Holographic phases of QCD

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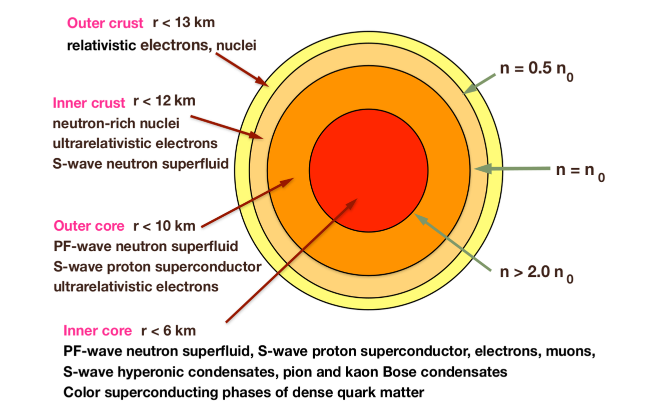
Introduction

The QCD phase diagram features a variety of phases in the $\mu_B - T$ plane. Nonetheless, determining the equations of state and phase-transition lines from the first-principles QCD is difficult.



Sedrkian : 2018

Non-perturbative methods:

- *EFTs:* low densities and low temperature
- *Lattice QCD:* vanishing densities
- Holographic QCD
  - **Top-down:** String theory e.g. D3/D7, D4/D8 ...
  - **Bottom-up:** Phenomenology e.g. Hardwall, Softwall, VQCD ...

Holographic Hardwall Model

$$S_0 = \frac{N_c^2}{8\pi^2 L^3}\int d^5x\sqrt{g}\,(R-2\Lambda) - \frac{\theta N_f N_c}{24\pi^2 L}\int d^5x\sqrt{g}\frac{F^2}{4}$$

L is the radius of AdS.
Parameter θ: Dilaton coupling and different choices of compactification.
The cosmological constant Λ: Asymptotical AdS_5 geometries.

Methodology

- An **IR cutoff $\mathbf{z_0}$** is implemented to break the conformal symmetry.
- The equations of motion are solved with suitable **boundary conditions**.
- The holographic renormalization procedure is applied to determine the on-shell action.
- The phase with the lowest free energy and the equations of state are identified.
- Phenomenological methods are employed to establish the phase diagram in **physical units.**

Solutions/Phases

The diagonal metric ansatz:

$$ds^2 = -\frac{g(z)}{h(z)}L^2dt^2 + \frac{L^2}{z^2}d\vec{x}^2 + \frac{L^2}{g(z)}dz^2$$

and the gauge field ansatz $A_0 = \phi(z)$. There are four boundary conditions: one each for g and h, and two for ϕ. There are three possible geometries:

1. **Thermal AdS:** zero denisty, confined phase.
2. **Charged Black Hole:** finite density, deconfined phase.
3. **Charged AdS:** finite density, confined phase.

Charged AdS

The charged AdS is a **horizonless geometry** in the bulk that represents a finite-density confined phase of the boundary theory.

$$g = z^2\left(1-\left(1-\frac{g_0}{z_0^2}\right)\frac{z^4}{z_0^4} + \frac{\theta N_f}{9N_c}Q^2(z^6 - z^4z_0^2)\right); \quad h = z^4; \quad \phi(z) = \pm\mu + Qz^2;$$

Scalar Glueball mass: $\implies z_0^{-1} = 290$ MeV Rinaldi: 2018

Two free parameters: Q and $g_0 = g(z_0)$

ρ-meson mass: $\implies \bar{g}_0 \simeq 6.$

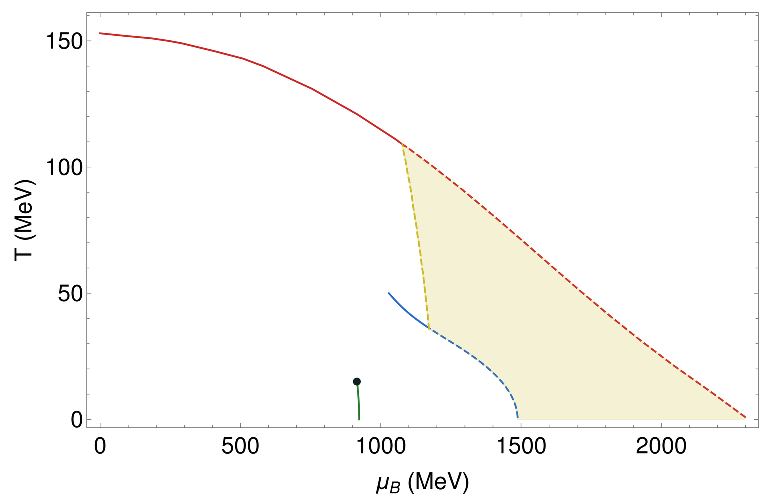
Perturbation of the Vector field, with boundary conditions given by $a_\nu(k,0) = \partial_z a_\nu(k,z_0) = 0.$

$$z\partial_z\left(\frac{g}{z^3}\partial_z V(z)\right) + k^2V(z) = 0$$

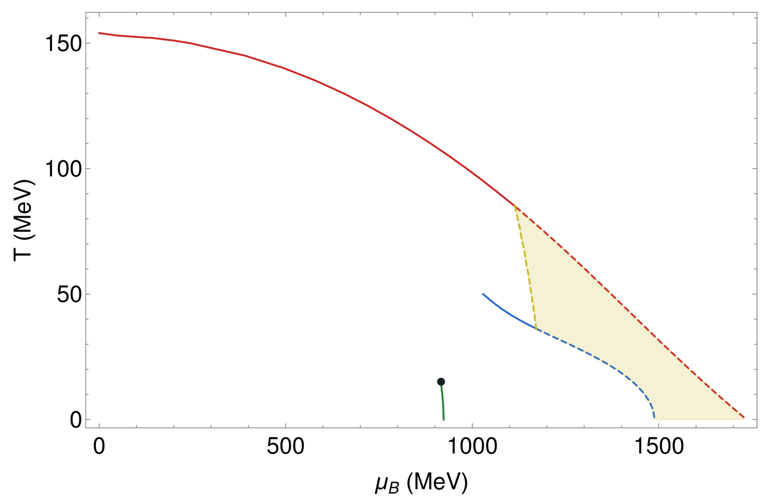
The equation of state relating charge density Q with μ is determined by:

- **Simple boundary condition:** $\phi(z_0) = 0 \implies Q = \frac{\mu}{z_0^2}$
  The deconfinement phase transition at $T = 0$
  $$\left(\frac{\mu\alpha_g}{M_g}\right)^2 > \frac{6N_c}{\theta N_f}\left(1+\sqrt{4+3\alpha_\rho\left(\frac{M_g}{M_\rho}\right)^2}\right)$$
  The equation of state: $\varepsilon = p - \frac{N_c^2}{4\pi^2 z_0^6}(z_0^2 + g_0)$
- **Physically motivated boundary condition:**
  $$\rho = \frac{\theta N_f N_c}{12\pi^2}Q \qquad g_0 = \bar{g}_0 + \frac{8\pi^2}{N_c^2}\left(p - \frac{2\pi^2 z_0^2}{\theta N_f N_c}\rho^2\right)$$
  - **Nambu–Jona-Lasinio:** Temperature and chemical potential dependent constituent quark mass. Klevansky:1992
  - **Van der Waals:** Isospin-symmetric nuclear matter (in-medium ChPT). Fiorilla:2012
    The equation of state: $\varepsilon = \alpha + \beta\sqrt{p} + \gamma p$

Phase diagram



(a) $\theta = 10.4$



(b) $\theta = 13.5$

cf: Costa: 2019

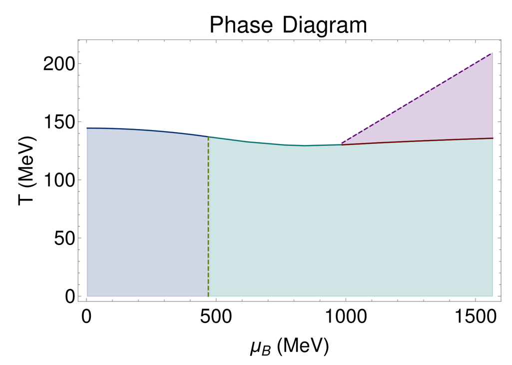
A similar analysis with a softwall model suggests low density confined phase.

Condensate solutions

A complex scalar field ψ is included to break $U(1)_B$ symmetry spontaneously through the action.

$$S = S_0 - \lambda_s\int d^5x\sqrt{g}\left(|D\psi|^2 + m^2|\psi|^2\right)$$

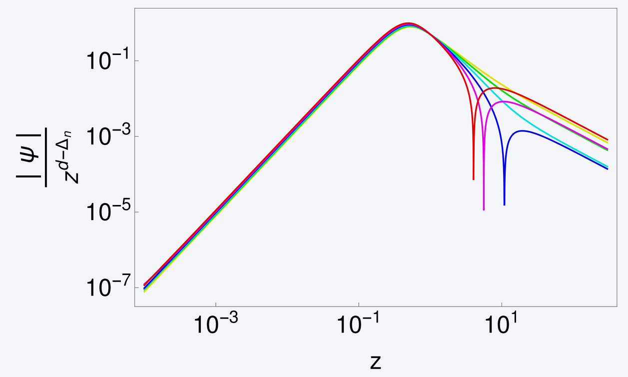
- Scaling dimension Δ $\quad m^2L^2 = \Delta(\Delta - d)$
- Asymptotic analysis $\quad \psi \sim \psi_+ z^{\Delta_+} + \psi_- z^{\Delta_-} + \ldots$
- Breaking symmetry spontaneously $\quad \psi_- = 0$; $\psi_+ \neq 0$



A phase diagram for $\phi(z_0) = 0$

Condensate vanishes at high density

Phase	sign	Onset μ (MeV)	Offset μ (MeV)
VdW	-ve	1426.8	1487.7
VdW	+ve	1235.4	1287.6
NJL	-ve	-	-
NJL	+ve	1139.7	1287.6



Conclusions and Discussion

- A simple 5-D holographic hardwall model provided low density confined phase resulting in a complete phase diagram.
- The probe analysis of baryon condensates in the confined phase at zero temperature identifies a critical chemical potential beyond which the condensate disappears.
- An ongoing study for backreacted condensate solutions with physical boundary conditions is suggesting a similar result for vanishing condensate at high chemical potential.
- Improve the model by incorporating the Running coupling, Isospin, and Chiral condensate.
- D3/D7 hardwall model with backreaction can provide finite density confined phase with baryons as solitons.

References

[1] Akash Singh and K. P. Yogendran. Confined phases at finite density in the Hardwall model. *arxiv*, 2407.xxxxx.

[2] Akash Singh and K. P. Yogendran. Phases of nuclear matter from AdS Hardwall models. *arxiv*, 24xx.xxxx.

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