Genova, seminar, 13th September 2023

EFT, Positivity, and the Electroweak Hierarchy

Tevong You

2308.06226 Davighi, Melville, Mimasu, TY

Outline

Naturalness

Effective Field Theory (EFT)

Positivity bounds on EFT

Positivity bounds on scalar potentials

Positively light Higgs

Naturalness

Take aesthetic problems seriously.

Example 1

$$F = m_{inertia} a \qquad F \propto \frac{q_1 q_2}{r^2}$$

Inertial mass and charge have nothing to do with each other, and yet for gravity we arbitrarily set by hand

$$\mathrm{q} = m_{inertia}$$

Solution to this equivalence problem took centuries: Newtonian gravity → GR

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

<u>Example 2</u>

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\text{Coulomb}}. \qquad \Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}.$$

Avoiding cancellation between “bare” mass and divergent self-energy in classical electrodynamics requires new physics around

$$e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8 \times 10^{-13} \text{ cm}$$

Indeed, the positron and quantum-mechanics appears just before!

$$\Delta E = \Delta E_{\text{Coulomb}} + \Delta E_{\text{pair}} = \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}$$

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Example 3

Divergence in pion mass: $$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\Lambda^2$$

Experimental value is $m_{\pi^\pm}^2 - m_{\pi_0}^2 \sim (35.5\,\mathrm{MeV})^2$

Expect new physics at $\Lambda \sim 850$ MeV to avoid fine-tuned cancellation.

ρ meson appears at 775 MeV!

Naturalness

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Example 4

Divergence in Kaons mass difference in a theory with only up, down, strange:

$$m_{K_L^0} - m_{K_S^0} = \simeq \frac{1}{16\pi^2} m_K f_K^2 G_F^2 \sin^2\theta_C \cos^2\theta_C \times \Lambda^2$$

Avoiding fine-tuned cancellation requires $\Lambda < 3$ GeV.

Gaillard & Lee in 1974 predicted the charm quark mass!

Naturalness

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1307.7879 G. Giudice - Naturalness after LHC

Higgs?

Higgs also has a quadratically divergent contribution to its mass

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda\right)$$

Avoiding fine-tuned cancellation requires $\Lambda < O(100)$ GeV??

As Λ is pushed to the TeV scale by null results, tuning is around 10% - 1%.

Note: in the SM the Higgs mass is a parameter to be measured, not calculated. What the quadratic divergence represents (independently of the choice of renormalisation scheme) is the fine-tuning in an underlying theory in which we expect the Higgs mass to be calculable.

Naturalness

Gauge theories have the quality we seek in a satisfying theory.

In contrast, everything to do with the Higgs in the SM is arbitrary; more like a *parametrisation* than an explanation of electroweak symmetry breaking.

We seek to better understand the origin of the Higgs in an underlying theory from which it emerges, where we can calculate its potential in terms of more fundamental principles (*c.f.* condensed-matter Higgs)

Avoiding fine-tuning in underlying theory = expect such new physics to appear close to the weak scale!

Naturalness

Take fine-tuning problems seriously. However, new physics appears not to be at the weak scale.

Symmetry may not be the answer. Cosmological dynamics? *e.g.* 1504.07551 Graham, Kaplan, Rajendran, 2105.08617 Giudice, McCullough, TY

UV-IR relation? *e.g.* 1909.01365 Craig, Koren

When past successful approaches fail, *we have a chance to learn something genuinely new.*

This talk: Perhaps we live in a fine-tuned corner of EFT parameter space where the Higgs *must* be light!

2308.06226 Davighi, Melville, Mimasu, TY

Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Effective Field Theory

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Effective Field Theory

$$\mathcal{L} = \overbrace{\Lambda^4 + \Lambda^2 \mathcal{O}^{(2)}}^{\text{Naturalness?}} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

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Effective Field Theory

e.g. QED as an EFT includes Fermi theory and Euler-Heisenberg dimension-8 operators

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi}\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Fermi theory
(1933)

$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2} (\bar{\Psi} \Gamma \Psi)(\bar{\Psi} \Gamma \Psi)$$

$$\Gamma = \{ \mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} \}$$

Euler-
Heisenberg
(1936)

$$+ \frac{c_8^{(1)}}{\Lambda^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} + \ldots$$

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Euler-Heisenberg (1936)

$$+ \frac{\boxed{c_8^{(1)}}}{\Lambda^4} (F_{\mu\nu}F^{\mu\nu})^2 + \frac{\boxed{c_8^{(2)}}}{\Lambda^4} F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + \dots$$

Wilson coefficients generated by UV physics

Effective Field Theory

The SM *is* an Effective Field Theory; SMEFT is the Fermi theory of the 21st century

$$\mathcal{L}_{SM}^{EFT} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} \\
\mathcal{L}_H &= (D^L_\mu\phi)^\dagger(D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d\bar{Q}_L\phi q^d_R + y_u\bar{Q}_L\phi^c q^u_R + y_L\bar{L}_L\phi l_R + \text{h.c.} \qquad ,
\end{aligned}$$

Explore heavy BSM physics in this framework

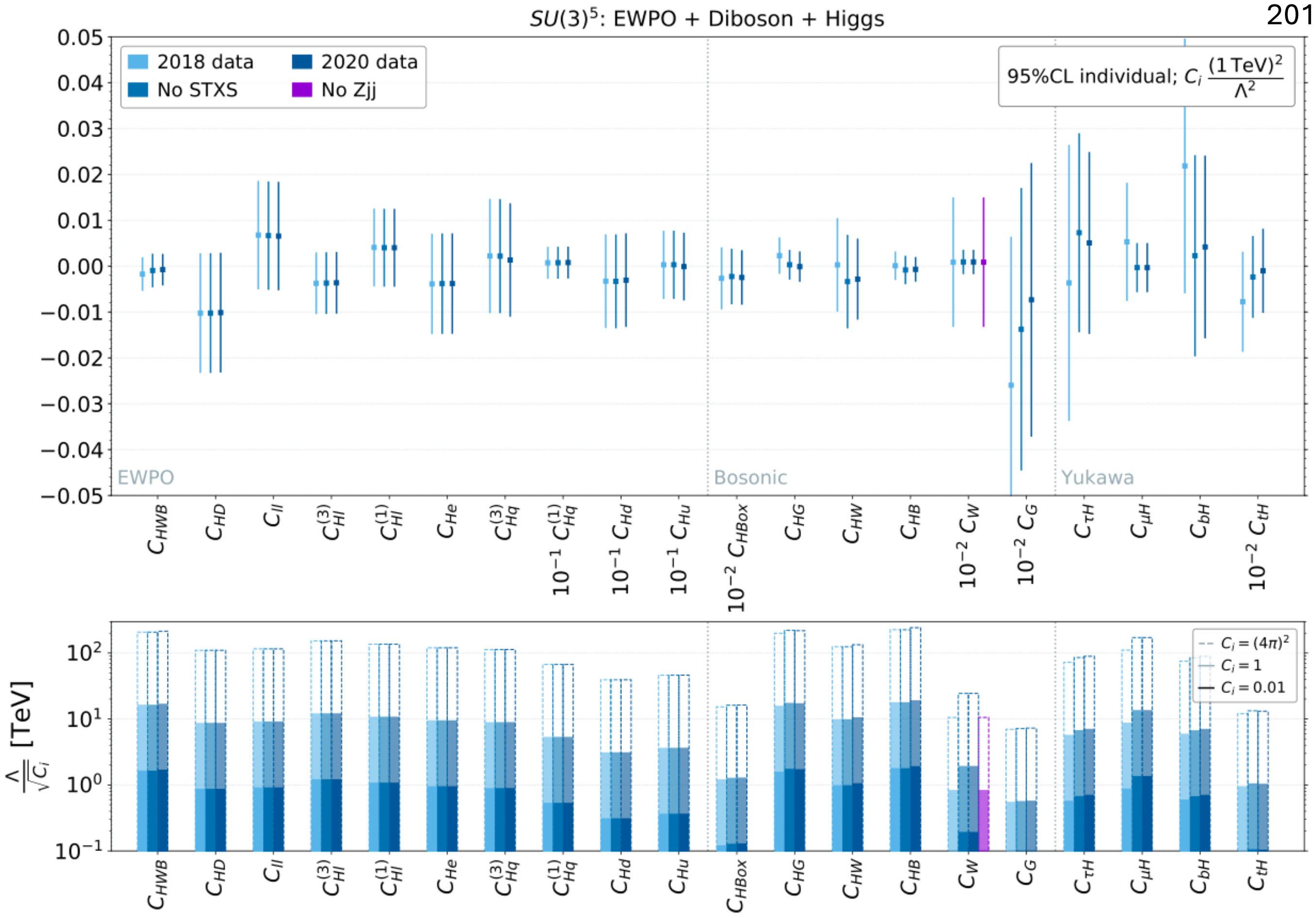
This does not exclude the possibility of light new physics; just add those fields in as part of the EFT if desired or discovered.

Non-linear chiral electroweak lagrangian + singlet scalar is a more general EFT framework (known as HEFT).

Effective Field Theory

The SM *is* an Effective Field Theory; SMEFT is the Fermi theory of the 21st century

2012.02779 Ellis, Madigan, Mimasu, Sanz, TY



For virtually all particles of the SM indirect evidence preceded direct discovery. May be true of BSM!

Effective Field Theory

Energy

Λ $\qquad \mathcal{L}_{UV} = ?$

$E < M_W$

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi}\Psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$+ \sum_i \boxed{\frac{C_6^{(i)}}{\Lambda^2} (\bar{\Psi} \Gamma^i \Psi)(\bar{\Psi} \Gamma^i \Psi)} \qquad \Gamma^i = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$

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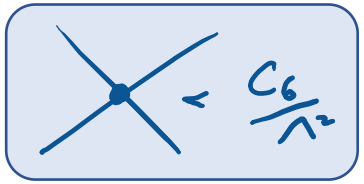

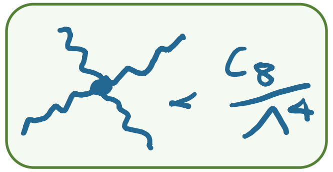

Effective Field Theory

Energy

$\boldsymbol{\Lambda}$

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \qquad ,$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R \\
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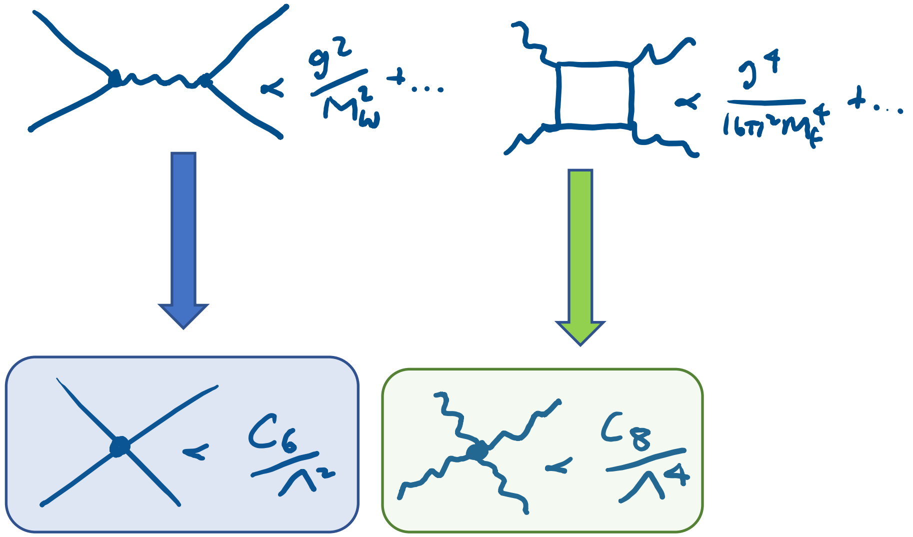

Effective Field Theory

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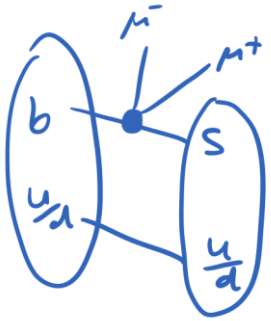

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Effective Field Theory

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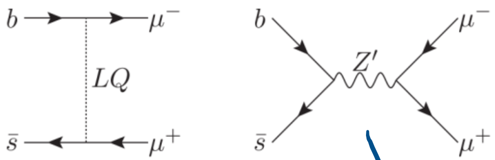

$\boldsymbol{\Lambda}$

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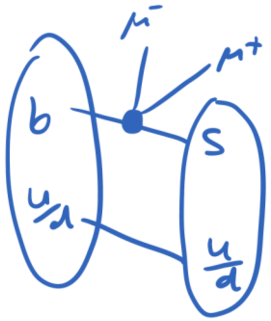

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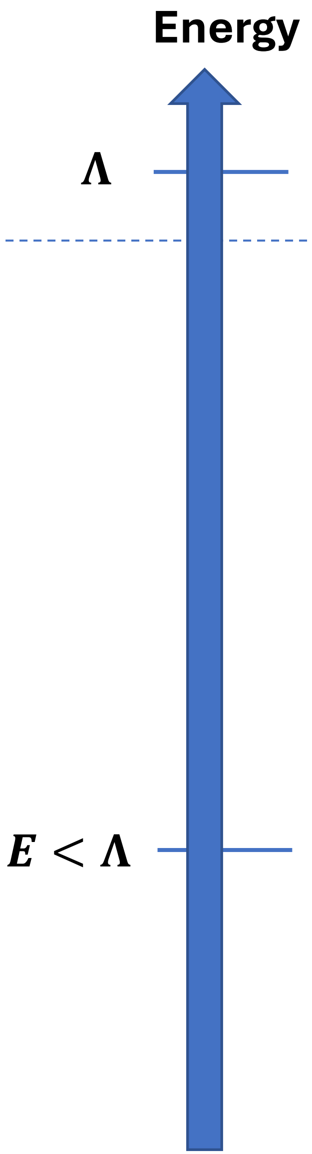
EFT Positivity Bounds



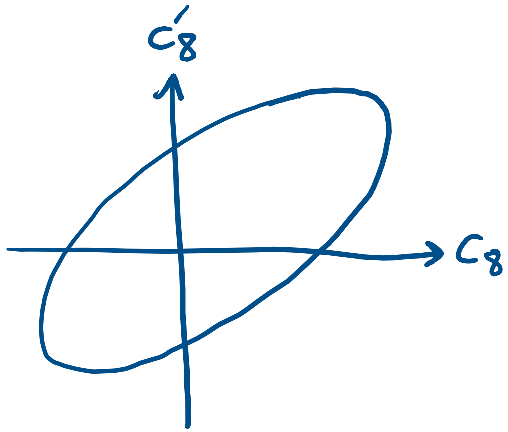
$$\mathcal{L}_{UV} = ?$$

$$\mathcal{L}_{IR} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \dots$$

EFT Positivity Bounds

Energy

$\boldsymbol{\Lambda}$ $\mathcal{L}_{UV} = ?$



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EFT Positivity Bounds

Energy

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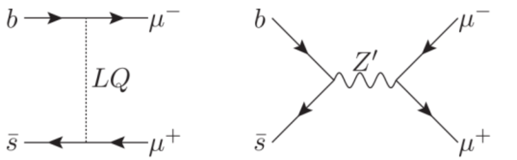

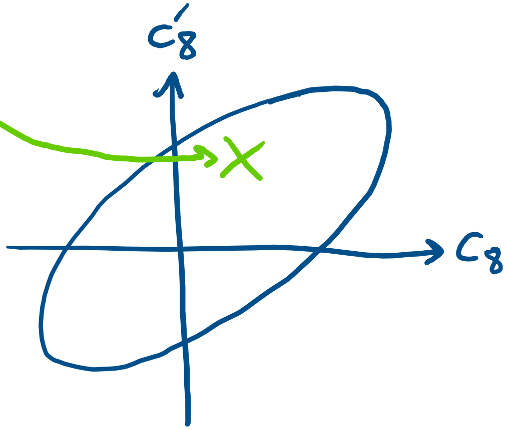

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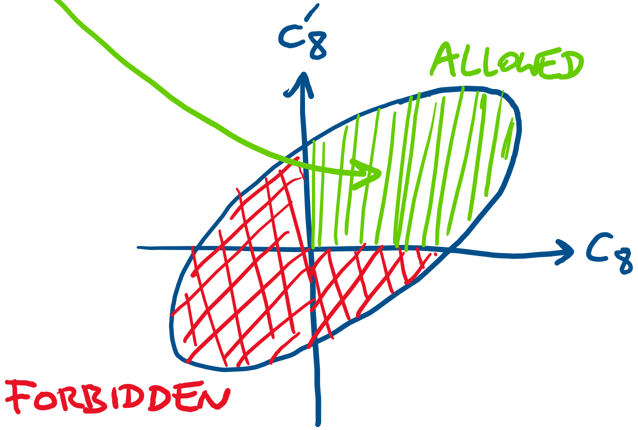
Matching explicit UV models populates a subspace of Wilson coefficient space

EFT Positivity Bounds

Energy

Λ $\mathcal{L}_{UV} = ?$

Unitarity | Locality | Causality | ...



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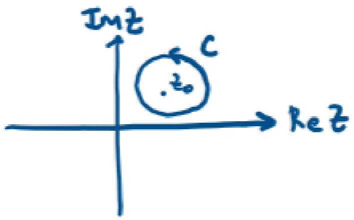
Positivity bounds forbid signs of Wilson coefficients *assuming only general principles* in the UV

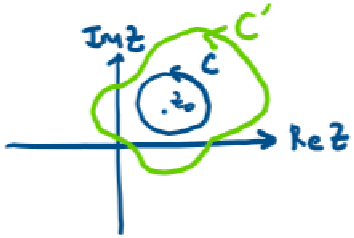
EFT Positivity Bounds

Contour integral isolates coefficient of simple pole

- $f(z) = a_n (z-z_0)^n + \dots + a_1 (z-z_0) + a_0 + \frac{b_1}{z-z_0} + \frac{b_2}{(z-z_0)^2} + \dots$



$$\Rightarrow \oint_C \frac{dz}{2\pi i} f(z) = b_1$$

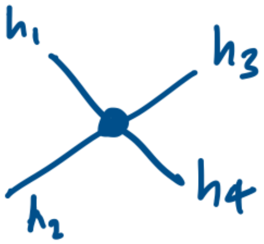


$$\Rightarrow \oint_{C'} \frac{dz}{2\pi i} f(z) = b_1$$

Analyticity allows contour deformation

EFT Positivity Bounds

Analytically continue 2-to-2 scattering amplitude $A(s)$ to complex s



$$= \hat{A}_{h_1 h_2 h_3 h_4}(s,t) = \sum_{i,j=0} C_{i,j} \frac{s^i t^j}{\Lambda^{2i+2j}}$$

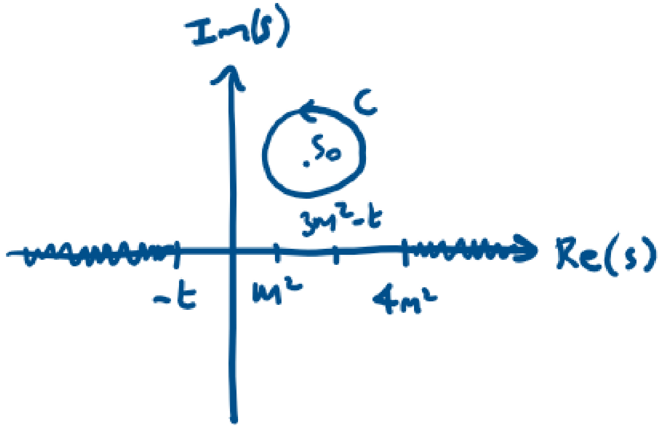
e.g. $\mathcal{L}_{\text{dim-8}} = \dfrac{\bar{c}_8}{\Lambda^4} (\Psi \partial \Psi)^2 \Rightarrow \hat{A} \sim \dfrac{\bar{c}_8 s^2}{\Lambda^4}$

Higher-dimensional operators contribute to amplitude at different powers of s

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1 h_2 h_3 h_4}(s,t)}{(s-s_0)^{2N+1}}$$



Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

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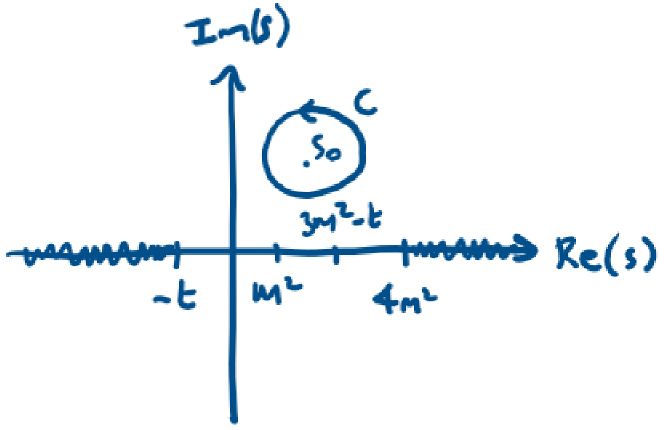

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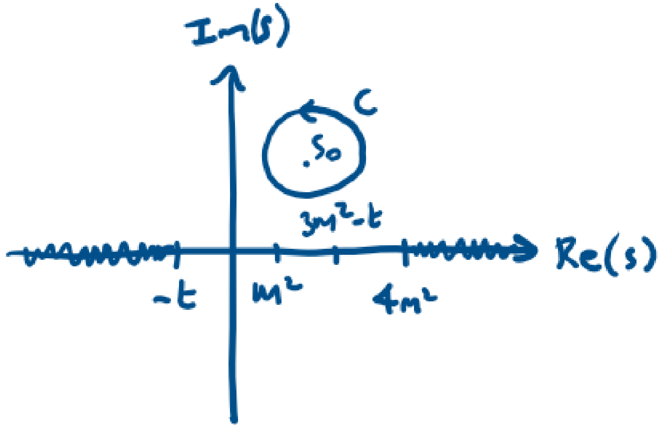

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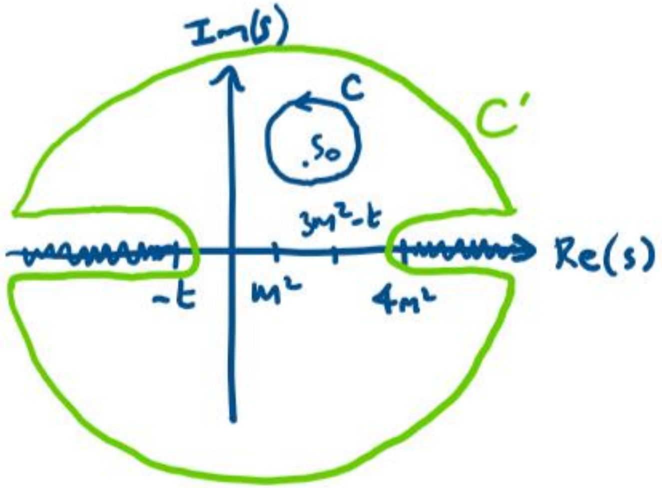

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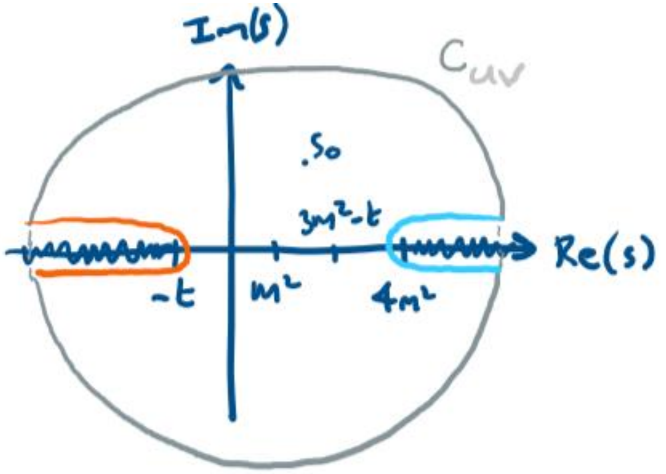
$$= \underbrace{I_{uv}}_{\text{locality}} + \underbrace{I_s + I_u}_{\text{unitarity}} > 0 \ ?$$



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EFT Positivity Bounds

e.g. 2203.06805 Snowmass review

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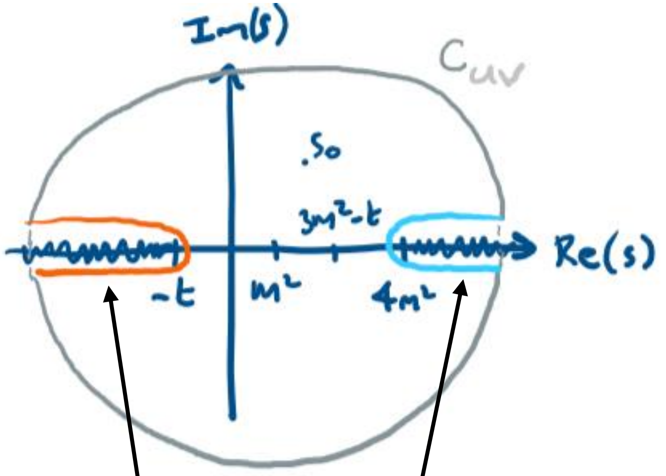


$$\lim_{|s|\to\infty} \hat{A}(s) < s^2$$

$$\mathbb{1} = S^\dagger S = (\mathbb{1} + iT)^\dagger(\mathbb{1} + iT) \Rightarrow i(T^\dagger - T) = |T|^2 \geq 0$$

Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

EFT Positivity Bounds

e.g. 2203.06805 Snowmass review

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\Rightarrow \quad \partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

EFT Positivity Bounds

e.g. Contour integral isolates dimension-8 operator contributions for N = 1

$$\mathcal{L}_{\text{EFT}}[v] = \bar{c}_8 \frac{\mathcal{O}_8}{\Lambda^4}$$

$$\bar{c}_8 \sim \partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

Potential Positivity Bounds

Scalar potentials with a stable vev can contribute to positivity bounds

2308.06226 Davighi, Melville, Mimasu, TY

$$\mathcal{L}_{\mathrm{EFT}}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

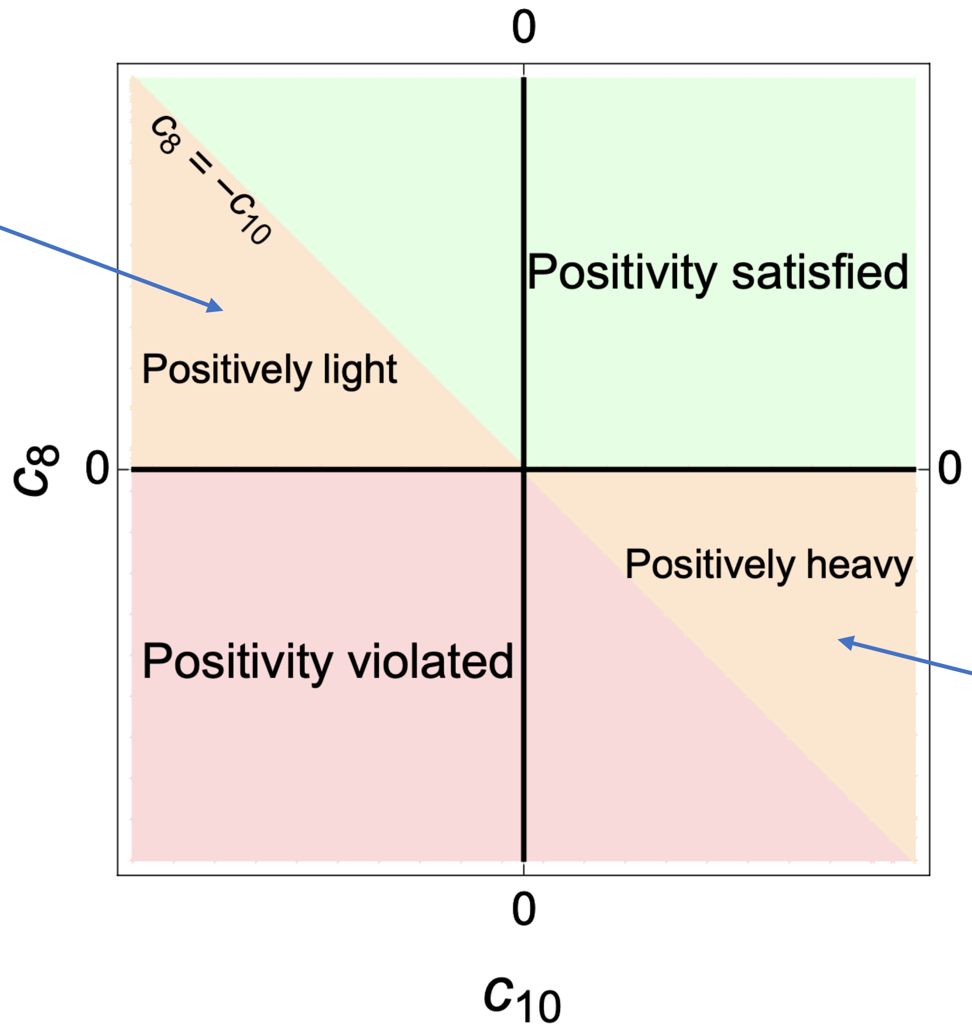
$$c_8 + \frac{v^2}{\Lambda^2} c_{10} \geq 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

Positively light Higgs

A unitary, local, and causal UV theory that lives in $|c_8| \ll |c_{10}|$ EFT parameter space *necessarily* has restricted vev v

$$\boxed{\frac{v^2}{\Lambda^2} < \frac{|c_8|}{|c_{10}|}}$$

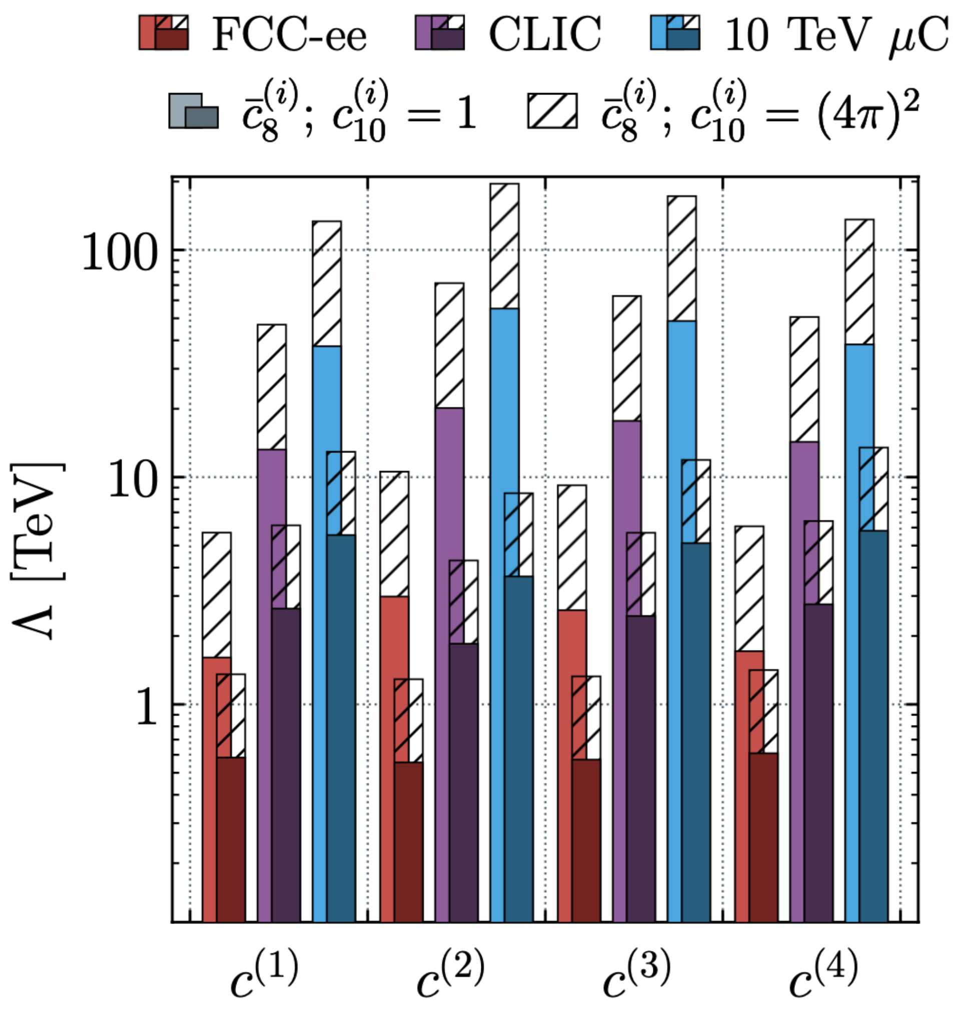


$$c_8 + \frac{v^2}{\Lambda^2} c_{10} \geq 0$$

$$\boxed{\frac{v^2}{\Lambda^2} > \frac{|c_8|}{|c_{10}|}}$$

Positively light Higgs

This scenario could in principle be established experimentally for a little hierarchy up to O(10) TeV

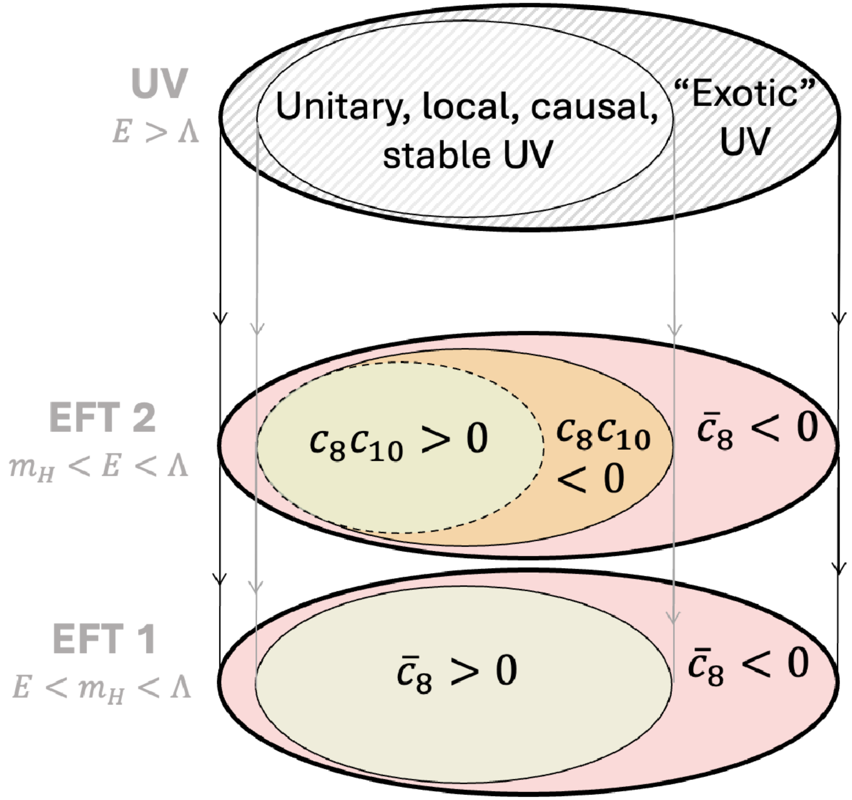


$$\mathcal{L}_{\rm EFT}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

$$\begin{aligned}
\mathcal{O}_8^{(1)} &= \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{e}_i \gamma_\mu e_i\right) \ , \\
\mathcal{O}_8^{(2)} &= \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) \ , \\
\mathcal{O}_8^{(3)} &= D^\nu \left(\bar{e}_i L_i\right) D_\nu \left(\bar{L}_i e_i\right) \ , \\
\mathcal{O}_8^{(4)} &= \partial^\nu \left(\bar{L}_i \gamma^\mu L_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) \ ,
\end{aligned}$$

Conclusion

There exists a region of EFT parameter space where positivity is conditional upon a scalar vev hierarchy



Connects an *a priori* unrelated IR observable to a restricted Higgs vev through general UV assumptions

(*c.f.* Fifth force and Weak Gravity Conjecture = light Higgs) [1407.7865 Cheung & Remmen]

Conclusion

Everything about the SM Higgs potential coefficients are highly non-generic:

Dimension-0 operator (cosmological constant) balanced between implosion and explosion

Dimension-4 operator (Higgs quartic) on boundary of vacuum stability and instability

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

Dimension-2 operator (Higgs mass) tuned between unbroken and phase phases

Higher-dimensional operator coefficients may also place us on the edge of positive and non-positive theory space!

Backup

Experimental sensitivity projections

Collider	Runs: Energy [GeV] (Luminosity [ab^{-1}])			
FCC-ee	161 (10),	240 (5),	350 (0.2),	365 (1.5)
CLIC	380 (0.5),	1500 (2),	3000 (4)	
μC	10000 (10)			

Collider	E_{CM} [GeV]	$\cos(\Delta\theta_{\ell^+\ell^-})$ bin edges
FCC-ee	161	10 equally spaced bins in $[-1, 1]$
	240	[-1. , -0.38, 1.]
	350	[-1. , -0.31, 0.49, 1.]
	365	[-1. , -0.28, 0.54, 1.]
CLIC	380	[-1. , -0.26, 0.59, 1.]
	1500	[-1. , -0.95, -0.9 , 1.]
	3000	[-1. , -0.95, -0.9 , -0.85, -0.8 , 1.]
μC	10000	[-1. , -0.95, -0.9 , 1.]