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Constraints on cosmological sources of gravitational waves with pulsar timing arrays

 $14/02/2023$ - Rome - GW Probes of Fundamental Physics -

- Brief intro on recent Pulsar Timing Array (PTA) observations
- Constraints on cosmological sources:
	- (Model-independent) role of QCD thermal history
	- Second-order induced GWs and primordial black hole bounds
- Estimate future sensitivity to sub-dominant GW backgrounds

Pulsar Timing Array observations

"Recent" Pulsar Timing Array (PTA) observations

Correlation between pulsar pairs: Hellings-Downs curve

 $\mathcal{L}(\mathcal{$

 0.9

Evidence for HD found most recent datasets

PULSA

- Results compatible with each other IPTA [arXiv:2309.00693]
	- NANOGrav 15 currently giving the most stringent constraint -14.4

GWs sources in the nHz frequency range

Astrophysics:

SMBH binaries formed in galaxy mergers

Image: NASA, ESA L. Mayer *et al.* Science **316** (2007), 1874-1877 [arXiv:0706.1562]

Hubble+Keck observations Numerical simulations of galaxy mergers

"New" physics: GW backgrounds of cosmological origin

Role of the QCD thermal history

and coincidence of scales: PTA and *QCD* scale factor (today). From the the thermodynamical relation \mathcal{F}_c relations \mathcal{F}_c scale factor (today). From the thermodynamical relationship in the thermodynamical relationship in the theory
The thermodynamical relationship in the thermodynamical relationship in the theory of the theory of the theory

• Relevant scale for cosmological GWs: Hubble sphere $R_H =$ 1 *H*(*T*) \bullet R \mathbf{f} = \mathbf{f} + $\mathbf{H}(\mathbf{I})$ \bullet Relevant scale for cosmological GWs: Hubble sphere $\mathbf{1}$ $\mathbf{H}(\mathbf{r})$

(i) the GW energy density ⇢gw(*f*) in the CT is

for a generic *w*. This scaling is additionally a↵ected by

Frequencies of modes crossing the Hubble sphere $r_{\rm F}$ \ddot{o} \ddot{o} \ddot{o} \ddot{o} cies of modes crossing the full one sphere $J =$ Frequencies of m entry, and then proceed with under-damped oscillations The set of $f = k/2\pi = aH/2\pi$ in terms of temperatures reads (see App. A) and the temperatures reads (see App. A) and the temperatures reads
A) and the temperatures reads (see App. A) and the temperature reads (see App. A) and the temperature reads (
A

$$
f \simeq 3.0 \text{ nHz} \cdot \left(\frac{g_{*,s}(T)}{20}\right)^{1/6} \left(\frac{T}{150 \text{ MeV}}\right)
$$

PTA frequency range \longrightarrow epoch of confinement of QC App. B), accounting for the temperature dependence of We solve the equations of motion for *hk*(*t*) (see PTA frequency range \longleftrightarrow epoch of confinement of QCD

QCD effects on cosmological evolution

10-1

1.05

QCD crossover affects $\frac{1}{2}$ ¹¹⁰ cosmological evolution primordial General Gen
1980 - Johann General cosmological evolution: $\frac{3}{5}$ 80

$$
s \sim g_{*,s} T^3
$$

$$
\rho \sim g_* T^4
$$

Evolution of the equation of state of the universe **for the universe f** the universe \mathbf{f} the frequency of \mathbf{f} entering the Hubble radius when

$$
w(T) = \frac{4}{3}(g_{*,s}(T)/g_*(T)) - 1
$$

$$
\hbox{RD: } w=1/3
$$

1

tensor modes [42]. After the GW source shuts o↵,

d⇢gw(*f*)

that GW_s deviation from perfect ranger induced by OCD crossover \mathcal{C} correlation length. The source \mathcal{C} sol alexiation from perfe T 30% deviation from perfect radiation 10^{-9} PTA optimal induced by QCD crossover

Class of GW spectra: transient sources of the GW spectrum consists of the waves whose period and wavelength are much longer than the source of the spatial correlations respectively. In other words, 1/, $\frac{1}{\sqrt{2}}$ energy density of that sector, and the normalization of ⇧*ij* follows that of [46] with a prefactor of 32°^C/3. We will also both *h*^{*ij*} and *J*^{*i*} and *J*^{*i*} on the two independent (+*,* α *)* points, α

- Model independent effect of QCD thermal history on transient sources M assumed that the GMD and a time scale fast compared to Hubble solution and the Hubble solution of Hubble solution \mathcal{L} Through independent ences of we cheft mother instery on trainsient sourc

and assume that the respective amplitudes *J* and *h* of the two polarisations are equal. We

• Source active for timescale τ_*

$$
\partial_{\tau}^2 h_{ij} + 2\mathcal{H} \partial_{\tau} h_{ij} + k^2 h_{ij} = a^2 \frac{32\pi G\rho}{3} \Pi_{ij} \equiv J_{ij}
$$

where is the wavelength and 1/ is the wavelength and 1/ is the duration of the process generating the process g
The process generation of the process generation of the GWs. The gward the process generating the GWs. The gwa

 A nisotropic stress

⌧ *^h* + 2*H*@⌧*^h* ⁺ *^k*2*^h* = 0 *,*

Over longer time-scales: $J(k, \tau') = J_{\star}(k)\delta(\tau' - \tau_{\star})$

Example of transient sources: Example of transient sources:

- First-order cosmological phase transitions
	- Second-order GWs with peaked spectrum

@2

• Spectator fields

 W will be interested in the initial conditions right after α 0.000 μ 0.000 μ 0.000 μ

• ….

• Source active for timescale *R^H* Using conformal time ⌧ and conformal Hubble rate *H* = *a*⁰ *hk /a*, the equation of motion for a comoving mode *k* of the graviton *hij* is (we follow a notation close to the one of [46]) ⇧*ij* ⌘ *Jij ,* (2.1) where ⇧*ij* is the dimensionless anisotropic stress of the sector generating the GWs, ⇢ is the energy density of that sector, and the normalization of ⇧*ij* follows that of [46] with a prefactor The solution to Eq. (2.1), assuming *h*(⌧) = 0 for ⌧ *<* ⌧?, can be found using Green's ^p*k*² *^H*² *J*(*k,* ⌧)*.* (2.2) = ⌧? + ✏ assuming that *J*

e.g. bubbles collisions from PTs

= ⌧? + ✏ assuming that *J*

GW spectra for transient sources

Model independent feature of transient cosmological sources: ['03 Seto, Yokoyama; '05 Boyle, Steinhardt; '06 Watanabe, Komatsu; '09

"Causality Tail"

Caprini, Durrer, Konstandin, Servant; '18 Caprini, Figueroa; '18 Saikawa, Shirai; '18 Cui, Lewicki, Morrissey, Wells; '19 D'Eramo, Schmitz;…]

Derivation of the causality tail is found to be always valid on superhorizon superhorizon superhorizon superhorizon scales where α the source disappears (k ≪ (ausuality tail−1). The source disappears (k \sim 1). This causality tail is found to be a perivation of the causality tail is found to be ex-
is found to be always valid on the Causality valid on superhorizon scales where \sim

• On scales: $1/k > 1/k_*$ at the horizon crossing of $1/k_*$ • On scales: $1/k > 1/k_*$ is found to be always valid on superhorizon superhorizon superhorizon superhorizon scales where α \bullet On scales: $1/k > 1/k_0$. \bullet On scales. $1/N \geq 1/N$ #

 \int_{0}^{R} finds $N = (k/k)^{3}$ independent patches t_{max} and the tensor perturbation is have the tensor in the tensor is the tensor in the tensor is have the tensor in the tensor is the tensor of $\sqrt{2}$ nds $N = (k_*/k)^3$ independent patches \bigwedge $\sum_{n=1}^{\infty}$ and $\sum_{n=1}^{\infty}$ $(k/k)^3$ independent patches one $\lim_{n \to \infty} s_n = (n \times n)$ independent patches one finds $N = (k_*/k)^3$ independent patches one finds $N = (k_*/k)^3$ independent patches $\bigcup_{k=1}^{\infty}$ \mathcal{C} at a characteristic scale 1/k∗. For an infrared 1/k∗. For

 TW amplitude over super-Hubble scales is $\sqrt{2}$ which means on scale of $\frac{1}{k}$, he is the scale source *r* amplitude over super-Hubble scales is $\sqrt{2\pi}$ \bullet The GW amplitude over super-Hubble scales is random variable, h∗(i) obeys the Poisson distribution, $\bullet\,$ The GW amplitude over super-Hubble scales is are $\frac{1}{3}$ causally disconnected patches where $\frac{1}{3}$ causally disconnected patches where $\frac{1}{3}$

condition that integral (20) is positive and finite can be integral (20) is positive and finite can be integra
In the condition of the can be integral (20) is positive and finite can be integral (20) in the can be integra

the source disappears (^k [≪] (aeHe)−1). This can be ex-

$$
h_k = \sum_i h_{*(i)}/N
$$

 SW two point function: $\mathcal{L}_\mathcal{D} = \mathcal{L}_\mathcal{D} = \mathcal{L}_\mathcal{D}$ \mathcal{N} two point function: • The GW two point function: (k/k∗)³|h[∗][|] 2.6 km stays point failed on superhorizon superhor point function of the tensor perturbation at the forma-

$$
\langle h_k h_k \rangle = N^{-2} \sum_{ij} \langle h_{*(i)} h_{*(j)} \rangle = (k/k_*)^3 |h_*|^2
$$

dvnamics: constant on super-Hubble scales (over-damped), t obtain the scales (degraduate scales degraduate scales degraduate α namics: constant on super-Hubble scales (over-damped), then decays as $1/a$: $\frac{1}{\alpha}$ $\log y$ by $\cos 1/w$. ϵ CW dynamics: constant on super Hubble scales (c • Gw dynamics: constant on super-пubble scales (с ble scales (over-damped), then decays as $1/a$: $\sum_{i=1}^{n}$ is called $\sum_{i=1}^{n}$ of the definition of the energy $\sum_{i=1}^{n}$ of the energy $\sum_{i=1}^{n}$ \bullet GW dynamics: constant on super-Hubble scales (o $h at present by a redshift factor at present by a redshift factor and $\mathcal{A}^{\mathcal{A}}$$ \mathcal{L} • GW dynamics: constant on super-Hubble scales (over-damped), then decays as $1/a$: $h^{(1)}$ then decenses $1/x$ α , onch accays as $\frac{1}{w}$.

$$
\rho_{\rm GW} \sim \langle \dot{h}_k^2 \rangle \sim (k/a)^2 \langle h_k^2 \rangle
$$

\n $\Omega_{\rm GW} \sim \rho_{\rm GW} \sim \begin{cases} k^3 & \text{Radiation Domination} \\ k & \text{Matter Domination} \end{cases}$ (super-Hubble scales)

guaranteed easily if the source is spikely if the source is spikely if the source is spikely and transient. If
If the source is spikely if the sou

QCD crossover affects the causality tail in the PTA band

G.Franciolini, D.Racco and F.Rompineve, **PRL** [arXiv:2306.17136]

entropy injection to SM bath only

Relevant modulation for all .
ive before early-universe sources of GWs active before QCD crossover.

QCD crossover affects the causality tail in the PTA band

• Effect already relevant for signal interpretation

 $\log_{10}\mathcal{B}_{f^3}^{\text{CT}}=1.6$

f3 $\frac{1}{2}$ **f**3 data Significant difference in evidence between approximated cubic and actual causality tail

QCD crossover affects the causality tail in the PTA band

G.Franciolini, D.Racco and F.Rompineve, **PRL** [arXiv:2306.17136]

posteriors for a power-law model, the law model, t
The law model, the l

QCD crossover. As highlighted in this paper, this

Second-order induced GWs

Induced GWs at second-order

Emission of induced GWs at second-order

 $h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} \approx \mathcal{S}_{ij}(\zeta\zeta)$

K. Tomita, Prog. Theor. Phys. 54, 730 (1975).

S. Matarrese, O. Pantano, and D. Saez, Phys. Rev. Lett. 72, 320 (1994), [arXiv:9310036].

V. Acquaviva, *et al.* Nucl. Phys. B 667, 119 (2003), [arXiv:0209156].

S. Mollerach, D. Harari, and S. Matarrese, Phys. Rev. D 69, 063002 (2004), [arXiv:0310711]. K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev. D 75, 123518 (2007), [arXiv:0612013]. …

Characterization of the spectrum

At second-order in comoving curvature perturbation, after averaging over the fast oscillating pieces:

$$
\Omega_{\rm GW}(\eta, k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2} \int \frac{d^3p}{(2\pi)^3} \frac{p^4 \left[1 - \mu^2\right]^2}{p^3 \left|\vec{k} - \vec{p}\right|^3} \mathcal{P}_{\zeta}(p) \mathcal{P}_{\zeta}\left(|\vec{k} - \vec{p}|\right) \mathcal{I}^2\left(\vec{k}, \vec{p}\right)
$$

- $\bullet\,$ Double peak feature for narrow spectra \sim
- Universal IR slope (super-Hubble modes) \overline{R} s al
- UV model dependent: $\propto \mathcal{P}_{\zeta}^2(k \gg k_*)$ $\frac{1}{2}$

SIGW can fit PTA data well $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ \bm{A} data well peaks encountered, for in-

Assuming BPL curvature spectrum:

$$
\mathcal{P}_{\zeta}^{\text{BPL}}(k) = A \frac{(\alpha + \beta)^{\gamma}}{\left(\beta \left(k/k_*\right)^{-\alpha/\gamma} + \alpha \left(k/k_*\right)^{\beta/\gamma}\right)^{\gamma}}
$$

nario can fit well the data The IR tail of a SIGW scenario can fit well the data $(7.92^{+0.89}_{-0.62}, 8.19^{+0.65}_{-1.20})$ has $\frac{1}{100}$. The parameter is the parameterize the parameters the parameters the parameterizes the parameterizes the parameterizes the parameterize the parameterizes the parameterizes of the parameterizes the parameter SIGW-BPL EPTADR2New $(0.86^{+0.79}_{-0.65}, -0.49^{+0.47}_{-1.18})$ NANOGrav15 °0*.*5 *A* °1*.*0 log_{10} $\lfloor k \rfloor$ °1*.*5 ⇣ (*k*) = *^A* $[\text{Mpc}^{-1}]$
 10^7 °2*.*0 $(4.50^{+3.33}_{-3.37}, 4.53^{+3.30}_{-3.46})$ 6 $\frac{1}{2}$ $\left[\begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \right]$ Æ \mathcal{P} 10^{-6} ϵ \sim SIGW-BPL γ $(5.25_{-4.80}^{+4.53}, 5.09_{-4.67}^{+4.68})$ $_{\mathrm{SIGW-LN}}$ 10^{-7} $h^2 \Omega_{\rm GW}(f)$ θ 10^{-8} 6 Ø β 10^{-9} \cap $(5.02^{+4.75}_{-4.68}, 5.03^{+4.74}_{-4.67})$ 10^{-10} $^{\circ}$ 6 10^{-11} ∞ \mathcal{P} NANOGrav15 ባ, 10^{-12} $\frac{1}{10^{-8}}$ 10^{-9} γ 66*.*6 ⁷*.*² ⁷*.*⁸ 8*.*4 λ γ \bowtie 6V & G & °2*.*0 °1*.*5 °1*.*0 °0*.*5 $^{\circ}$ $\log_{10} A$ $\log_{10}(k_*/\mathrm{Mpc}^{-1})$ β α f [Hz] γ

 10^{-7}

Large curvature perturbations are required

Large curvature perturbations: primordial black hole formation

Mass and frequency related by the Hubble scale at formation

…

R.Saito and J.Yokoyama, Phys. Rev. Lett. **102** (2009), 161101 [arXiv:0812.4339] J. Garcia-Bellido, M. Peloso and C. Unal, JCAP **09** (2017), 013 [arXiv:1707.02441] N. Bartolo, *et al*, Phys. Rev. Lett. **122** (2019) no.21, 211301 [arXiv:1810.12218]

Complementary constraints from PBH overproduction

Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]

GW probes of FP - 23

Detailed computation of the PBH abundance

0.54

M. Shibata and M. Sasaki, Phys. Rev. D **60**, 084002 (1999) [arXiv:gr-qc/9905064] T. Harada, C. M. Yoo and K. Kohri, Phys. Rev. D **88**, no.8, 084051 (2013) [arXiv:1309.4201] C. Germani and I. Musco, Phys. Rev. Lett. **122**, no.14, 141302 (2019) [arXiv:1805.04087] … \sum

Formation criterion: $\delta \ge \delta_c \; (\approx 0.5)$ where we introduced *^F*(↵s) = ⁿ ¹ ² ⁵ *^e*1*/*↵s↵¹5*/*2↵^s ^s */* [(5*/*2↵s) (5*/*2↵s*,* 1*/*↵s)] $\lambda > \lambda$ / \sim 11 \sim 1 15*e*1*/*↵^s ↵¹5*/*2↵^s

Threshold for collapse sensitive to the following:

• Shape of the collapsing peak (controlled by curvature spectrum) $\frac{0.4}{\mathbb{S}}$

0.660

• QCD era: softening of the equation of state • QCD era: softening of the equation of state $\sum_{0.28}$ and \sum_{δ_0} and \sum_{δ_1} and \sum_{δ_2}

….

I. Musco, K. Jedamzik, and S. Young $\left[\text{arXiv:}2303.07980 \right]$

 \sim • Non-linearities: $\delta(\vec{x}, t) = -\frac{8}{9a^2H^2}e^{-5\zeta(\vec{x})/2}$ $\frac{6}{9a^2H^2}e^{-5\zeta(\vec{x})/2}\nabla^2e^{\zeta(\vec{x})/2}+\ldots$ $\frac{\mathcal{S}(\vec{x})}{\mathcal{S}}$ and $\frac{-5\zeta(\vec{x})}{2\pi^2\sqrt{(\vec{x})}}$ $\sigma(\omega,\iota)=-\frac{1}{9a^2H^2}e^{\frac{2\pi i}{3\omega^2H^2}}$ with κ of τ . These traced back to equal cases in the series of Cases in the literature of α is κ . Museo and C. T. Byrnes, JCAP 11 (2019), 012 [arXiv:1904.00984] Non-linearities: ^G + *...*

S. Young, I. Musco and C. T. Byrnes, JCAP 11 (2019), 012 $\text{[arXiv:1904.00984]}$ U.De Luca, G.Franciolini, A.Kehagias, M.Peloso, A.Riotto and C.Ünal, JCAP 07 (2019), 048 [arXiv:1904.00970] summarized in the schematic below. $904.00984]$ ⇣ ⁼ ²

 -1 • **Primordial non-Gaussianities:** e.g. different models predict local-type NGs $\zeta = F(\zeta_G)$ $\arctan y$ pc roll $\arctan y$

²⁵ *g*NL⇣³

⁵ *f*NL⇣²

Curvaton model: $\zeta = \log [X(r_{\text{dec}}, \zeta_{\text{G}})]$ \overline{U} α \overline{O} *DI*^{\overline{C}}

$$
\text{USR models:} \quad \zeta = -\left(\frac{6}{5}f_{\text{NL}}\right)^{-1} \log\left(1 - \frac{6}{5}f_{\text{NL}}\zeta_{\text{G}}\right)
$$

In the course of this paper, and in particular in section III, we will describe the main results of our analysis using the See talk by I. Musco on Friday

GW probes of FP - 24 ⇣ ⁼ ²

(3)

….

|h|

^F(↵s)[1 + *^F*(↵s)]↵^s ⁼ ¹

1 + *r^m*

Non-Gaussian effects on the SGWB Γ The Higher-order corrections to the GW power spectrum can then be written as well as the written as well as the
Then be written as well as wel Appendix A: Second-order induced gravitational waves with non-Gaussianities density fraction spectrum of SIGW at the current epoch is the current epoch

 W summarize here the formulas used to compute the spectrum of S when \mathcal{A} are included, and included, and \mathcal{A} referring the reader to Refs. [314–321] for more details. We assume that the curvature perturbation can be written in *k* = *O*(*k*⇤) are parameters that depend mildly on the shape of the curvature power spectrum.

where *A*˜ and ˜

P. Adshead, K.D. Lozanov and Z.J. Weiner, JCAP 10 (2021), 080 [arXiv:2105.01659]		
$\zeta = \zeta_{G} + F_{\rm NL}\zeta_{G}^{2}$	$\Omega_{\rm GW}(T) = \Omega_{\rm GW}^{(0)}(T) + \Omega_{\rm GW}^{(1)}(T) + \Omega_{\rm GW}^{(2)}(T)$	
A^{2}	$A^{3}F_{\rm NL}^{2}$	$A^{4}F_{\rm NL}^{4}$
$\frac{10^{6}}{\pi r_{\rm N}^{(2)}/(A^{4}r_{\rm M}^{2})}$	$\frac{10^{-2}}{\pi - 0.4}$	
$\frac{10^{-3}}$	$\frac{10^{-3}}{\pi - 0.4}$	$\frac{10^{-3}}{\pi}$
$\frac{10^{-3}}$	$\frac{10^{-1}}{\pi}$	
$\frac{10^{-1}}{\pi}$	$\frac{10^{6}}{\pi}$	$\frac{10^{6}}{\pi}$
$\frac{10^{-1}}{\pi}$	$\frac{10^{6}}{\pi}$	$\frac{10^{6}}{\pi}$
$\frac{10^{-1}}{\pi}$	$\frac{10^{6}}{\pi}$	$\frac{10^{6}}{\pi}$

- \bullet No additional footures generated by NCs in the IR α in ϕ • No additional features generated by NGs in the IR $\Omega_{\rm GW}(k \ll k_*) \propto k^3 (1 + \tilde{A} \ln^2(k/\tilde{k}))$ $k))$, s genera
20-20ste 10° 11. $\Omega_{\text{GW}}(k \ll k_*) \propto k^{\circ} (1 + A \ln^2(k/k)),$
- centered at *k*⇤ that is characterised by a width = 0*.*4. • A- f_{NL} degeneracy in the posterior distribution

(

GW(*T*) =2*F*²

2

Z ¹

Z ¹

- Expansion parameter $(A\times F_{\rm NL}^2)$ small in realistic models considered distribution
hall in realistic models considered

NGs can alleviate PBH overproduction bounds

G. Franciolini, A. Iovino, Junior., V. Vaskonen and H. Veermae, PRL [arXiv:2306.17149]

- SIGWs amplitude constrained by PBH overproduction, possible tension if it is the **only** source
- One needs scenarios in which the PBH abundance is suppressed
- On the other hand, current observations do not constrain PBHs in the (sub-)solar mass range

* beware of remaining uncertainties on PBH formation (De Luca et al [arXiv: 2307.13633])

SMBHs vs CGWs I: Poisson noise \mathbf{S} *MRH*_e *ve* \mathcal{C} *CW*_e *I*. where \mathcal{O} *initiation and* \mathcal{O} *is the SMBHB,* \mathcal{O} D_{α} iggan noiga α and a Markov chain Monte Carlo (MCMC) sampler tai-tai-chain mentioned, the SMBH signal mentioned, the SMBH sig

mon (spatially) uncorrelated red noise (CURN) signal present in all pulsars. More details on these noise modplyed SMBH mergers *i* (*m*1*m*2)³*/*⁵*/*(*m*¹ + *m*2)¹*/*⁵ is a combination of the black • Resolved SMBH mergers pear because and the signal strain is produced and strain in the second strain is produced in the signal strain is produced $\overline{}$

$$
h_0 = \frac{2\mathcal{M}^{5/3} (\pi f_{\rm GW})^{2/3}}{d_L} \qquad \begin{array}{c} \frac{1}{100} \\ \frac{
$$

sky location and the distance of the pulsar; and *F* ⁺*,*⇥ is the antenna pattern function, which depends on the etral fuctuations an individual binary, following previous searches (see Spectral fluctuations pecially for high *M* and *f*GW values. Note that both *M* • Spectral fluctuations Γ respond to the probability of an Γ \bullet *Spectral nuctuations*

 $s_{\rm ct,i} \equiv P(-\Delta^{2}\Omega(f_{i}) > \Omega_{\rm th}(f_{i}))$ \overline{a} and \overline{b} are related are related as \overline{b} $P_{\text{fluct},i} \equiv P(-\Delta^2 \Omega(f_i) > \Omega_{\text{th}}(f_i))$

- of \mathcal{C} MRH_s models prodict spectral fluctuations due to Poisson effects $\text{rel}\rightleftharpoons \text{rel}\rightleftharpoons \text{rel}\rightleftharpoons$ (most relevant at high-f) • SMBHs models predict spectral
- *^d^L* !(*t*)¹*/*³ 2 cos 2(*t*) cos◆*,* (4) software package that builds on the enterprise \mathbb{R} \sum over the relevant PTA scales tinguish SMBH binaries from cosmological GW sources, \bullet Cosmological signals mostly smooth • Cosmological signals mostly smooth

is the luminosity distance to the source, and *M* ⌘

SMBHs vs CGWs II: Anisotropies In the spherical-harmonic basis (Allen & Ottewill and allen as a multipole (the *line of the line of the <i>line of the line of the line of* the monopole $\mathbf{f}_{\mathbf{t}}$ and $\mathbf{f}_{\mathbf{t}}$ will have $\mathbf{f}_{\mathbf{t}}$ will have $\mathbf{f}_{\mathbf{t}}$

Predicted 95 % C.I. for anisotropies SMBHs pop

14 NANOGRAFIJA I PODIJELA U SRADILJE U SRADILJE U SRADILJE U SRADILJE. U SRADILJE U SRADILJE U SRADILJE U SRAD
Dogodki

- all anisotronies et al. 2023b). The distribution of the distribution of the distribution of the distribution o • Cosmological SIGW background predicts very small anisotropies If σ id iy background predicts very shiah amsoliopics Following the notation of [45], one can define the two-point function as
	- s plating to large scale modes) consider \bullet small scales at emission (unless large local NGs correlating to large scale modes)
- propagation effects (analogue of the SW effect) propagation enects (analogue of the

$$
\sqrt{\frac{\ell(\ell+1)}{2\pi} C_{\ell,I+S} (k)} \simeq 2.8 \cdot 10^{-4} \left| \frac{1 + \tilde{f}_{\rm NL}(k)}{10} \right| \left(\frac{\mathcal{P}_{\zeta_L}}{2.2 \cdot 10^{-9}} \right)^{1/2}
$$

third data release (DR3), currently under preparation,

(3.21)
<u>(3.21)</u>
(3.21)

Other probes of large SIGWs: sub-solar mergers

G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]

Forecast future PTA sensitivity

Disentangle multiple contributions to the spectrum?

• Steep growth of SNR with time in the weak signal limit, slow afterwords ϵ Steep growth of SNR with time in the weak st *The stochastic background: scaling laws and time to detection for pulsar timing arrays*6

 \mathcal{B}_max in figures 6 and 10, the median softwares 6 and 10, the median software medians of all 50

population of eccentric binaries (Taylor et al. 2017; Chen

The argument of the hypergeometric function for the second integral of the second integral of the \mathcal{H}_S

 ADII_2 are achieves to seemale ricel COWB (Ω = Ω) Simulated injection of SMBHs spectrum + cosmological SGWB ($\Omega_{\rm PGW}/\Omega_{\rm SMBHB} = 0$. $\rm p \rm_{\rm \,M\,S}$ ($\rm \Omega_{\rm \,PGW}/\Omega_{\rm \,SMBHB} \; = \; 0.5$) $\mathop{\rm gcd\,}\nolimits$ injectron of planning spectrum $\mathop{\rm Res}\nolimits$ of *f*_L μ ^L μ / μ / σ is the correct dependence on the various PTA properties. In this plot we took *T* ! 1*.*55*T* for both Eq. 33 and

in a simple free PL despite the WN at higher free P

and 95% Ci (vertically striped orange shaded region), and the right panel, with its median (dashed red line) and the α

 Λ D V_{eisen} Λ other in Λ 0.00 (0.00) A. R. Kaiser, Astrophys. J. 938 (2022) no.2, 115 [arXiv:2208.02307] \mathbf{h} in the free spectrum (blue dots and lines) in \mathbf{h} in the left panel \mathbf{h}

(↵ = 2*/*3, = 13*/*3) at an amplitude of ASMBHB*,*Inj =

For each pulsar we assume Gaussian and stationary data: \tilde{d} $d = \tilde{s} + \tilde{n}$ $pulsar$ we assume Gaus *ssian and sta* $\frac{1}{2}$ ∂ \mathbf{r} , \mathbf{r} $\tilde{d} = \tilde{s} + \tilde{n}$

which takes the analytic expression \mathcal{L}_max

$$
\langle \tilde{d}^2 \rangle = \langle \tilde{s}^2 \rangle + \langle \tilde{n}^2 \rangle = \mathcal{R}P_h + P_n
$$

GW power spectrum:
$$
\Omega_{\text{GW}} = \frac{1}{3H_0^2 M_p^2} \frac{d\rho_{\text{GW}}}{d \ln f} = \frac{4\pi^2}{3H_0^2} f^3 P_h
$$

Response function: $\mathcal{R}(f) = \frac{1}{12\pi^2 f^2}$

in the vector *◊* = *{◊s, ◊n}* both parameters for the signal

Simplified future PTA sensitivity forecasts

k running over frequency bins *fk*, described only by their S. Babak, M. Falxa, G. Franciolini, M. Pieroni, to appear

$$
\text{White likelihood:} \quad -2\ln\mathcal{L}(\tilde{d}|\theta) \propto \sum_{k,IJ} \ln\left[C_{IJ}(f_k,\theta)\right] + \tilde{d}_I^k C_{IJ}^{-1}(f_k,\theta)\tilde{d}_J^{k*}
$$

 \mathcal{H} summarise the main ingredients of main ingredients of main ingredients of \mathcal{H} x estimates on measurement uncertainties. integral over the frequency range as Γ is her matrix estimates or Fisher matrix estimates on measurement uncertainties:

$$
F_{\alpha\beta} \equiv T_d \sum_{f_k} \frac{\partial \log C}{\partial \theta^{\alpha}} \frac{\partial \log C}{\partial \theta^{\beta}} \qquad \sigma_{\alpha} = \sqrt{F_{\alpha\alpha}^{-1}}
$$

^Ph(*f*) = *^R*(*f*)*Sh*(*f*)*,* with *^R*(*f*) = ¹

$$
\sigma_\alpha = \sqrt{F_{\alpha\alpha}^{-1}}
$$

how likely it is to detect a particular type of GW signal.

f GW probes of FP - 33 \mathcal{S} is shown depend not only only only on the properties of the properties of the noise of the nois in the detector, but also on the type of signal that α we propose of $\mathbb{F}P$ $\mathcal{P}(\mathcal{O})$

we parameterise them as a power-law. Spectral

⌃*n,I* by ⌃*^I* ⌘ *^G^T*

diagonal (auto-correlated) terms. But we will replace

Recover results with "state-of-the-art" PTA data analyses

Power-law model:

$$
\Omega_{\rm GW} h^2 = 10^\alpha \left(\frac{f}{f_{\rm yr}}\right)^{n_T}
$$

\mathcal{P} \rightarrow Ω Δ $\Delta\alpha_*$ °3*.*0 °1*.*5 ⁰*.*⁰ \sim ⁵ ¢ *n T* MCMC EPTA10 Fisher EPTA10 Current data: EPTADR2New 10 yrs, 25 Pulsars

Fisher estimate (Time-domain) full Bayesian analyses

S. Babak, M. Falxa, G. Franciolini, M. Pieroni, to appear

Simulated EPTA 20, 50 p.

Simulated SKA 10, 50 p.

GW Probes of FP - 34

Future sensitivity to subdominant CGWB

$$
S_{\text{eff}}(f) = \left(C_{IJ}^{-1} C_{KL}^{-1} \chi_{JK} \chi_{LI} \mathcal{R}^2\right)^{-1/2}
$$

(If the dominant SGWB is astro, can one subtract resolved mergers?)

Multiple SGWB signals at future observatories

Power-law model (e.g. SMBHs) + lognormal bump (e.g. CGWB)

$$
\Omega_{\rm GW} h^2 = 10^{\alpha} \left(\frac{f}{f_{\rm yr}} \right)^{n_T} + 10^{\alpha_{\rm LN}} \exp \left[-\frac{1}{2\rho^2} \ln^2 \left(\frac{f}{f_{\star}} \right) \right]
$$

Future SKA: 10yrs, 50 pulsars

S. Babak, M. Falxa, G. Franciolini, M. Pieroni, to appear

Conclusions

- PTA observations provide unprecedented ways to test the GW spectrum at nHz
- Astrophysical models can fit the data, with constraining power for new physics
- New physics could also fit the data well, no smoking gun signature detected so far
- QCD thermal history could leave detectable imprints, if there was a transient source of GW at slightly larger temperatures
- Primordial black hole overproduction constrains the amplitude of SIGWs, complementarity with ground based detectors bounds
- Future sensitivities will be limited by the observed foregrounds (can this be reduced?)

Gabriele Franciolini

Thanks!

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