The QCD equation of state from Heavy ion collisions and neutron star mergers

Jan Steinheimer-Froschauer

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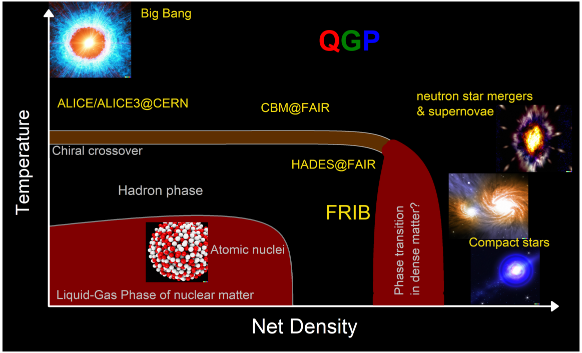
The essence of high energy nuclear physics: The QCD phase diagram



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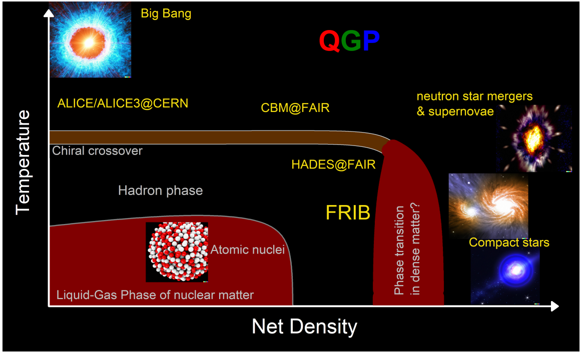

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- Established $T_{cep} \lessapprox 120$ MeV.

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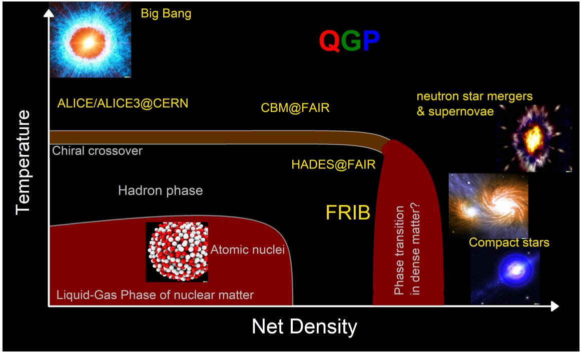

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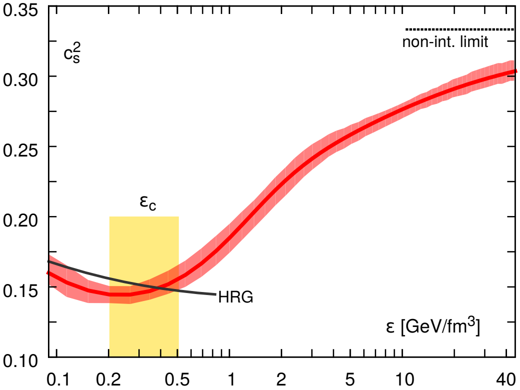
- There is a continuation.
- Direct QCD simulations face the sign problem and expansions break down for $\mu_B/T \gtrsim 3-4$.
- Results at low density: Crossover is now confirmed.
- Established $T_{cep} \lesssim 120$ MeV.
- High density: room for speculations.
- We have to rely on effective model descriptions of the EoS and experimental data from HIC and astrophysics.

What (we think) we know

Direct approach: lattice QCD

- Ideally one would like input from first principle calculations.
- Available only for small baryon densities.
- Still useful as we see very smooth behavior of the speed of sound.



(HotQCD Collaboration) Phys. Rev. D 90, 094503

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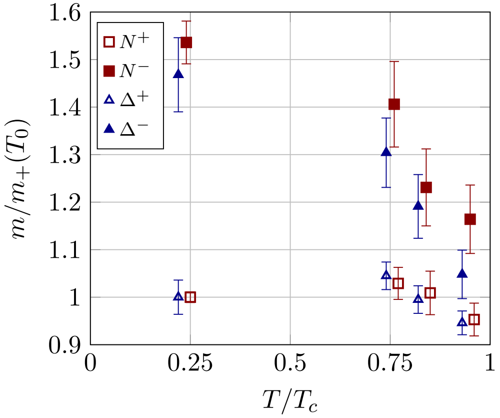
- The behavior and origin of the nucleon mass is essential for high density nuclear physics as well.
- Lattice QCD: effective mass of baryons is not created by coupling to the (scalar) chiral field.



JHEP 06 (2017) 034

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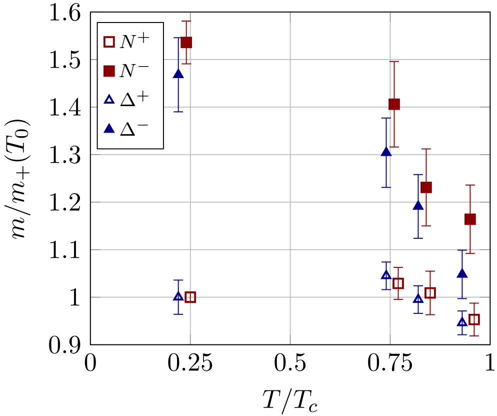
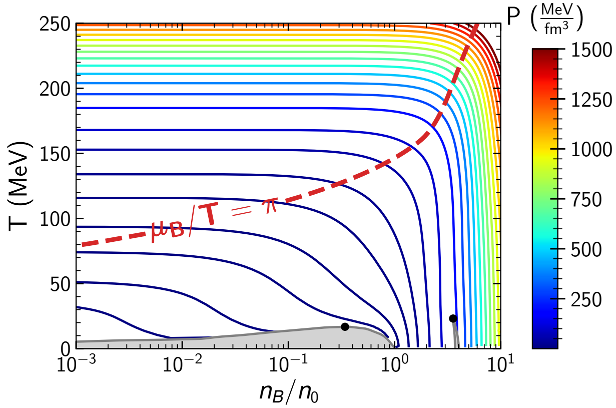
- The behavior and origin of the nucleon mass is essential for high density nuclear physics as well.
- Lattice QCD: effective mass of baryons is not created by coupling to the (scalar) chiral field.
- Useful to constrain the repulsive couplings of deconfined quarks → very high densities.



JS
and S. Schramm, Phys. Lett. B **696**, 257-261 (2011)

The baryonic problem



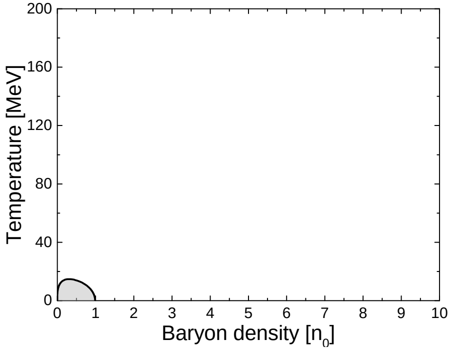
Why does the method break down around $\mu_B/T \approx \pi$

- Sudden change of isobaric lines at this point.
- From Boson (mesons/gluons) dominated matter to fermionic matter (nucleons/quarks).
- Calculations seem to fail for matter where (multi-) baryonic interactions become important.
- Positive: for the region of interest a density dependent EoS may be enough.

A. Motornenko, JS, V. Vovchenko, S. Schramm and H. Stoecker, Nucl. Phys. A 1005 (2021), 121836

Regions of access to the PD - cold NeutronStars

- Starting from the phase diagram in Temperature and density.



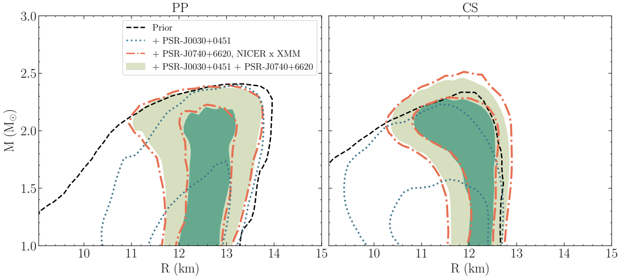
Disclaimer: At the moment we will ignore any isospin dependence → will discuss that later

Regions of access to the PD - cold NeutronStars

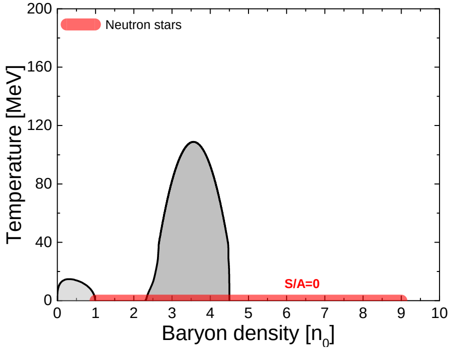
- Starting from the phase diagram in Temperature and density.
- For $T = 0$ we can use the mass-radius relation of observed stars.
- For example NICER Data $\rightarrow$ not highly-constrained. Model dependence!

TOV equation:

$$\frac{dP}{dr} = -(P+\rho)\frac{m+4\pi r^3 P}{r(r-2m)}$$



G. Raaijmakers, et.al., Astrophys. J. Lett. **918**, no.2, L29 (2021)



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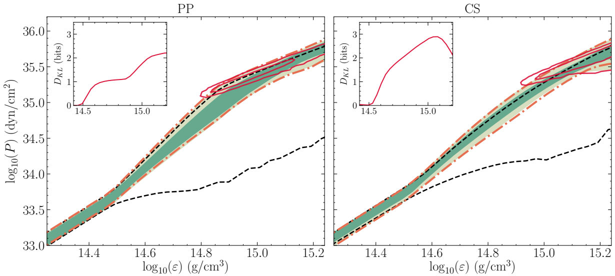

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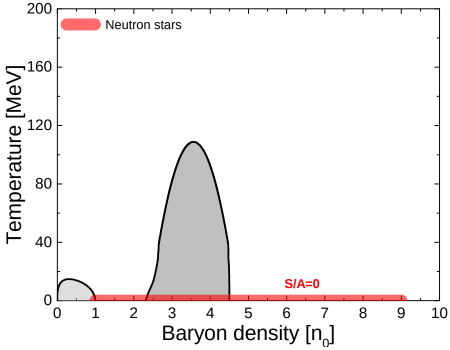

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- Constraints from neutron star mergers (pre-merger).

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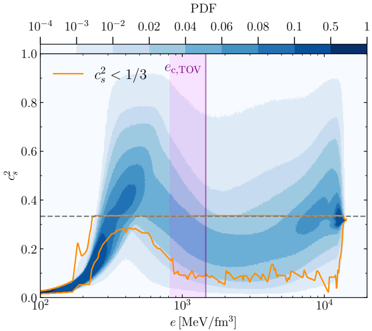
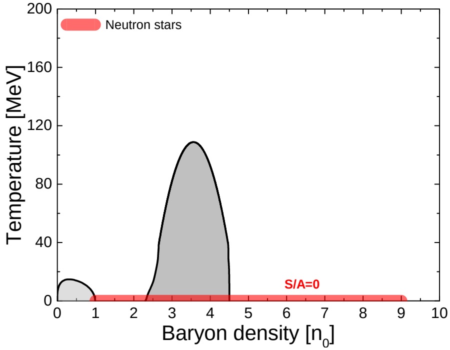


S. Altiparmak, C. Ecker and L. Rezzolla, Astrophys. J. Lett. **939**, no.2, L34 (2022).



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Ways to study the EoS

1. Pion production, assuming equilibration at different temperatures (e.g. R. Stock et.al.)

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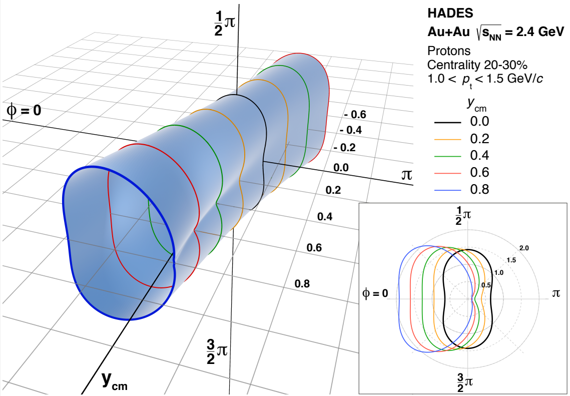
Ways to study the EoS

1. Pion production, assuming equilibration at different temperatures (e.g. R. Stock et.al.)
2. Kaon production below threshold: compression allows more secondary scatterings. IMHO: Highly model dependent!
3. The complex 3D structure of the system gives rise to a complex shape in momentum space.
4. Since this shape is a result of pressure gradients its sensitive to the EoS.
5. Collective flow: Fourier coefficients of the azimuth distributions are analyzed.
6. Lots of high quality recent data and possibly in the future (FRIB?)



J. Adamczewski Musch *et al.* [HADES], Phys. Rev. Lett. **125** (2020), 262301

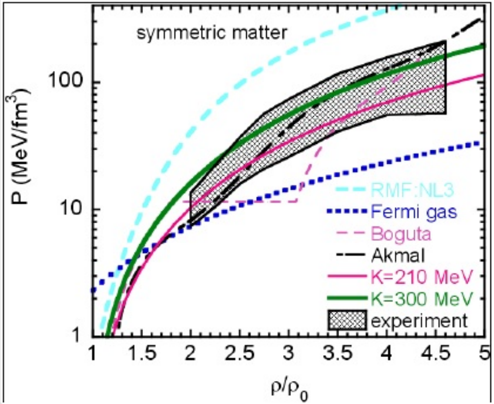
Previous results from flow

- Of course I have to mention the Danielewicz, Lacey, Lynch results from flow.
- Comparison with transport simulation. Conclusions: Soft equation of state.
- Already then: Some tension in the data observed + large uncertainties.

Skyrme model was used that is based on a 2-term expansion in density:

$$U(n_B) = \alpha \cdot n_B + \beta \cdot n_B^{\gamma} \tag{1}$$

Problem: Once saturation density and binding energy is fixed, only 1 d.o.f. left and EoS likely becomes unphysical. No structure above saturation!



Science 298 (2002) 1592-1596

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- What does 'Soft EoS' mean?

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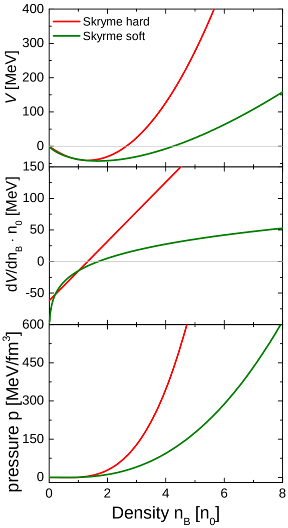

Our first strategy for Heavy Ion Collisions

Combining our knowledge

We want to understand QCD matter, not neutron star matter or heavy ion collision matter.

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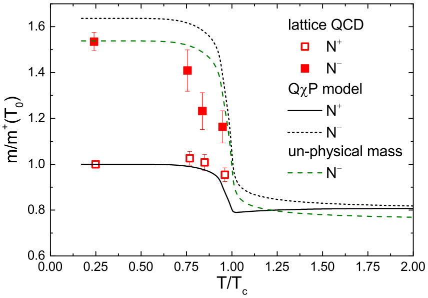
4. Cross check with astrophysical observations.
5. Reject unlikely EoS.

An effective model for everything: the Chiral Mean Field model (CMF)

It's what holds it all together.

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!



A. Mukherjee, S. Schramm, **JS** and V. Dexheimer, Astron. Astrophys. **608**, A110 (2017)

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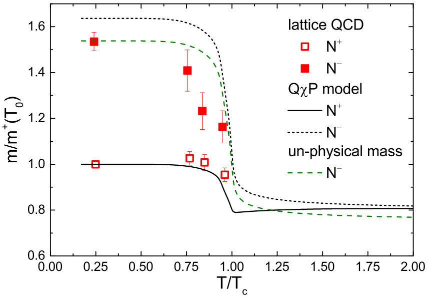

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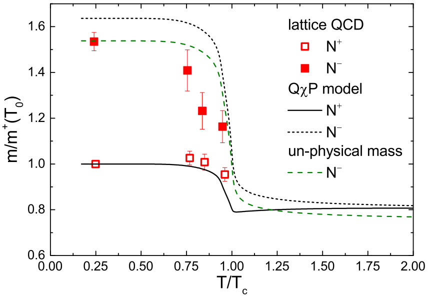

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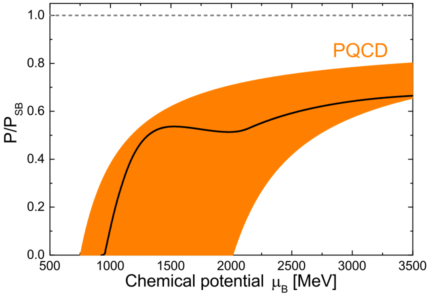
- Quarks and gluons are included in a Polyakov Loop inspired approach
- $U = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\ln[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2]$,
- Transition appears naturally through excluded volume of hadrons.

Pressure at $T = 0$



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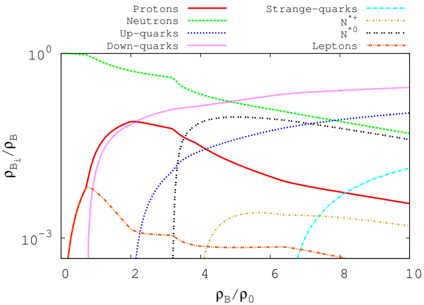
Chemical composition of neutron star matter.
A smooth transition from hadrons to quarks.



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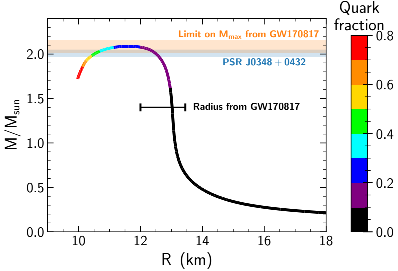
Mass-Radius relation satisfies constraints.



A. Motornenko, **JS** , V. Vovchenko, S. Schramm and H. Stoecker, Phys. Rev. C **101**, no.3, 034904 (2020)

No consistent description of crust-dense transition. Yet. FRIB!

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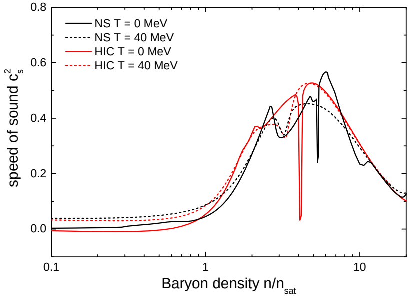
Symmetry energy at high densities decreases!

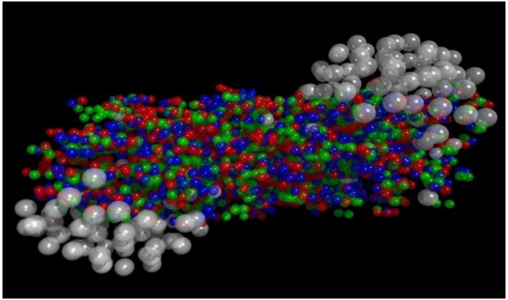


Finite temperature naturally included and is consistent with lattice QCD data!

UrQMD for the description

UrQMD is a microscopic transport model

- In cascade mode: Particles follow a straight line until they scatter.
- Only $2 \leftrightarrow 2$, $2 \leftrightarrow 1$, $2 \rightarrow N$ and $1 \rightarrow N$ interactions allowed.
- Resonance excitation and decays according to PDG values + guesstimates.
- EoS resembles a hadron resonance gas.



Any EoS in UrQMD

To implement any density dependent EoS in UrQMD:
In UrQMD the real part of the interaction is implemented by a density dependent potential energy $V(n_B)$.
Once the potential energy is known, the change of momentum of each baryon is calculated as:

$$\dot{\mathbf{p}}_i = -\frac{\partial \langle H \rangle}{\partial \mathbf{r}_i} = -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r}_i}\right) - \left(\sum_{j \neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial \mathbf{r}_i}\right) , \tag{2}$$

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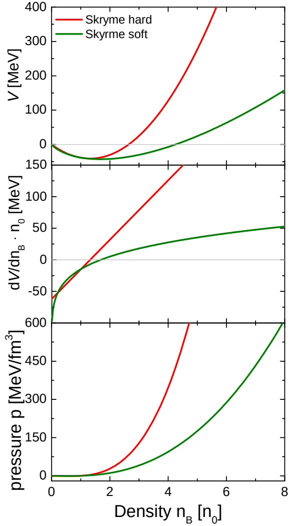
For the potential energy V often a Skyrme model was used that is based on a 2-term expansion in density:

$$U(n_B) = \alpha \cdot n_B + \beta \cdot n_B^{\gamma} \quad \text{with} \quad U(n_B) = \frac{\partial(n_B \cdot V(n_B))}{\partial n_B} \tag{3}$$

Problem: Once saturation density and binding energy is fixed, only 1 d.o.f. left and EoS likely becomes unphysical. No phase transition possible.



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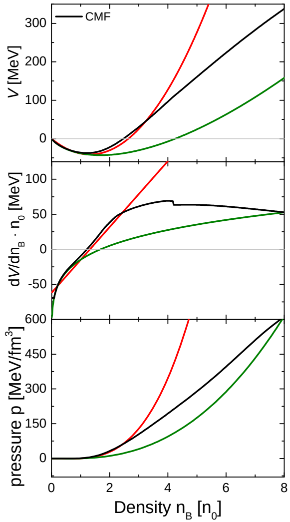
In CMF we can simply use the effective field energy per baryon E_{field}/A calculated from the CMF model:

$$V_{CMF} = E_{\text{field}}/A = E_{\text{CMF}}/A - E_{\text{FFG}}/A \, , \quad (5)$$

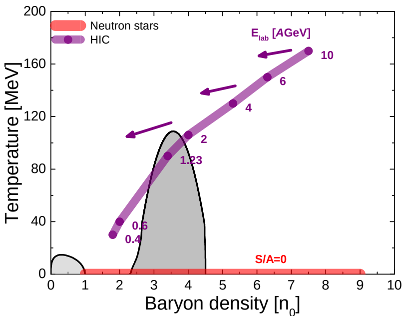
A phase transition can be simply included by adding another minimum in the potential energy: leading to (meta-)stable solutions at high density.

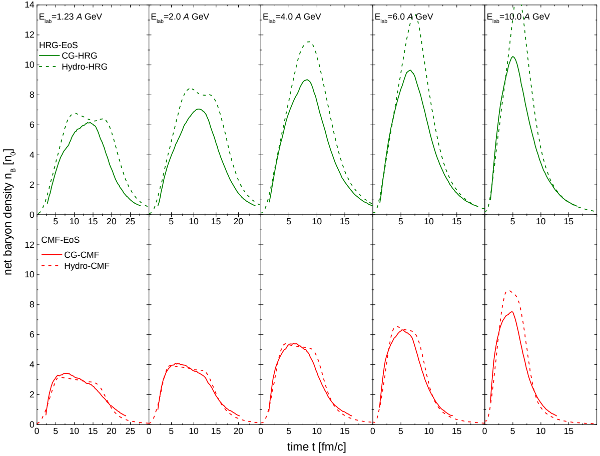


JS , A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, Eur. Phys. J. C 82, no.10, 911 (2022).

HIC regions of access

- Including the CMF EoS in UrQMD vs. a hadron resonance gas baseline.
- Bulk evolution consistent with 3+1D hydro + CMF
- Initial compression from CMF model in UrQMD



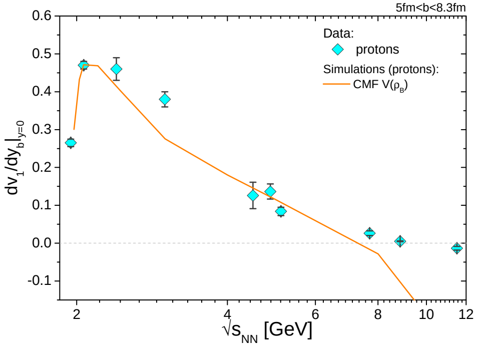


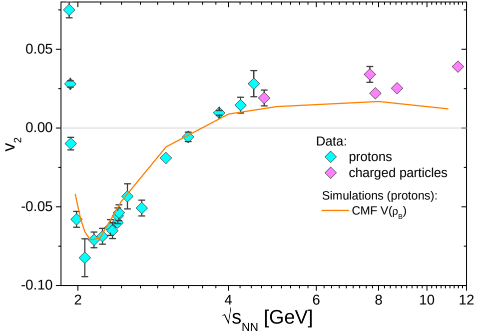
M. Omana Kuttan, A. Motornenko, **JS**, H. Stoecker, Y. Nara and M. Bleicher, Eur. Phys. J. C **82** (2022) no.5, 427

Results on flow

- The CMF EoS gives good results on flow coefficients.
- Sensitivity up to $\approx 4n_0$.

- $v_1 = p_x/p_T$
- $v_2 = (p_x^2 - p_y^2)/p_T^2$

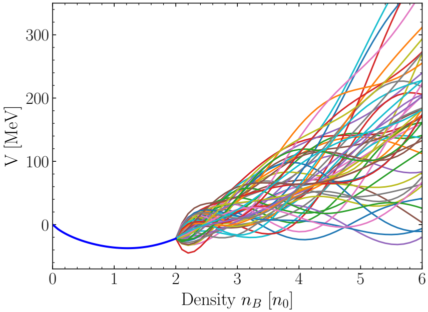




JS, A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, [arXiv:2208.12091 [nucl-th]].

Statistical analysis of available flow data

- Using Bayesian inference methods we can try to constrain the EoS from flow data
- Use UrQMD as described but parameterize $V(n_B)$ with a seventh order polynomial.



M. Omana Kuttan, JS, K. Zhou and H. Stöcker, Phys. Rev. Lett. 131, no.20, 202303 (2023).

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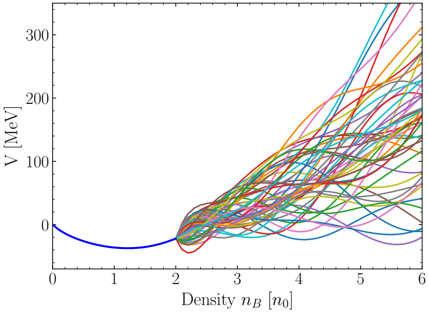
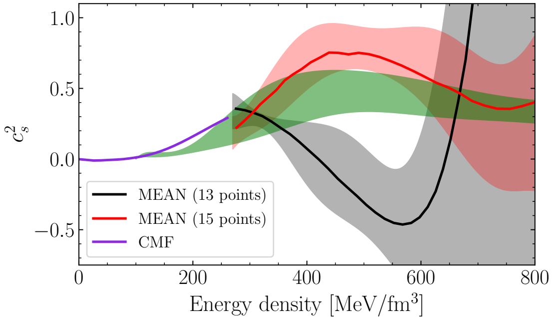


- Results depend strongly on the data used.
- If all data on the mean m_T and v_2 are used, constraints are similar to those from astrophysics (NS and BNSM).



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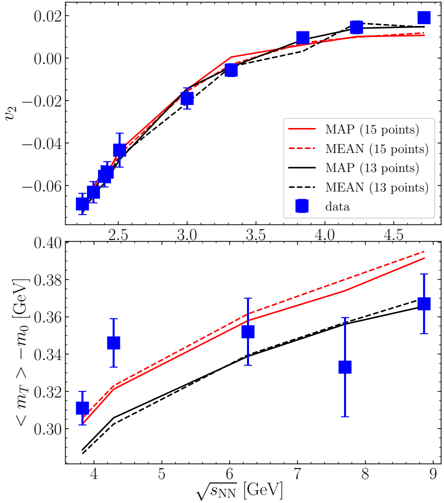
Closure test

- Sensitivity to few data points from AGS experiments.



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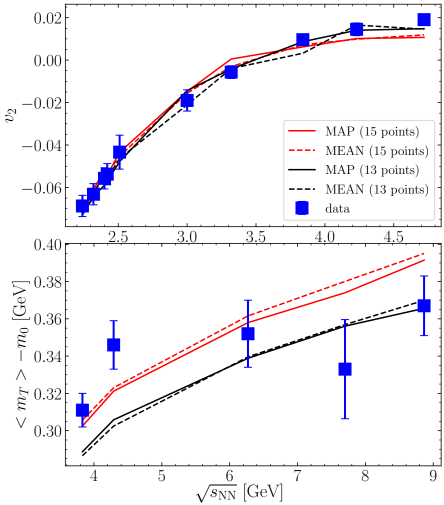
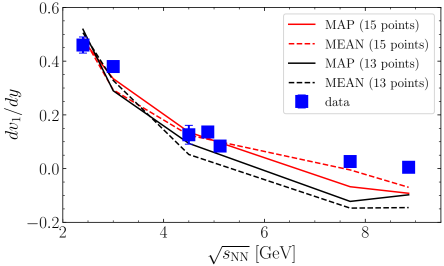


- However: directed flow also consistently described
- Data is from STAR BES and HADES, AGS data is not shown.
- Do we underestimate our systematic errors? And how to deal with it?



Other observables

The advantage of using an event generator like UrQMD: we can now study a multitude of observables:

Interferometry

The study of hadron two particle correlations allows for the study of the system size at freeze out which is sensitive to the EoS.

P. Li, JS, T. Reichert, A. Kittiratpattana, M. Bleicher and Q. Li, Sci. China Phys. Mech. Astron. **66** (2023) no.3, 232011

Dileptons

The study of electromagnetic probes (di-electrons) provides direct access to the hot and dense phase and the lifetime of the fireball.

O. Savchuk, A. Motornenko, JS, V. Vovchenko, M. Bleicher, M. Gorenstein and T. Galatyuk, [arXiv:2209.05267 [nucl-th]].

Fluctuations

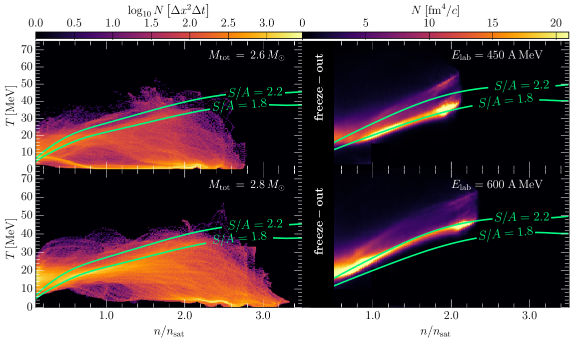
Fluctuations of conserved charges are (not) sensitive to the formation of clusters at a phase transition.

O. Savchuk, R. V. Poberezhnyuk, A. Motornenko, JS, M. I. Gorenstein and V. Vovchenko, [arXiv:2211.13200 [hep-ph]].

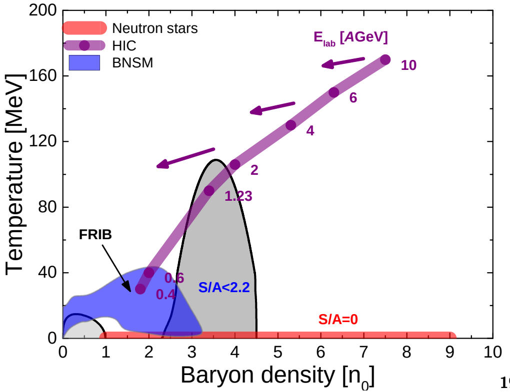
All observables indicate a rather stiff EoS.

Regions of access to the PD - BNSM

- Using BNSM we can also turn on the heat to connect to HIC and study the symmetry energy!
- During the post-merger $T < 40$ MeV is reached
- Observables: GW
- No smoking gun $\rightarrow$ quantitative studies necessary which are expensive.



E. R. Most, A. Motornenko, JS, V. Dexheimer, M. Hanauske, L. Rezzolla and H. Stoecker, Phys. Rev. D **107**, no.4, 043034 (2023).



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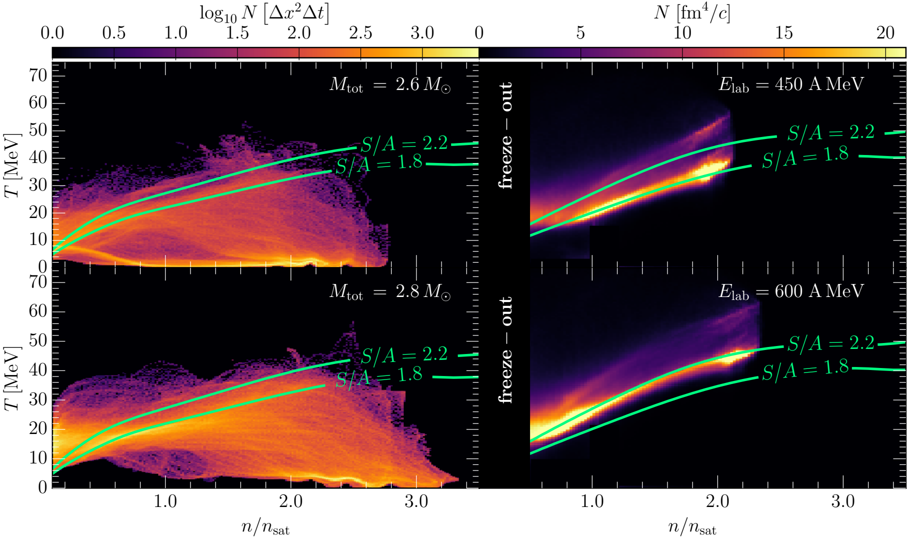

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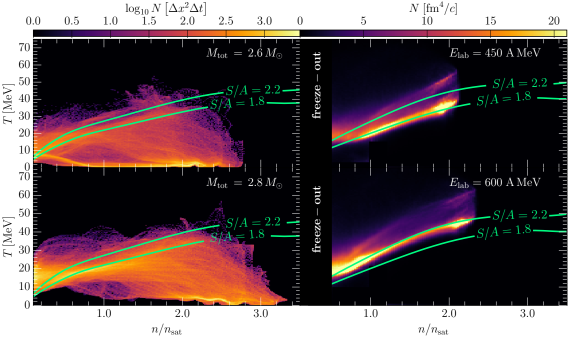

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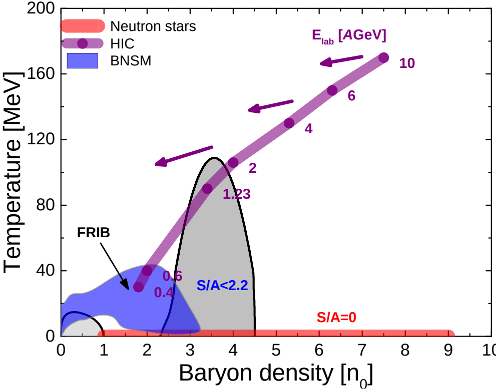
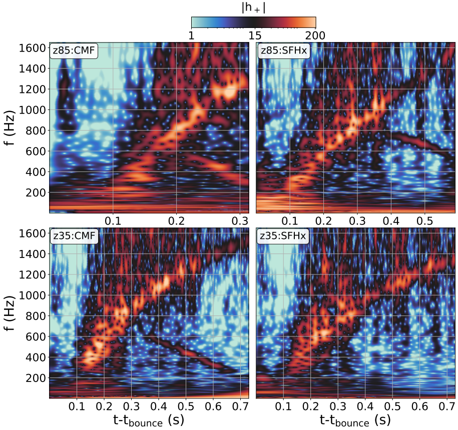
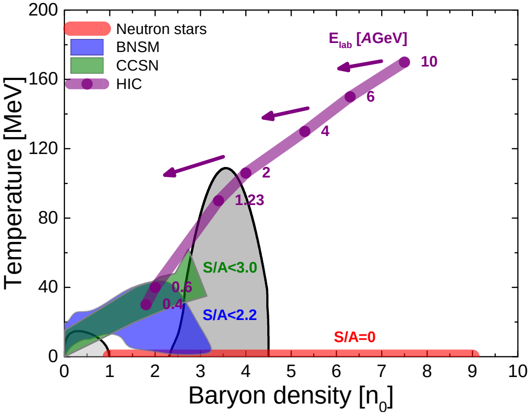


Regions of access to the PD - CCSN

- Core Collapse Supernovae (CCSN) can reach even higher S/A
- GR Hydro simulation with same EoS (CMF model): Some GW in the ${}^{2}g_{1}$-mode are sensitive to the EoS up to 2 n_0.

1



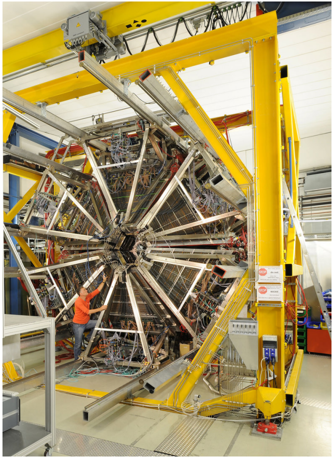


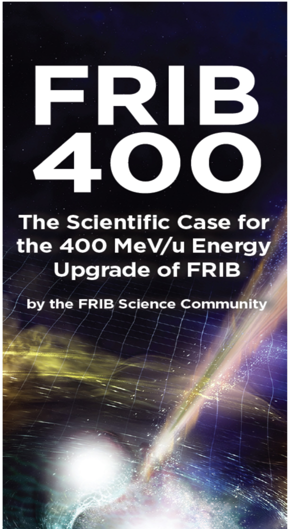
P. Jakobus, B. Mueller, A. Heger, A. Motornenko, JS and H. Stoecker, Phys. Rev. Lett. **131**, no.19, 191201 (2023).

The sensitivity on the EoS in the sub 1GeV beam energy range

Several competing/complimentary facilities will measure in this energy range

HADES@GSI: 400 and 600 MeV/u



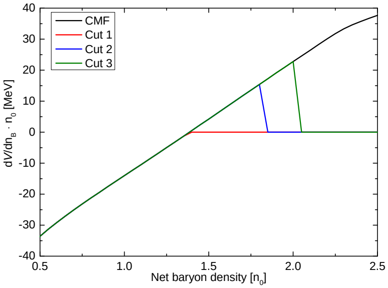


HIAF in china: up to 800 MeV/u from 2027.

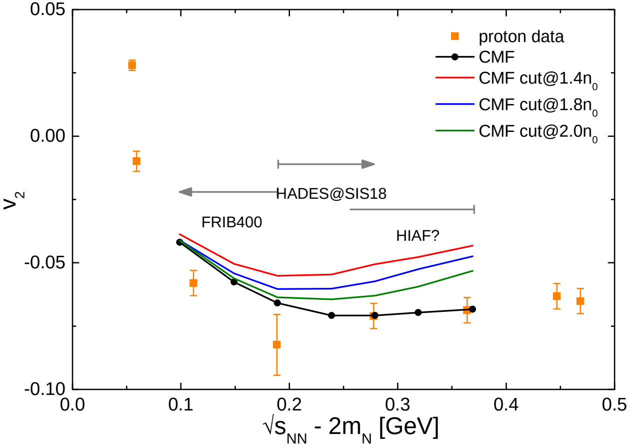
The sensitivity on the EoS in the sub 1GeV beam energy range

Which density ranges can we expect to cover at these beam energies?

Density dependent potential
(actually QMD force)



Elliptic flow of protons



- We can nicely see which ranges beam energy can be used to study which densities.

Summary and conclusions of part 1

- Can use HIC and GW astrophyics to scan the high density QCD PD.
- Especially for HIC in thelow to intermediate beam energies new ideas/methods for old and new models are necessary.
- Effects of the EoS don't occur at the same beam energy: **Need consistent modeling!!**
- Showed an example on how statistical analyses of large datasets available now and in the future can be constructed.
- Still much room and need for further development on how to connect the different regimes and observables.
- Especially the low-intermediate energy range is a big missing piece in the puzzle.

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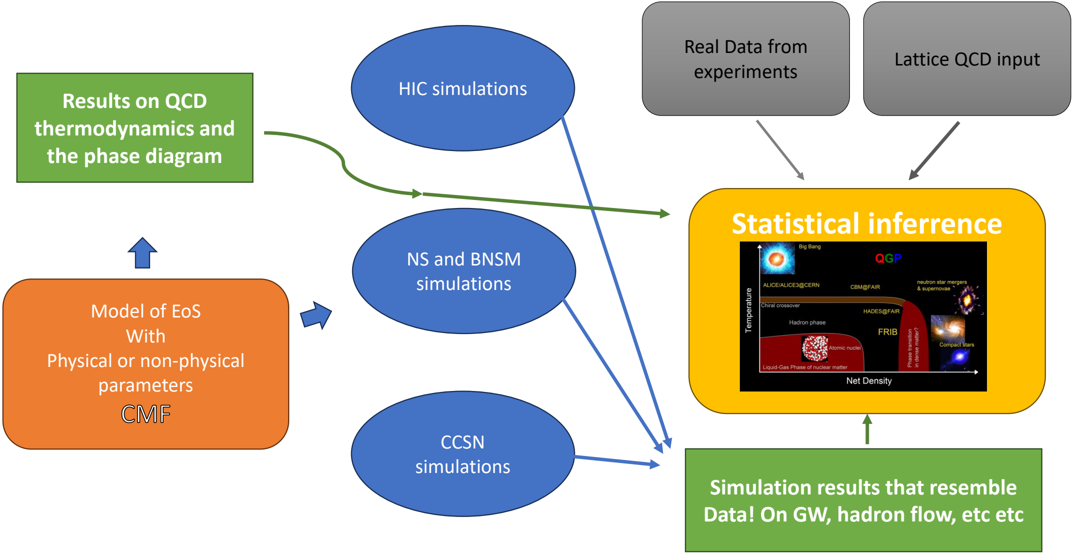
Part 2: Future challenges and ways to approach them

1. How do (future) experiments (at FRIB) contribute to the EoS?
2. Momentum dependence of potentials and the value of a good CMF.
3. The role of the Δ at finite temperature and in the isospin dependence.
4. Relativistic effects.
5. How to combine all that in a quantitative statistical analysis (or inferrence).

How to possible combine everything into a big ragout?



More open challenges

- Sensitivity to fluctuations and correlations in the nucleus - nuclear structure.
- Which observables should be used to connect the isospin dependence in HIC to GW observables?
- Pions depend on Δ-interaction which do not appear in cold NS.
- We use classical Hamiltonian dynamics. Clearly wrong. But how wrong?
- Proper relativistic QMD description is difficult to achieve (no interaction theorem).
- How can the finite T EoS be implemented?
- Interaction length scale at high density? Density dependence of the QMD-range parameter?
- Can we even think about changing d.o.f. at the phase transition?

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- Good thing: Enough to do for a several year research program.

(Why and how) Introducing a momentum dependent potential?

- For years the importance of including a momentum dependent potential has been highlighted
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What convinced me that there is at least something

The definition of the single particle potential is:

$$U_i = E_i^* - E_i \tag{6}$$

where E_i is simply the non-interacting single particle energy $E_i = \sqrt{m_i^2 + p_i^2} - \mu_i$.
and E_i^* is the interacting one $E_i^* = \sqrt{m_i^{*2} + p_i^2} - \mu_i^*$.
It is easy to see that U_i will have a momentum dependent part (from the scalar interactions) and a non momentum dependent part (from the vector interactions). [a]

[a] See also recent work by Yasushi Nara.

Momentum dependence from CMF model

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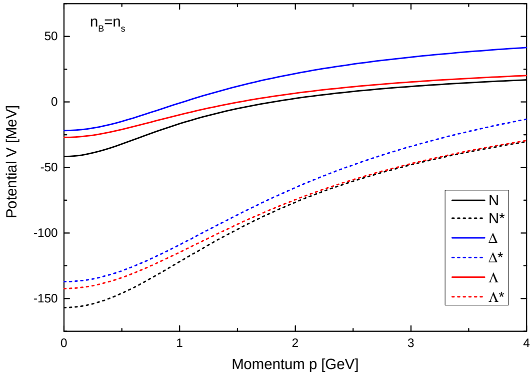
- Parity partners are deeply bound at saturation density. Leads to enhanced correlations and other possibly interesting effects.



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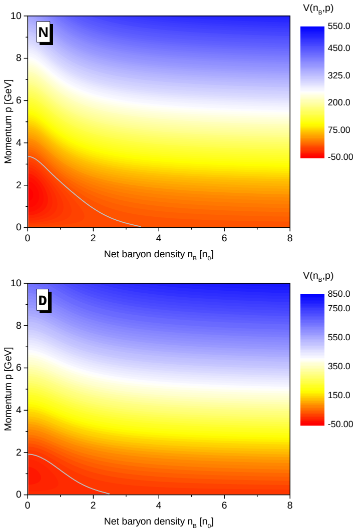
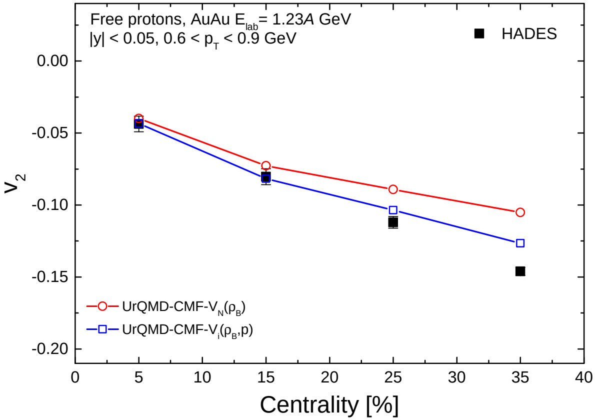
- Parity partners are deeply bound at saturation density. Leads to enhanced correlations and other possibly interesting effects.
- In addition we have all the potentials as function of density + momentum



Momentum dependence effects

- Effects with respect to centrality dependence of v_2 (HADES data)



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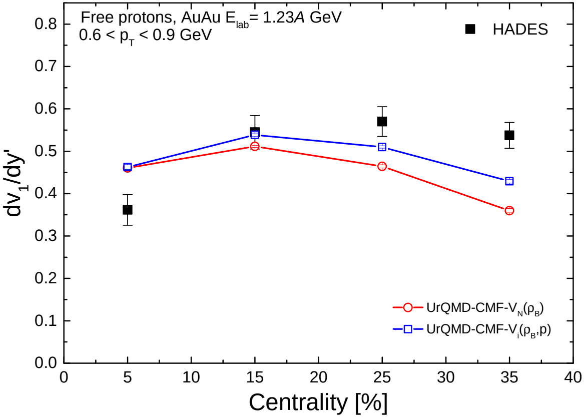
Momentum dependence effects

- Effects with respect to centrality dependence of v_2 (HADES data)
- And v_1.



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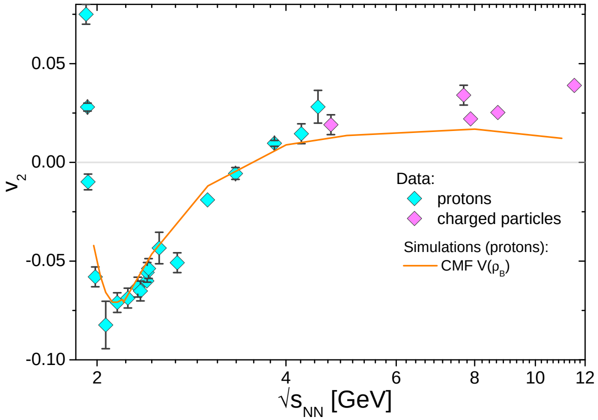
Momentum dependence effects

- The integrated flow without momentum dependence.



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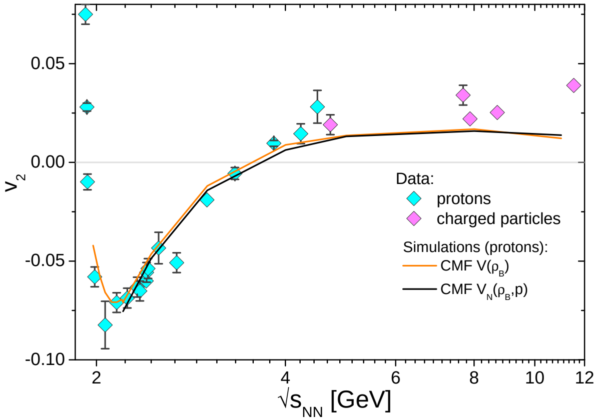
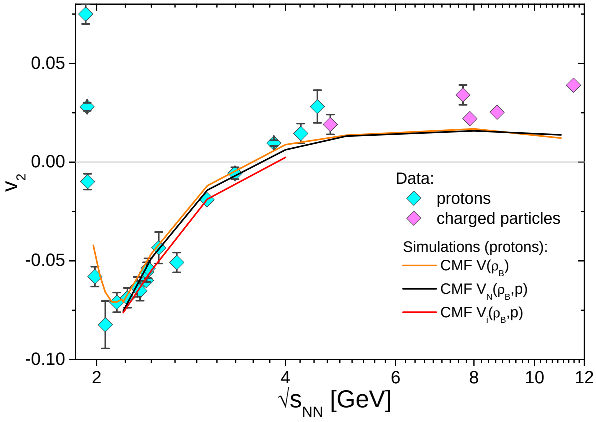


Momentum dependence effects

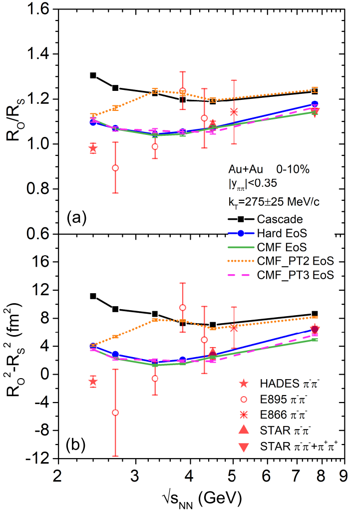
- The integrated flow without momentum dependence.
- The integrated flow with momentum dependence as for nucleons.
- The integrated flow with momentum dependence for all.
- Effect from Delta potential more important.
- Effect from only nucleon probably small because the momentum dependence is consistent with the overall EoS.
- Picture changes at low beam energies. Also keep in mind effect on pions and isospin.



HBT

- Hanbury-Brown-Twiss (HBT) correlations for charged pions are a tool to measure the freezeout volume and time.
- Pions that are emitted close in coordinate space are correlated in momentum space.
- Simulation with a PT show a clear maximum.
- 'Old' data seem inconclusive, newest STAR data have much smaller error and favor the no-PT scenario.
- Sensitivity only up to $\approx 4n_0$.

P. Li, T. Reichert, A. Kittiratpattana, JS, M. Bleicher, Q. Li



Dileptons

- Hydro simulations have suggested a strong increase (of factor 2) of the dilepton yield for a phase transition:
  F. Seck, T. Galatyuk, A. Mukherjee, R. Rapp, JS and J. Stroth, [arXiv:2010.04614 [nucl-th]].
- A significant increase of the low mass dilepton yield is observed when a phase transition is included in the UrQMD-CMF model.

O. Savchuk, A Motornenko, JS, V. Vovchenko, M. Bleicher, M. Gorenstein, T. Galatyuk, [arXiv:2209.05267 [nucl-th]].