The Status of Analytical Theory: Methods and Bench-Marks

Gravitational Wave Probes of Fundamental Physics

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What is the definition of ``fundamental'' physics?

We have the known unknowns:

e.g. What is the equation of state for dense nuclear matter? We know the action and we know something about the quantum number of the ground state, but we dont know the Free energy as a function of say the extensive variables.

We have the unknown unknowns:

e.g. is GR really valid at all scales down to the Planck scale? This would imply the existence of terms in the action aside from those in the effective field theory of gravity coupled to the standard model, say like a graviton mass, or Axion stars, ``BSM''.

Theory plays a dual role in searching for insights into ``fundamental” physics

1) Given an action calculate within a systematic expansion with a well defined notion of theoretical error. Calculate to sufficiently high order in the relevant expansion parameter. These calculations may be indirect (i.e. backgrounds) or direct. e.g. testing an equation of state for a neutron star.

2) Finding new actions that are a) well motivated b) mathematically consistent. e.g. deviations from GR, extra degrees of freedom. Or new types of compact objects (boson/axion stars) or environmental effects due say to dark matter.

In this talk I will only talk about 1) making precision predictions, focusing on direct effects, though the indirect effects are ``matched'' to the direct (end of talk).

Consider probing the EOS of a star. We must be able to disentangle the non-linear GR effects from the effects due to the tidal distortion of the star.

DIfficult problem: solve Einsteins equation sourced by a dynamical compact objects

$$G_{\mu\nu} = T_{\mu\nu} \qquad S_{matter} = \int d^4x L(g_{\mu\nu}(x), \phi_i(x)) \qquad T_{\mu\nu} = \langle \Psi \mid T_{\mu\nu}(x) \mid \Psi \rangle$$

$\mid \Psi \rangle$ Describes two localized clumps of stress-energy

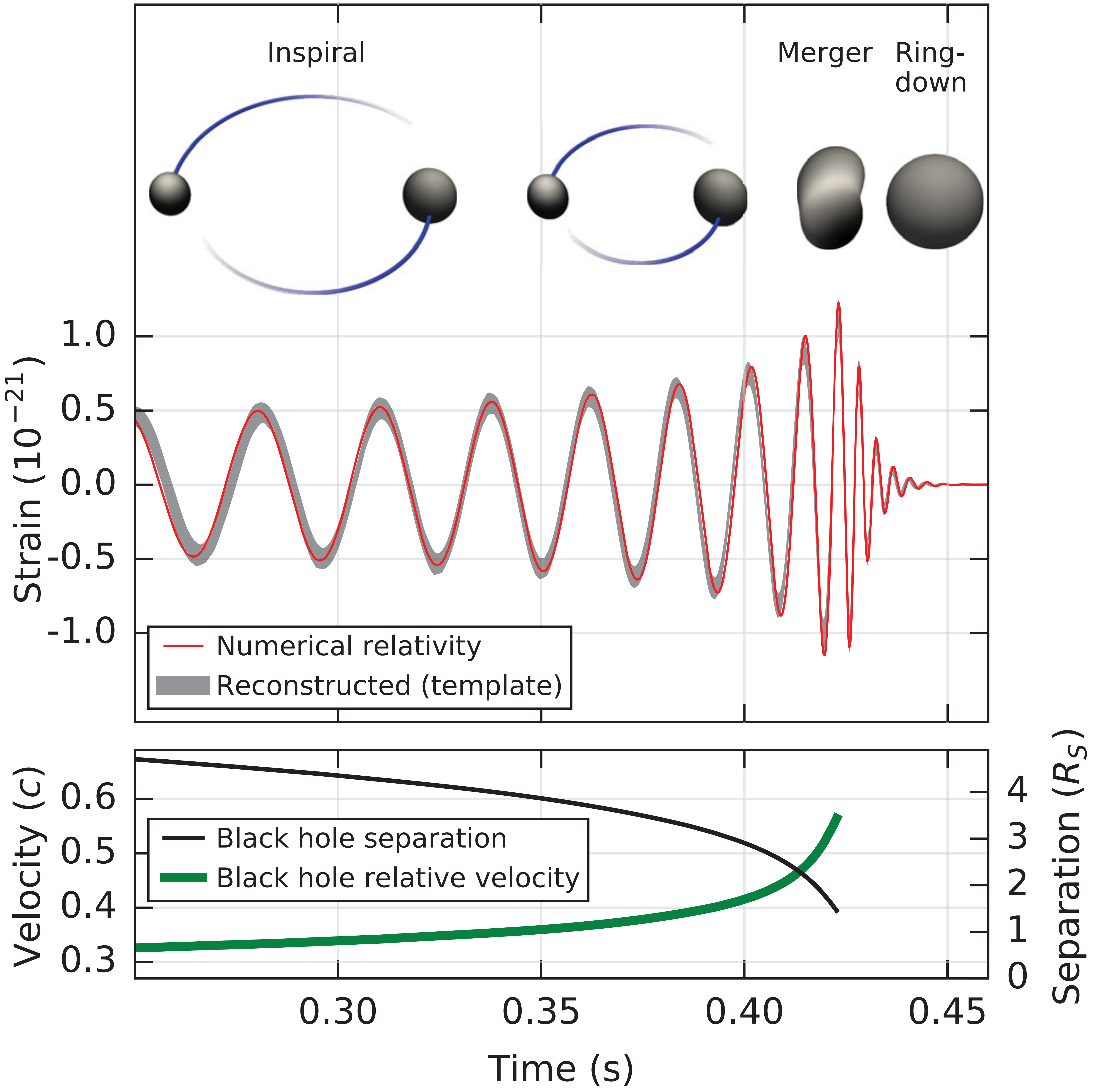
1) When can we say with confidence that we have found something beyond the SM?

2) How well can we test an EOS for a NS?

1) In particle physics to claim a discovery one need 5 sigma deviations from null hypothesis. Assumes that theoretical errors are well defined. Not just varying parameters in a model, but there should be an expansion parameter such that when it approaches zero the prediction becomes exact. The community will have to decide the proper criteria

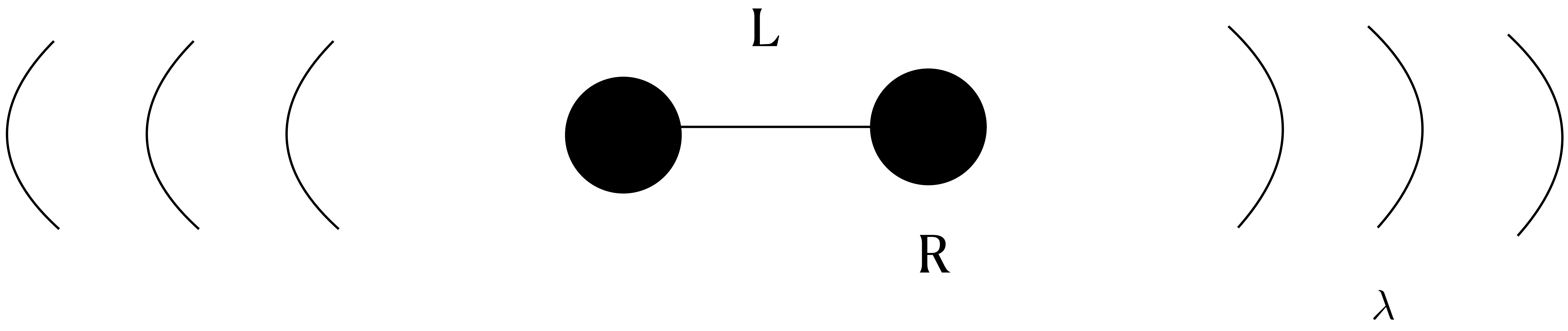
2) This is much tougher to quantify.

How we calculate:



Analytic PN Region | Interpolation region | NR Region | BH PT Region

Inspiral Phase further complicated by multi scale nature of the problem



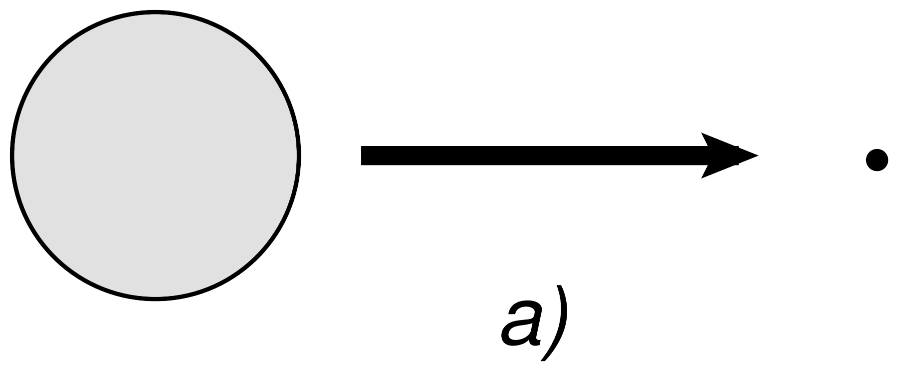
$$R \ll L \ll \lambda$$

We have a tool that is built for such multi-scale problems: Effective Field Theory (EFT)

Divide and conquer: Treat one scale at a time in expansion in the ratio of scales

EFT is a QFT tool: goal to utilize this methodology efficiently

Treat the smallest scale first: Point particle limit



Eliminates need to solve boundary value problem (not completey)

$$S^0 = -M\int d\tau - 2M_{pl}^2\int d^4x\sqrt{g}R$$

Note: at this point we have not used the PN expansion, simply a multipole expansion. R/r expansion

Systematically include higher order (finite size) effects

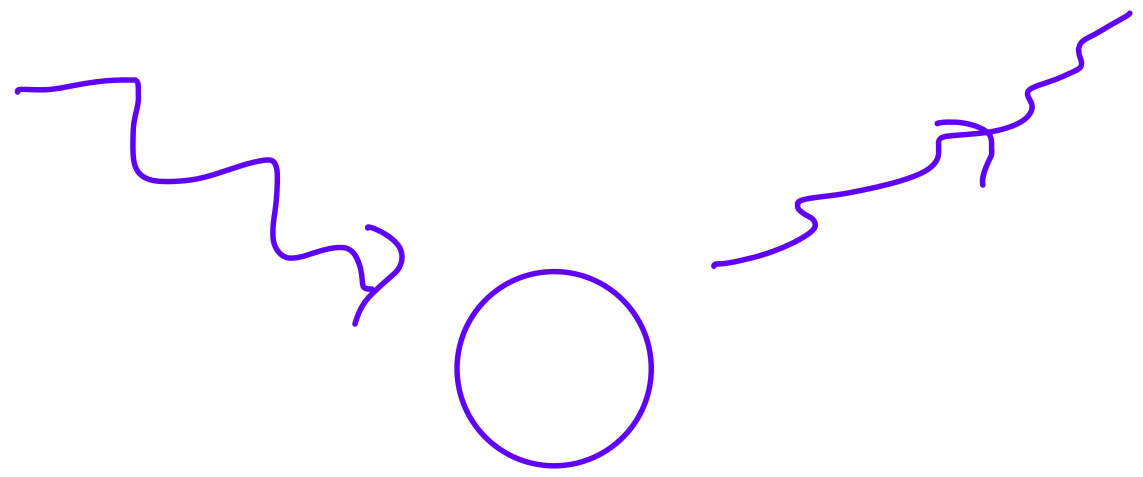
$$S_{E+B} = \int d\tau (C_E E_{\mu\nu}E^{\mu\nu} + C_B B_{\mu\nu}B^{\mu\nu}) + \ldots\ldots$$

Static Love Numbers. Higher order effects include dynamic Love numbers as well as non-linear response terms

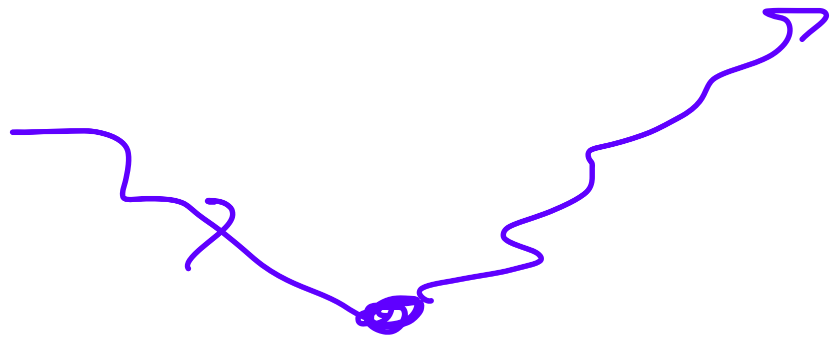
BH Love numbers

Perform matching calculation for black holes

Full Theory:



EFT:



To extract Love number we must subtract the EFT from the Full theory, separates the non-linearities due to gravity from the effect of interest

Static Love numbers vanish (Damour/Nagar, Biinigton/Poisson,Kol/Smolkin)

From point of view of QFT, this is a fine tuning problem that is searching for a symmetry explaination.

For Scalar Perturbations it has been shown that the near horizon geometry has a conformal symmetry that would forbit the World line coupling. See Hui et. Al. 2010.00593

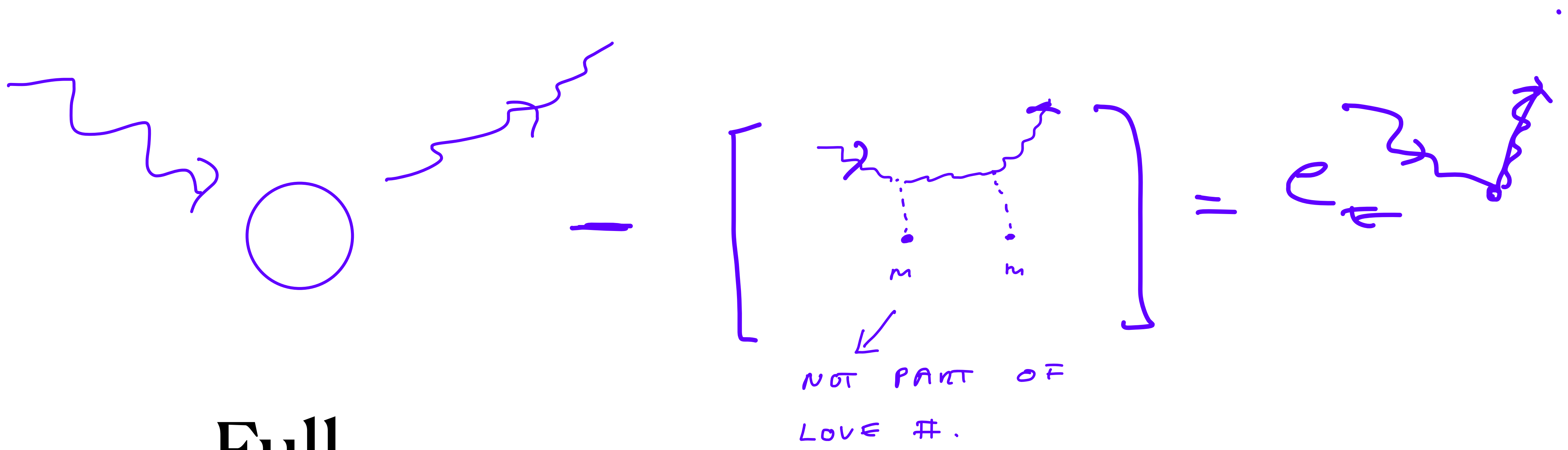
Interestingly enough a new result also finds that the non-linear static Love number vanishes

$$S^{E}_{NL} = C_{E^3} \int d\tau E^3$$

(Riva, Santoni, Savic, Vernizzi) : $C_{E^3} = 0.$

What do we do about Neutron Stars?

Here we see why its crucial to subtract the EFT from the full theory calculations



Full

Need some model for the full theory side

$$S = \int d^4x L(\phi, g) \qquad \phi = \phi_0 + \delta\phi, g = g_0 + \delta g$$

Extract Response function at Gaussian level

See e.g.Chakrabarti, T. Delsate, and J. Steinhoff

$$G(\omega) = \sum_n \frac{A_n}{\omega^2 - \omega_n^2} \sim \frac{A_0}{\omega_0^2} + \ldots$$

Love number Guaranteed to be gauge invariant

Including Dissipative Effects

When we integrate out the short distance physics we implicitly assume there are no gapless modes, which fails in the case for which we have dissipation. We need to integrate back in these modes by introducing new world line d.o.f

$$Q^{ab}(\tau), M_{ab}(\tau)$$

$$S_{dis} = \int d\tau (Q_{ab}E^{ab} + M_{ab}B^{ab})$$

Assuming the ground state is spherically symmetric $\langle Q^{ab}(\tau) \rangle = 0$

Linear Response information is contained in the retarded Greens function

$$G^{E/B;ret}_{\mu\nu,\rho\sigma}(s - s') = -i\theta(s - s')\langle [Q^{E/B}_{\mu\nu}(s), Q^{E/B}_{\rho\sigma}(s')] \rangle$$

Matching for Black Holes

e.g. match graviton absorption cross section

Since the response function has no long time tails we can expand in small omega

$$G_R(\omega) = A_1 + i\omega B_1 + A_2\omega^2 + \ldots$$

Now we can extract the coefficients B1 by comparing to the graviton absorption cross section using the relations

$$\sigma_{abs}(\omega) = \frac{\omega^3}{2M_{pl}^2}\left(A^{E+}(\omega) + A^{B+}(\omega)\right)$$

$$\sigma_{abs} = \frac{4\pi r_s^6\omega^4}{45}$$

(Page,Starobinski)

$$\theta(\omega)A^+(\omega) - \theta(-\omega)A_-(\omega) = 2ImG_{ret}(\omega)$$

Add Spin

(R. Porto)

Arbitrary scalar build from (p, S, g)

$$S_{pp} = -\int dx^\mu p_a e^a{}_\mu + \frac{1}{2}\int d\lambda \; S^{ab}\Omega_{ab} + \frac{1}{2}\int ds\,(p_a p^a - m^2) + \int ds \lambda_a S^{ab} p_b + \cdots \,,$$

$$ds = e(\lambda) d\lambda$$

Higher order finite size effects again follow from writing down all Diff and RPI invariant terms

We can add dissipation along the lines of the spinless case. Dissipation is enhanced in the spinning case. Incoming wave sees enhancement of frequency.

Aside: Can quantum effects/Hawking Radiation get enhanced such that its measurable?

If we assume that EFT of GR obeys the standard axioms of QFT,

all vacuum effects (Hawking) will be suppressed by powers of M_pl.

(Goldberger,IZR) 2023

Roughly: All classical observables can only depend upon

$$G_R(x,x') = -i\theta(t-t')\langle\Psi|[\phi(x),\phi(x')]|\Psi\rangle$$

Independent of vacuum up to corrections suppressed by the Planck Mass

Unruh, Hartle-Hawking and Boulware all give the same result.

So far we have shown that we can reproduce the interactions of a finite size object in a gravitational background using a point particle approximation in a multiple expansion. But the pay off of the EFT is that we have traded a boundary value problem for a much simpler one because the action is universal. Dont need to resolve it.

Next we coarse grain at the level of the orbital radius to reduce the two body system to a dynamical one body. We will do so in the PN expansion.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} \to H_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} \neq 0 \quad k < 1/r$$

$$H_{\mu\nu} \neq 0 \quad k > 1/r$$

Split the fluctuations into long (radiation, h) and short (potential,H) wavelengths

H and h have definate scalings in v, and the action can be written as power expansion such that every term scales homogeneously.

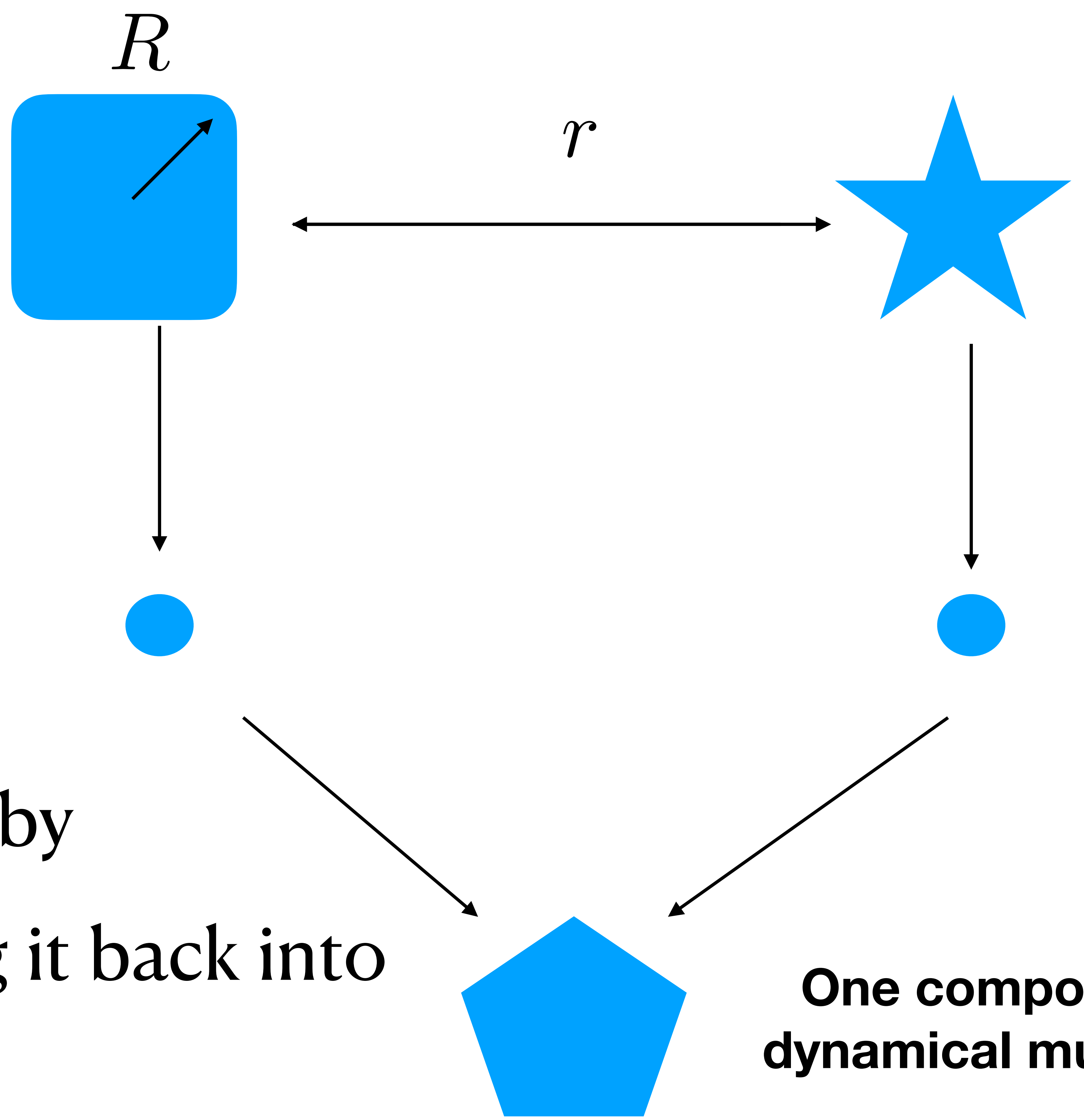
$$H \sim v^2 \qquad\qquad h \sim v^{5/2}$$

In this way we learn that the static Electric Love number terms scale as v^10 (5PN).

``Effacement Theorem''. (Damour)

Though for Neutron Stars we expect a numerical enhancement (factor of 10 since the radius is not fixed by the mass).

Next Step in Coarse Graining Procedure



This step is accomplished by

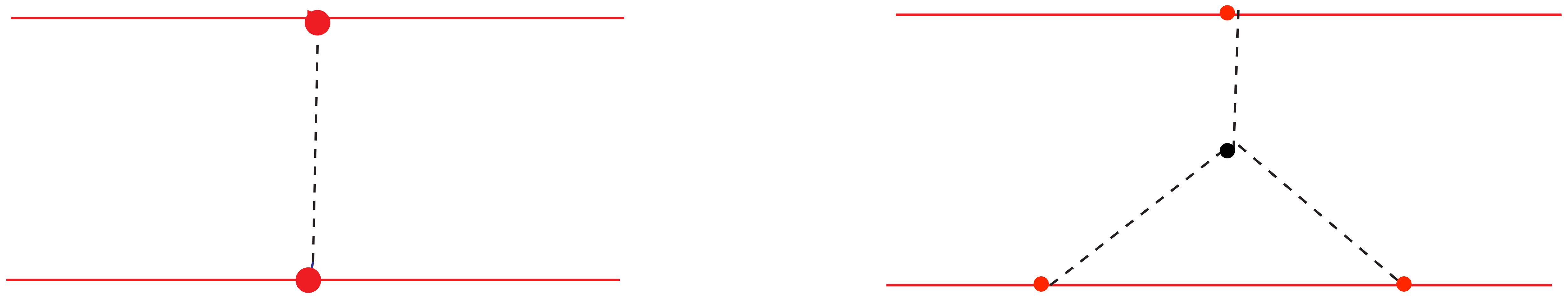
Solving for H and plugging it back into

The action.

One composite object with dynamical multipole moments

This is equivalent to calculating all Feynman diagrams within no external graviton lines. Generates a set of potentials in the PN expansion.

Order v^2 Calculation of Potential (EIH)



$$V(r) = \frac{iG_N m_1 m_2}{2|\mathbf{x_1} - \mathbf{x_2}|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7\mathbf{v}_1 \cdot \mathbf{v}_2 - (\mathbf{v}_1 \cdot \mathbf{n})(\mathbf{v}_2 \cdot \mathbf{n})\right]$$

In this way one can calculate Feynman diagrams up to any required order.

The PM Expansion

QFT was developed to treat theories relativistically, so why not perform an expansion in G and keep all orders in v? This corresponds to the ``Post-Minkowksian" expansion which corresponds to an expansion in $\lambda = \frac{GM}{b}$ (Eikonal limit).

Despite the fact that PM is not systematic for the problem of interest it is useful:

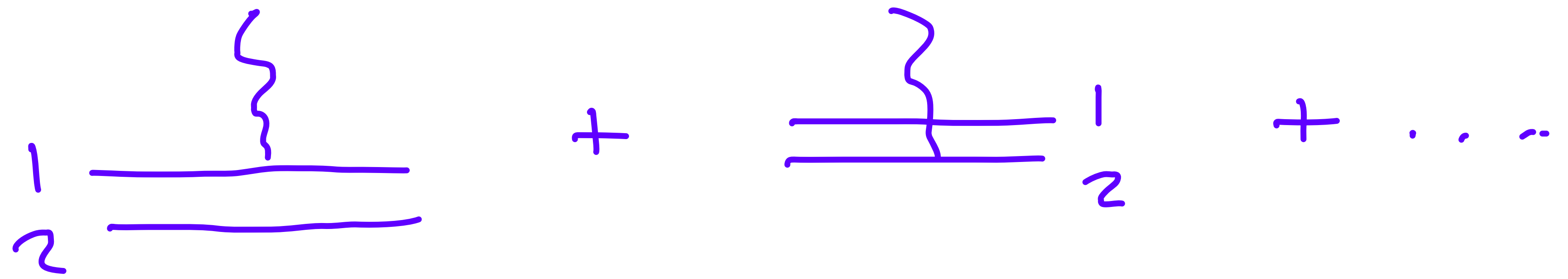
1) Reduce the number of diagrams needed

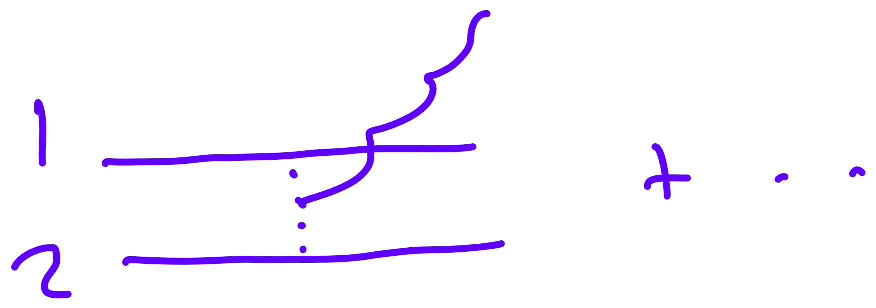
2) Used to fit parameters in EOB

World line EFT has been used to calculate to 4 PM (Porto et al)

Once we have integrated out the potential modes we are left with a theory of one particle with dynamical multipole moments which we use to calculate the radiation.

Calculate moments of the stress energy Tensor





$$S = \int Q \cdot E + \int O \cdot \partial E + \cdots$$

$$Q = Q(\dot{x}_i, r)$$

	BH	NS	Exotic Object
Leading Love number	5PN	4PN ish	?
Leading Dissipative	4PN (S=0) (2.5 max S)	<?4PN	?

Distinguishing Neutron Star from an exotic object will be a challenge. We must first gain confidence that we know whats going on with NS. Look for unique signature of exotics (.e.g. parity violating effects E.B finite size effects). (Modrekiladze)

Where do we stand in calculating background to finite size effects?

Potentials: 5 PN (Bernard,Blanchet, Bohe,Faye, Marchant, Marsat. Blumlein, Maier, Marquard and Schaefer)
5PN spin(Levi+Yin)

Radiation: 4Pn (Blanchet,Bernard,Blanchet, Bohe,Faye, Marchant, Marsat), 4Pnspin (Cho, Porto,Yang)

Reaching 5Pn was a great achievement of the community, that involved many people who dedicated signficant time and energy on these tough calculations. Foffa and Sturani, Damour, Jaranowski.

Environmental Effects

The motion of a finite body in a fluid in GR is computationally intensive. Utilize EFT techniques to simplify the problem.

(With Beka Modekiladze)

Again point particle approximation, but now we have the caveat that the fluid gradients must be small compared to the radius of the objects.

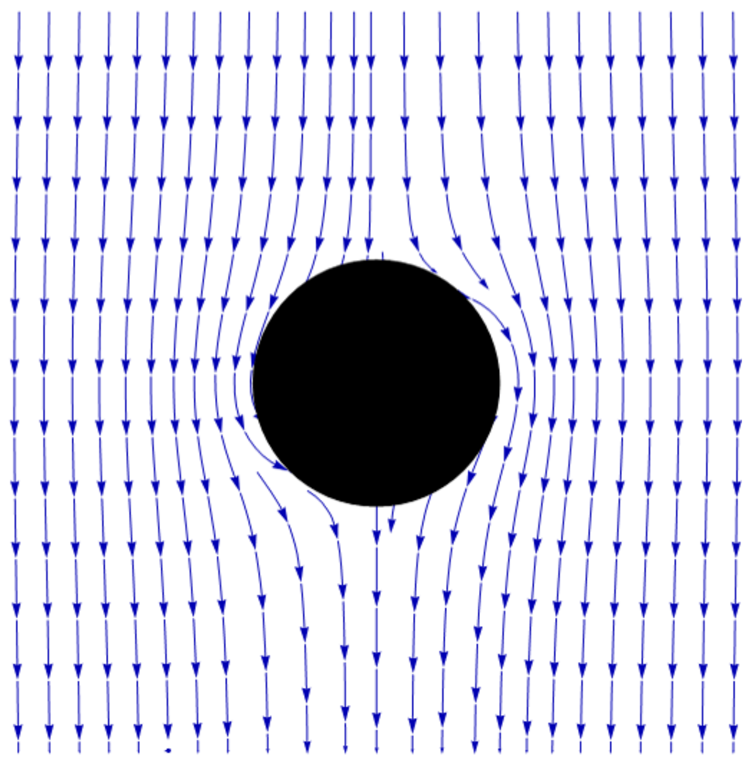
$$S = -m \int dt \sqrt{\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}} F\left(1 - \frac{(\dot{x}^\alpha u^\beta g_{\alpha\beta})^2}{\dot{x}^\rho \dot{x}^\sigma g_{\rho\sigma}}\right)$$

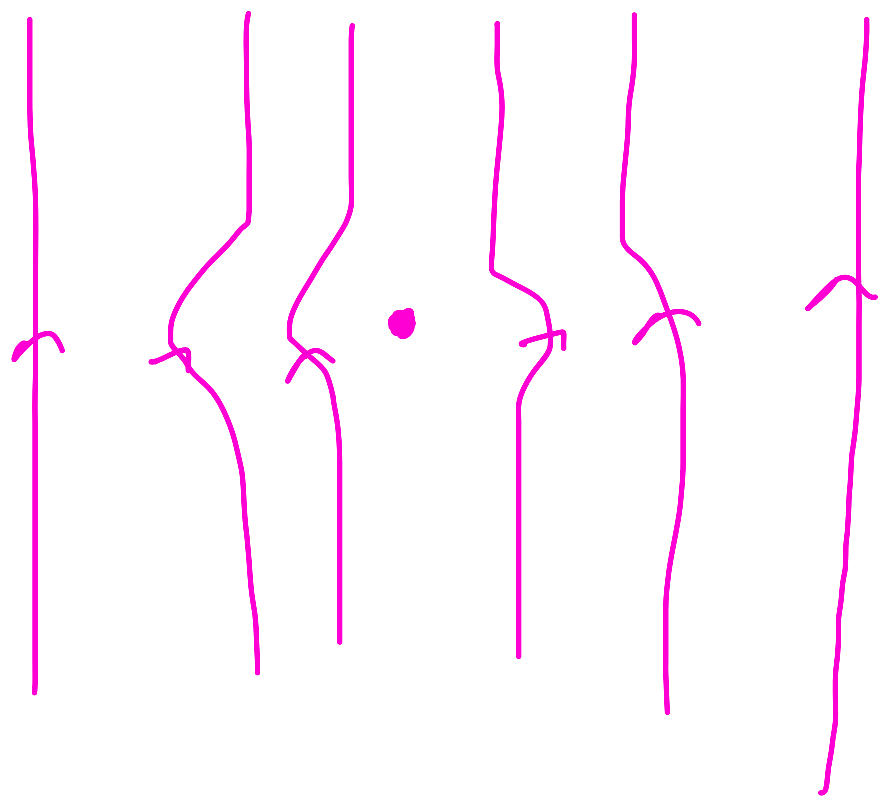
$$L_{v^0} = -\frac{M}{2M_{pl}} (1 - 2C) H_{00}$$

$$L_{v^1} = -\frac{M}{M_{pl}} \left(v^i - 2Cu^i\right) H_{0i}$$

$$L_{v^2} = -\frac{M}{4M_{pl}} \left(v^2 - C(2\mathbf{u} + \mathbf{v}) \cdot \mathbf{u}\right) H_{00} + \frac{M H_{00} H_{00}}{8M_{pl}^2} (1 + 4C) - \frac{M}{2M_{pl}} \left(v^i v^j (1 + 2C) - 2C(v^i u^j + u^i v^j)\right) H_{ij}$$

Need to match full theory to effective theory to determine C



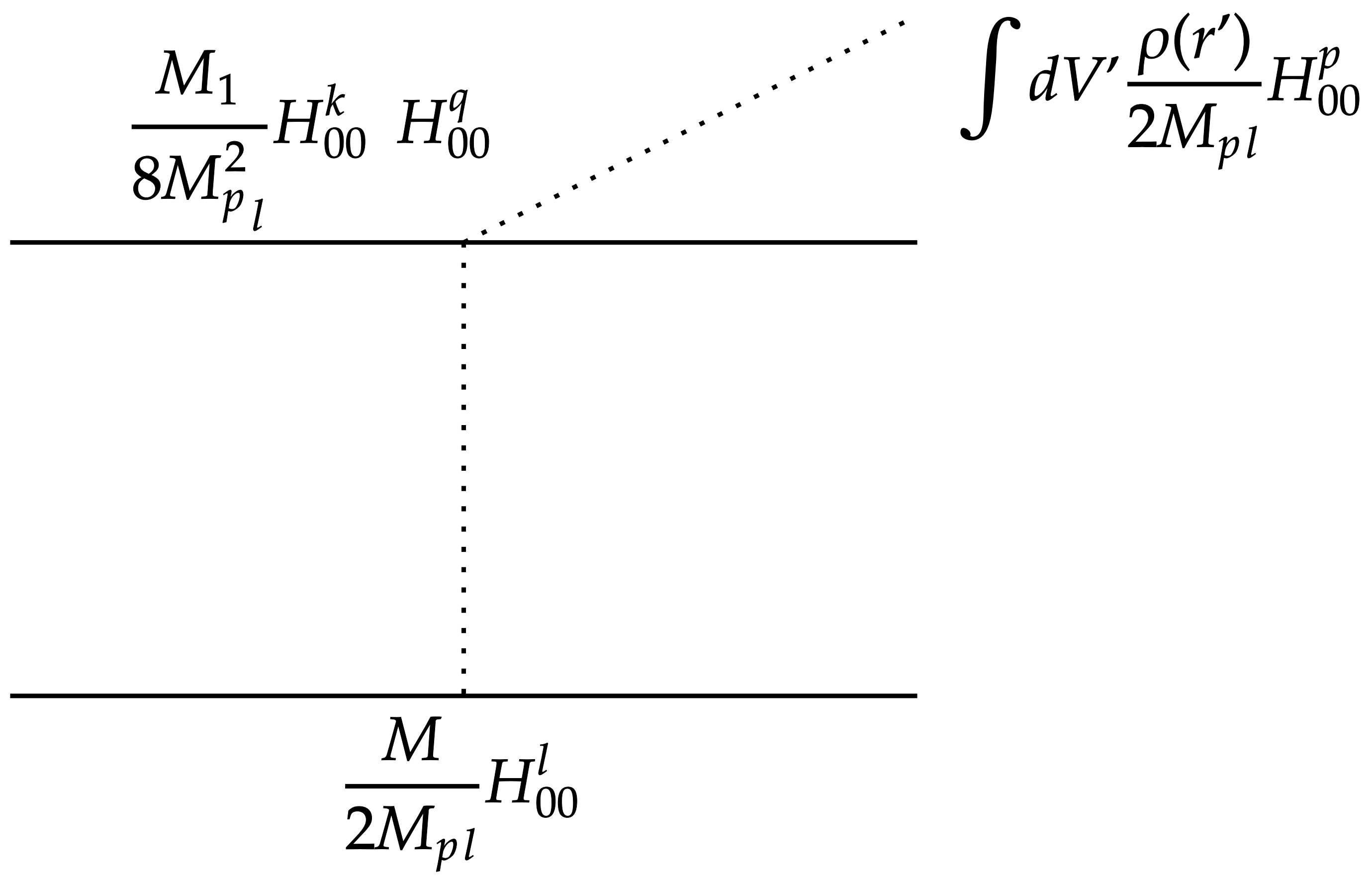


For an incompressible fluid one can solve for C in the case of a rigid BC's analytically

Effects the Fluid EOM as well as the PN potentials (drag force negligible in most realistic cases, though dynamical friction will not be)

Calculate new set of Potentials and Multipole moments due to effects of fluid

At the 1PN order we also have the following binary interactions with fluid



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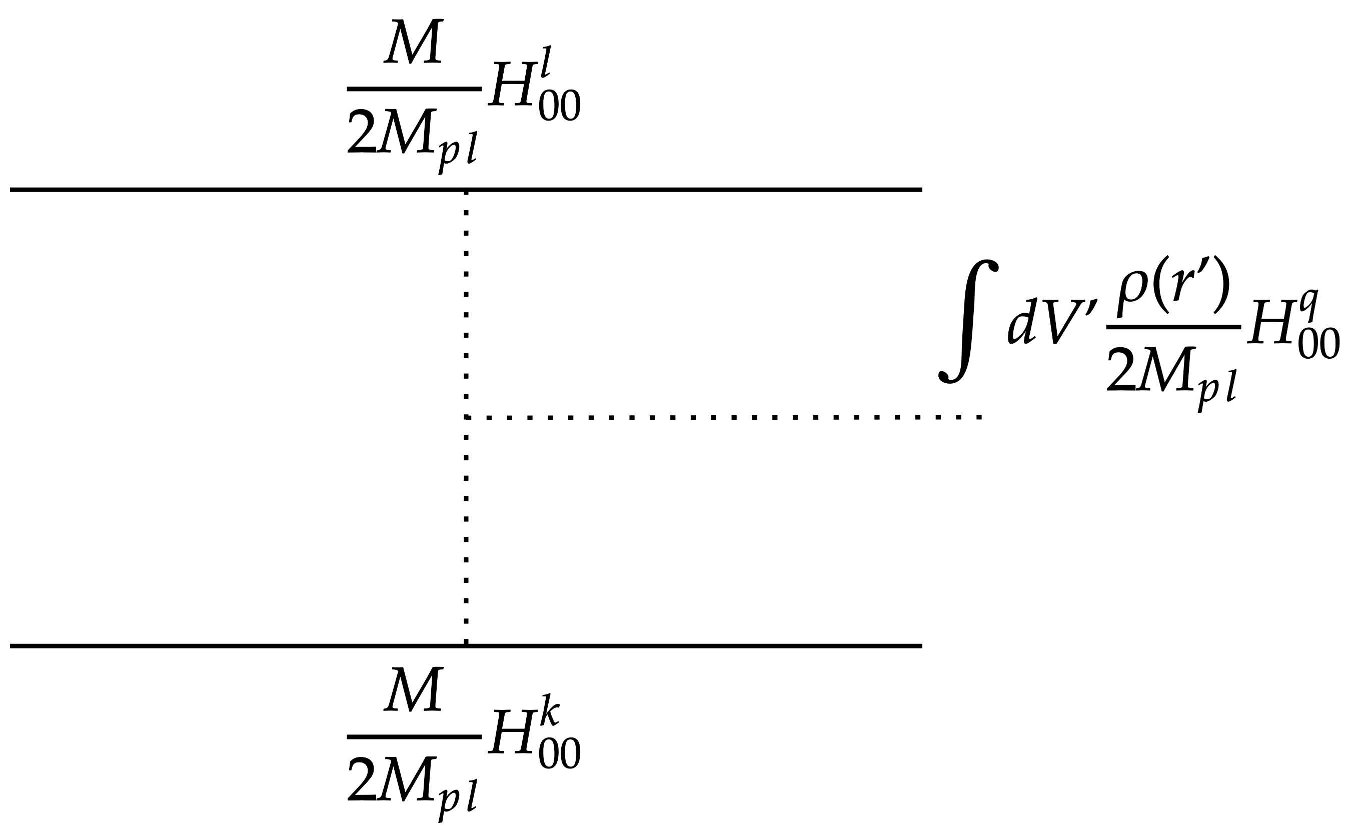

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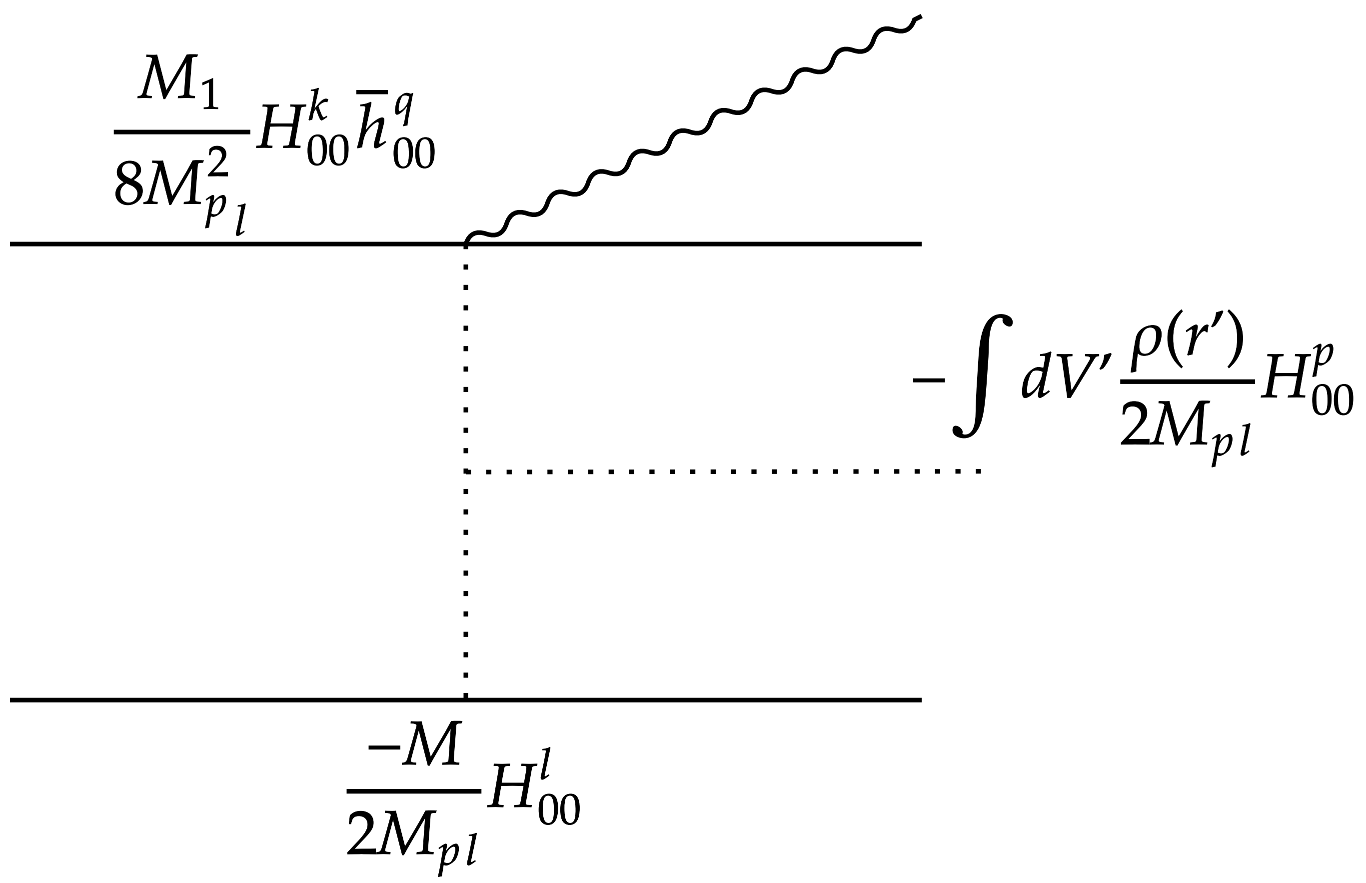

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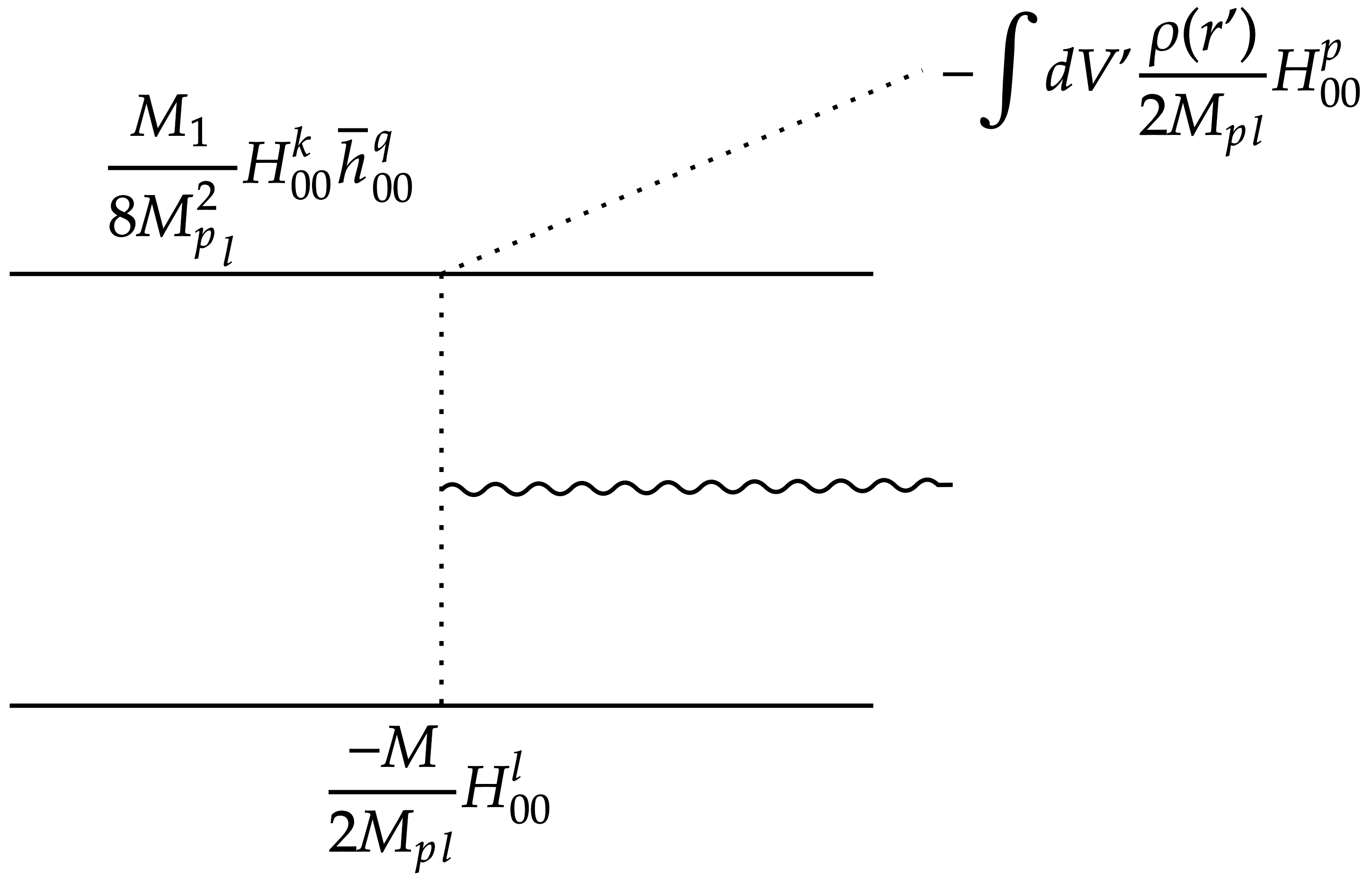

(b)

In order to find new physics or corroborate some models of collective phenomena we still have much work to do. For neutron stars need to start by feeling confident about smoking guns for qualitatively distinct models.

A further challenge arises from the fact that we can extract tidal parameters in a systematic way from the merger data, but must restrict ourselves to the early stages of the inspiral.