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Two-meson scattering and tetraquarks in two-dimensional QCD

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Two-dimensional QCD

Two-dimensional QCD, in the large-N_c limit, first introduced by 't Hooft (1974), is an efficient tool to probe many properties of hadrons related with the confinement of quarks.

- In two dimensions, confinement is a natural property of QCD.
- In the large-N_c limit, inelasticity and pair creation effects are damped.
- In noncovariant gauges (light-cone, axial), gluon self-interactions and ghosts disappear.
- In the many-body case, the interactions are free of long-range van der Waals forces.

We investigate, in this framework, the properties of the theory in the four-body sector, made of two quarks and two antiquarks with different flavors, and search for the possible existence of tetraquark bound states. The investigation is concentrated on the properties of two-meson scattering amplitudes.

Definitions

Quark fields in the fundamental representation of the color-gauge group $SU(N_c)$.

Light-cone gauge: $A_- = 0$.

Because of the particular structure of the quark-gluon vertex in the light-cone gauge, where only the γ_- matrix appears, the Dirac matrices can be factorized from the N_c-leading expressions of all dynamical quantities.

Coupling constant g: has dimension of mass. Definition of the string tension:

$$\sigma \equiv \frac{g^2 N_c}{4}.$$

The gluon propagator:

$$D(q) \equiv D_{++}(q) = \frac{i}{q_-^2} \longrightarrow \frac{1}{q_-^2 + \lambda^2}, \quad \Lambda \equiv \frac{\sigma}{\lambda}.$$

λ is a small mass parameter, which plays the role of an infrared cutoff.

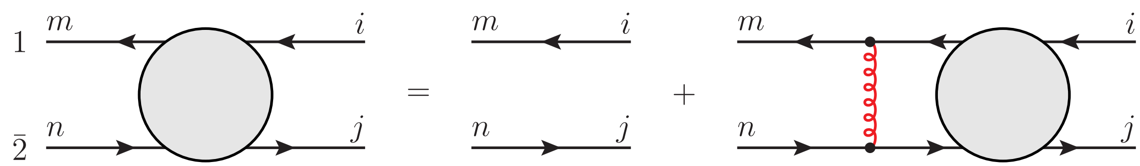
Quark propagator (self-energy included):

$$S(p) = \frac{ip_-}{\Big[2p_+p_- - |p_-|\Lambda - m'^2 + i\epsilon\Big]},$$

$$m'^2 = m^2 - \frac{2\sigma}{\pi}$$

Physical quantities should be independent of the infrared cutoff Λ.

Quark-antiquark bound-state equation



$$G^{m,\ j}_{n,\ i} = \delta^m_{\ n}\delta^j_{\ i}\,G_s + (\delta^m_{\ i}\delta^j_{\ n} - \frac{1}{N_c}\delta^m_{\ n}\delta^j_{\ i})\,G_{adj},$$

$$G_s = \frac{1}{N_c}G_0 \ + \ G_0(4K)G_s, \quad G_0 = S_1S_2, \quad K = (-2\sigma D).$$

Bound-state equation. Interaction is lightlike instantaneous. Define

$$\varphi(r, p_-) = \int \frac{dp_+}{2\pi}\,\phi(r, p).$$

$\Longrightarrow$ spectral conditions on the quark and antiquark momenta:

$$\varphi(r, p_-) = \varphi(r, p_-)\theta(p_-)\theta(r_- - p_-).$$

Defining

$$x = \frac{p_-}{r_-}, \qquad 0 \leq x \leq 1, \qquad y = \frac{p''_-}{r_-}, \qquad 0 \leq y \leq 1,$$

the bound-state equation becomes

$$\left[r^2 - \frac{m_1^2}{x} - \frac{m_2^2}{(1-x)} \right] \varphi(x) = -(\frac{2\sigma}{\pi}) \int_0^1 dy \frac{(\varphi(y) - \varphi(x))}{(y-x)^2}.$$

The infrared cutoff has disappeared. This is the 't Hooft equation.

Tower of meson states with Regge trajectory.

The complete wave function:

$$\phi_n(r,p) = 4iG_0 \Big[r_+ - \Lambda - \frac{m_1'^2}{2p_-} - \frac{m_2'^2}{2(r_- - p_-)} \Big] \varphi_n(r,p_-) \equiv \frac{2}{\sqrt{N_c}} G_0 \, \widetilde{\phi}_n(r,x).$$

The quark-antiquark scattering amplitude

The scattering amplitude $\mathcal{T}$ satisfies the integral equation

$$\mathcal{T} = \frac{1}{N_c} K + 4iKG_0\mathcal{T}.$$

Equation solved by Callan, Coote, Gross (1976):

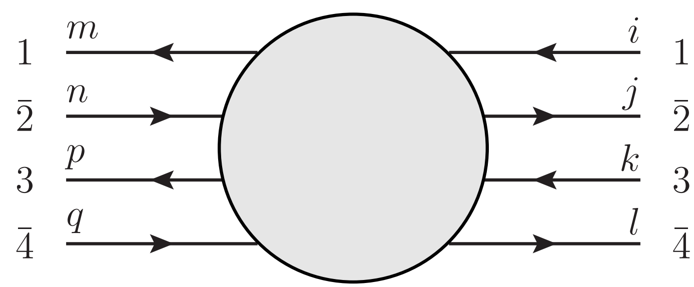
$$\mathcal{T}(r; x, x') = -\frac{2\sigma}{N_c}\frac{1}{r_-^2 (x - x')^2} + \frac{1}{N_c}\sum_n \frac{\widetilde{\phi}_n(r, x)\widetilde{\phi}_n^*(r, x')}{(r^2 - r_n^2)}.$$

$\mathcal{T}$ is of order $1/N_c$. Is cutoff Λ-dependent through the complete wave functions $\widetilde{\phi}_n$.

Two-quark–two-antiquark systems

The quark flavors are assumed to be different from each other; this case avoids mixing problems with ordinary meson states. The quarks are denoted with the labels $\mathbf{1}$ and $\mathbf{3}$, with masses m_1 and m_3 and the antiquarks with labels $\bar{\mathbf{2}}$ and $\bar{\mathbf{4}}$, with masses m_2 and m_4.

The Green's function corresponding to the four ingoing and outgoing particles made of the quarks $\mathbf{1}$ and $\mathbf{3}$ and the antiquarks $\bar{\mathbf{2}}$ and $\bar{\mathbf{4}}$:



(Latin indices, $i, j, \ldots$, correspond to color indices.)

We are interested in the color-singlet sectors of the incoming and outgoing particles in the Green's function, describing scattering processes of mesons.

We have two sets of mesonic clusters: $(\mathbf{1\bar{2}})(\mathbf{3\bar{4}})$ and $(\mathbf{1\bar{4}})(\mathbf{3\bar{2}})$.

One can then distinguish four different channels in the scattering process, two of which will be called direct, the ingoing and outgoing clusters remaining the same, and the two others recombination, the outgoing clusters having undergone a quark exchange. The four different processes are:

$$\begin{aligned}
&\mathbf{(1\bar{2}) + (3\bar{4}) \longrightarrow (1\bar{2}) + (3\bar{4}),} \quad \text{direct channel } \mathbf{1} \quad \boldsymbol{(D1),}\\
&\mathbf{(1\bar{2}) + (3\bar{4}) \longrightarrow (1\bar{4}) + (3\bar{2}),} \quad \text{recombination channel } \mathbf{1} \quad \boldsymbol{(R1),}\\
&\mathbf{(1\bar{4}) + (3\bar{2}) \longrightarrow (1\bar{4}) + (3\bar{2}),} \quad \text{direct channel } \mathbf{2} \quad \boldsymbol{(D2),}\\
&\mathbf{(1\bar{4}) + (3\bar{2}) \longrightarrow (1\bar{2}) + (3\bar{4}),} \quad \text{recombination channel } \mathbf{2} \quad \boldsymbol{(R2).}
\end{aligned}$$

One obtains from the equations satisfied by the four-particle Green's functions those satisfied by the scattering amplitudes.

One finds the behaviors

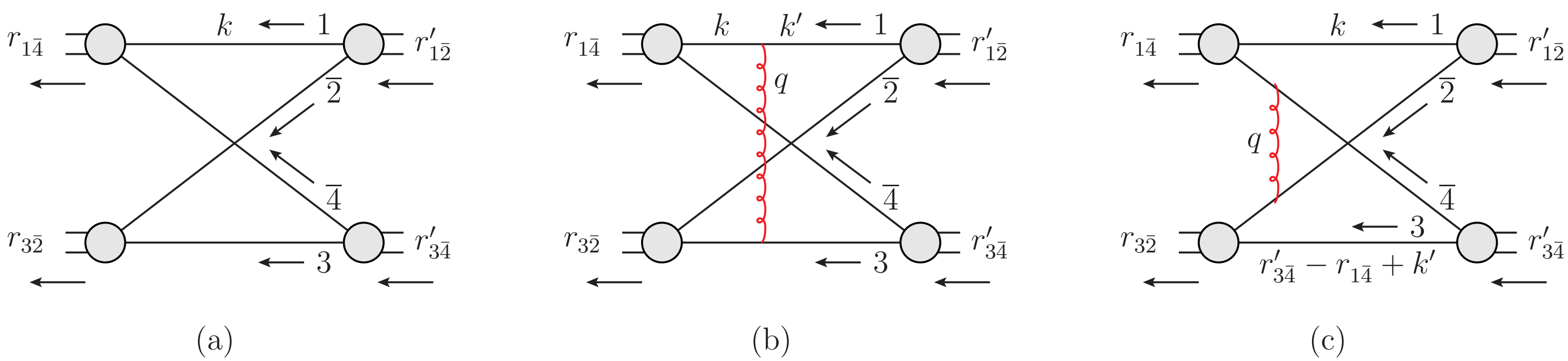
$$\begin{aligned}
\mathcal{T}_{R1} &= O(N_c^{-1}) + O(N_c^{-3}), \\
\mathcal{T}_{D1} &= O(N_c^{-2}) + O(N_c^{-4}).
\end{aligned}$$

The on-mass shell scattering amplitudes are obtained by projecting the off-mass shell scattering amplitudes on the ingoing and outgoing (complete) wave functions.

Finiteness of the scattering amplitudes to order $1/N_c^2$

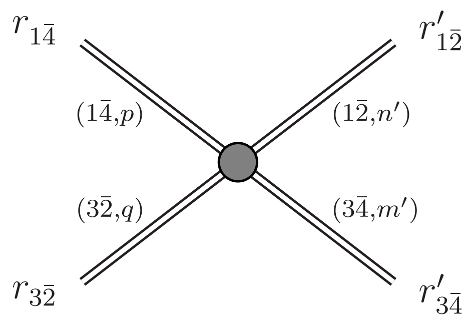
An important test for the theory is the verification that the meson-meson on-mass-shell scattering amplitudes are finite quantities, independent of the infrared cutoff Λ. The latter is present in the quark propagators, the gluon propagator and the external wave functions.

We first consider the recombination channel scattering amplitude at leading order $O(1/N_c)$. It is composed of three contributions.



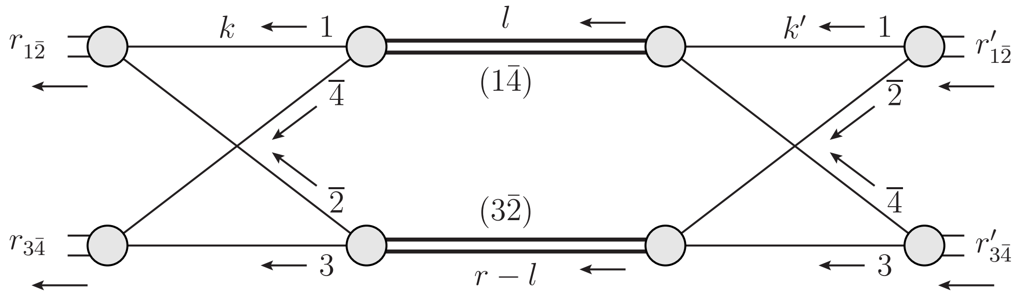
Each diagram diverges like $O(\Lambda)$, but the sum of the three is finite.

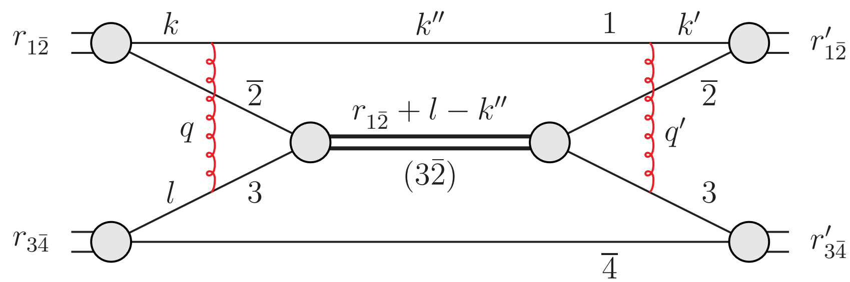
The finite remaining part represents an effective four-meson contact term, having weak dependence on the external momenta. The expression of the contact term is given by an overlapping function of the four external wave functions.

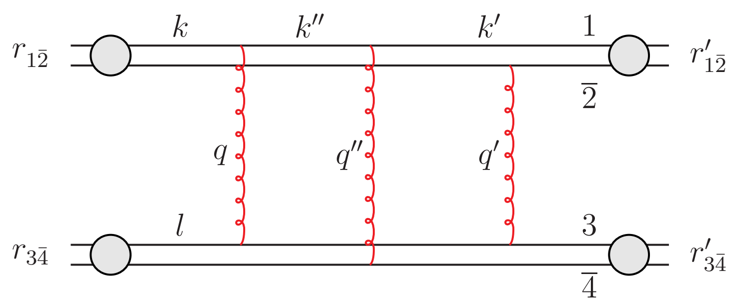


$O(1/N_c)$

We next consider the direct channel scattering amplitude at leading order $\boldsymbol{O(1/N_c^2)}$. Receives contributions from $\mathbf{65}$ diagrams. They can be grouped into $\mathbf{5}$ categories, each having a specific structure.


$r_{1\bar{2}}$
k
1
l
$(1\bar{4})$
k'
1
$r'_{1\bar{2}}$
$\bar{4}$
$\bar{2}$
$\bar{2}$
$\bar{4}$
$(3\bar{2})$
$r_{3\bar{4}}$
3
$r - l$
3
$r'_{3\bar{4}}$


$r_{1\bar{2}}$
k
k''
1
k'
$r'_{1\bar{2}}$
$\bar{2}$
$r_{1\bar{2}} + l - k''$
$\bar{2}$
q
q'
l
3
$(3\bar{2})$
3
$r_{3\bar{4}}$
$\bar{4}$
$r'_{3\bar{4}}$


$r_{1\bar{2}}$
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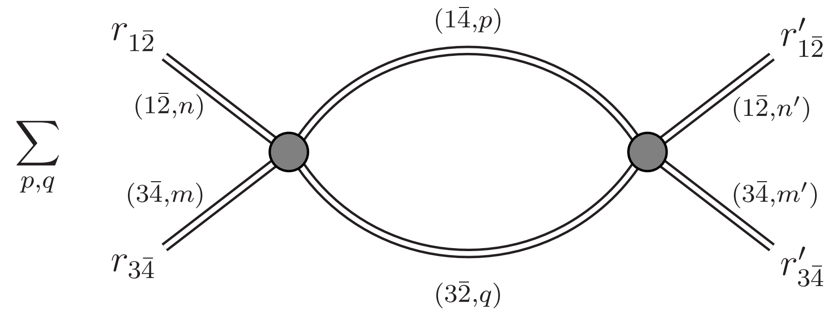

$O(1/N_c^2)$

To order $\mathbf{1/N_c^2}$, the meson-meson scattering amplitudes have been reduced, after the infrared cutoff $\boldsymbol{\lambda}$ has been sent to zero, to that of an effective theory of mesons with contact interactions in the recombination channels and unitarity diagrams generated by them in the direct channels.

Next step: Unitarize completely the scattering amplitudes by iteration.

Unitarized scattering amplitudes

Coupled-channel formalism between the recombination and direct channels.

Approximate the contact term by a constant, designated by K_R. (Unitarization requires K_{R1}=K_{R2}.)

$$\mathcal{T} = \begin{pmatrix} \mathcal{T}_{D1} & \mathcal{T}_{R2} \\ \mathcal{T}_{R1} & \mathcal{T}_{D2} \end{pmatrix}, \quad K = \begin{pmatrix} 0 & K_R \\ K_R & 0 \end{pmatrix}, \quad J = \begin{pmatrix} J_{1\bar{2},3\bar{4}} & 0 \\ 0 & J_{1\bar{4},3\bar{2}} \end{pmatrix}.$$

($J_{1\bar{2},3\bar{4}}$ and $J_{1\bar{4},3\bar{2}}$ are the two-meson loop functions.) One has

$$\begin{aligned} \mathcal{T} &= (1 - iKJ)^{-1}K \\ &= \frac{1}{\left(1 + K_R J_{1\bar{4},3\bar{2}} K_R J_{1\bar{2},3\bar{4}}\right)} \begin{pmatrix} iK_R J_{1\bar{4},3\bar{2}} K_R & K_R \\ K_R & iK_R J_{1\bar{2},3\bar{4}} K_R \end{pmatrix}. \end{aligned}$$

Tetraquarks

Tetraquark bound states might occur at the zeros of the denominator of $\mathcal{T}$.

$$1 + K_R J_{1\bar{4},3\bar{2}} K_R J_{1\bar{2},3\bar{4}} = 0.$$

The bound-state equation has generally one bound-state solution, which lies very close to the lowest two-meson threshold. The reason is the smallness of $(2\sigma/\pi)^2$ (which governs the scale of K_R^2) in front of the product of the four meson masses.

The binding energies come out, with rough estimates, of the order of 1-10 MeV.

Formalism can also be applied to the case of three open flavors, in which case the recombination and direct channels are the same. One has the simpler case of one channel. Here also, one has one bound state lying very close to the two-meson threshold. The binding energy is in this case of the order of 1 MeV.

Conclusion

Two-dimensional QCD at large N_c allows a consistent treatment of the confining interactions.

- Physical quantities come out independent of the infrared cutoff, needed in intermediate calculations.

- In the two-meson sector, the theory is reduced to an effective theory of mesons, where $1/N_c$ plays the role of a perturbation parameter.

- Tetraquark bound states appear, in the cases of four and three open flavors, very close to the lowest two-meson threshold. The smallness of the binding energy is understood as a consequence of the small ratio of the string tension to the meson masses squared.

- Apart from the string tension and the quark masses, there are no adjustable parameters.