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Hagedorn temperature in confining gauge theories from holography

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- JHEP 01 (2023) 034, with Tommaso Canneti and Aldo L. Cotrone
- JHEP 08 (2023) 185, with Tommaso Canneti and Wolfgang Mück
- JHEP 10 (2023) 056, with Tommaso Canneti and Aldo L. Cotrone
- Ongoing works with T. Canneti, F. Castellani, A. L. Cotrone, W. Mück, J.M. Penin Ascariz

Hagedorn temperature

- Definition: a quantum system displays a Hagedorn behavior if at high energies E the density of states grows exponentially like the energy

$$\rho(E) \sim E^{\alpha} e^{E/T_H}$$

- T_H = Hagedorn temperature
- The thermal partition function of such a system…

$$Z[\beta] \sim \int dE \rho(E) e^{-\beta E} \,, \quad \beta \equiv 1/T$$

- …diverges for $T \to T_H^-$

- Examples include both QFTs (as Yang-Mills or quenched QCD) and string theory

Hagedorn temperature

- First example: strong interactions
- Rolf Hagedorn 1965: *Statistical thermodynamics of strong interactions at high energies*

- Argued for asymptotically exponential hadron mass spectrum and a limiting (Hagedorn) temperature

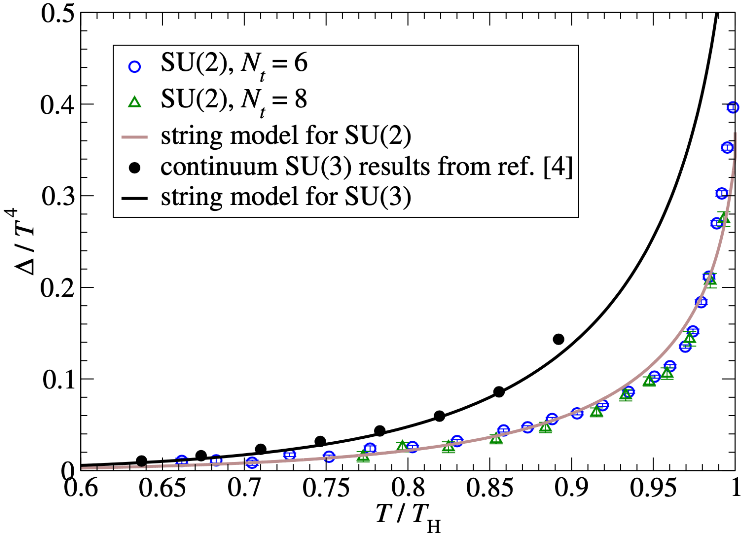
$$\rho(m) \xrightarrow[m \to \infty]{} \text{const.}\, m^{-5/2} \exp\left(\frac{m}{T_0}\right).$$

- Today we would say that this is expected to hold for the confining phase of Yang-Mills or quenched (large N) QCD

- These theories display first order deconfinement transition at T_c
- In these cases $T_H > T_c$

Hagedorn temperature

- Lattice: no first principle computation of T_H yet.
- Assuming T_H to be that of effective string theory, can match high $T<T_c$ behavior of observables. For SU(2), SU(3) YM see [Caselle, Nada, Panero, 15]



$$T_H \sim \sqrt{T_s}$$

T_s = string tension

(Δ=trace anomaly)

- At large N (e.g. SU(12)) focusing on metastable confining phase at $T>T_c$ indirect hints for $T_H \sim 1.16(9)\, T_c$ [Bringoltz, Teper 2005]

Motivations

- Second famous example: string theory
- In flat spacetime the string Hagedorn temperature is exactly known
- What about string theory on curved backgrounds?
- Interesting question in view of the string/gauge theory correspondence
- This maps large N, large ‘t Hooft coupling ($\lambda \gg 1$) confining gauge theories to weakly coupled string theories on curved gravity backrounds with fluxes.
- As a corollary: T_H (QFT) = T_H (String)
- Can we compute the Hagedorn temperature of strongly coupled large N confining gauge theories from the holographic dual description?
- Can we cross-check the holographic results with QFT ones?

Strings in flat space

- Consider type II superstrings in flat 10-dim spacetime
- Number of particle species with mass less than M asymptotes to

$$d(M) \sim M^{-9} e^{2\pi\sqrt{2\alpha'}M}$$

- Hence: $T_H = \sqrt{\frac{T_s}{4\pi}}\,, \qquad T_s = \frac{1}{2\pi\alpha'}$

- Z(T): torus (one-loop) partition function on $R^9 \times S^1$
- The one-loop contribution to Z(T) is determined completely by its spectrum, in particular it diverges if the spectrum includes a tachyon.
- Is there a tachyon appearing when $T > T_H$?

Strings in flat space

- Strings winding the thermal circle have tachyonic mode when the size of the circle is small enough [Sathiapalan 87; Kogan 87; Atick,Witten, 88]

- In fact, the ground state with winding ± 1 has mass

$$m_W^2 = \frac{2}{\alpha'}\left(\frac{1}{8\pi^2\alpha' T^2} - 1\right)$$

- This becomes tachyonic for $T > T_H$
- For T close to T_H this state corresponds to an almost massless complex scalar field in target space which can be accounted for in the low energy effective action.
- We extend all this to curved spacetimes dual to confining gauge theories.

Strings on confining backgrounds

- Finite T confining phase of (q+1)-dim QFTs is holographically dual to closed string theory on backgrounds with metric asymptotically going as

$$ds^2 \approx 2\pi\alpha' T_s \left(1 + \frac{r^2}{l^2}\right) \underbrace{(dt^2 + \eta_{ij} dx^i dx^j)}_{\text{q+1}} + \underbrace{dr^2 + r^2 d\Omega^2_{d-1}}_{\text{flat d-dim space}} + ds^2_{\mathcal{M}}$$

- r<<l dual to IR regime, $i, j = 1 \cdots q$; T_s = QFT confining string tension
- Euclidean metric, $t \sim t + \beta\,, \quad \beta = 1/T$
- $\mathcal{M}$: compact (9-q-d)-dimensional space
- Backgrounds also include RR forms, NSNS form B_{MN} and dilaton ϕ(r)
- Let me assume $B_{tM} = 0$ for simplicity (general case can be treated as well)

Strings on confining backgrounds

- Consider a closed string at r=0 and at a point in $\mathcal{M}$ winding once along t

$$x^0 \equiv t = \frac{\beta}{2\pi}\sigma + \xi^0(\tau,\sigma)\,.$$

- Get quantum spectrum of quadratic fluctuations of worldsheet around this
- Ground state mass m_w tachyonic at $T>T_H$ with T_H solution of

$$\frac{T_s}{2}\,\beta_H^2 = 2\pi\,[\,\Delta(\mu) + \Delta\mathcal{E}\,] \qquad\qquad \mu = \frac{\beta_H}{2\pi}\frac{\sqrt{2\pi\alpha' T_s}}{l}$$

- $\Delta(\mu)$: finite zero-point energy of the world-sheet sigma model

$$\Delta(\mu) = 1 - \frac{d}{2}\mu + d\,\mu^2 \log 2 + \mathcal{O}(\mu^4)$$

- $\Delta\mathcal{E}$: quartic and higher order contributions of bosonic zero modes accounted for by the effective action for the thermal scalar field.

Strings on confining backgrounds

- We can determine T_H in a perturbative expansion in

$$\frac{\sqrt{\alpha'}}{l} \sim \lambda^{-k} \sim \frac{M_{gl}}{\sqrt{T_s}} \qquad k > 0 \quad \lambda \gg 1$$

- To leading order we get [FB, Canneti, Cotrone, 2022]

$$T_H = \sqrt{\frac{T_s}{4\pi}}$$

- To NLO we get [FB, Canneti, Muck, 2023]

$$T_H = \sqrt{\frac{T_s}{4\pi}} \left[1 + \frac{d}{2\sqrt{2}} \frac{\sqrt{\alpha'}}{l} \right]$$

- This result coincides with the one obtained by the effective action for the winding string scalar mode [Urbach 2023].
- Can go beyond NLO using worldsheet + effective approach.

Global AdS backgrounds

- As an example, let us consider global AdS_{d+1} cases
- Dual to CFTs on compact S^d spheres of radius R. Here $T_c \sim 1/R$
- In R=1 units ($g \sim \lambda^k$) we get [FB, Canneti, Cotrone 2023; Ekhammar et al 2023]

$$T_H = \sqrt{\frac{g}{2\pi}} + \frac{d}{8\pi} + \frac{d^2 + d - 8d\log 2}{32\sqrt{2}\pi^{3/2}\sqrt{g}} + \frac{4d^3 + 7d^2 - 2d}{1024\pi^2 g} + \mathcal{O}(g^{-3/2}) .$$

- T_H parametrically larger than T_c (general feature in holographic limit)
- In two cases QFT results for T_H at strong coupling are available, via integrability and quantum spectral curve methods [Harmark, Wilhelm, 2021; Ekhammar, Minahan, Thull 2023]
- Thus we can check string theory results against QFT ones

Global AdS backgrounds

- d=4: $\mathcal{N}$ =4 SYM theory on S^3 ($g = \lambda/4\pi$)
- String theory result:

$$T_H = \sqrt{\frac{g}{2\pi}} + \frac{1}{2\pi} + \frac{5 - 8\log 2}{8\pi\sqrt{2\pi}\sqrt{g}} + \frac{45}{128\pi^2 g} + \mathcal{O}(g^{-3/2})$$
$$\approx 0.39894...\sqrt{g} + 0.15916... - \frac{0.00865...}{\sqrt{g}} + \frac{0.0356...}{g} + \mathcal{O}(g^{-3/2})$$

- Gauge theory result:

$$T_H^{\text{[Ekhammar et al.'23]}} \approx (0.39894 \pm 0.00001)\sqrt{g} + (0.15916 \pm 0.00001) - \frac{(0.00865 \pm 0.00001)}{\sqrt{g}} + \frac{(0.0356 \pm 0.0001)}{g} + ...$$

Global AdS backgrounds

- d=3: *ABJM* theory on S^2
- String theory result:

$$T_H = \sqrt{\frac{g}{2\pi}} + \frac{3}{8\pi} + \frac{3(1-2\log 2)}{8\sqrt{2}\pi^{3/2}\sqrt{g}} + \frac{165}{1024\pi^2 g} + \mathcal{O}(g^{-3/2})$$

$$\approx 0.39894...\sqrt{g} + 1.1781... - \frac{0.0183954...}{\sqrt{g}} + \frac{0.0163262...}{g} + \cdots$$

- Gauge theory result:

$$T_H^{\text{Ekhammar et al 2023}} \approx \sqrt{\frac{g}{2\pi}} + \frac{3}{8\pi} - \frac{0.0183...}{\sqrt{g}} + \frac{0.01626...}{g} + \cdots$$

Conclusions and ongoing works

- Computed T_H for holographic confining gauge theories
- When QFT results available, remarkable agreement (to NNNLO) between the latter and string theory ones: highly non trivial tests of holography.

- Add N_f flavors: $T_H / T_s^{1/2}$ decreases (at fixed `t Hooft coupling)
- Add Yang-Mills θ angle: $T_H / T_s^{1/2}$ increases (at fixed `t Hooft coupling)
- Include non perturbative (instanton-like) corrections
- Explore large string coupling regime going to M-theory
- Extrapolating our results in a specific model (Witten's large N Yang-Mills) to SU(3) Yang-Mills, we get $T_H/T_c \sim 1.19$
- Model dark sectors with holographic confining theories: in cosmological first order phase transitions it is important to know T_H

Thank you for your time