Gabriele Franciolini

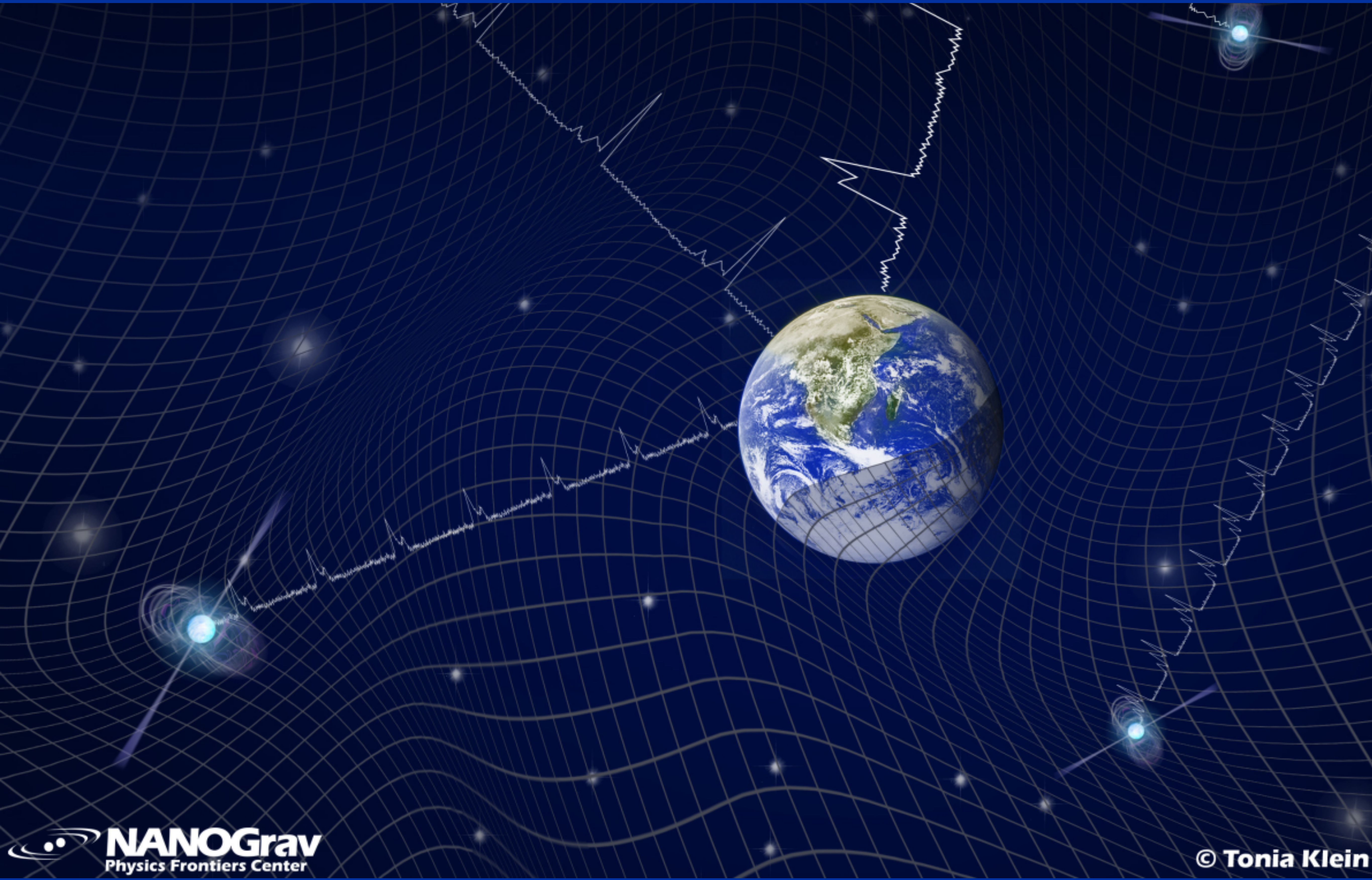
Constraints on cosmological sources of gravitational waves with pulsar timing arrays

14/02/2023 - Rome - GW Probes of Fundamental Physics -

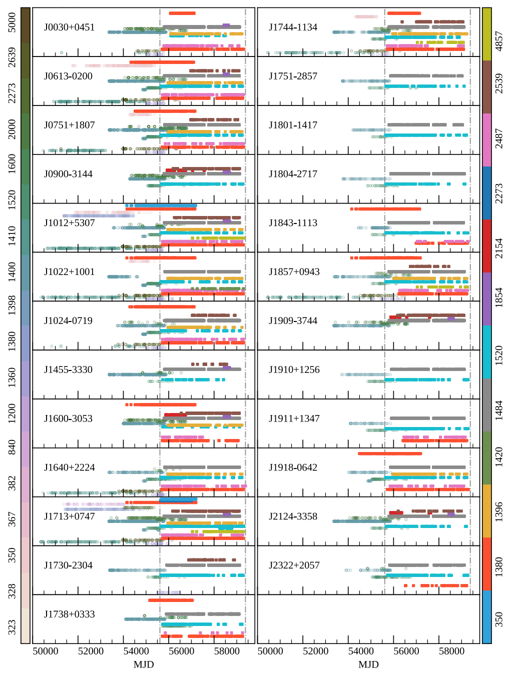
Outline

- Brief intro on recent Pulsar Timing Array (PTA) observations
- Constraints on cosmological sources:
    - (Model-independent) role of QCD thermal history
    - Second-order induced GWs and primordial black hole bounds
- Estimate future sensitivity to sub-dominant GW backgrounds

Pulsar Timing Array observations

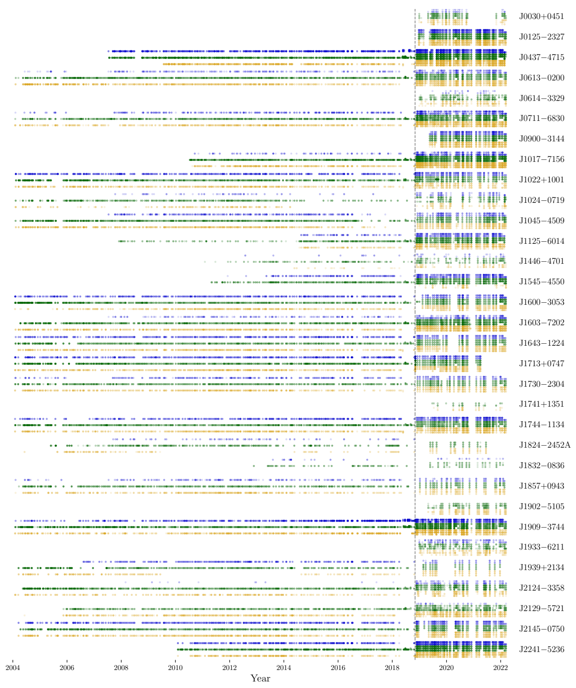


“Recent” Pulsar Timing Array (PTA) observations



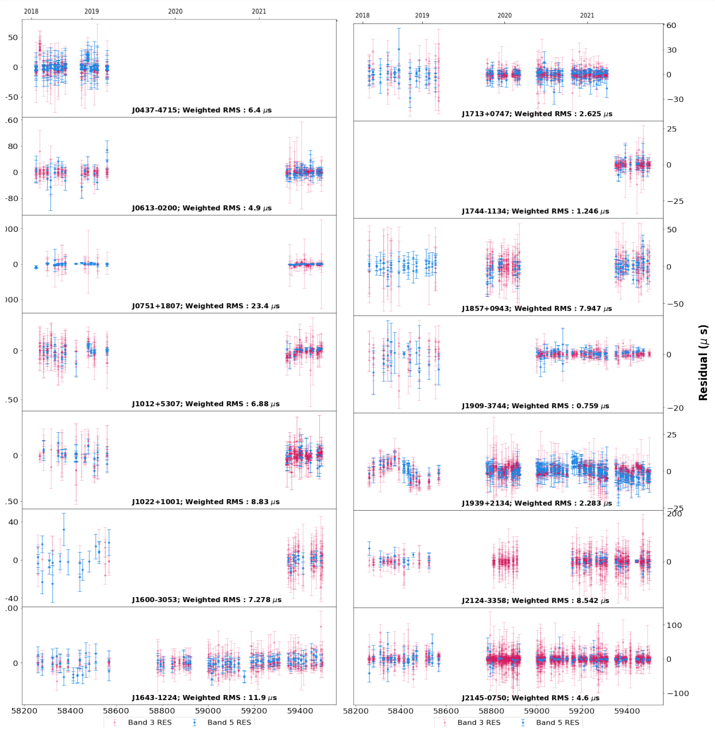
Antoniadis et al. [arXiv:2306.16224]

EPTA



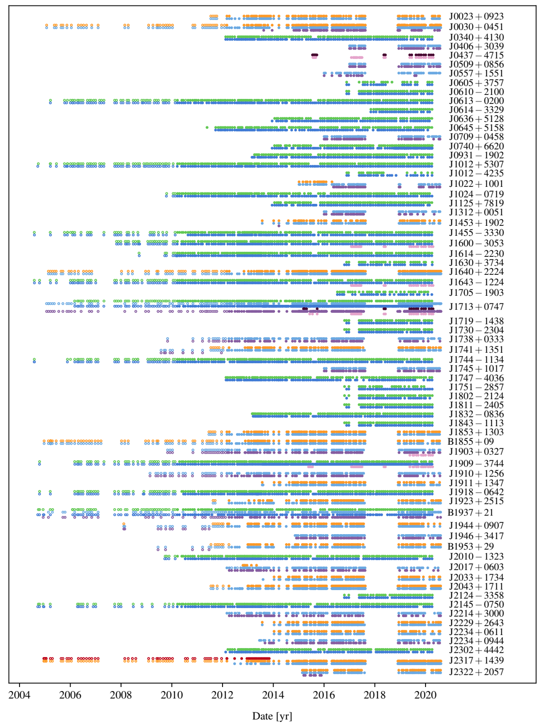
Zic et al. [arXiv:2306.16230]

PPTA



Tarafdar et al. [arXiv:2206.09289]

InPTA



Agazie et al. [arXiv:2306.16217]

NANOGrav

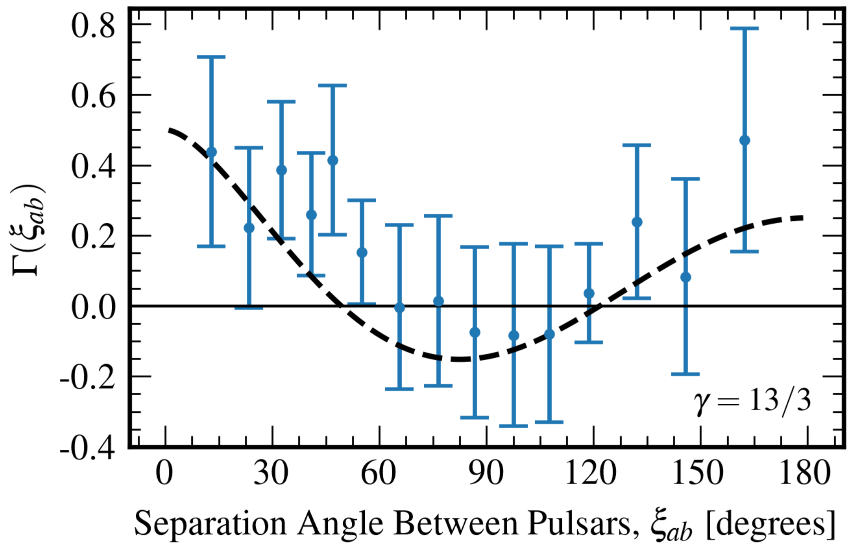
- Pulse frequency variation induced by GWs

$$\frac{\delta\nu}{\nu} = -H^{ij}\left[h_{ij}(t,\vec{x}_\mathrm{d}) - h_{ij}(t-D,\vec{x}_\mathrm{p})\right]$$

>3 sigma evidence for HD in PTA datasets

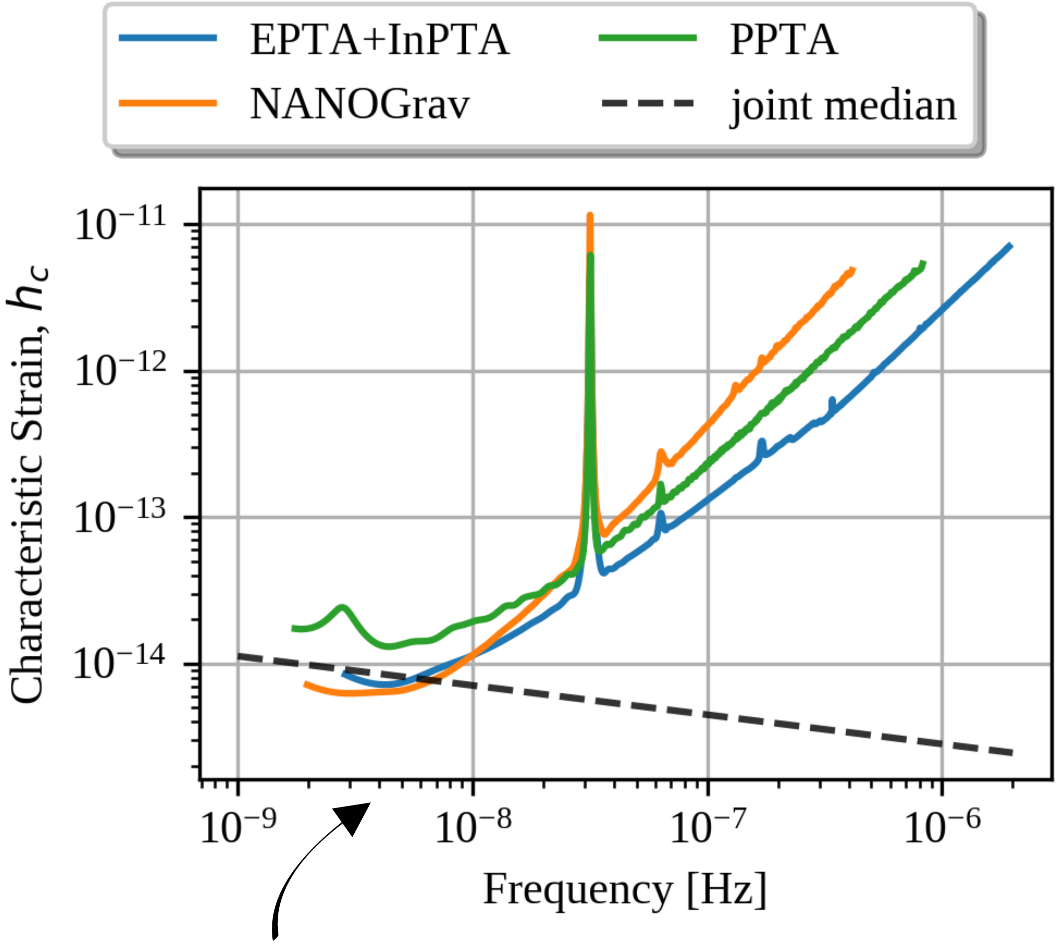
G. Agazie *et al.* [NANOGrav], Astrophys. J. Lett. **951** (2023) no.1, L8 [arXiv:2306.16213]

Correlation between pulsar pairs: Hellings-Downs curve

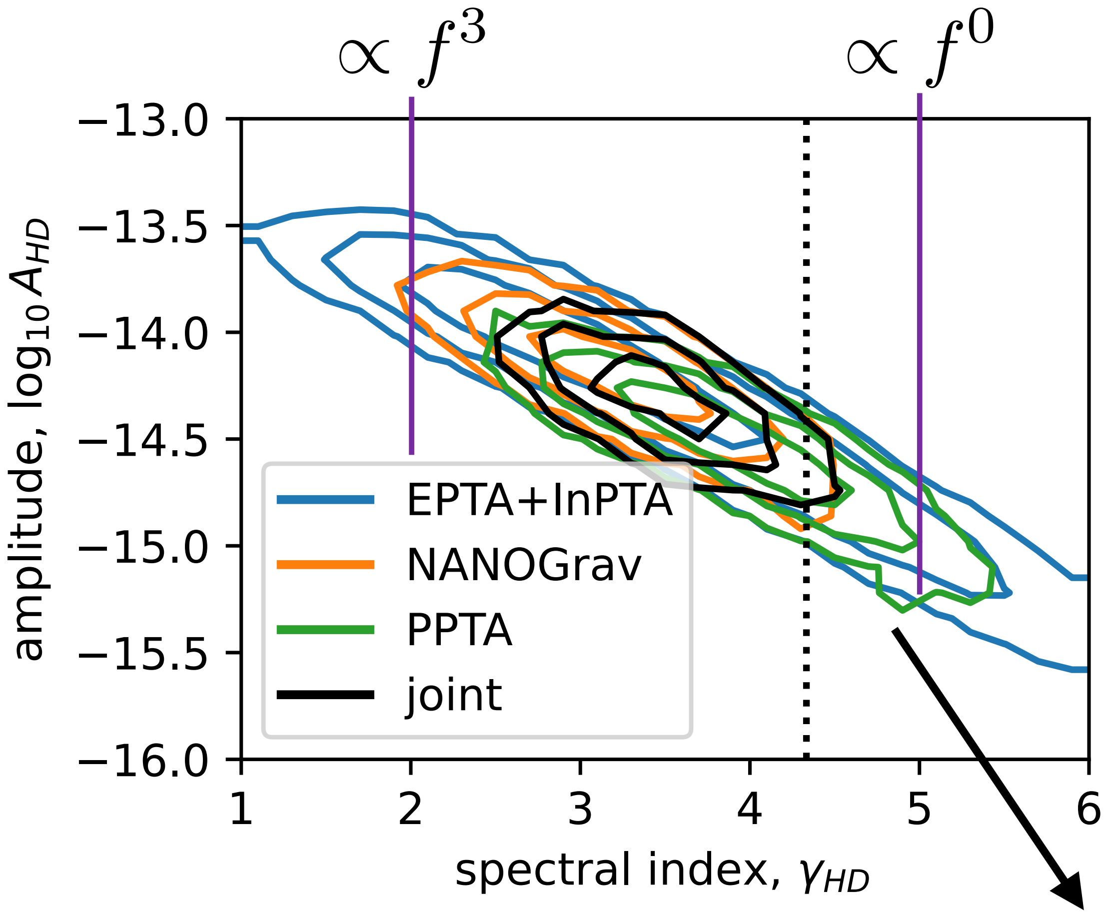


Evidence for HD found most recent datasets

- Results compatible with each other IPTA [arXiv:2309.00693]
- NANOGrav 15 currently giving the most stringent constraint



PTA optimal range



$n_T = 2/3$

(See also Hazboun et al [arXiv:1907.04341])

SGWB spectrum: $\Omega_{\rm GW} \propto (f/f_{\rm yr})^{n_T}$ SGWB tilt: $n_T = 5 - \gamma.$

GWs sources in the nHz frequency range

Astrophysics:

SMBH binaries formed in galaxy mergers

Hubble+Keck observations

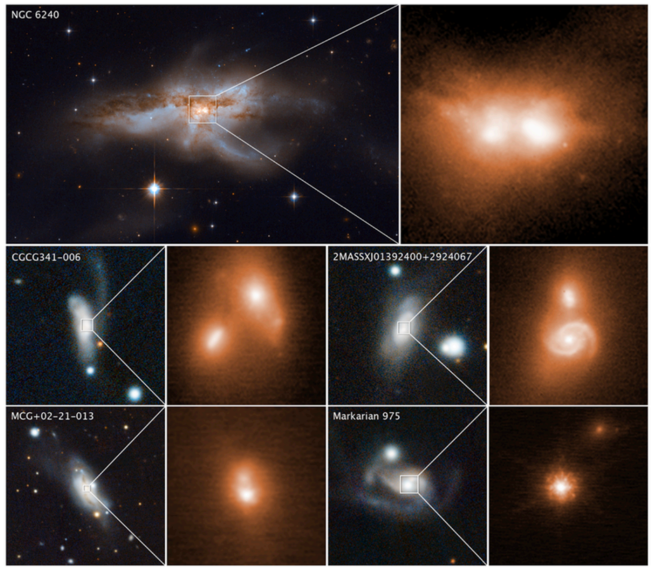
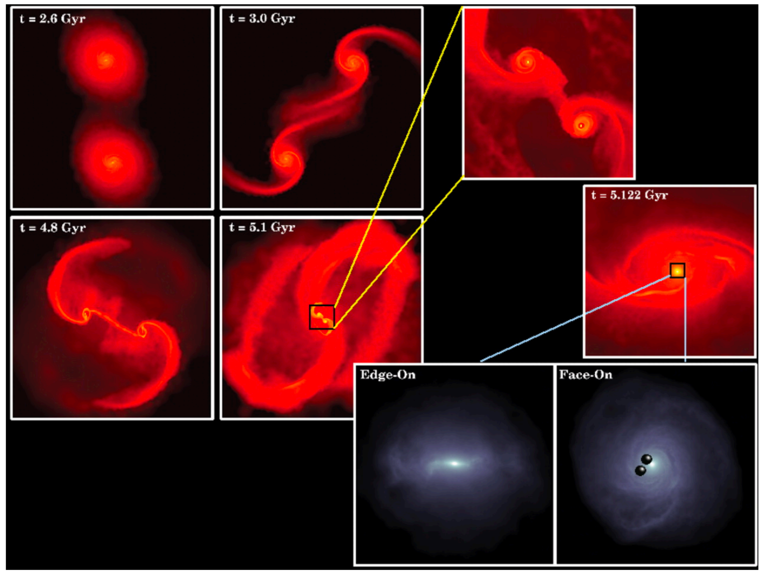


Image: NASA, ESA

Numerical simulations of galaxy mergers

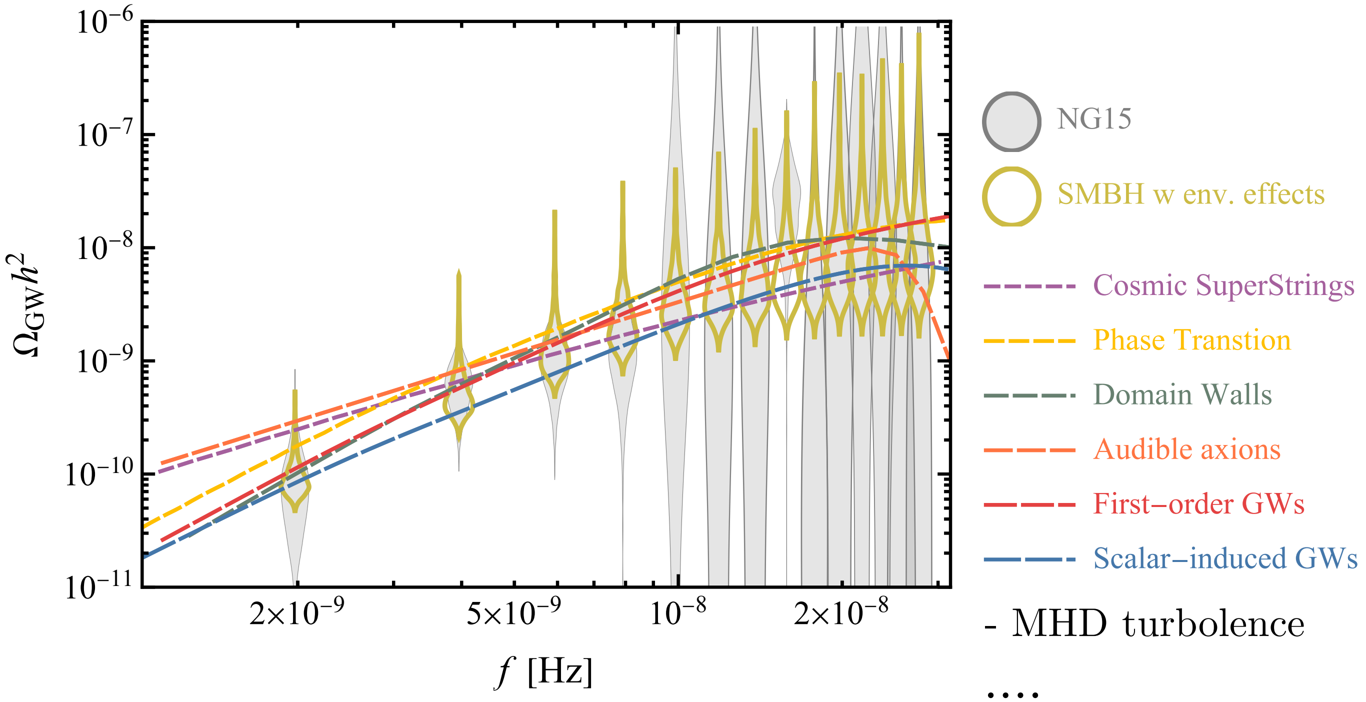


L. Mayer *et al.* Science **316** (2007), 1874-1877 [arXiv:0706.1562]

“New” physics:

GW backgrounds of cosmological origin

J. Ellis, **G.F.** , et al. [arXiv:2308.08546]



- MHD turbolence

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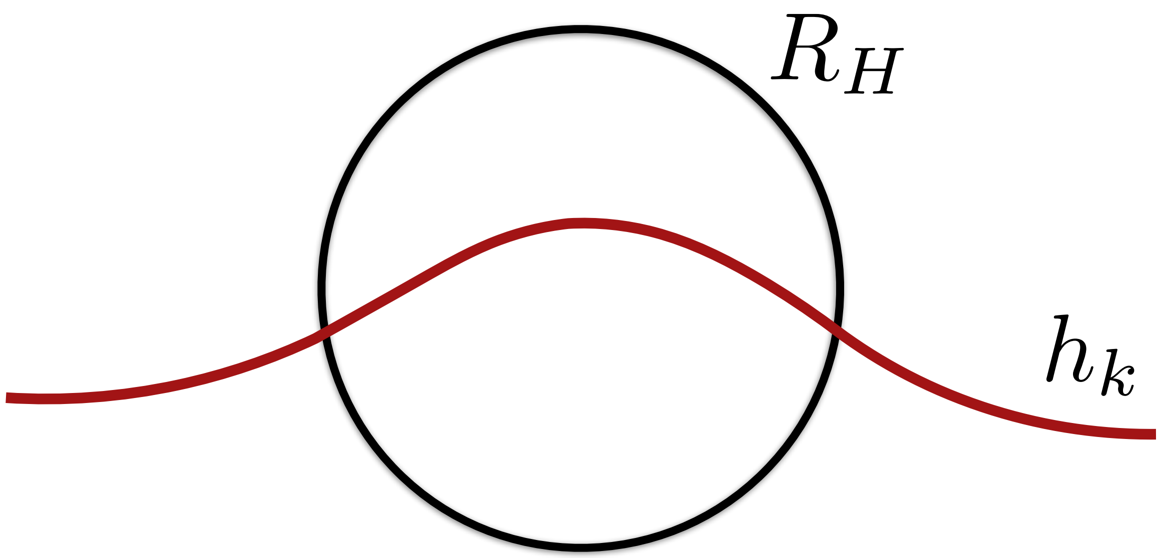
Role of the QCD thermal history

Coincidence of scales: PTA and QCD

- Relevant scale for cosmological GWs: Hubble sphere $R_H = \dfrac{1}{H(T)}$

$$H \sim \frac{T^2}{M_p}$$



GW mode

Frequencies of modes crossing the Hubble sphere $f \; = \; k/2\pi \; = \; aH/2\pi$

$$f \simeq 3.0\,\mathrm{nHz} \cdot \left(\frac{g_{*,\mathrm{s}}(T)}{20}\right)^{1/6} \left(\frac{T}{150\,\mathrm{MeV}}\right)$$

PTA frequency range ⟷ epoch of confinement of QCD

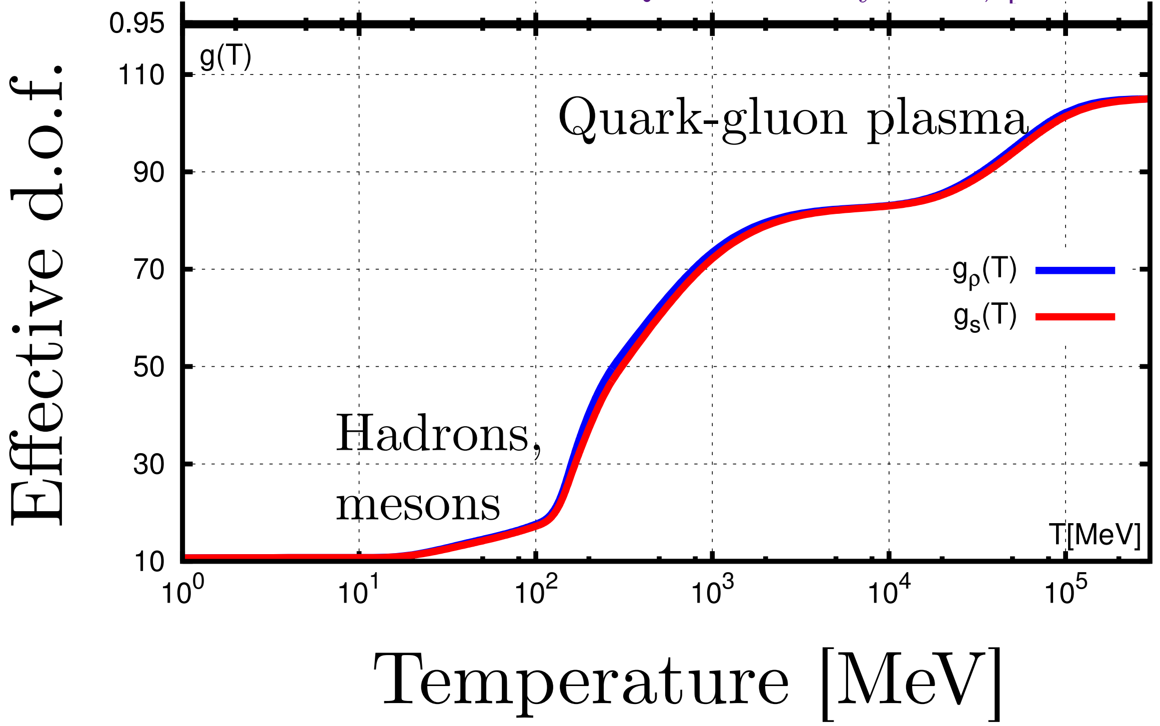
QCD effects on cosmological evolution

QCD crossover affects cosmological evolution

$$s \sim g_{*,s} T^3$$

$$\rho \sim g_* T^4$$

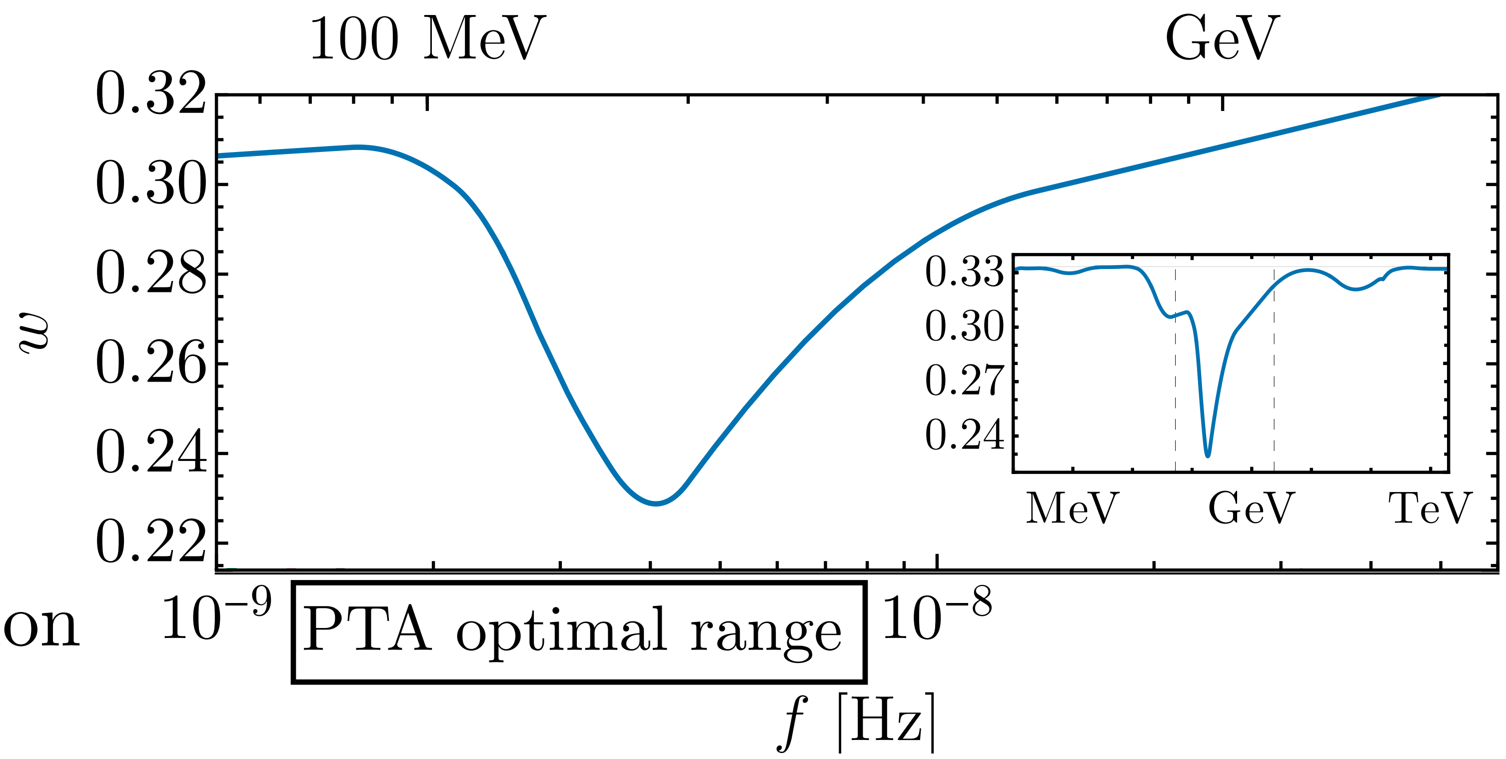
Lattice QCD. Borsanyi et al, [1606.07494]



Evolution of the equation of state of the universe

$$w(T) = \frac{4}{3}(g_{*,s}(T)/g_*(T)) - 1$$

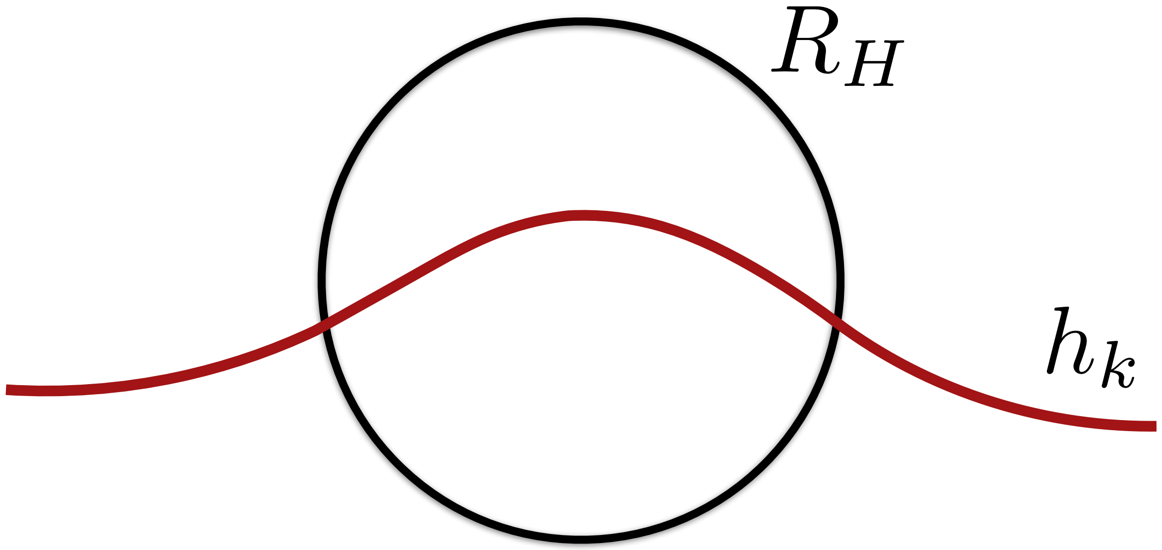
RD: $w = 1/3$



30% deviation from perfect radiation induced by QCD crossover

Class of GW spectra: transient sources

- Model independent effect of QCD thermal history on transient sources
- Source active for timescale τ_*

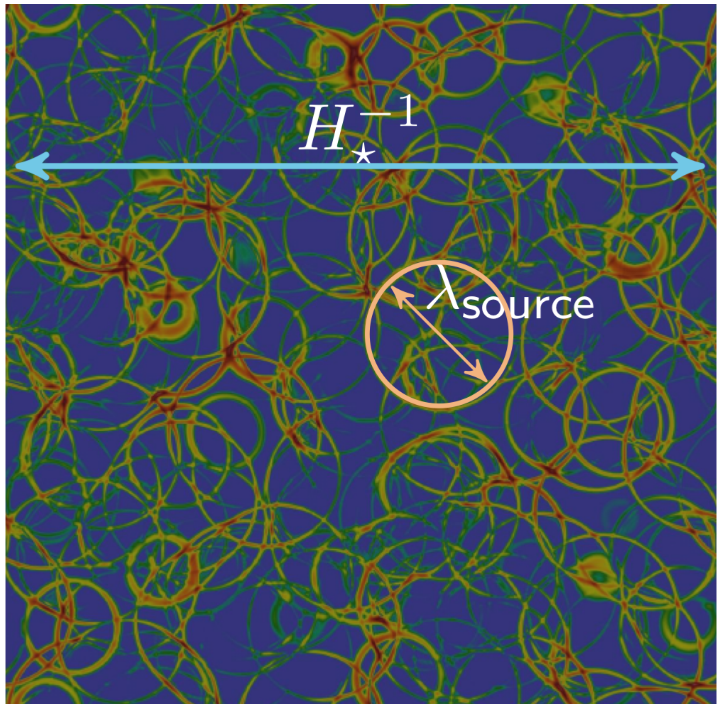


$$\partial_\tau^2 h_{ij} + 2\mathcal{H}\partial_\tau h_{ij} + k^2 h_{ij} = a^2 \frac{32\pi G\rho}{3}\Pi_{ij} \equiv J_{ij}$$

Anisotropic stress

Over longer time-scales: $J(k,\tau') = J_\star(k)\delta(\tau' - \tau_\star)$

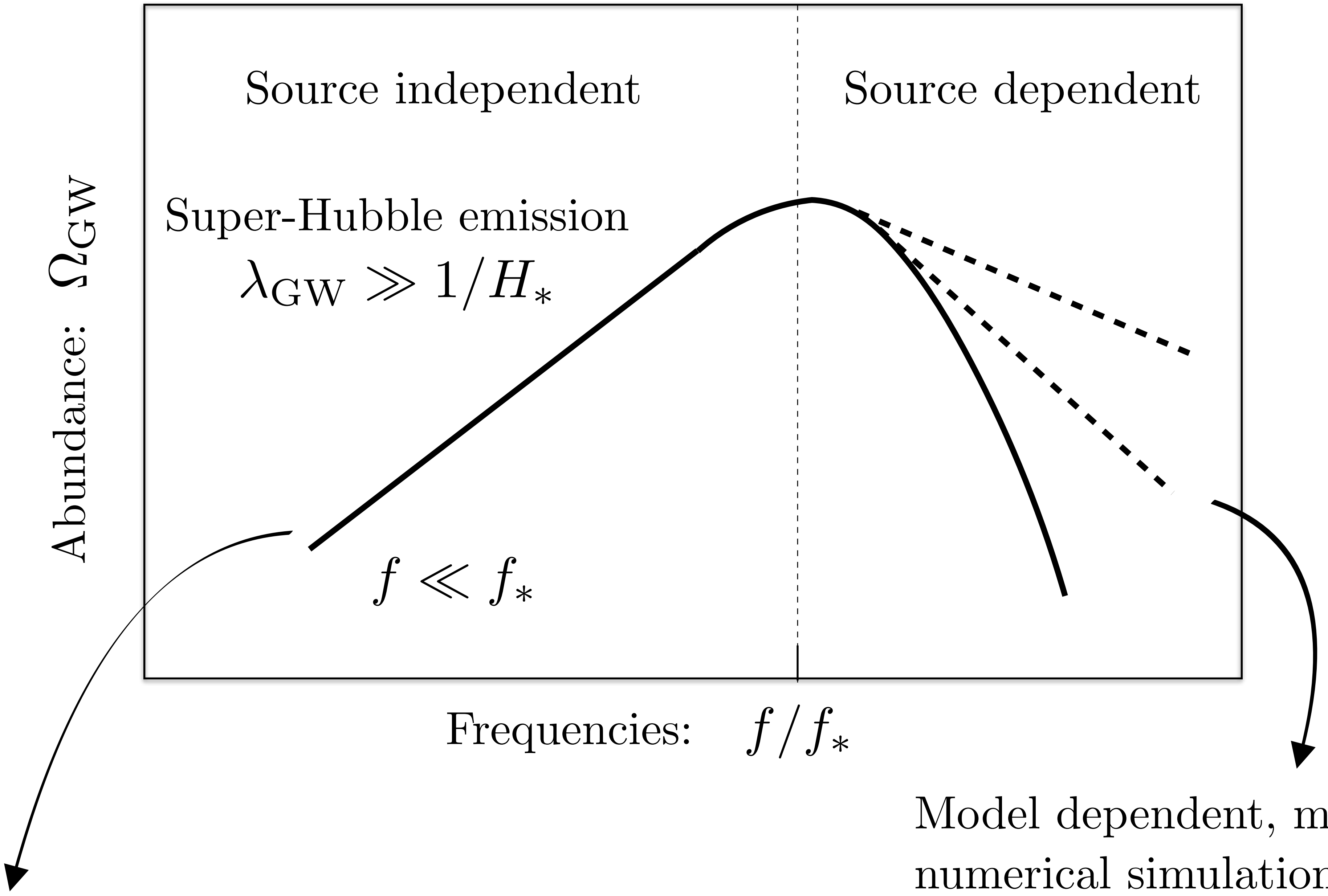
e.g. bubbles collisions from PTs



Example of transient sources:

- First-order cosmological phase transitions
- Second-order GWs with peaked spectrum
- Spectator fields
- ....

GW spectra for transient sources



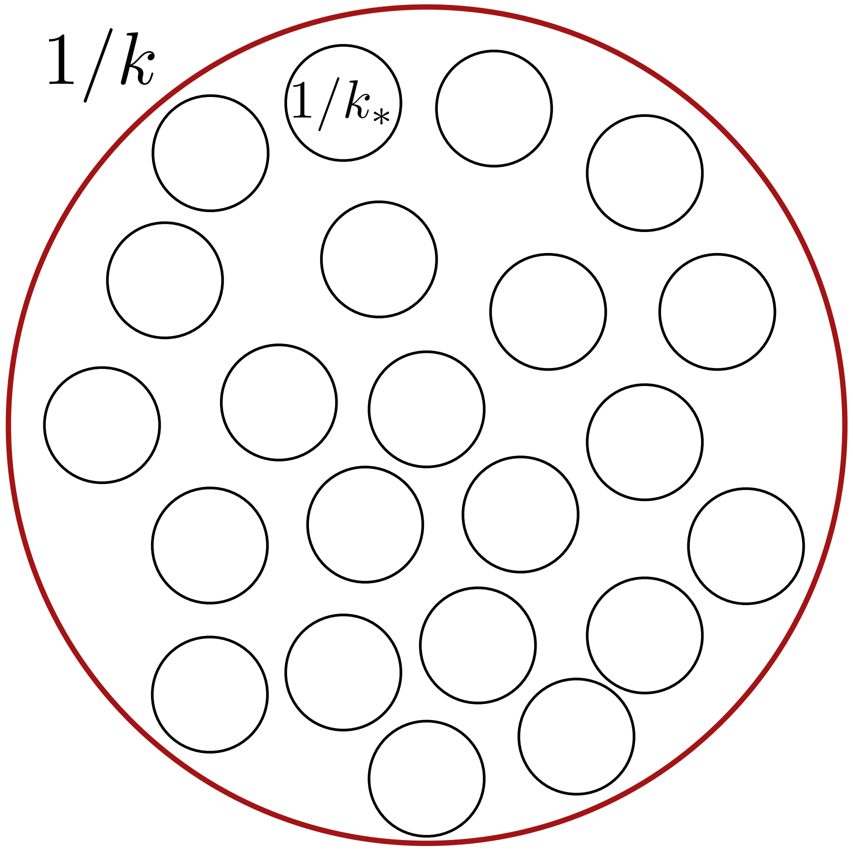
Model dependent, may require numerical simulations, potential degeneracy with other sources

Model independent feature of transient cosmological sources:

“Causality Tail”

[‘03 Seto, Yokoyama; ‘05 Boyle, Steinhardt; ‘06 Watanabe, Komatsu; ‘09 Caprini, Durrer, Konstandin, Servant; ‘18 Caprini, Figueroa; ‘18 Saikawa, Shirai; ‘18 Cui, Lewicki, Morrissey, Wells; ‘19 D’Eramo, Schmitz;...]

Derivation of the causality tail



- On scales: $1/k > 1/k_*$

  one finds $N = (k_*/k)^3$ independent patches

- The GW amplitude over super-Hubble scales is

$$h_k = \sum_i h_{*(i)}/N$$

- The GW two point function:

$$\langle h_k h_k \rangle = N^{-2} \sum_{ij} \langle h_{*(i)} h_{*(j)} \rangle = (k/k_*)^3 |h_*|^2$$

- GW dynamics: constant on super-Hubble scales (over-damped), then decays as $1/a$:

$$\rho_{\rm GW} \sim \langle \dot{h}_k^2 \rangle \sim (k/a)^2 \langle h_k^2 \rangle$$

$$\Omega_{\rm GW} \sim \rho_{\rm GW} \sim \begin{cases} k^3 & \text{Radiation Domination} \\ k & \text{Matter Domination} \end{cases} \quad \text{(super-Hubble scales)}$$

QCD crossover affects the causality tail in the PTA band

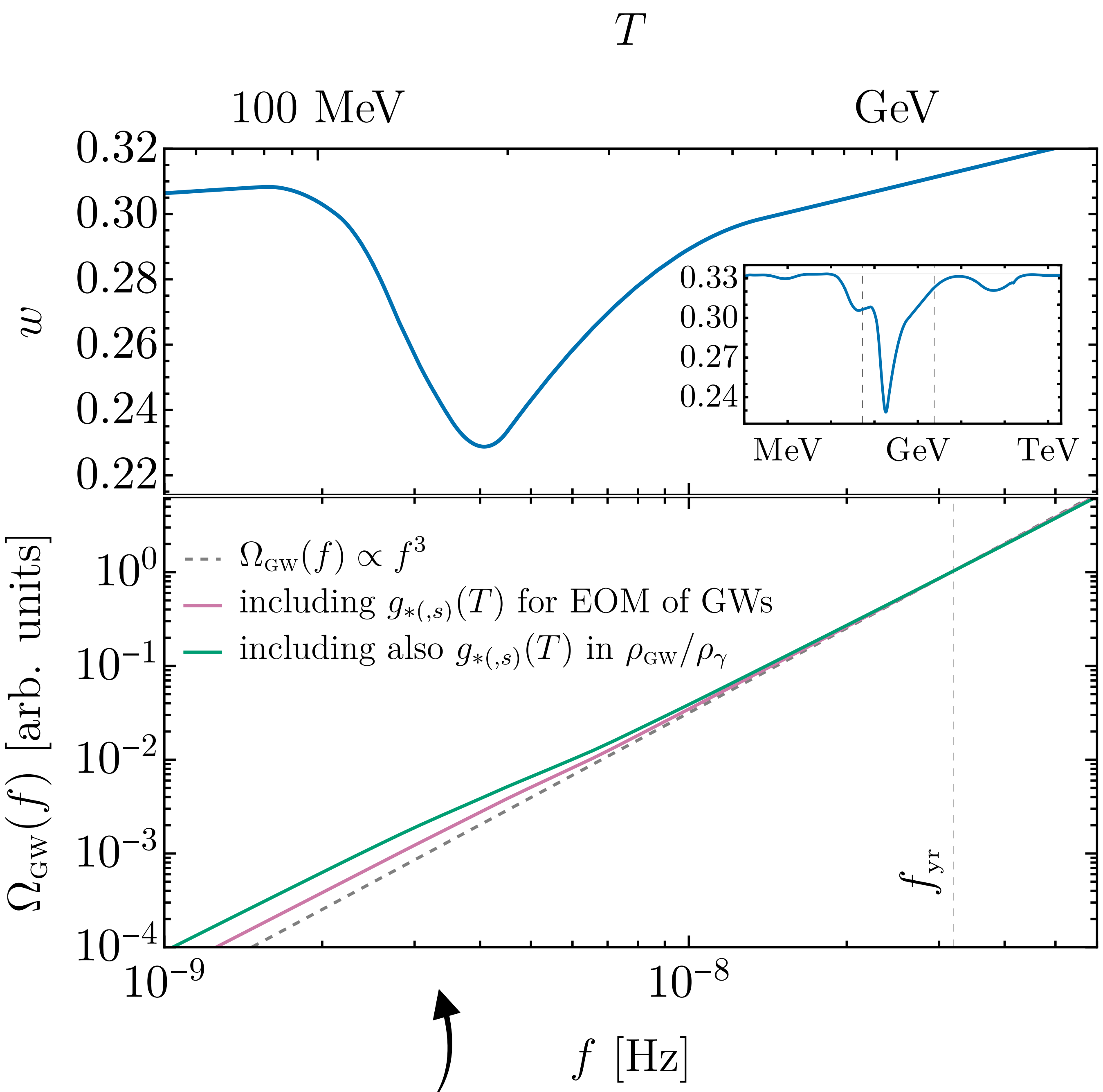
G.Franciolini, D.Racco and F.Rompineve, **PRL** [arXiv:2306.17136]

Change of background equation of state

$$\frac{d\Omega_{\rm GW}}{d\ln f} = \underbrace{\frac{1}{\rho_c}}_{\propto\, g_{*,s}^{-4/3} g_*} \times \overbrace{\frac{d\rho_{\rm GW}}{d\ln f}}^{\propto\, f^{3+2\frac{3w-1}{3w+1}}}$$

entropy injection to SM bath only

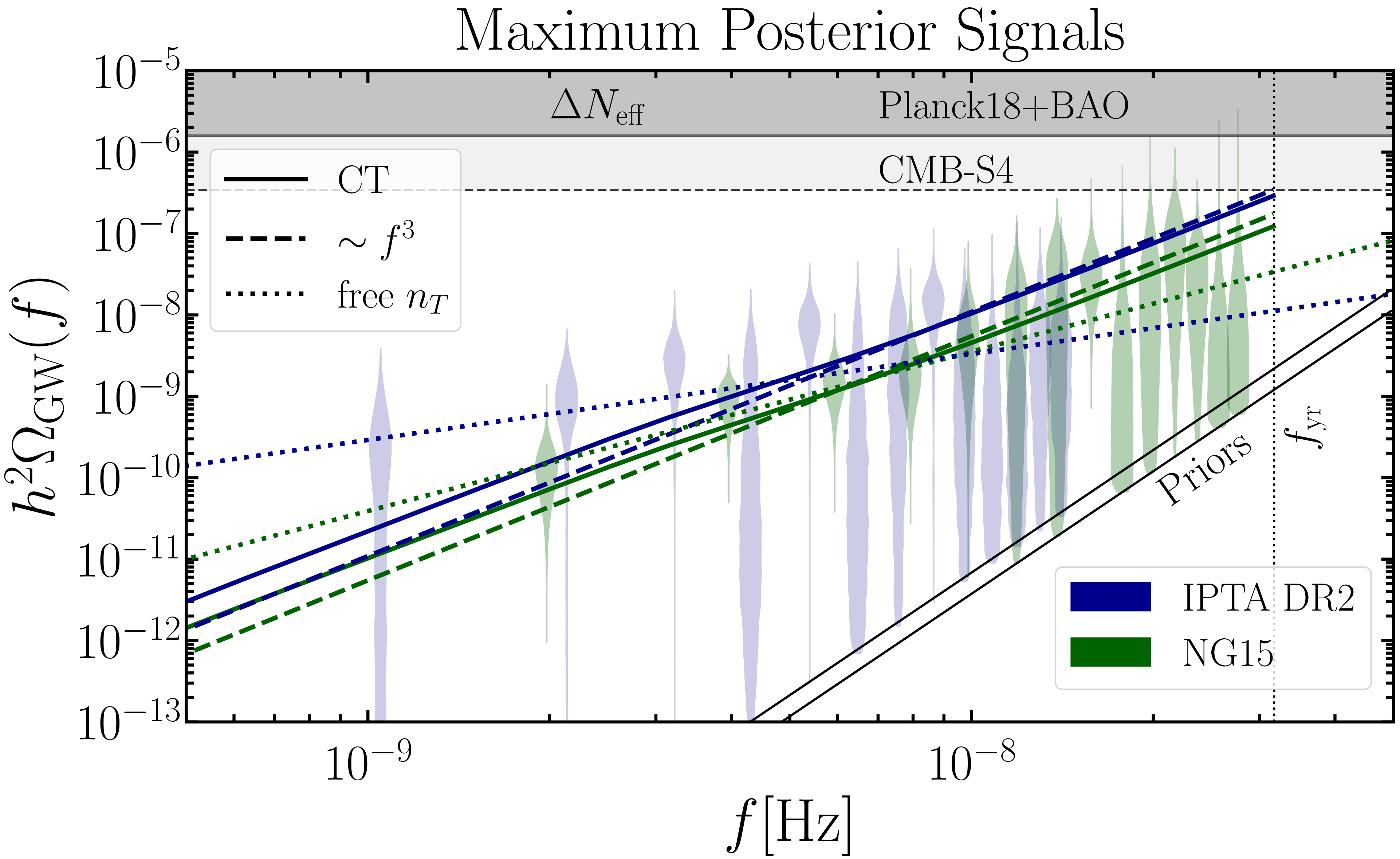
Relevant modulation for all early-universe sources of GWs active before QCD crossover.



PTA optimal range

QCD crossover affects the causality tail in the PTA band

G.Franciolini, D.Racco and F.Rompineve, **PRL** [arXiv:2306.17136]



- Effect already relevant for signal interpretation

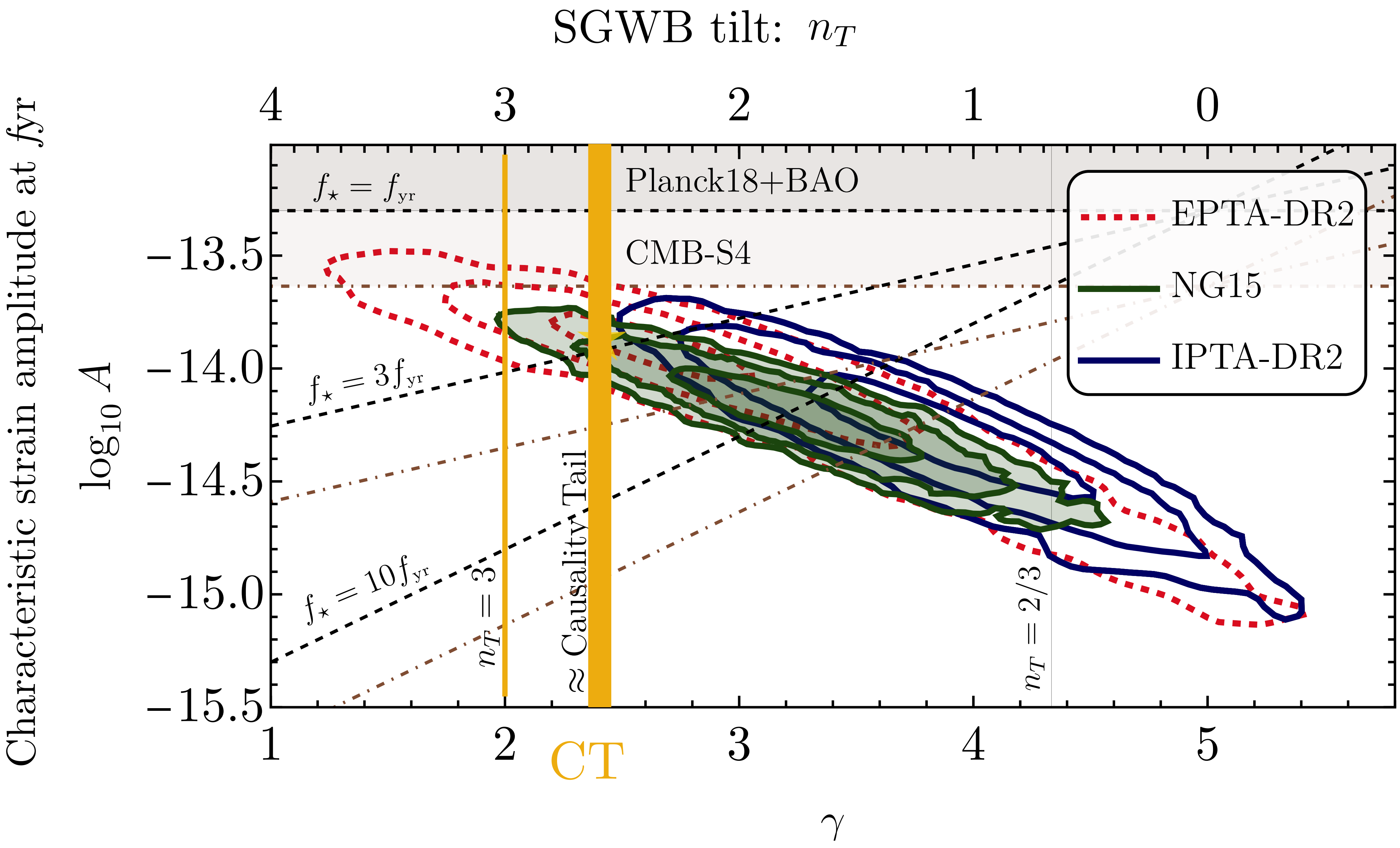
$$\log_{10} \mathcal{B}^{\rm CT}_{f^3} = 1.6$$

NG15 data

Significant difference in evidence between approximated cubic and actual causality tail

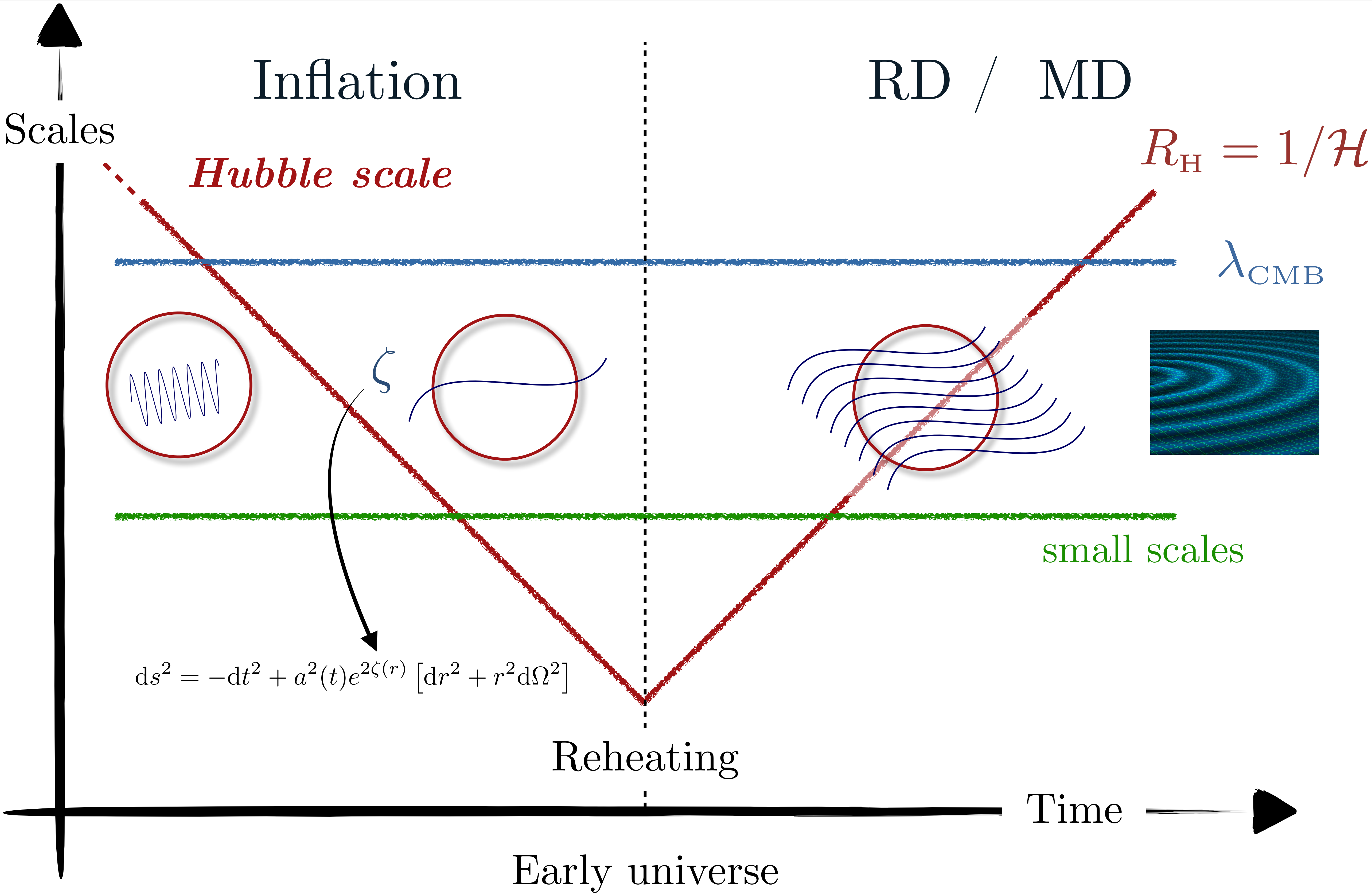
QCD crossover affects the causality tail in the PTA band

G.Franciolini, D.Racco and F.Rompineve, **PRL** [arXiv:2306.17136]



Second-order induced GWs

Induced GWs at second-order

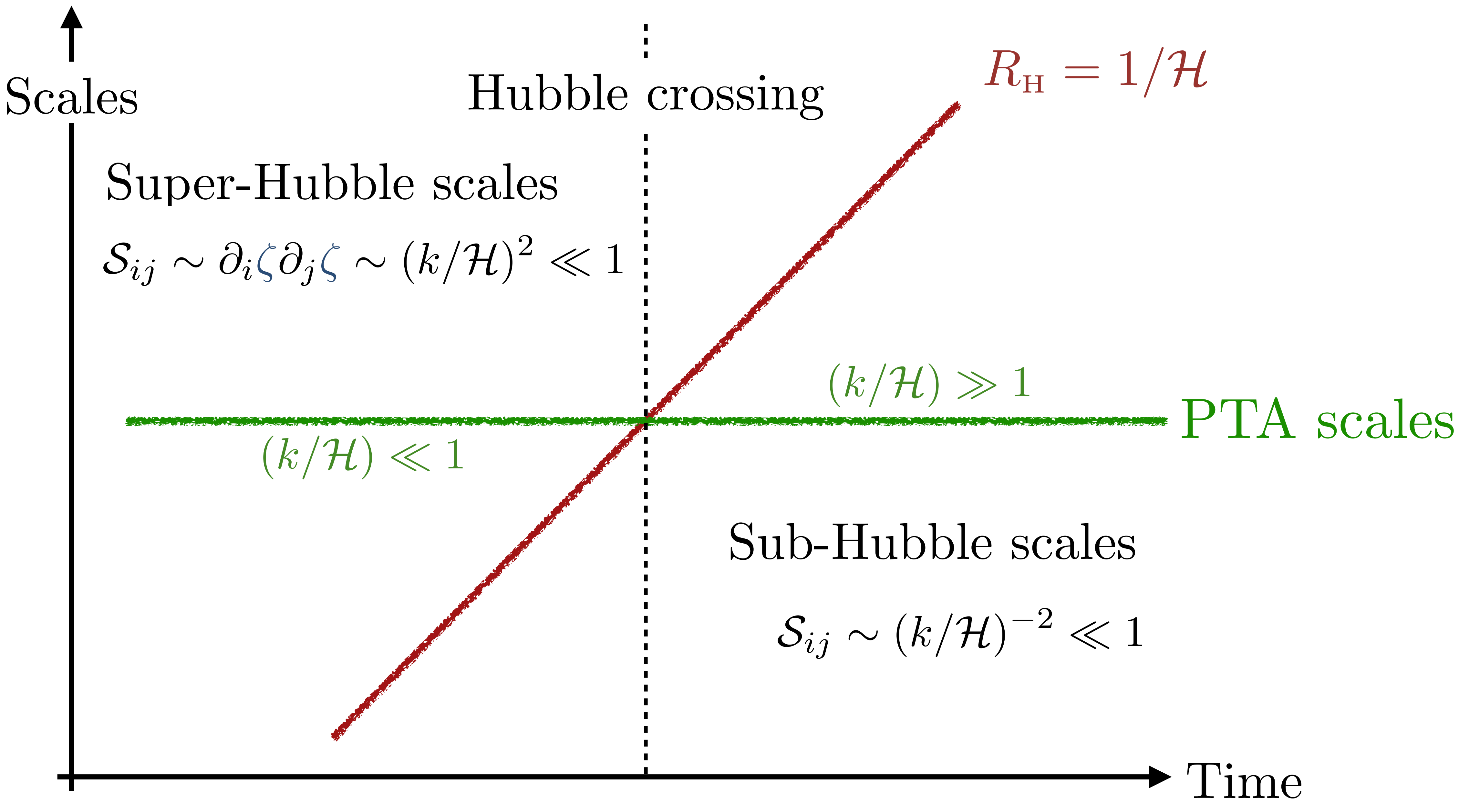


$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)e^{2\zeta(r)}\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right]$$

Emission of induced GWs at second-order

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \approx \mathcal{S}_{ij}\left(\zeta\zeta\right)$$

K. Tomita, Prog. Theor. Phys. 54, 730 (1975).
S. Matarrese, O. Pantano, and D. Saez, Phys. Rev. Lett. 72, 320 (1994), [arXiv:9310036].
V. Acquaviva, *et al.* Nucl. Phys. B 667, 119 (2003), [arXiv:0209156].
S. Mollerach, D. Harari, and S. Matarrese, Phys. Rev. D 69, 063002 (2004), [arXiv:0310711].
K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev. D 75, 123518 (2007), [arXiv:0612013].
...



Scales

Hubble crossing

$R_{\rm H} = 1/\mathcal{H}$

Super-Hubble scales

$\mathcal{S}_{ij} \sim \partial_i\zeta\partial_j\zeta \sim (k/\mathcal{H})^2 \ll 1$

$(k/\mathcal{H}) \gg 1$

PTA scales

$(k/\mathcal{H}) \ll 1$

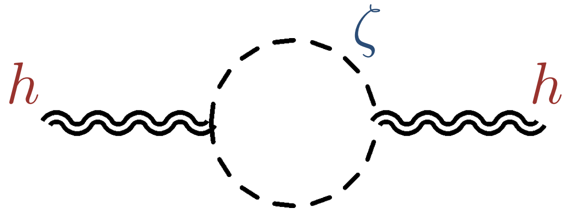
Sub-Hubble scales

$\mathcal{S}_{ij} \sim (k/\mathcal{H})^{-2} \ll 1$

Time

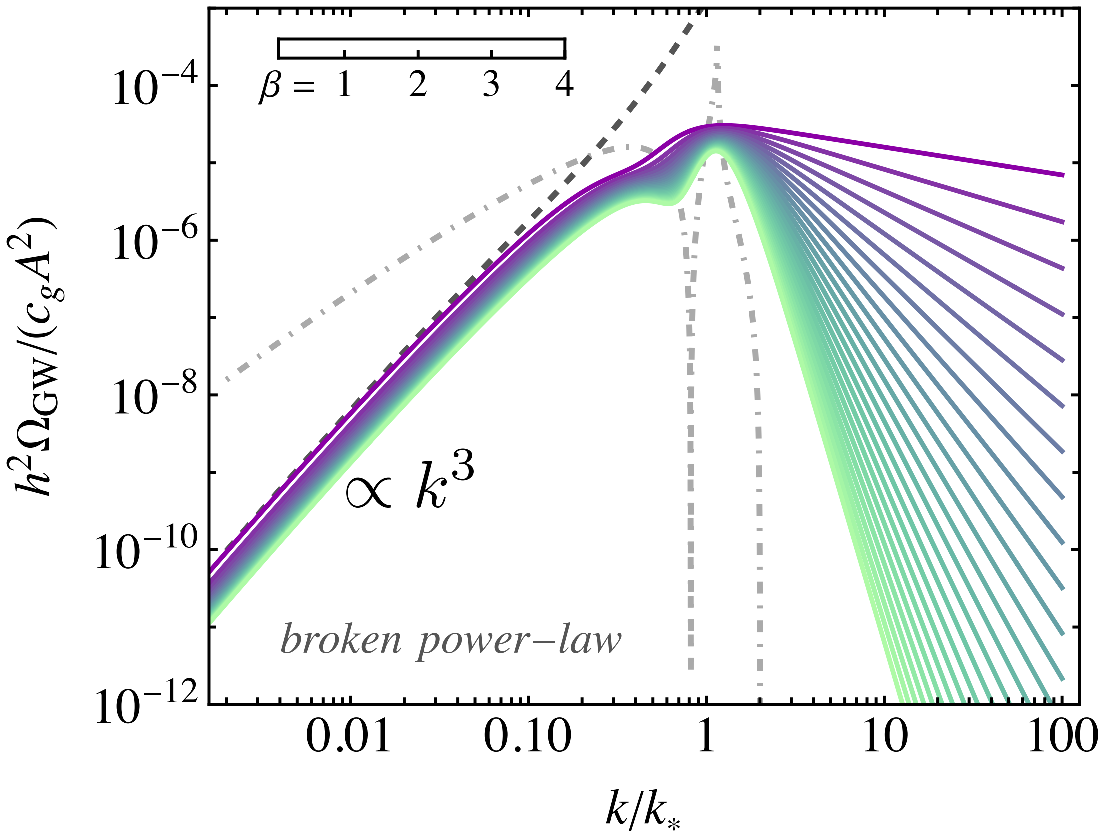
Characterization of the spectrum

At second-order in comoving curvature perturbation, after averaging over the fast oscillating pieces:



$$\Omega_{\rm GW}\left(\eta,k\right)=\frac{\pi^2}{243\mathcal{H}^2\eta^2}\int\frac{d^3p}{(2\pi)^3}\,\frac{p^4\left[1-\mu^2\right]^2}{p^3\left|\vec{k}-\vec{p}\right|^3}\boxed{\mathcal{P}_\zeta\left(p\right)\mathcal{P}_\zeta\left(\left|\vec{k}-\vec{p}\right|\right)}\,\boxed{\mathcal{I}^2\left(\vec{k},\,\vec{p}\right)}$$

- Double peak feature for narrow spectra
- Universal IR slope (super-Hubble modes)
- UV model dependent: $\propto\mathcal{P}_\zeta^2(k\gg k_*)$

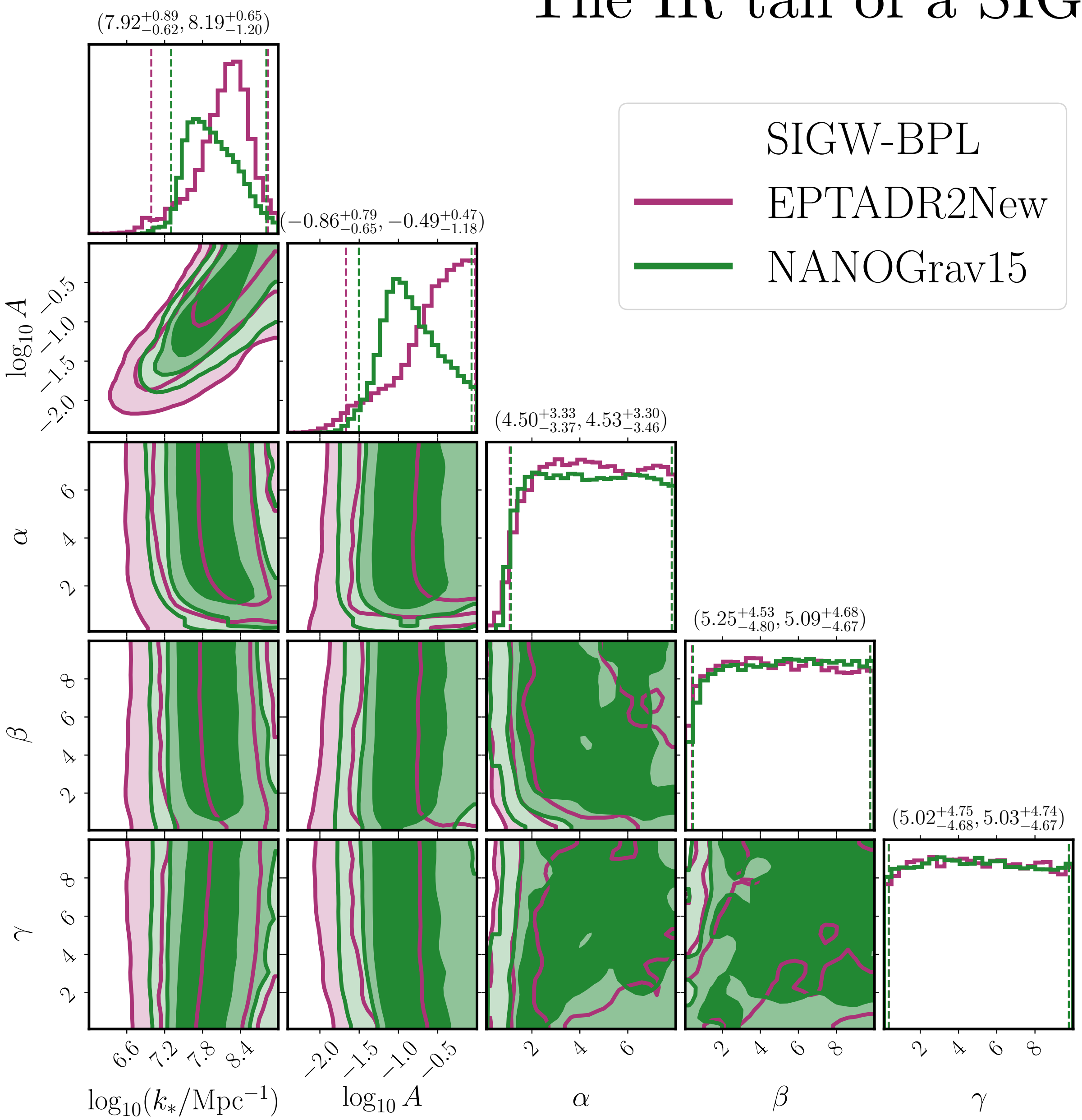


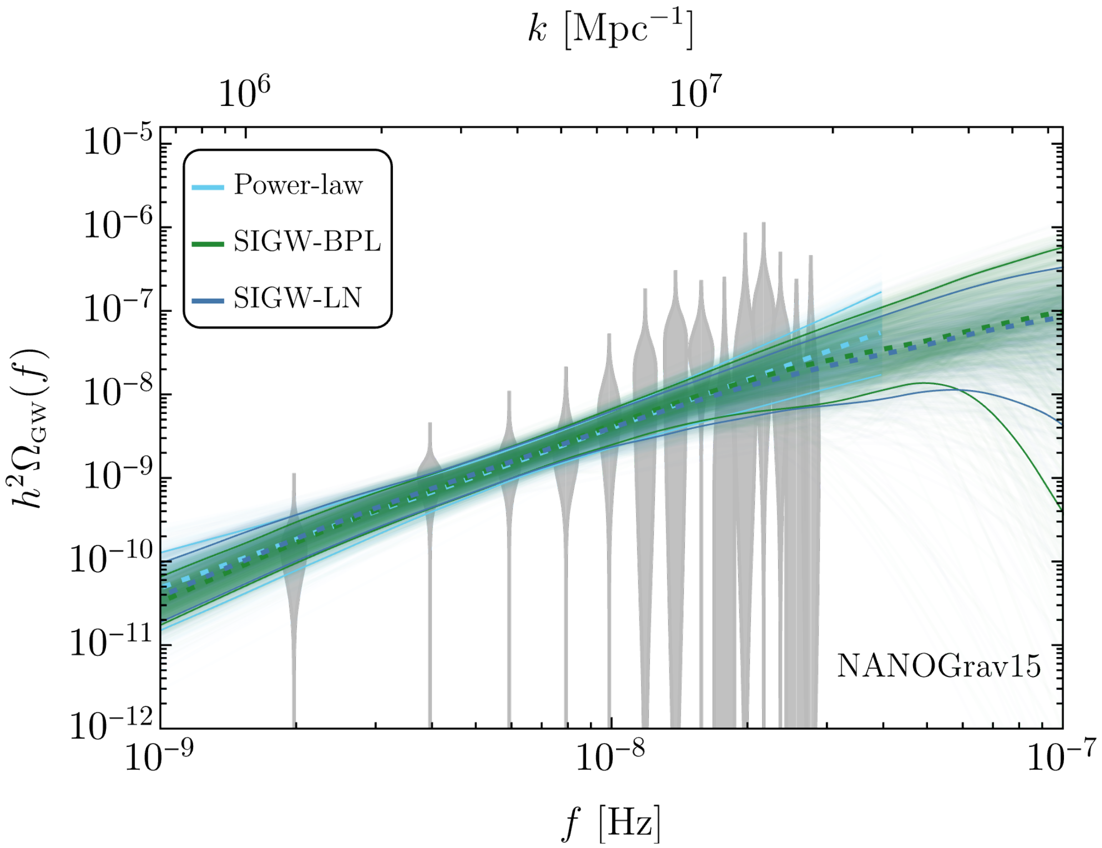
SIGW can fit PTA data well

Assuming BPL curvature spectrum:

$$\mathcal{P}_\zeta^{\mathrm{BPL}}(k) = A\frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma}+\alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$

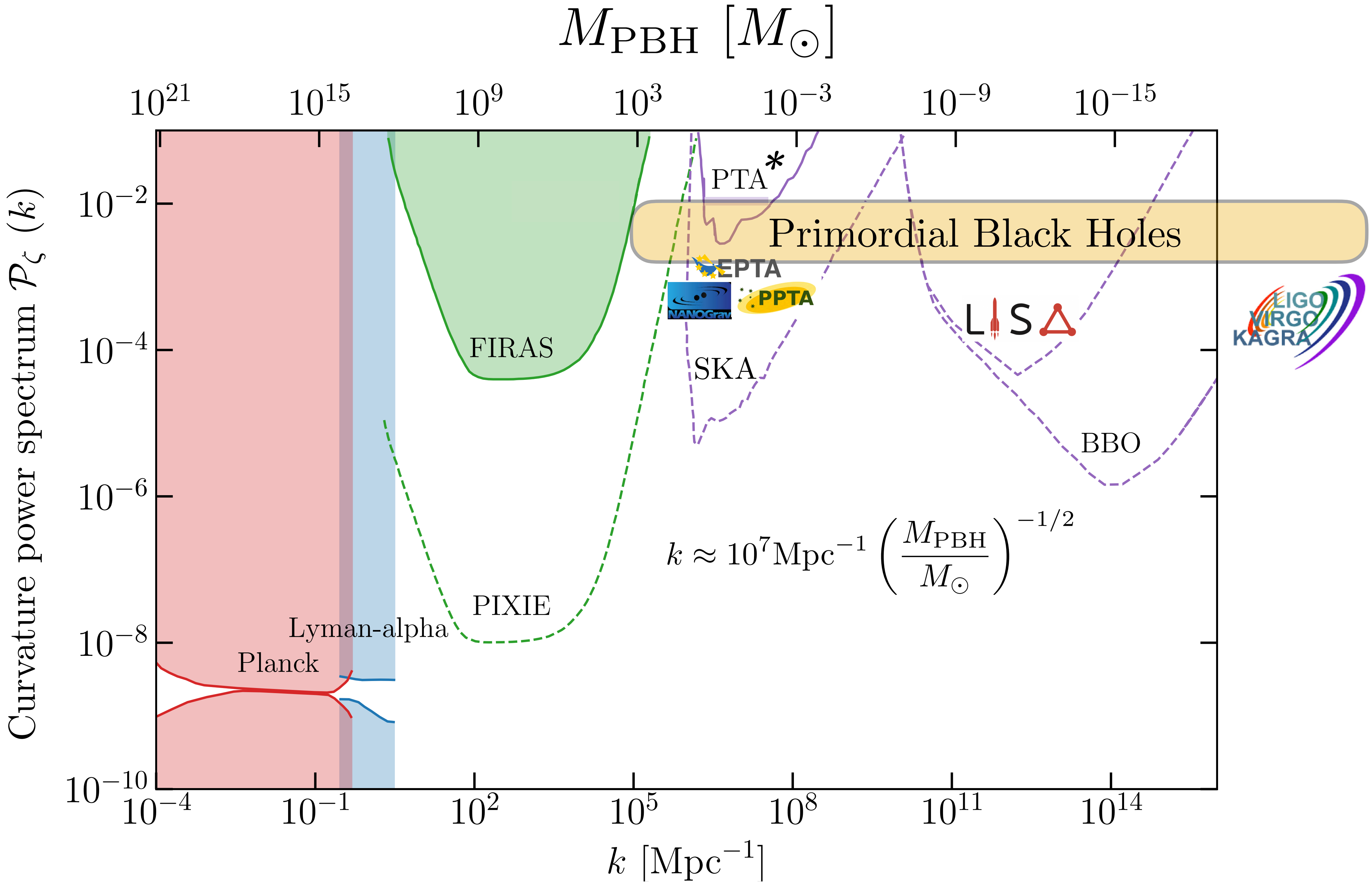
The IR tail of a SIGW scenario can fit well the data



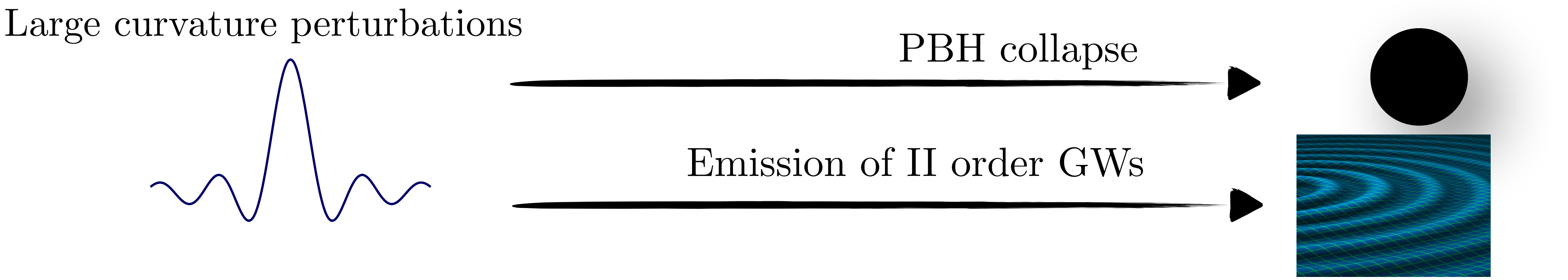


Large curvature perturbations are required

$$\boxed{\Omega_{\rm GW} \propto \Omega_{r,0} \times \zeta^4}$$



Large curvature perturbations: primordial black hole formation

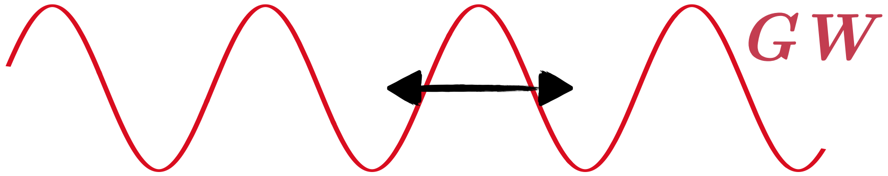


Large curvature perturbations

PBH collapse

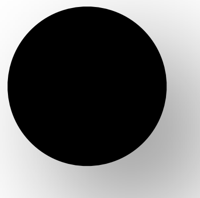
Emission of II order GWs

Mass and frequency related by the Hubble scale at formation



$$f_{\rm GW} \approx 3 \cdot 10^{-9} {\rm Hz} \left(\frac{m_{\rm PBH}}{M_\odot} \right)^{-1/2}$$

PBH



R.Saito and J.Yokoyama, Phys. Rev. Lett. **102** (2009), 161101 [arXiv:0812.4339]

J. Garcia-Bellido, M. Peloso and C. Unal, JCAP **09** (2017), 013 [arXiv:1707.02441]

N. Bartolo, *et al*, Phys. Rev. Lett. **122** (2019) no.21, 211301 [arXiv:1810.12218]

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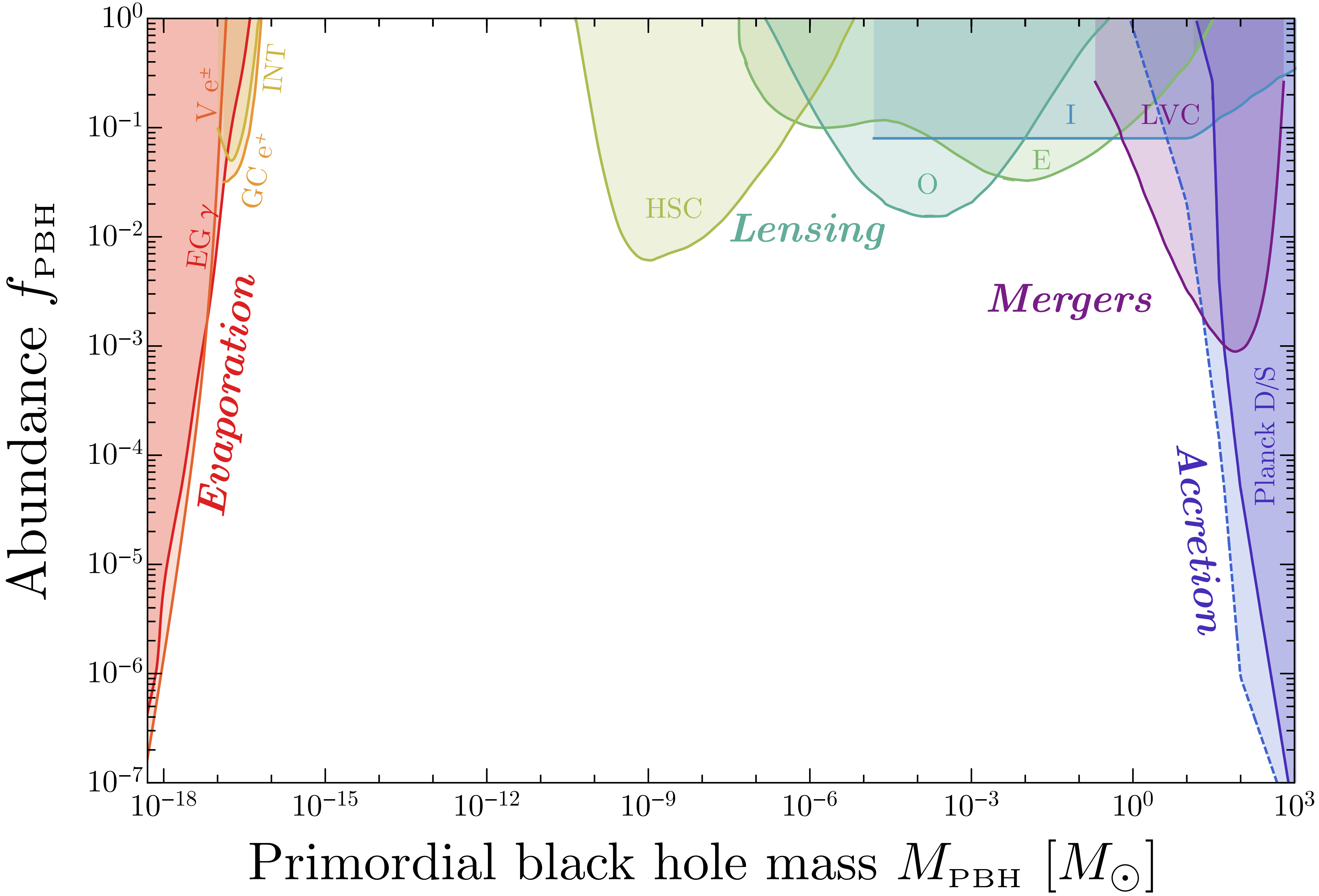
Complementary constraints from PBH overproduction

Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]



Detailed computation of the PBH abundance

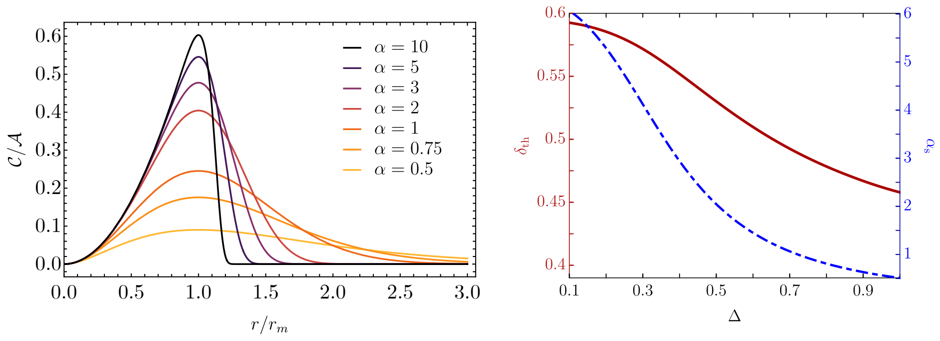
M. Shibata and M. Sasaki, Phys. Rev. D **60**, 084002 (1999) [arXiv:gr-qc/9905064]
T. Harada, C. M. Yoo and K. Kohri, Phys. Rev. D **88**, no.8, 084051 (2013) [arXiv:1309.4201]
C. Germani and I. Musco, Phys. Rev. Lett. **122**, no.14, 141302 (2019) [arXiv:1805.04087]
...

Formation criterion: $\delta \geq \delta_c \ (\approx 0.5)$

Threshold for collapse sensitive to the following:

- Shape of the collapsing peak (controlled by curvature spectrum)

  I. Musco, V. De Luca, G. Franciolini and A. Riotto, Phys.Rev.D **103** (2021) no.6, 063538 [arXiv:2011.03014]



- QCD era: softening of the equation of state

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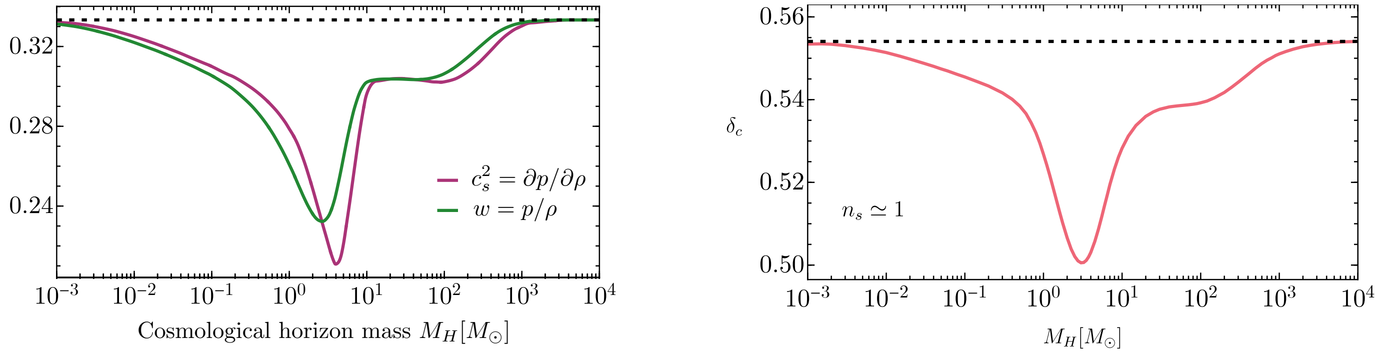
  I. Musco, K. Jedamzik, and S. Young [arXiv:2303.07980]



- Non-linearities: $$\delta(\vec{x},t) = -\frac{8}{9a^2H^2}e^{-5\zeta(\vec{x})/2}\nabla^2 e^{\zeta(\vec{x})/2} + \dots$$

S. Young, I. Musco and C. T. Byrnes, JCAP **11** (2019), 012 [arXiv:1904.00984]
V.De Luca, G.Franciolini, A.Kehagias, M.Peloso, A.Riotto and C.Ünal, JCAP **07** (2019), 048 [arXiv:1904.00970]
....

- **Primordial non-Gaussianities:** e.g. different models predict local-type NGs $\zeta = F(\zeta_{\rm G})$

Curvaton model: $\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$

USR models: $\zeta = -\left(\frac{6}{5}f_{\rm NL}\right)^{-1}\log\left(1 - \frac{6}{5}f_{\rm NL}\zeta_{\rm G}\right)$

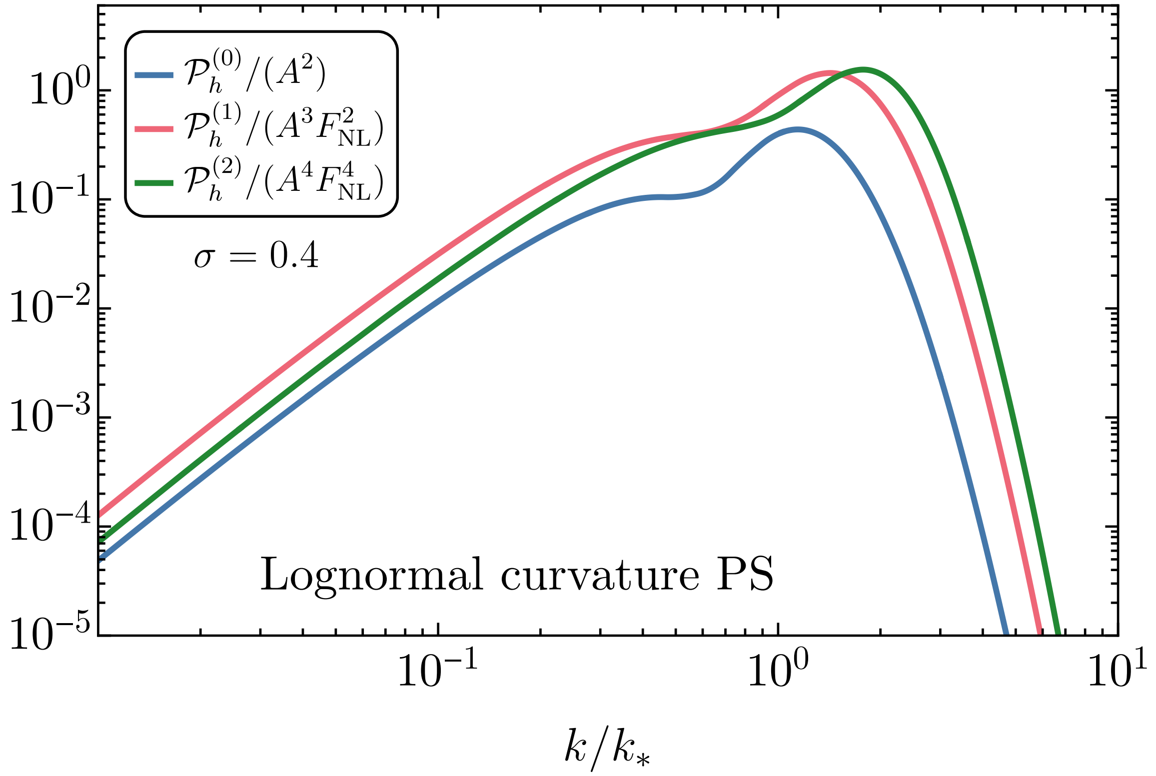
See talk by I. Musco on Friday

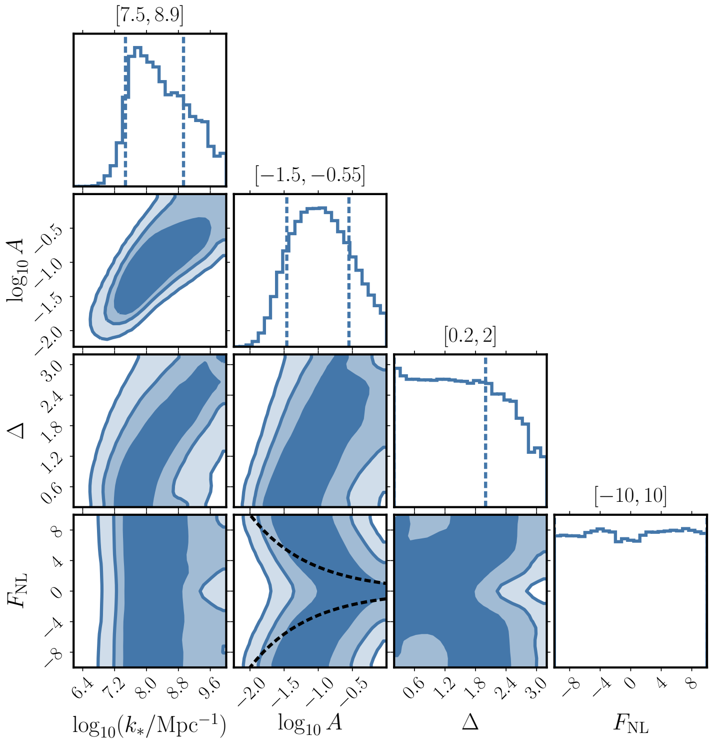
Non-Gaussian effects on the SGWB

P.Adshead, K.D.Lozanov and Z.J.Weiner, JCAP 10 (2021), 080 [arXiv:2105.01659]

$$\zeta = \zeta_{\rm G} + F_{\rm NL}\zeta_{\rm G}^2 \qquad \Omega_{\rm GW}(T) = \underset{A^2}{\Omega^{(0)}_{\rm GW}(T)} + \underset{A^3 F_{\rm NL}^2}{\Omega^{(1)}_{\rm GW}(T)} + \underset{A^4 F_{\rm NL}^4}{\Omega^{(2)}_{\rm GW}(T)}$$

J. Ellis, **G.F.** , et al. [arXiv:2308.08546]

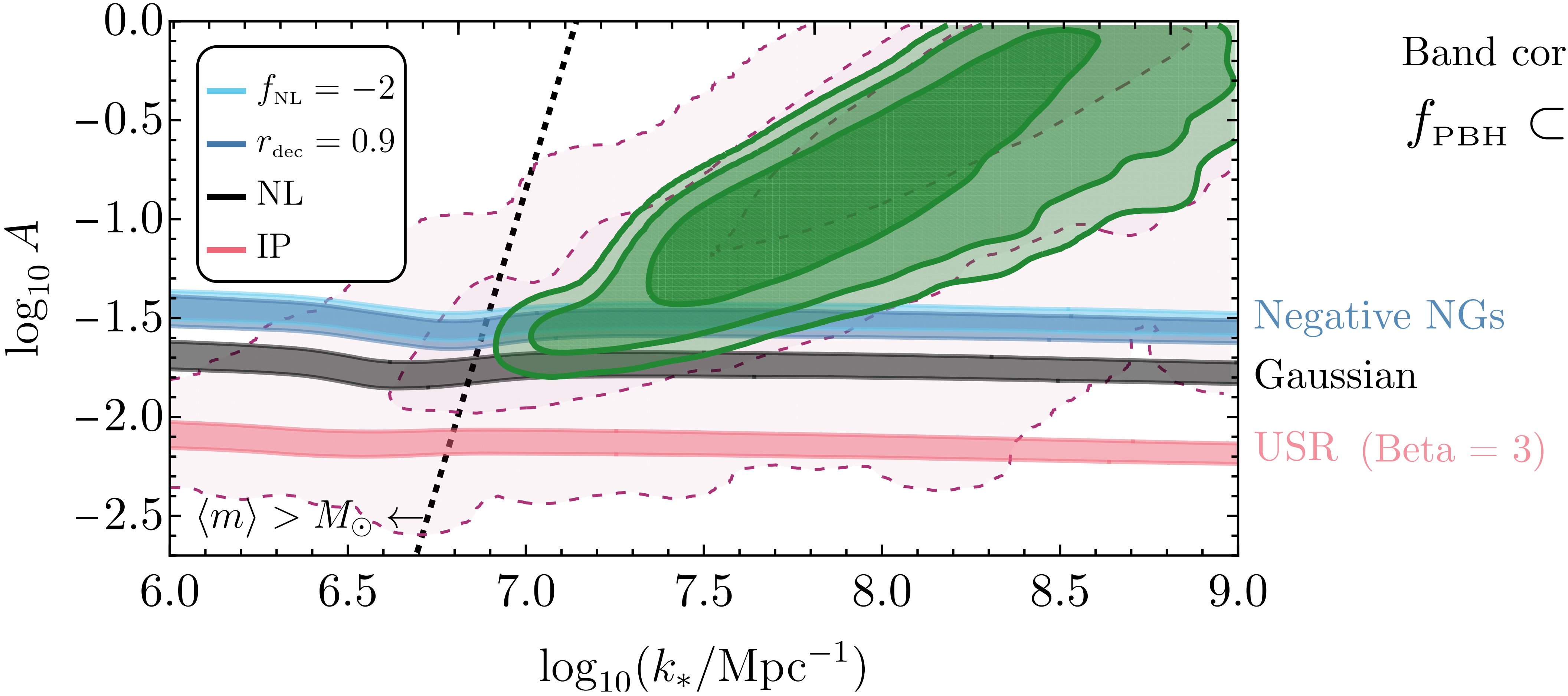




- No additional features generated by NGs in the IR $\quad \Omega_{\rm GW}(k \ll k_*) \propto k^3(1 + \tilde{A}\ln^2(k/\tilde{k}))\,,$
- A-f$_{\rm NL}$ degeneracy in the posterior distribution
- Expansion parameter $(A \times F_{\rm NL}^2)$ small in realistic models considered

NGs can alleviate PBH overproduction bounds

G. Franciolini, A. Iovino, Junior., V. Vaskonen and H. Veermae, PRL [arXiv:2306.17149]



Band corresponds to

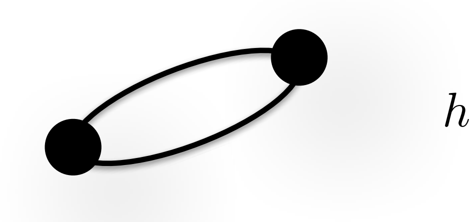
$f_{\rm PBH} \subset [10^{-3}, 1]$

- SIGWs amplitude constrained by PBH overproduction, possible tension if it is the **only** source
- One needs scenarios in which the PBH abundance is suppressed
- On the other hand, current observations do not constrain PBHs in the (sub-)solar mass range

* beware of remaining uncertainties on PBH formation (De Luca et al [arXiv: 2307.13633])

SMBHs vs CGWs I: Poisson noise

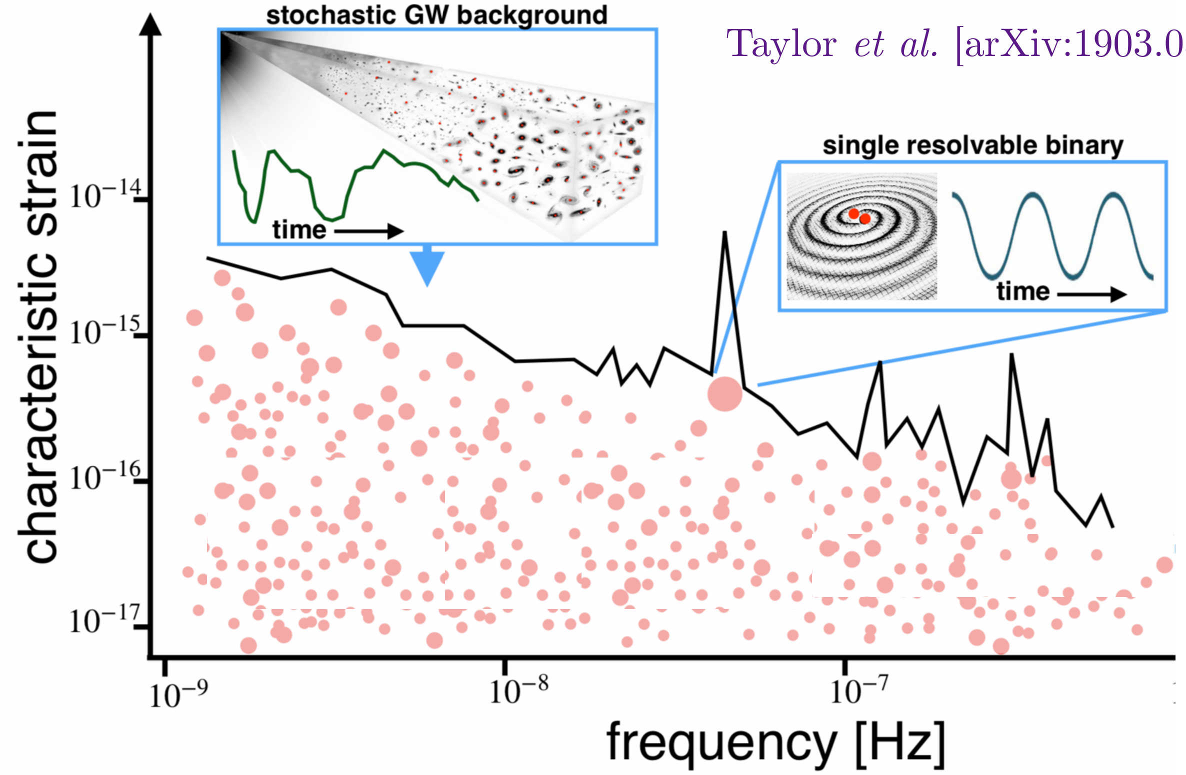
- Resolved SMBH mergers



$$h_0 = \frac{2\mathcal{M}^{5/3}(\pi f_{\rm GW})^{2/3}}{d_L}$$

$$\omega(t) = \omega_0 \left[1 - \frac{256}{5}\mathcal{M}^{5/3}\omega_0^{8/3}(t - t_0)\right]^{-3/8}$$

Taylor *et al.* [arXiv:1903.08183]

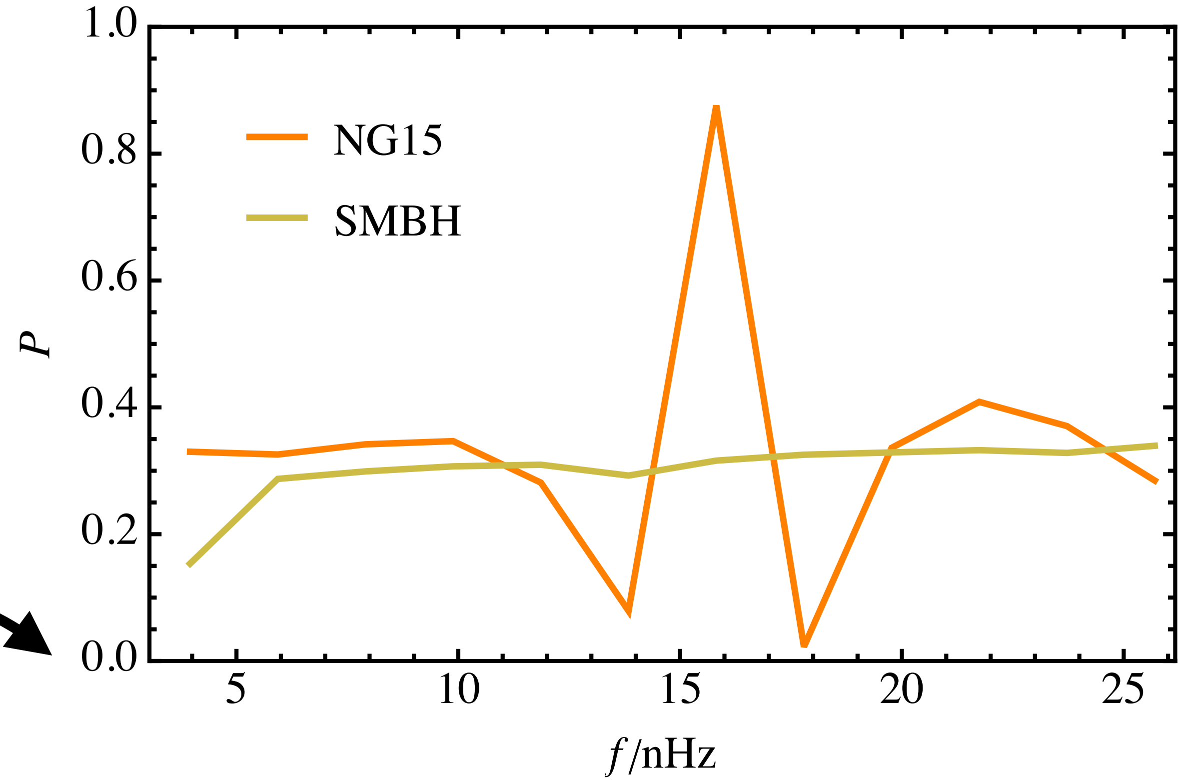


- Spectral fluctuations

$$P_{\rm fluct,i} \equiv P(-\Delta^2\Omega(f_i) > \Omega_{\rm th}(f_i))$$

- SMBHs models predict spectral fluctuations due to Poisson effects (most relevant at high-f)
- Cosmological signals mostly smooth over the relevant PTA scales

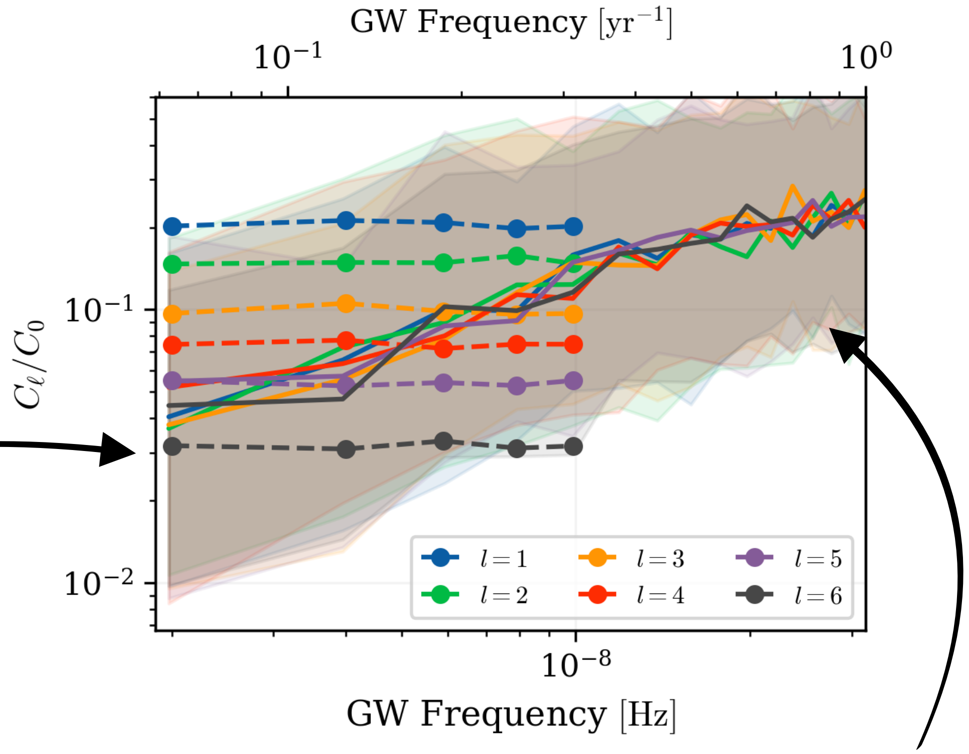
J. Ellis, **G.F.** , et al. [arXiv:2308.08546]



SMBHs vs CGWs II: Anisotropies

- **Astrophysical** SGWB is anisotropic, due to finite number of emitting sources

$$P(\hat{\Omega}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\hat{\Omega}) \qquad \ell_{\max} \sim \sqrt{N_{\rm psr}}$$



NANOGrav [arXiv:2306.16221]

NANOGrav 15 upper bound on anisotropies normalized w.r.t. monopole

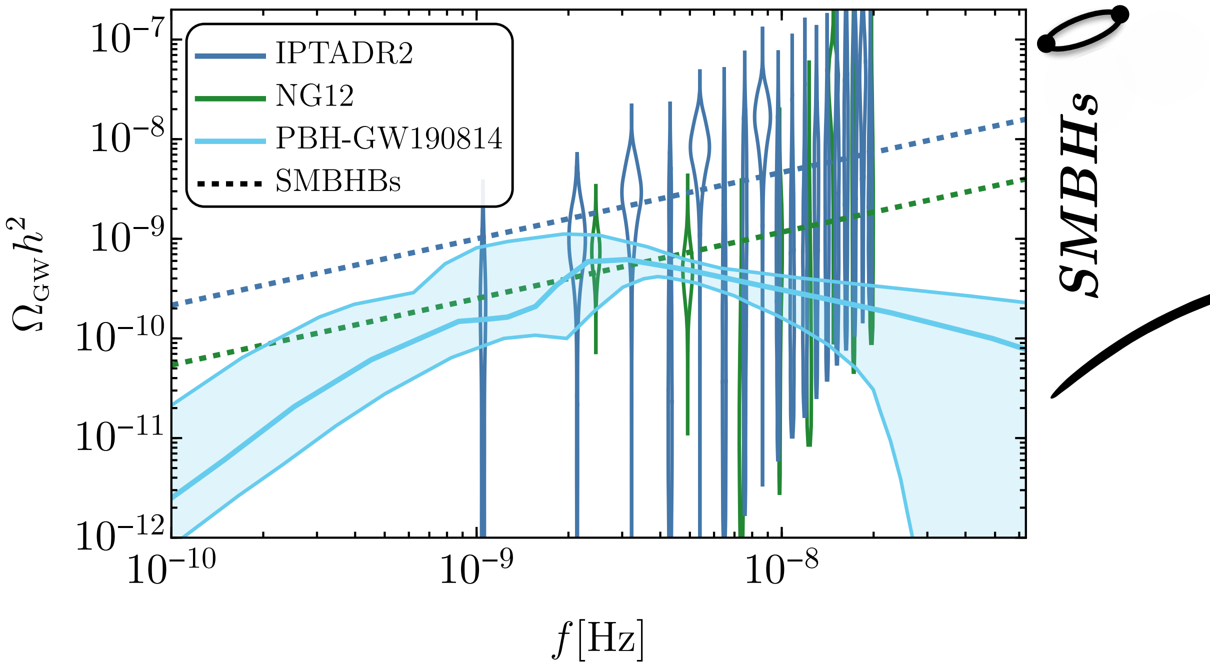
Predicted 95 % C.I. for anisotropies SMBHs pop

- **Cosmological** SIGW background predicts very small anisotropies
  - small scales at emission (unless large local NGs correlating to large scale modes)
  - propagation effects (analogue of the SW effect)

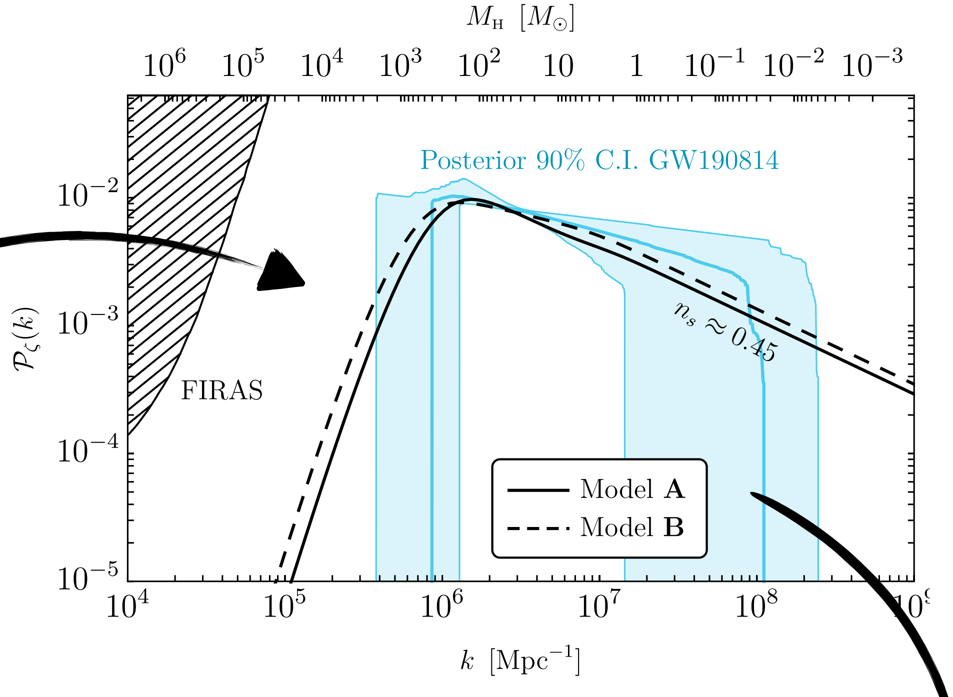
$$\sqrt{\frac{\ell\left(\ell+1\right)}{2\pi}\,C_{\ell,I+S}\left(k\right)} \simeq 2.8\cdot 10^{-4}\left|\frac{1+\tilde{f}_{\rm NL}\left(k\right)}{10}\right|\left(\frac{\mathcal{P}_{\zeta_L}}{2.2\cdot 10^{-9}}\right)^{1/2}$$

Other probes of large SIGWs: sub-solar mergers

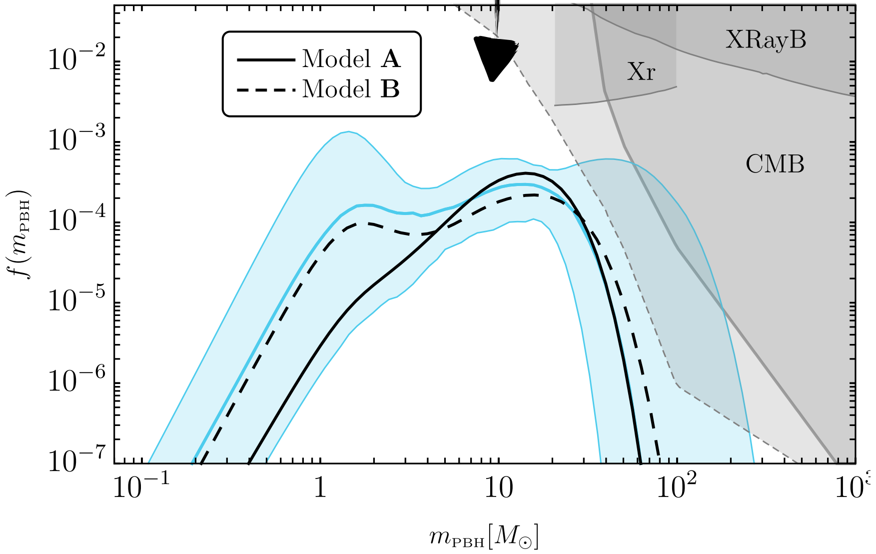
Induced nHz SGWB



Curvature spectrum



PBH abundance



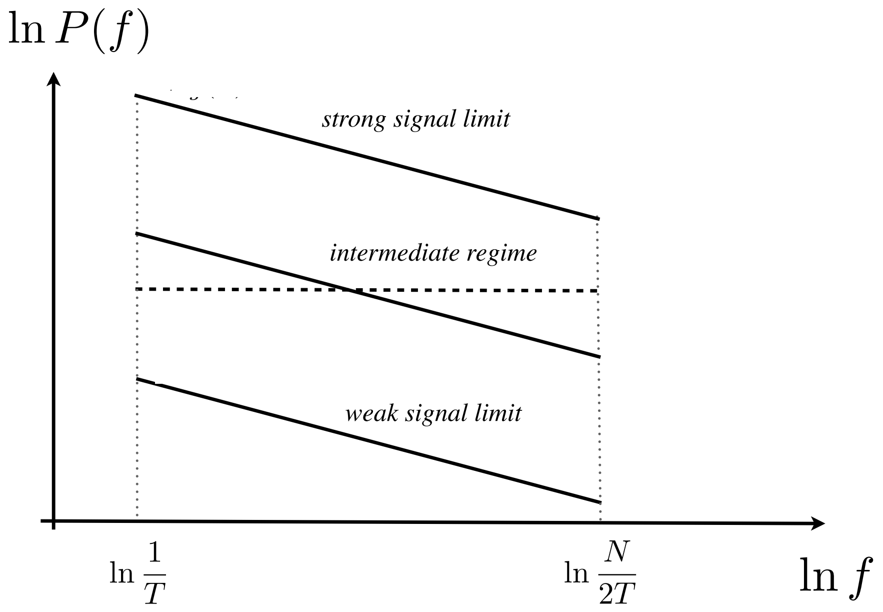
- Assuming no large non-Gaussianities in the model
- Mass distribution that could be tested with ground based detectors searching for PBH mergers
- Cyan band compatible with some PBHs in the LVK band

G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]

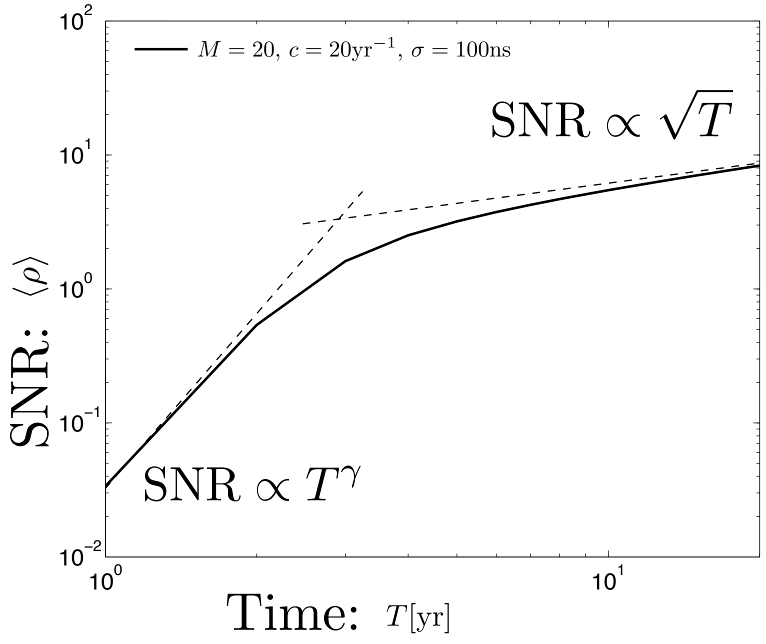
Forecast future PTA sensitivity

Disentangle multiple contributions to the spectrum?

- Steep growth of SNR with time in the weak signal limit, slow afterwords

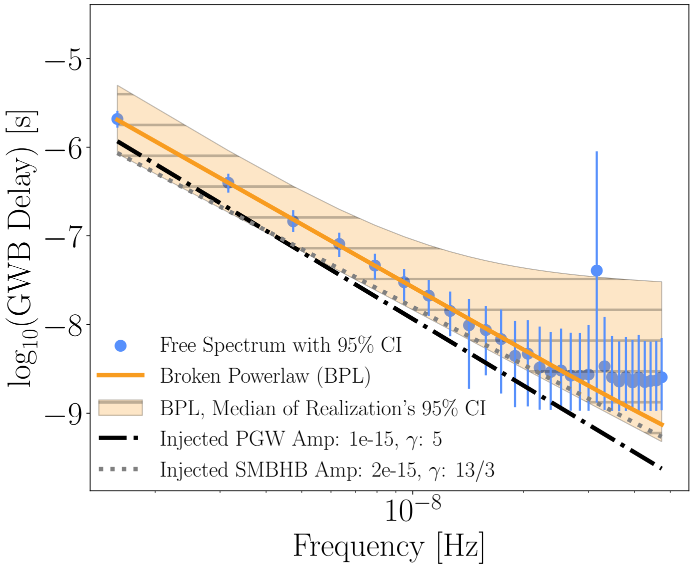


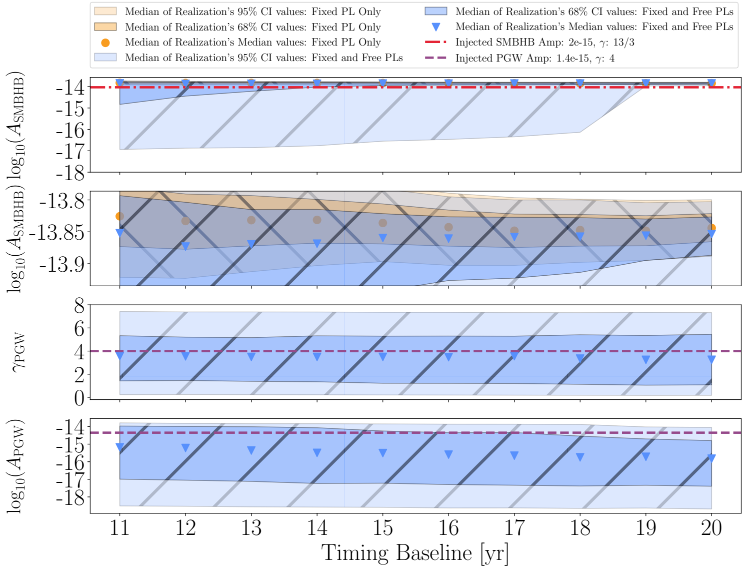
Siemens et al [arXiv:1305.3196]



+ scaling with n. pulsars

Simulated injection of SMBHs spectrum + cosmological SGWB $\left(\Omega_{\text{PGW}}/\Omega_{\text{SMBHB}} = 0.5 \right)$





A. R. Kaiser, Astrophys. J. 938 (2022) no.2, 115 [arXiv:2208.02307]

Simplified future PTA sensitivity forecasts

For **each pulsar** we assume Gaussian and stationary data: $\tilde{d} = \tilde{s} + \tilde{n}$

$$\langle \tilde{d}^2 \rangle = \langle \tilde{s}^2 \rangle + \langle \tilde{n}^2 \rangle = \mathcal{R} P_h + P_n$$

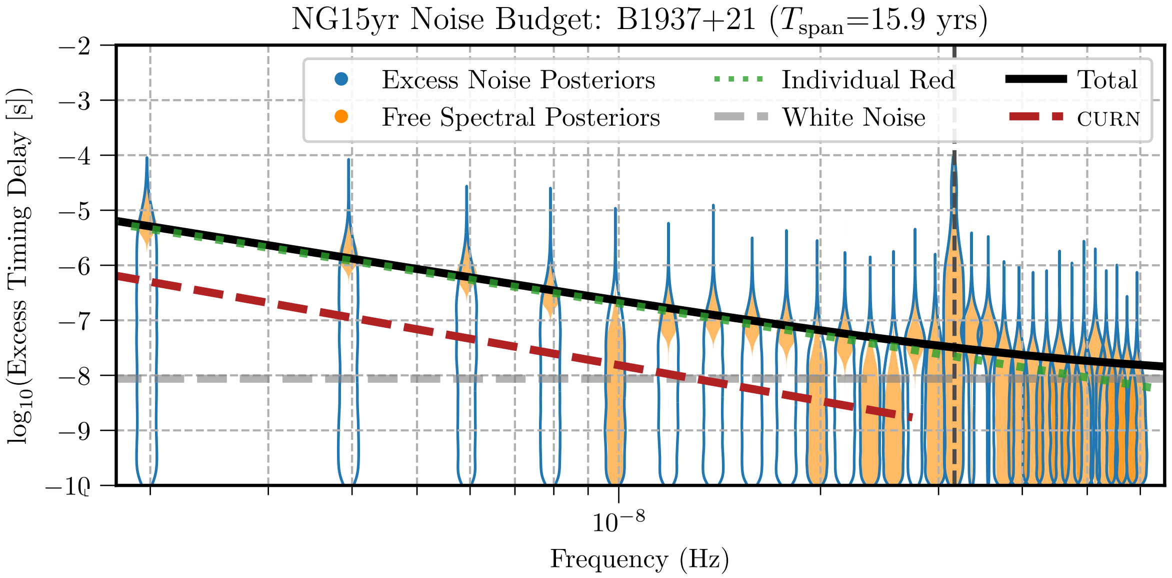
GW power spectrum: $\Omega_{\rm GW} = \dfrac{1}{3H_0^2 M_p^2} \dfrac{d\rho_{\rm GW}}{d\ln f} = \dfrac{4\pi^2}{3H_0^2} f^3 P_h$

Response function: $\mathcal{R}(f) = \dfrac{1}{12\pi^2 f^2}$

Noise power spectrum: WN + RN + DM + ...

e.g. pulsar from NG:

NG15 [arXiv:2306.16218]



Simplified future PTA sensitivity forecasts

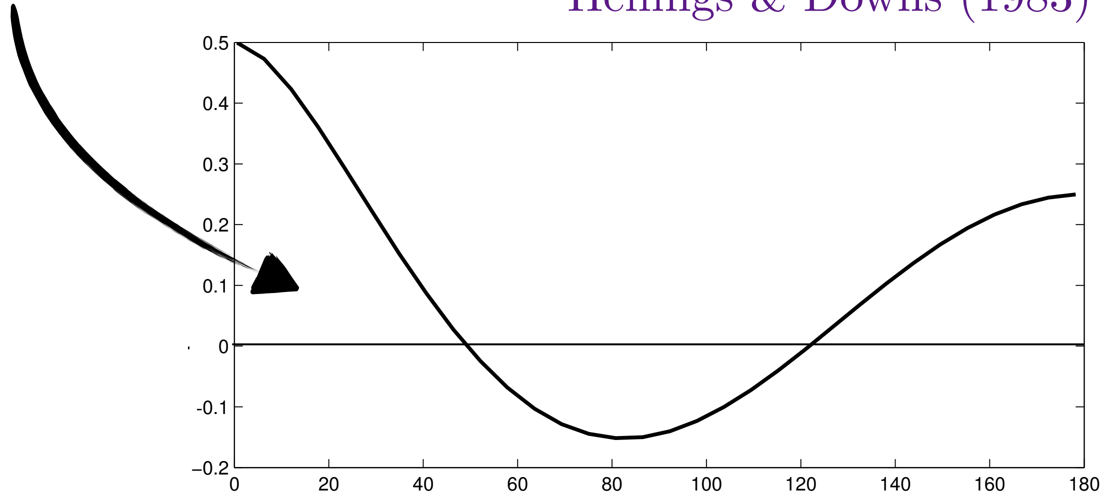
S. Babak, M. Falxa, G. Franciolini, M. Pieroni, to appear

Whittle likelihood: $$-2\ln\mathcal{L}(\tilde{d}|\theta) \propto \sum_{k,IJ} \ln\left[C_{IJ}(f_k,\theta)\right] + \tilde{d}_I^k C_{IJ}^{-1}(f_k,\theta)\tilde{d}_J^{k*}$$

Covariant matrix: $$C \simeq P_{n,IJ} + \chi_{IJ}\,\mathcal{R}\,P_h$$

(Pulsar indices *I*,*J*)

Hellings & Downs (1983)



Angle: θ_{IJ}

Fisher matrix estimates on measurement uncertainties:

$$F_{\alpha\beta} \equiv T_d \sum_{f_k} \frac{\partial \log C}{\partial \theta^\alpha}\frac{\partial \log C}{\partial \theta^\beta}$$

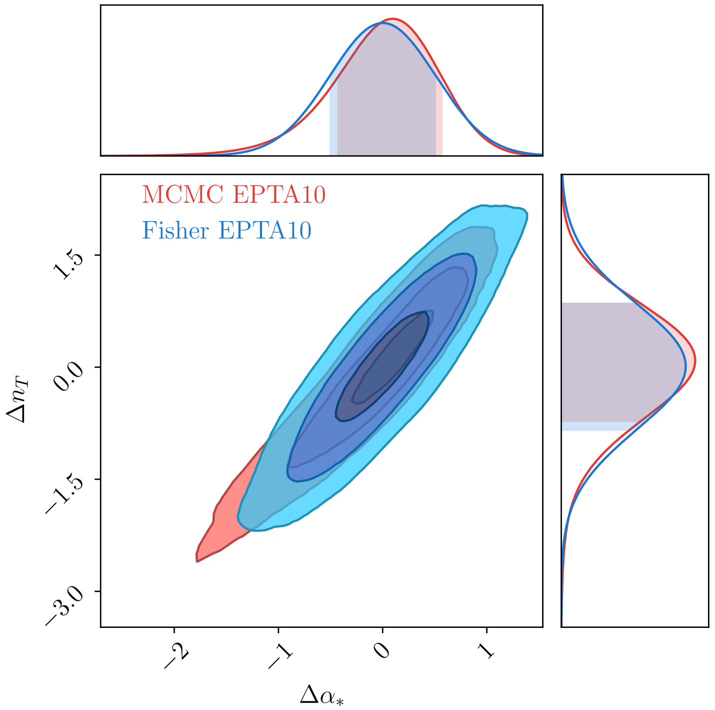
$$\sigma_\alpha = \sqrt{F_{\alpha\alpha}^{-1}}$$

Recover results with "state-of-the-art" PTA data analyses

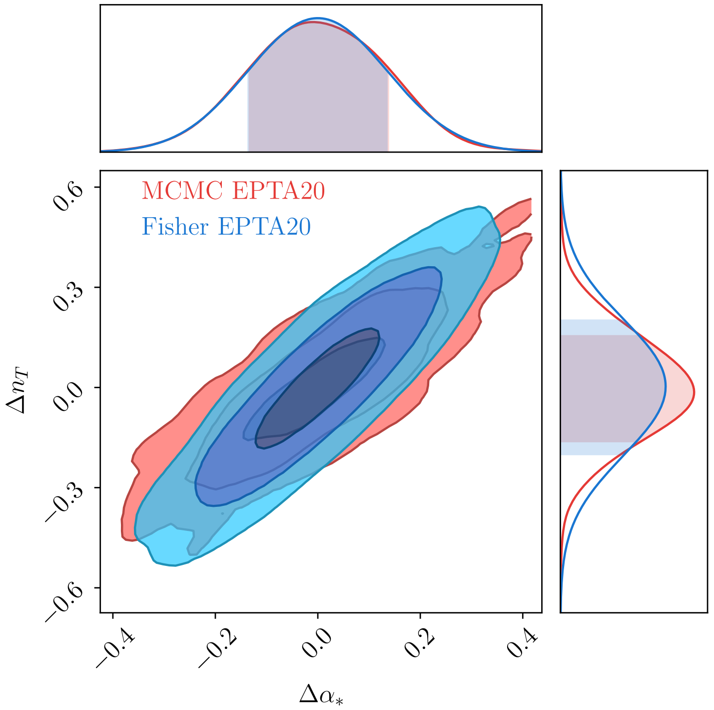
Power-law model: $$\Omega_{\rm GW} h^2 = 10^{\alpha}\left(\frac{f}{f_{\rm yr}}\right)^{n_T}$$

Current data: EPTADR2New

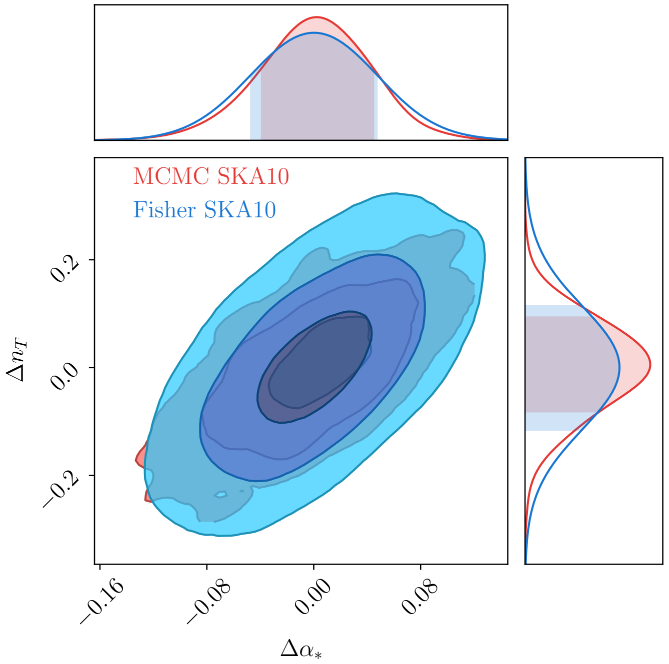
10 yrs, 25 Pulsars



Simulated EPTA 20, 50 p.



Simulated SKA 10, 50 p.



Fisher estimate

(Time-domain) full Bayesian analyses

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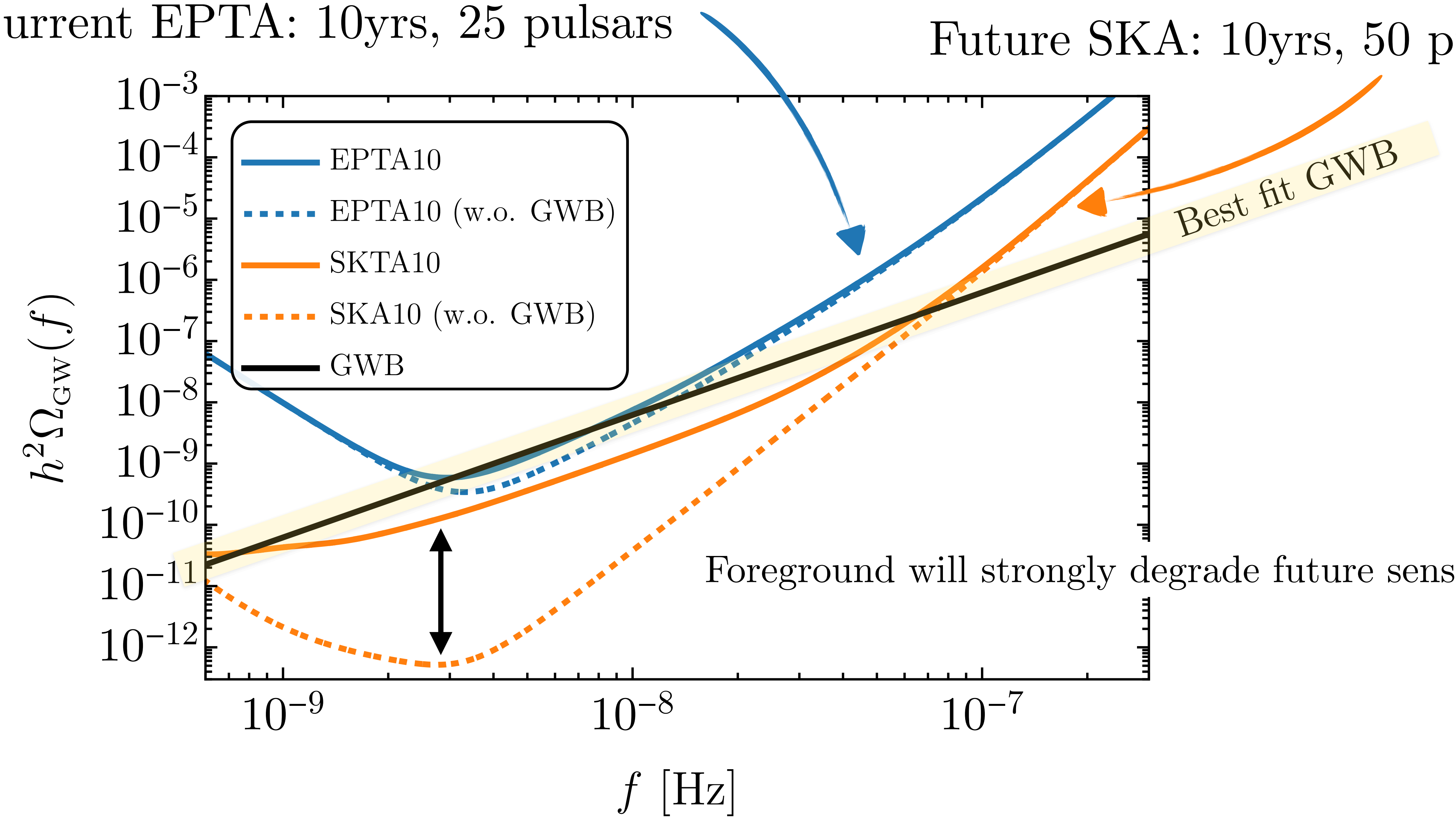
Future sensitivity to subdominant CGWB

$$S_{\mathrm{eff}}(f) = \left(C^{-1}_{IJ}\, C^{-1}_{KL}\, \chi_{JK}\, \chi_{LI}\, \mathcal{R}^2\right)^{-1/2}$$

Current EPTA: 10yrs, 25 pulsars

Future SKA: 10yrs, 50 pulsars



Foreground will strongly degrade future sensitivity!

(If the dominant SGWB is astro, can one subtract resolved mergers?)

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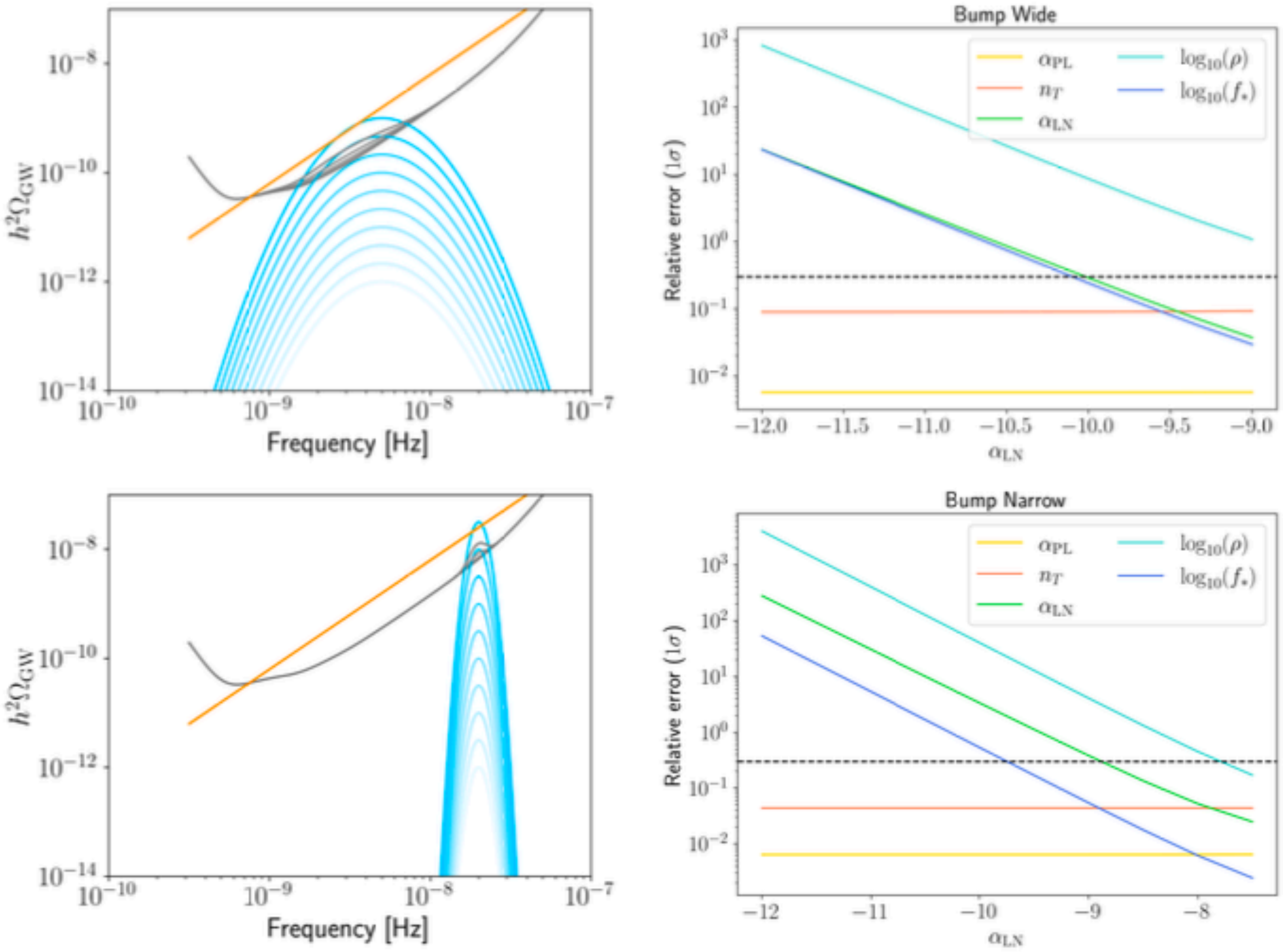
Multiple SGWB signals at future observatories

Power-law model (e.g. SMBHs) + lognormal bump (e.g. CGWB)

$$\Omega_{\rm GW} h^2 = 10^{\alpha}\left(\frac{f}{f_{\rm yr}}\right)^{n_T} + 10^{\alpha_{\rm LN}} \exp\left[-\frac{1}{2\rho^2}\ln^2\left(\frac{f}{f_\star}\right)\right]$$



Future SKA: 10yrs, 50 pulsars

S. Babak, M. Falxa, G. Franciolini, M. Pieroni, to appear

Conclusions

- PTA observations provide unprecedented ways to test the GW spectrum at nHz
- Astrophysical models can fit the data, with constraining power for new physics
- New physics could also fit the data well, no smoking gun signature detected so far
- QCD thermal history could leave detectable imprints, if there was a transient source of GW at slightly larger temperatures
- Primordial black hole overproduction constrains the amplitude of SIGWs, complementarity with ground based detectors bounds
- Future sensitivities will be limited by the observed foregrounds (can this be reduced?)

Gabriele Franciolini

Thanks!