Low Energy Description of Single Flavor Baryons

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Plan

- No Skyrmions with one flavor
- A proposal for the solution
- Realization in Holographic QCD

No Skyrmion with one flavor

Low-energy effective action for QCD for $N_f \geq 2$

Pion matrix $$U = e^{i \sum_{a=1}^{N_f^2-1} \frac{\pi^a(x) T^a}{f_\pi}}$$

$$\mathcal{L}_{\text{eff}} = -\frac{f_\pi^2}{4} Tr\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2} Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Chiral Lagrangian with Skyrme term (postulated)

No Skyrmion with one flavor

Low-energy description of baryons

Skyrme term allows for solitonic solutions, e.g. for $N_f = 2$

$$U = e^{i \frac{f(r)}{r} x^a \sigma^a}$$

(with known $f(r)$): a "Skyrmion".

If $f(0) = \pi k \, , k \in \mathbb{Z}$, then solution has winding number k for

$$\Pi_3(SU(N_f = 2)) = \mathbb{Z}$$

where the 3-sphere is in coordinate space.

This is interpreted as the baryon number

But: $$\Pi_3(U(N_f = 1)) = 0$$

No Skyrmions with one flavor

A proposal for the solution

In single-flavor low-energy QCD, baryons are charged sheets
[Komargodski 2018 and many others before]

- With a single flavor, low energy Lagrangian is just Lagrangian for η' (light at large N_c)

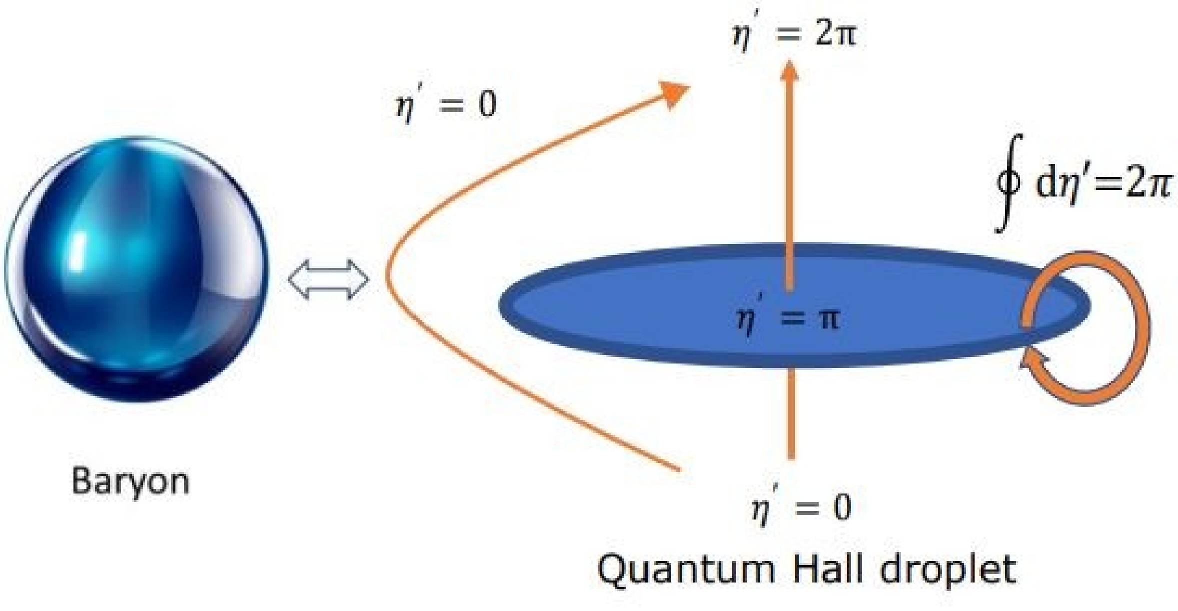
$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2}m\Lambda_{QCD}\cos{(\eta')} - \frac{1}{2}m_{WV}^2 Min_{k\in\mathbb{Z}}(\eta' + 2\pi k)^2$$

- Potential is singular at $\eta' = \pi$, extra gluonic d.o.f. are localized there: "sheet"
- η' has non-trivial monodromy through the sheet
- Infinite sheet is similar to a domain-wall but no charge associated: unstable
- But can be stabilized by baryonic charge!

A proposal for the solution

Sheet hosts a $U(1)_{N_c}$ Chern-Simons theory on its world-volume
"Quantum Hall Droplet"

- If it has a circular boundary can have a chiral edge mode: baryonic charge
- Charge forbids shrinking of the "pancake-shaped" sheet



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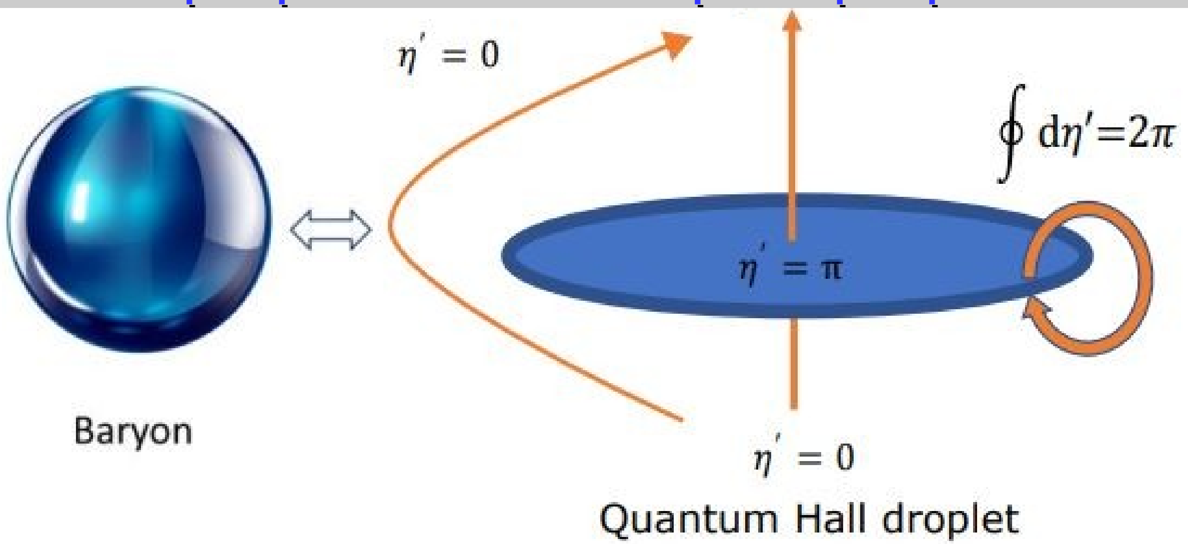
Can we test the proposal and compute properties of this baryon?

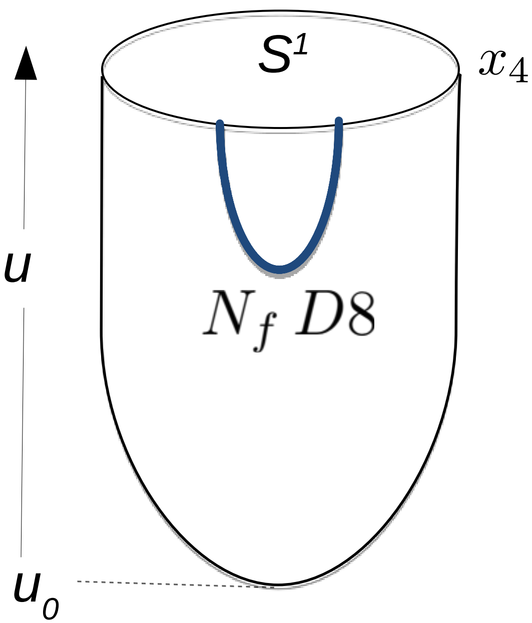


Realization in Holographic QCD

Holographic QCD

[Witten 1997, Sakai-Sugimoto 2004]

- IIA background from N_c D4-branes wrapped on S^1 + N_f D8/anti-D8-brane pairs
- Low energy: dual to non-susy 4d $SU(N_c)$ YM + KK modes + N_f chiral quarks
- Confinement, mass gap, chiral symmetry breaking



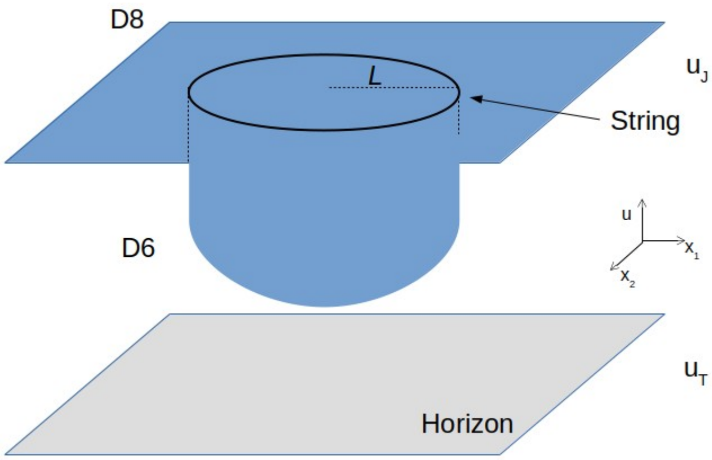
Baryons: [Hata et al. 2007]

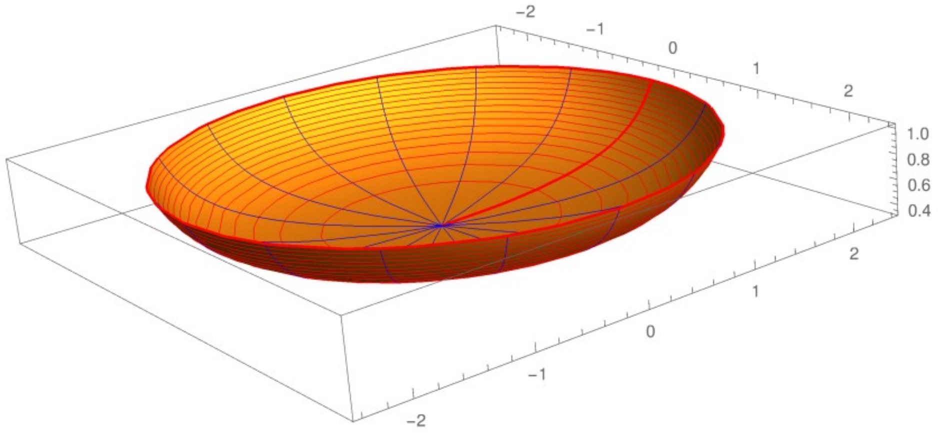
Instantonic configurations of gauge field on D8-branes, but only for $N_f \geq 2$!

How do we describe the sheet?

Realization in Holographic QCD

- Behavior of η' through the sheet is realized by D6-brane
- D6-brane hosts a $U(1)_{N_c}$ Chern-Simons theory on its world-volume [Acharya-Vafa 2001, Argurio et al. 2018]
- So, sheet is a D6 attached to the D8:





D6-brane constitutes the gluonic core of the baryon (equivalent of the string junction)

Realization in Holographic QCD

- D6 ending on D8 is a magnetic source for D8 fields F describing η' and charge
- A problem in 5d Maxwell-Chern-Simons theory on curved space
- In flat space limit:

$$\rho\partial_r(rF_{tr}) + r\partial_\rho(\rho F_{t\rho}) - \tau(F_{\rho\psi}F_{r\theta} - F_{r\psi}F_{\rho\theta}) = 0$$
$$\frac{1}{\rho}\partial_r(rF_{r\psi}) + r\partial_\rho(\frac{1}{\rho}F_{\rho\psi}) - a\tau(F_{tr}F_{\rho\theta} - F_{t\rho}F_{r\theta}) = 0$$
$$\frac{1}{r}\partial_\rho(\rho F_{\rho\theta}) + \rho\partial_r(\frac{1}{r}F_{r\theta}) - a\tau(F_{t\rho}F_{r\psi} - F_{tr}F_{\rho\psi}) = 0$$
$$dF = -\sqrt{2}\delta(r)\delta(\rho - L)d\rho \wedge dr \wedge d\theta$$

- Not easy!

D8-brane gauge field constitutes the mesonic shell of the baryon

Realization in Holographic QCD

We found a (approximate) solution in deconfined phase such that:

- It is an equilibrium configuration of size $L_{eq} \sim \sqrt{\frac{\Lambda_{QCD}}{\lambda T^3_{\chi SB}}}$

  'Ooft coupling — Temperature of chiral symmetry breaking

- Its mass is $M \sim \frac{\lambda^2 N_c T^3_{\chi SB}}{\Lambda^2_{QCD}}$
- It has unit baryon number $n_B = \frac{1}{12\pi^2} \int d^4x (F_{\rho\phi} F_{r\theta} - F_{r\phi} F_{\rho\theta})$
- It has spin $\frac{N_c}{2}$ (for a single flavor all spins are aligned)
- Weak point: dependence on UV cut-off

Realization in Holographic QCD

Done:

- Realized Komargodski's proposal for low-energy description of single flavor baryon
- Computed basic properties

To do:

- Numeric solution?
- Compute other observables
- Extend solution to confined phase
- Construction has application for axionic dark matter

Thank you for your time

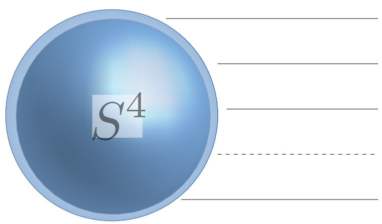
Review of Holographic QCD

Baryons

- Microscopic point of view: D4-branes wrapped on four-sphere + strings [Witten 1998]

$$S_{D4} = -T_4 \int d^5x\, e^{-\Phi} \sqrt{-g_5} = -m_B \int dt \qquad \text{with} \quad m_B = \frac{\lambda N_c}{27\pi} M_{KK}$$

D4 →



N_c strings to account for flux of F_4

"Baryon vertex"

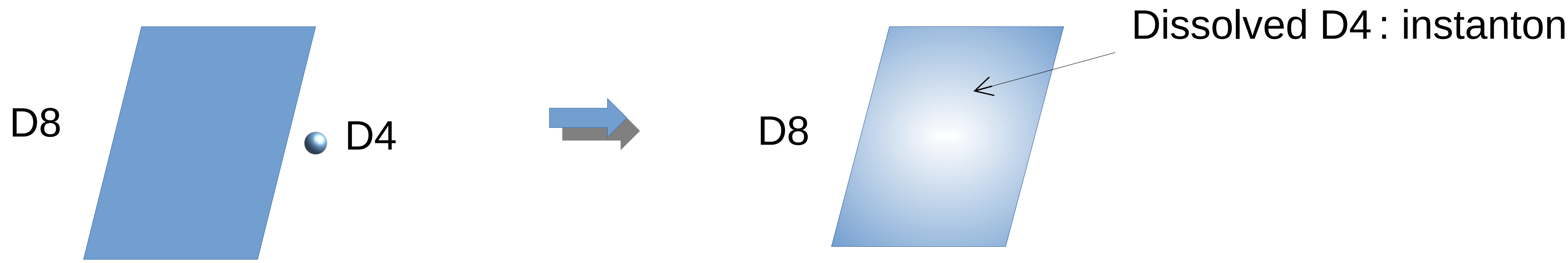
"...I still suspect it will be D-branes that Joe [Polchinski] is remembered for. Because — as I've tried to make clear — they're real. Really real. There's one in every proton, one in every neutron."

Matt Strassler, "In Memory of Joe Polchinski, the Brane Master", 2018

Review of Holographic QCD

Baryons

- Macroscopic point of view: instantonic configurations on D8-brane world-volume
  [Hata et al. 2007, Hong et al. 2007]



N$_f$*=2: s*tandard instanton solution in directions $x_{i=1,2,3}, z$ describing nucleon

$$\mathcal{A}_M^{nAb} = -if(\xi) g \partial_M g^{-1} \qquad \mathcal{A}_0^{Ab} = \frac{N_c}{8\pi^2 \kappa} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right]$$

$$f(\xi) = \frac{\xi^2}{\rho^2 + \xi^2} \qquad g = \frac{(z - Z)\mathbf{1} - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi} \qquad \xi^2 = (\vec{x} - \vec{X})^2 + (z - Z)^2$$

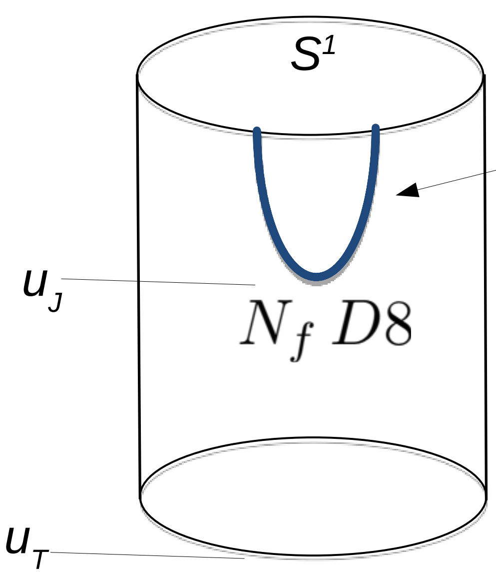
$$U = Exp\left[i\pi \frac{\vec{\tau} \cdot \vec{x}}{|\vec{x}|} \left(1 - \frac{1}{\sqrt{1 + \rho^2/|\vec{x}|^2}} \right) \right]$$

Energy coincides with baryon mass m_B

Review of Holographic QCD

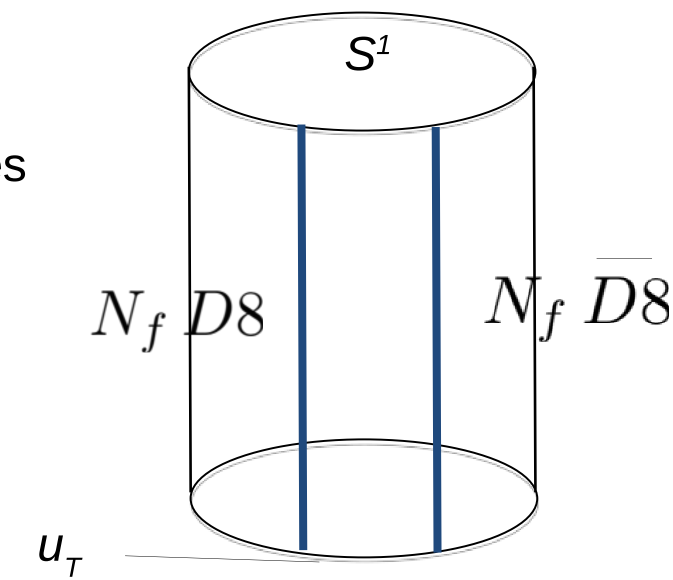
Deconfined phase

- In Holographic QCD: chiral symmetry breaking can persist if D8 non-antipodal [Aharony-Sonnenschein-Yankielowicz 2006]
- Chiral symmetry breaking transition at new scale $f_a \sim u_J$
- Chiral symmetry broken phase:



Embedding determines a function $\gamma_T(z)$

- Chiral symmetry unbroken phase:



Review of Holographic QCD

Low-energy effective action for N_f>1

- Define pion matrix

$$U = \mathcal{P} e^{i\int \mathcal{A}_z} = e^{i\Pi(x)/f_\pi}$$

$$f_\pi = 2\sqrt{\frac{\kappa}{\pi}}$$

$$\mathcal{F}_{\mu z} \approx U^{-1}\partial_\mu U \qquad \mathcal{F}_{\mu\nu} \approx \left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]$$

- Get

$$e = -\frac{1}{2.5\kappa}$$

$$\mathcal{L}_{\text{eff}} = -\frac{f_\pi^2}{4} Tr\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2} Tr\left[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Chiral Lagrangian with Skyrme term, derived from gravity!

Review of Holographic QCD

How does the WSS model perform?

Some meson masses:
(table from [Rebhan 14])

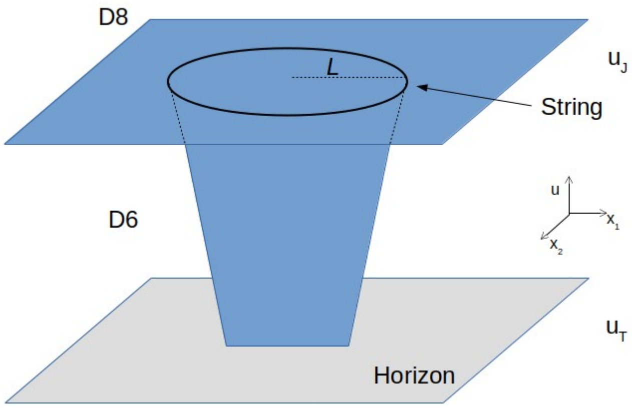
Isotriplet Meson	$\lambda_n = m^2/M^2_{KK}$	m/m_ρ	$(m/m_\rho)^{exp.}$	m/m_ρ [30]
0^{-+} (π)	0	0	0.174 | 0.180	0
1^{--} (ρ)	0.669314	1	1	1
1^{++} (a_1)	1.568766	1.531	1.59(5)	1.86(2)
1^{--} ($\rho*$)	2.874323	2.072	1.89(3)	2.40(4)
1^{++} (a_1*)	4.546104	2.606	2.12(3)	2.98(5)

Some nucleon properties:
(table from [Hashimoto et al 08])

	WSS	Skyrmion	experiment
$\langle r^2\rangle^{1/2}_{I=0}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2\rangle^{1/2}_{M,I=0}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2\rangle_{E,p}$	$(0.742\ \text{fm})^2$	∞	$(0.875\ \text{fm})^2$
$\langle r^2\rangle_{E,n}$	0	$-\infty$	$-0.116\ \text{fm}^2$
$\langle r^2\rangle_{M,p}$	$(0.742\ \text{fm})^2$	∞	$(0.855\ \text{fm})^2$
$\langle r^2\rangle_{M,n}$	$(0.742\ \text{fm})^2$	∞	$(0.873\ \text{fm})^2$
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	−	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	−1.34	−1.31	−1.91
$\left\|\frac{\mu_p}{\mu_n}\right\|$	1.63	1.43	1.46
g_A	0.734	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	−	4.2 ~ 6.5

Closed strings and walls

- D6 ending on D8 is a magnetic source for D8 fields *F:* macroscopic point of view
- A problem in 5d Maxwell-Chern-Simons on curved space



- For circular D6 boundary of radius $\rho = L$ in (x_1, x_2) plane

$$dF = -2\pi\sqrt{2}\,\delta(\rho - L)\delta(x_3)\delta(z)d\rho \wedge dx_3 \wedge dz$$

- Solve equations of motion as before with

$$H_T = \sum_{n=0}^{\infty} \zeta_{T,n}(0)\zeta_{T,n}(z)J_{T,n}(\rho, x_3; L)\,, \qquad \rho = \sqrt{x_1^2 + x_2^2}$$

$$J_{T,n}(\rho, x_3, L) = \int \frac{dk}{2\pi} e^{ikx_3}\hat{J}_n(\rho, k, L)$$

$$\hat{J}_n(\rho, k, L) = \begin{cases} 2\pi\sqrt{2}LI_1\left(L\sqrt{k^2 + \frac{9T_a^2}{4c_a^2}\tau_n}\right)\rho K_1\left(\rho\sqrt{k^2 + \frac{9T_a^2}{4c_a^2}\tau_n}\right) & \rho > L \\ 2\pi\sqrt{2}LK_1\left(L\sqrt{k^2 + \frac{9T_a^2}{4c_a^2}\tau_n}\right)\rho I_1\left(\rho\sqrt{k^2 + \frac{9T_a^2}{4c_a^2}\tau_n}\right) & \rho < L \end{cases}$$

Closed strings and walls

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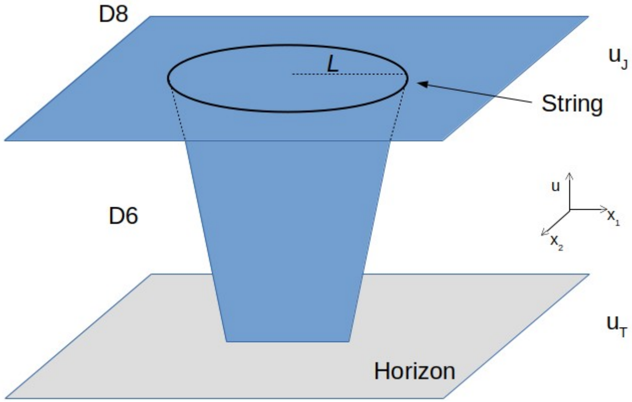
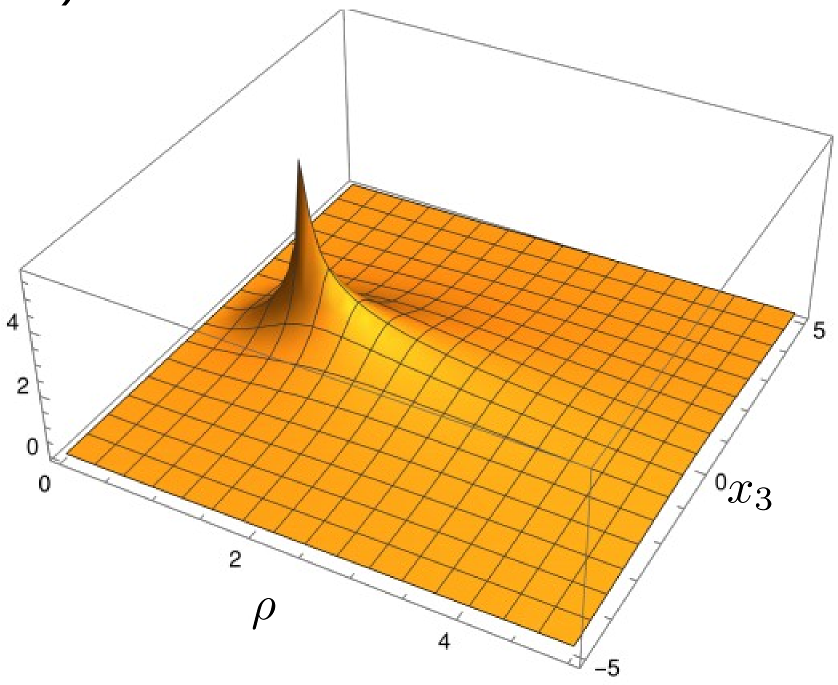


- Solution (one mode)

$L = 1$



- No divergence in tension at large distance!