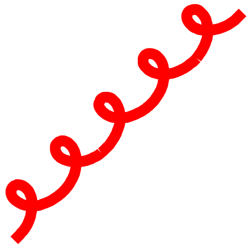
Landau damping is a competitive phenomenon
M. Laine et al. JHEP 0703, 054 (2007)

See Thursday talks

Modeling the strong interaction

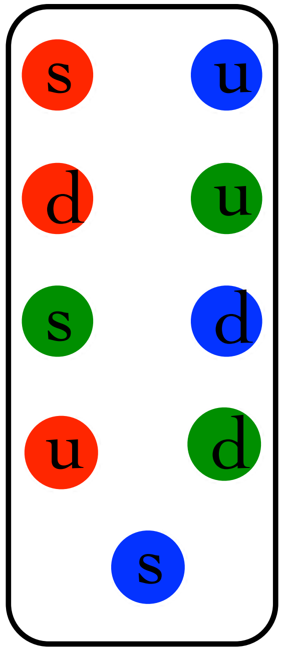
Any description of QCD should
1) reproduce confinement and asymptotic freedom
2) have the right particle content

8 Gluons



exchange color and glue quarks inside baryons

colored quarks



Gluons are responsible for confinement and asymptotic freedom

UV freedom, IR confinement

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}(i\gamma^\mu D_\mu + \mu\gamma_0 - M)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a}$$

quark fields: $\psi_{\alpha,i}$

$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, \ldots, 6$ **flavor indices**

gluon gauge fields: $\mathbf{A}^a$

$a = 1, \ldots, 8$ **adjoint color index**

QCD non-Abelian gauge theory, non-perturbative at energy scales below $\Lambda_{\mathrm{QCD}} \sim 200\ \mathrm{MeV}$

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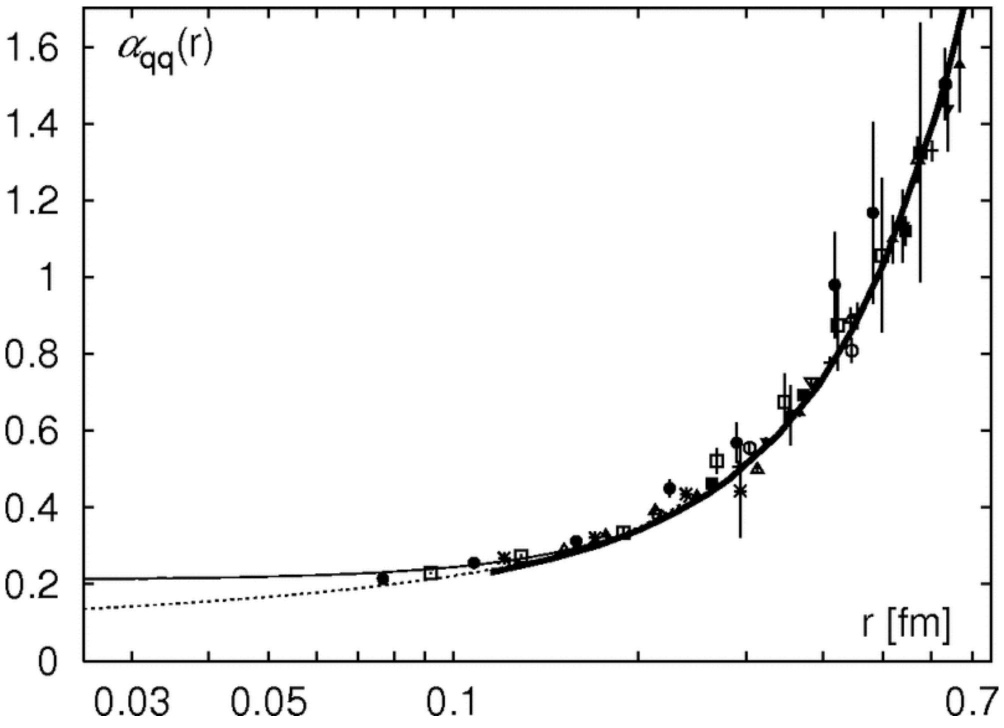
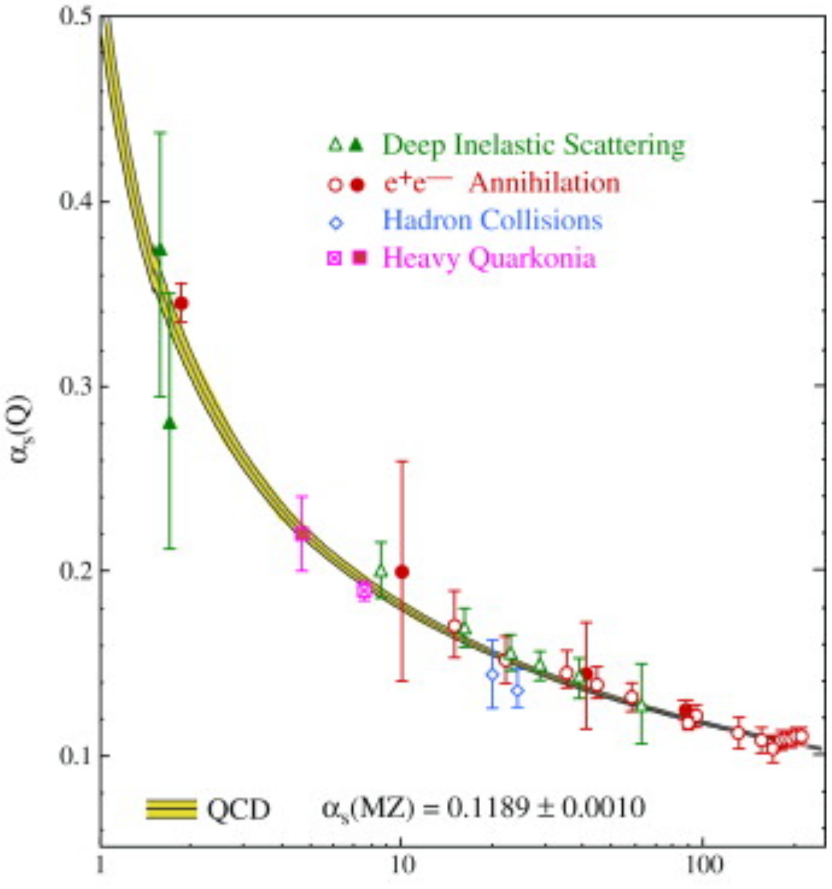


Kaczmarek and Zantow
Physical Review D 71(11):114510



S. Bethke,
Prog.Part.Nucl.Phys. 58 (2007) 351-386

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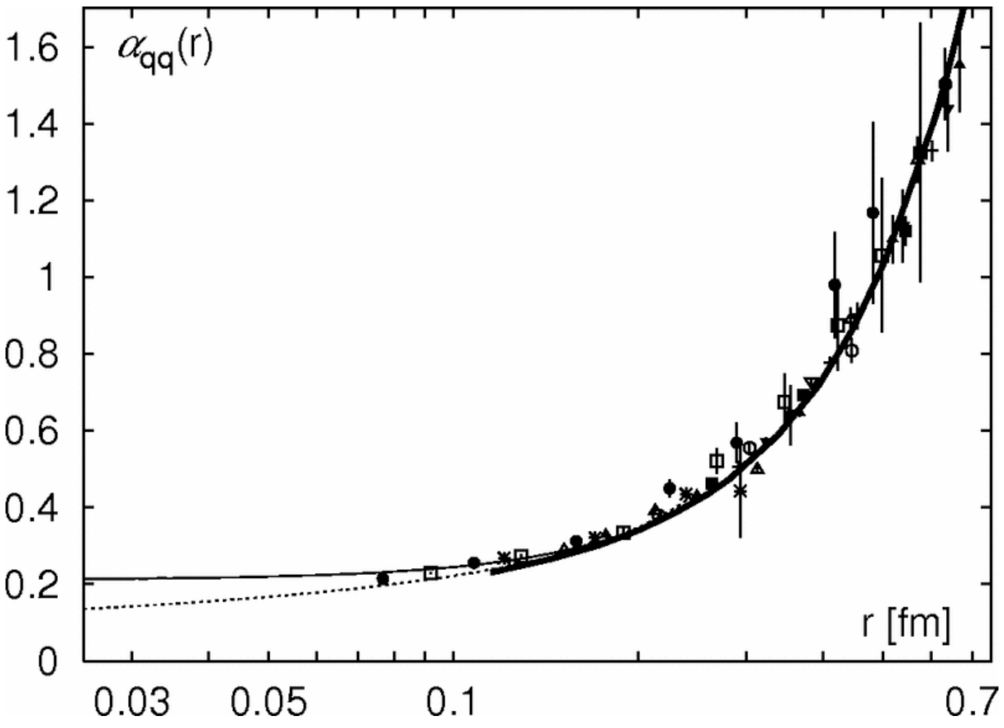

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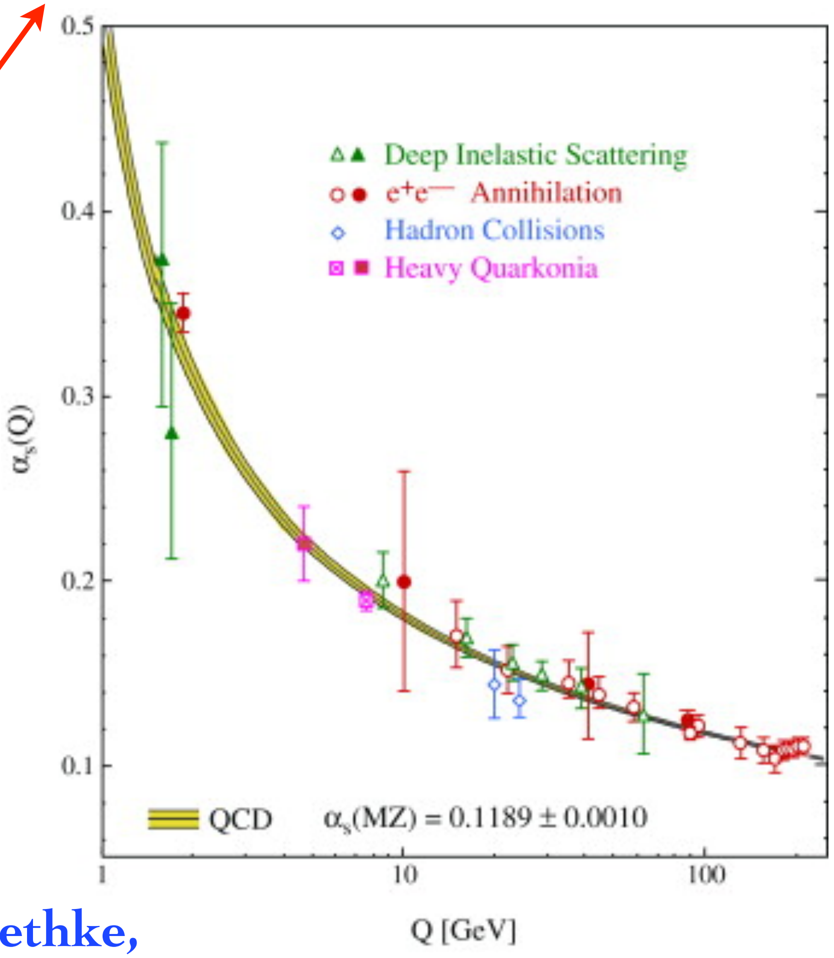
confinement



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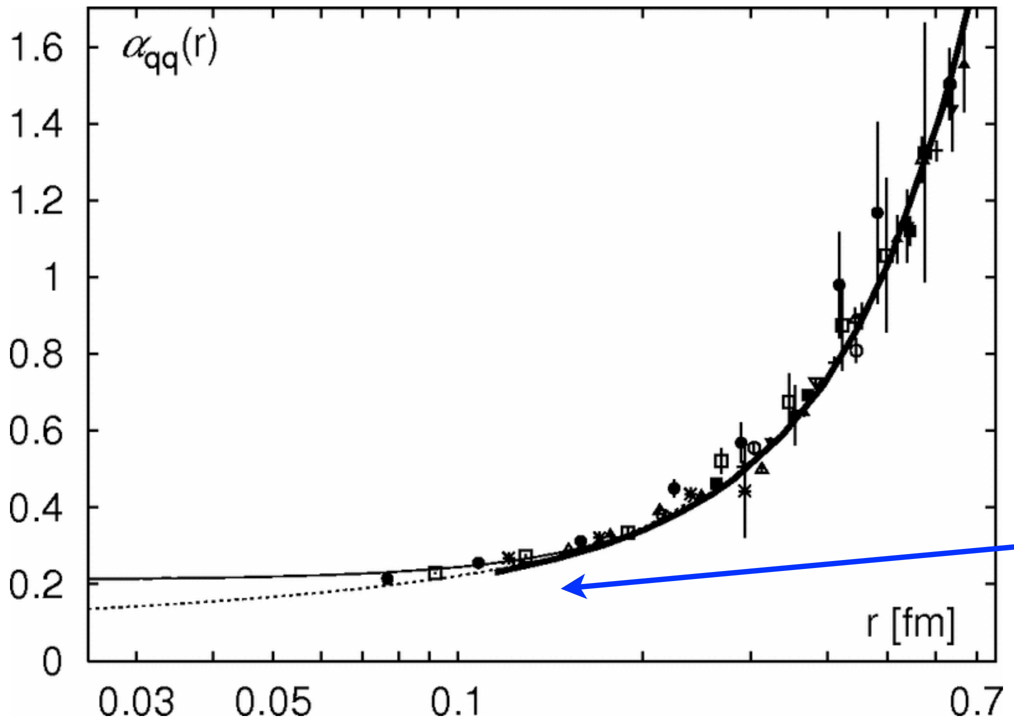
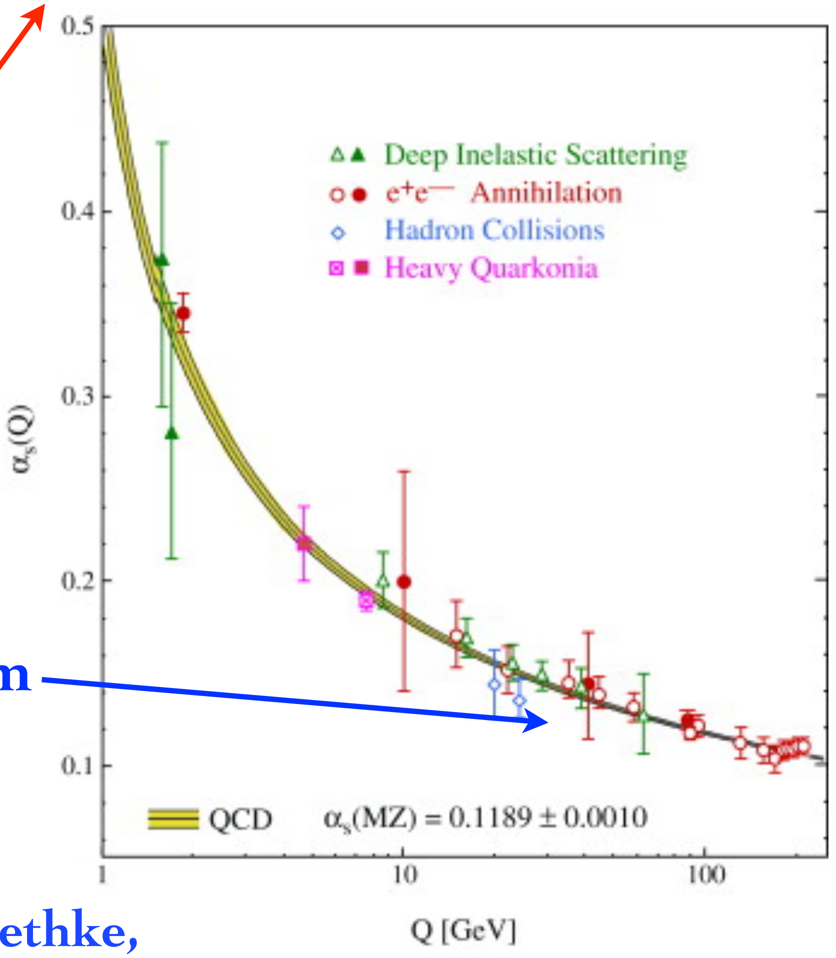


confinement

asymptotic freedom



Kaczmarek and Zantow
Physical Review D 71(11):114510

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Prog.Part.Nucl.Phys. 58 (2007) 351-386

Symmetries

$m = 0$

Three flavor massless quark matter

gauge group

global chiral symmetry

global baryonic number

$$SU(3)_{\mathrm{c}} \times \underbrace{SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}}_{\supset [U(1)_{\mathrm{e.m.}}]} \times U(1)_{\mathrm{B}}$$

$m \to \infty$

Quenched QCD (pure Yang-Mills)

Polyakov loop

$$L = \mathcal{P} \exp \left[i \int_0^\beta dx_4 A_4 \right]$$

remarkably $e^{-\beta F_q} = \langle L \rangle$

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Chiral symmetry amounts to rotate independently the left- and right-handed quark fields

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These rotations can be locked by the $\langle\bar{\psi}\psi\rangle$ condensate

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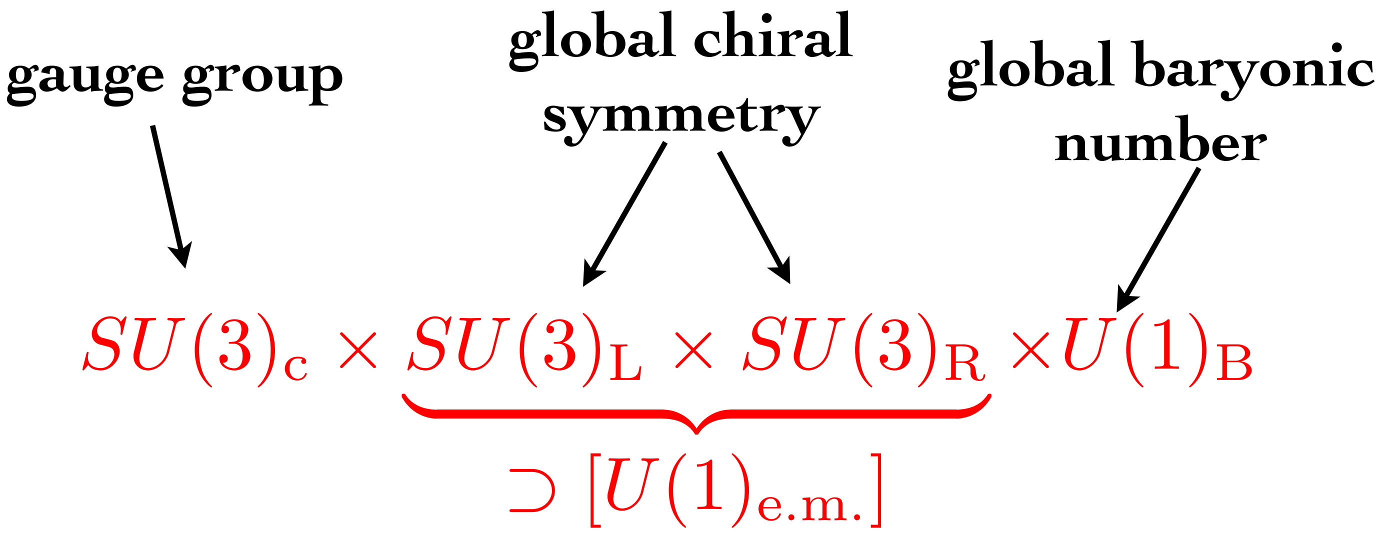

gauge group

global chiral symmetry

global baryonic number

$$SU(3)_{\mathrm{c}} \times \underbrace{SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}}_{\supset\, [U(1)_{\mathrm{e.m.}}]} \times U(1)_{\mathrm{B}}$$

Chiral symmetry amounts to rotate independently the left- and right-handed quark fields

These rotations can be locked by the $\langle \bar{\psi}\psi \rangle$ condensate

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Quenched QCD (pure Yang-Mills)

Polyakov loop

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Deconfinement and chiral symmetry breaking

m: mass of quark fields

m

Quenched QCD (pure Yang-Mills) $m \to \infty$

Center symmetry: $Z(N_c)$, broken at T_D

Order parameter for deconfinement: <Polyakov loop>

Chiral limit $m = 0$

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QCD

T_D and T_χ are pseudo-critical temperatures

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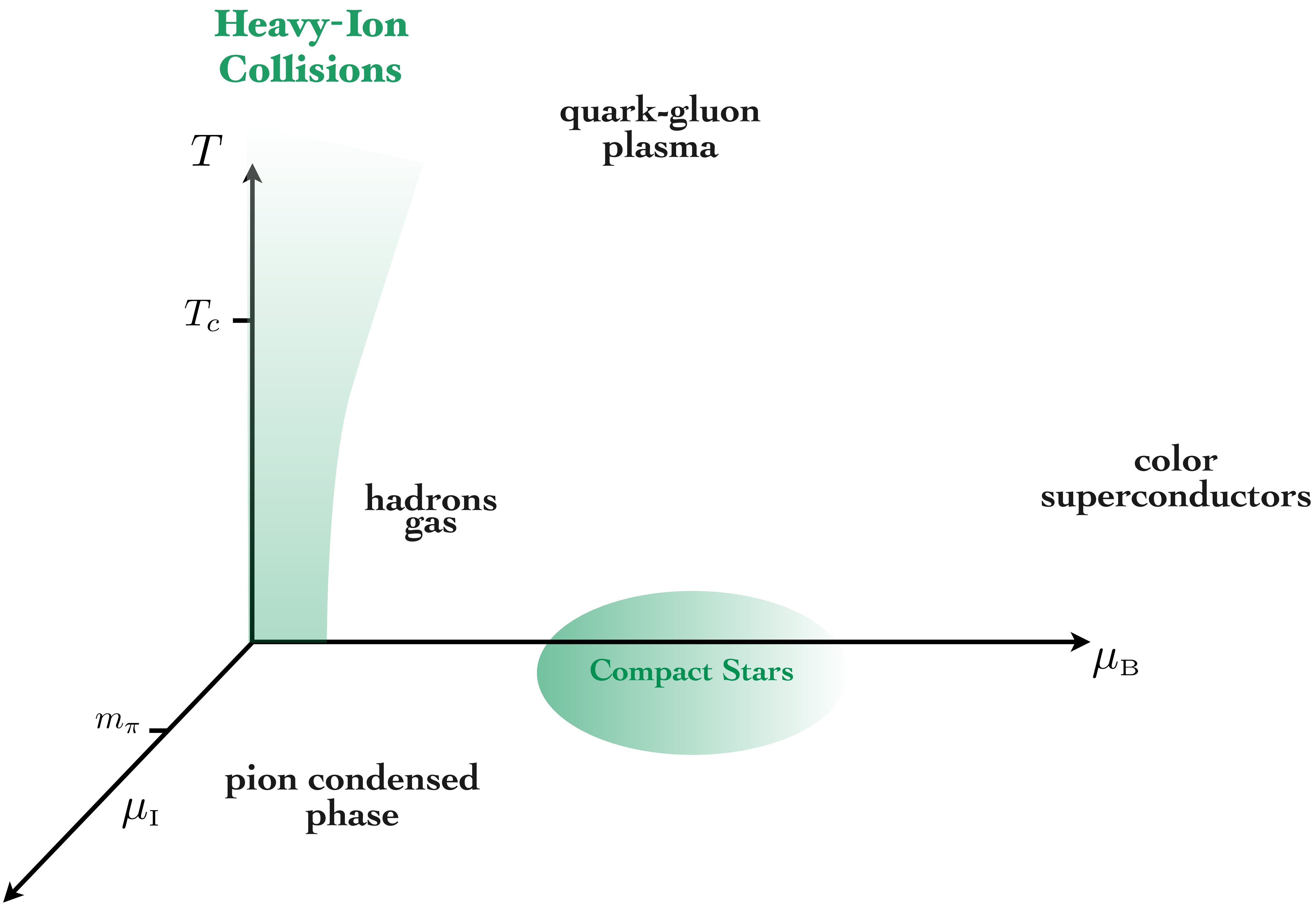
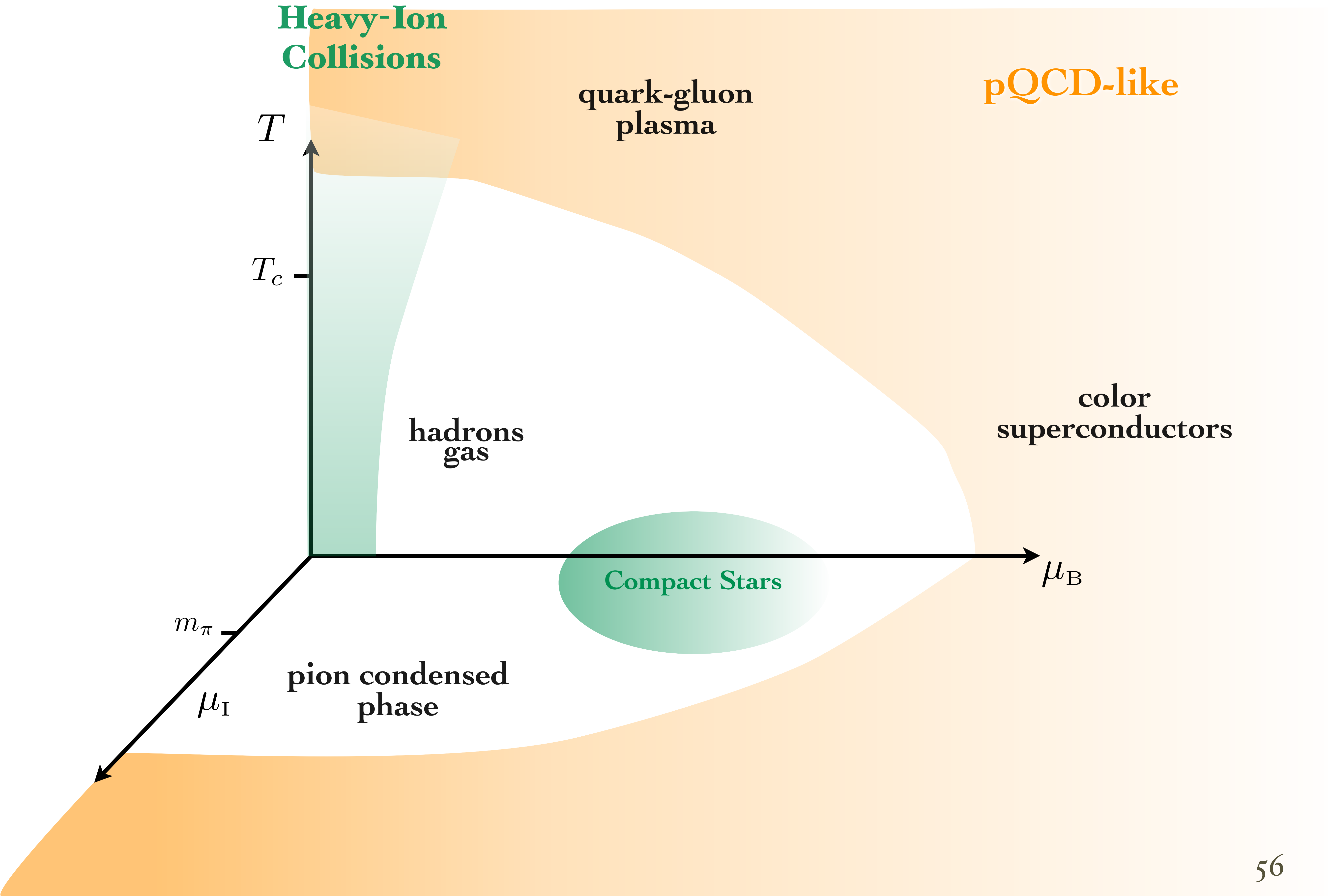
3) Apart from these theory group arguments, it is important to have a phenomenological description of confinement (and chiral symmetry breaking) as associated to a change of degrees of freedom.

Methods


Heavy-Ion Collisions
quark-gluon plasma
T
T_c
hadrons gas
color superconductors
Compact Stars
μ_{B}
m_π
μ_{I}
pion condensed phase

Methods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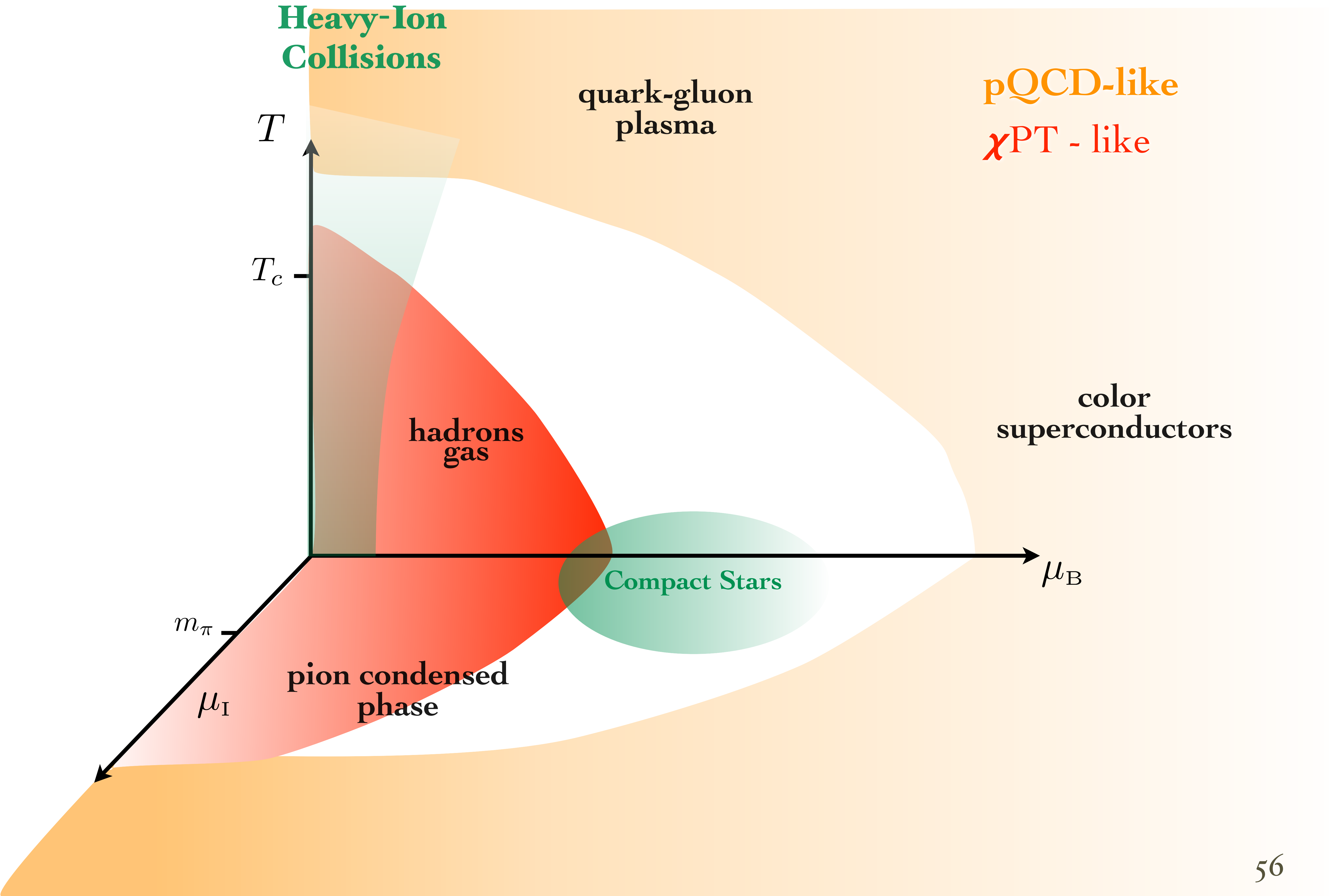
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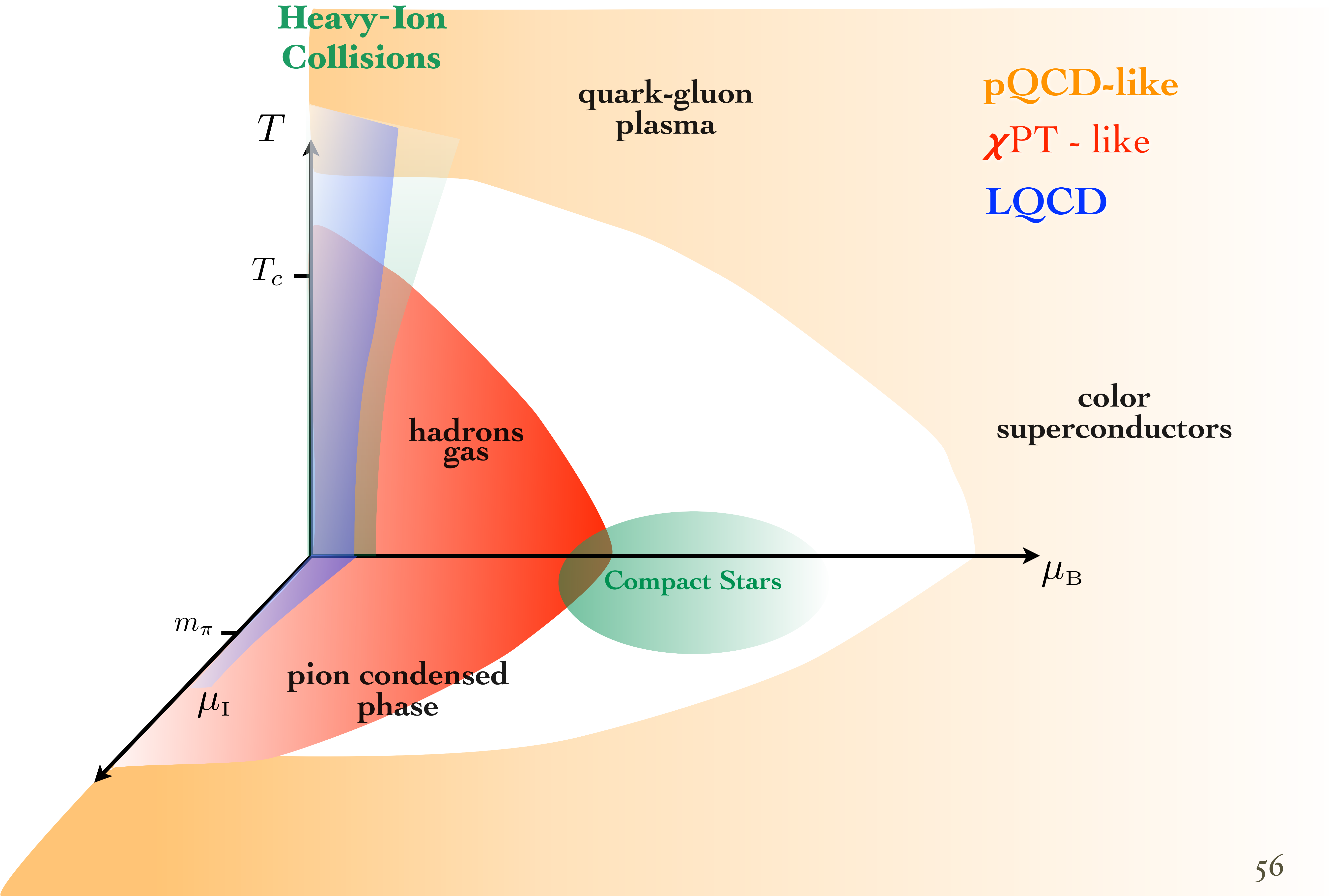
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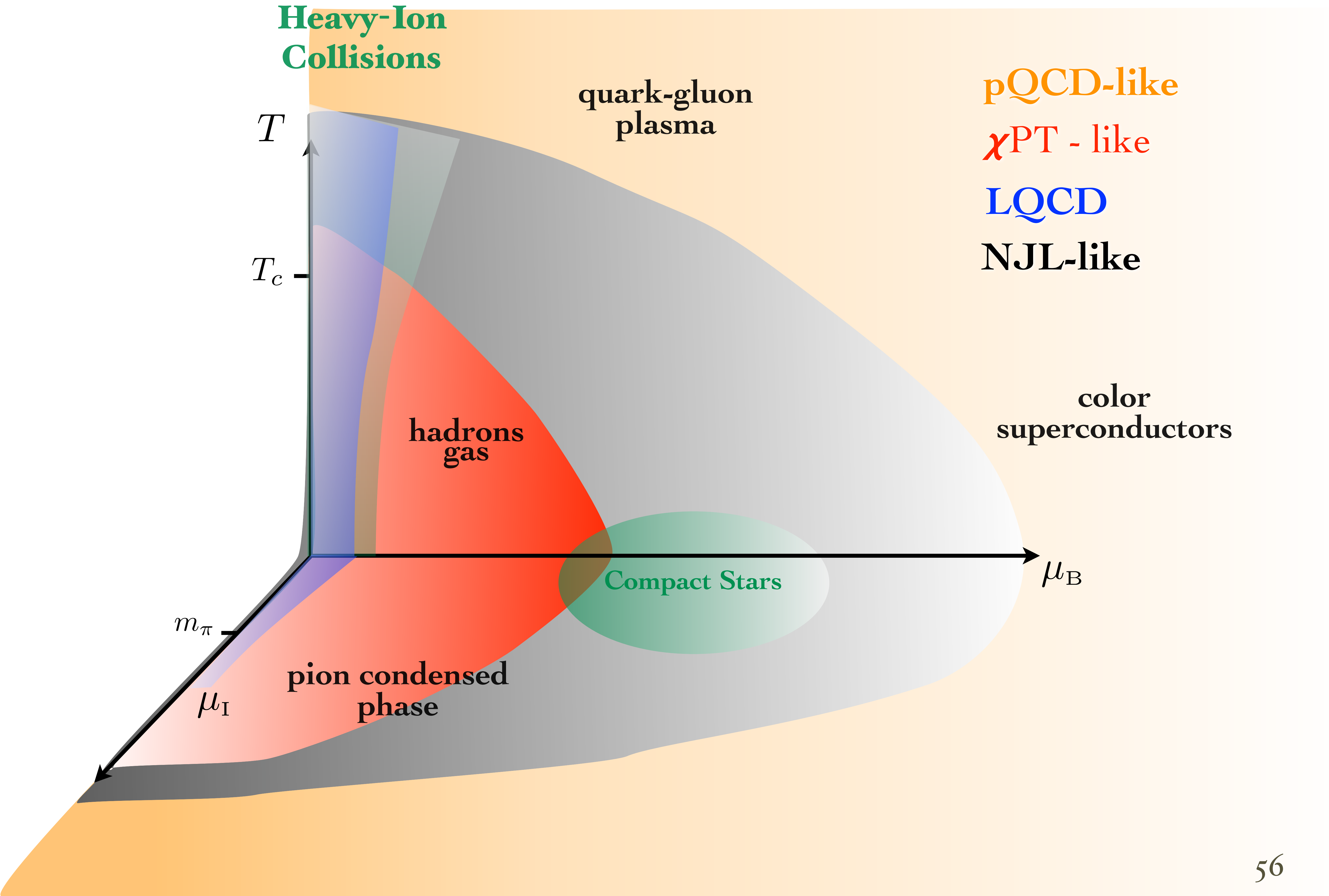
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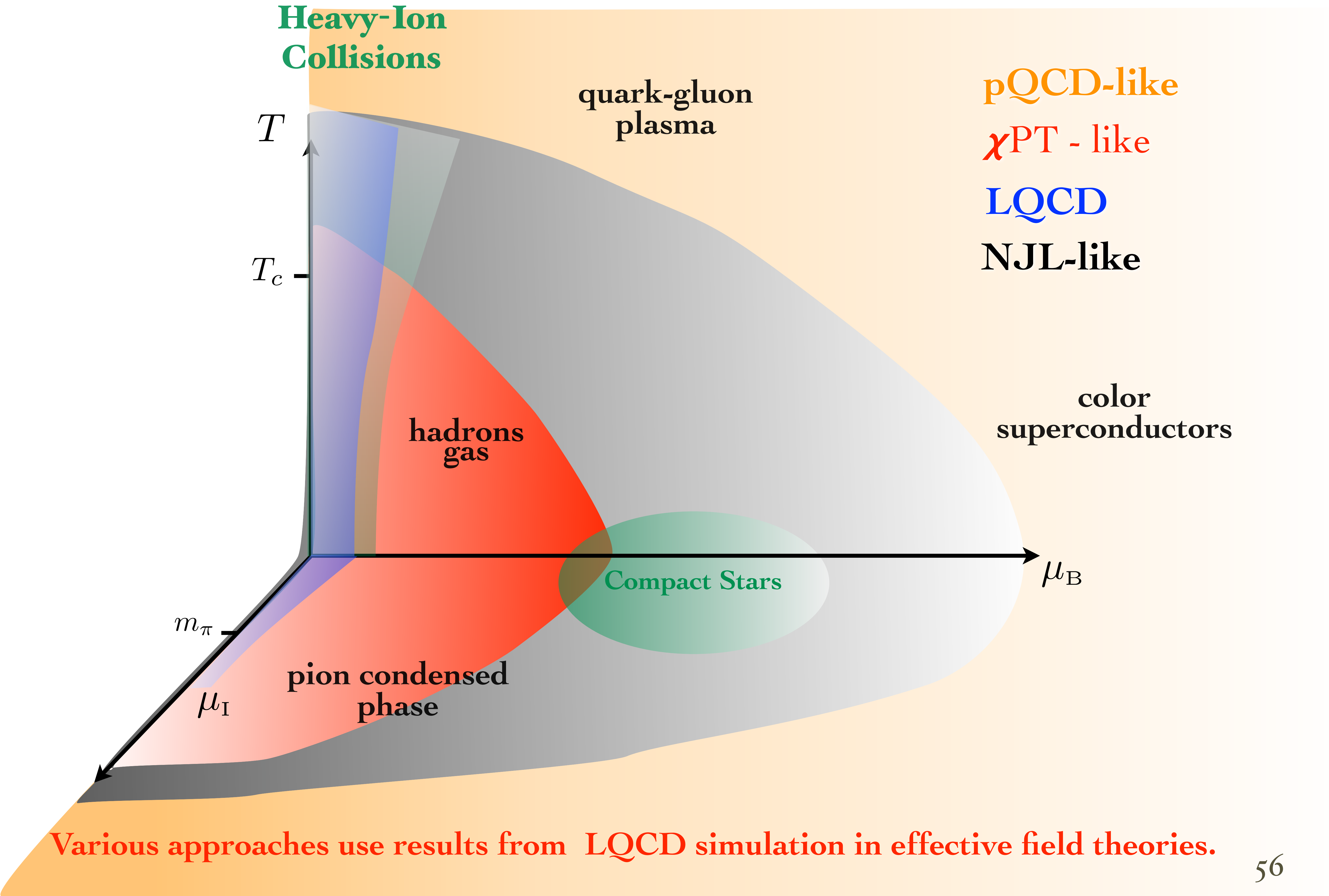

Various approaches use results from LQCD simulation in effective field theories.

Effective field theory: two perspectives

Schematically, two approaches to matter in extreme conditions

1) Understanding the (astro)physical phenomena related to high chemical potential and temperature
2) Understanding QCD in a region in which the correct degrees of freedom are quarks and gluons

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Schematically, two approaches to matter in extreme conditions

1) Understanding the (astro)physical phenomena related to high chemical potential and temperature
2) Understanding QCD in a region in which the correct degrees of freedom are quarks and gluons

The two perspectives are not mutually exclusive.

However, for those who are interested in (astro)physical phenomena, it is enough to have an effective theory which mimics/reproduces the strong interaction in a sufficiently accurate way.

Who is interested to understand QCD wants an effective theory that in a well defined limit is QCD

In medium heavy quarks

“Brownian motion” of Heavy Quarks (HQs) in the QGP
While propagating in the QGP heavy quarks interact with in-medium quarks and gluons

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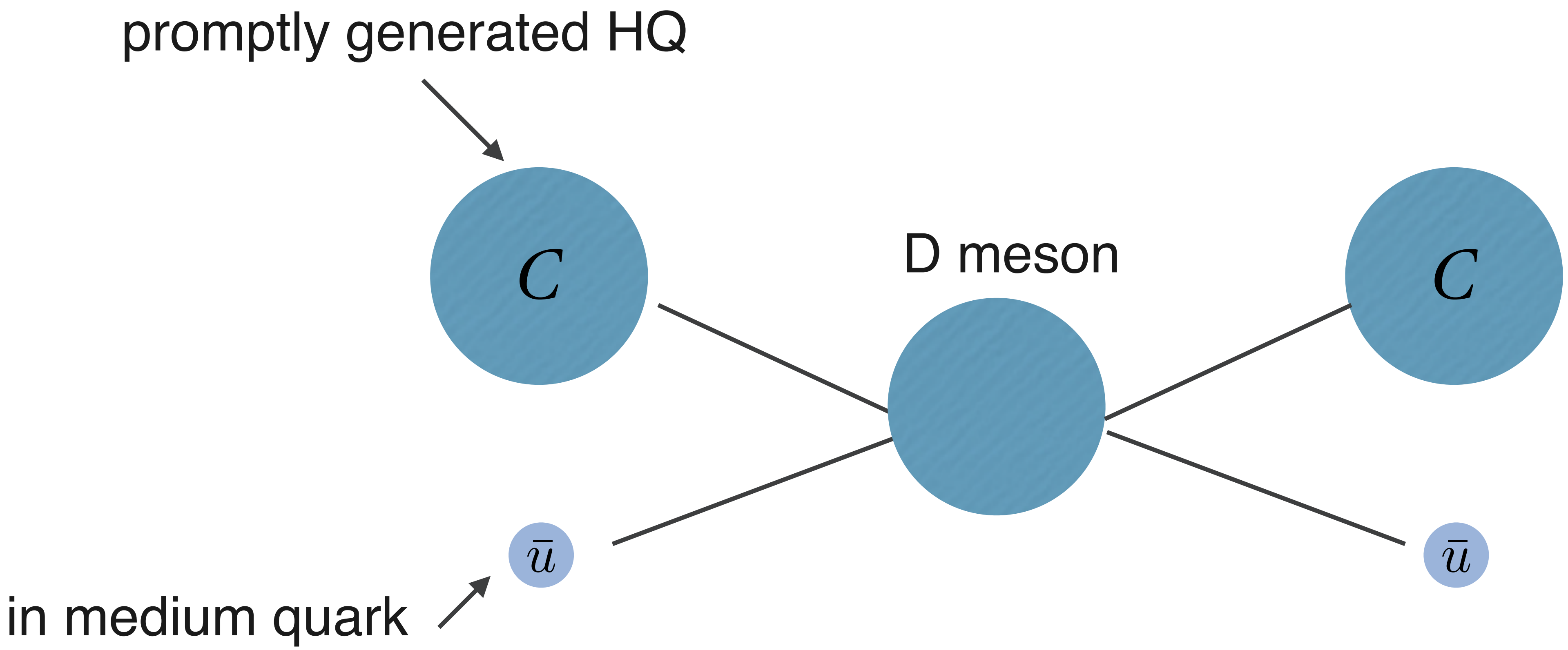

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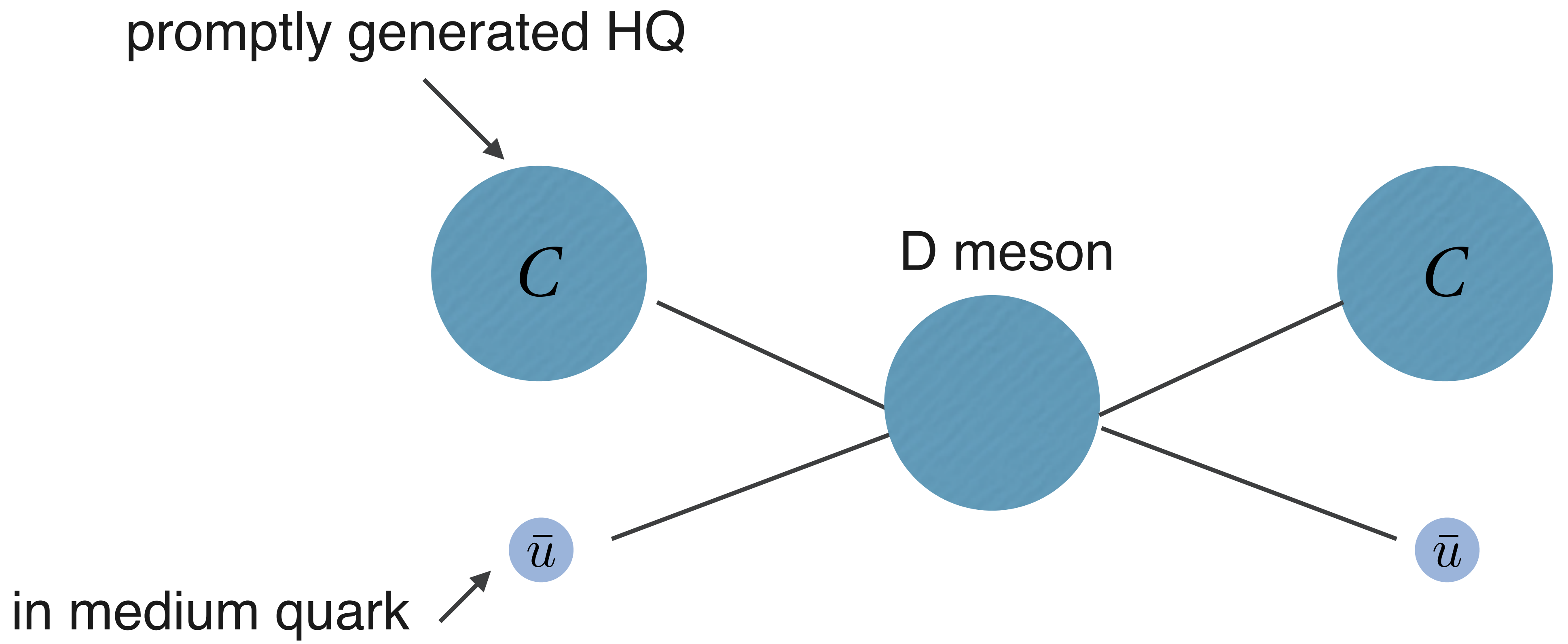

Replacing the light quark with a heavy quark, one has quarkonia generation Thews et al. (2001)

A symmetry breaking path (two flavor quark matter)

$\mu_I = m_{u,d} = 0$

$\psi_L \to U_L \psi_L$

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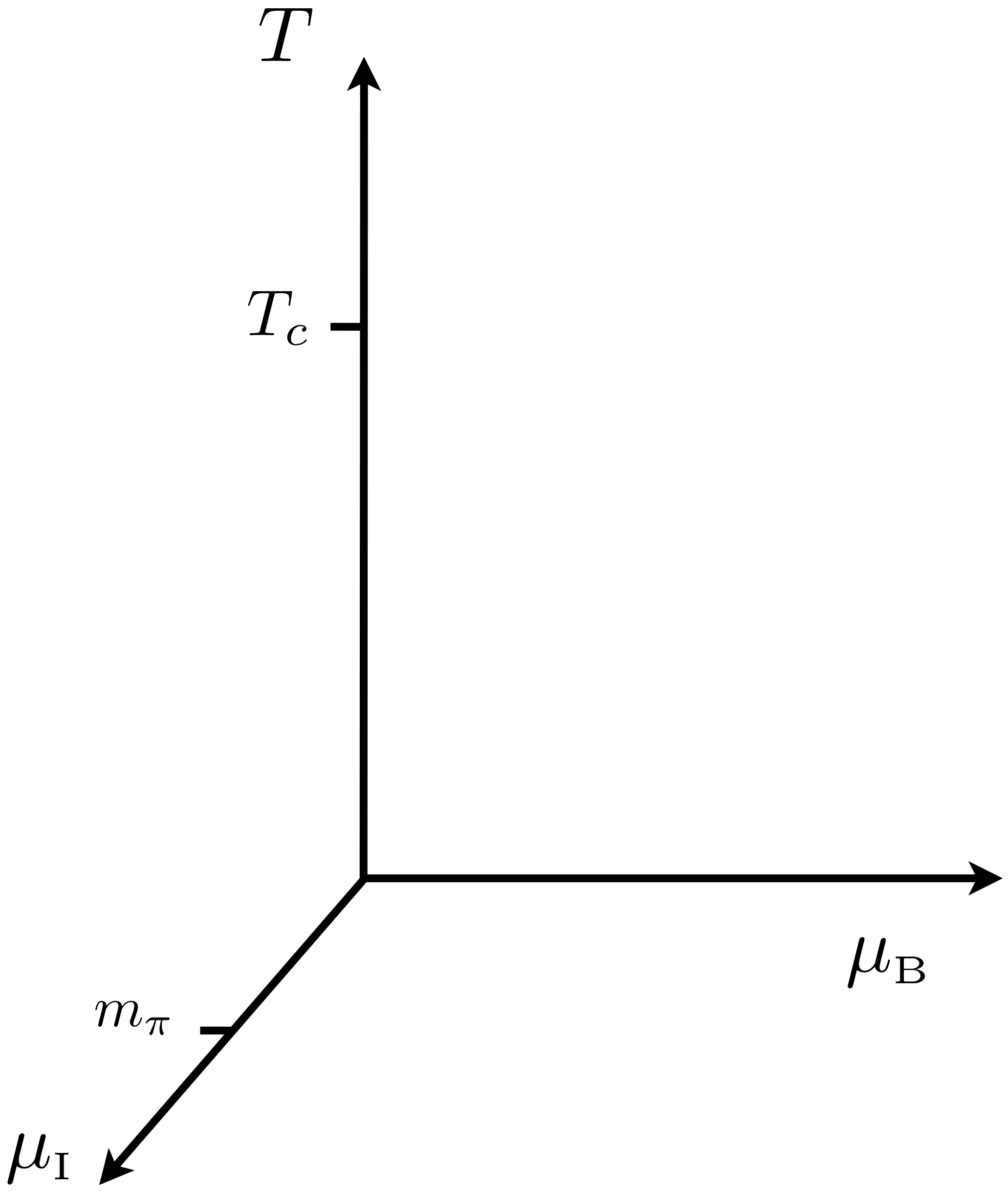

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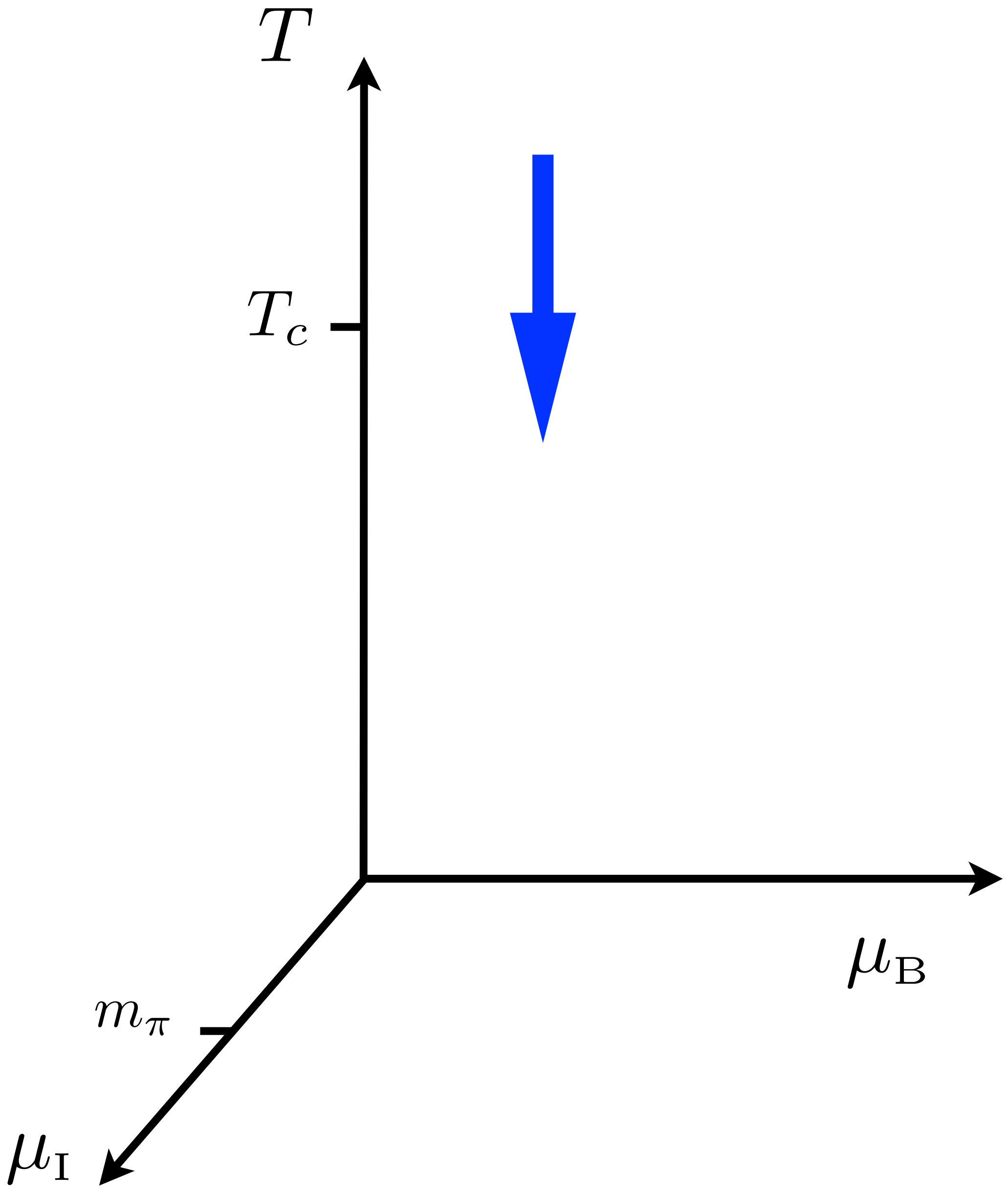

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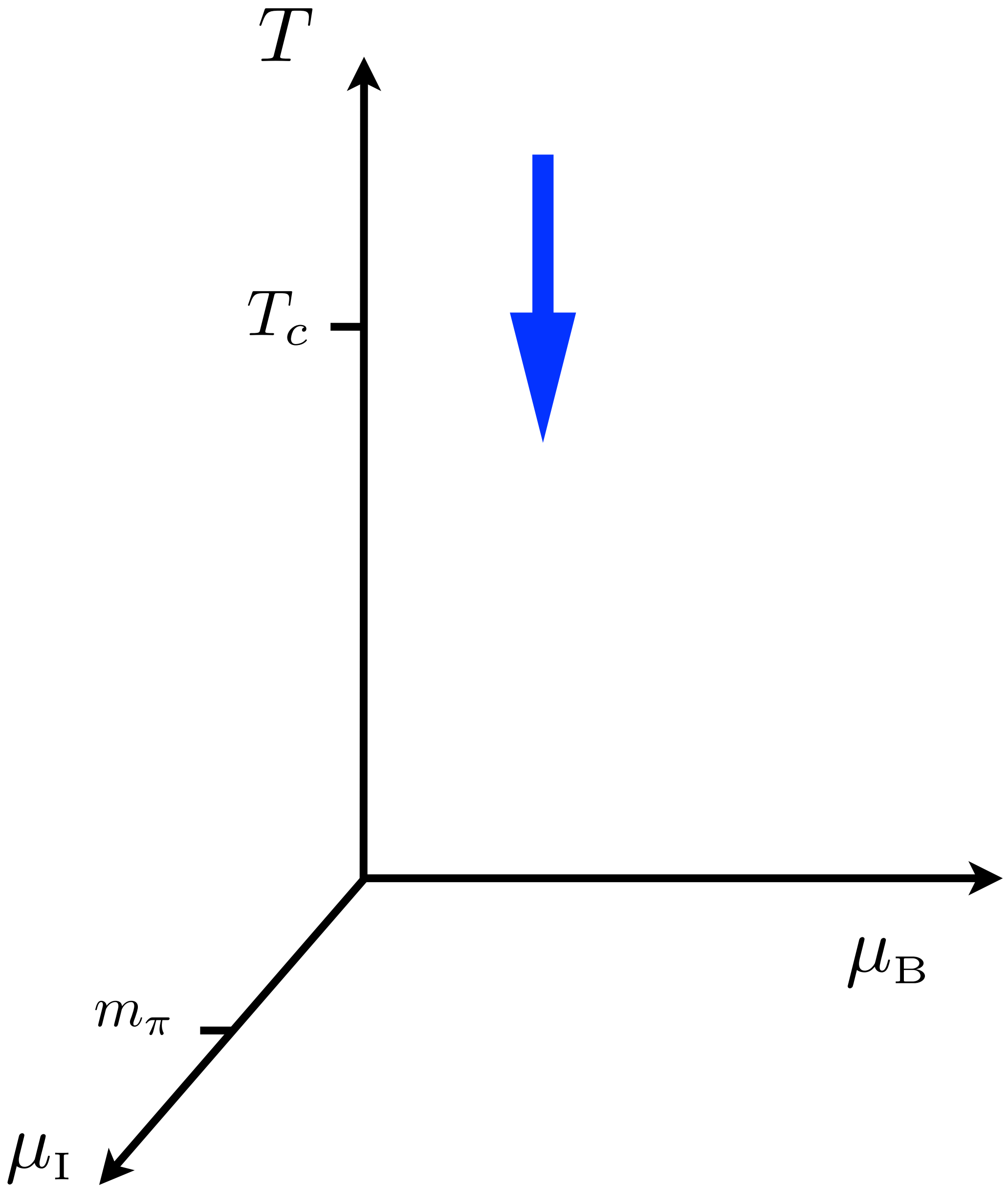

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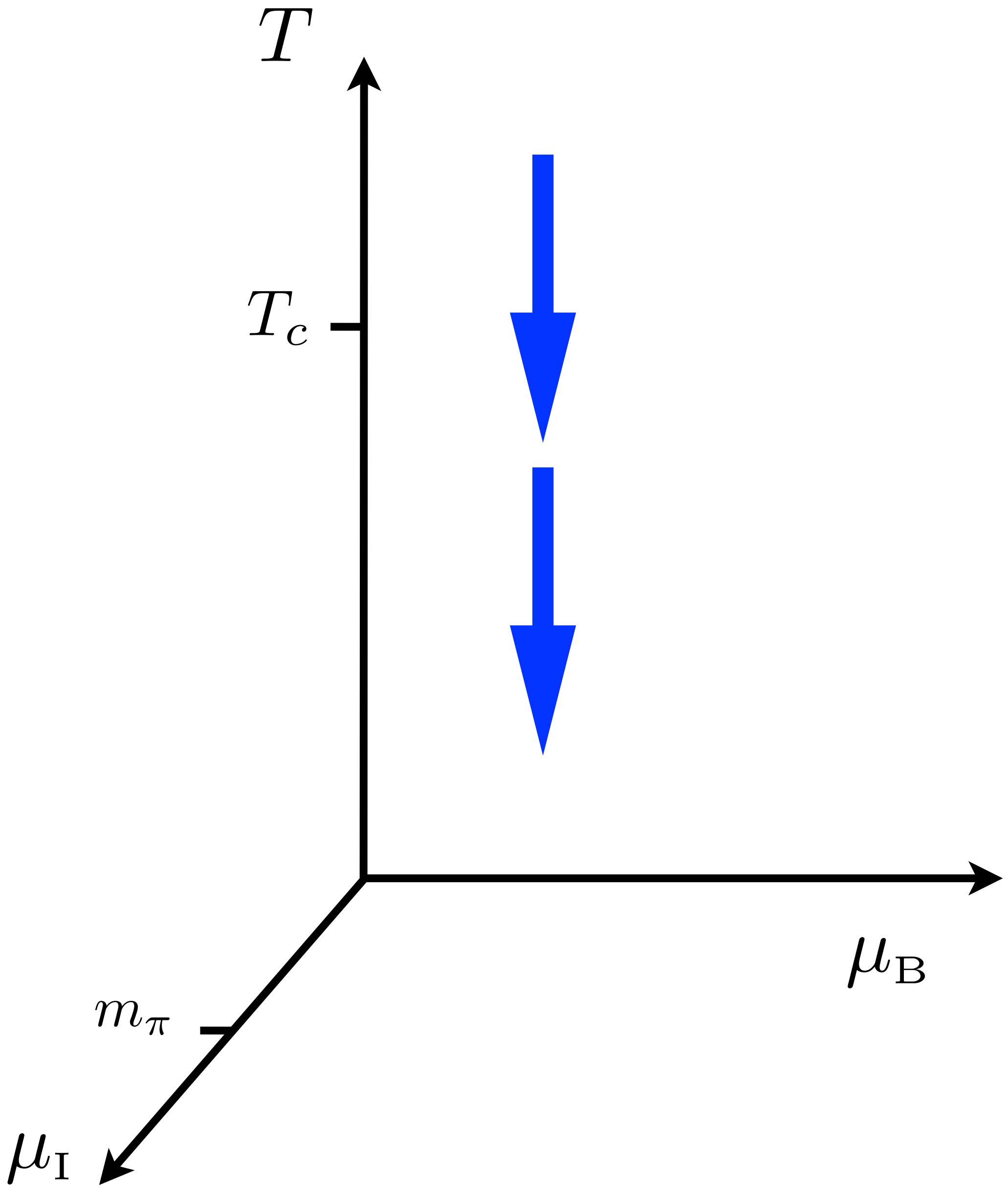

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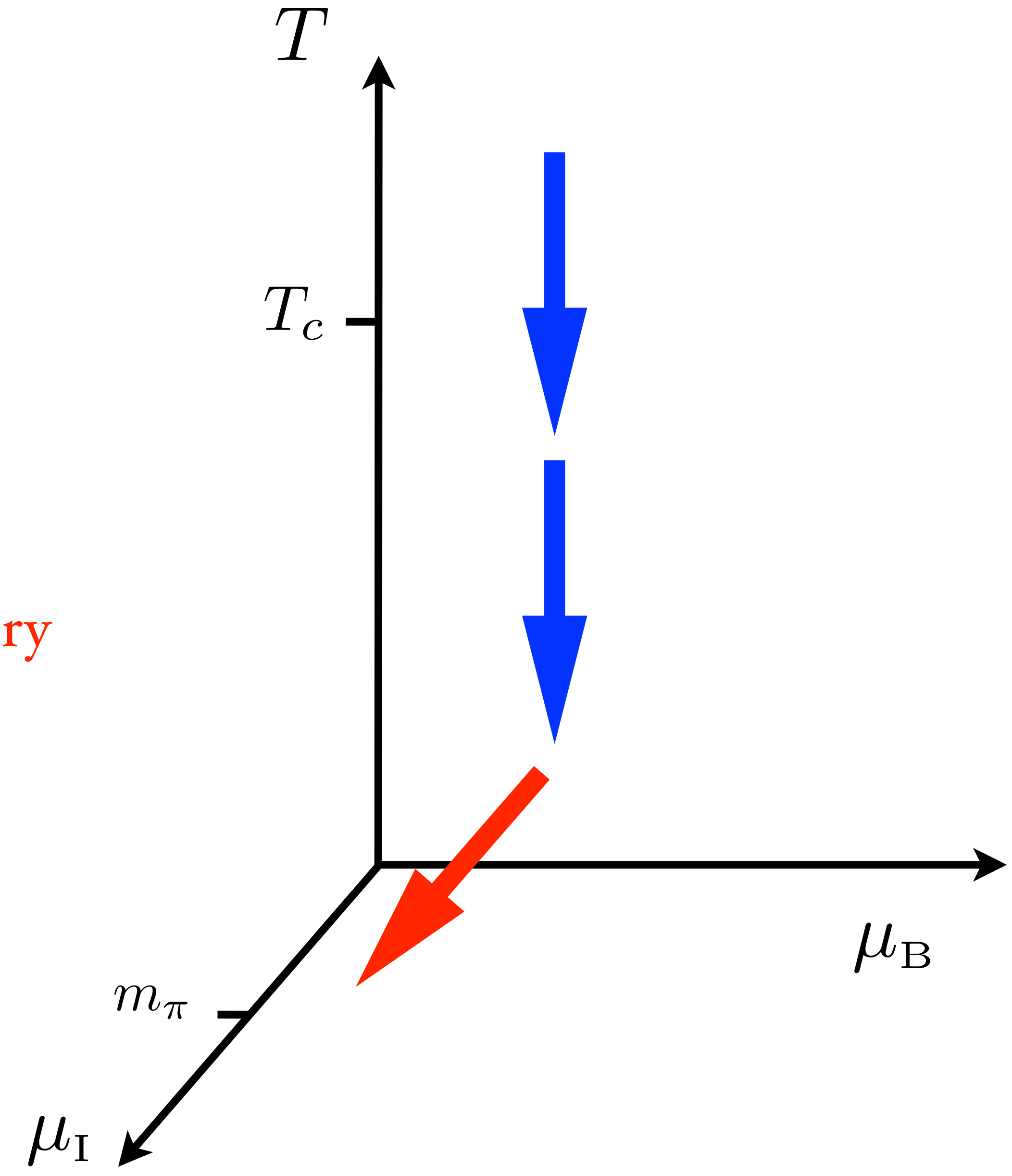

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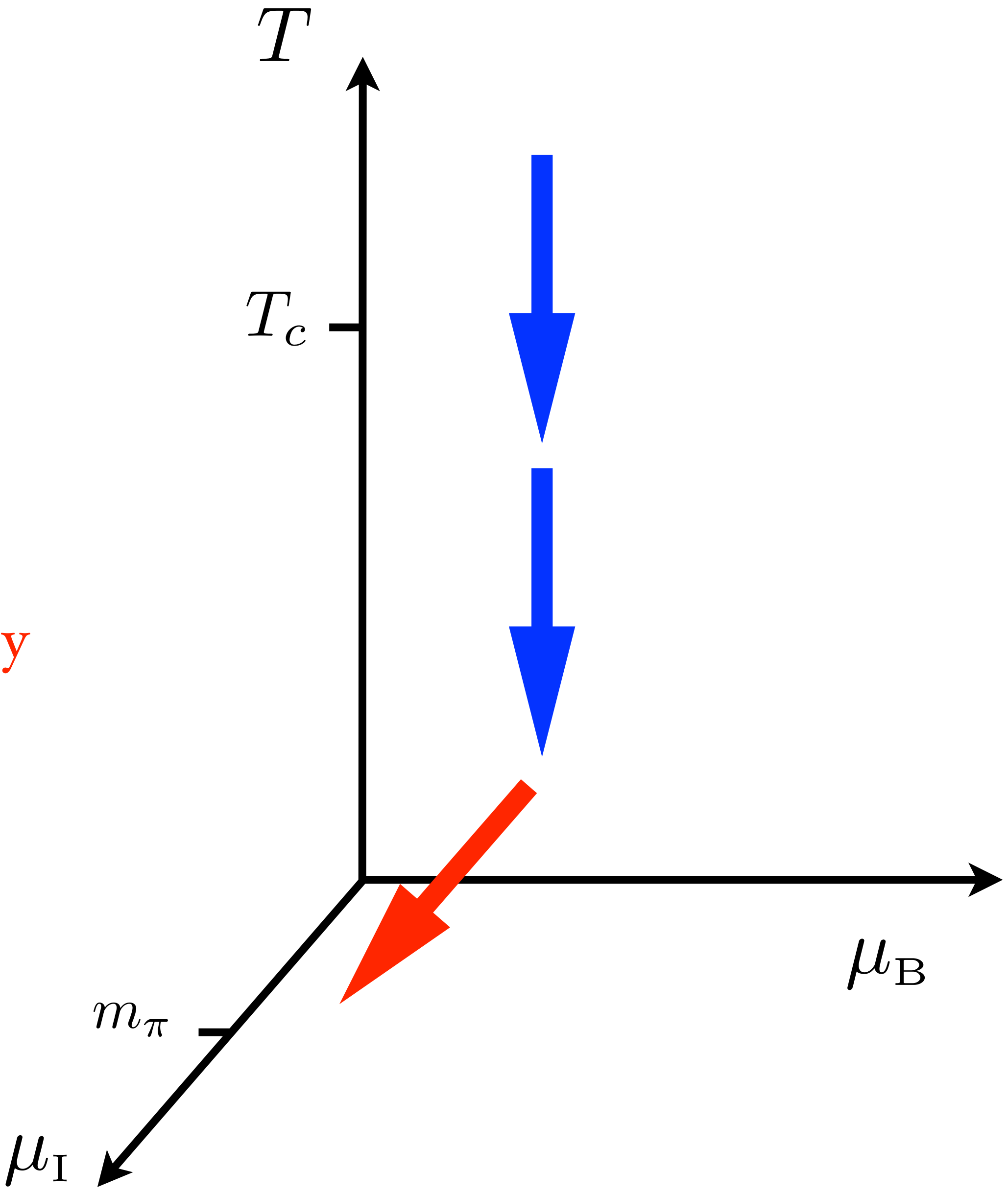

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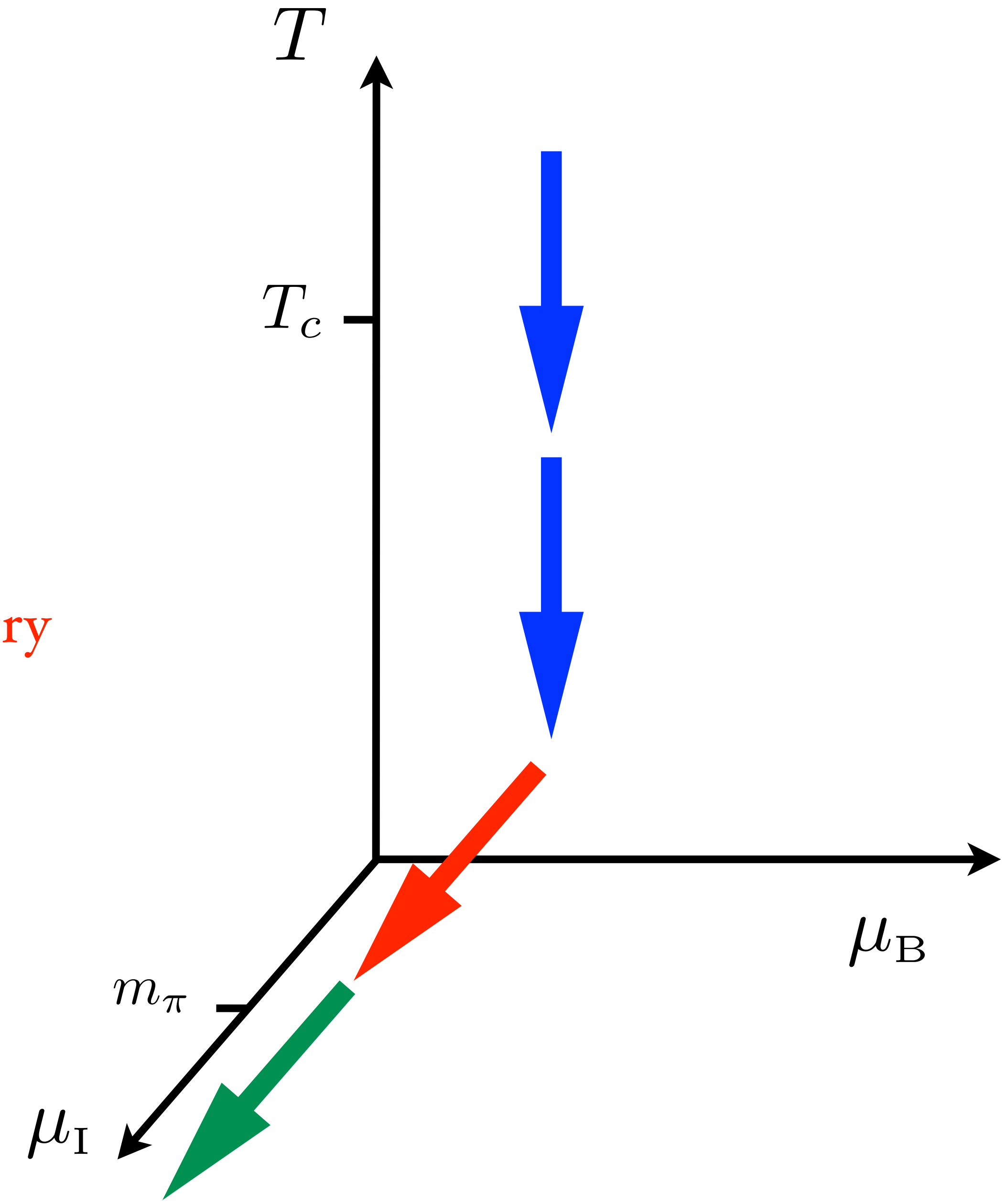

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Spontaneous symmetry breaking (meson condensation)

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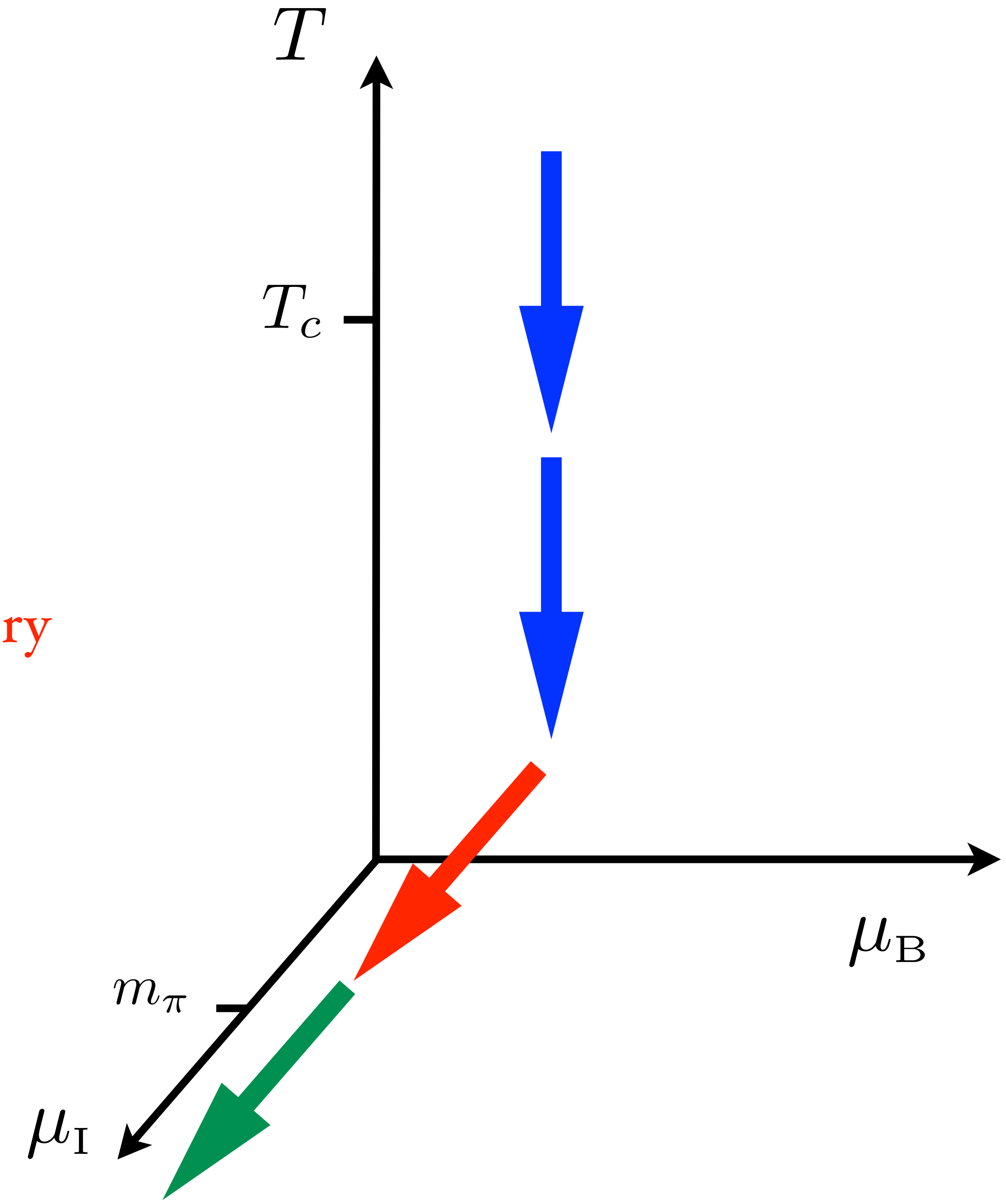
$\langle \bar\psi \sigma_2 \gamma_5 \psi \rangle \neq 0$

$$\underbrace{U(1)_{\rm B}}_{\not\supset [U(1)_{e.m.}]}$$

One NGB



Baryon masses

Protons and neutrons are made of quarks u, d and gluons

$$m_u \simeq 2.3 \text{ MeV}$$
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$$m_g = 0$$

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A nucleon is not a bound state of quarks and gluons

The interaction model

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One possibility is a NJL-like model with the same global symmetries of QCD

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Free Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - M + \mu\gamma_0)\psi$$

+

Contact interaction

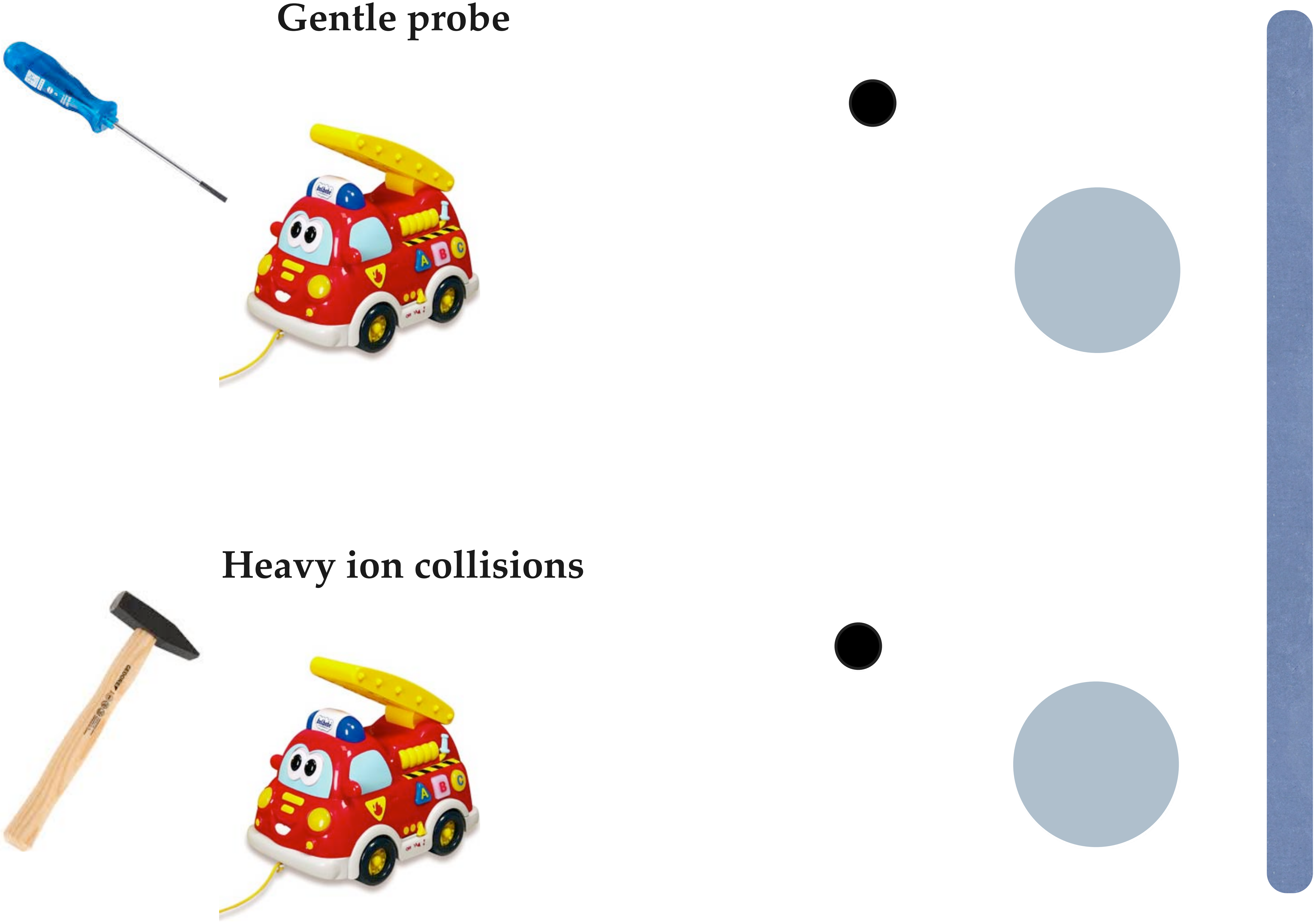
$$\mathcal{L}_{\text{int}} = -g\,\bar{\psi}\gamma_\mu \lambda^A \psi \bar{\psi}\gamma^\mu \lambda^A \psi$$

coupling constant

spin, color, flavor structure

$$M = \text{diag}(m, m, m_S)_{ij}$$

$$\mu \equiv \mu_{ij,\alpha\beta}$$



detector

Gentle probe

Heavy ion collisions

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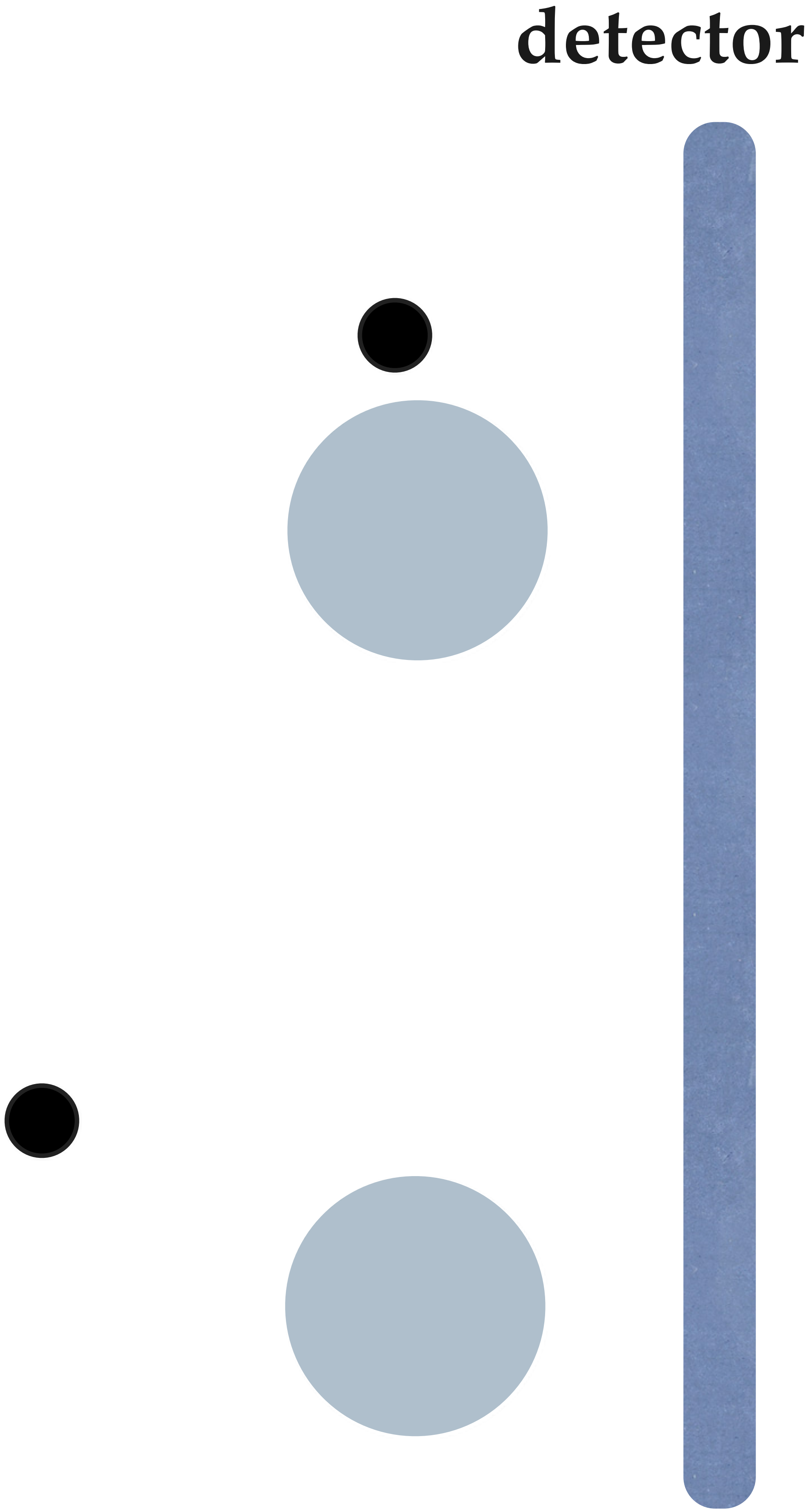

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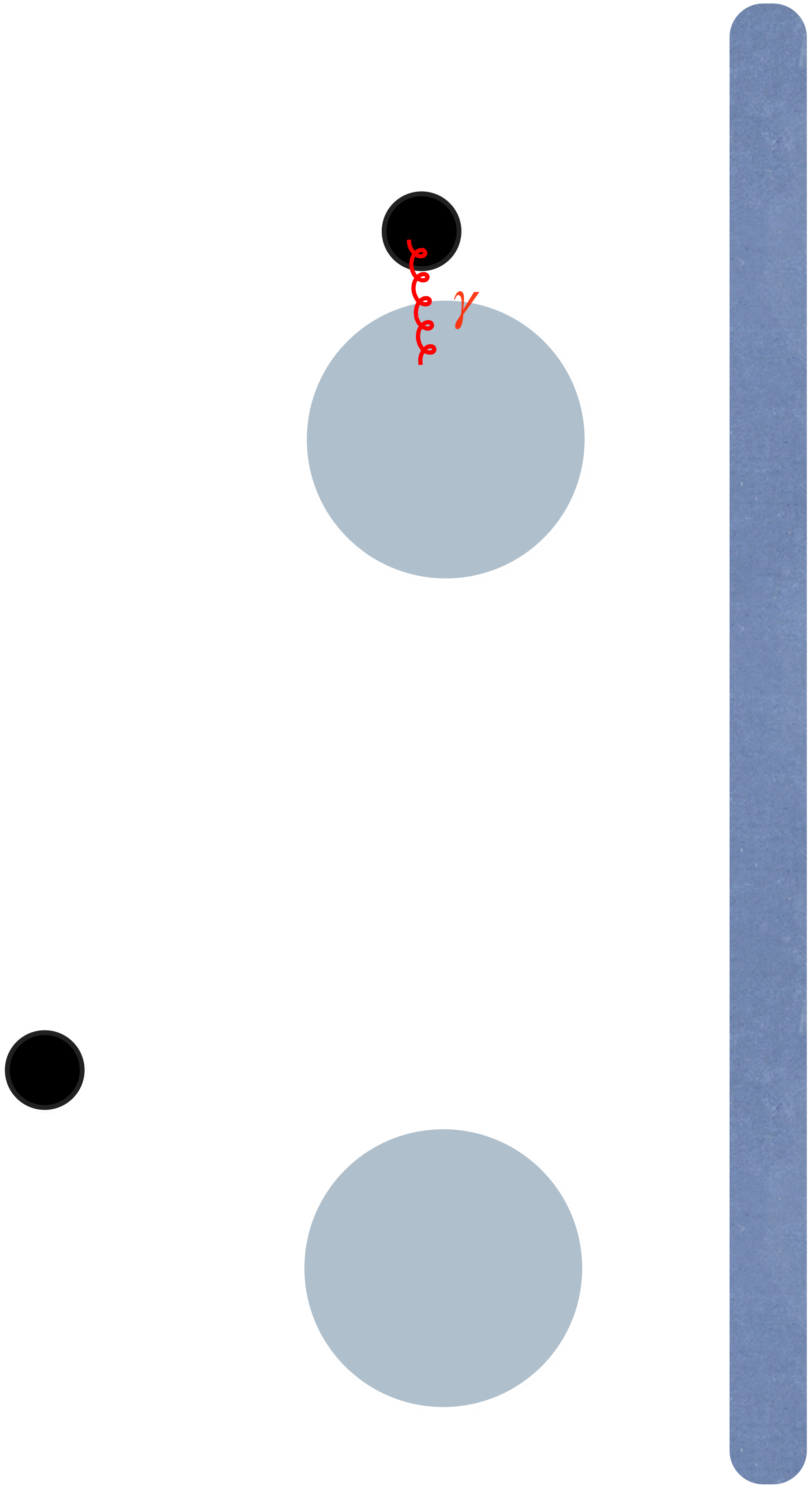

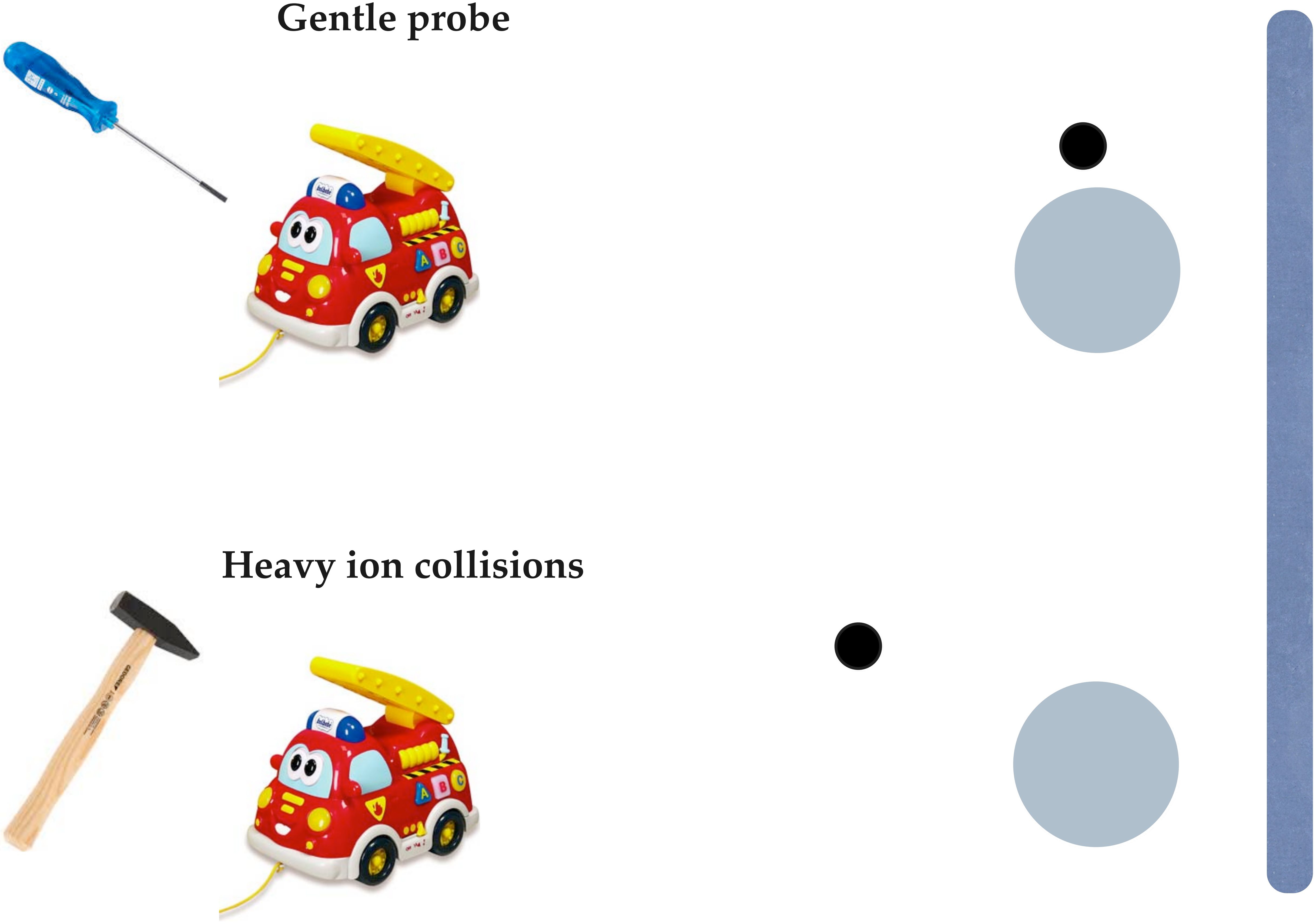

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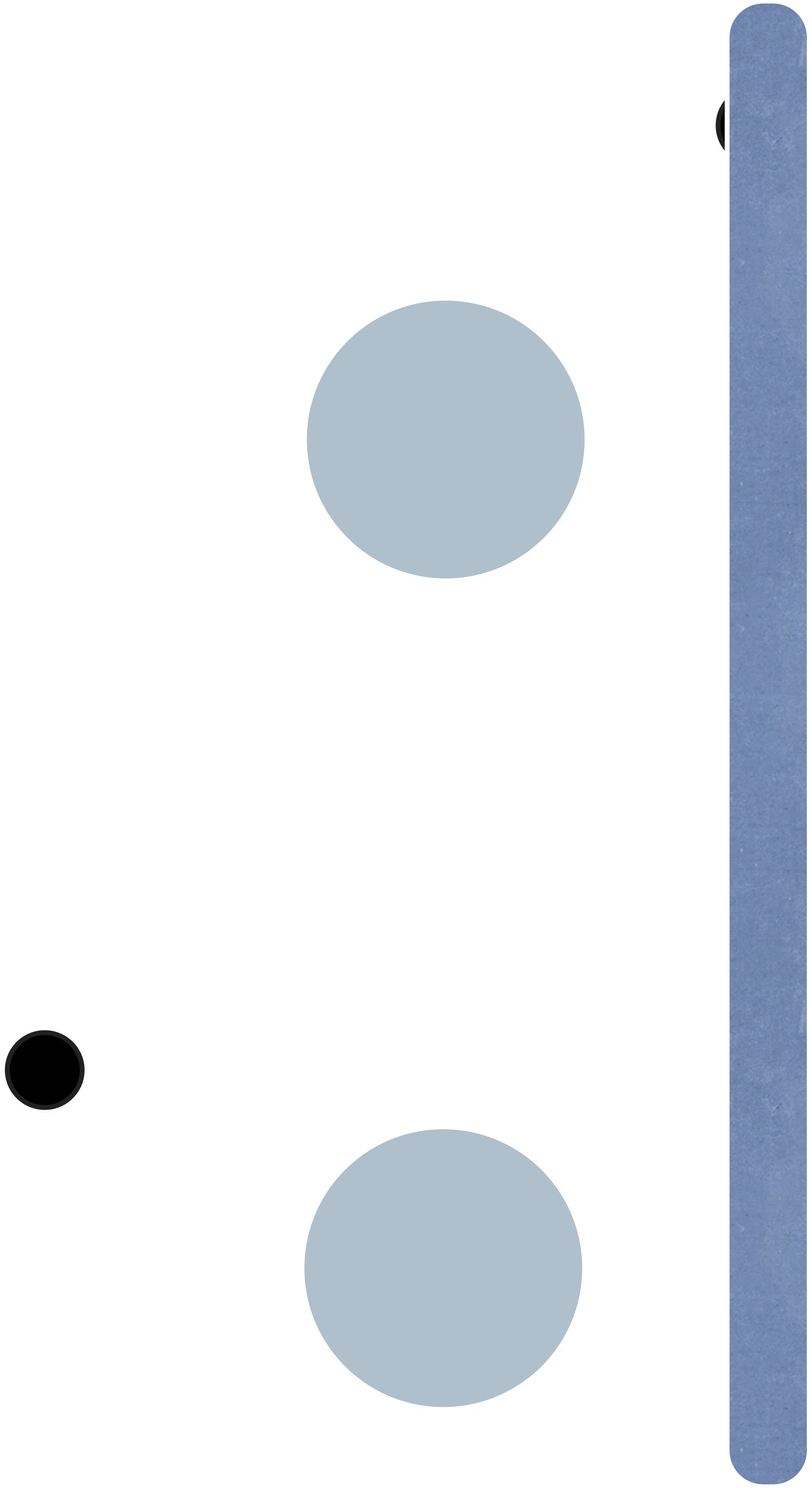

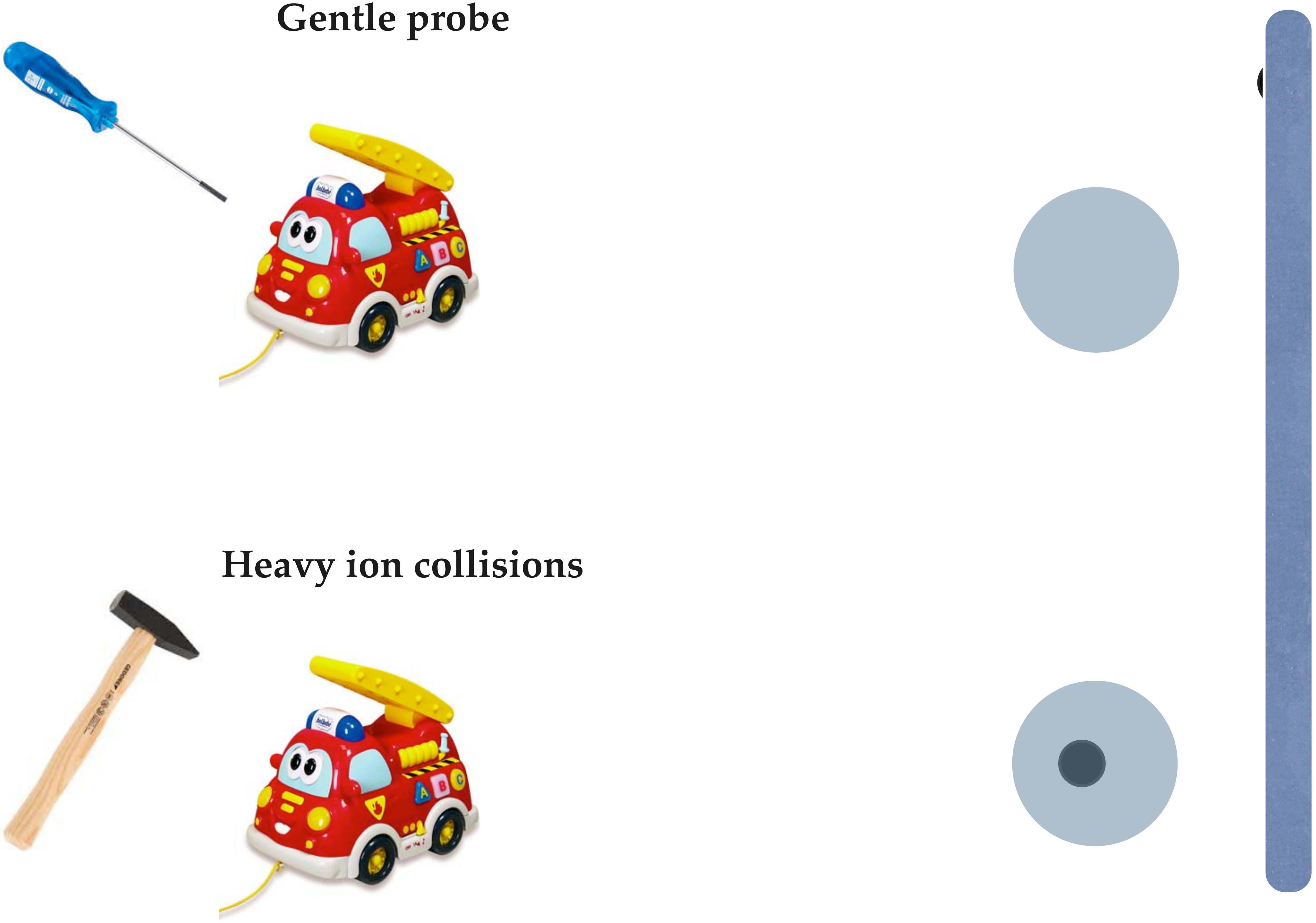

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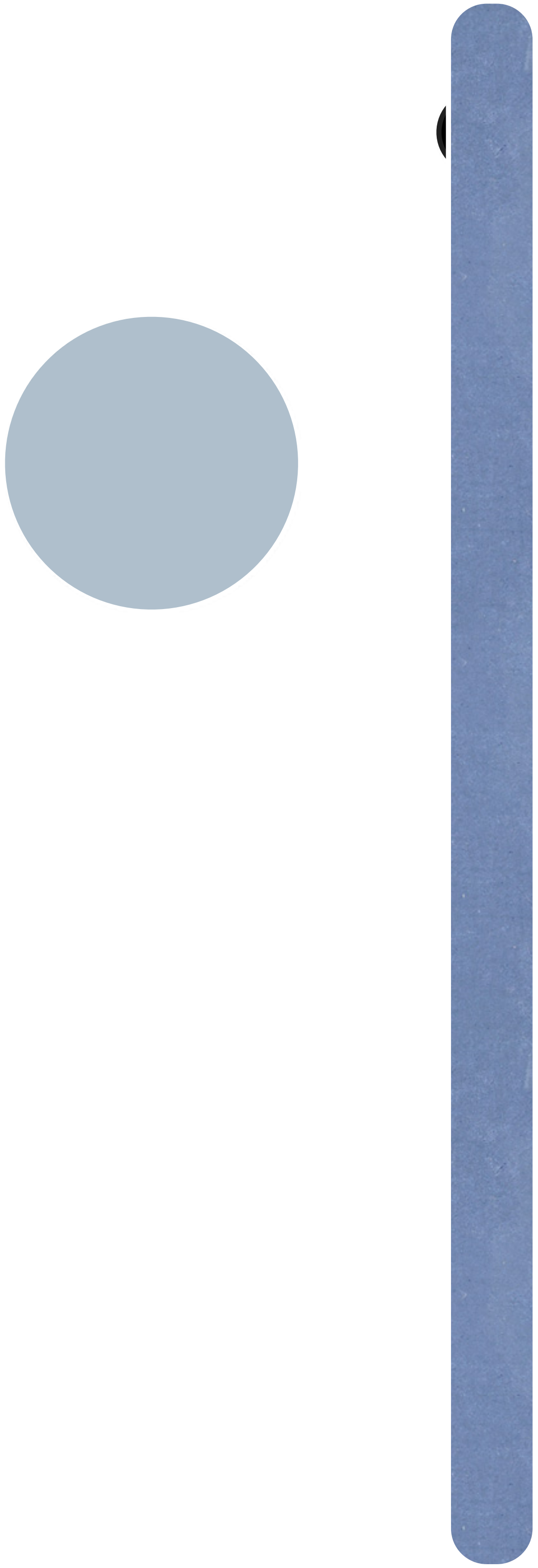
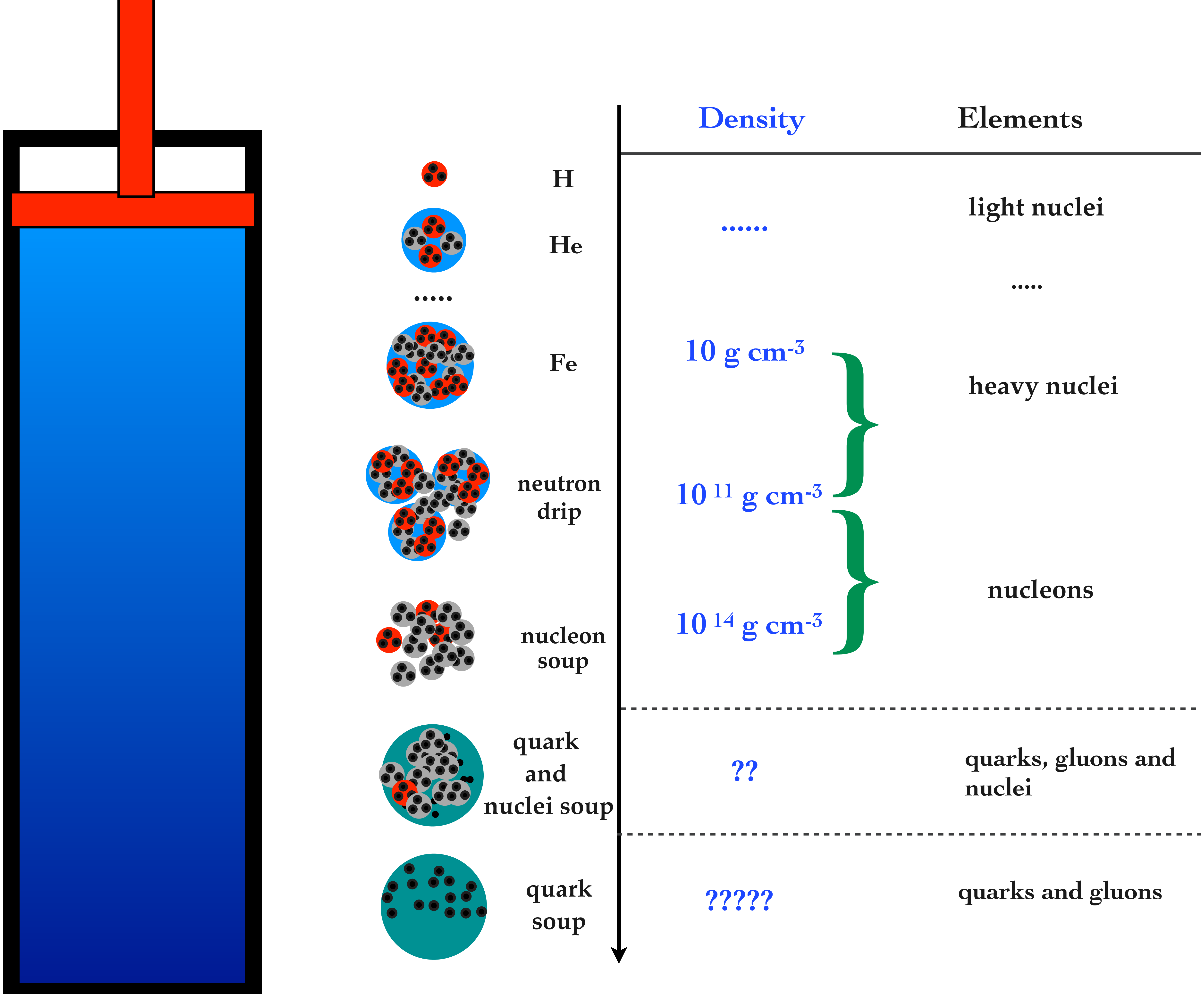




H

He

......

Fe

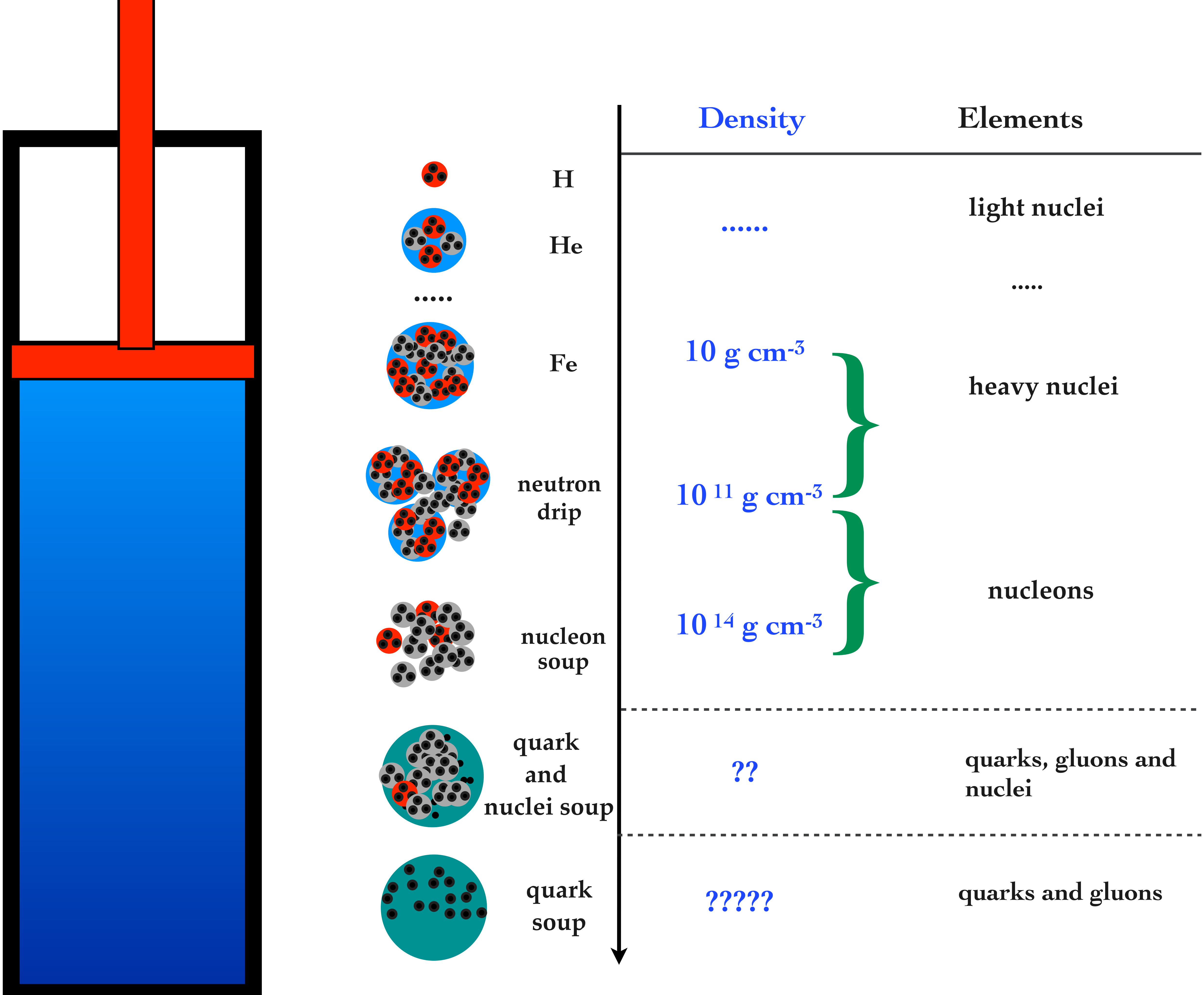
neutron drip

nucleon soup

quark and nuclei soup

quark soup

Density	Elements
......	light nuclei
	.....
$10\ \mathrm{g\ cm^{-3}}$	heavy nuclei
$10^{11}\ \mathrm{g\ cm^{-3}}$	
$10^{14}\ \mathrm{g\ cm^{-3}}$	nucleons
??	quarks, gluons and nuclei
?????	quarks and gluons



	Density	Elements
H		light nuclei
He	......	
......		.....
Fe	10 g cm^{-3}	heavy nuclei
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Diquark Condensate

Quark fields $\psi_{\alpha i}$

$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, 2, 3$ **flavor indices**

Mixture of **9 different fermions**. Six of them are relativistic, three are non relativistic

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$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, 2, 3$ **flavor indices**

Mixture of **9 different fermions**. Six of them are relativistic, three are non relativistic

General color superconducting condensate

color structure

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \sum_{I=1}^{3} \Delta_{\mathrm{I}} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

flavor structure

gap parameters

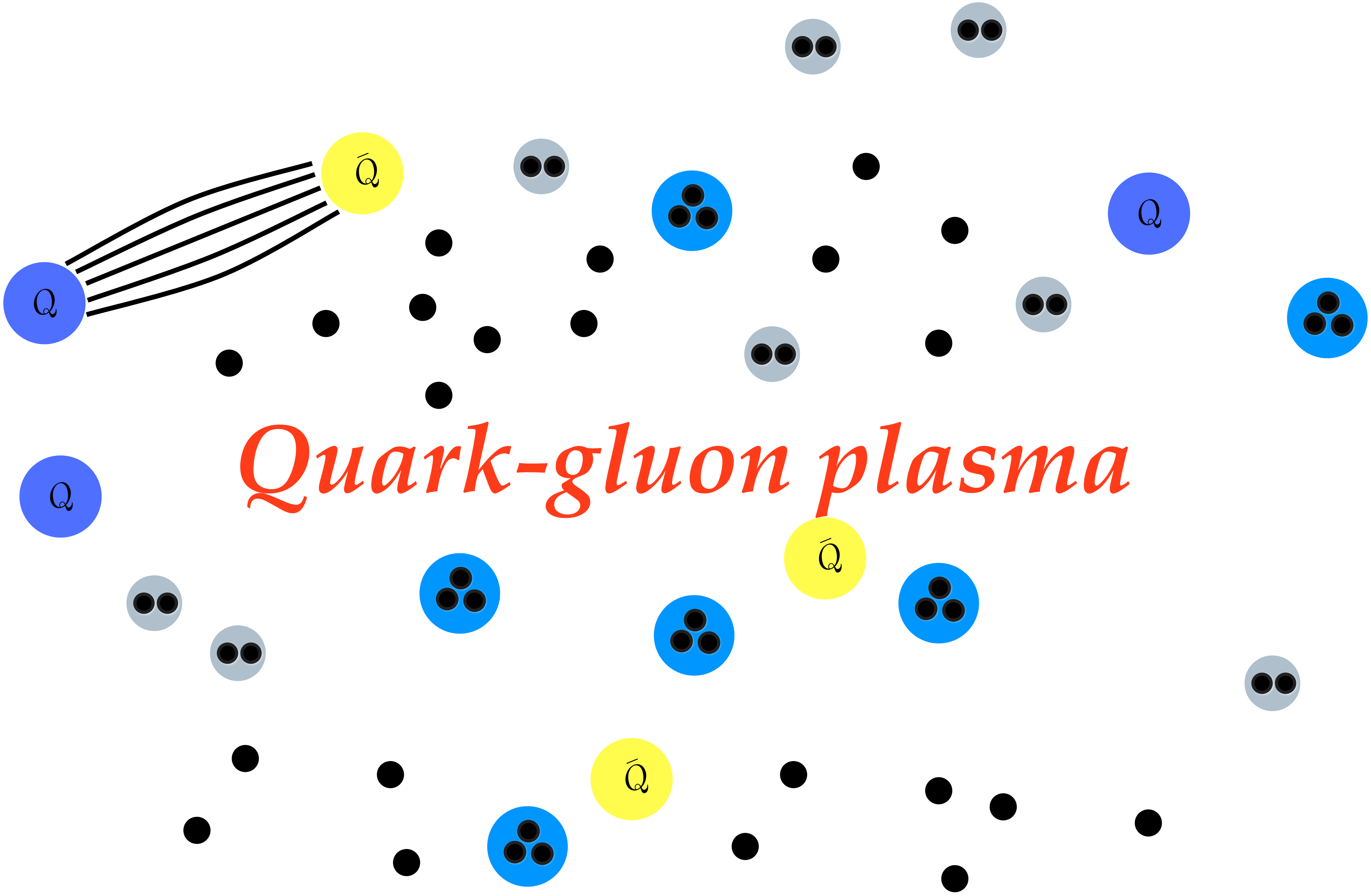
It has a color charge

It has a flavor charge

It has a baryonic charge

The corresponding symmetries are broken, locked or mixed

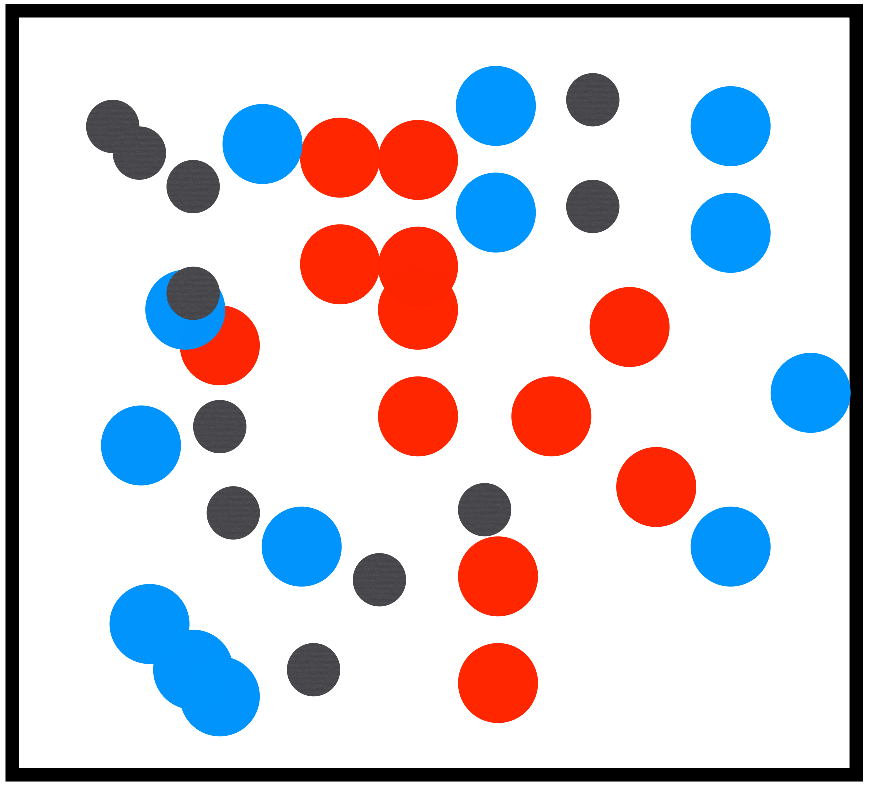
Quark-gluon plasma



Deconfinement by increasing temperature

Mesons ● Baryons 🔴 Anti Baryons 🔵

Increasing T
Fixed low μ_B



At high temperature, matter interacts so strongly to producea large number of mesons and baryons

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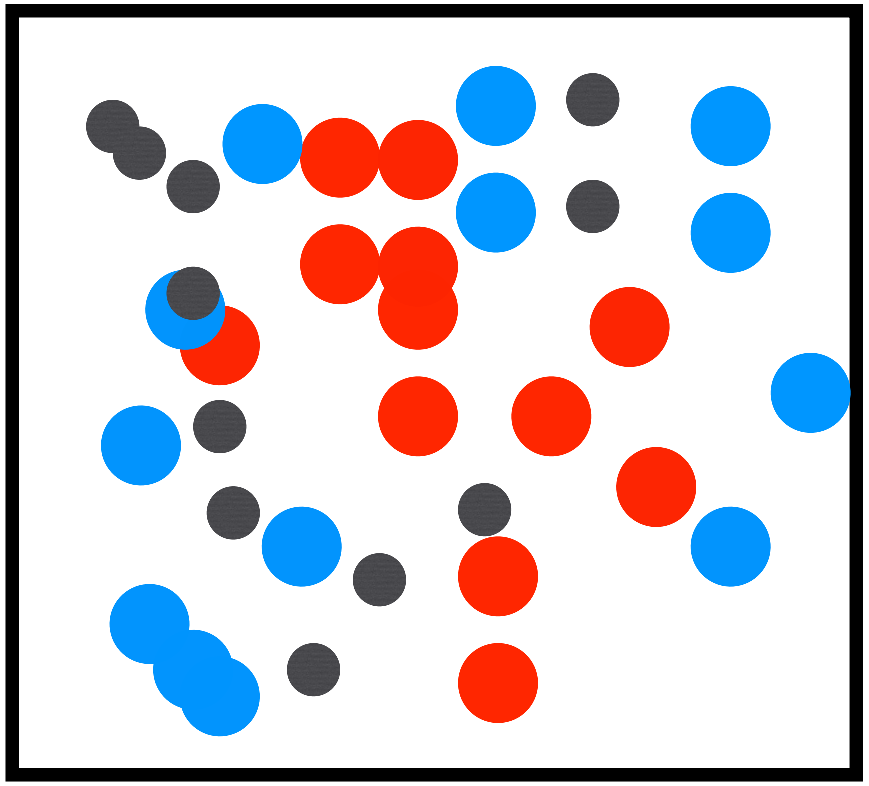

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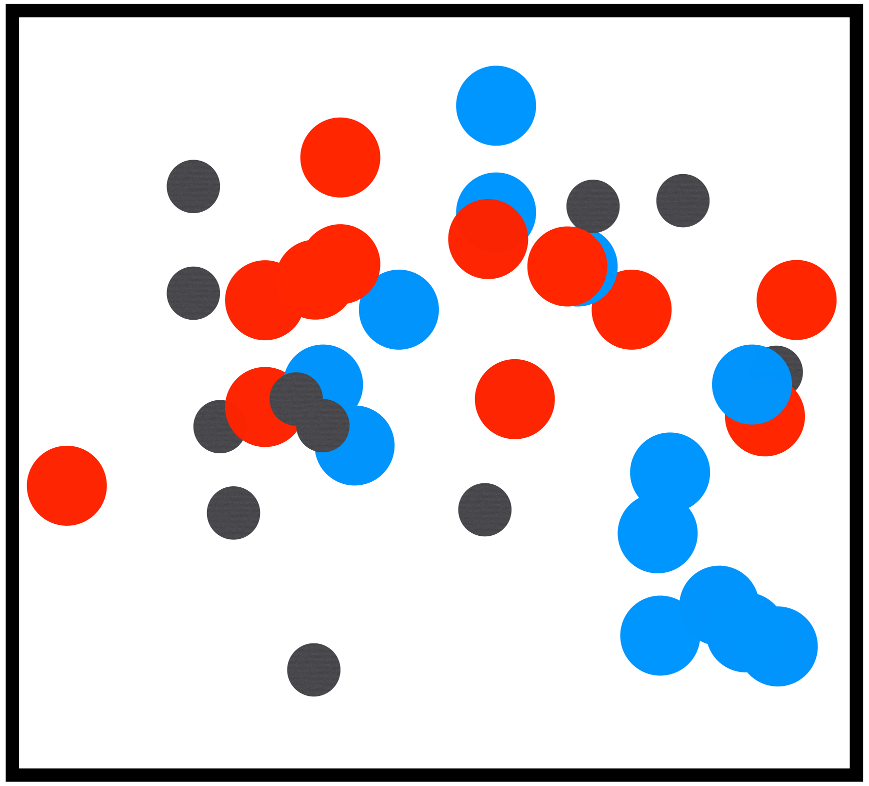

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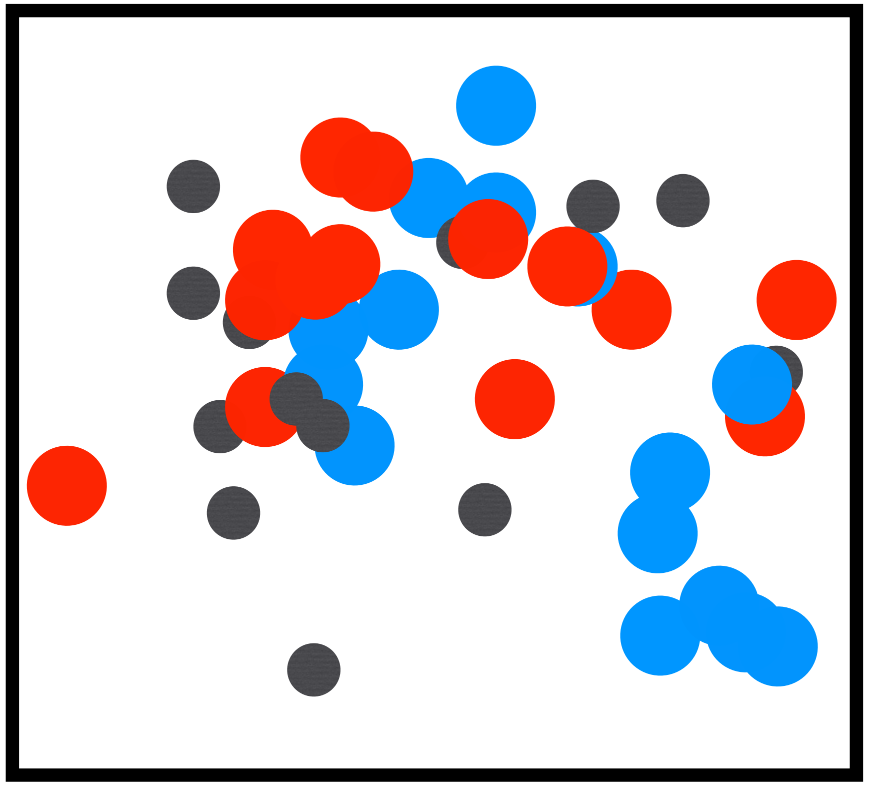

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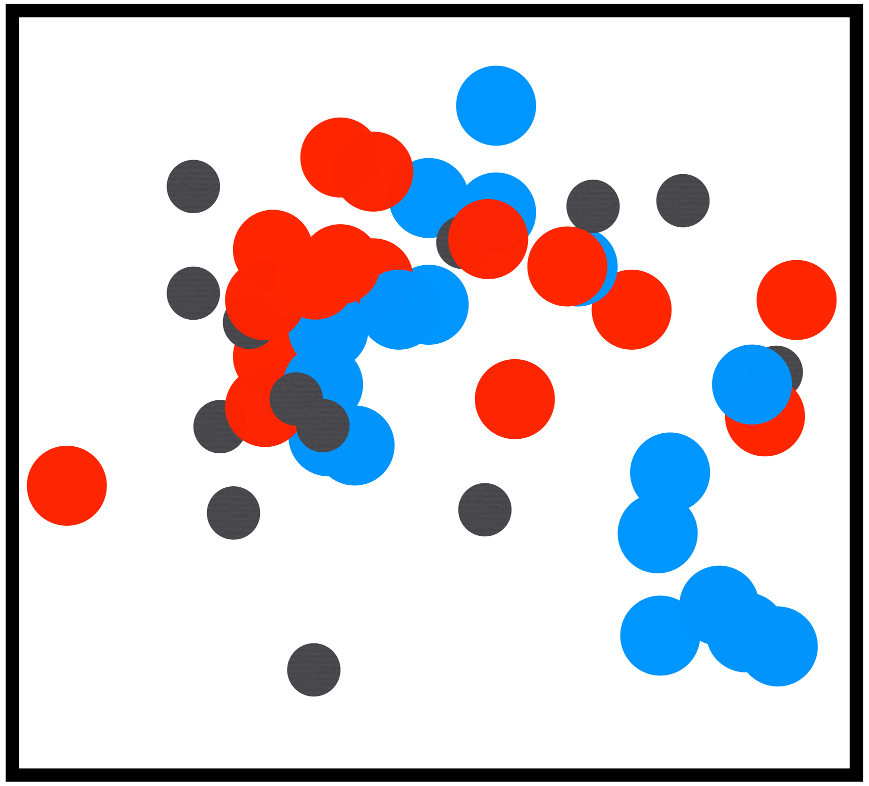

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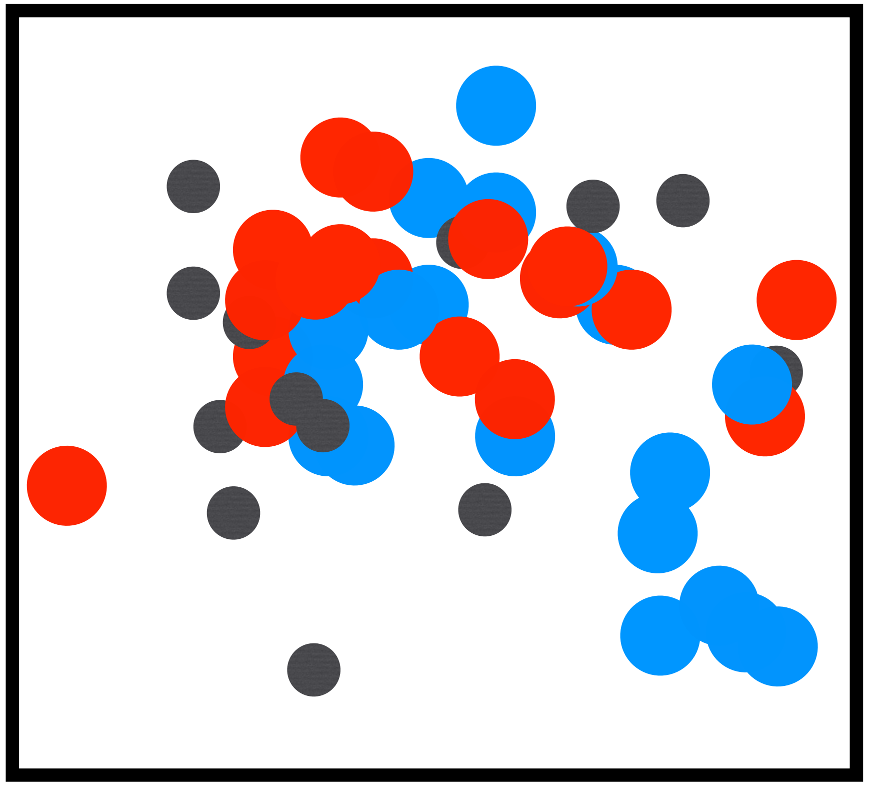

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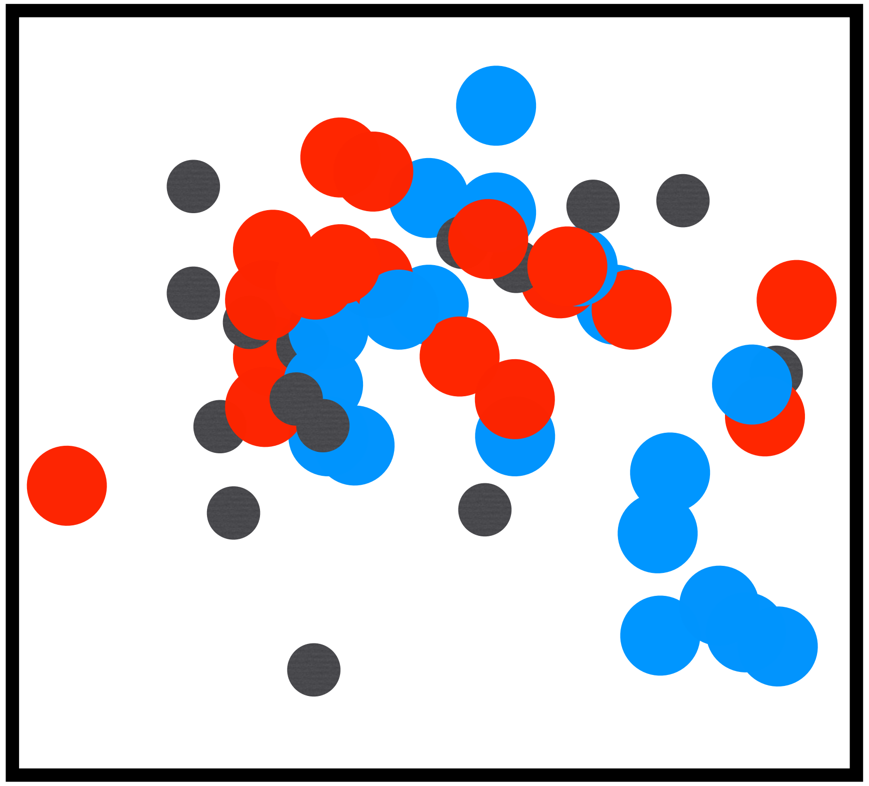

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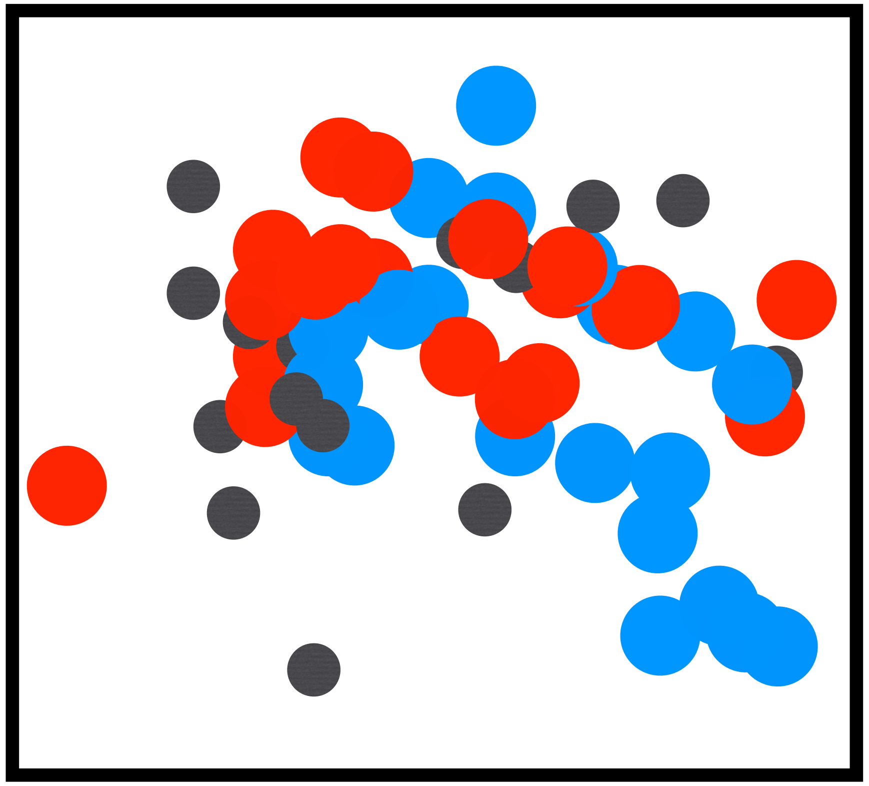

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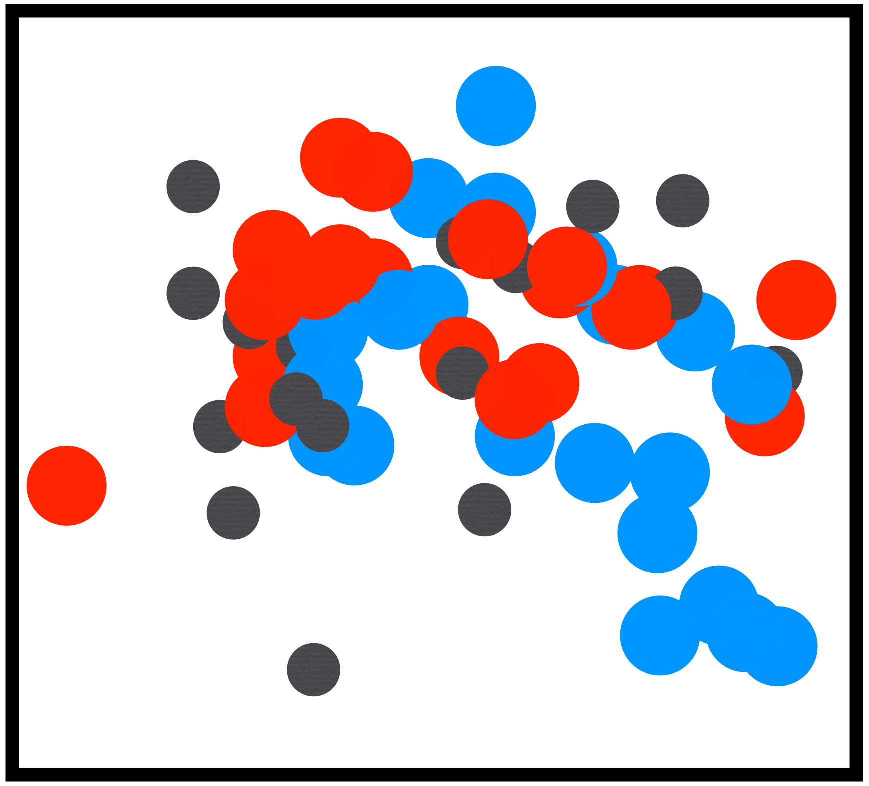

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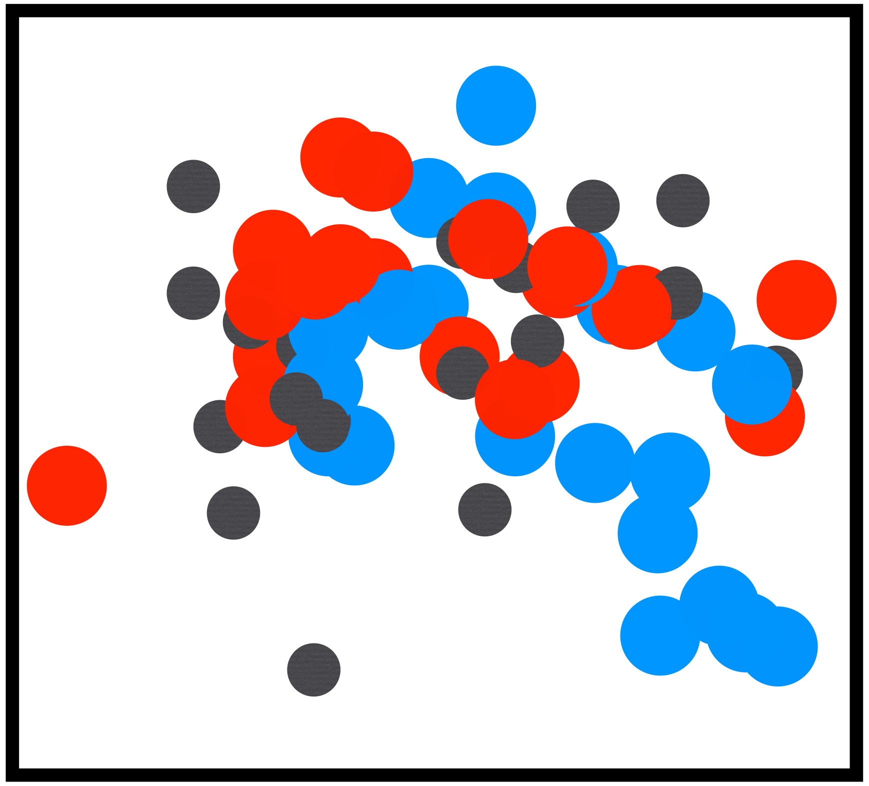

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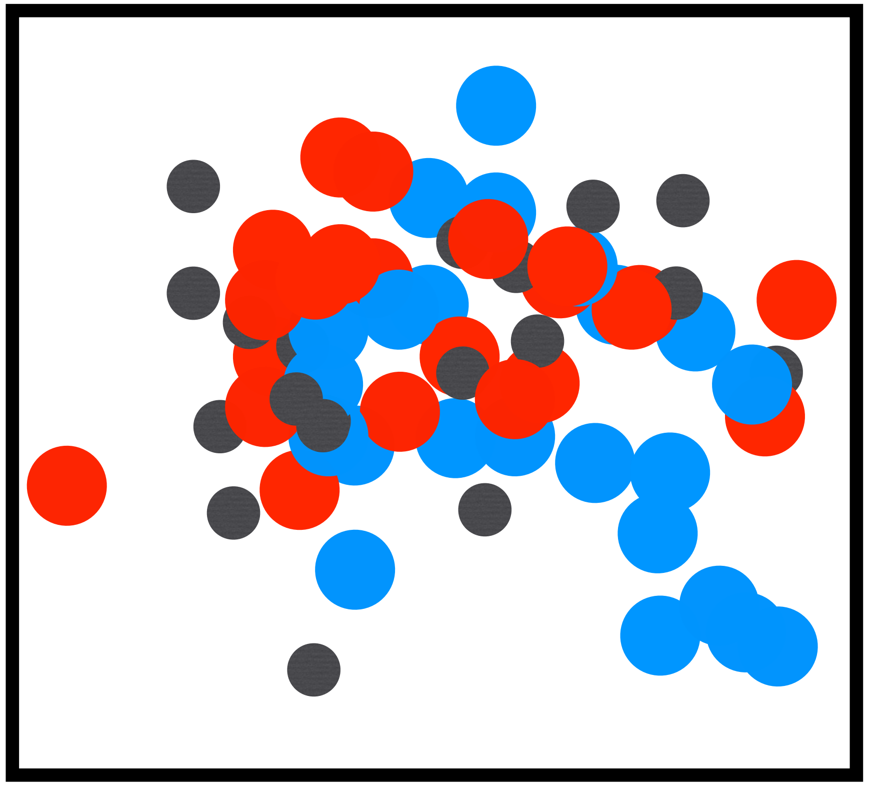

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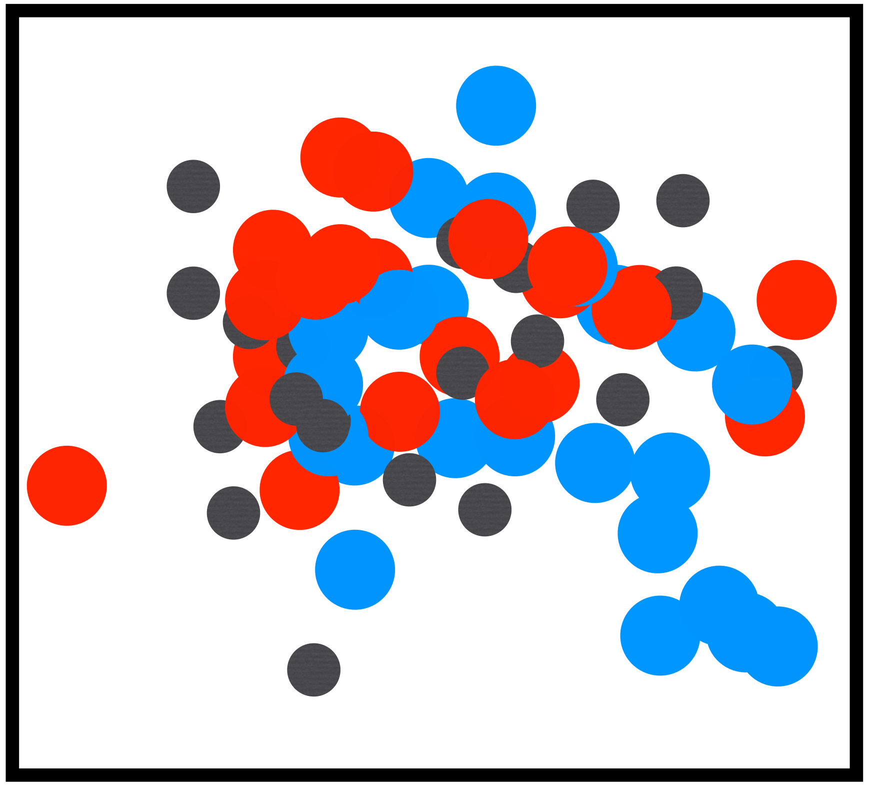

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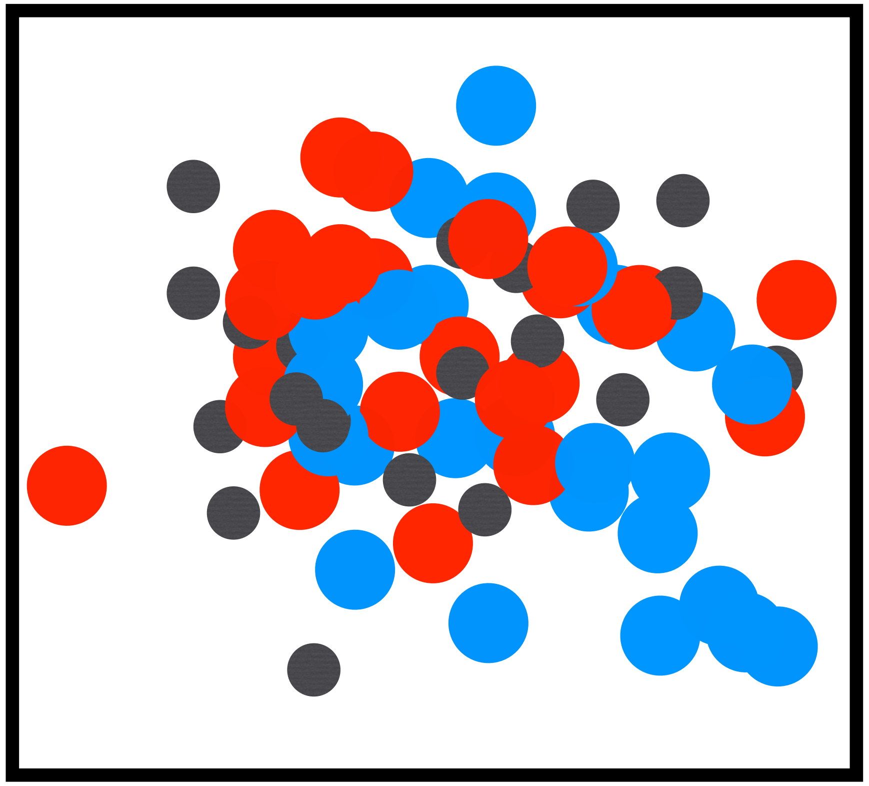

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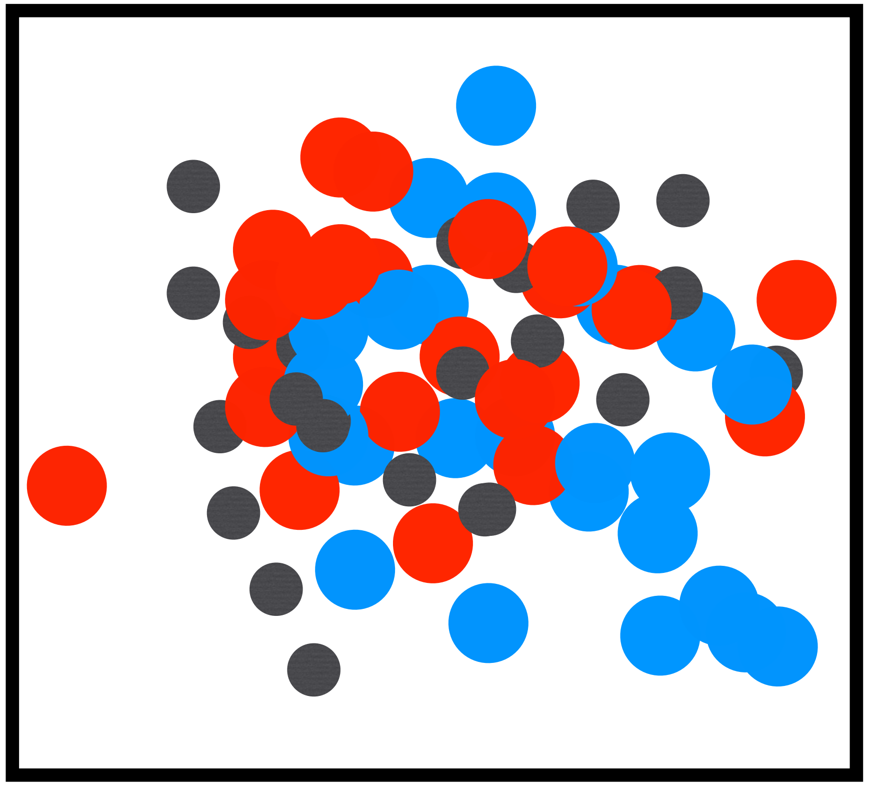

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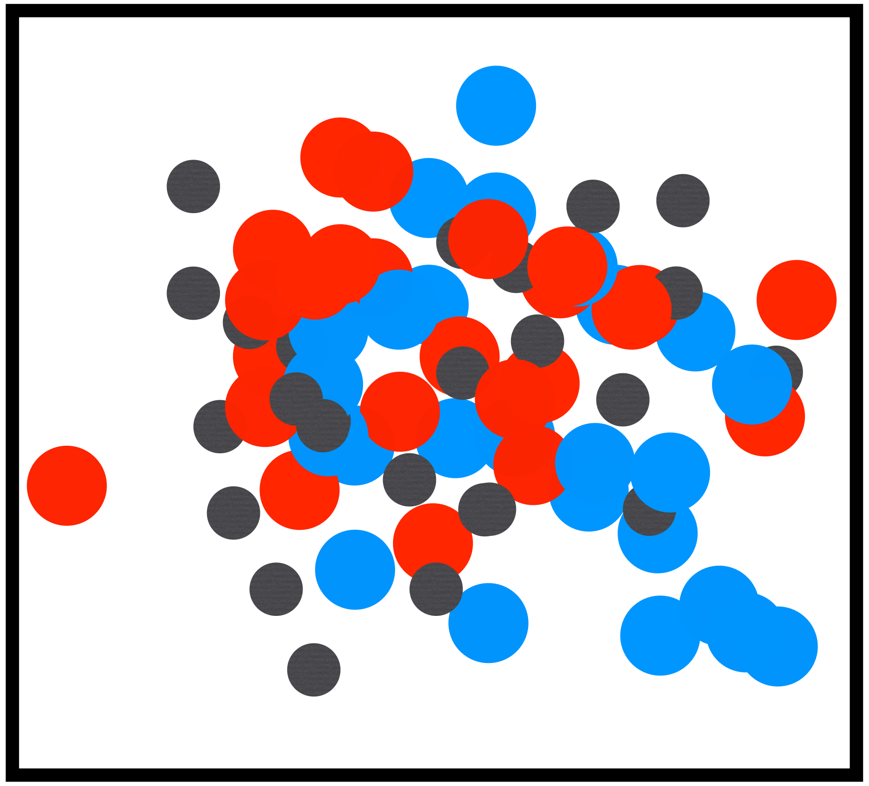

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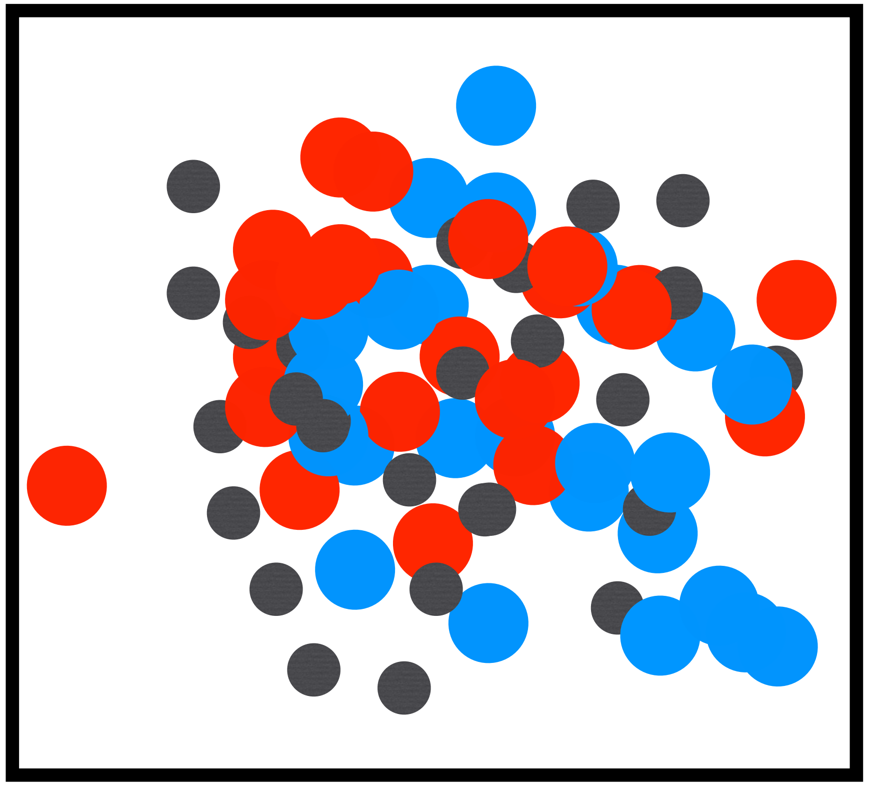

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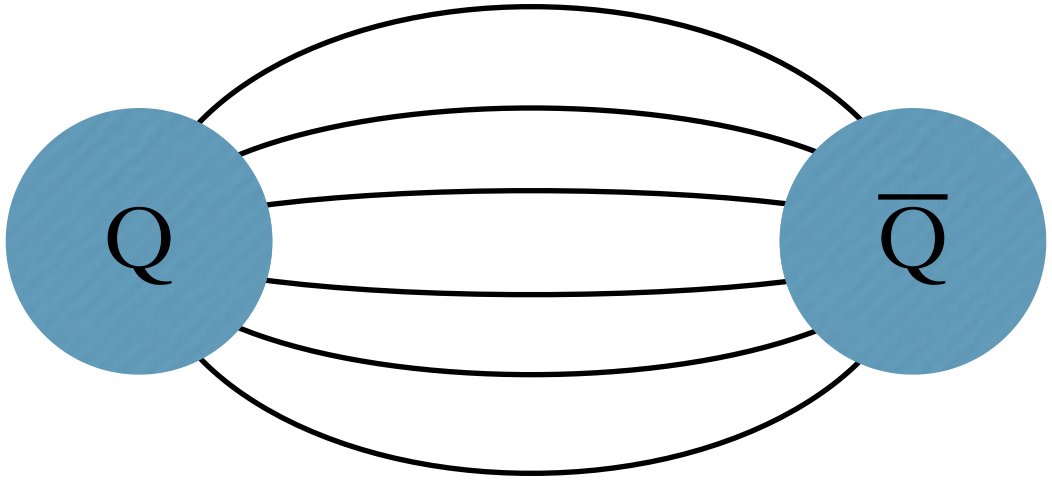
Heavy quarkonia as probes



VACUUM
Anti-screening

It is a bound state
Potential models can be used

$$V(r) \sim \sigma r - \frac{\alpha}{r}$$

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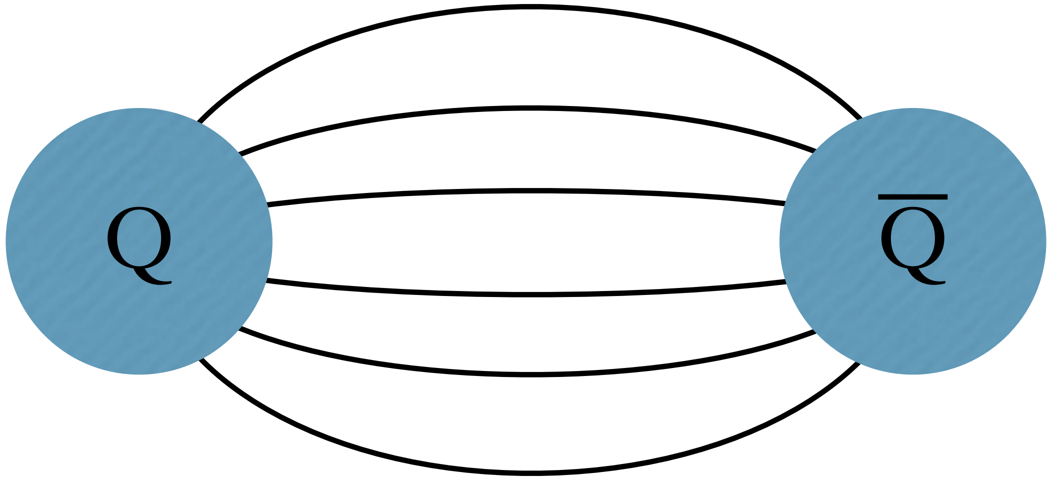

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Increasing T and/or μ_B they can dissociate by the combination of different effects

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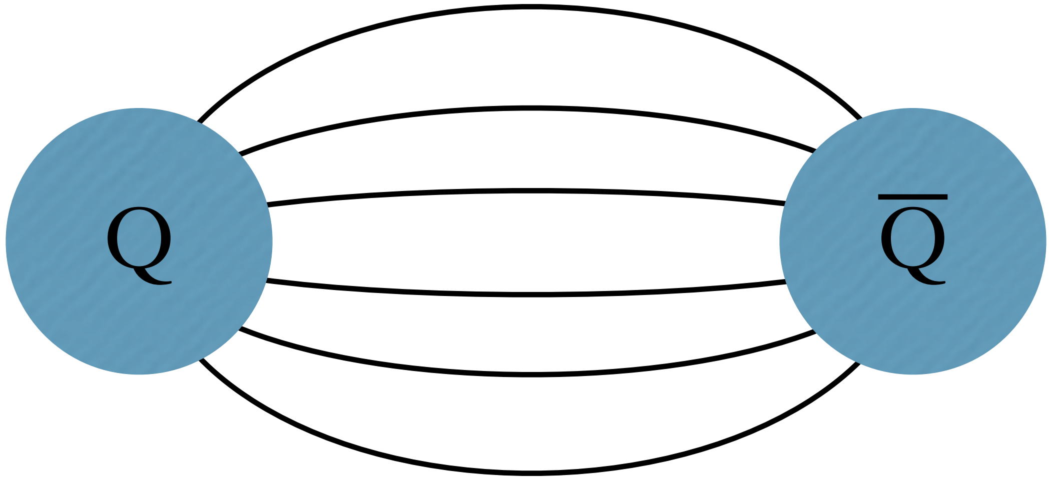

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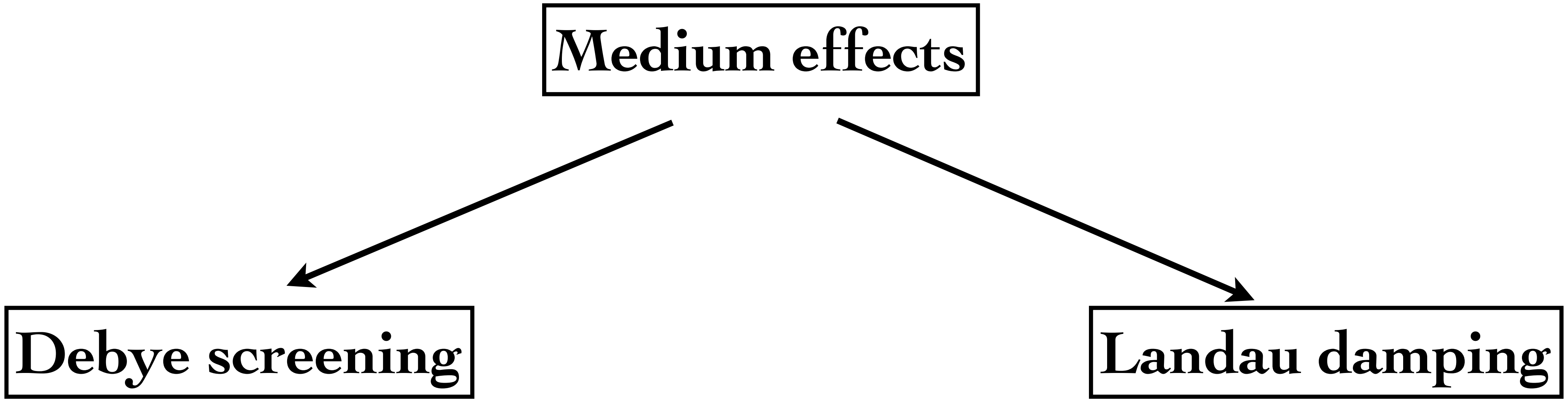
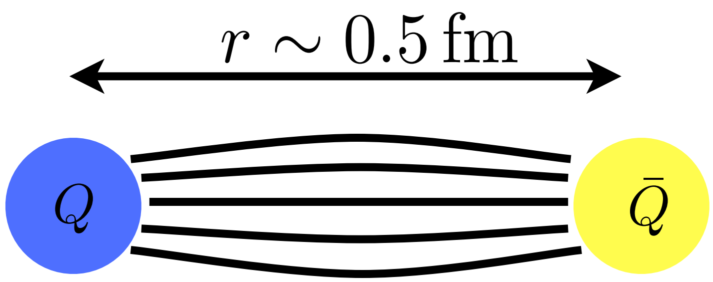


Medium effects

Debye screening

Landau damping

Debye Screening



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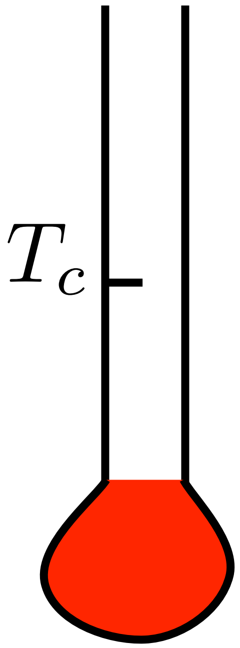

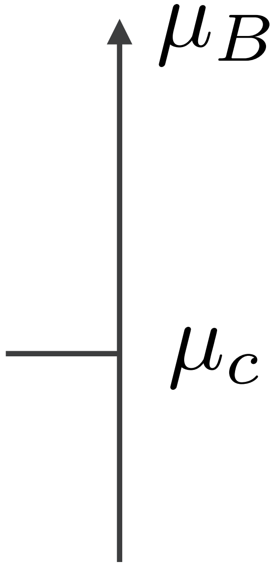

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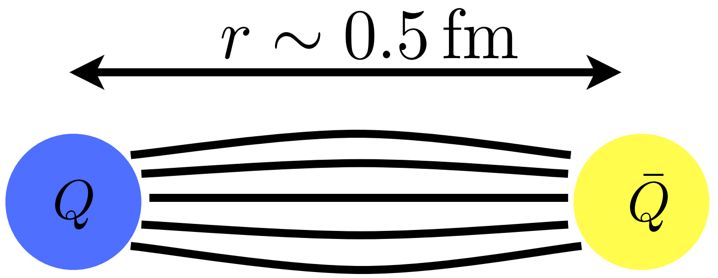

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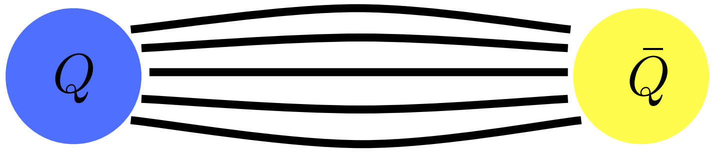

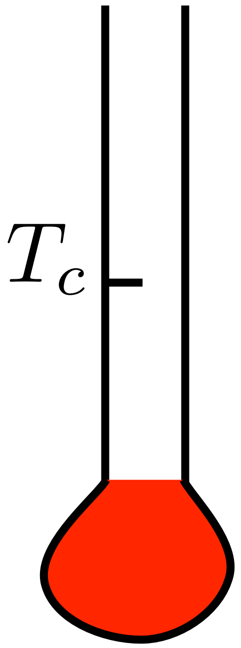

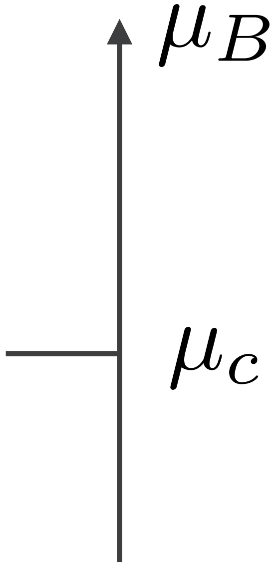

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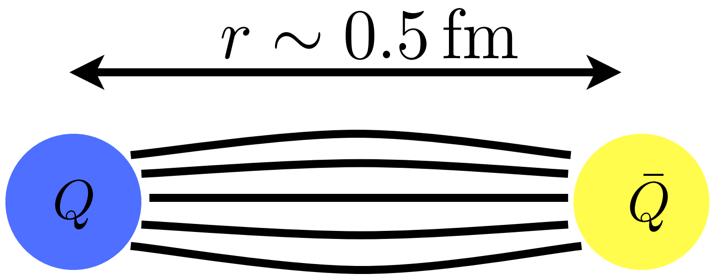

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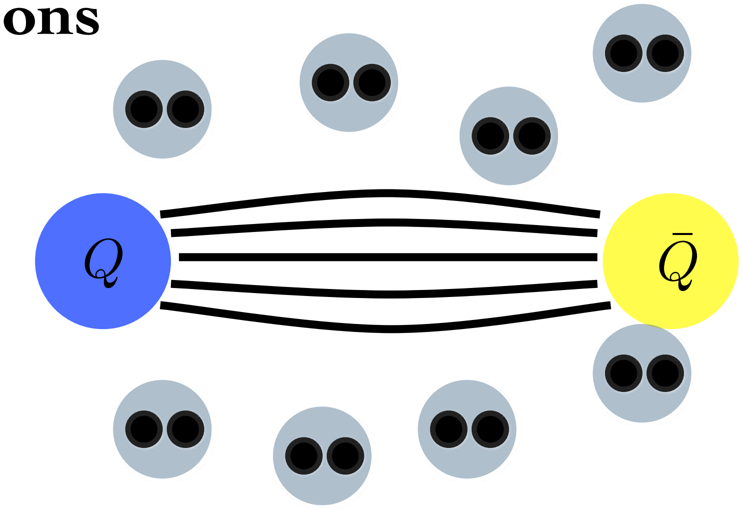
mesons



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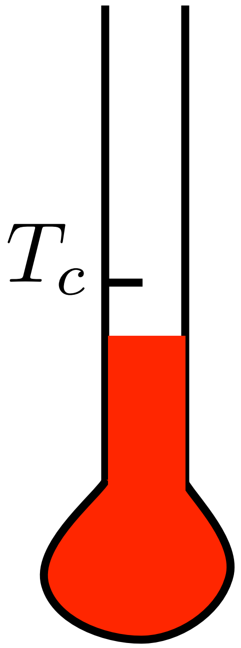

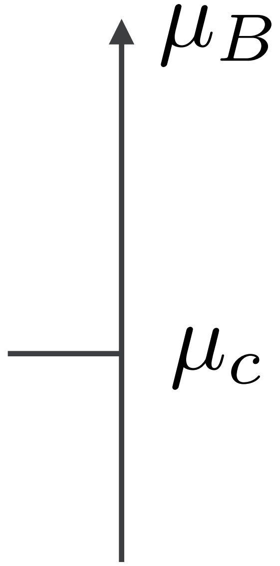

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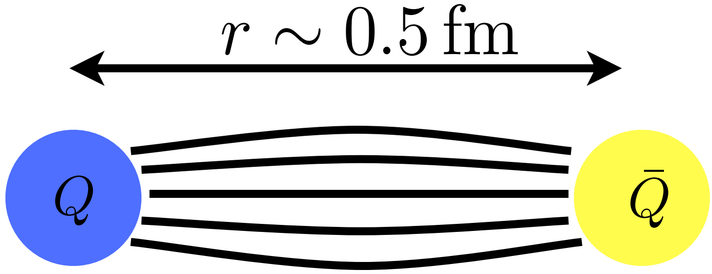

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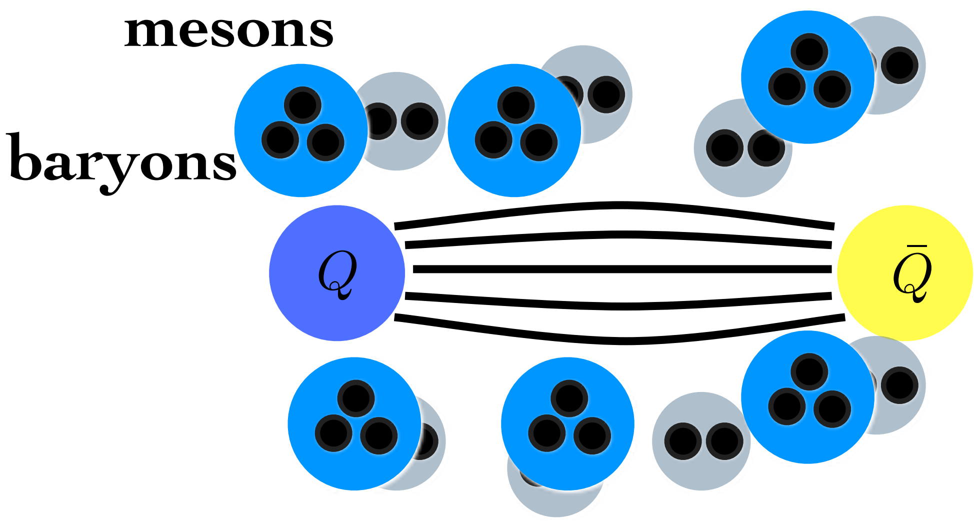

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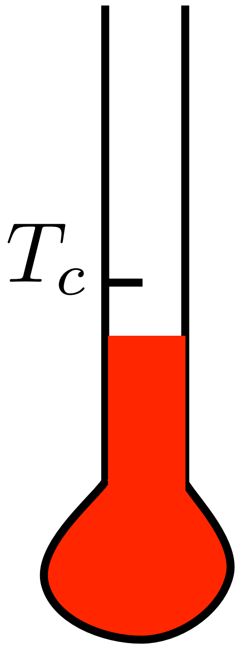

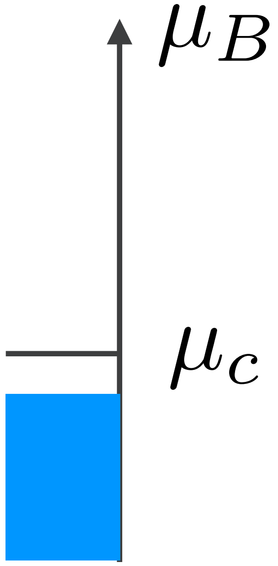

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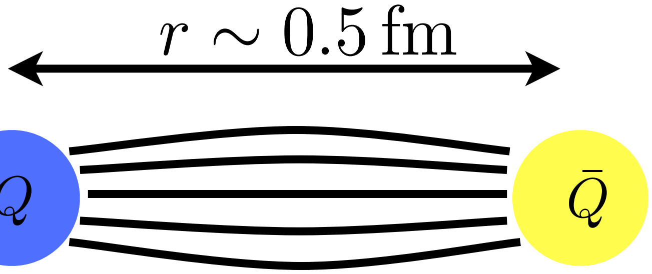

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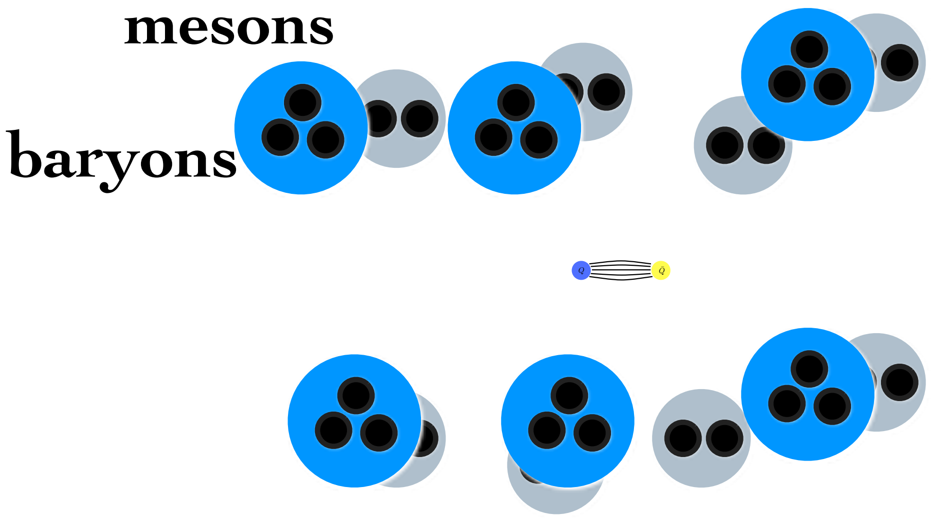

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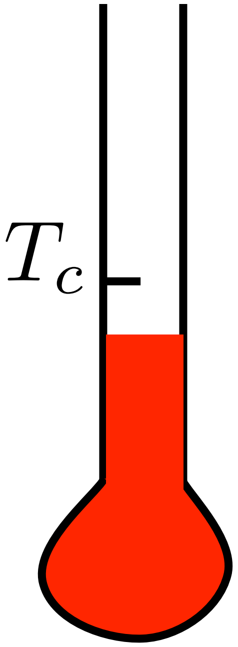

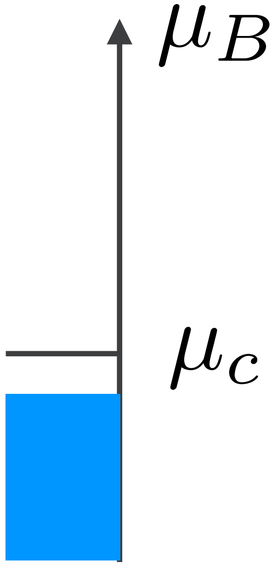

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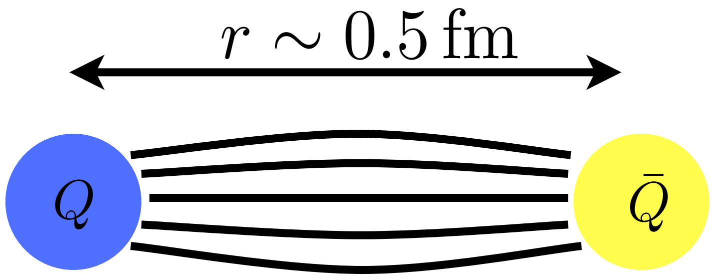

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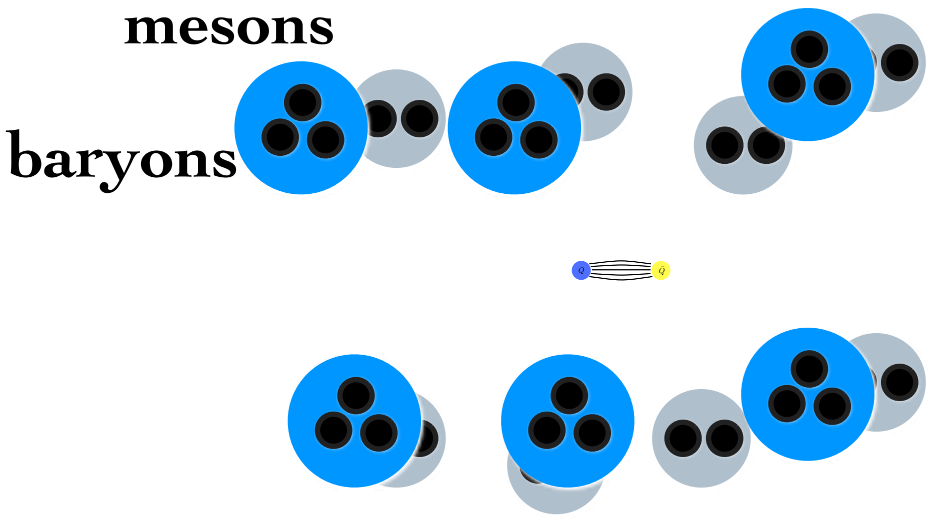

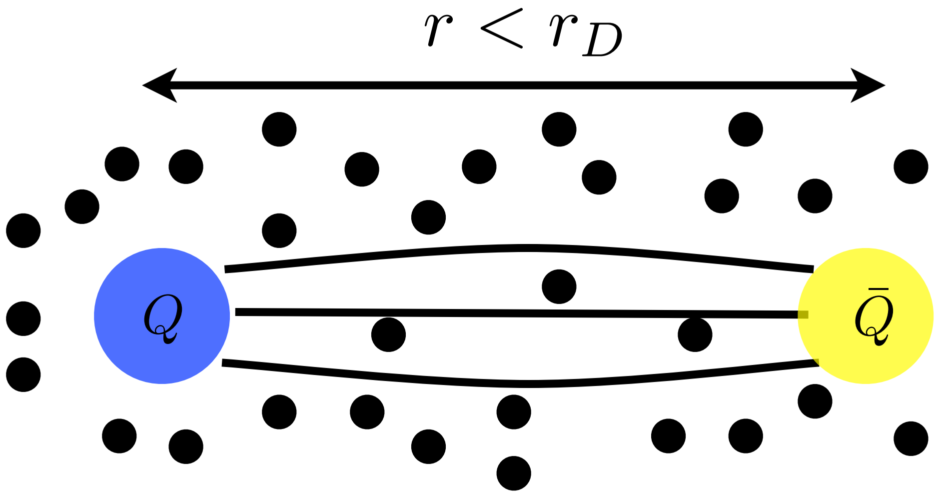

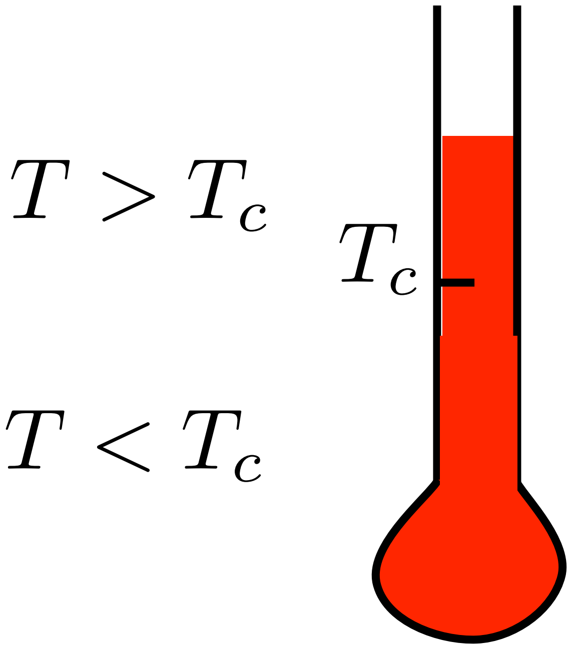

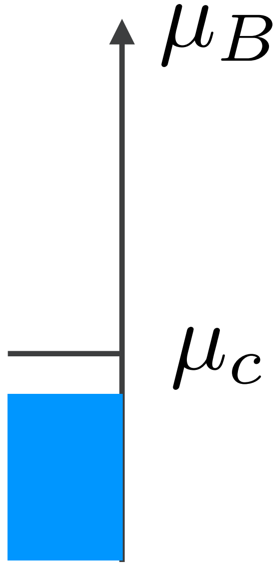

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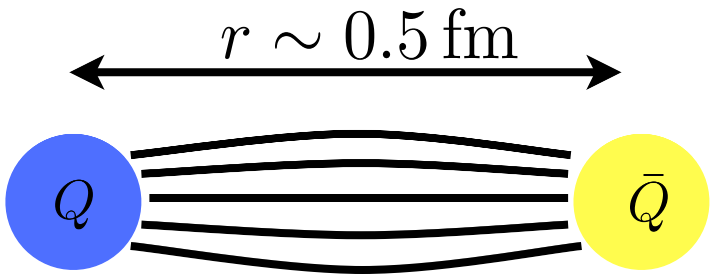

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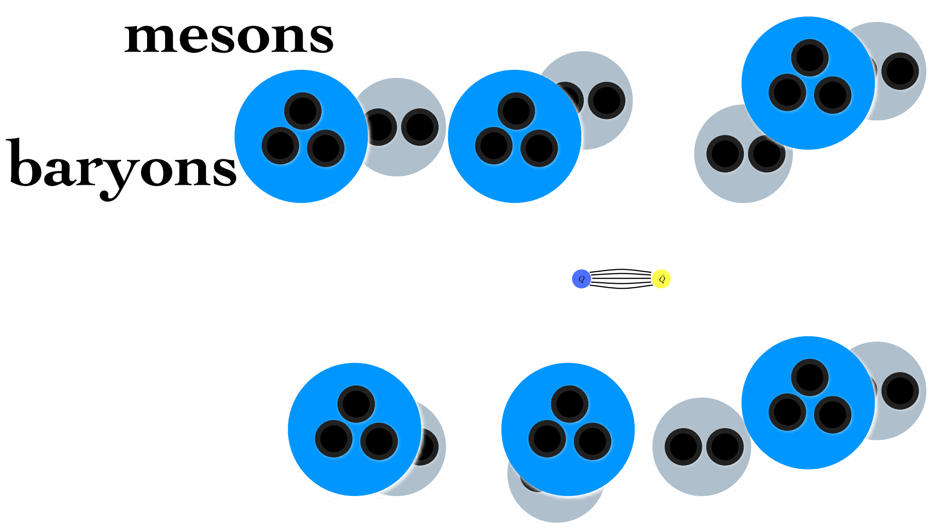
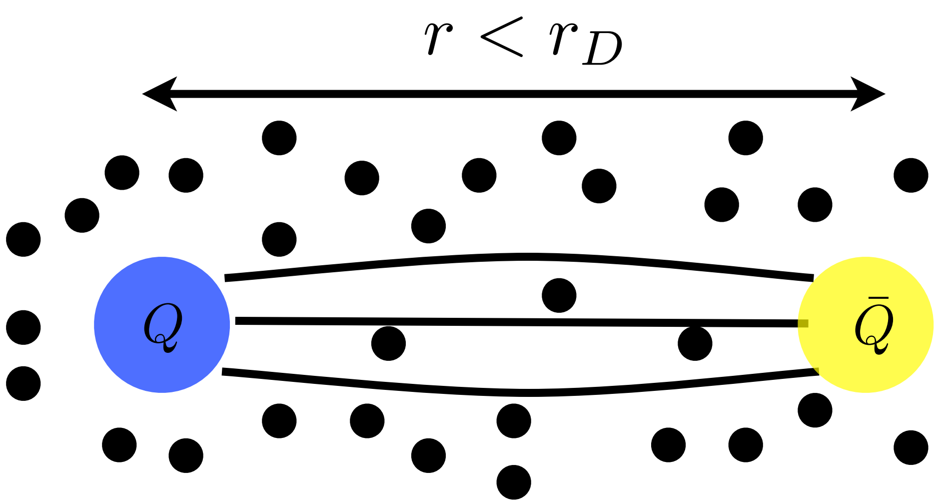
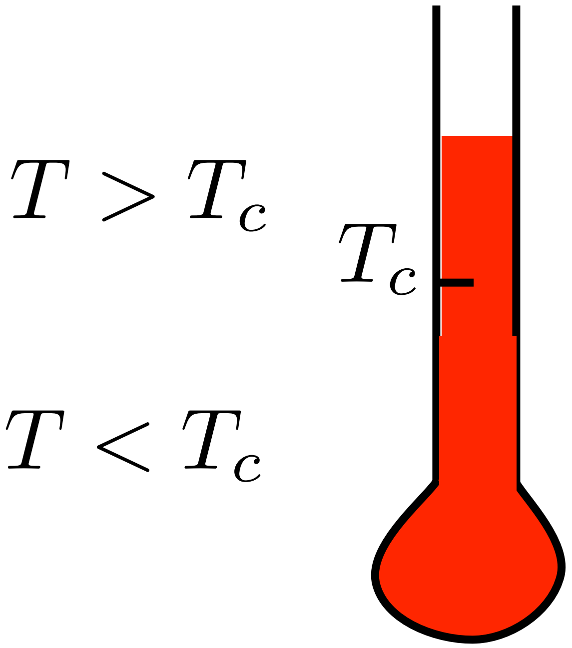
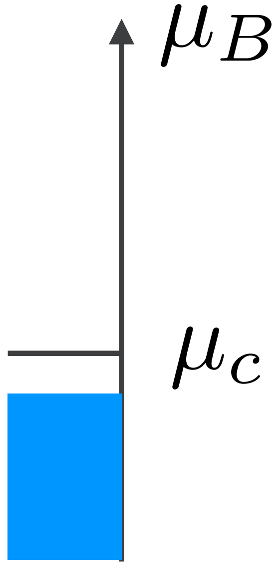




$$V(r) = -\frac{\alpha_{\text{eff}}}{r} e^{-r/r_D}$$





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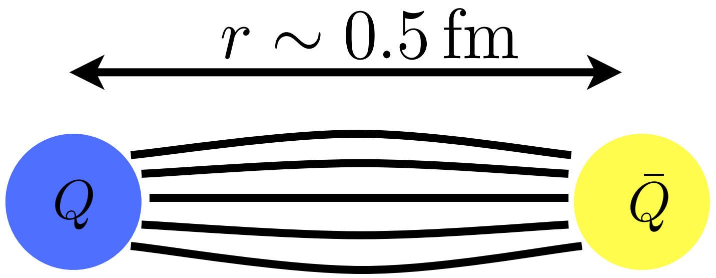

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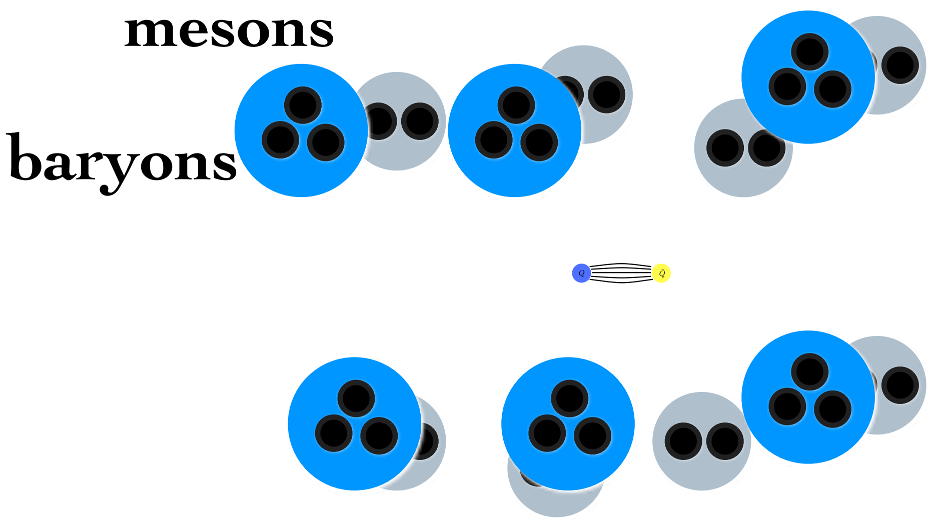

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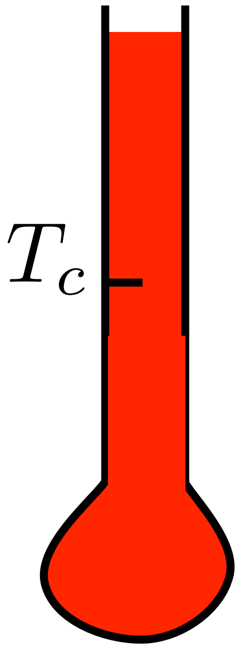
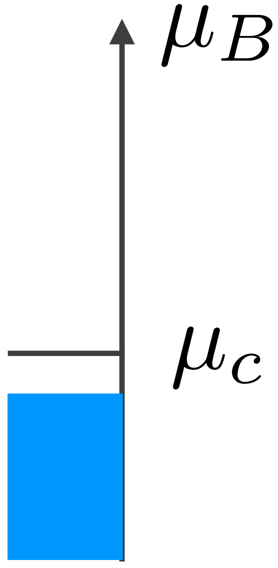
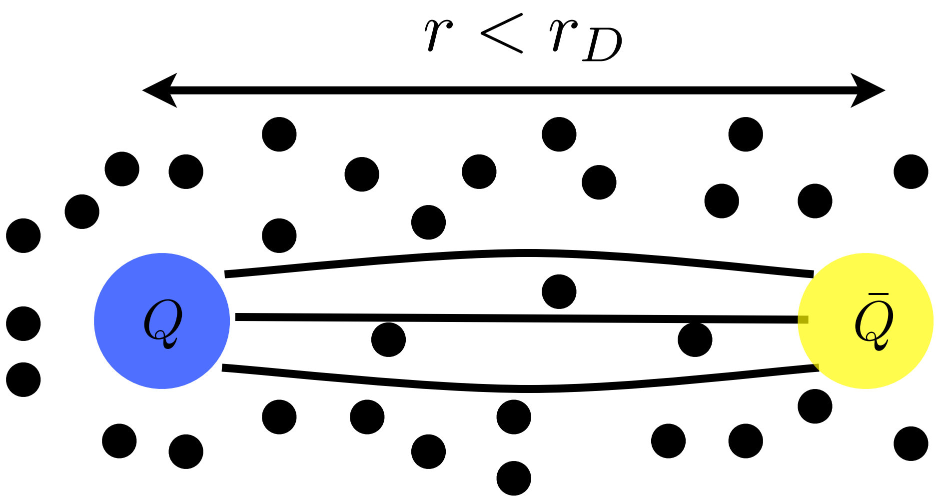
$T > T_c$

$T < T_c$







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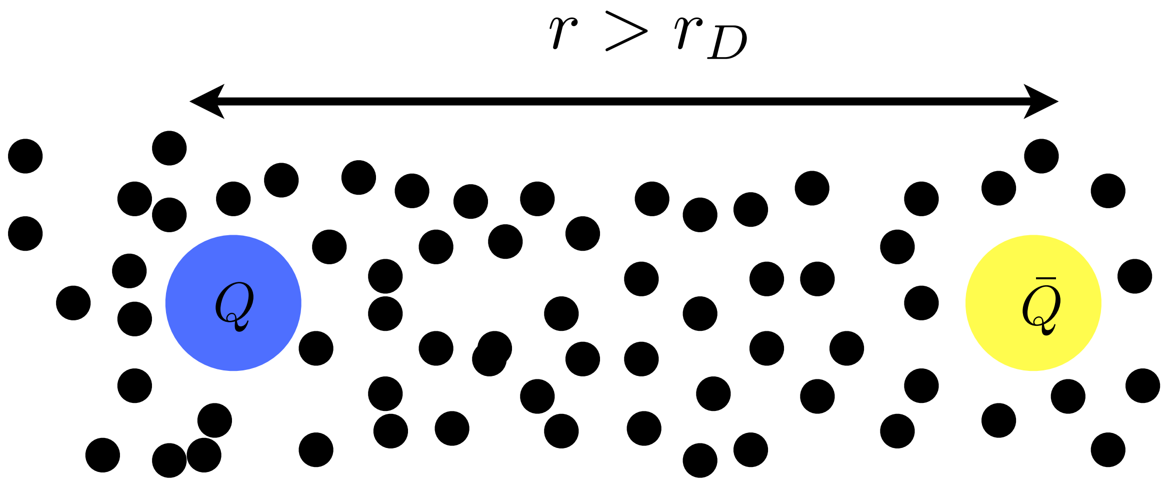

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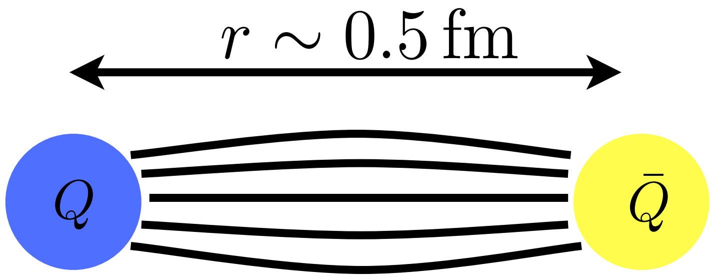

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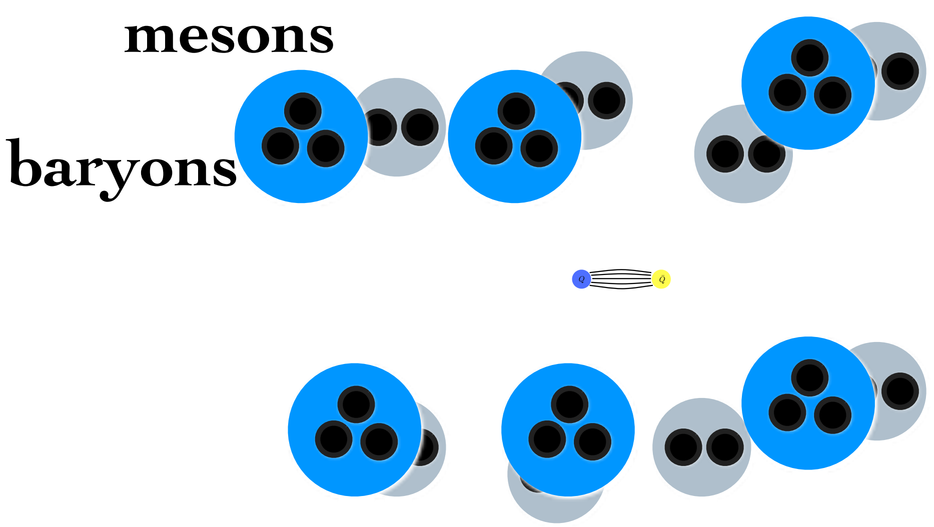

$T > T_d$

$T > T_c$

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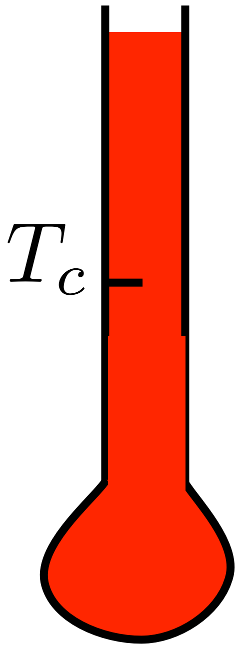
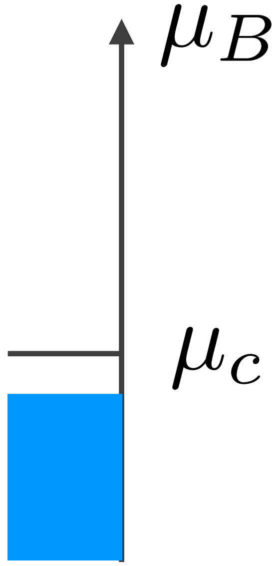
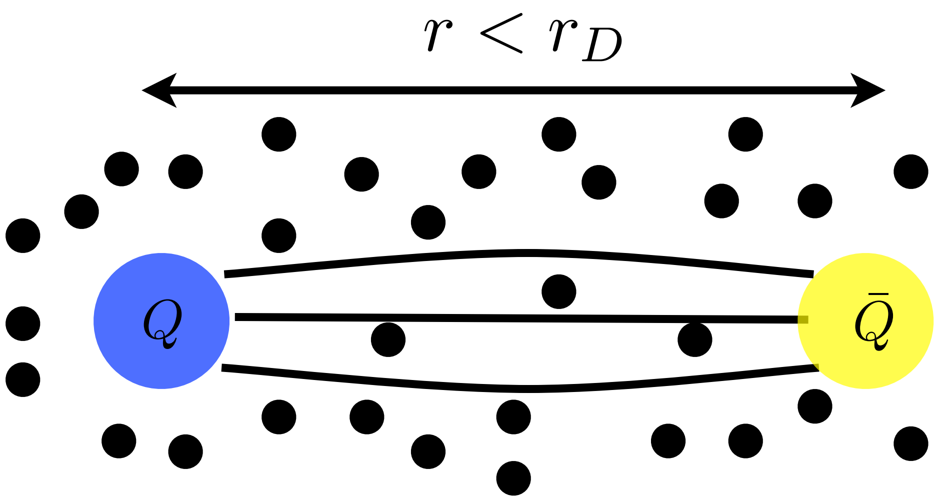
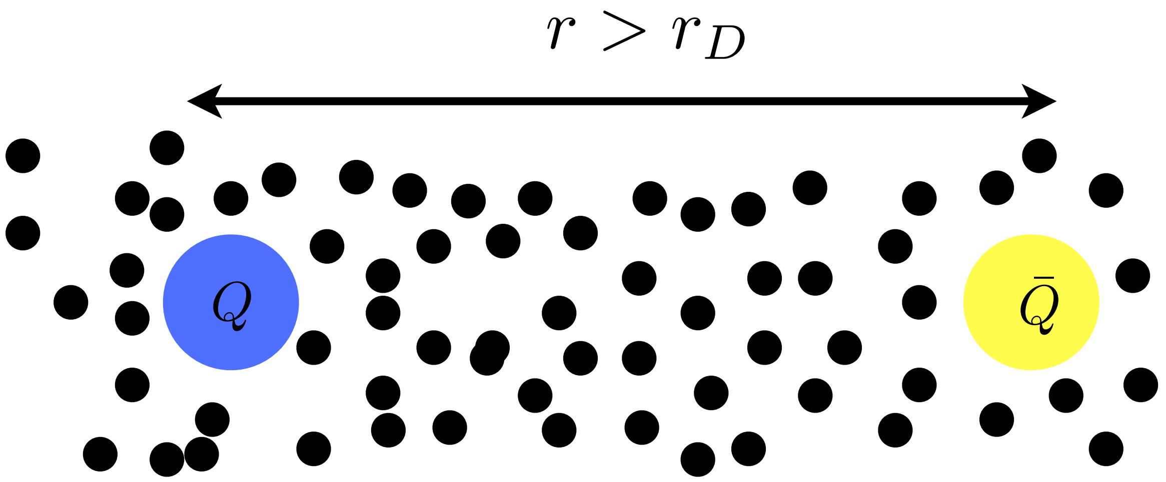




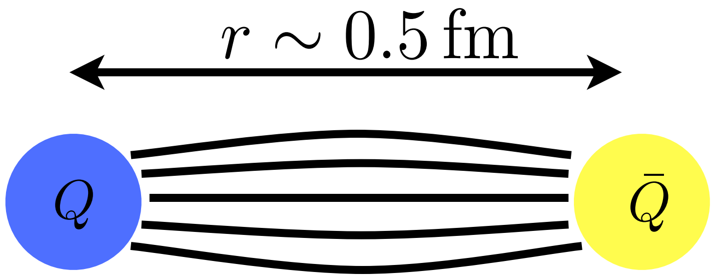


$$V(r) = -\frac{\alpha_{\mathrm{eff}}}{r} e^{-r/r_D}$$



The plasma screens chromo-electric fields: thermal unbinding Matsui and Satz, (1986).

Debye Screening



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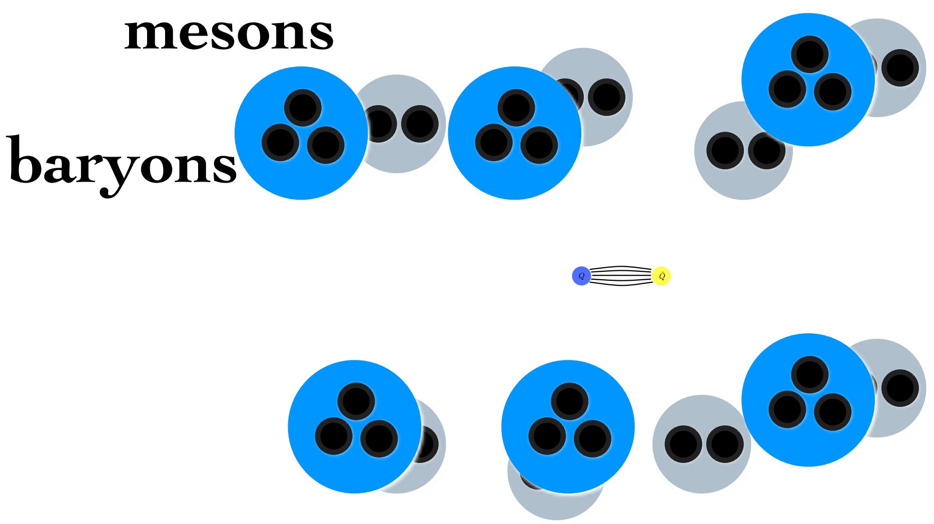

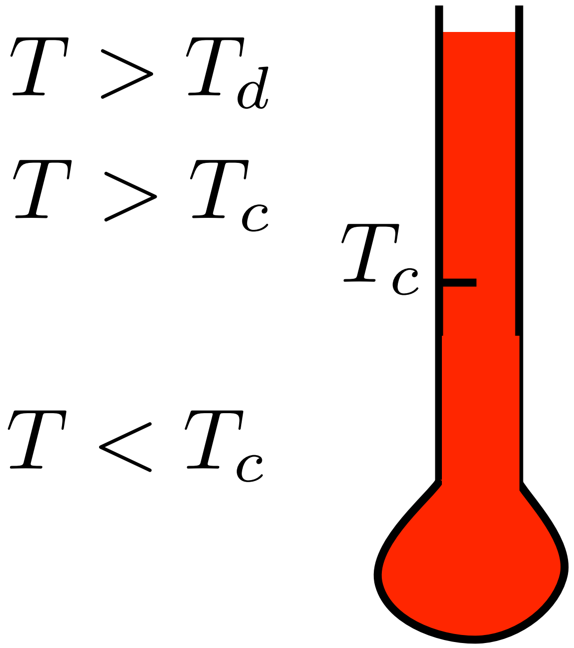

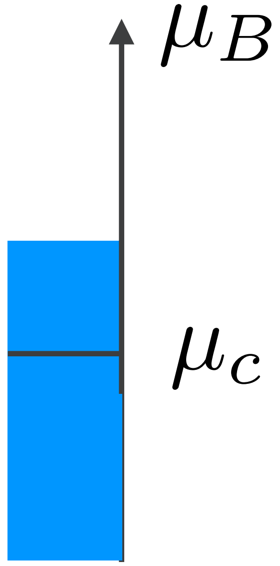

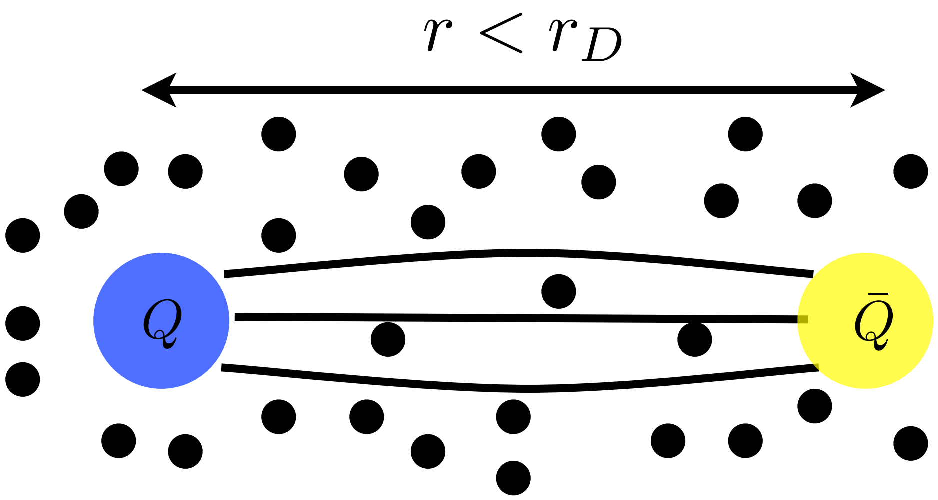

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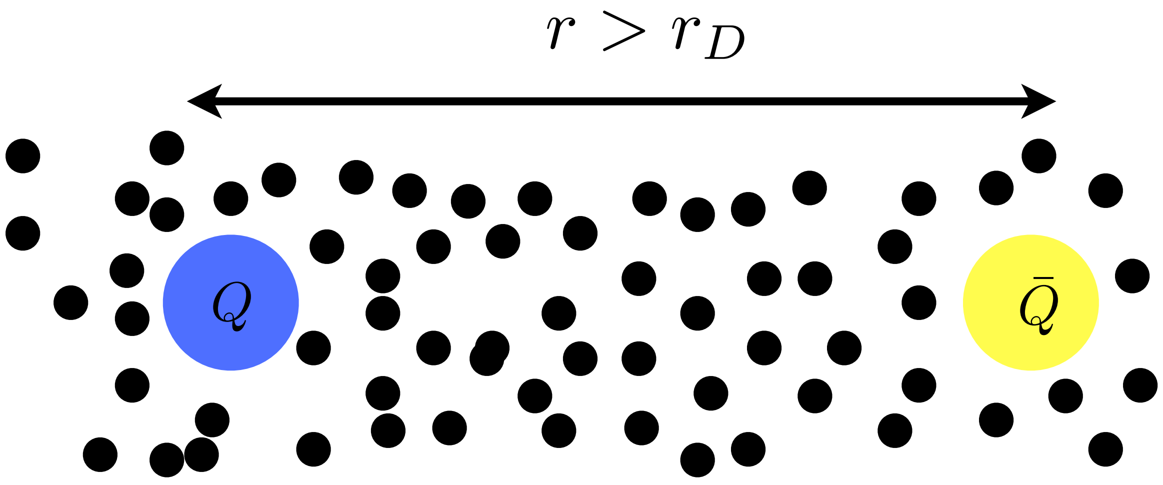
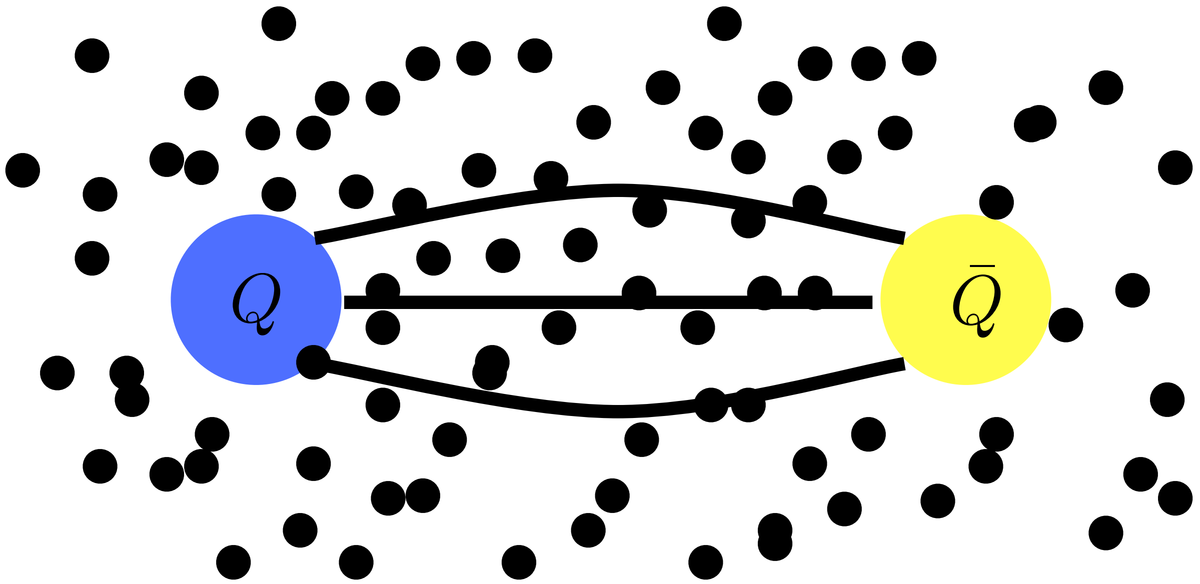


The plasma screens chromo-electric fields: thermal unbinding **Matsui and Satz, (1986)**.

Expect similar effect by μ_B: **Kakade and Patra, (2015), Carignano and Soto, (2020)**.

Landau damping

In a thermal medium, no strictly stationary bound state exists.
Medium interactions imply a finite lifetime for all states.



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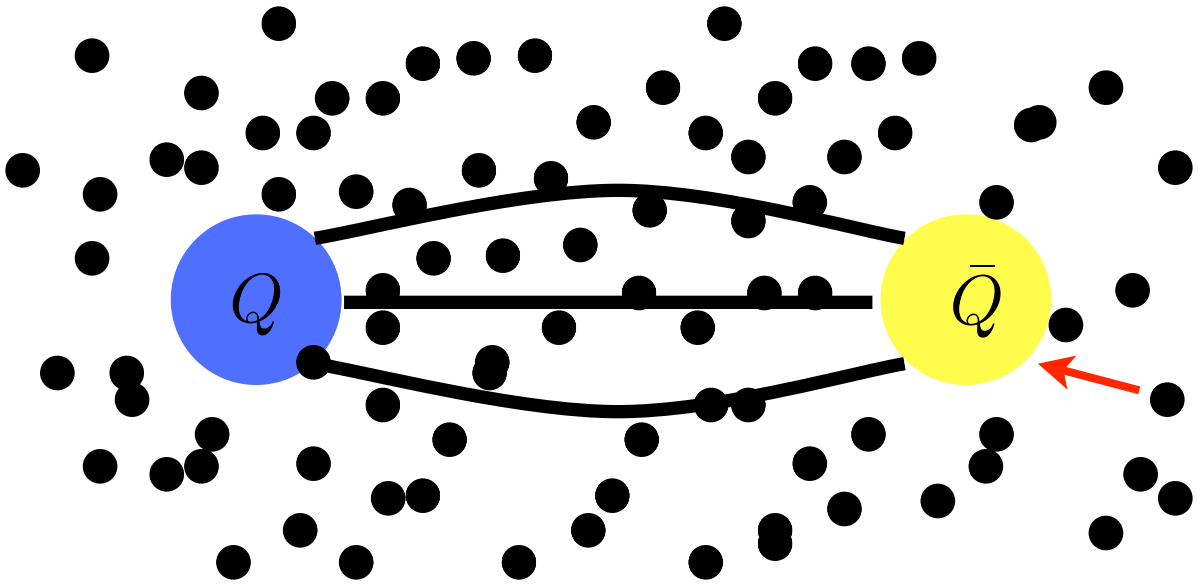

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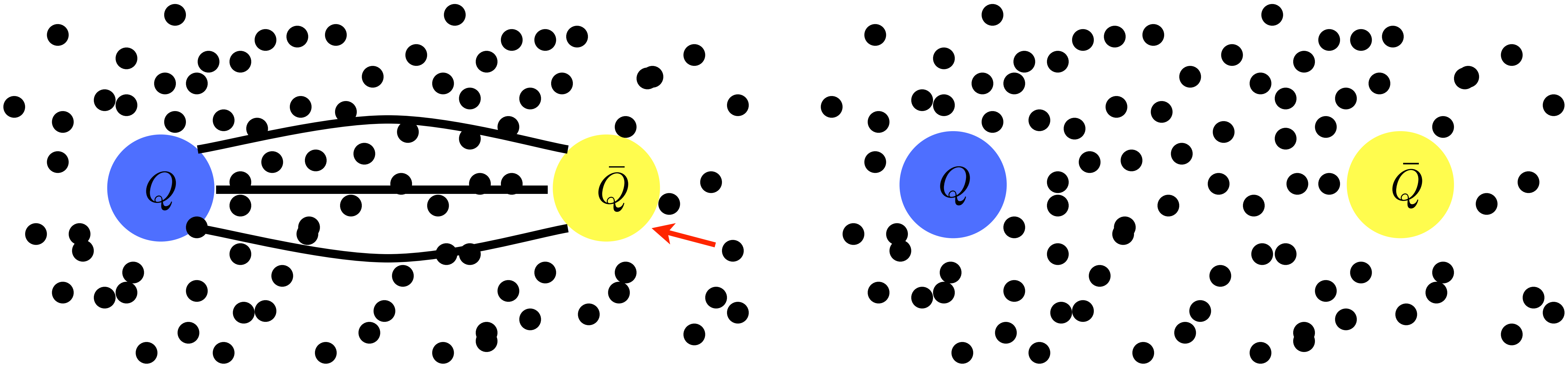

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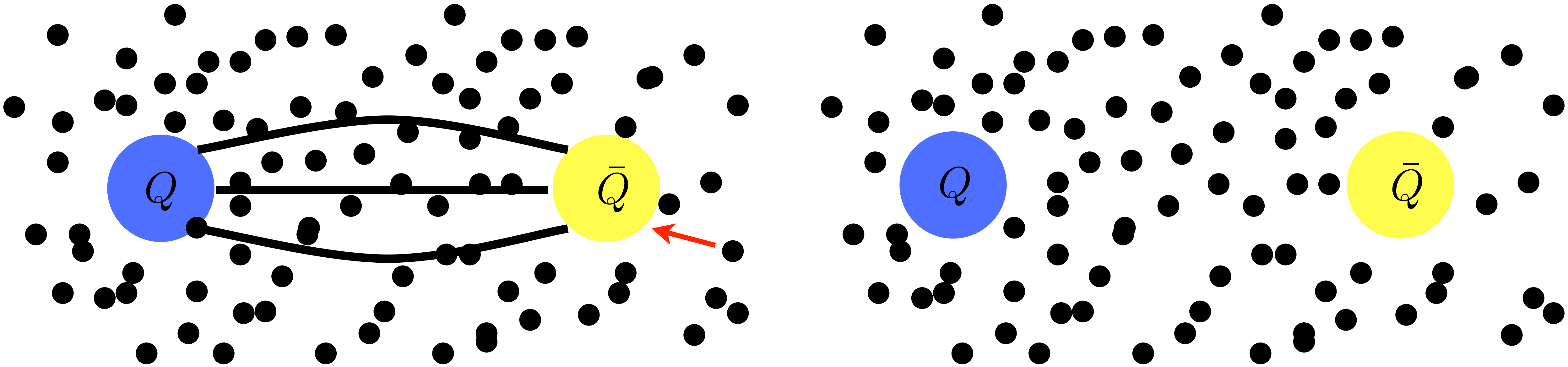

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M. Laine et al. JHEP 0703, 054 (2007)

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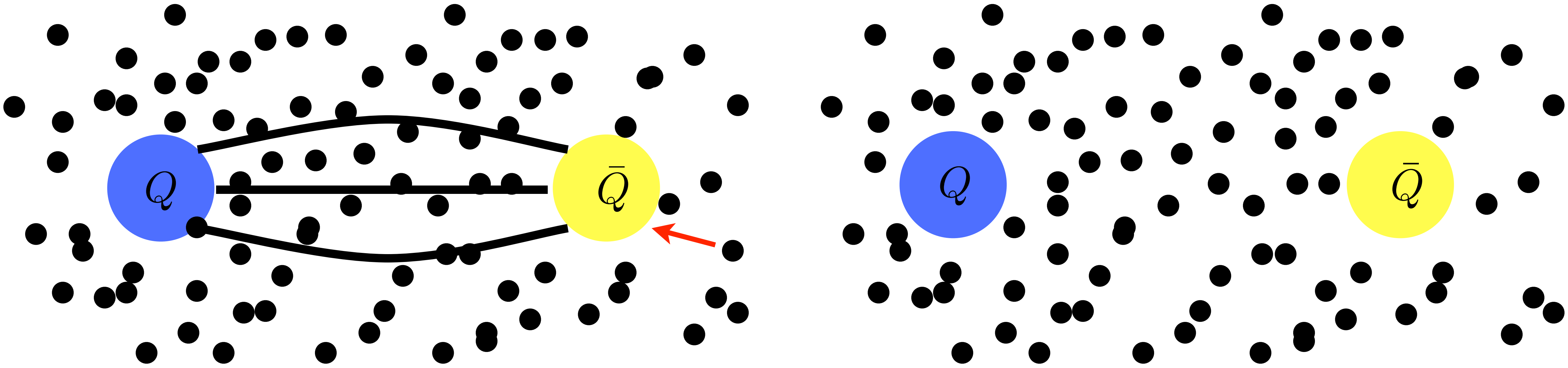

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Analogous to photodissociation of molecules like

$$O_2 + \gamma \rightarrow 2O$$

in a heat bath.

The FFLO-phase analog

Inhomogeneous superconductor with a spatially modulated condensate

P. Fulde, R.A Ferrell "Superconductivity in a Strong Spin-Exchange Field". Phys. Rev. 135 (3A): A550–A563 (1964).
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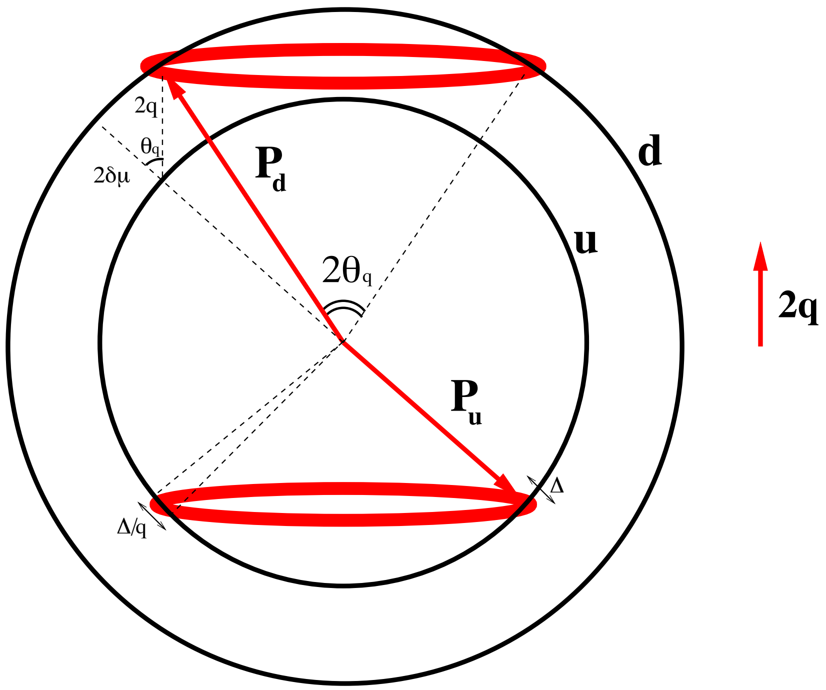

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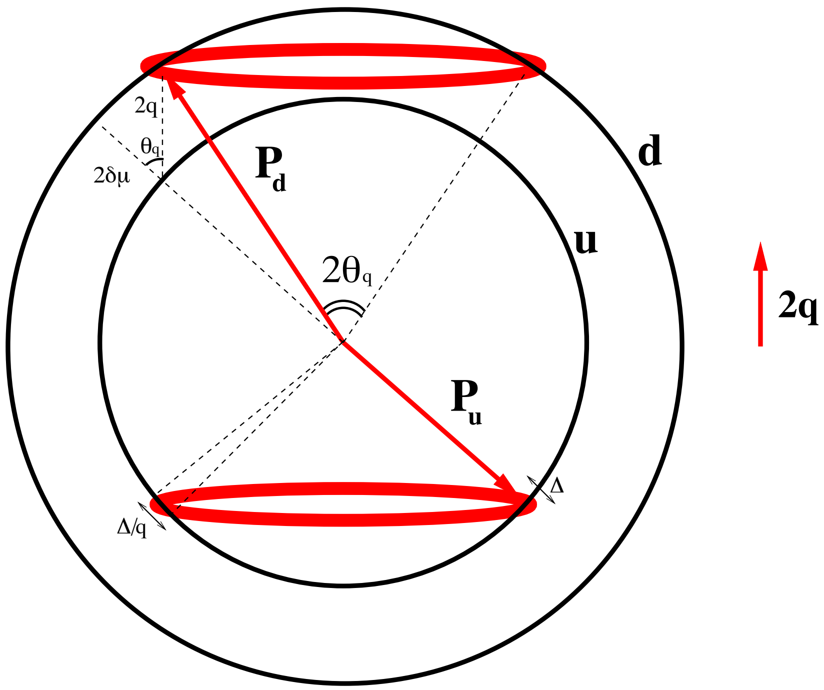

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$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\, \delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

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$$< \psi(\boldsymbol{x})\psi(\boldsymbol{x}) > \sim \Delta\, e^{i2\boldsymbol{q}\cdot\boldsymbol{x}}$$

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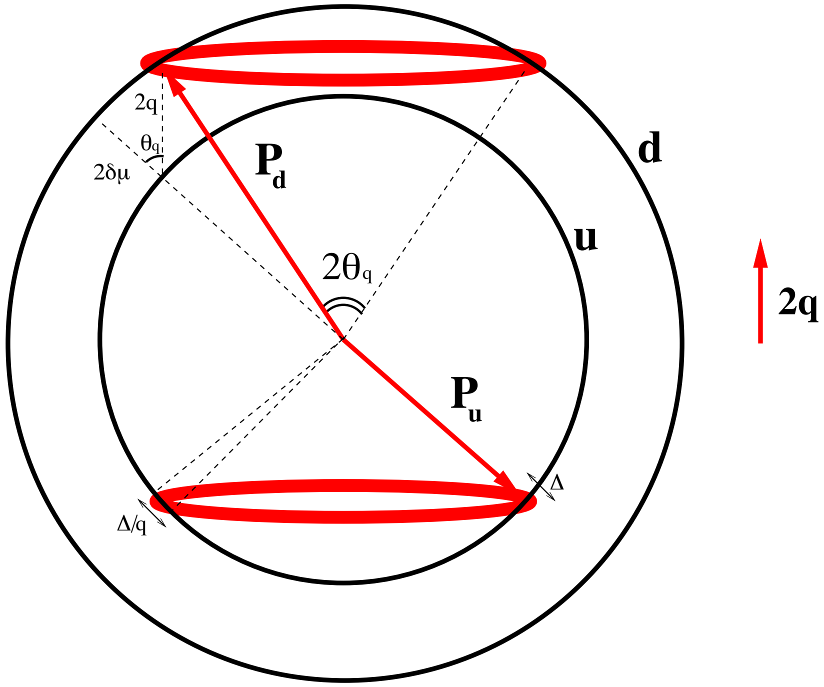

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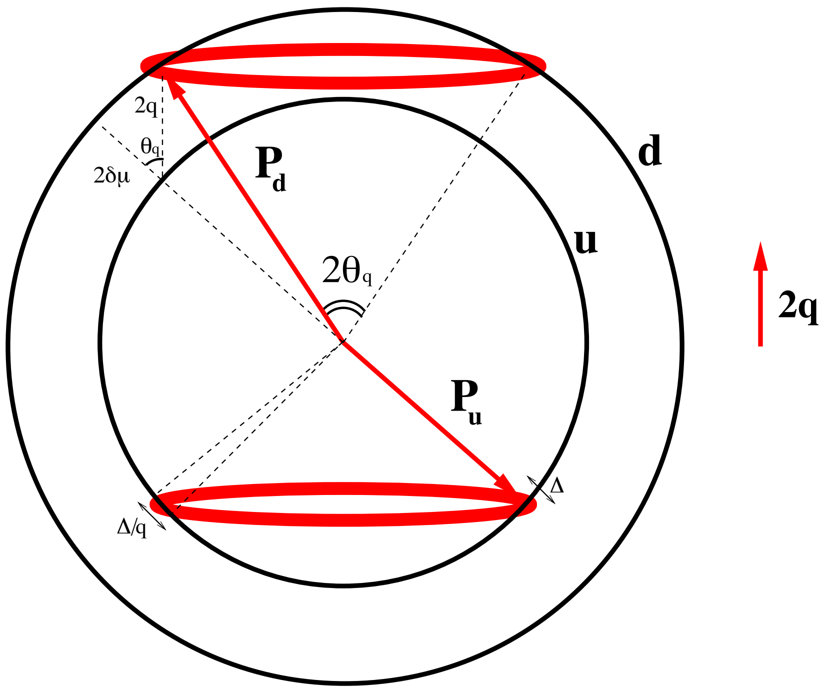

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For two flavors in weak coupling

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$

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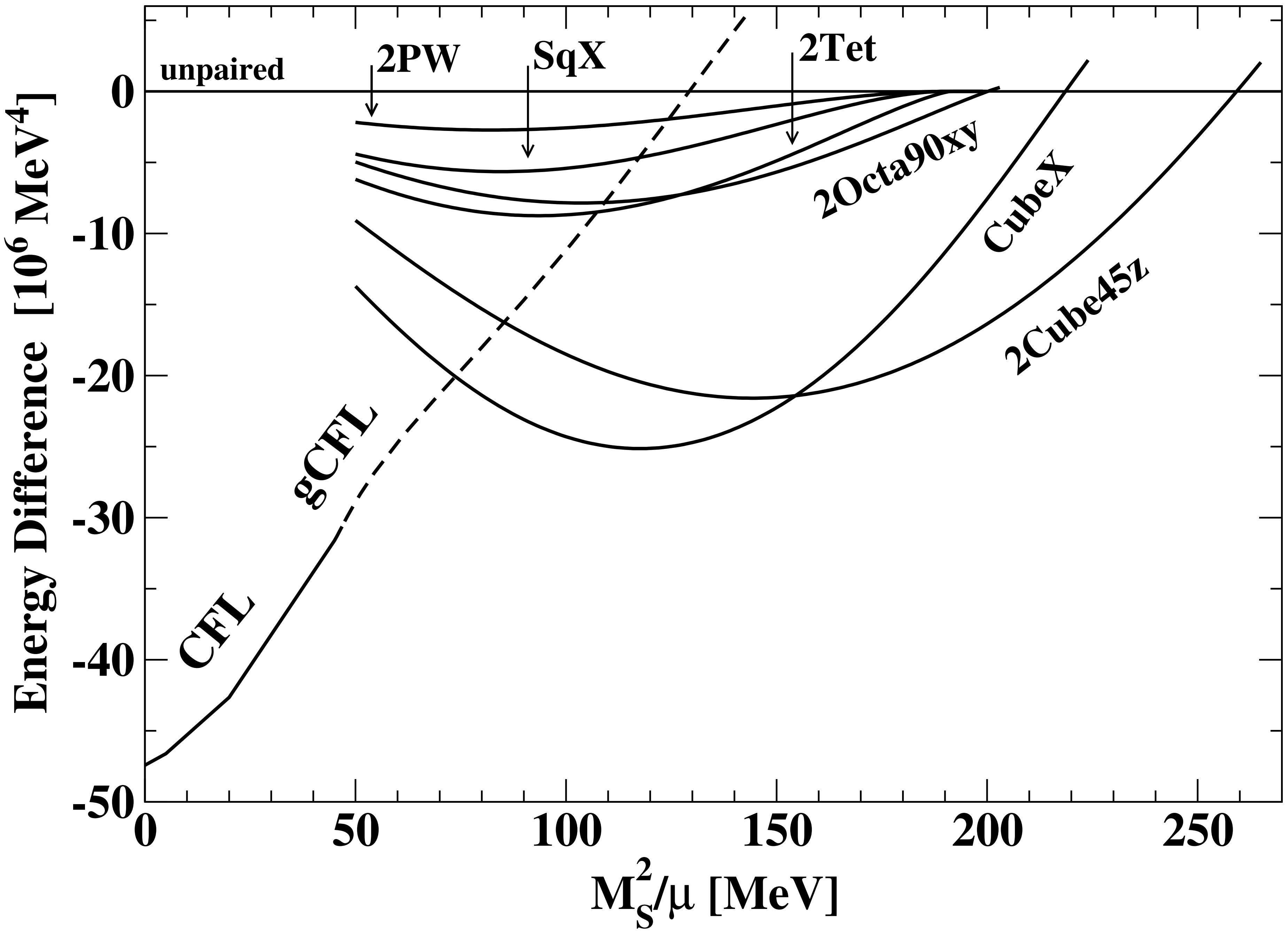
See **Redlich and Satz, e-Print: 1501.07523 [hep-ph]**
for more on Hagedorn's work.

Free energy estimate

The used modeling: NJL + GL expansion!!



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019
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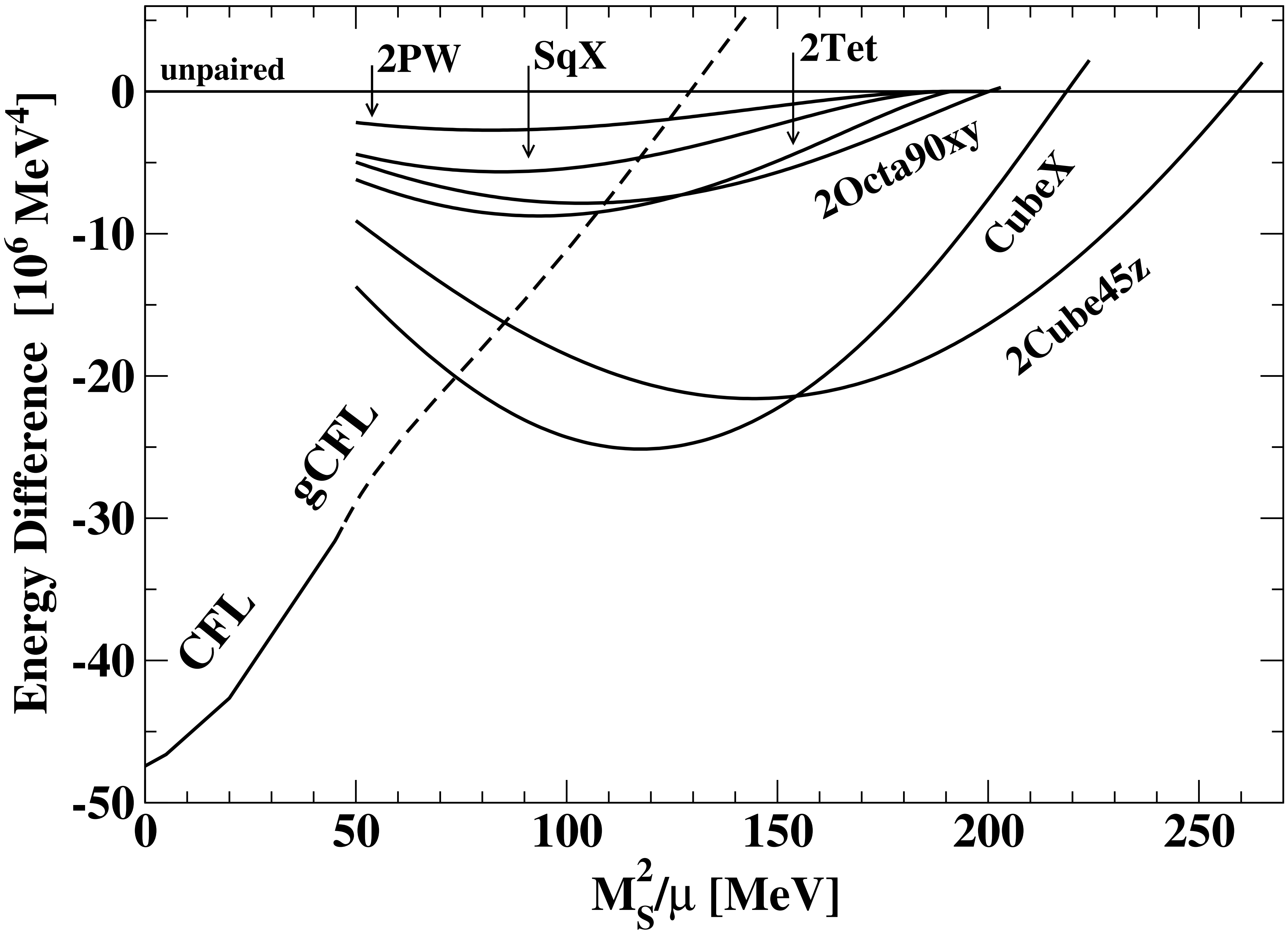

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019
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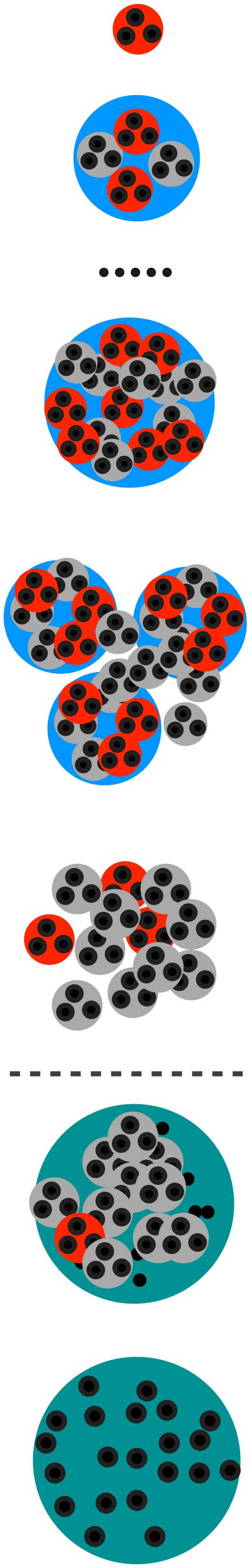
Improved GL expansion S.Carignano, MM, O.Benhar and F.Anzuini Phys.Rev.D 97 (2018) 3, 036009

Increasing the density



	Density	NS part	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H				light nuclei
He	10^{-4} g cm^{-3}	atmosphere		
…..				…..
Fe	10 g cm^{-3}	outer crust	confining	heavy nuclei
neutron drip	10^{11} g cm^{-3}	inner crust		
neutron proton soup	10^{14} g cm^{-3}	core		neutrons and protons
quark drip	very large		strong coupling	Cooper pairs of quarks? quarkyonic matter? instantons? glueballs?
quark soup	extreme		weak coupling	quarks and gluons

ρ