Imaging calorimetry in scintillating media for high energy physics (GRAIN) and tomography

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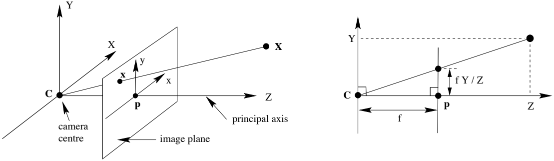
Outline

Aims of the Project for the Lecce Unit (Phase 1)

1. Development of general-purpose analytical reconstruction algorithms
2. Development of mathematical framework for analytical reconstruction
3. Development of general-purpose numerical reconstruction algorithms
4. Development of mathematical framework for numerical reconstruction
5. Focus on Multiple Coded Masks
6. Multiple Optical devices (Lenses and Coded Masks)

The Camera Model

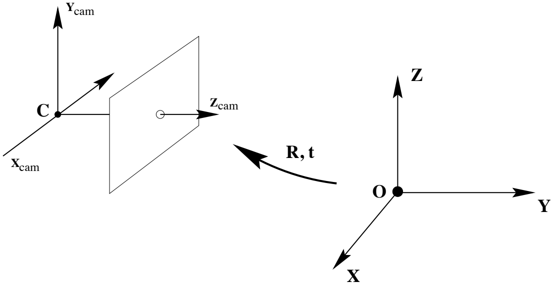


The Pinhole Camera Model

$$\lambda\, \mathbf{x} = P\mathbf{X}, \quad \forall\, \lambda \in \mathbb{R}_{/0}, \qquad P: \; \mathbf{X} \in \mathbb{P}^3 \rightarrow \mathbf{x} \in \mathbb{P}^2$$

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1\mathsf{T}} \\ \mathbf{P}^{2\mathsf{T}} \\ \mathbf{P}^{3\mathsf{T}} \end{bmatrix}$$

The Camera projection matrix



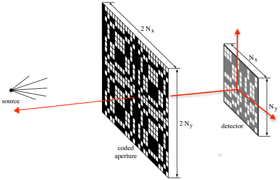
$$P = K\,[R|\mathbf{t}]\,,$$

$R \in SO(3)$ and $\mathbb{R}^3 \ni \mathbf{t} = -R\,\mathbf{c}$: $\qquad P\,\mathbf{C} = P\begin{pmatrix}\mathbf{c}\\ 1\end{pmatrix} = 0$

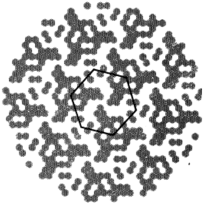
Camera calibration matrix $\qquad K = \begin{pmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}$

Can the projective description applicable to the Coded Masks?

1. A Coded Mask is **not** a centered optical system !
2. Approximations are needed and to be verified!



Mosaic of 2×2 cyclic coded aperture imaging system



An HURA of order 79.

0-order Approximation: determining P

n source points

$$\mathbf{X}_i \leftrightarrow \mathbf{x}_i \;\Leftrightarrow\; \exists\, \lambda_i : \; \lambda_i\, \mathbf{x}_i = \; \lambda_i\; (x_i,\; y_i,\; w_i)^T = P\, \mathbf{X}_i \quad i = 1, \ldots, n$$

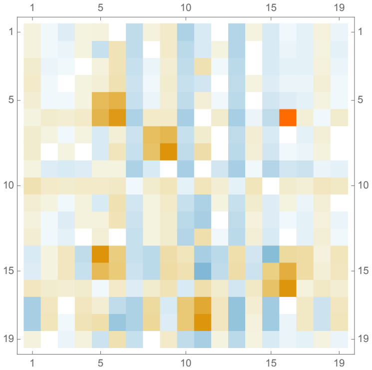
$$\mathbf{x}_i \wedge P\, \mathbf{X}_i = 0 \Leftrightarrow A \begin{pmatrix} P^1 \\ P^2 \\ P^3 \end{pmatrix} = 0$$

$A \in \mathbb{R}^{2n \times 12} \quad \det A = 0,\ \operatorname{rank} A = 11$

P is defined by $n \geq 6$ points (modulo a scale factor)

Application to a MURA mask

A 19×19 MURA mask, with parameters
$a \to 15, b \to 1.3637, p_m \to 0.11, p_d \to 0.12 \;\; (cm)$
6 point-like sources (minimal constraint)



$$\Rightarrow \qquad A_{ex}, \quad \det A_{ex} \neq 0, \quad A_{ex}\, P_{ex} = 0$$

Solving for P

Direct Linear Algorithm

- Assemble $A \in \mathbb{R}^{2n \times 12}$
- Performe the Singular Valued Decomposition $A = U\ D\ V^T$, with $D = \operatorname{diag}(\lambda_{max}, \ldots, \lambda_{min}) \in \mathbb{R}^{12 \times 12},\ \lambda_i > 0$
- $P = V_{\lambda_{min}}$

λ_{min} of $A_{ex} = 0.05042$

$$P_{ex} = \begin{pmatrix} -0.081 & -0.404 & 0.480 & 0.174 \\ 0.209 & -0.839 & -0.534 & 1.851 \\ -1.074 & 0.136 & 1. & -0.920 \end{pmatrix}$$

$$P_{ex}.\text{Sorgenti} \longrightarrow \frac{\Delta\text{Immagini}}{Immagini} = \begin{pmatrix} 0.005 & 0.005 \\ 0.02 & 0.01 \\ 0.01 & 0.02 \\ 0.02 & 0.03 \\ 0.03 & 0.01 \\ 0.003 & 0.01 \end{pmatrix}$$

Calibration matrix

$$K = P_{ex}^{1-3} R^{-1} = \begin{pmatrix} 0.281 & 0.223 & 0.236 \\ 0 & 0.560 & -0.402 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = I_y \; R_z(0.22) \; R_y(0.82) \; R_z(0.12) \qquad \mathbf{c} = (0.33, 1.62, 1.10)$$

- In first approximation a Coded Mask is a Projective Camera
- Different focal lengths $f_x \neq f_y$
- $(x_0, y_0) \neq \mathbf{0} \leftrightarrow$ image plane Origin $\neq$ principal point
- Non vanishing skew parameter s
- Rotation relative to the world frame
- Traslated camera center

Optimization

for $n \geq 6$ find the Maximum Likelihood estimate of P by a Standard Algorithm

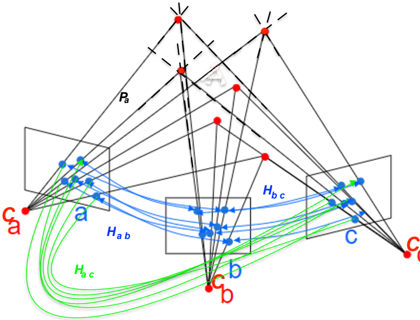
1. Estimate P by a linear procedure
2. Normalize source and image points : $\tilde{\mathbf{X}}_i = U\,\mathbf{X}_i, \quad \tilde{\mathbf{x}}_i = T\,\mathbf{x}_i,$

   $$\text{such that } \sum_i \tilde{\mathbf{X}}_i = \mathbf{0},\ \sum_i \tilde{\mathbf{x}}_i = \mathbf{0}\ \frac{\sum_i |\tilde{\mathbf{X}}_i|^2}{n-1} = 3,\ \frac{\sum_i |\tilde{\mathbf{x}}_i|^2}{n-1} = 2$$

3. Generate $A\left(\left\{\tilde{\mathbf{X}}_i\right\}, \{\tilde{\mathbf{x}}_i\}\right)$ and take its normalized eigenvector $\widetilde{\mathbf{p}}_{min} \leftrightarrow \lambda_{min} \neq 0 \ \Rightarrow\ \tilde{P}$
4. Minimize recursively $\min_P \left(\sum_{i=1}^{n} |\mathbf{x}_i - \tilde{P}\mathbf{X}_i|^2\right)$
5. Back in the original coordinates : $P = T^{-1}\,\tilde{P}\,U$

Multiple Views

- $P_i : \mathbb{P}^3 \to \mathbb{P}^2$
- $\lambda_{i\,j}\, \mathbf{x_j} = P_i \mathbf{X}_j, \quad \forall\, \lambda_{i\,j} \in \mathbb{R}_{/0}$
- $P_i = K_i\,[R_i|\mathbf{t_i}] \quad P_i\, \mathbf{C_i} = 0$
- $\mathbf{C_i} = \begin{pmatrix} \mathbf{c_i} \\ 1 \end{pmatrix} = \begin{pmatrix} -R_i^T\ \mathbf{t_i} \\ 1 \end{pmatrix}$
- Transition f. $h_{a\,b} : \mathbb{P}^2 \leftrightarrow \mathbb{P}^2$
  $$\begin{cases} h_{a\,a} & = \mathbb{I} \\ h_{a\,b}\, h_{b\,a} & = \mathbb{I} \\ h_{a\,b}\, h_{b\,c}\, h_{c\,a} & = \mathbb{I} \end{cases}$$
  cocycle identity



The Projective Reconstruction Theorem

The triplet $\left(P,\ P',\ \{\mathbf{X}_\alpha\}_{\alpha\geq 8}\right)$ is called a 3D reconstruction of the images $\{\mathbf{x}_\alpha\}$ and $\{\mathbf{x}'_\alpha\}$ if it satisfies the relations

$$\lambda_\alpha\,\mathbf{x}_\alpha = P\,\mathbf{X}_\alpha, \quad \lambda'_\alpha\,\mathbf{x}'_\alpha = \mathbf{P}'\,\mathbf{X}_\alpha$$

Th. A 3D reconstruction is unique up to a homography of $\mathbb{P}^3$.

- $\left(\tilde{P},\ \tilde{P}',\ \left\{\tilde{\mathbf{X}}_\alpha\right\}_{\alpha\geq 8}\right)$ is another reconstruction iff
  $\tilde{P} = P\,H^{-1}, \quad \tilde{P}' = P'\,H^{-1}, \quad \tilde{\mathbf{X}}_\alpha = H\,\mathbf{X}_\alpha, H : \mathbb{P}^3 \leftrightarrow \mathbb{P}^3$ linear
- For any Camera Stereo Rig $P_1, P_2, \ldots$ its canonical form is

$$\tilde{P}_1 = [\mathbb{I}|\mathbf{0}]\,, \tilde{P}_2 = K_2 R_2 \left(K_1 R_1\right)^{-1} [\mathbb{I}\mid \det\left(K_1\right) K_1 R_2 \left(\mathbf{c}_1 - \mathbf{c}_2\right)]\,, \ldots$$

$$H = \begin{pmatrix} K_1 R_1 & -K_1 R_1 \mathbf{c}_1 \\ \mathbf{0}^T & 1/\det\left(K_1\right) \end{pmatrix}$$

- Equally calibrated cameras
  $P_0 = K\ [\mathbb{I}|\mathbf{0}]\,,\ P_\beta = P_0\,H_{0,\beta}\,, H_{0,\beta} = \begin{pmatrix} R_\beta & \mathbf{t}_\beta \\ \mathbf{0}^\mathbf{T} & 1 \end{pmatrix},\ H_{\alpha\,\beta} = H_{0\,\alpha}^{-1}\,H_{0\,\beta}.$
- $H_{0\,\alpha}$ are the generators of a Borel subgroup of $GL\left(4\right)$.

Pseudo-Inverse of P and Reconstruction formula in double-view

$$P = K\,[R|\mathbf{t}] \qquad \mathbf{x} = P\,\mathbf{X}$$

$$P^{+} = \begin{bmatrix} R^{T} \\ \mathbf{t}^{T} \end{bmatrix} \left(\mathbb{I} - \frac{\mathbf{t} \otimes \mathbf{t}}{1 + |\mathbf{t}|^{2}} \right) K^{-1} = \begin{bmatrix} \mathbb{I} \\ -\mathbf{c}^{T} \end{bmatrix} \left(\mathbb{I} - \frac{\mathbf{c} \otimes \mathbf{c}}{1 + |\mathbf{c}|^{2}} \right) R^{T} K^{-1}$$

$$\Rightarrow \qquad \mathbf{X} = P^{+}\mathbf{x} + \mu\,\mathbf{C} \qquad \mu \in \mathbb{R}$$

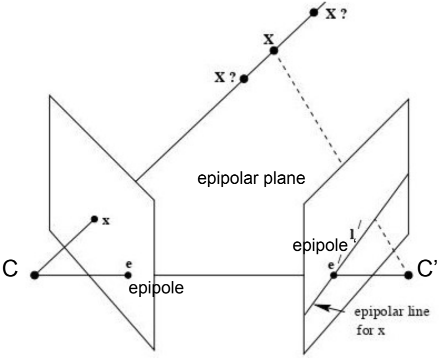
Corresponding point

$$\lambda'\,\mathbf{x}' = P'\,\mathbf{X} \quad \lambda' \in \mathbb{R}_{/0}$$

$$\mathbf{x}' \wedge \mathbf{x}' = 0 \Rightarrow \boxed{\mathbf{X} = P^{+}\mathbf{x} + \frac{(P'\,P^{+}\,\mathbf{x} \;\wedge\; \mathbf{x}') \cdot (\mathbf{x}' \;\wedge\; P'\,\mathbf{C})}{(\mathbf{x}' \;\wedge\; P'\,\mathbf{C})^{2}}\,\mathbf{C}}$$

The Fundamental Matrix

- Epipoles
  $\mathbf{e}' = P'\,\mathbf{C}, \quad \mathbf{e} = P\,\mathbf{C}'$
- $\mathbf{x}' = P'\,\mathbf{X} = P'\,P^{+}\mathbf{x} + \mu\,\mathbf{e}'$
- $\mathbf{l}' = \mathbf{e}' \wedge \mathbf{x}' = \mathbf{e}' \wedge P'\,P^{+}\,\mathbf{x}$
- $\boxed{F = [\mathbf{e}']_{\wedge}\ P'\,P^{+}}$
  rank 2 $\mathbb{P}^2 \to \mathbb{P}^2$, $\det F = 0$
- Corresponding image points
  $\mathbf{x}' \cdot \mathbf{l}' = \boxed{\mathbf{x}'^{T}\ F\ \mathbf{x} = 0}$
- $\sum_{i\,j} x'_i\,F_{i\,j}\,x_j =$
  $\det \begin{pmatrix} P' & \mathbf{x}' & 0 \\ P & 0 & \mathbf{x} \end{pmatrix} = 0$
- $F = [\mathbf{e}']_{\wedge}\,K'\,R'\,K^{-1} = K'^{-T}\,R'\,K^{T}\,[\mathbf{e}]_{\wedge}$



Computing F from a pair of Camera Matrices - Examples I

Equally calibrated c.s $K' = K$

- Translated Camera :

  $$F_{\mathbf{t}} = [K\mathbf{t}]_\wedge, \quad \mathbf{x}' = \mathbf{x} + \lambda K\ \mathbf{t}, \quad \mathbf{X} = \begin{pmatrix} K^{-1}\mathbf{x} \\ -\frac{x-x'}{st_y+\alpha t_x+t_z(x_0-x')} \end{pmatrix}$$

- Roto-traslated Camera: $F = [K\ \mathbf{t}]_\wedge\ K\ R\ K^{-1}$,

  $$\mathbf{x}' = KRK^{-1}\,\mathbf{x} + \mu K\mathbf{t}, \quad \mathbf{X} = \begin{pmatrix} K^{-1}\mathbf{x} \\ \frac{\left(KRK^{-1}\ \mathbf{x}\wedge\mathbf{x}'\right)\cdot\left(\mathbf{x}'\wedge K\mathbf{t}\right)}{(\mathbf{x}'\wedge K\mathbf{t})^2} \end{pmatrix}$$

- front-to-front stereo rig:

  $$P' = K\ I_z\ [\mathbb{I}, -\mathbf{c}],\ \mathbf{c} = (0,0,c)^T,\ I_z = \mathrm{diag}\,(1,1,-1)$$
  $$\mathbf{e}' = c\,(x_0, y_0, 1)^T = c\mathbf{k}_3, \quad F_{ff-z} = [\mathbf{k}_3]_\wedge\, KI_zK^{-1}$$
  $$\mathbf{x}'^T F_{ff-z}\mathbf{x} = (y'-y_0)\,x + (y_0-y)\,x' + (y-y')\,x_0 = 0$$

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  $$x_S = \frac{2x_Ax_B}{x_A+x_B}, \quad z_S = \frac{2z_Az_B}{z_A+z_B}, \quad y_S = a\frac{x_A-x_B}{x_A+x_B} = a\frac{z_A-z_B}{z_A+z_B}$$

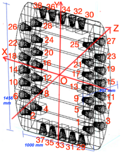
Computing F from a pair of Camera Matrices - Examples II

- Cameras at right angle:
  $P = K\,[\mathbb{I}, \mathbf{0}]\,, \quad P' = K\;R^y_{-\pi/2}\,\left[\mathbb{I}, -r\;R^y_{\pi/4}\hat{\mathbf{z}}\right]$
  $\mathbf{e} = r\;K\;R^y_{\pi/4}\hat{\mathbf{z}}, \quad \mathbf{e}' = -r\;K\;R^y_{-\pi/4}\hat{\mathbf{z}}$

$$F_\perp = \left[K\;R^y_{-\pi/4}\;\hat{\mathbf{z}}\right]_\times\;K\;R^y_{-\pi/2}\,K^{-1} = \begin{pmatrix} 0 & 1 & -y_0 \\ -1 & 0 & x_0 - \alpha \\ y_0 & -\alpha - \;x_0 & 2\alpha\;y_0 \end{pmatrix}$$

Non anti-symmetry $\Rightarrow$ Horopter: $\mathbf{x}^T\frac{F_\perp + F_\perp^T}{2}\mathbf{x} = 0$ is degenerate conic of the image points the coordinates of which coincide.

GRAIN as a multiple-view (lenses) system



Set of lenses centered at

$$\mathbf{c}_m^{\epsilon_1,\,\epsilon_2,\,\epsilon_3} = \begin{pmatrix} \xi_m^{\epsilon_2,\,\epsilon_3}\; h \\ \psi_m^{\epsilon_1,\,\epsilon_3}\; b \\ \zeta_m^{\epsilon_2,\,\epsilon_3}\; \ell \end{pmatrix} = \begin{pmatrix} \left[(1-\epsilon_3)\,\delta_m - \frac{(-1)^{\epsilon_2}}{2}\,\epsilon_3\right]\; h \\ -\,(1-\epsilon_3)\,(-1)^{\epsilon_1}\;\beta_m\; b \\ \left[-\,(1-\epsilon_3)\,\frac{(-1)^{\epsilon_2}}{2} + \lambda\;\epsilon_3\; m\right]\;\ell \end{pmatrix}$$

for $\epsilon_i = 0,\,1$ and $0 \le |m| \le 3$

$\delta_{-m} = -\delta_m,\; \delta_0 = 0$ and $\beta_{-m} = \beta_m$

Projection Matrices

$$P_m^{\epsilon_1\;\epsilon_2\;\epsilon_3} = K\;\left(R_\pi^x\right)^{\epsilon_2(1-\epsilon_3)}\;\left(R^y_{(-1)^{\epsilon_2}\,\frac{\pi}{2}}\right)^{\epsilon_3}\;\left[\mathbb{I},\,-\mathbf{c}_m^{\epsilon_1,\,\epsilon_2,\,\epsilon_3}\right]$$

Fundamental Matrices

$$F_{m\;m'}^{\epsilon_1\;\epsilon_2\;\epsilon_3\;\epsilon'_1\;\epsilon'_2\;\epsilon'_3} = \left(P_{m'}^{\epsilon'_1\;\epsilon'_2\;\epsilon'_3}\right)^{+\;T}\;\left(P_m^{\epsilon_1\;\epsilon_2\;\epsilon_3}\right)^T\;\left[\mathbf{e}_{m\;m'}^{\epsilon_1\;\epsilon_2\;\epsilon_3\;\epsilon'_1\;\epsilon'_2\;\epsilon'_3}\right]$$

Reconstruction by double-Views

- Minimal hypotesis :
  $K = \mathrm{diag}\left(-f, -f, 1\right)\ \forall m, \epsilon_1, \epsilon_2, \epsilon_3\ ,\ f = 100mm$
- Compute $P_m^{\epsilon_1\ \epsilon_2\ \epsilon_3}$ and $F_{m\ m'}^{\epsilon_1\ \epsilon_2\ \epsilon_3\ \epsilon_1'\ \epsilon_2'\ \epsilon_3'}$
- Each point is detected/seen by $N \leq 38$ cameras
- Centroids of the photon distribution as image coordinates .
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-views
- Check the consistency conditions
  $\mathbf{x}_{\epsilon_1'\ \epsilon_2'\ \epsilon_3'}^{m'\ T} \cdot F_{m\ m'}^{\epsilon_1\ \epsilon_2\ \epsilon_3\ \epsilon_1'\ \epsilon_2'\ \epsilon_3'} \cdot \mathbf{x}_{\epsilon_1\ \epsilon_2\ \epsilon_3}^{m} \approx 0$
- Perform M reconstructions by the 3D formula
- take the mean value of the M possible reconstructions for each coordinate $X = \frac{\sum_{i<j}^{N} X_{ij}}{M},\ Y = \frac{\sum_{i<j}^{N} Y_{ij}}{M},\ Z = \frac{\sum_{i<j}^{N} Z_{ij}}{M}$
- $\Delta X_i = X_{calc\ i} - X_{true\ i}$:

$\overline{\Delta X} \approx 10^{-2}\mathrm{mm}\quad \overline{\Delta X^2}^{1/2} \approx 5\mathrm{mm},\quad \overline{\Delta Y} \approx 10^{-2}\mathrm{mm}\quad \overline{\Delta Y^2}^{1/2} \approx 3\mathrm{mm},\quad \overline{\Delta Z} \approx 10^{-2}\mathrm{mm}\quad \overline{\Delta Z^2}^{1/2} \approx$

Errors in computing the corresponding points

$$S = \{\mathsf{x}_1, \ldots, \mathsf{x}_N\}\,,\; S' = \left\{\mathsf{x}'_1, \ldots, \mathsf{x}'_{N'}\right\} \rightarrow S \times S' = \left\{\left(\mathsf{x}_\alpha, \mathsf{x}'_\beta\right)\right\}$$

Corresponding points : $S_{cp} = \left\{\left(\mathsf{x}_\alpha, \mathsf{x}'_\beta\right) : \mathsf{x}'_\beta\; F\; \mathsf{x}_\alpha = 0\right\}$

Empirical data $0 \leq E_{\alpha,\beta} = |\; \mathsf{x}'_\beta\; F\; \mathsf{x}_\alpha \;| \leq\; \epsilon$

Bounded spread criterion

$$\left(\mathsf{x}_\alpha, \mathsf{x}'_\beta\right) = \left(\mathsf{x}^0_\alpha + \Delta\mathsf{x}, \mathsf{x}'^0_\beta + \Delta\mathsf{x}'\right)$$

$$|\Delta\mathsf{x}| \leq \frac{\epsilon}{|\mathsf{x}'^0_{\bar{\beta}}|\;||F||},\; |\Delta\mathsf{x}'| \leq \frac{\epsilon}{|\mathsf{x}^0_{\bar{\beta}}|\;||F||}$$

The Fundamental matrix from data

The set S_{cp} of the corresponding points is known and $\#S_{cp} \geq 8$
$\mathbf{x}'_\alpha F \mathbf{x}_\alpha = 0 \quad \Rightarrow \quad A\mathbf{F} = 0$ with $A \in \mathbb{R}^{n\times 9}$
$\mathrm{rank}\,(A) = 8 \Rightarrow \lambda\, F$ is determined
$\mathrm{rank}\,(A) = 9$ a least square solution is found by solving
$Min_A ||A\mathbf{F}||$ subject to $||\mathbf{F}|| = 1$

- Singular Value Decomposition : $F = U\ D\ V^T$ with $D = \mathrm{diag}\,(p, q, \epsilon)\,, \epsilon \ll q < p$, U, V orthogonal
- Introduce the matrices $Z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- Compute $S = UZU^T$ and $M = UW^TDV^T$, then $F = SM$
- Associated to F a pair of cameras is $P = [\mathbb{I}|\mathbf{0}]\,, \quad P' = [M|U_{.3}]$

Three view reconstruction I

$\lambda_\alpha \, \mathbf{x}_\alpha = P \, \mathbf{X}_\alpha, \quad \lambda'_\alpha \, \mathbf{x}'_\alpha = \mathbf{P}' \, \mathbf{X}_\alpha, \quad \lambda"_\alpha \, \mathbf{x}"_\alpha = \mathbf{P}" \, \mathbf{X}_\alpha$

$$\begin{pmatrix} P & \mathbf{x}_\alpha & 0 & 0 \\ P' & 0 & \mathbf{x}'_\alpha & 0 \\ P" & 0 & 0 & \mathbf{x}"_\alpha \end{pmatrix} \begin{pmatrix} \mathbf{X}_\alpha \\ -\lambda_\alpha \\ -\lambda'_\alpha \\ -\lambda"_\alpha \end{pmatrix} = 0$$

Since a solution exists, then all 7×7 sub-matrices have determinant $= 0$ $\Rightarrow$

$$\sum_{i,j,k,q,r=1}^{3} x^i \, x'^j \, x"^k \epsilon_{jqu} \epsilon_{krv} \mathcal{T}_i^{qr} = 0$$

4 independent eq.s
9 trilinear conditions involving the trifocal tensor (27 components)

$$\mathcal{T}_i^{qr} = (-1)^{i+1} \det \left(P_{\hat{i}}, P'_q, P"_r \right)$$

- 7 triplets of corresponding points completely determine $\mathcal{T}_i^{qr}$
- Known $\mathcal{T}_i^{qr}$, three views allow to reconstruct $\mathbf{X}$

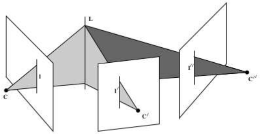
Three view reconstruction II

- Duality point-line :

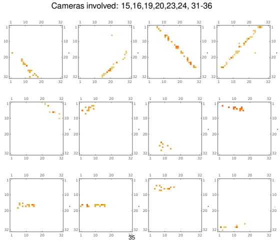
$$l_p \; l'_q \; l''_r \mathcal{T}_i^{qr} = 0$$

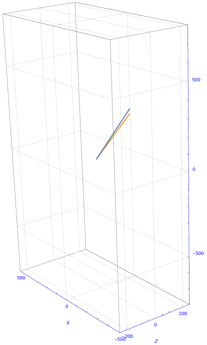
- Line Transfer :

$$l_p \; = \; l'_q \; l''_r \mathcal{T}_p^{qr}$$



Three view reconstruction III





$\Delta\Omega \approx 10^{-3}\pi$ st.rad.

Conclusions I

- Different reconstruction algorithms are in development.
- A coded mask can be treated as a projective camera at 0-order approximation
- Camera projection matrices, Fundamental matrices and trifocal tensors are common tools in a multiple - view treatment.
- Camera projection matrices, fundamental matrices and trifocal tensors can be derived from constructive design data
- 3D Reconstruction formulas are displayed for generic and special arrangements of cameras.
- Alternatively, they can be derived from calibration methods, exploiting a minimal finite number of empirical data.
- Optimization methods in the above calculations are already at our disposal.

Conclusions II

- Several tests addressed to evaluate the capability of 3D reconstruction point like sources in different regions of GRAIN have been performed.
- Adopting the trifocal tensor approach, point and line sources can be treated at the same foot.
- Generalized methods in presence of more than three view should be developed.
- Reconstruction of a single muon trace with origin within the GRAIN volume;
- Reconstruction of two tracks from a vertex within GRAIN;
- Development of criteria for the association of images of different simultaneous tracks on different sensors;
- Image Transfer.
- Further Ideas