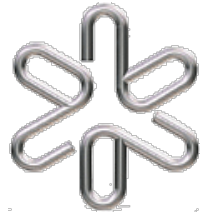



Exotic charmonium production in heavy ion collisions

F.S. Navarra
University of São Paulo



IFUSP
São Paulo
Brazil

L.M. Abreu, F.S. N, H.P.L. Vieira, arXiv:2401.11320



QCD@Work
International Workshop on QCD
Theory and Experiment

Exotic Charmonium

Not a state like: $c\bar{c}$

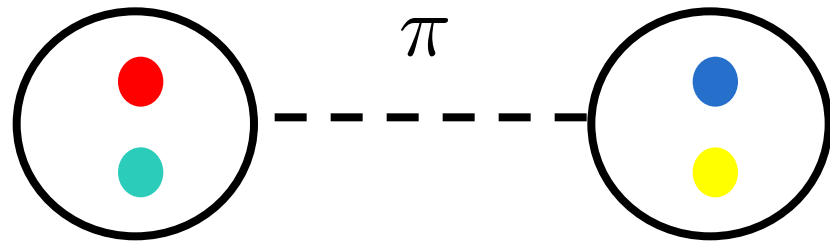
Multiquark state: $c\bar{c}q\bar{q}'$

Meson molecule

Large object: $\sim 5 - 10$ fm

Loosely bound

Meson exchange

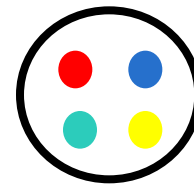


Tetraquark

Compact object: $\sim 0.5 - 1$ fm

Deeply bound

Color exchange



Use heavy ion collisions to determine the structure of X(3872)

Exotic Hadrons from Heavy Ion Collisions[☆]

Sungtae Cho^a, Tetsuo Hyodo^b, Daisuke Jido^c, Che Ming Ko^d, Su Houn Lee^e, Saori Maeda^f,
 Kenta Miyahara^g, Kenji Morita^b, Marina Nielsen^h, Akira Ohnishi^b, Takayasu Sekiharaⁱ,
 Taesoo Song^j, Shigehiro Yasui^f, Koichi Yazaki^k,
 (ExHIC Collaboration)

ExHIC, Prog. Part.Nucl.Phys. (2017) arXiv:1702.00486

Why heavy ion collisions ?

Production of a large number of heavy quarks

Medium can act as a "filter" to separate different structures

Evidence for X(3872) in Pb-Pb Collisions and Studies of its Prompt
 Production at $\sqrt{s_{NN}}=5.02$ TeV

CMS Collaboration • Albert M. Sirunyan (Yerevan Phys. Inst.) et al. (Feb 25, 2021)

Published in: *Phys.Rev.Lett.* 128 (2022) 3, 032001 • e-Print: [2102.13048](https://arxiv.org/abs/2102.13048) [hep-ex]

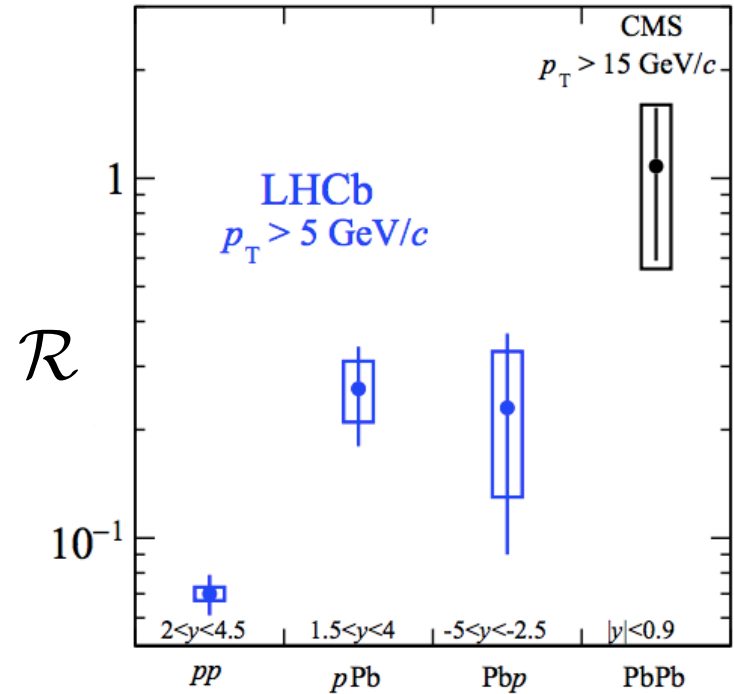
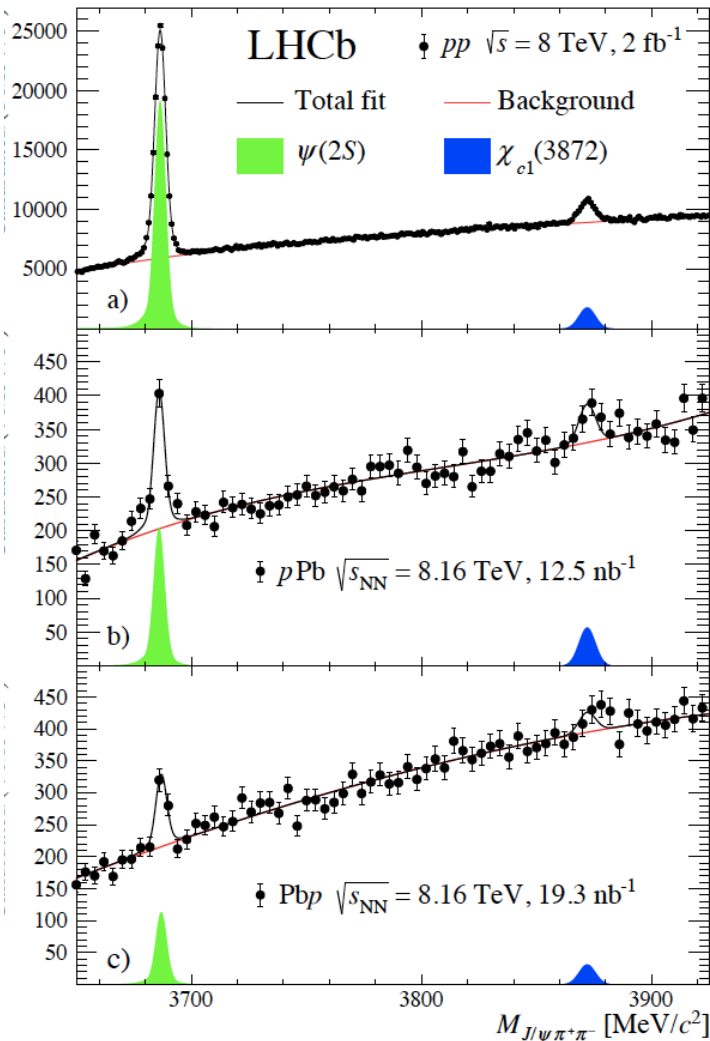
The data

$$\psi_{2S}(3686) \rightarrow J/\psi \pi^+ \pi^-$$

$$X(3872) \rightarrow J/\psi \pi^+ \pi^-$$

LHCb-CONF-2022-001
Quark Matter 2022

$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\psi(2S)}}$$

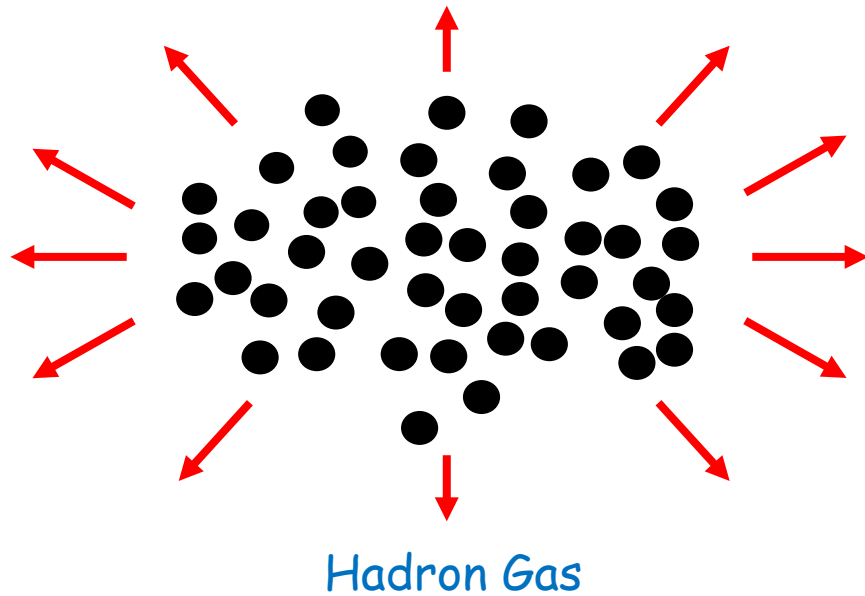
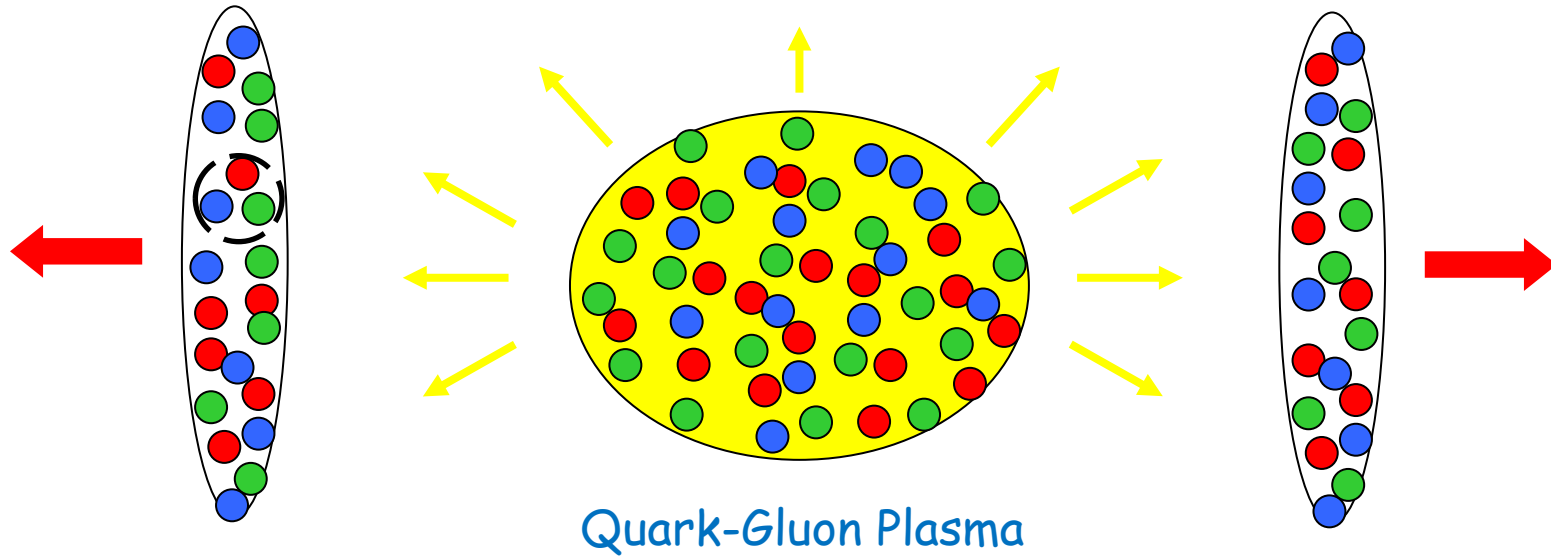


Medium effects:

Formation at the end of QGP

Interactions in the hadron gas

Nucleus - Nucleus Collisions

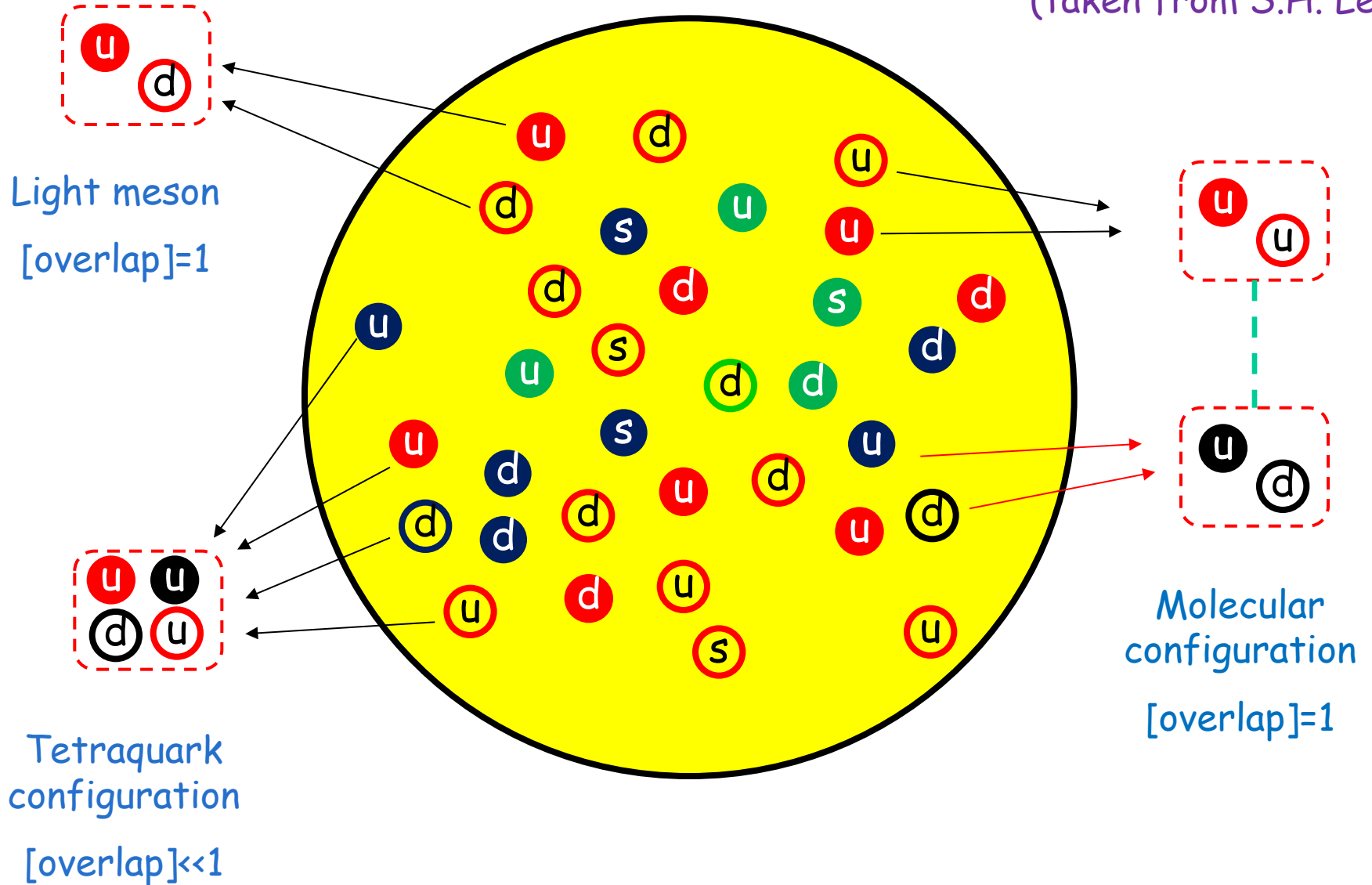


Thermal equilibrium

Hydrodynamical expansion

Lifetime ~ 10 fm

(taken from S.H. Lee)



Molecular configuration favored !

Coalescence model : geometry and dynamics of bound states

ExHIC, Prog. Part.Nucl.Phys. (2017) arXiv:1702.00486

$$N_h \approx g_h \prod_{j=1}^n \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{\frac{3}{2}}}{V(\tau)(1 + 2\mu_i T(\tau)\sigma_i^2)} \left[\frac{4\mu_i T(\tau)\sigma_i^2}{3(1 + 2\mu_i T(\tau)\sigma_i^2)} \right]^{l_i}$$

Pb - Pb collisions

$$\sqrt{s} = 5.02 \text{ TeV}$$

$$\left\{ \begin{array}{l} N_{\psi(2S)}(\tau_H) \approx 1.8 \times 10^{-4}, \\ N_X^{(4q)}(\tau_H) \approx 1.8 \times 10^{-4}, \\ N_X^{(Mol.)}(\tau_H) \approx 1.1 \times 10^{-2}, \end{array} \right.$$

At the end of QGP

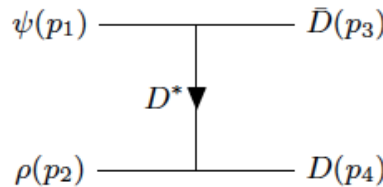
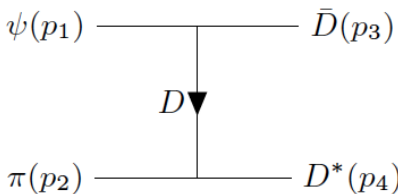
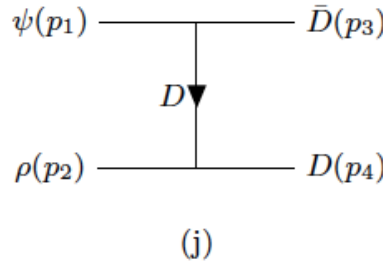
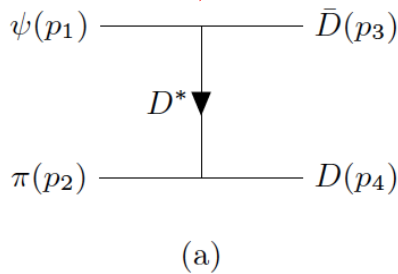
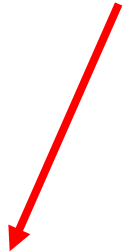
Before passing
the hadron gas

$$\left\{ \begin{array}{l} \frac{N_X^{(mol)}}{N_{\psi(2S)}} \simeq 50 \\ \frac{N_X^{(4q)}}{N_{\psi(2S)}} \simeq 1 \end{array} \right.$$

The interactions of J/Psi

Y.s. Oh, T. Song and S.H. Lee, arXiv:nucl-th/0010064

$$\begin{aligned} \mathcal{L}_{\psi DD} &= g_{\psi DD} \psi_{\mu} (\partial^{\mu} D \bar{D} - D \partial^{\mu} \bar{D}), \\ \mathcal{L}_{\psi D^* D^*} &= -i g_{\psi D^* D^*} [\psi^{\mu} (\partial_{\mu} D^{*\nu} \bar{D}_{\nu}^* - D^{*\nu} \partial_{\mu} \bar{D}_{\nu}^*) \\ &\quad + (\partial_{\mu} \psi_{\nu} D^{*\nu} - \psi_{\mu} \partial_{\nu} D^{*\nu}) \bar{D}^{*\mu} \\ &\quad + D^{*\mu} (\psi^{\nu} \partial_{\mu} \bar{D}_{\nu}^* - \partial_{\mu} \psi_{\nu} \bar{D}^{*\nu})], \\ \mathcal{L}_{\psi DD^*} &= -g_{\psi DD^*} \epsilon^{\mu\nu\alpha\beta} \partial_{\mu} \psi_{\nu} (\partial_{\alpha} D_{\beta}^* \bar{D} + D \partial_{\alpha} \bar{D}_{\beta}^*); \end{aligned}$$



Form factors and couplings from QCD sum rules:

M.E. Bracco, M. Chiapparini, F.S. Navarra and M. Nielsen, arXiv:1104.2864

Form factors:

$$g_{abc}(Q^2) = \frac{A}{Q^2 + B}$$

$$g_{abc}(Q^2) = A e^{-\left(\frac{Q^2+B}{c}\right)}$$

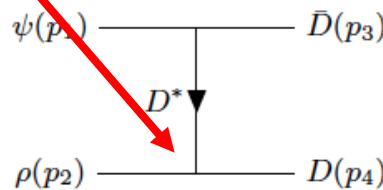
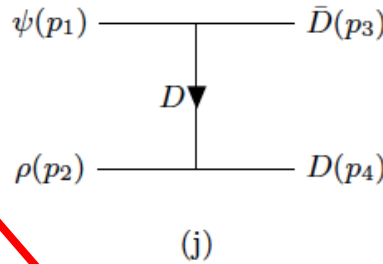
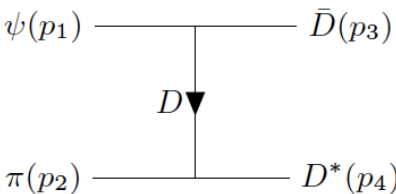
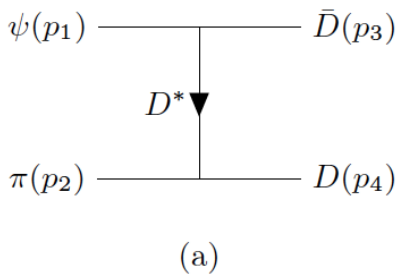
Couplings:

$$g_{abc} = g_{abc}(Q^2 = -m_c^2)$$

The interactions of J/Psi

Y.s. Oh, T. Song and S.H. Lee, arXiv:nucl-th/0010064

$$\begin{aligned}\mathcal{L}_{\pi DD^*} &= ig_{\pi DD^*} D^{*\mu} \vec{\tau} \cdot (\bar{D} \partial_\mu \vec{\pi} - \partial_\mu \bar{D} \vec{\pi}) + h.c., \\ \mathcal{L}_{\rho DD} &= ig_{\rho DD} (D \vec{\tau} \partial_\mu \bar{D} - \partial_\mu D \vec{\tau} \bar{D}) \cdot \vec{\rho}^\mu, \\ \mathcal{L}_{\rho D^* D^*} &= ig_{\rho D^* D^*} [(\partial_\mu D^{*\nu} \vec{\tau} \bar{D}_\nu^* - D^{*\nu} \vec{\tau} \partial_\mu \bar{D}_\nu^*) \cdot \vec{\rho}^\mu \\ &\quad + (D^{*\nu} \vec{\tau} \cdot \partial_\mu \vec{\rho}_\nu - \partial_\mu D^{*\nu} \vec{\tau} \cdot \vec{\rho}_\nu) \bar{D}^{*\mu} \\ &\quad + D^{*\mu} (\vec{\tau} \cdot \vec{\rho}^\nu \partial_\mu \bar{D}_\nu^* - \vec{\tau} \cdot \partial_\mu \vec{\rho}^\nu \bar{D}_\nu^*)], \\ \mathcal{L}_{\pi D^* D^*} &= -g_{\pi D^* D^*} \epsilon^{\mu\nu\alpha\beta} \partial_\mu D_\nu^8 \pi \partial_\alpha \bar{D}_\beta^*, \\ \mathcal{L}_{\rho DD^*} &= -g_{\rho DD^*} \epsilon^{\mu\nu\alpha\beta} (D \partial_\mu \rho_\nu \partial_\alpha \bar{D}_\beta^* + \partial_\mu D_\nu^* \partial_\alpha \rho_\beta \bar{D})\end{aligned}$$



Form factors and couplings from QCD sum rules:

M.E. Bracco, M. Chiapparini, F.S. Navarra and M. Nielsen, arXiv:1104.2864

Form factors:

$$g_{abc}(Q^2) = \frac{A}{Q^2 + B}$$

$$g_{abc}(Q^2) = A e^{-\left(\frac{Q^2+B}{C}\right)}$$

Couplings:

$$g_{abc} = g_{abc}(Q^2 = -m_c^2)$$

The interactions of Psi(2S)

$$r_{\psi(2S)} \simeq 2 r_{J/\psi} \quad \longrightarrow \quad \sigma_{\psi(2S)} \simeq 4 \sigma_{J/\psi}$$

$$\sigma_{\psi(2S)} \propto \left| \begin{array}{c} \psi(p_1) \text{---} \overset{2g}{\text{---}} \bar{D}(p_3) \\ \quad \quad \quad \downarrow D \\ \pi(p_2) \text{---} \text{---} D^*(p_4) \end{array} \right|^2 \propto 4g^2$$

$$g_{\psi(2S)D\bar{D}} = [g_{J/\psi D\bar{D}}, 2g_{J/\psi D\bar{D}}] \quad \text{main source of uncertainty}$$

$$\sigma_{ab \rightarrow cd} = \frac{1}{64\pi^2 s g_a g_b} \frac{|\vec{p}_{cd}|}{|\vec{p}_{ab}|} \int d\Omega \sum_{S,I} |\mathcal{M}_{ab \rightarrow cd}(s, \theta)|^2$$

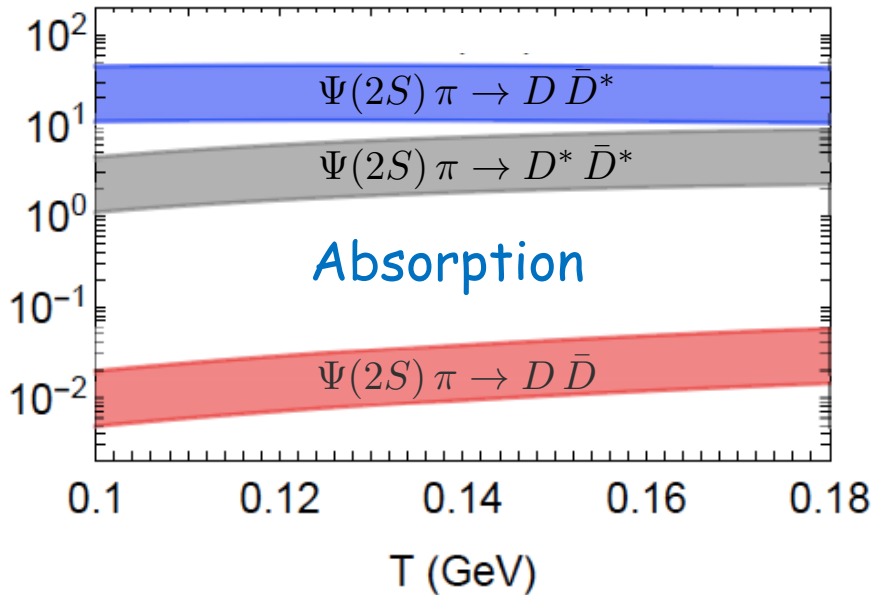
$$\sigma_{cd \rightarrow ab}(s) = \frac{g_a g_b}{g_c g_d} \frac{|\vec{p}_{ab}|}{|\vec{p}_{cd}|} \sigma_{ab \rightarrow cd}(s) \quad \text{inverse reaction}$$

$$\langle \sigma_{ab \rightarrow cd} v_{ab} \rangle = \frac{\int d^3 \mathbf{p}_a d^3 \mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b) \sigma_{ab \rightarrow cd} v_{ab}}{\int d^3 \mathbf{p}_a d^3 \mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b)}$$

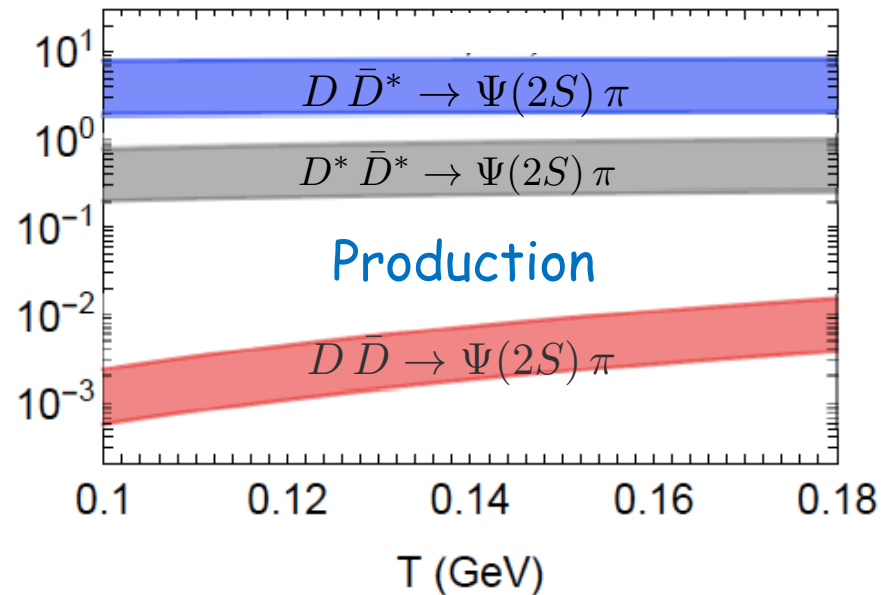
$f_i(\mathbf{p}) =$ thermal distribution

$v_{ab} =$ relative velocity

$\sigma(mb)$



$\sigma(mb)$



$$\frac{dN_\psi}{d\tau} = \langle \sigma(D\bar{D} \rightarrow \Psi(2S)\pi) \rangle n_D(\tau) N_{\bar{D}}(\tau) - \langle \sigma(\Psi(2S)\pi \rightarrow D\bar{D}) \rangle n_\pi(\tau) N_\psi(\tau)$$

$$\frac{dN_h(\tau)}{d\tau} = \sum_{\substack{\bar{c}=\bar{D},\bar{D}^*; \\ c=D,D^*; \\ \varphi=\pi,\rho}} [\langle \sigma_{\bar{c}c \rightarrow \varphi h} v_{\bar{c}c} \rangle n_{\bar{c}}(\tau) N_c(\tau) - \langle \sigma_{\varphi h \rightarrow \bar{c}c} v_{\psi\varphi} \rangle n_{\varphi}(\tau) N_h(\tau)]$$

gain
loss

$$n_i(\tau) \approx \frac{1}{2\pi^2} \gamma_i g_i m_i^2 T(\tau) K_2 \left(\frac{m_i}{T(\tau)} \right)$$

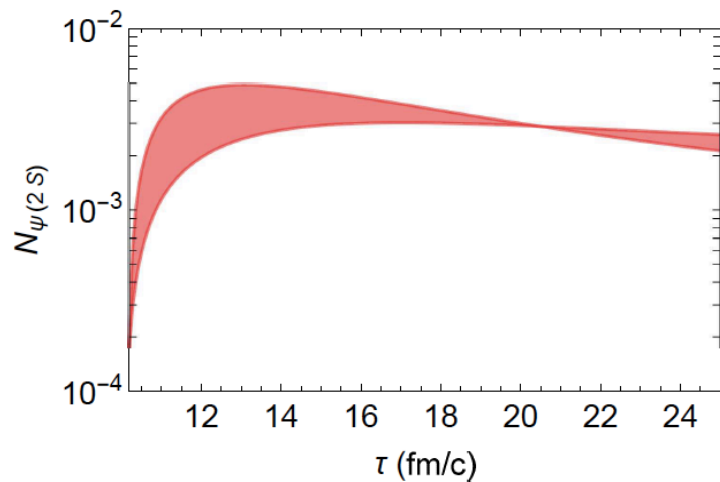
$$N_c(\tau) = n_c(\tau) \cdot V(\tau)$$

$$V(\tau) = \pi \left[R_C + v_C (\tau - \tau_C) + \frac{a_C}{2} (\tau - \tau_C)^2 \right]^2 \tau c$$

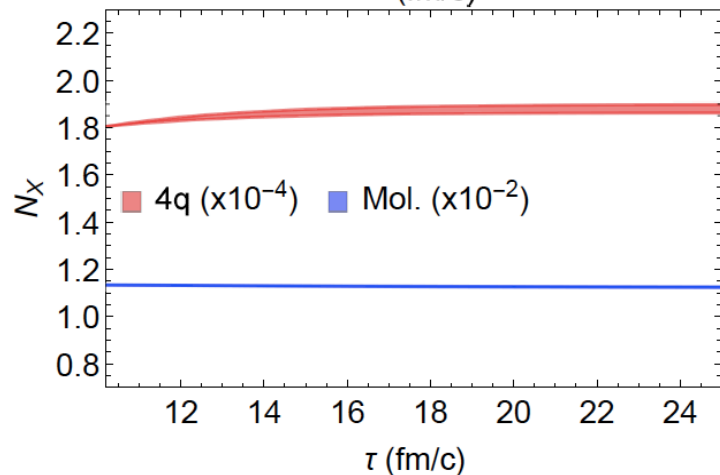
$$T(\tau) = T_C - (T_H - T_F) \left(\frac{\tau - \tau_H}{\tau_F - \tau_H} \right)^{\frac{4}{5}}$$

Parameters fixed in:

ExHIC Collaboration
 Prog. Part.Nucl.Phys. (2017)
 arXiv:1702.00486

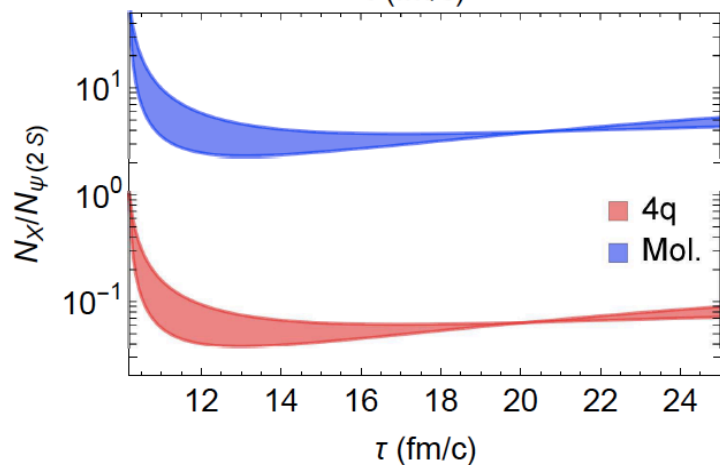


grows by a factor 10 !



A. Martinez Torres et al. ,
arXiv:1405.7583

S. Cho and S.H. Lee
arXiv:1405.7583



before the hadron gas :

$$\frac{N_X^{(mol)}}{N_{\psi(2S)}} \simeq 5$$

$$\frac{N_X^{(mol)}}{N_{\psi(2S)}} \simeq 50$$

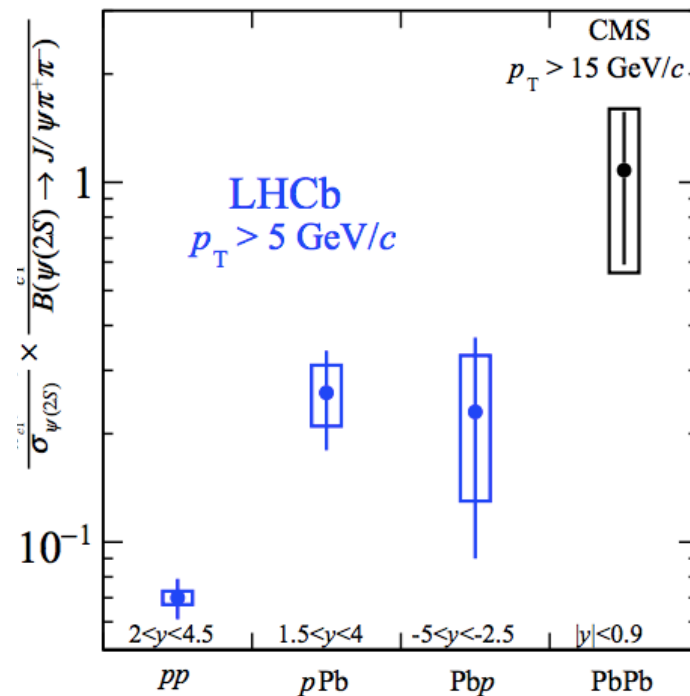
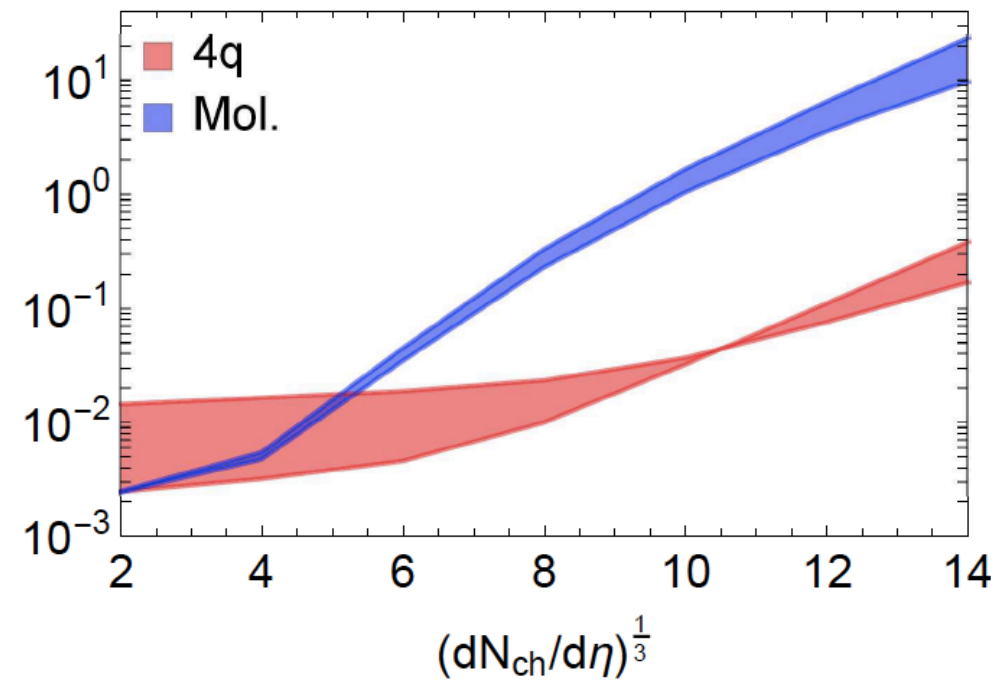
$$\frac{N_X^{(4q)}}{N_{\psi(2S)}} \simeq 1$$

$$\frac{N_X^{(4q)}}{N_{\psi(2S)}} \simeq 0.1$$

System size dependence

L.M. Abreu, F.S. N, H.P.L. Vieira, arXiv:2401.11320

$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\psi(2S)}}$$

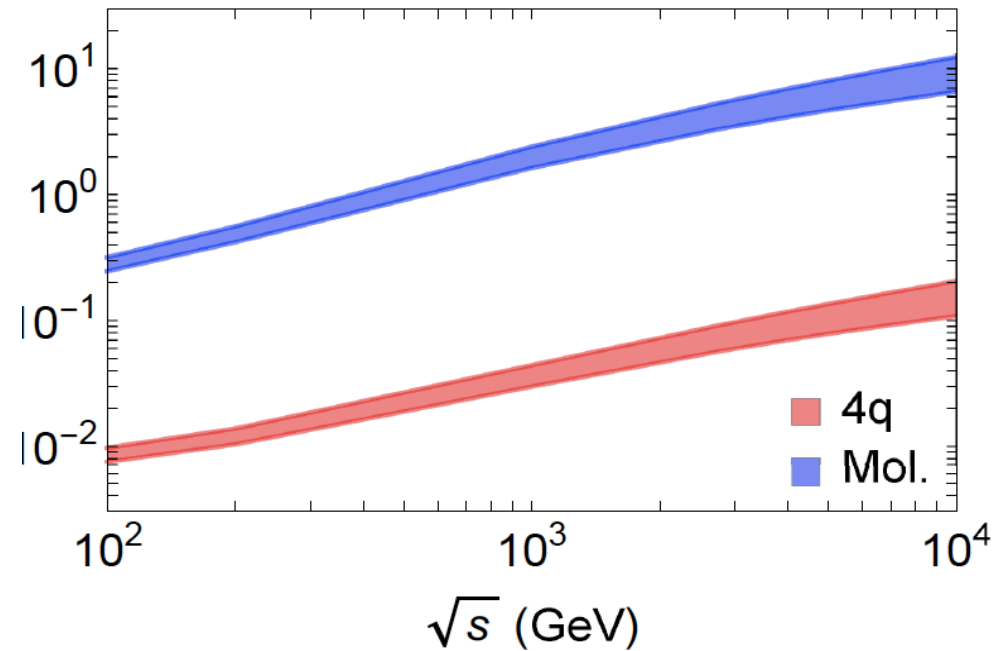
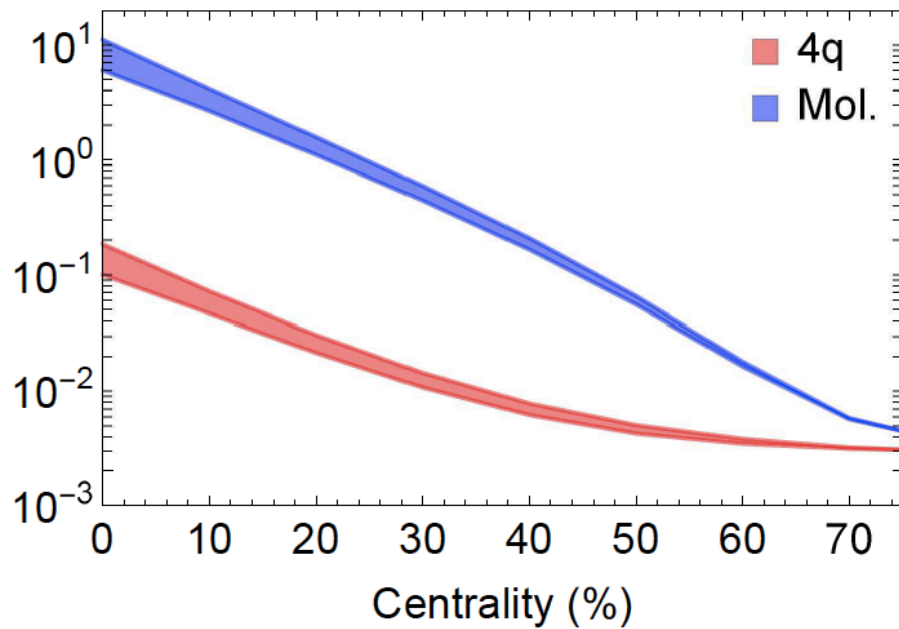


Centrality and energy dependence

L.M. Abreu, F.S. N, H.P.L. Vieira, arXiv:2401.11320

$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\psi(2S)}}$$

Pb-Pb collisions



Production and absorption of $X(3872)$ and $\Psi(2S)$ in a hadron gas

Initial conditions from QGP given by the Coalescence Model

Effective Lagrangians for the interactions

Thermal cross sections to solve rate equations

Multiplicities change with time.

$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\Psi(2S)}}$$

Increases with time

Increases with system size

Increases with energy

Decreases with centrality

Prediction for ALICE:

$$\frac{N_X}{N_{\Psi(2S)}} \simeq 5 \quad \text{molecules}$$

$$\frac{N_X}{N_{\Psi(2S)}} \simeq 0.1 \quad \text{tetraquarks}$$

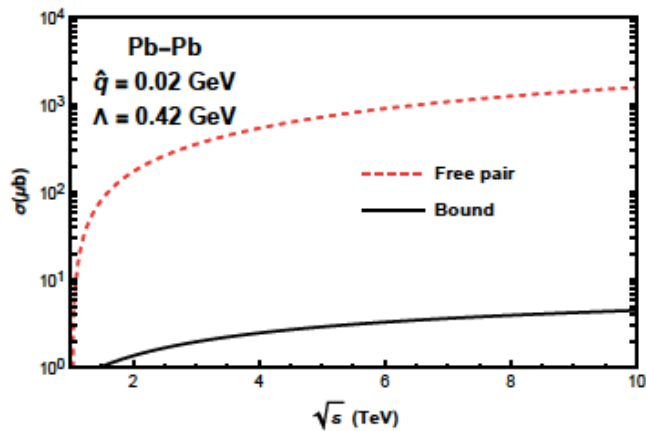
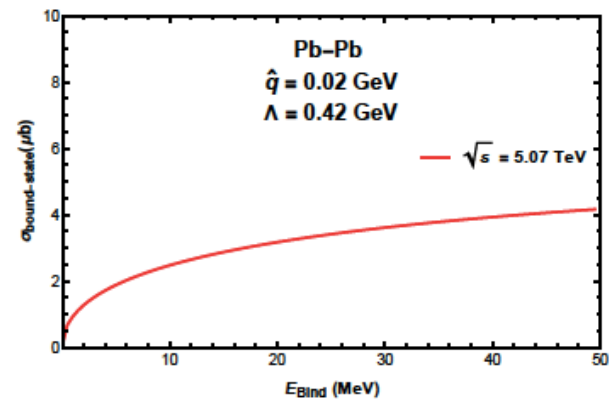
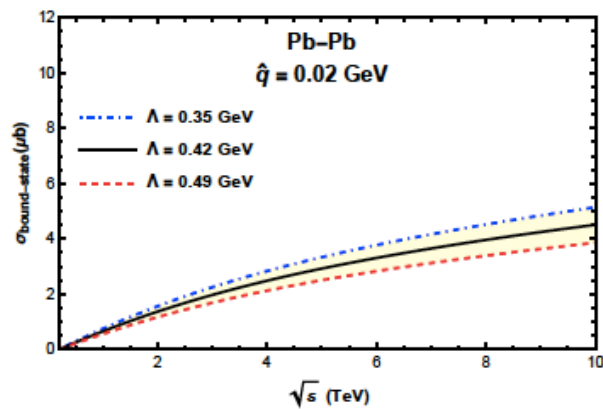
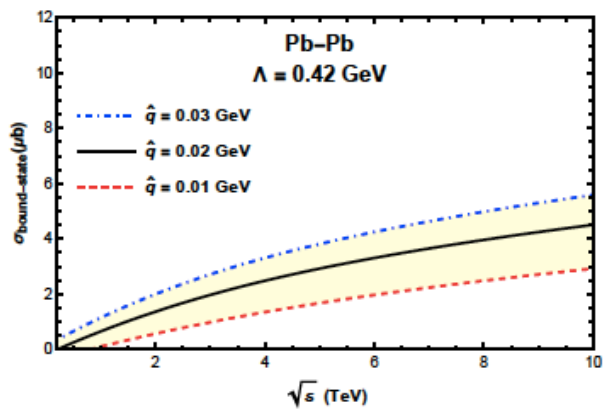
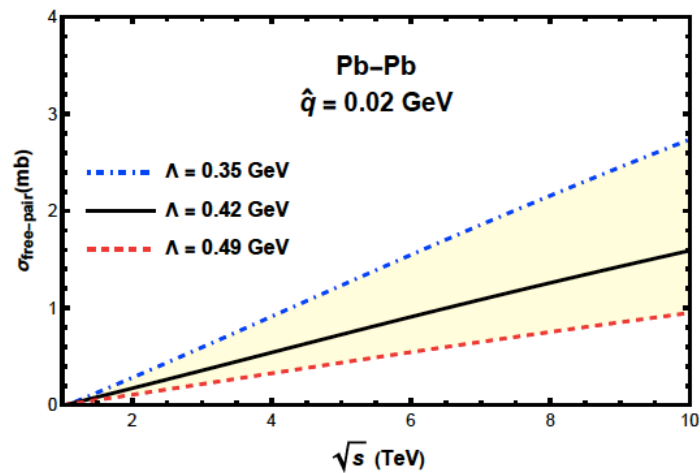
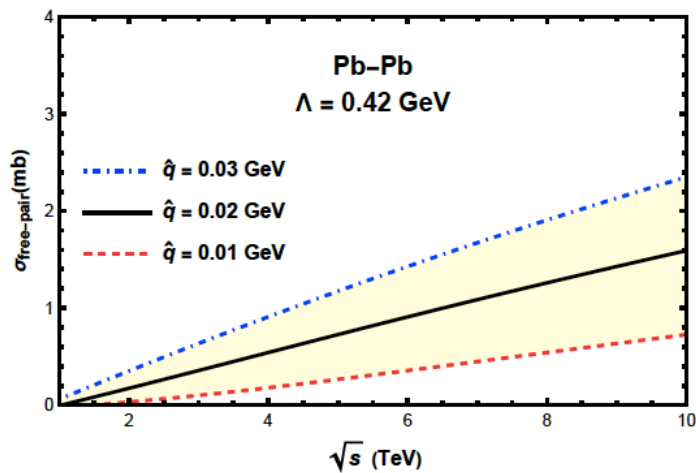


Thank you !

Prediction for ALICE:

(which includes low transverse momenta)

$$\frac{N_X}{N_{\psi(2S)}} \simeq 5 \quad \text{for molecules}$$
$$\frac{N_X}{N_{\psi(2S)}} \simeq 0.1 \quad \text{for tetraquarks}$$



$$\sigma(Pb Pb \rightarrow Pb Pb D^+ D^-) = 0.75_{-0.4}^{+0.4} \text{ mb}$$

$$\sigma_{exclusive}(Pb Pb \rightarrow Pb Pb c \bar{c}) = 1.5_{-0.4}^{+0.4} \text{ mb.}$$

$$\sigma_{QED}(Pb Pb \rightarrow Pb Pb c \bar{c} X) \approx \sigma_{QCD}(pp \rightarrow pp c \bar{c} X)$$

$$\sigma_{inclusive}(pp \rightarrow c \bar{c} X) = 8.43_{-1.16}^{+1.05} \text{ mb}$$

$$\sigma(Pb Pb \rightarrow Pb Pb B) = 3.0_{-1.2}^{+0.8} \mu\text{b}$$

$$5 \leq \sigma(Pb Pb \rightarrow Pb Pb R) \leq 11 \mu\text{b}$$

$$I) \quad g_{M_1 M_2 M_3} = \frac{A}{Q^2 + B},$$

$$II) \quad g_{M_1 M_2 M_3} = A e^{-\left(\frac{B+Q^2}{C}\right)}$$

M_1	M_2	M_3	Form	A	B	C
ψ	D	D	II	5.8	20	15.8
ψ	D^*	D	II	20	27	18.6
ψ	D	D^*	II	13	26	21.2
ψ	D^*	D^*	II	6.2	0	3.55
π	D	D^*	I	126	11.9	-
π	D^*	D	I	126	11.9	-
ρ	D	D	I	37.5	12.1	-
ρ	D^*	D^*	II	4.9	0	13.3
π	D^*	D^*	II	4.8	0	6.8
ρ	D	D^*	I	234	44	-
ρ	D^*	D	I	234	44	-

System size dependence

$$\mathcal{N} = \left[\frac{dN}{d\eta}(\eta = 0) \right]^{1/3}$$

$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\psi(2S)}}$$

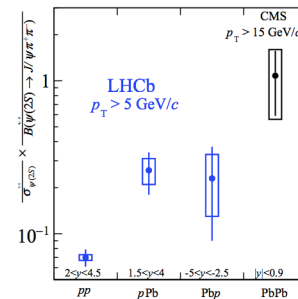
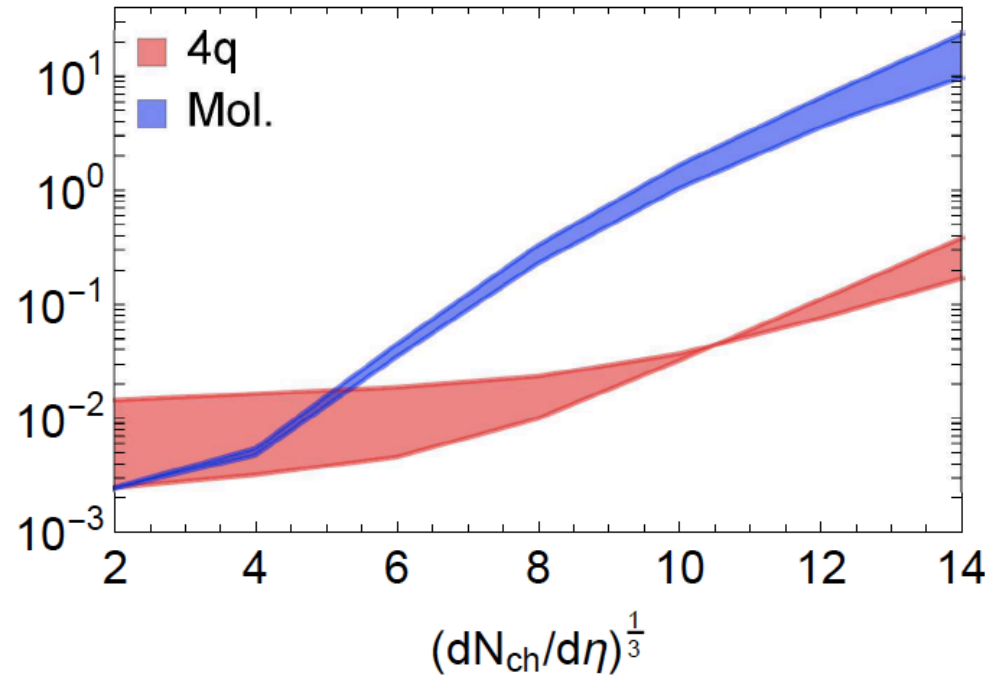
$$T_F = T_{F0} e^{-b\mathcal{N}}$$

$$\tau_F = \tau_H \left(\frac{T_H}{T_{F0}} \right)^3 e^{3b\mathcal{N}}$$

$$V = 2.82\mathcal{N}^3$$

$$N_c = 7.9 \times 10^{-5} \mathcal{N}^{4.8}$$

$$N_u = N_d = 0.37\mathcal{N}^3.$$

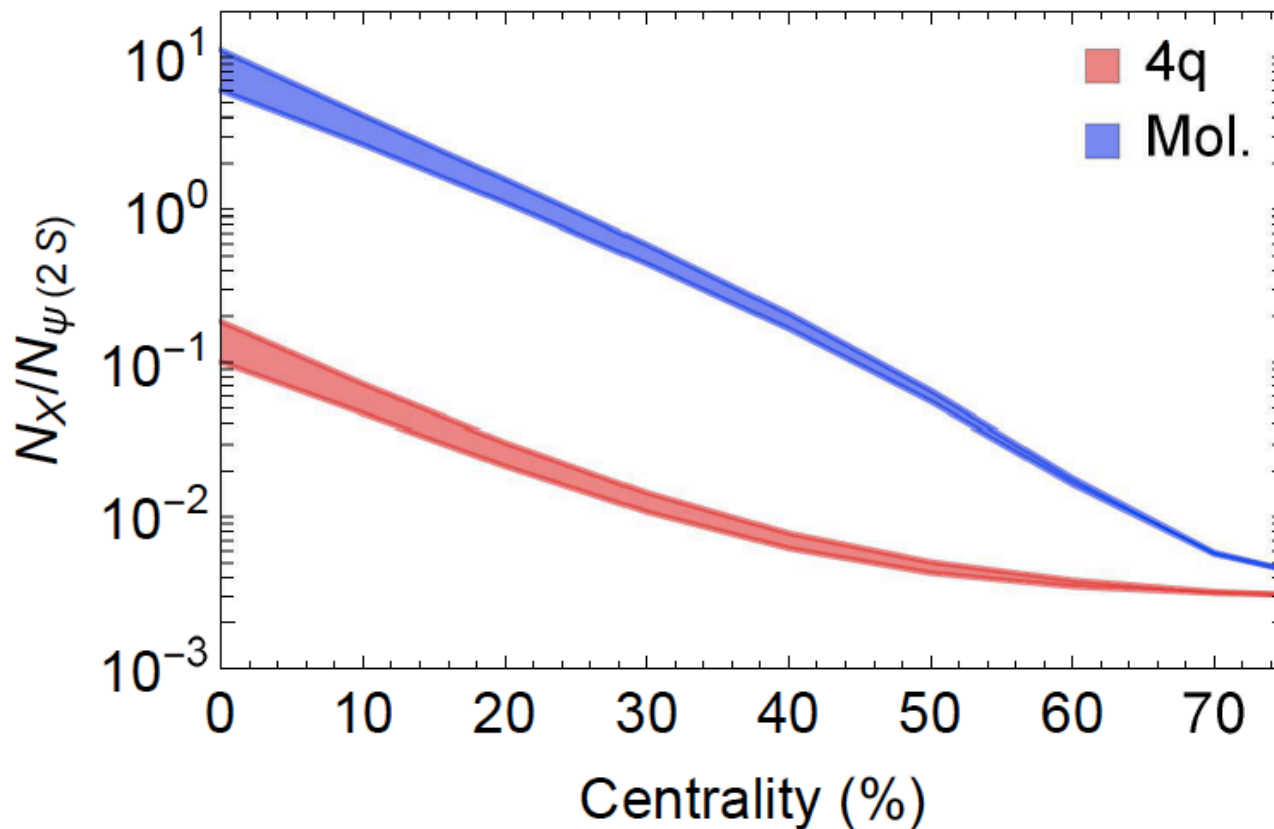


Centrality dependence

$$\left. \frac{dN_{ch}}{d\eta} \right|_{|\eta| < 0.5} = 2142.16 - 85.76x + 1.89x^2 - 0.03x^3$$

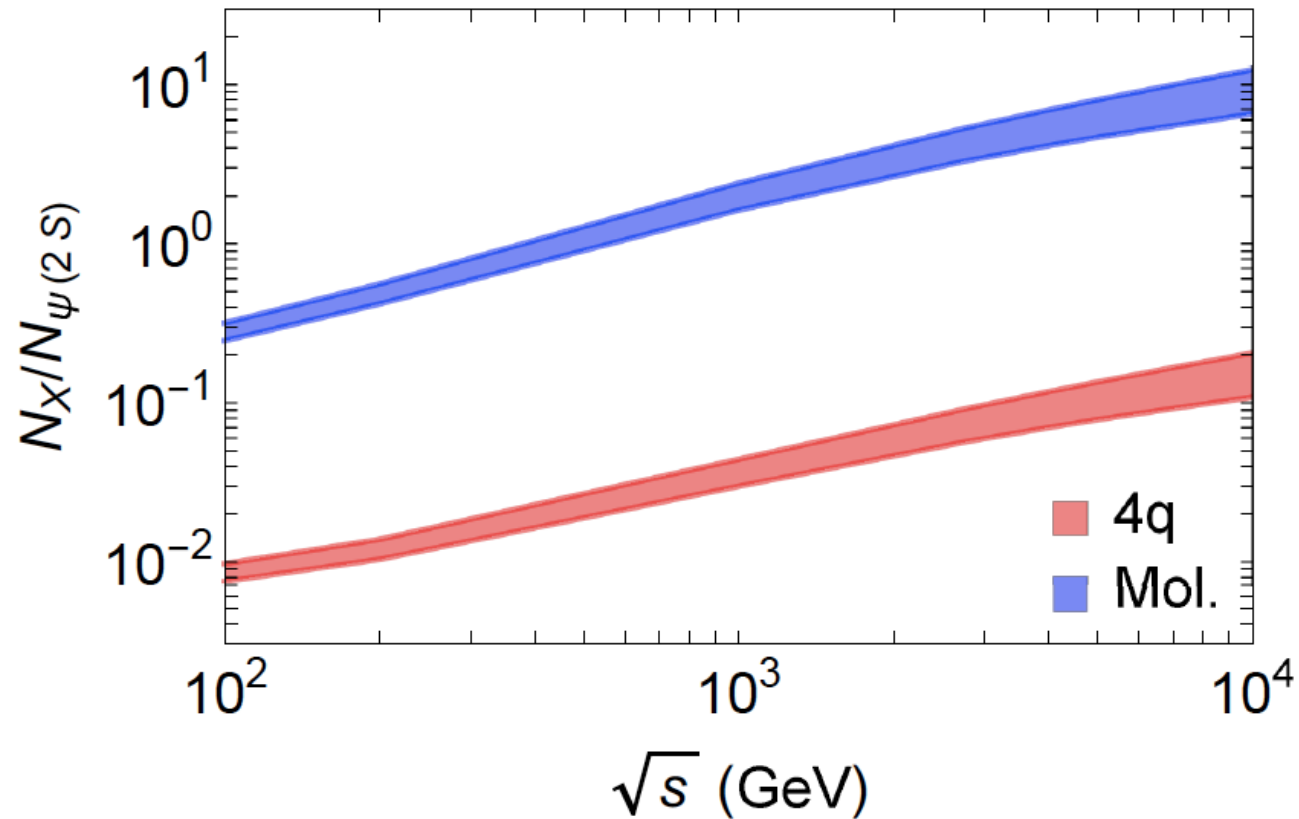
$$+ 3.67 \times 10^{-5} x^4 - 2.24 \times 10^{-6} x^5$$

$$+ 5.25 \times 10^{-9} x^6, \quad (17)$$

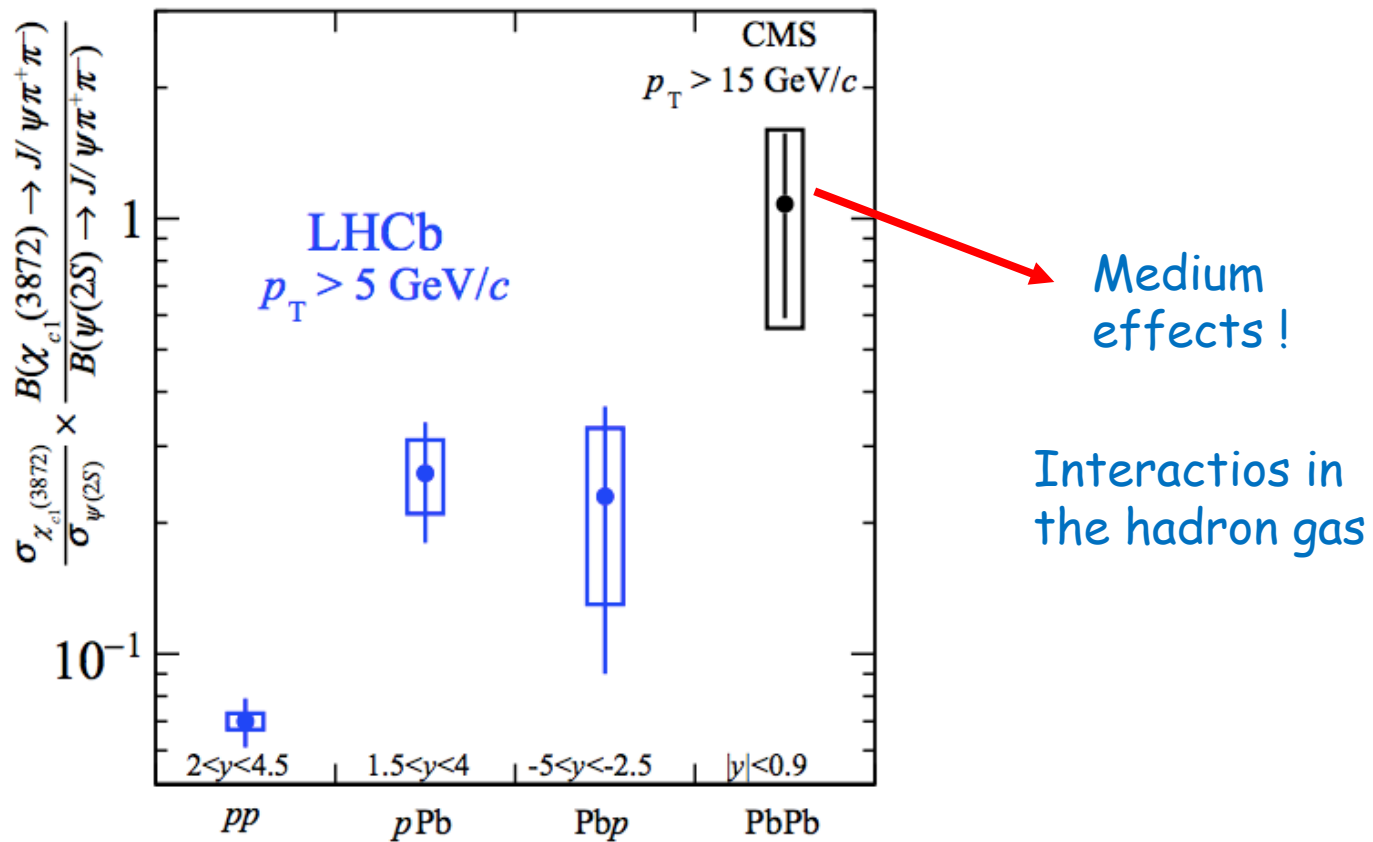


Energy dependence

$$\left. \frac{dN_{ch}}{d\eta} \right|_{|\eta| < 0.5} = -2332.12 + 491.69 \log(220.06 + \sqrt{s})$$



$$\mathcal{R} = \frac{N_{X(3872)}}{N_{\psi(2S)}}$$



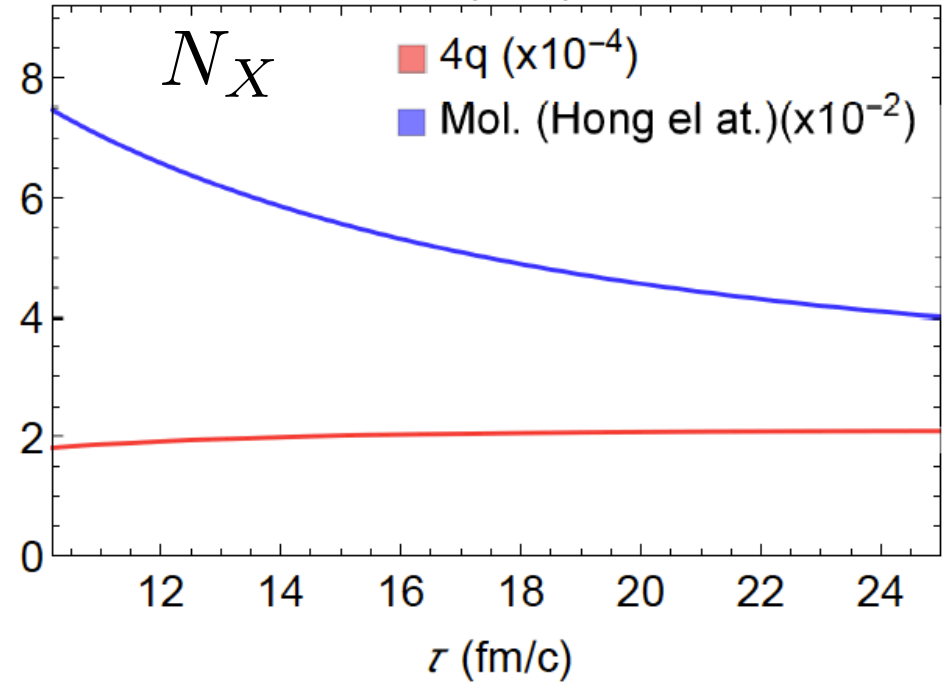
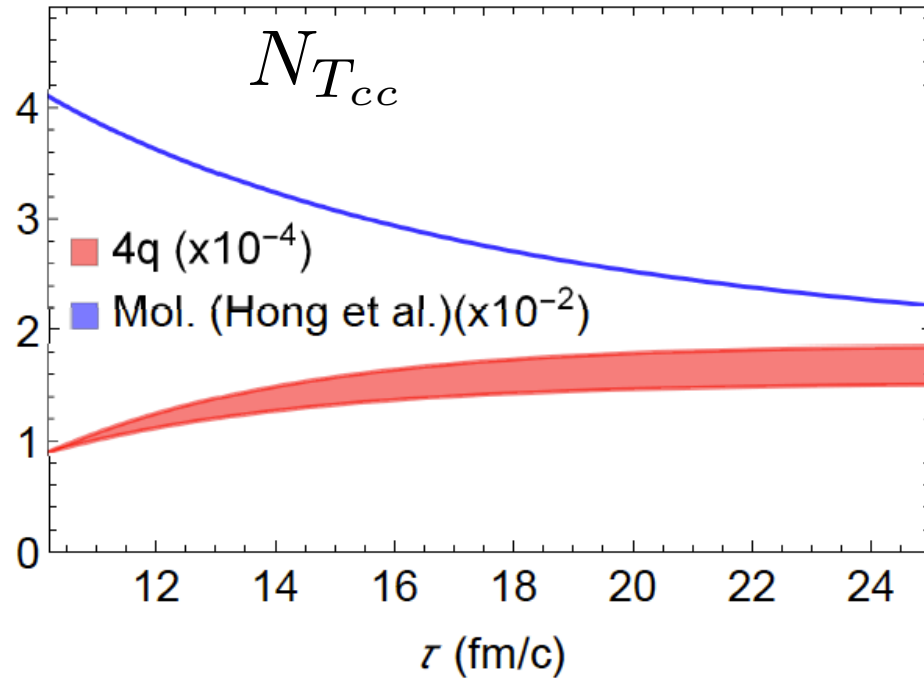
LHCb, arXiv:2402.14975

Multiplicities

Abreu, Navarra, Vieira, PRD (2022), arXiv:2202.10882

$$\frac{dN_{T_{cc}}(\tau)}{d\tau} = \sum_{\substack{c,c'=D,D^* \\ \varphi=\pi,\rho}} [\langle \sigma_{cc' \rightarrow T_{cc}\varphi} v_{cc'} \rangle n_c(\tau) N_{c'}(\tau) - \langle \sigma_{\varphi T_{cc} \rightarrow cc'} v_{T_{cc}\varphi} \rangle n_{\varphi}(\tau) N_{T_{cc}}(\tau)]$$

Pb - Pb collisions at $\sqrt{s} = 5.02$ TeV



Difference of multiplicities decreases but remain large !

Tetraquarks: Thermal Cross Sections and Rate Equation

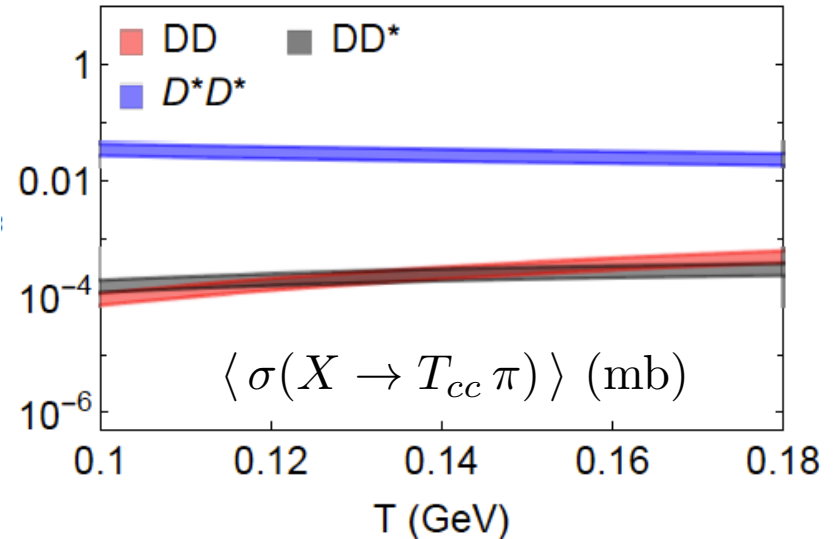
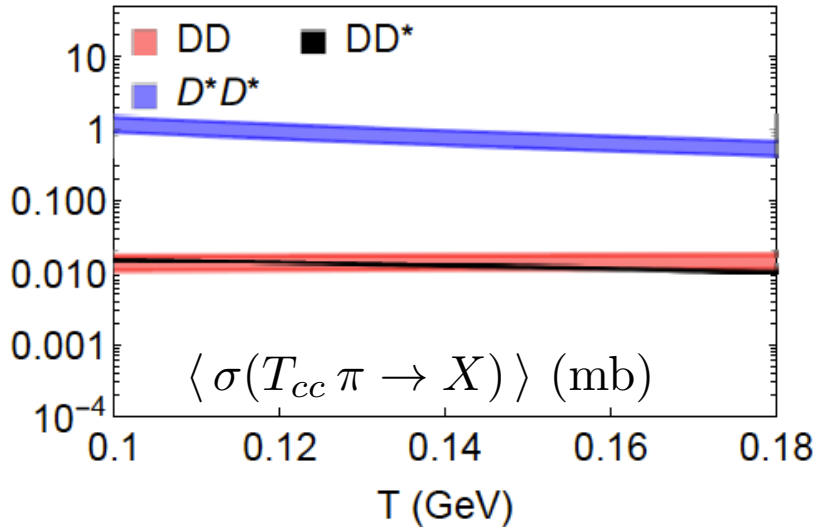
Abreu, Navarra, Vieira, PRD (2022), arXiv:2202.10882

$$\langle \sigma_{ab \rightarrow cd} v_{ab} \rangle = \frac{\int d^3 \mathbf{p}_a d^3 \mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b) \sigma_{ab \rightarrow cd} v_{ab}}{\int d^3 \mathbf{p}_a d^3 \mathbf{p}_b f_a(\mathbf{p}_a) f_b(\mathbf{p}_b)}$$

$f_i(\mathbf{p}) =$ thermal distribution

$v_{ab} =$ relative velocity

Inverse processes with detailed balance: $g_a g_b |\vec{p}_{ab}|^2 \sigma_{ab \rightarrow cd}(s) = g_c g_d |\vec{p}_{cd}|^2 \sigma_{cd \rightarrow ab}(s)$



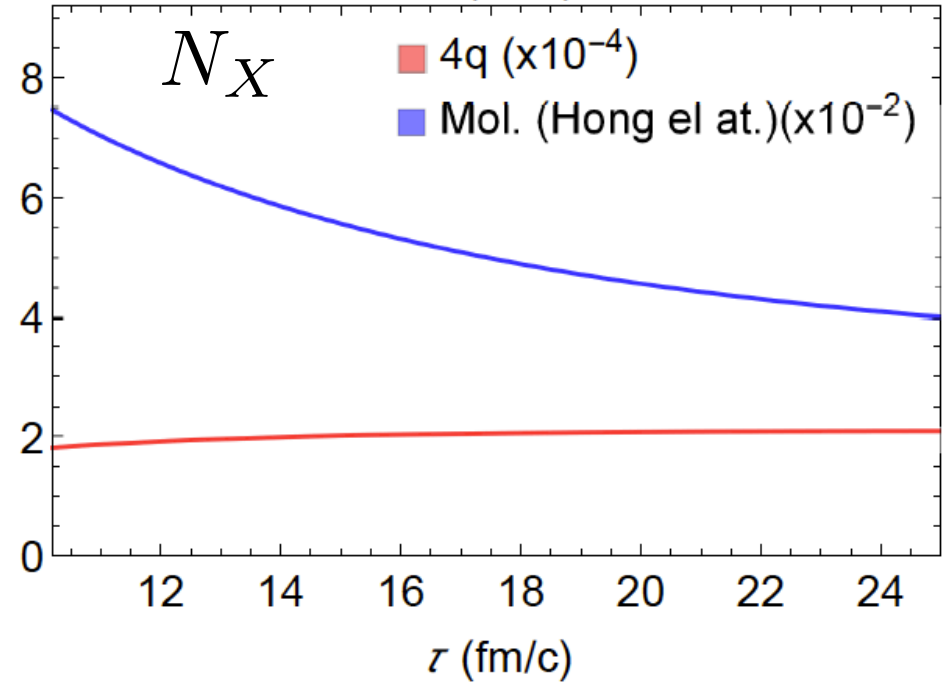
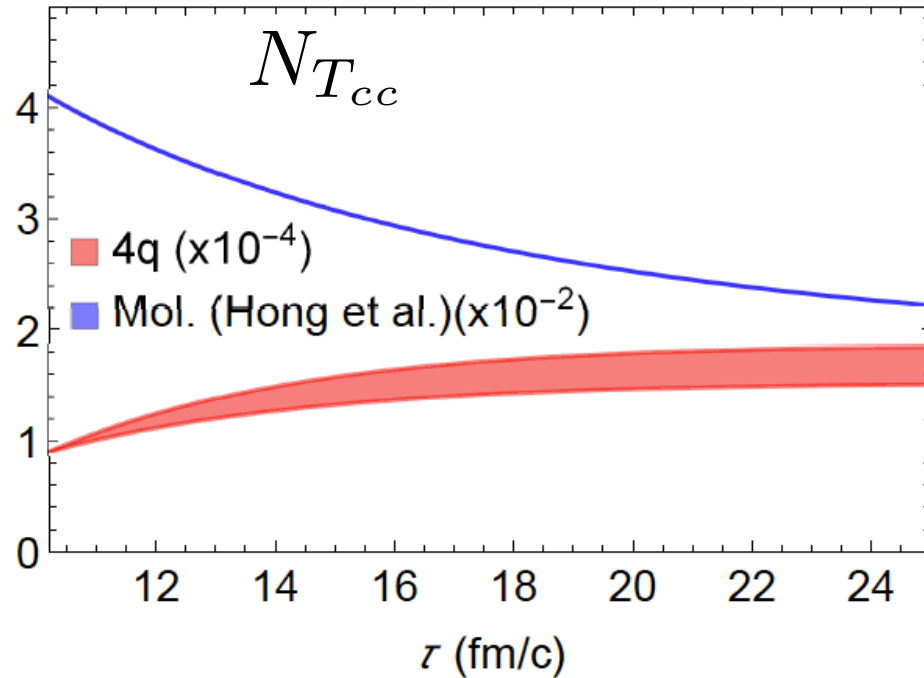
$$\frac{dN_{T_{cc}}(\tau)}{d\tau} = \sum_{\substack{c, c' = D, D^* \\ \varphi = \pi, \rho}} [\langle \sigma_{cc' \rightarrow T_{cc} \varphi} v_{cc'} \rangle n_c(\tau) N_{c'}(\tau) - \langle \sigma_{\varphi T_{cc} \rightarrow cc'} v_{T_{cc} \varphi} \rangle n_{\varphi}(\tau) N_{T_{cc}}(\tau)]$$

Multiplicities

Abreu, Navarra, Vieira, PRD (2022), arXiv:2202.10882

$$\frac{dN_{T_{cc}}(\tau)}{d\tau} = \sum_{\substack{c,c'=D,D^* \\ \varphi=\pi,\rho}} [\langle \sigma_{cc' \rightarrow T_{cc}\varphi} v_{cc'} \rangle n_c(\tau) N_{c'}(\tau) - \langle \sigma_{\varphi T_{cc} \rightarrow cc'} v_{T_{cc}\varphi} \rangle n_{\varphi}(\tau) N_{T_{cc}}(\tau)]$$

Pb - Pb collisions at $\sqrt{s} = 5.02$ TeV



Difference of multiplicities decreases but remain large !



Thank you !

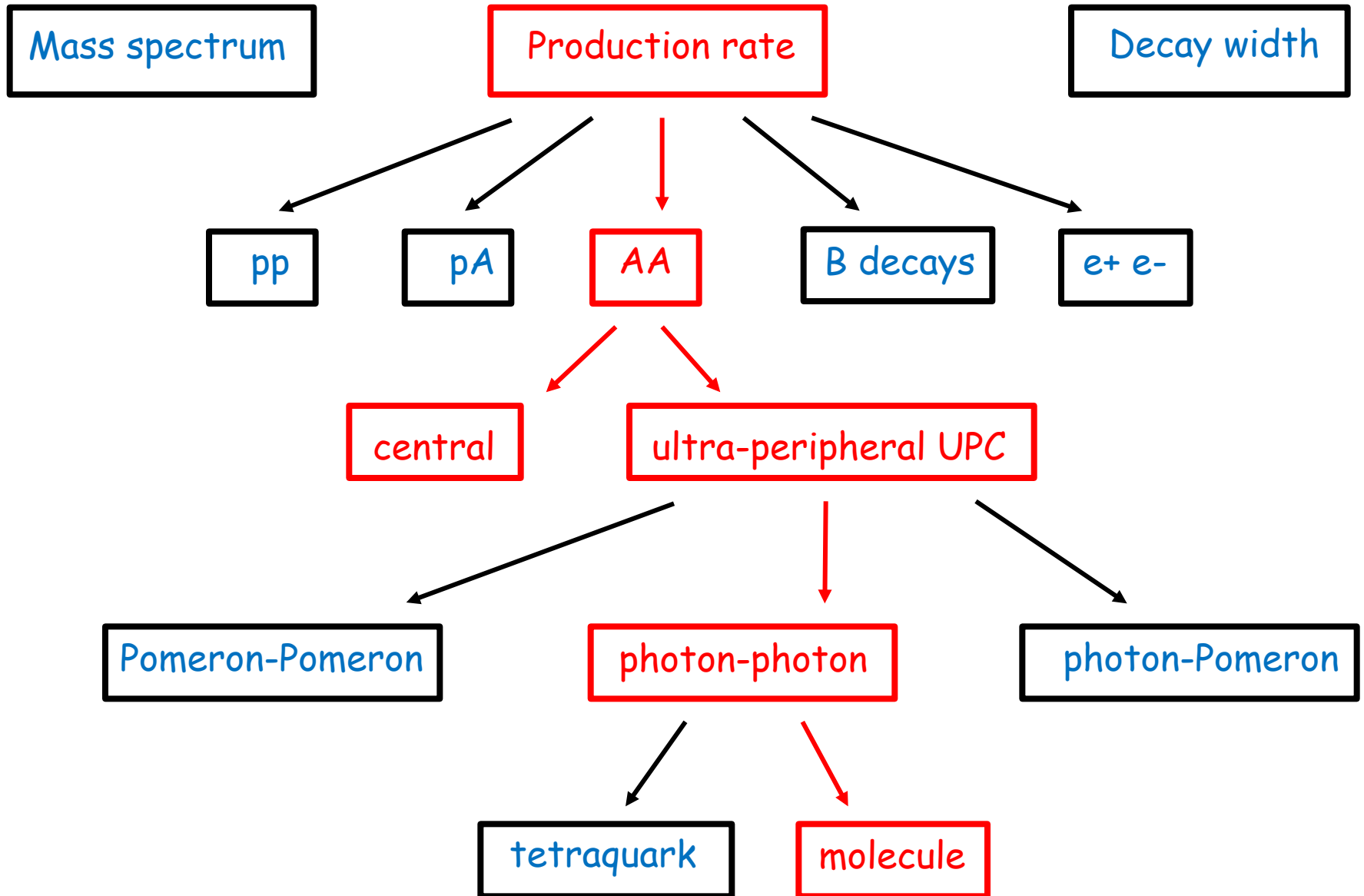
I - Production of exotic charmonium in ultra-peripheral collisions (UPC)

F.C. Sobrinho, L.M. Abreu, C. A. Bertulani, F.S. Navarra, arXiv:2405.02645

II - Production of exotic charmonium in central collisions

L.M. Abreu, F.S. Navarra, H.P.L. Vieira, arXiv:2401.11320

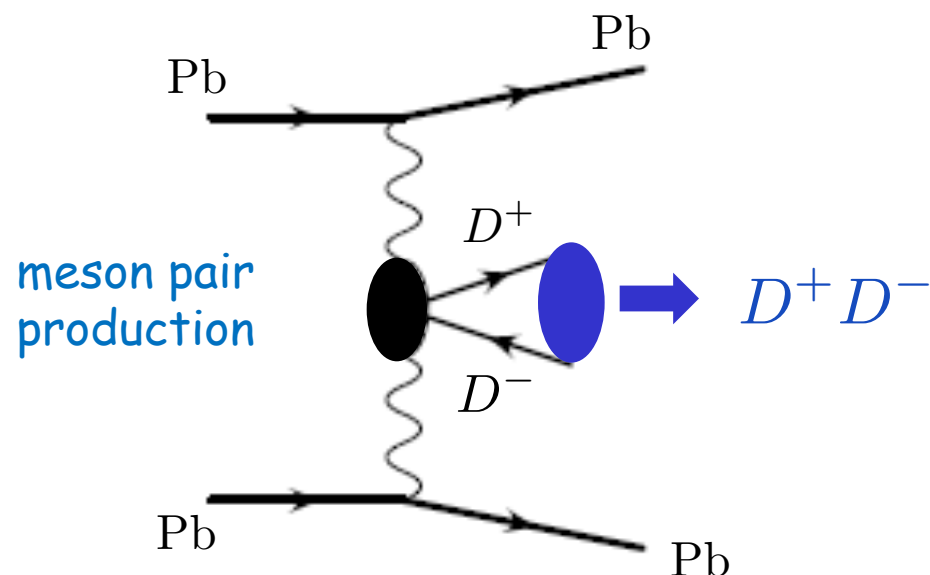
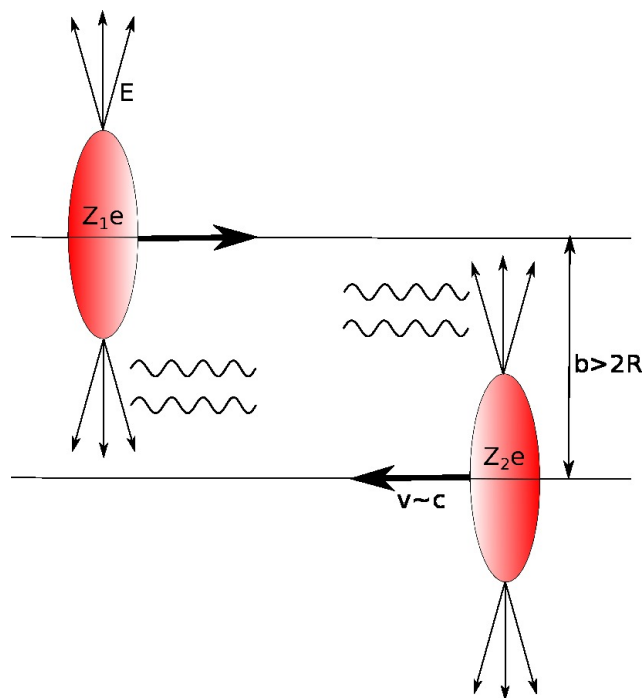
Studying Exotic Charmonium



Production of charm molecules in Ultra-Peripheral Collisions

UPC:

The lightest charm molecule : $D^+ D^-$



C. A. Bertulani, S. R. Klein, and J. Nystrand,
 Ann. Rev. Nucl. Part. Sci. 55, 271 (2005),
 arXiv:nucl-ex/0502005.

$$\sqrt{s_{NN}} = 5.02 \text{ TeV}$$

Lightest Charm Molecule : scalar D^+D^- bound state

5

Known as $X(3700)$ $M \approx 3723$ MeV

Predicted in:

D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D 76, 074016 (2007).

C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D 87, 076006 (2013).

J. Nieves and M. P. Valderrama, Phys. Rev. D 86, 056004 (2012).

S. Prelovsek, S. Collins, D. Mohler, M. Padmanath and S. Piemonte, JHEP 06, 035 (2021).

Experimental evidence (not conclusive) in:

S. Uehara et al. (Belle Collaboration), Phys. Rev. Lett. 96, 082003 (2006).

B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 81, 092003 (2010).

Recent works:

M.~Ablikim et al. [BESIII], Phys. Rev. D 108, 052012 (2023)

P. C. S. Brandao, J. Song, L. M. Abreu and E. Oset, Phys. Rev. D 108, 054004 (2023)

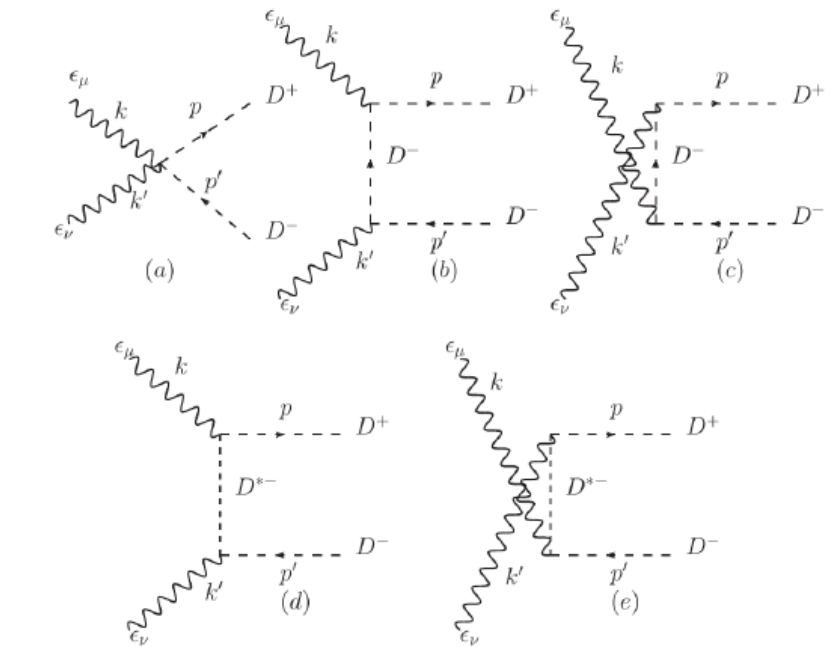
$$\mathcal{L} = -ig_{\gamma D^+ D^* -} F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} (D_\alpha^{*-} \overset{\leftrightarrow}{\partial}_\beta D^+ + D^- \overset{\leftrightarrow}{\partial}_\beta D_\alpha^{*+})$$

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - m_D^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Form factors in the vertices:



$$F(q^2) = \frac{\Lambda^2 - m_D^2}{\Lambda^2 - q^2}$$

$$F(m_D^2) = 1$$

Parameter !

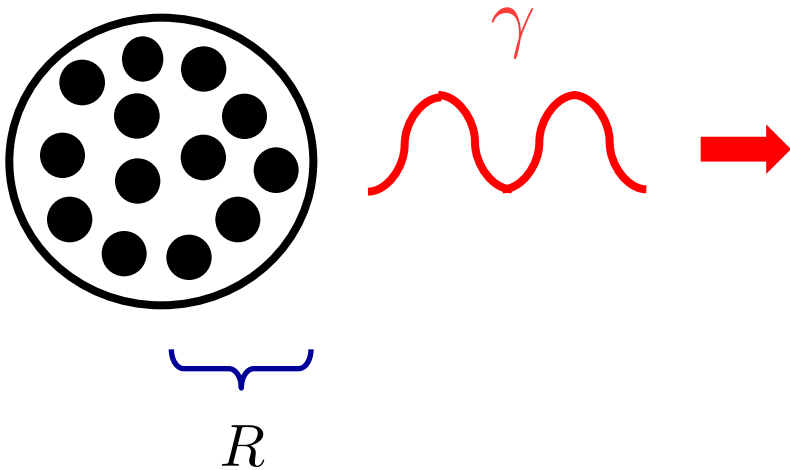
$$\sigma_P = \frac{1}{64\pi^2} \frac{1}{\hat{s}} \sqrt{1 - \frac{4m_D^2}{\hat{s}}} \int |M(\gamma\gamma \rightarrow D^+ D^-)|^2 d\Omega$$

The Number of Equivalent Photons

F.C. Sobrinho, L.M. Abreu, C. A. Bertulani, F.S. Navarra, arXiv:2405.02645

$$n(\vec{q})d^3q = \frac{Z^2\alpha}{\pi^2} \frac{(\vec{q}_\perp)^2}{\omega q^4} d^3q = \frac{Z^2\alpha}{\pi^2\omega} \frac{(\vec{q}_\perp)^2}{((\vec{q}_\perp)^2 + (\omega/\gamma)^2)^2} d^3q$$

Coherent emission: the photon does not resolve the charges in the source



minimum wavelength : $2R$

maximum transverse momentum :

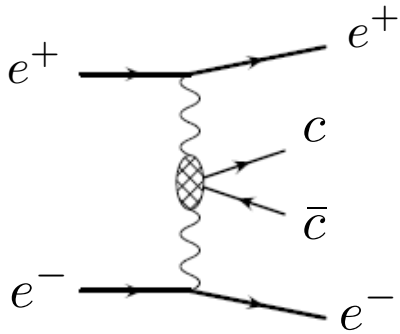
$$\hat{q} = \frac{1}{2R} \quad \text{Parameter !}$$

Pb : $\hat{q} = 0.014 \text{ GeV}$

proton : $\hat{q} = 0.1 \text{ GeV}$

$$n(\omega)d\omega = \frac{2Z^2\alpha}{\pi} \ln\left(\frac{\hat{q}\gamma}{\omega}\right) \frac{d\omega}{\omega}$$

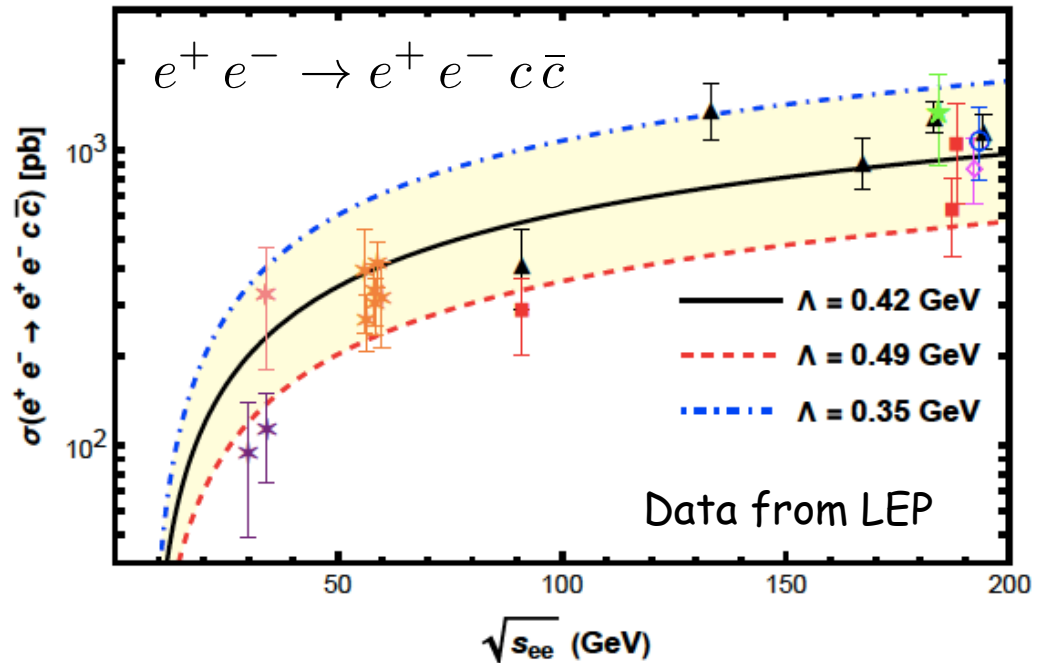
Fixing the cut-off with LEP data



$$\sigma_P = \frac{1}{64\pi^2} \frac{1}{\hat{s}} \sqrt{1 - \frac{4m_D^2}{\hat{s}}} \int |M(\gamma\gamma \rightarrow D^+ D^-)|^2 d\Omega$$

$$\sigma(e^+ e^- \rightarrow e^+ e^- D^+ D^-) = \int d\omega_1 d\omega_2 n(\omega_1) n(\omega_2) \sigma_P$$

One parameter: Λ



Take the average over the bound state wave function :

(QFT, Peskin,
page 150)

$$\frac{M(\gamma\gamma \rightarrow B)}{\sqrt{2E_B}} = \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}^*(\vec{q}) \frac{1}{\sqrt{2E_{D^+}}} \frac{1}{\sqrt{2E_{D^-}}} M(\gamma\gamma \rightarrow D^+ D^-)$$

If relative momentum q is small: $M(\gamma\gamma \rightarrow B) = \psi^*(0) \sqrt{\frac{2}{E_B}} M(\gamma\gamma \rightarrow D^+ D^-)$

$$d\sigma = \frac{1}{H} \frac{d^3p_B}{(2\pi)^3} \frac{1}{2E_B} (2\pi)^4 \delta^{(4)}(k + k' - p_B) |M(\gamma\gamma \rightarrow B)|^2$$

The Wave Function at the Origin

D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D 81, 014029 (2010)

Bethe-Salpeter Equation for a two (heavy) particle system:

$$\psi(0) = \frac{g}{(2\pi)^{3/2}} G \left\{ \begin{array}{l} G = -8\mu\pi \left(\Lambda_0 - \gamma \arctan \left(\frac{\Lambda_0}{\gamma} \right) \right) \\ g^2 = \frac{\gamma}{8\pi\mu^2 \left(\arctan \left(\frac{\Lambda_0}{\gamma} \right) - \frac{\gamma\Lambda_0}{\gamma^2 + \Lambda_0^2} \right)} \end{array} \right. \quad \gamma = \sqrt{2\mu E_b}$$

$$\left\{ \begin{array}{l} \Lambda_0 \\ \mu = m_D/2 \\ E_b \end{array} \right. \begin{array}{l} \text{cut-off} \\ \text{reduced mass} \\ \text{binding energy} \end{array} \quad \longrightarrow \quad \psi(0) = f(E_b)$$

$$\sigma_P(AA \rightarrow AA D^+ D^-) = \int_{m_D^2/\hat{q}\gamma}^{\hat{q}\gamma} d\omega_1 \int_{m_D^2/\omega_1}^{\hat{q}\gamma} d\omega_2 \sigma_P(\omega_1, \omega_2) n(\omega_1) n(\omega_2)$$

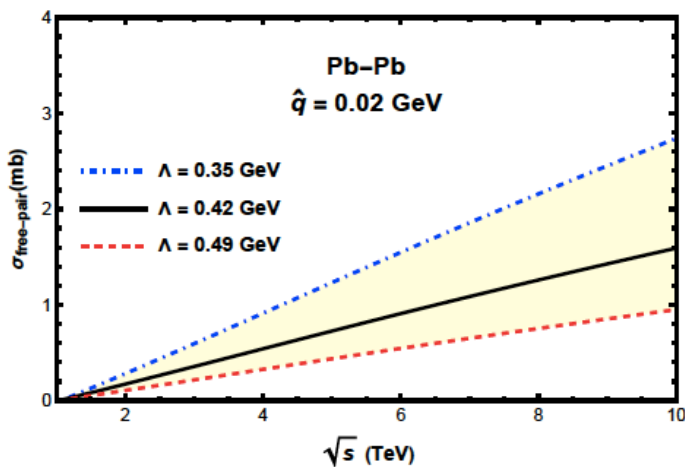
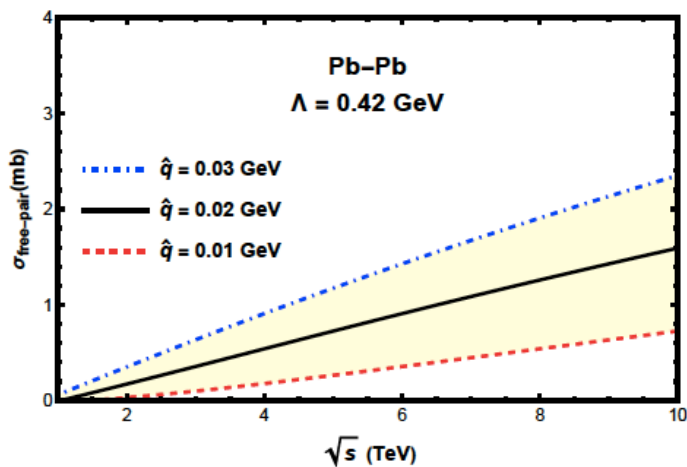
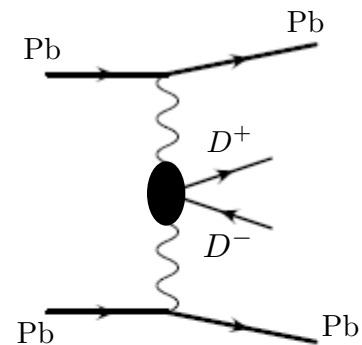
Two parameters: Λ \hat{q}

$$\sigma_B(AA \rightarrow AA B) = \int_{m_D^2/\hat{q}\gamma}^{\hat{q}\gamma} d\omega_1 \int_{m_D^2/\omega_1}^{\hat{q}\gamma} d\omega_2 \sigma_B(\omega_1, \omega_2) n(\omega_1) n(\omega_2)$$

Three parameters: Λ \hat{q} $\psi(0)$

Results

Free pair production



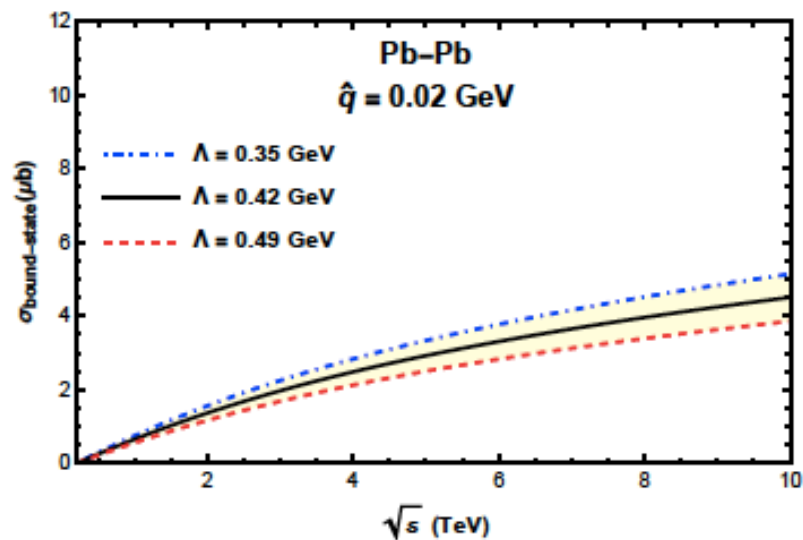
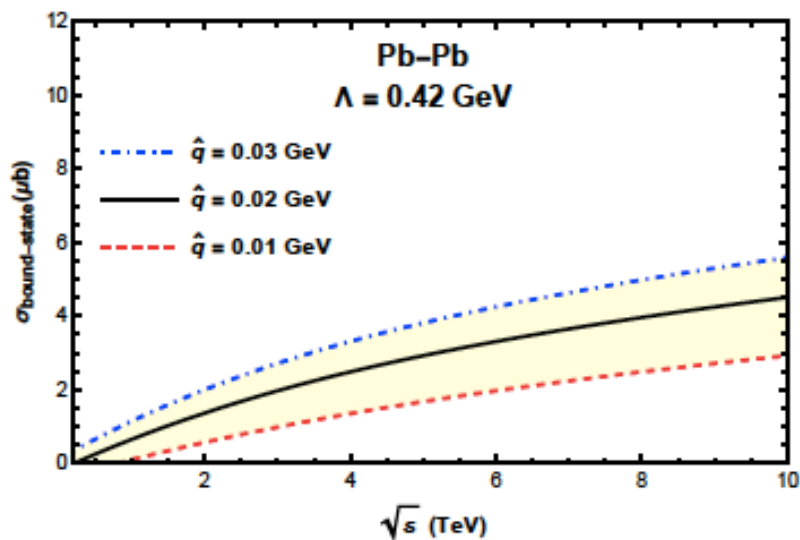
$$\sqrt{s} = 5.02 \text{ TeV} \longrightarrow$$

$$\sigma(Pb Pb \rightarrow Pb Pb D^+ D^-) = 0.75^{+0.4}_{-0.4} \text{ mb}$$

Bound state production in Pb Pb

$$E_b = 17 \text{ MeV}$$

$$|\psi(0)|^2 \simeq 0.008 \text{ GeV}^3$$



$$\sqrt{s} = 5.02 \text{ TeV}$$



$$\sigma(\text{Pb Pb} \rightarrow \text{Pb Pb B}) = 3.0^{+0.8}_{-1.2} \mu\text{b}$$

Comparison with previous results

$$\sigma = \int N(\omega_1, b_1) N(\omega_2, b_2) \hat{\sigma}(\gamma\gamma \rightarrow R) S(b) d^2b_1 d^2b_2 d\omega_1 d\omega_2$$

b-dependent equivalent photon spectrum

Form factor

$$N(\omega, b) = \frac{Z^2 \alpha_{em}}{\pi^2 b^2 \omega} \left[\int du u^2 J_1(u) \boxed{F\left(\sqrt{\frac{(b\omega/\gamma)^2 + u^2}{b^2}}\right)} \frac{1}{(b\omega/\gamma)^2 + u^2} \right]^2$$

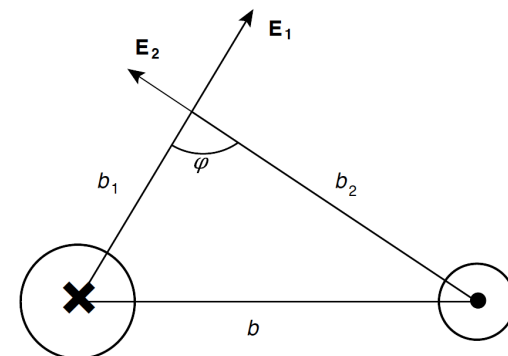
Photon fusion cross section : Low formula

Decay width

$$\hat{\sigma}(\gamma\gamma \rightarrow R) = 8\pi^2 (2J + 1) \frac{\boxed{\Gamma_{R \rightarrow \gamma\gamma}}}{M_R} \delta(4\omega_1\omega_2 - M_R^2)$$

Geometric factor

$$S(b) = \Theta(|\mathbf{b}_1 - \mathbf{b}_2| - R_1 - R_2)$$



pointlike

monopole

Klusek-Gawenda, Szczurek,
arXiv:1004.5521

State	Mass	$\Gamma_{\gamma\gamma}^{theor}$ (keV)	$\sigma_{b_{min}}$ (μb)			σ_F (μb)			σ_R (μb)		
			2.76 TeV	5.5 TeV	39 TeV	2.76 TeV	5.5 TeV	39 TeV	2.76 TeV	5.5 TeV	39 TeV
X(3940), 0^{++}	3943	0.33	4.2	8.2	31.6	6.5	11.8	40.9	5.7	10.8	39.6
\bar{X} (3915), 0^{++}	3919	0.20	2.6	5.1	19.8	4.0	7.3	25.3	3.6	6.7	24.5

↑
molecules
↓

Moreira, Bertulani, Gonçalves, FSN,
arxiv:1610.06604

State	Mass	$\Gamma_{\gamma\gamma}$ [keV]	σ [μb]		
			2.76 TeV	5.02 TeV	39 TeV
X(3940), 0^{++}	3943	0.33	5.5	9.7	32.5
χ_{c0} (3915), 0^{++}	3919	0.20	3.4	6.0	20.1

Fariello, Bhandari, Bertulani, F.S.N.,
arXiv:2306.10642

$$5 \leq \sigma \leq 11 \mu\text{b}$$

Summary-I

Formalism to create charm meson molecules in photon-photon collisions

Use effective Lagrangians and the bound state wave function

Three parameters: Λ \hat{q} $\psi(0)$ Can be better constrained!

Production of the molecule $D^+ D^-$ $\sigma \simeq 3.0 \mu b$

Production of similar states with a similar method $5 \leq \sigma \leq 11 \mu b$

Production of the equivalent tetraquark $\sigma \simeq 0.18 \mu b$

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