

# Timing the cosmic expansion with populations of gravitational waves dark sirens

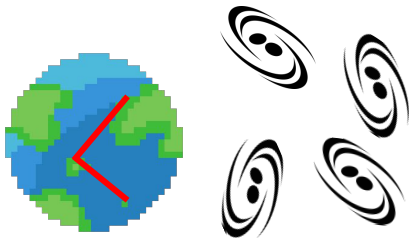
“What can go wrong?”

S. Mastrogiovanni



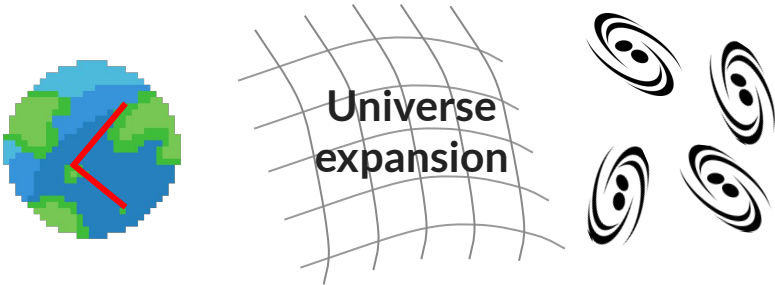
# Gravitational Waves from cosmic distances

## Source frame



$$h(t) \propto \frac{\overset{\text{Chirp mass}}{\mathcal{M}^{5/4}} \overset{\text{Source time}}{(t_c - t)^{-1/4}}}{\underset{\text{Physical distance}}{d}} e^{\phi(t)}$$

## Detector frame



$$h(t^{\text{det}}) \propto \frac{\overset{\text{Redshifted chirp mass}}{[\mathcal{M}(1+z)]^{5/4}} \overset{\text{Detector time}}{(t_c^{\text{det}} - t^{\text{det}})^{-1/4}}}{\underset{\text{Luminosity distance}}{d_L}} e^{\phi(t)}$$

# Gravitational Waves from cosmic distances

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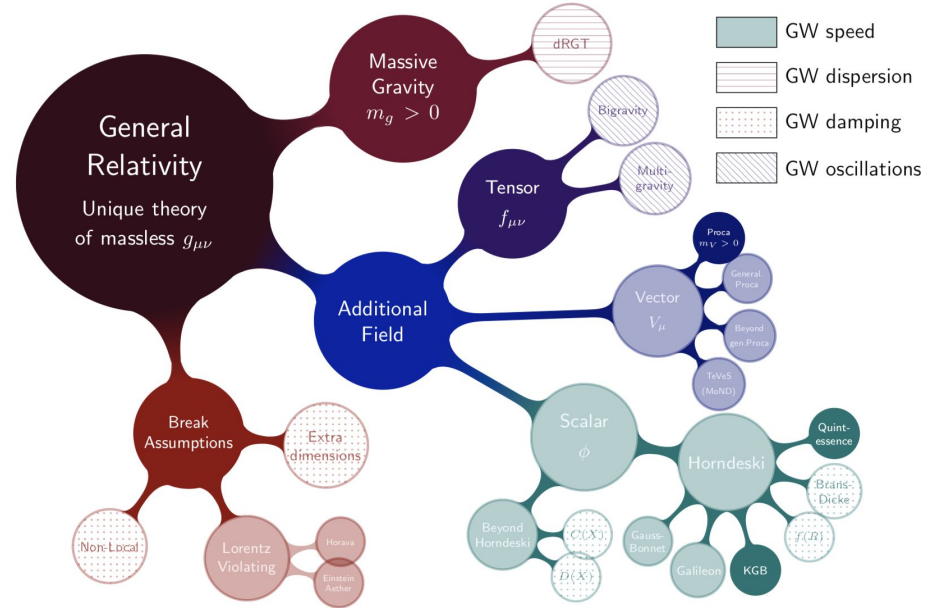
1. Gravitational Waves **directly** provide the luminosity distance of the source.
2. Even if we do not observe electromagnetic counterparts, we can invoke **dark sirens** methods (see overview talk) to do cosmology.
3. With Einstein Telescope and Cosmic Explorer we will have millions of detections, statistics is on our side.

“What can go wrong?”

# What can go wrong? General Relativity

- Alternative GR theories are possible solutions to open issues in Standard cosmological model, e.g. dark energy, Hubble constant tension.
- We want to understand how Standard Cosmology parameters mix to GR deviation parameters.

Modified gravity roadmap



J. M. Ezquiaga+, *Front. Astron. Space Sci.* 5:44 (2018)

# What can go wrong? General Relativity

M. Lagos, Phys. Rev. D 99, 083504 (2019)

$$h'' + 2[1 + \alpha_M(\eta)] \frac{a'}{a} h' - c_T \nabla^2 h = 0$$

GW friction

Dispersion  
relation

## Dispersion relation:

- GWs group velocity depends on the frequency.
- GWs modes arrive off-phased at the detector.
- GWs modes show a time delay w.r.t EM counterparts.

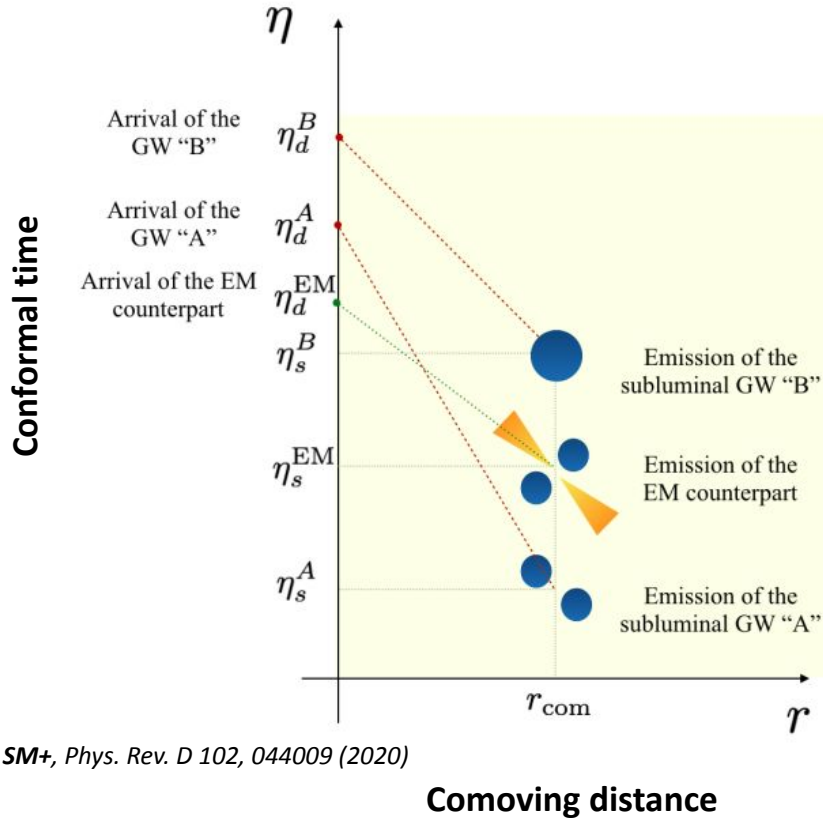
*Horava gravity, massive gravity, scalar tensor theories with field derivative couplings*

## GW friction:

- GWs show an additional energy leakage as they travel.

*extra energy dissipation terms, e.g. a running Planck mass, 4+n dimensional gravity, scalar-tensor theories*

# What can go wrong? General Relativity



SM+, Phys. Rev. D 102, 044009 (2020)

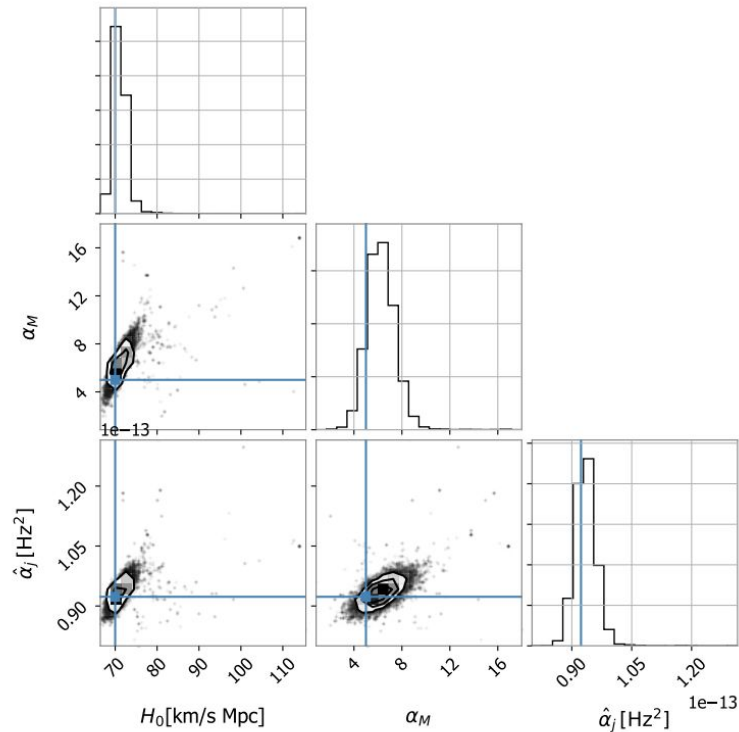
## Dispersion relation:

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## GW friction:

- GWs show an additional energy leakage as they travel.

# What can go wrong? General Relativity



- To understand the interplay between cosmology ( $H_0$ ), GW friction and GW speed, we simulate the observation of 100 BNS mergers with observed EM counterparts.
- From the GW luminosity distance, galaxy redshift and GW-GRB time delay it is possible to measure  $H_0$ , GW friction and speed of gravity.
- **Correlations** among  $H_0$  and modified gravity appears at the percent-level precision.

We need to model Modified Gravity to measure  $H_0$  at a sub-percent precision.

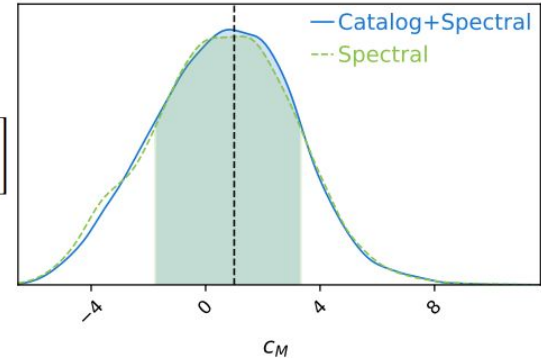
SM+, Phys. Rev. D 102, 044009 (2020)

# What can go wrong? General Relativity

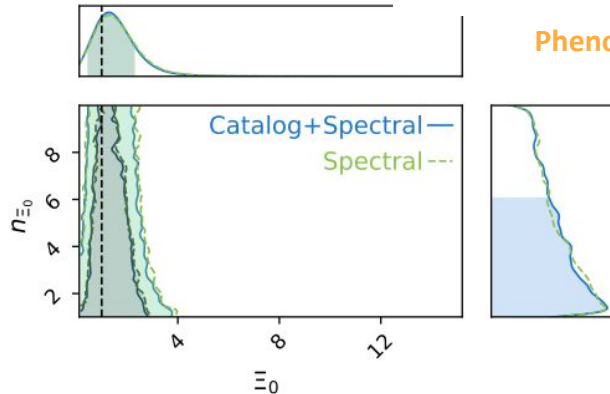
- We looked to GW friction using BBHs from GWTC-3 using the [spectral sirens](#) and [galaxy catalog](#) method.
- By marginalizing over  $H_0$ , it is possible to obtain interesting constraints on the GW friction.
- With current GW events, other population-level parameters does not strongly correlate with the determination of Modified Gravity.
- Bayes factors strongly (factor of 10) prefer classical GR, [too many degrees of freedom](#).

$$d_L^{\text{GW}} = d_L^{\text{EM}} \exp \left[ \frac{c_M}{2} \int_0^z \frac{1}{(1+z')E^2(z')} dz' \right]$$

Running Planck mass



Phenom. Parametrization



$$d_L^{\text{GW}} = d_L^{\text{EM}} \left( \Xi_0 + \frac{1 - \Xi_0}{(1+z)^n} \right)$$

*SM et al Phys. Rev. D 102, 044009 (2020), Leyde et al SM JCAP 09012 L (2022), Mancarella M. et al Phys. Rev. D 105, 064030 (2022).*



# What can go wrong? Astrophysics

The likelihood for an inhomogeneous Poisson process in presence of selection biases, for a **constant rate in detector time**, is (see [Mandel+ 2018 MNRAS](#), [Vitale+ 2020](#))

[Link to icarogw  
function](#)



Logo possible thanks to:  
LIGO-Virgo-KAGRA / Aaron Geller /  
Northwestern

$$\mathcal{L}(x|\Lambda) \propto e^{-N_{\text{exp}}} \prod_{i=1}^{N_{\text{obs}}} T_{\text{obs}} \int \mathcal{L}_n(x|\theta, \Lambda) \frac{dN}{dt d\theta} d\theta$$

Noise process

Rate

**Expected  
number of  
detections**

$$N_{\text{exp}} = T_{\text{obs}} \int p_{\text{det}}(\theta, \Lambda) \frac{dN}{dt d\theta} d\theta.$$

# What can go wrong? Astrophysics

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To write down the hierarchical likelihood it is crucial to answer the question:

**What is the probability of observing a CBC at a given redshift  
and in a given sky position?**

# What can go wrong? Astrophysics

The CBC likelihood is often parametrized in terms of redshift and **source-frame time**

$$\frac{dN_{\text{CBC}}(\Lambda)}{d\vec{m}d\vec{\chi}d\Omega dz dt_s} = R_0$$

Rate of CBC [#mergers Gpc<sup>-3</sup> yr<sup>-1</sup>]

$$\psi(z; \Lambda)$$

Rate evolution function, e.g. (1+z)<sup>gamma</sup>. Two models available

$$p_{\text{pop}}(\vec{m}, \vec{\chi} | \Lambda)$$

Probabilities for source-frame masses and spins, 8 models for masses, 2 for spins

$$\frac{dV_c}{dz d\Omega}$$

Comoving volume (depends on cosmology)

# What can go wrong? Astrophysics

The CBC likelihood is often parametrized in terms of redshift and **source-frame time**

CBC per galaxy per  
year

$$\frac{dN_{\text{CBC}}(\Lambda)}{dz d\vec{m} d\vec{\chi} d\Omega dt_s} = R_{\text{gal},0}^* \psi(z; \Lambda) p_{\text{pop}}(\vec{m}, \vec{\chi} | \Lambda) \times$$

Term similar to the vanilla  
rate

$$\left[ \frac{dV_c}{dz d\Omega} \phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}(M_{\text{thr}}), x_{\text{min}}) + \right.$$

Integral of Schechter function

Number density of galaxies  
per steradian (completeness  
correction)

$$\frac{1}{\Delta\Omega} \sum_{j=1}^{N_{\text{gal}}(\Omega)} \left[ f_L(M(m_j, z); \Lambda) p(z | z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) \right]$$

Number density of galaxies  
per steradian (catalog)

Sky pixel area

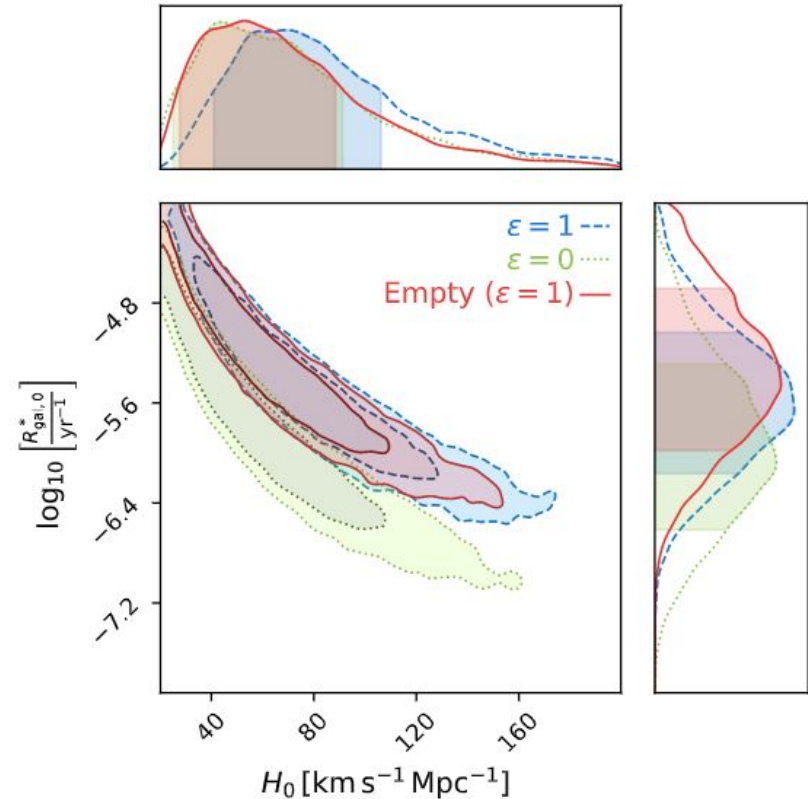
Luminosity weight

Galaxy localization in  
redshift

# What can go wrong? Astrophysics

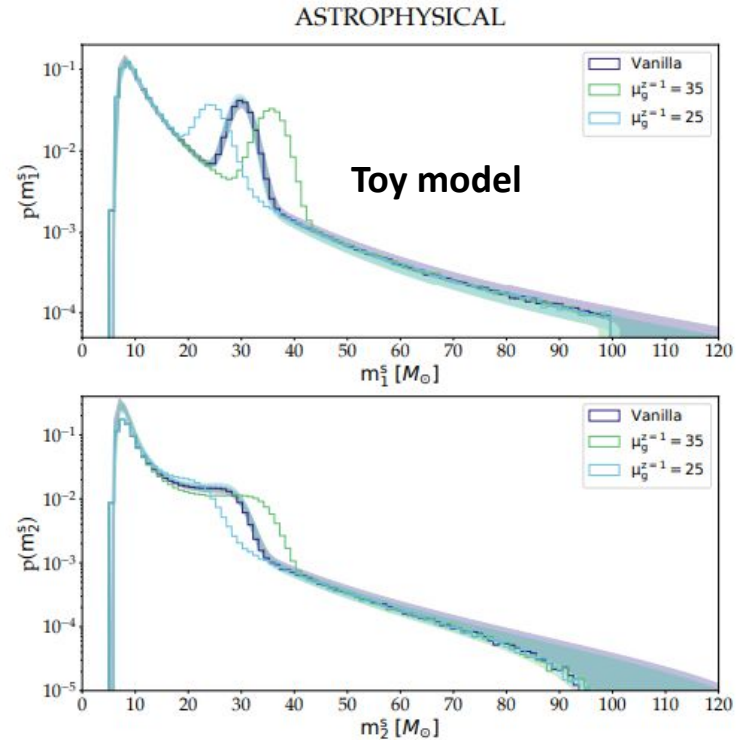
- With luminosity weights it is easier to exclude low  $H_0$  values (galaxy catalog complete).
- With no luminosity weights, the galaxy catalog is too incomplete.
- If brighter galaxies are more likely to emit GWs, the rate of CBC mergers per galaxies should be higher (very few luminous galaxies).
- Determination of CBC rate per galaxy degenerate with Hubble constant: Playing with the Hubble constant changes the galaxy number density in a fixed comoving volume.

SM+, Phys. Rev. D 102, 044009 (2020)



# What can go wrong? Astrophysics: BBH mass spectrum

- Current models for BBH mass spectrum does not evolve with redshift
- Astrophysical simulations indicate that the BBH mass spectrum *could* evolve in redshift  
[M. Mapelli *Handbook of GW astronomy* (2021), R. Srinivasan et al *SM MNRAS* 524 (2023)]
- Can an evolving BBH mass spectrum translate to a biased estimation of  $H_0$  with **spectral sirens**?

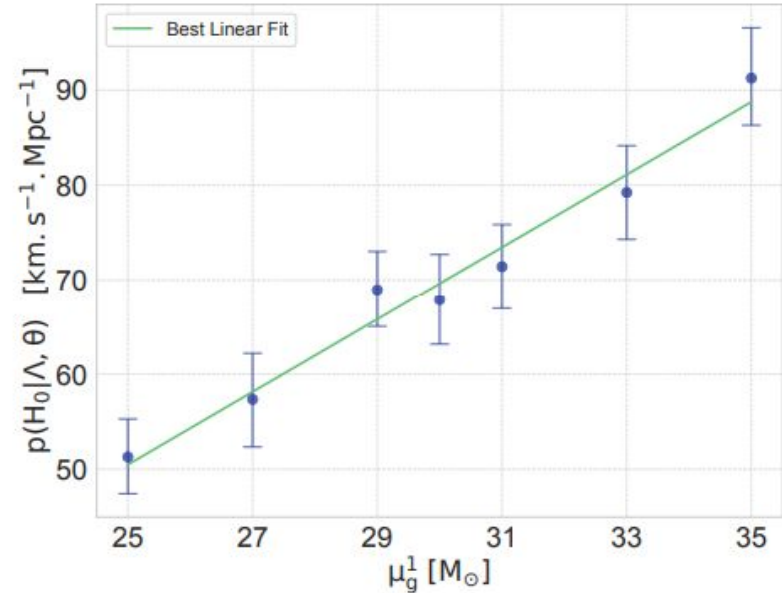


G. Pierra, *SM+*, S. Perries, M. Mapelli, arXiv 2312.11627

# What can go wrong? Astrophysics: BBH mass spectrum

- An evolution of the mass spectrum **can result** in a biased estimation of the Hubble constant.
- If the peak features evolves the bias could be significant.
- The reconstructed mass distribution are strongly degenerate.

## Simulation with 2000 GW detections



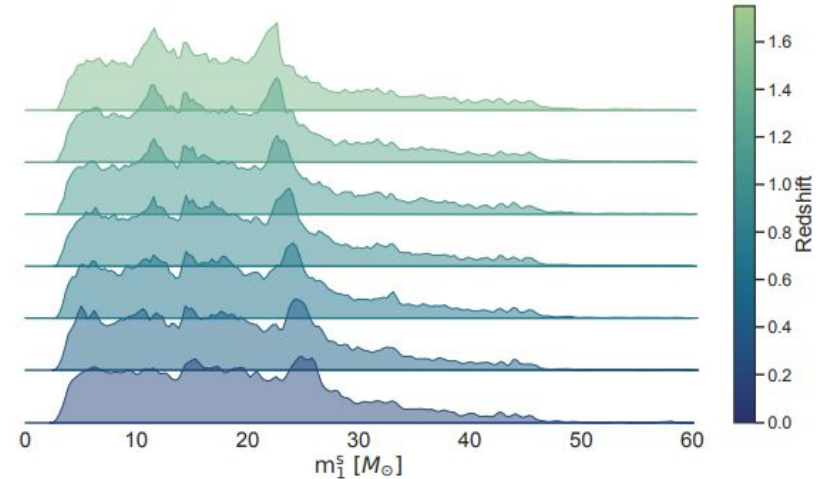
G. Pierra, *SM+*, S. Perries, M. Mapelli, arXiv 2312.11627

# What can go wrong? Astrophysics: BBH mass spectrum

- We use a synthetic BBH catalog containing 4 different formation channels: isolated binaries and hierarchical mergers in young, globular and nuclear star clusters.

[M. Mapelli et al MNRAS 511 (2022)]

- The BBH mass spectrum shows a mild evolution in redshift, in particular in the 10-30 solar mass region.
- We simulated 2000 GW detections from the BBH catalog and using simple redshift-independent mass models we inferred the value of  $H_0$ .

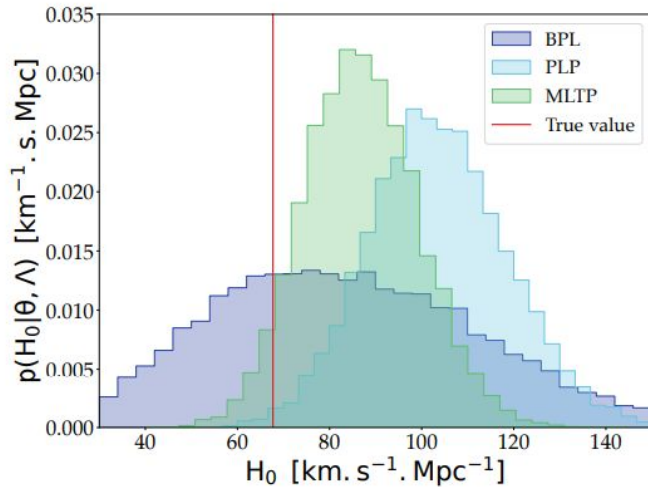


G. Pierra, *SM+*, S. Perries, M. Mapelli, arXiv 2312.11627

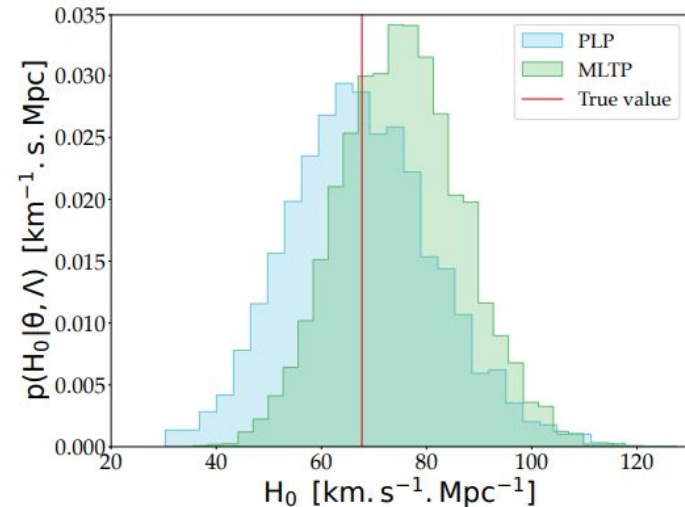


# What can go wrong? Astrophysics: BBH mass spectrum

- Redshift-independent mass models with mass features are more prone to systematics when inferring  $H_0$



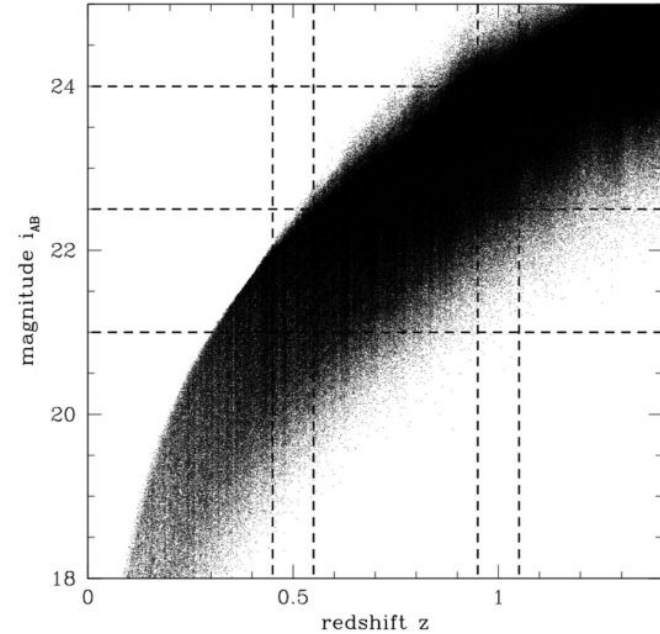
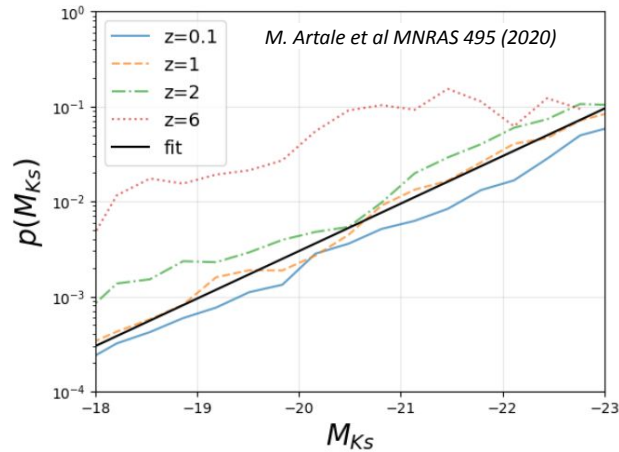
- The bias is removed when removing the mild redshift dependence from the mass spectrum



G. Pierra, *SM+*, S. Perries, M. Mapelli, arXiv 2312.11627

# What can go wrong? Astrophysics: Galaxy relations

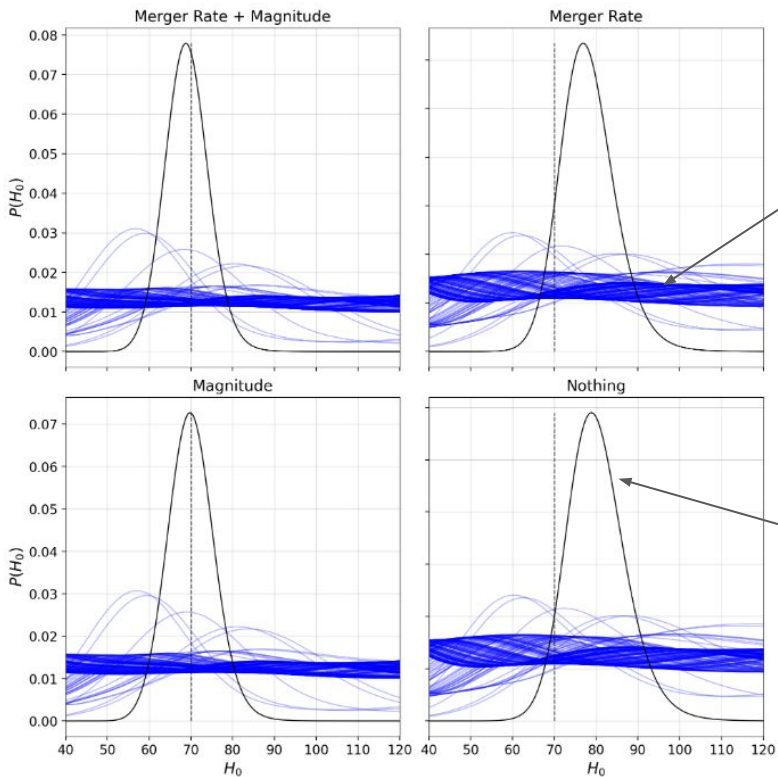
- The **MICE Grand Challenge lightcone** is a N-body cosmological simulation of 70 billion dark matter particles in which galaxies are formed.
- The MICE galaxies mimic all the properties of our Universe, e.g. clustering length, observational properties etc.
- We simulate GW detections from MICE assuming a relation between GW hosting probability and galaxies' luminosity.
- We use different detection horizons.



*M. Crocce et al MNRAS 453 (2015)*

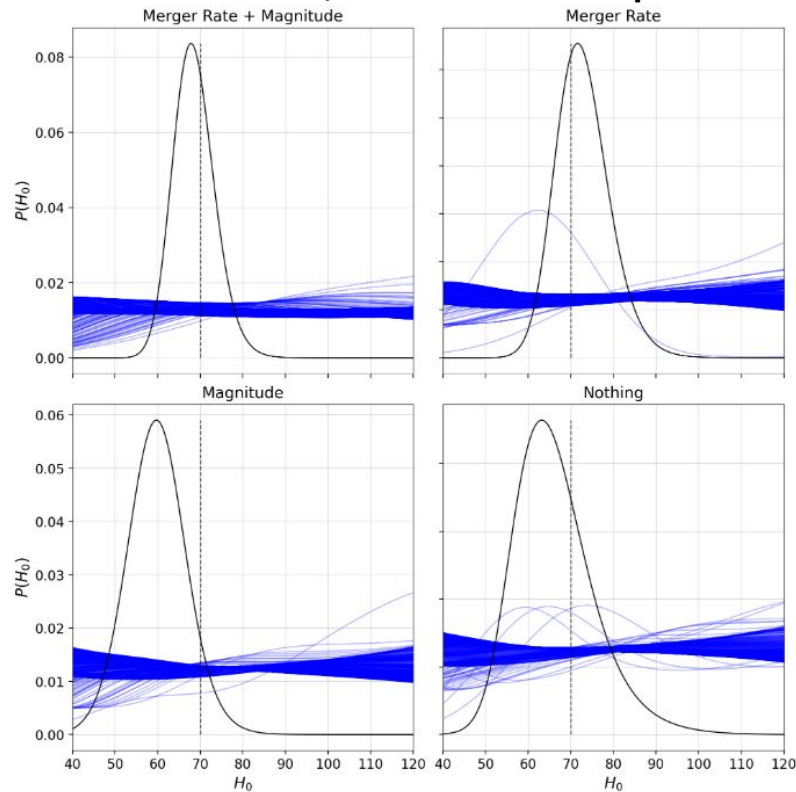
# What can go wrong? Astrophysics: Galaxy relations

500 GW events, horizon = 1550 Mpc



G. Perna, SM, A. Ricciardone in prep.

500 GW events, horizon = 4250 Mpc

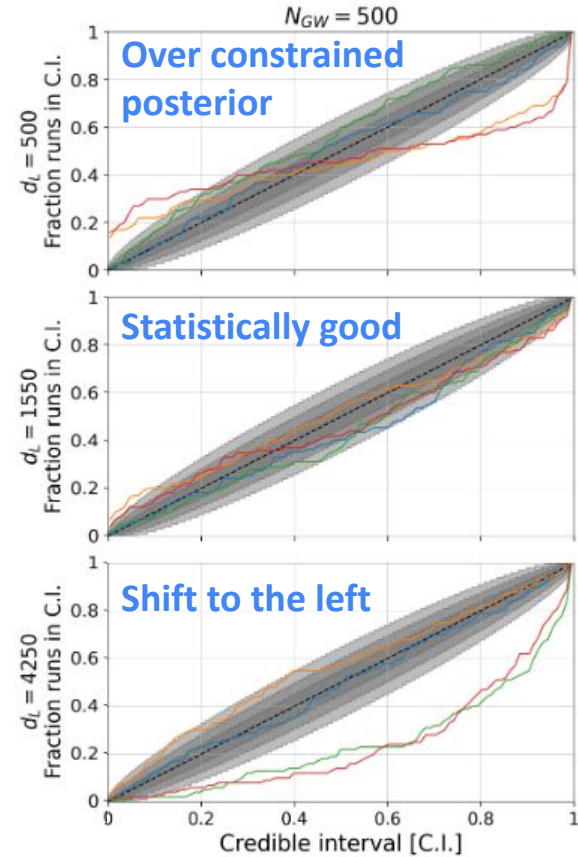


G. Perna, SM, A. Ricciardone in prep.

# What can go wrong? Astrophysics: BBH mass spectrum

- Parameter - Parameter (PP) plots are the definitive tool to check for presence of systematics bias in the inference (even with high statistical errors).
- If the **detection horizon is low (or high SNR cut)**, mismatching the galaxy/GW relation will result in a bias for  $H_0$ .
- If the **detection horizon is high (or low SNR cut)**, it is important to model accurately the CBC merger rate as function of redshift.

—	Merger_Rate_plus_Magnitude
—	Merger_Rate
—	Magnitude
—	Nothing



# Conclusions

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- Gravitational Wave Cosmology is entering in its systematics era
- Modifications to General Relativity on cosmological scales (GW friction and speed), could introduce a systematic in the estimation of  $H_0$  at [the percent-level](#) precision.
- Mismodeling of the BBH mass spectrum is [very likely](#) to introduce a bias in the estimation of  $H_0$  even with current BBH detections.
- Well-localized and close dark sirens can inherit a bias on  $H_0$  due to the unknown galaxy/GW relation.

*Don't Panic*

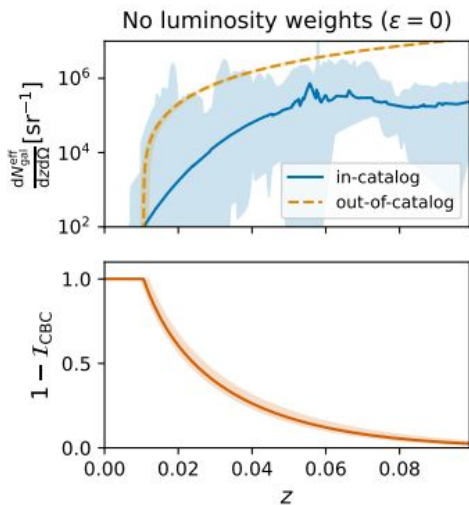
*Use Statistics*

*Know your biases*

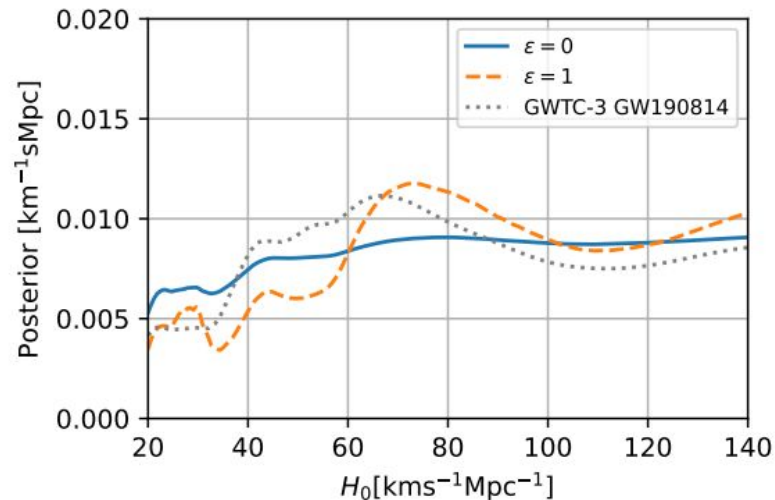
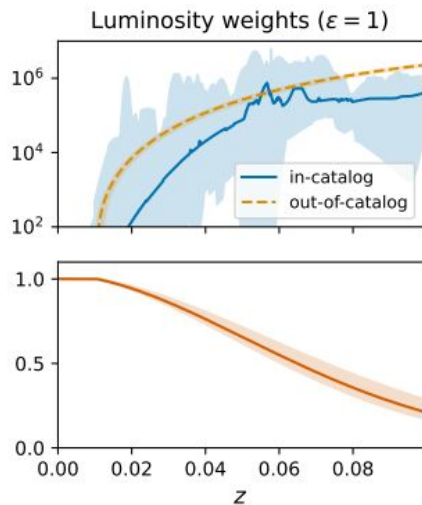
# Backup slides

# Dark sirens: Cosmology aided by galaxy surveys

## Equal host probability



## Red-luminous galaxies preferred hosts



[Mastrogiovanni+, PRD 2023]

GW hosting models will have an important impact for cosmology

# Observables for GR modifications

## The GW luminosity distance

$$d^{\text{GW}}(z) = d_{\text{EM}}(z) \exp \left[ \int_0^z \frac{\alpha_M(z)}{1+z} dz \right]$$

EM distance GW friction

## GW-GRB time delay

$$\Delta t_d^{\text{GW-EM}} = (1+z_s) \Delta t_s^{\text{GW-EM}} + \frac{f_{R,d}^j}{2} \mathcal{T}_j$$

GW-GRB Prompt delay GW Peak frequency

Mode-delay

$$\mathcal{T}_j = \int_0^{z_s} dz' \hat{\alpha}_j(z') \frac{(1+z')^j}{H_0 E(z')}$$

## GW Phase modes

$$\frac{\psi_{3j+8}(f_d) - \psi_{3j+8,\text{GR}}(f_d)}{\psi_{3j+8,\text{GR}}(f_d)} = \pi \frac{\mathcal{T}_j}{\beta_{3j+8}^{\text{PN}}(j+1)}$$

Measured PN coefficient GR PN coefficient

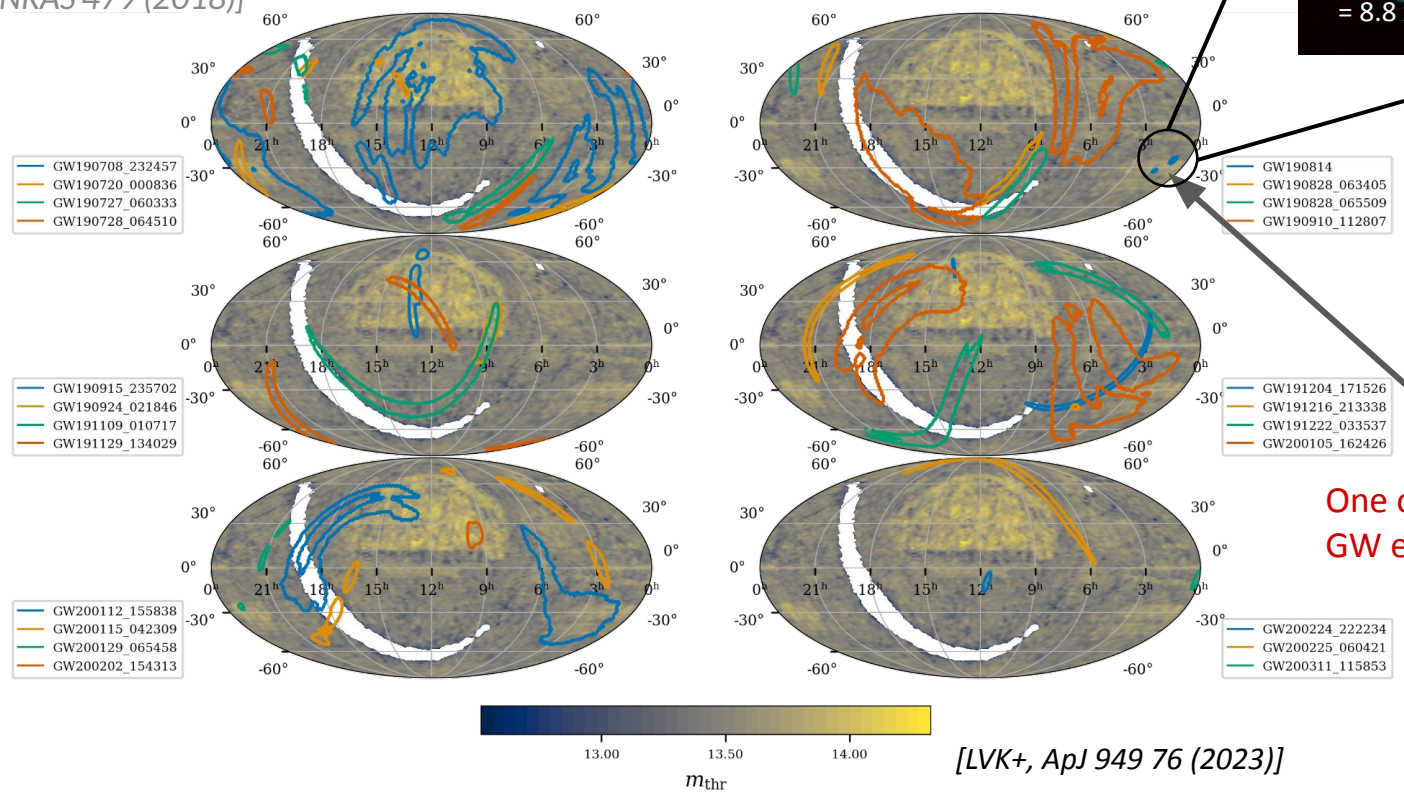
SM+, Phys. Rev. D 102, 044009 (2020)



# GW cosmology after GWTC-3: Dark sirens

etus GW190814  
 Sky area  $\sim 20$  sq. deg.  
 Localisation volume  
 $= 8.8 \times 10^{-7} \text{ Gpc}^3$

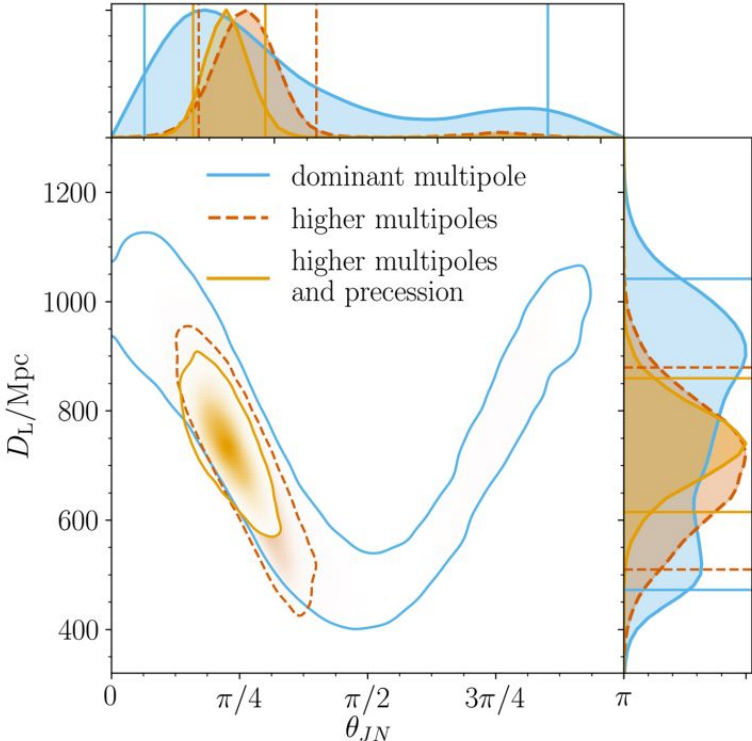
[Dalya+, MNRAS 479 (2018)]



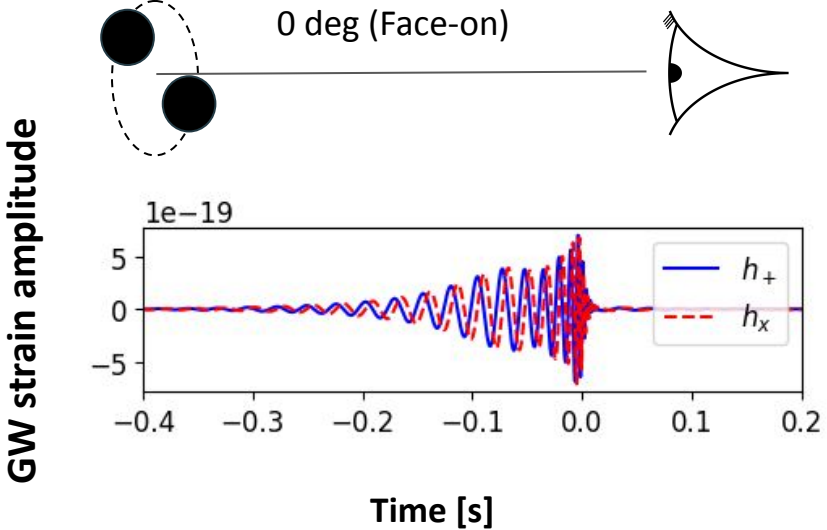
[LVK+, ApJ 949 76 (2023)]

One of the highly localised GW events

# Gravitational Wave sources at cosmological scales



[B. P. Abbott, PRD 102, 043015]



There are large uncertainties on the GW estimation of the luminosity distance. The precision can be improved with (i) extra EM information (ii) precession or higher order modes.