Recent developments in HQET

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Introduction

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  h_v is the heavy quark field and $D_\perp^\mu = D^\mu - (v \cdot D)v^\mu$
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- Expanding in powers of $iv \cdot D/2m_Q$ gives

$$\mathcal{L}_{\rm HQET} = \bar{h}_v iv \cdot D h_v - c_2 \bar{h}_v \frac{D_\perp^2}{2m_Q} h_v - c_F \bar{h}_v \frac{\sigma_{\alpha\beta} G^{\alpha\beta}}{4m_Q} h_v + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

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- The general matrix element: $\langle f(p_f)|O_n^j(\mu)|i(p_i)\rangle$

  $O_n^j(\mu)$ can be local or non-local; p_i, p_f independent or not

  List options in increased complexity

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- Non-diagonal matrix element of local operator: Form factor

$$\langle D(p_f)|\bar{c}\gamma^\mu b|\bar{B}(p_i)\rangle = f_+(q^2)(p_i+p_f)^\mu + f_-(q^2)(p_i-p_f)^\mu$$

where $p_f - p_i = q$

Non-local operators

- Non-local operator between vacuum and a state: LCDA

$$\langle H_v|\bar{h}_v(0)\not{n}\gamma_5\,[0,tn]\,q_s(tn)|0\rangle = -iF(\mu)\int_0^\infty d\omega\, e^{i\omega t}\phi_+(\omega,\mu)$$

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$$S(\omega) = \frac{1}{2\pi}\frac{1}{2M_B}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle \bar{B}(v)|\bar{b}(0)\,[0,tn]b(tn)|\bar{B}(v)\rangle$$

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- Non-diagonal matrix element of a non-local operator:
  Non-local Form factor

$$\langle K^{(*)}(p_f)|\bar{s}_L(0)\gamma^\rho \cdots \tilde{G}_{\alpha\beta} b_L(tn)|B(p_i)\rangle$$

[Khodjamirian, Mannel, Pivovarov, Wang, JHEP **09**, 089 (2010)]

Example: $|V_{cb}|$ and $\bar{B} \to X_c\, \ell\, \bar{\nu}_\ell$

- Semileptonic $b \to c$ transition

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} C_1(\mu) V_{cb}\, \bar{\ell}\gamma_\mu(1-\gamma^5)\nu_\ell\, \bar{c}\gamma^\mu(1-\gamma^5)b$$

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- c_n^j perturbative in α_s
- $\langle O_n^j \rangle$ are non perturbative, can be extracted from experiment
  - $\langle O_0 \rangle = \langle \bar{B} | \bar{b} b | \bar{B} \rangle = 1$
  - $\langle O_2^{\rm kin.} \rangle = \langle \bar{B} | \bar{b} (iD)^2 b | \bar{B} \rangle \Rightarrow \mu_\pi^2$
  - $\langle O_2^{\rm mag.} \rangle = \langle \bar{B} | \bar{b}\, \sigma_{\mu\nu} G^{\mu\nu} b | \bar{B} \rangle \Rightarrow \mu_G^2$ can be extracted from $M_{B^*} - M_B$

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- Orthogonality of v to μ_1, μ_n, λ [Mannel, PRD 50, 428 (1994)]
- Parity and Time reversal symmetry
- Hermitian conjugation
- Four dimensions
- Possible multiple color structures [Kobach, Pal PLB **772** 225 (2017)]

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- To decompose them in terms of the tensors
- v^{μ_i}, $\Pi^{\mu\nu} = g^{\mu\nu} - v^{\mu} v^{\nu}$, $\epsilon^{\rho\sigma\alpha\beta} v_{\rho}$

Dimension 9 HQET operators

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$$+a^{(9)}_{12,35}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6} + \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) +$$
$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} +$$
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$$\mathcal{L}^{\text{dim}=8}_{\text{NRQCD}} = \psi^\dagger \Big\{ \ldots c_{X1} g \frac{[\boldsymbol{D}^2, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{D}^i, \boldsymbol{E}^i]\}}{m_p^4} + c_{X3} g \frac{[\boldsymbol{D}^i, [\boldsymbol{D}^i, [\boldsymbol{D}^j, \boldsymbol{E}^j]]]}{m_p^4}$$

$$+ i c_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \{T^a, T^b\}\}}{2M^4} + i c_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b\, \delta^{ab}\}}{m_p^4} + i c_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{m_p^4}$$

$$+ i c_{X6} g \frac{\epsilon^{ijk} \sigma^i \boldsymbol{D}^j [\boldsymbol{D}^l, \boldsymbol{E}^l] \boldsymbol{D}^k}{m_p^4} + c_{X7a}\, g^2 \frac{\{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a T^a, [\boldsymbol{D}^i, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X7b}\, g^2 \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a [\boldsymbol{D}^i, \boldsymbol{E}^i]_a}{m_p^4}$$

$$+ c_{X8a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{m_p^4} + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4}$$

$$+ c_{X9b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_a}{m_p^4} + c_{X10a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_b T^b\}}{2M^4} + c_{X10b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_a}{m_p^4}$$

$$+ c_{X11a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X11b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_a}{m_p^4} + \tilde{c}_{X12a}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_b \{T^a, T^b\}}{2M^4}$$

$$+ \tilde{c}_{X12b}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_a}{m_p^4} + i c_{X13} g^2 \frac{[\boldsymbol{E}^i, [D_t, \boldsymbol{E}^i]]}{m_p^4} + i c_{X14} g^2 \frac{[\boldsymbol{B}^i, (\boldsymbol{D} \times \boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{D})^i]}{m_p^4}$$

$$+ i c_{X15} g^2 \frac{[\boldsymbol{E}^i, (\boldsymbol{D} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{D})^i]}{m_p^4} + c_{X16} g^2 \frac{[\boldsymbol{\sigma} \cdot \boldsymbol{B}, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X17} g^2 \frac{[\boldsymbol{B}^i, \{\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}\}]}{m_p^4} + c_{X18} g^2 \frac{[\boldsymbol{E}^i, \{\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i\}]}{m_p^4} \Big\} \psi$$

- 25 operators c_{Xib} start at $\mathcal{O}(\alpha_s)$

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- By mapping the problem to diagonal HQET local matrix elements
- List such operators, in principle, to *arbitrary* dimension
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- $\mathcal{O}(\alpha_s)$ operators are unknown but extremely small
  For example: $\alpha_s \left(\Lambda_{\rm QCD}/m_b\right)^4 \sim 0.2 \cdot (0.1)^4 \sim 10^{-5}$

Power corrections

- $1/m_b^4, 1/m_b^5$ matrix elements extracted from $\bar{B} \to X_c \ell \bar{\nu}_\ell$
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

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Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c$(3 GeV)	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
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$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044
$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
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$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3 \lvert V_{cb} \rvert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

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- "The higher power corrections have a minor effect on $|V_{cb}|$ ... There is a -0.25% reduction in $|V_{cb}|$"

State of the art: $|V_{cb}|$ and $\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

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- What is the current "state of the art"?

$$\Gamma \sim c_0 \langle O_0 \rangle + c_2^j \frac{\langle O_2^j \rangle}{m_b^2} + c_3^j \frac{\langle O_3^j \rangle}{m_b^3} + c_4^j \frac{\langle O_4^j \rangle}{m_b^4} + c_5^j \frac{\langle O_5^j \rangle}{m_b^5} + \cdots$$

- c_0 known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1), \mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^3)$ for selected observables
- c_2^j known at $\mathcal{O}(\alpha_s^0), \mathcal{O}(\alpha_s^1)$
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- Example: using four-loop relation between the pole and $\overline{\text{MS}}$ masses the extract HQET parameters from B and D meson masses
  [Takaura, EPJ Web Conf. **274**, 03003 (2022) arXiv:2212.02874]

Perturbative corrections: Four-loop HQET propagator

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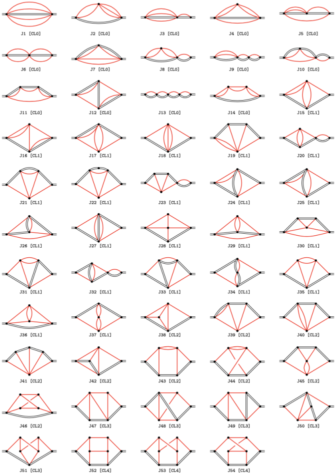

Red solid lines: massless propagators, double lines: HQET propagator

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$$
\begin{aligned}
\tilde{\gamma}_j(\alpha_s) = & -3C_F\frac{\alpha_s}{4\pi} + C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left[-C_F\left(\frac{8}{3}\pi^2-\frac{5}{2}\right)+\frac{C_A}{3}\left(2\pi^2-\frac{49}{2}\right)+\frac{10}{3}T_Fn_f\right] \\
& + C_F\left(\frac{\alpha_s}{4\pi}\right)^3\left[-C_F^2\left(36\zeta_3+\frac{8}{9}\pi^4-\frac{32}{3}\pi^2+\frac{37}{2}\right)\right. \\
& + \frac{C_FC_A}{3}\left(142\zeta_3-\frac{8}{15}\pi^4-\frac{592}{9}\pi^2-\frac{655}{12}\right)-\frac{C_A^2}{3}\left(22\zeta_3+\frac{4}{5}\pi^4-\frac{130}{9}\pi^2-\frac{1451}{36}\right) \\
& \left.-\frac{2}{3}C_FT_Fn_f\left(88\zeta_3-\frac{112}{9}\pi^2-\frac{235}{3}\right)+\frac{8}{3}C_AT_Fn_f\left(19\zeta_3-\frac{7}{9}\pi^2-\frac{64}{9}\right)+\frac{140}{27}(T_Fn_f)^2\right] \\
& + \left(\frac{\alpha_s}{4\pi}\right)^4\left[C_F^4\left(1200\zeta_5-168\zeta_3^2-\frac{896}{3}\pi^2\zeta_3+394\zeta_3+\frac{3884}{2835}\pi^6-\frac{4}{15}\pi^4+\frac{136}{3}\pi^2-\frac{691}{8}\right)\right. \\
& - C_F^3C_A\left(\frac{5660}{3}\zeta_5-192\zeta_3^2-\frac{4576}{9}\pi^2\zeta_3+1275\zeta_3+\frac{2659}{2835}\pi^6-\frac{119}{45}\pi^4+\frac{2398}{9}\pi^2-\frac{3991}{12}\right) \\
& + C_F^2C_A^2\left(\frac{434}{3}\zeta_5-42\zeta_3^2-\frac{1916}{9}\pi^2\zeta_3+\frac{39047}{27}\zeta_3+\frac{2087}{1890}\pi^6-\frac{2663}{90}\pi^4+\frac{41026}{243}\pi^2-\frac{189671}{324}\right) \\
& + C_FC_A^3\left(492\zeta_5+30\zeta_3^2+\frac{352}{9}\pi^2\zeta_3-\frac{14666}{27}\zeta_3-\frac{1439}{8505}\pi^6+\frac{23}{90}\pi^4-\frac{7246}{243}\pi^2+\frac{179089}{648}\right) \\
& + 8d_{FA}\left(30\zeta_5+\frac{106}{3}\pi^2\zeta_3-16\zeta_3-\frac{452}{567}\pi^6+\frac{29}{9}\pi^4+\frac{46}{3}\pi^2-8\right) \\
& + 4C_F^3T_Fn_f\left(\frac{580}{3}\zeta_5-\frac{224}{9}\pi^2\zeta_3-24\zeta_3-\frac{29}{45}\pi^4+\frac{68}{3}\pi^2-\frac{119}{3}\right) \\
& - \frac{C_F^2C_AT_Fn_f}{3}\left(1096\zeta_5-\frac{736}{3}\pi^2\zeta_3+\frac{18980}{9}\zeta_3-\frac{1138}{45}\pi^4-\frac{9404}{81}\pi^2-\frac{32093}{27}\right) \\
& - C_FC_A^2T_Fn_f\left(308\zeta_5+24\zeta_3^2+\frac{128}{9}\pi^2\zeta_3-\frac{20792}{27}\zeta_3-\frac{874}{8505}\pi^6+\frac{56}{27}\pi^4+\frac{5240}{243}\pi^2+\frac{27269}{162}\right) \\
& - 32d_{FF}n_f\left(15\zeta_5+\frac{8}{3}\pi^2\zeta_3-8\zeta_3-\frac{437}{2835}\pi^6+\frac{4}{9}\pi^4+\frac{20}{3}\pi^2-4\right) \\
& + \frac{16}{27}C_F^2(T_Fn_f)^2\left(326\zeta_3-\frac{11}{5}\pi^4+\frac{16}{9}\pi^2-\frac{206}{3}\right) \\
& - \frac{2}{27}C_FC_A(T_Fn_f)^2\left(2272\zeta_3-\frac{76}{5}\pi^4+\frac{32}{9}\pi^2-\frac{761}{3}\right) \\
& \left.-\frac{8}{9}C_F(T_Fn_f)^3\left(16\zeta_3-\frac{83}{9}\right)\right]+\mathcal{O}(\alpha_s^5) \qquad (3.3)
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For $n_f = 4 : \tilde{\gamma}_j = -\frac{\alpha_s}{\pi} - 2.487726 \left(\frac{\alpha_s}{\pi}\right)^2 - 6.292698 \left(\frac{\alpha_s}{\pi}\right)^3 - 13.878042 \left(\frac{\alpha_s}{\pi}\right)^4$

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- This anomalous dimension can be used to calculate f_B/f_D

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\left(\frac{\alpha_s^{(4)}(m_c)}{\alpha_s^{(4)}(m_b)}\right)^{-\frac{\tilde{\gamma}_{j0}}{2\beta_0^{(4)}}}\left\{1 + \cdot\cdot\alpha_s + \cdot\cdot\alpha_s^2 + \cdot\cdot\alpha_s^3 + [\sim 1\text{GeV}]\left(\frac{1}{m_c} - \frac{1}{m_b}\right) + \cdot\cdot\right\}$$

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Recent developments in HQET: Non-local matrix elements

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- [Beneke, Finauri, Vos, Wei, JHEP **09**, 066 (2023) arXiv:2305.06401]
- [Ishaq, Zafar, Rehman, Ahmed, arXiv:2404.01696]

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$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -iF_{\text{stat}}(\mu)\, n_+\cdot v \int_0^\infty d\omega\, e^{i\omega tn_+\cdot v}\varphi_+(\omega;\mu)$$

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- Factorization allows to resum large logs between
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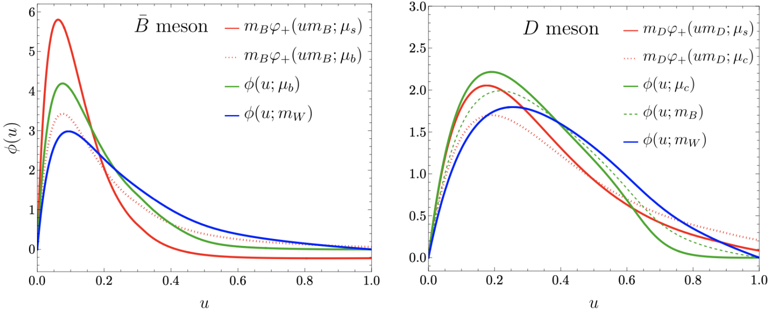
QCD LCDA of Heavy Mesons from boosted HQET



- Starting with HQET LCDA at soft scale $\mu_s = 1$ GeV (red, solid)

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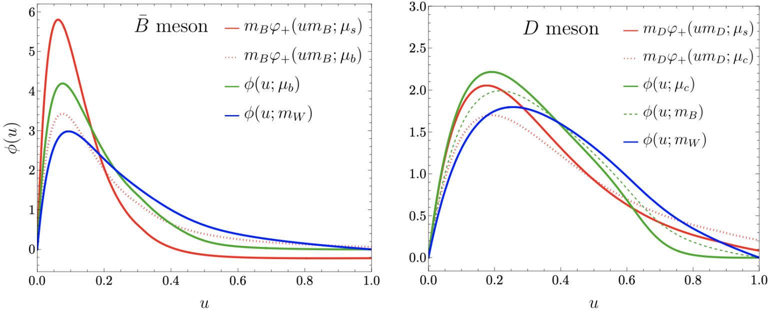

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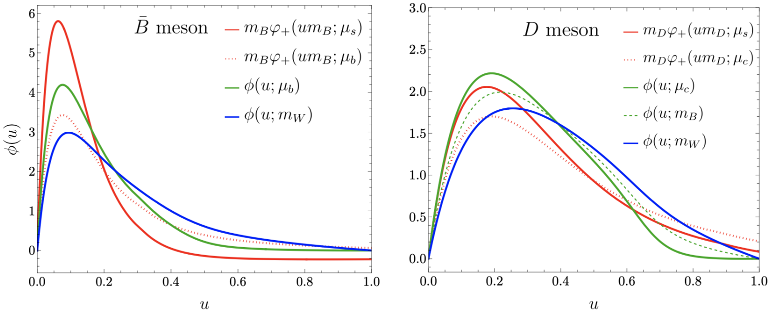

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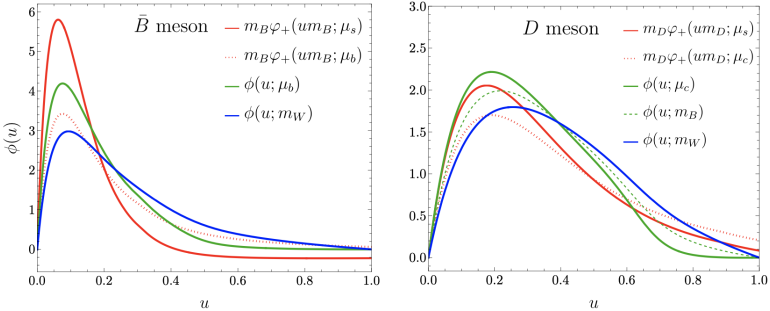

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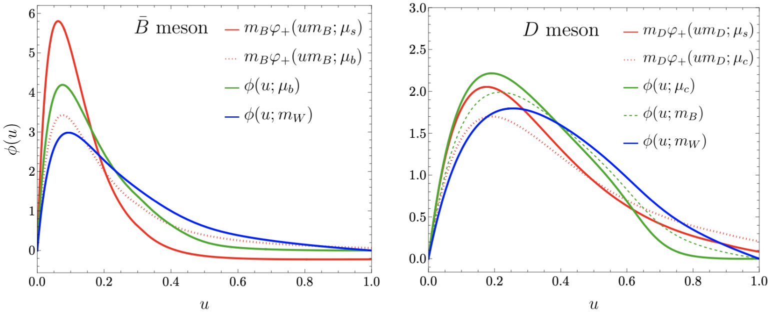

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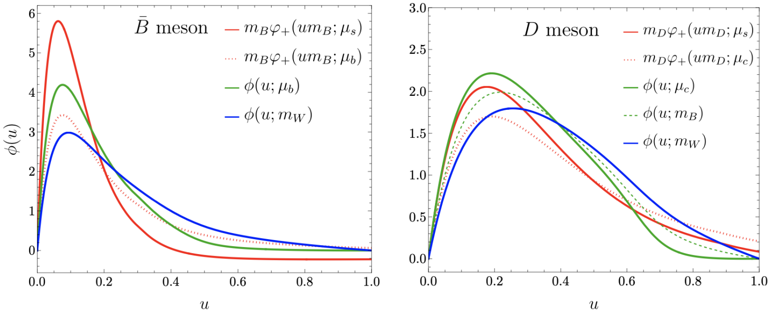
• The branching ratio

$$\mathrm{Br}(W \to B\gamma) = (2.58 \pm 0.21_{\text{in}}\, {}^{+0.05}_{-0.08}\,{}_{\mu_h}\, {}^{+0.05}_{-0.08}\,{}_{\mu_b}\, {}^{+0.18}_{-0.13}\,{}_{\delta}\, {}^{+0.61}_{-0.34}\,{}_{\beta}\, {}^{+2.95}_{-0.98}\,{}_{\lambda_B}) \cdot 10^{-12}$$

QCD LCDA of Heavy Mesons from boosted HQET



- Starting with HQET LCDA at soft scale $\mu_s = 1$ GeV (red, solid)
- Evolved in HQET to the matching scale μ (red, dotted)
- Matched to $\phi(u)$ (green, solid)
- Evolved in QCD to the hard scale m_W (blue, solid)

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dominated by low scale HQET LCDA parameters: λ_B, β

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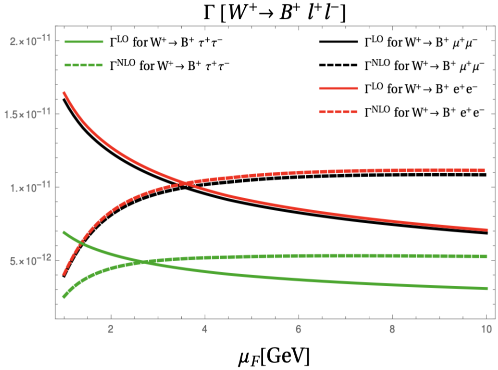

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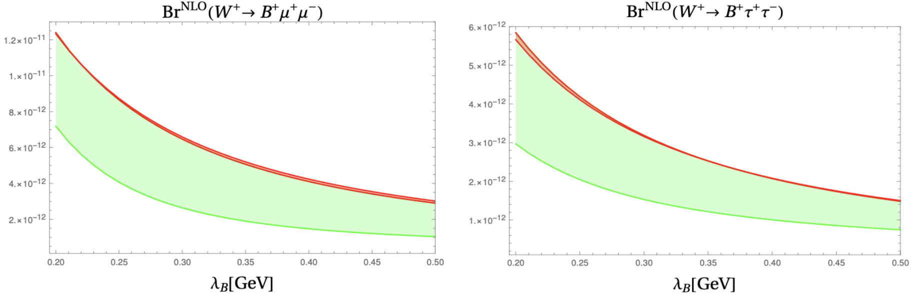

- Observing the process at the LHC could constrain λ_B

Recent developments in HQET: Local non-diagonal matrix elements (Form factors)

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Table from [Bernlochner, Ligeti, Papucci, Prim, Robinson, Xiong, PoS **ICHEP2022**, 758 (2022)]

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Recent developments in HQET: New directions

New theoretical framework for heavy quark resonances

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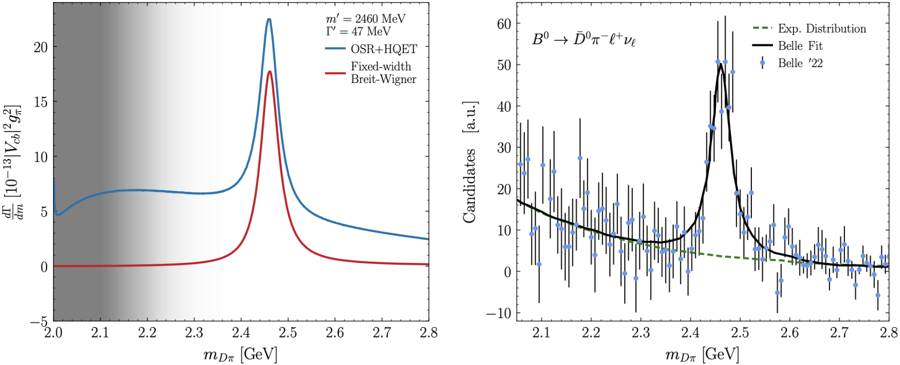

- Left: Toy example calculation in this framework
  Right: Belle data with a D_2^* resonance

Study of F-wave Bottom Mesons in HQET

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Table 2: Obtained masses for $1F$ bottom mesons

J^P	Masses of $1F$ Bottom Mesons (MeV)					
	Non-Strange			Strange		
	Calculated	[10]	[23]	Calculated	[10]	[23]
$2^+(1^3F_2)$	6473.6	6412	6387	6518.28	6501	6358
$3^+(1F_3)$	6478.93	6420	6396	6523.21	6515	6369
$3^+(1F_3')$	6447.76	6391	6358	6506.05	6468	6318
$4^+(1^3F_4)$	6450.14	6380	6364	6508.01	6475	6328

- Ref. [10] [Ebert, Faustov, Galkin, EPJ C **66**, 197-206 (2010)]
- Ref. [23] [Godfrey, Moats, Swanson, PRD **94**, 054025 (2016)]

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		$Q=c$			$Q=b$			
J^P	Baryons	Calculated	[17]	[5]	Calculated	[17]	[45]	[5]
$\frac{1}{2}^+$	Λ	$\mathbf{2766.6 \pm 2.4}$	2769		6093	6089		$\Lambda_b(6070)$
	Ξ	2942	2959	$\Xi_c(2970)$	6267	6266	6208	
	Σ	**2901**	2901		6246	6213		
	Ξ'	3028	2983		6369	6329	6328	
	Ω	3154	3088		6487	6450	6438	
$\frac{3}{2}^+$	Σ^*	2948	2936		6262	6226		
	Ξ'^*	3074	3026		6381	6342	6343	
	Ω^*	3190	3123		6507	6461	6462	

- Ref. [17] [Ebert, Faustov, Galkin, PRD **84**, 014025 (2011)]
- Ref. [45] [Kakadiya, Shah, Rai, IJMPA **37**, 2250053 (2022)]

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- More work to do!