QCD@Work
International Workshop on QCD
Theory and Experiment

Revealing nucleus properties by exploring ultra-central symmetric heavy-ion collisions

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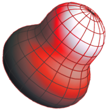
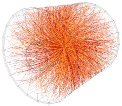
Based on: [arXiv:2406.13863]

Trani, Itally
June 21st, 2024

Introduction



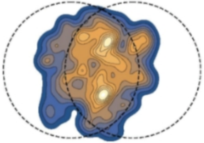
Low energy nuclear physics



High energy heavy-ion collision

* G. Giacalone, J. Jia, and C. Zhang, Impact of Nuclear Deformation on Relativistic Heavy-Ion Collisions, Phys. Rev. Lett. 127, 242301 (2021)

Introduction



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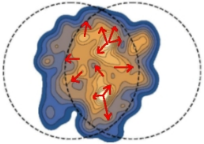

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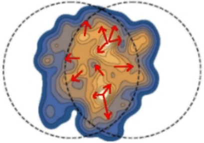
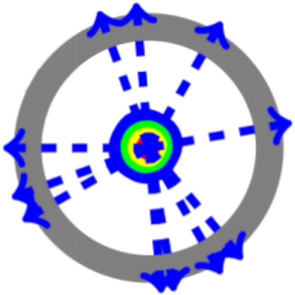


Collective evolution



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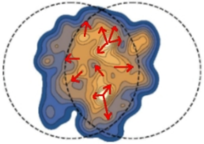

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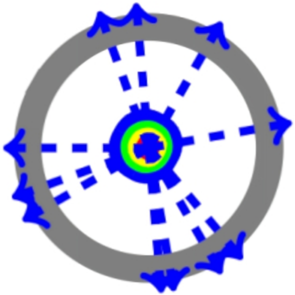
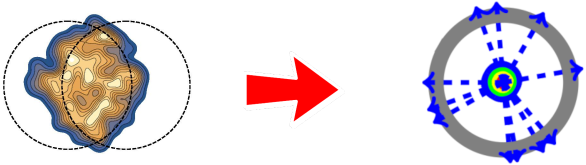


Initial spatial anisotropy $\longrightarrow$ Final momentum anisotropy

[Jean-Yves Ollitrault, Phys. Rev. D 46, 229, 1992]

Fluctuation

- **Nucleon position fluctuation:**



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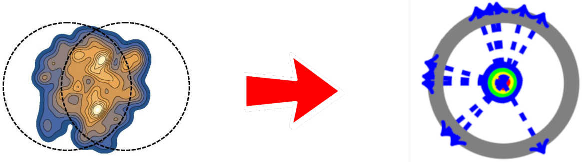

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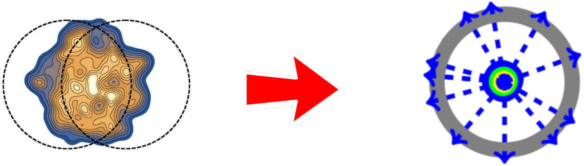
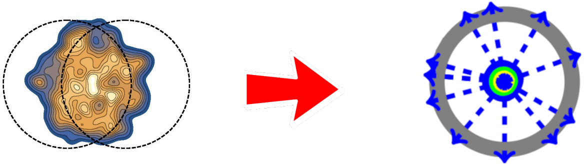
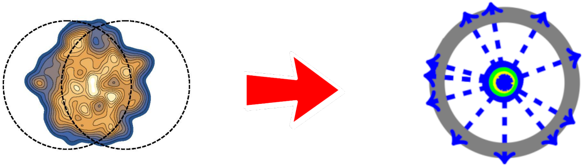


Fluctuation



- **Nucleon position fluctuation:**

Fluctuation

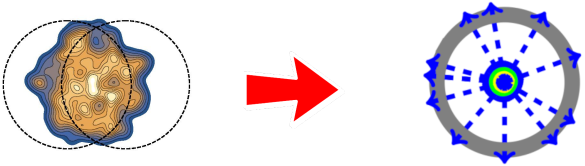


- **Nucleon position fluctuation:**

- **Constituent weight fluctuation:**

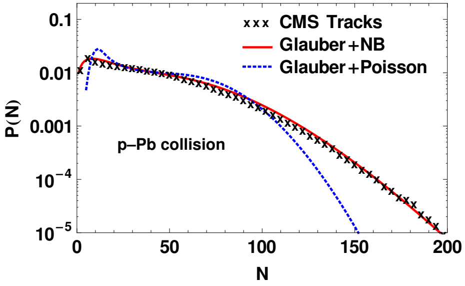
Fluctuation

- **Nucleon position fluctuation:**



- **Constituent weight fluctuation:**

[P. Bozek et. al., PRC 88, 014903 (2013)]



Fluctuation

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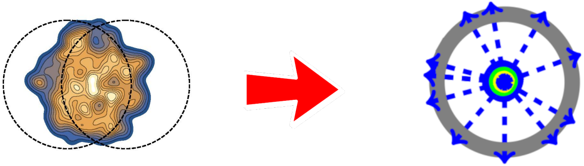
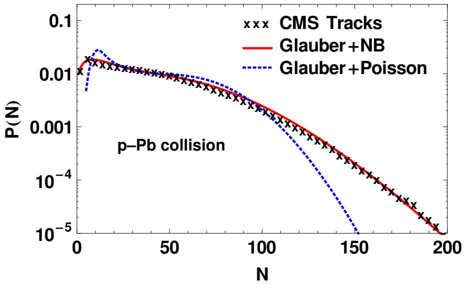


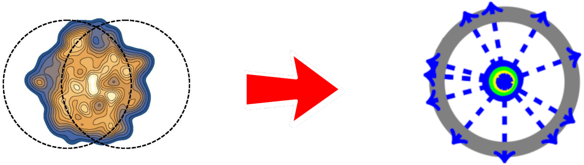
- **Constituent weight fluctuation:**

$$p_{\text{fluc}}(\gamma) = \frac{k^k}{\Gamma(k)}\gamma^{k-1}e^{-k\gamma}, \qquad k = 1/\sigma^2_{\text{fluc}}.$$

[P. Bozek et. al., PRC 88, 014903 (2013)]



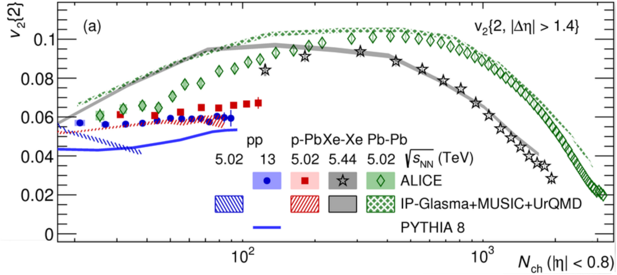
Fluctuation



- Nucleon position fluctuation:
- Constituent weight fluctuation:

$$p_{\text{fluc}}(\gamma) = \frac{k^k}{\Gamma(k)}\gamma^{k-1}e^{-k\gamma}, \qquad k = 1/\sigma_{\text{fluc}}^2.$$

[ALICE collaboration, PRL, 123, 142301 (2019)]



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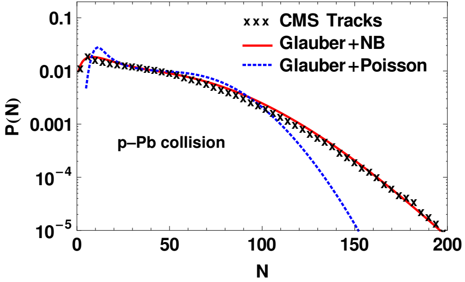
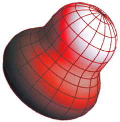
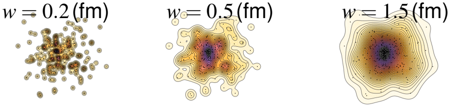


* $v_2^2\{2\} \equiv \langle v_2^2 \rangle \approx 2\sigma_v^2 + \bar{v}_2^2$

Initial State

Nuclear structure





- $\rho_{\text{WS}}(\vec{x}_i)$: Woods-Saxon distribution,

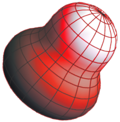
$$\rho_{\text{WS}}(\vec{x}) = \frac{\rho_0}{1+\exp\left[(|\vec{x}| - R(\theta,\phi))/a_0\right]},$$

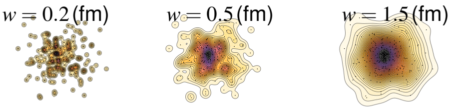
$$R(\theta,\phi) = R_0(1+\beta_2 Y_2^0(\theta,\phi) + + \cdots)$$

- $C_2(\vec{x}_1,\vec{x}_3)$: two-body correlation,

$$C_2(\vec{x}_1,\vec{x}_2) = \begin{cases} 0 & |\vec{x}_1 - \vec{x}_2| \le d_{\text{min}} \\ 1 & |\vec{x}_1 - \vec{x}_2| > d_{\text{min}} \end{cases}$$

Nuclear structure





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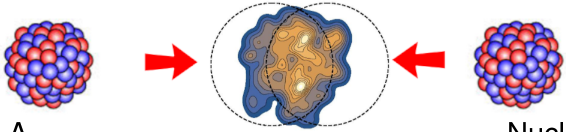
Parameter	Description
A	mass number
R_0	nuclear radius
a_0	skin thickness
β_2	quadrupole deformation
σ_{fluc}	Nucleon fluctuation
w	Nucleon width parameter
d_{min}	Minimum inter-nucleon distance

Entropy production

Entropy production



Nucleus A

Nucleus B

$T_A(x,y) = \int dz \rho_A(\vec{x})$

$N_{\text{ch}} \propto S = f(T_A, T_B)$

$T_B(x,y) = \int dz \rho_B(\vec{x})$

Constituents interact and produce a medium.

Functionality of $f(T_A, T_B)$

Functionality of $f(T_A, T_B)$

- **IP-Glasma**: (q is in the range 0.54 to 1.71) [1,2]: $$f^{3/2}(T_A, T_B) \sim (T_A T_B)^{3q/4}$$
- **CGC model** [3]: $$f^{3/2}(T_A, T_B) \propto \frac{T_A T_B}{(T_A + T_B)^{5/2}} \left[2T_A^2 + 7T_A T_B + 2T_B^2\right]$$
- **T_RENTo model** (p is a real parameter) [4] : $$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

[1] Schenke, Tribedy, Venugopalan, PRL. 108, 252301 (2012), Schenke, Tribedy, Venugopalan, PRC 86, 034908 (2012)
[2] Nijs, van der Schee, (2023), arXiv:2304.06191 [nucl-th]
[3] Borghini, Borrell, Feld, Roch , Schlichting, PRC 107 (2023) 3, 034905
[4] Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901
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T_RENTo, CGC model, (and IP-Glasma) are scale-invariant: $f(cT_A, cT_B) = cf(T_A, T_B)$.

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- **T$_R$ENTo model** (p is a real parameter) [4] : $$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$
- **MC-Glauber model** ($\alpha \sim 0.1$) [5]: $$f(T_A, T_B) = (T_A + T_B) + \alpha T_A T_B$$
- **Generalized T$_R$ENTo model** (p, q are real parameters) [2]: $$f^{3/2}(T_A, T_B) \propto E_{\text{ref}}^{2-2q} \left(\frac{T_A^p + T_B^p}{2}\right)^{q/p}$$

T$_R$ENTo, CGC model, (and IP-Glasma) are scale-invariant: $f(cT_A, cT_B) = cf(T_A, T_B)$.

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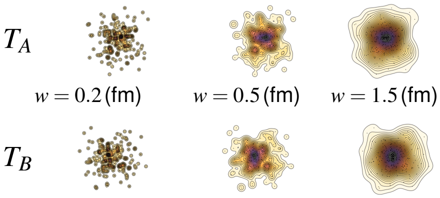
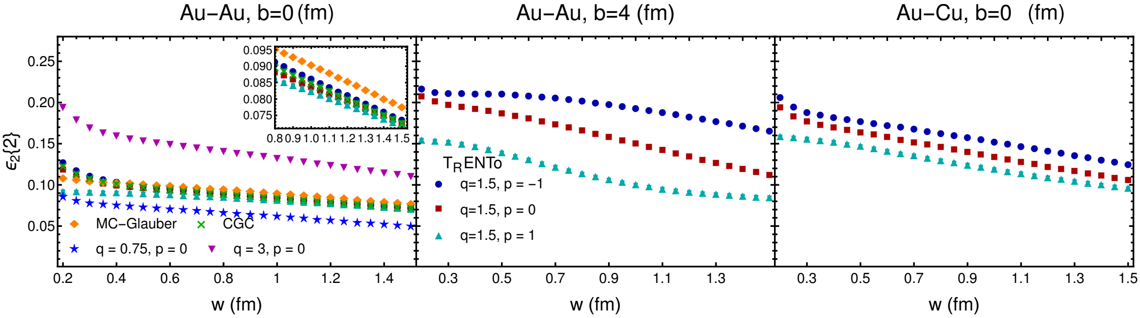
Ultra-central Symmetric Collisions



- Same behavior for other initial state moments.
- From data, $w \approx 0.7$(fm) is favored.

$$\varepsilon_2^2\{2\} \equiv \langle \varepsilon_2^2 \rangle$$

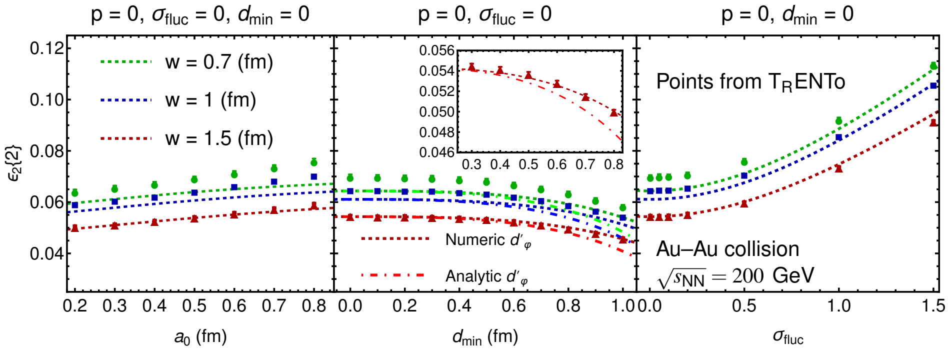


Ellipticity in ultra-central symmetric collisions

from an analogy with statistical mechanics

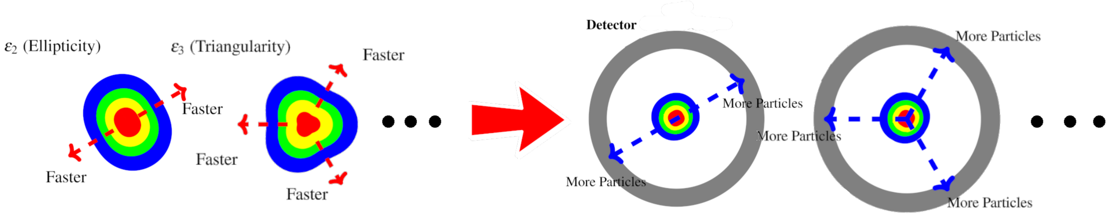
- Set of events $\leftrightarrow$ set of ensembles
- Short range correlations $\rightarrow$ cluster expansion method

$$\varepsilon_2^2\{2\} \approx \frac{5}{7A}\frac{1}{(1+(w/R_0)^2)^2}\left[1+\alpha_\varepsilon\left(\frac{a_0}{R_0}\right)^2 - \frac{A}{2}\left(\frac{d_{\mathrm{min}}}{R_0}\right)^3 + \sigma_{\mathrm{fluc}}^2 + b_{2,\varepsilon}\beta_2^2 + \cdots\right],$$



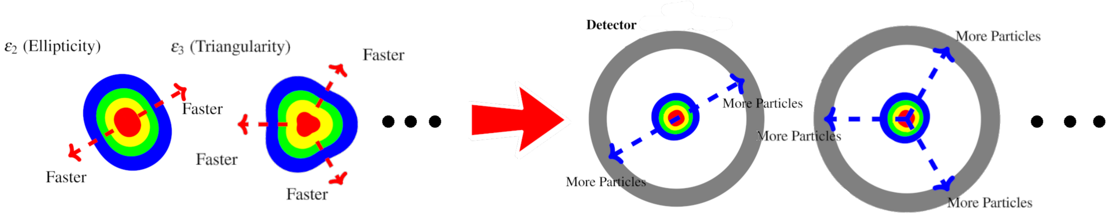
Collective expansion

Linear Response approximation



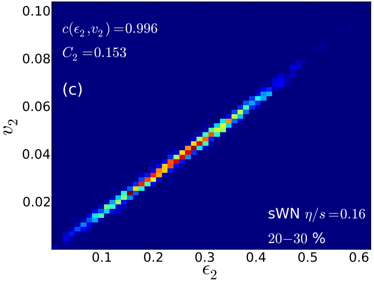
v_2 (Elliptic flow) v_3 (Triangular flow)

Linear Response approximation

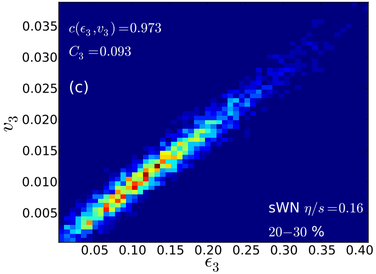


v_2 (Elliptic flow) v_3 (Triangular flow)

$v_2 \approx k_2 \varepsilon_2$



$v_3 \approx k_3 \varepsilon_3$



$v_n \approx k_n \varepsilon_n$

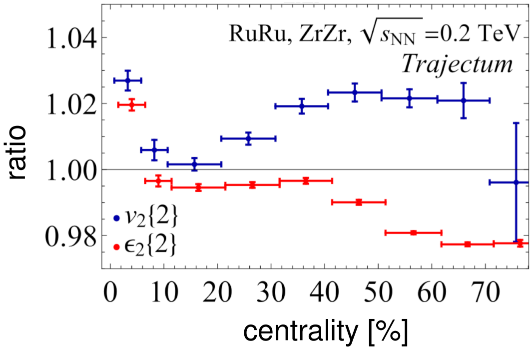
for $n = 2, 3$

[Niemi, Denicol, Holopainen, Huovinen, PRC 87 (2013) 5, 054901]

Deformed nuclei and isobar ratio

$${}^{96}_{40}Ru - {}^{96}_{40}Ru \qquad Vs \qquad {}^{96}_{44}Zr - {}^{96}_{44}Zr$$

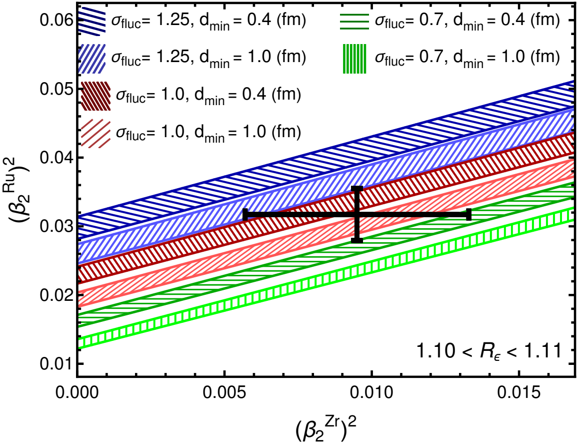
[Nijs, van der Schee, SciPost Phys. 15, 041 (2023)]



$$\frac{v_2^{\mathrm{Ru}}\{2\}}{v_2^{\mathrm{Zr}}\{2\}} \approx \frac{\varepsilon_2^{\mathrm{Ru}}\{2\}}{\varepsilon_2^{\mathrm{Zr}}\{2\}} \approx 1 + \frac{A/2}{1 - \frac{A}{2}\left(\frac{d_{\mathrm{min}}}{R_0}\right)^3 + \sigma^2_{\mathrm{fluc}} + \cdots}\left[(\beta_2^{\mathrm{Ru}})^2 - (\beta_2^{\mathrm{Zr}})^2\right]$$

- It was known that the ratio has the form $1 + R\left[(\beta_2^{\mathrm{Ru}})^2 - (\beta_2^{\mathrm{Zr}})^2\right]$ was known **[Zhang,Jia, PRL, 128, 022301]**.
- We now know how R depends on the initial state parameters.

Ru and Zr structure

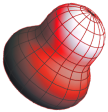


Equation $v_2^{Ru}\{2\}/v_2^{Zr}\{2\} =$ constant put a constraint on the nuclear structure parameters.

* Data point from: **[Pritychenko et al, Atom.Data Nucl.Data Tabl. 107, 1-139 (2016)]**

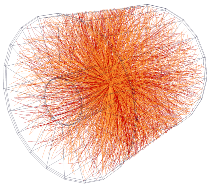
Summary

- Ultracentral symmetric collisions can be employed to study the nuclear structure.
- We showed that the isobar ratio in ultracentral collision is sensitive to nuclear structure including two body correlations and constituent weight fluctuation.



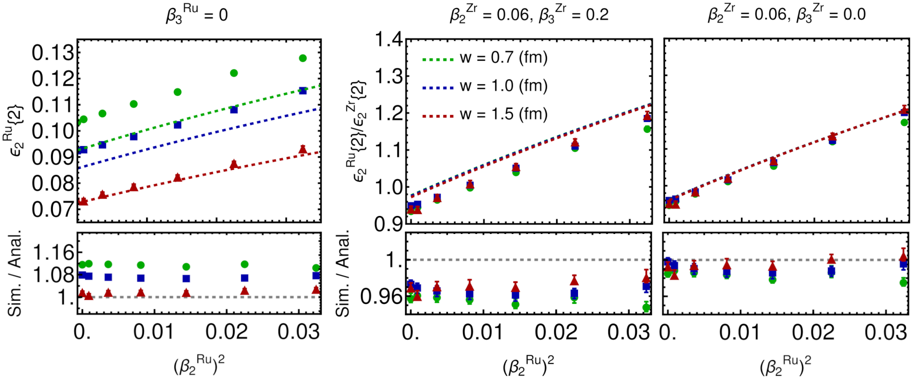
Outlook

- Find higher order moments and studying, higher harmonics $\varepsilon_2\{4\}$, $\varepsilon_3\{2\}$, ...
- Finding observables which are more sensitive to two-body correlations, disentangling σ_{fluc} and d_{min}?

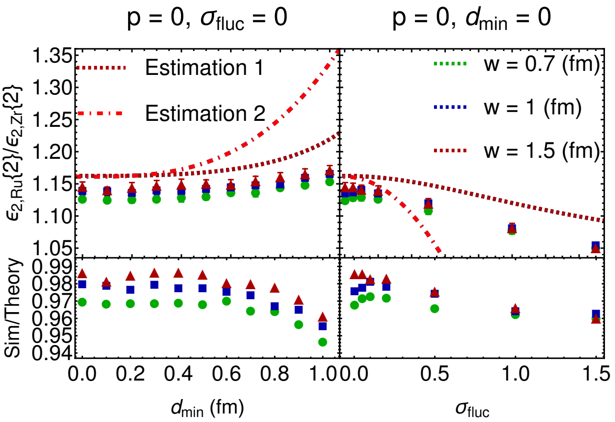


Thank You!

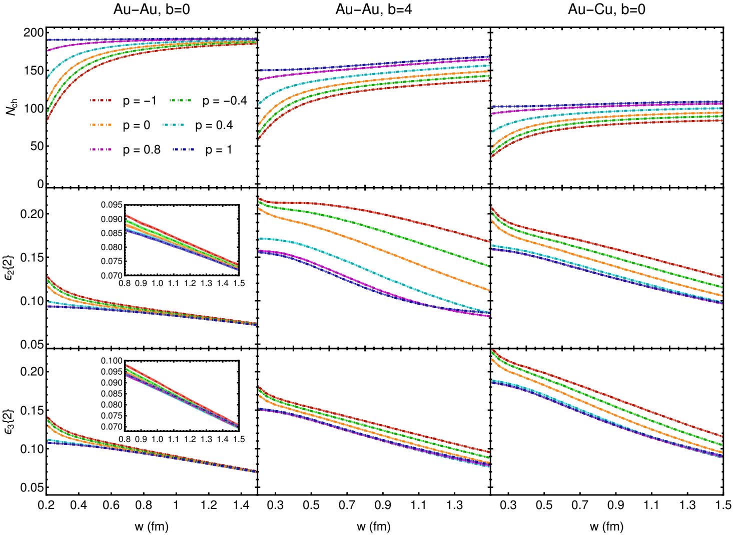
Backup Slides

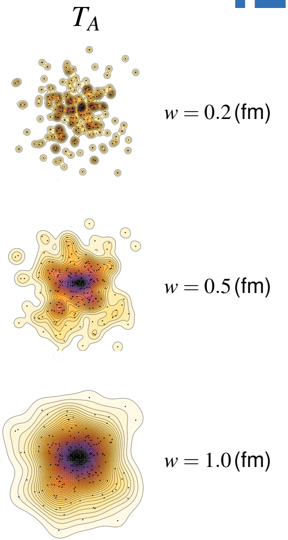


$\beta_3^{Ru} = 0$
$\beta_2^{Zr} = 0.06$, $\beta_3^{Zr} = 0.2$
$\beta_2^{Zr} = 0.06$, $\beta_3^{Zr} = 0.0$
$\epsilon_2^{Ru}\{2\}$
$\epsilon_2^{Ru}\{2\}/\epsilon_2^{Zr}\{2\}$
w = 0.7 (fm)
w = 1.0 (fm)
w = 1.5 (fm)
Sim. / Anal.
$(\beta_2^{Ru})^2$


p = 0, σ_{fluc} = 0
p = 0, d_{min} = 0
Estimation 1
Estimation 2
w = 0.7 (fm)
w = 1 (fm)
w = 1.5 (fm)
$\epsilon_{2,Ru}\{2\}/\epsilon_{2,Zr}\{2\}$
Sim/Theory
d_{min} (fm)
σ_{fluc}

The effect of scale-invariance in ultracentral symmetric collision





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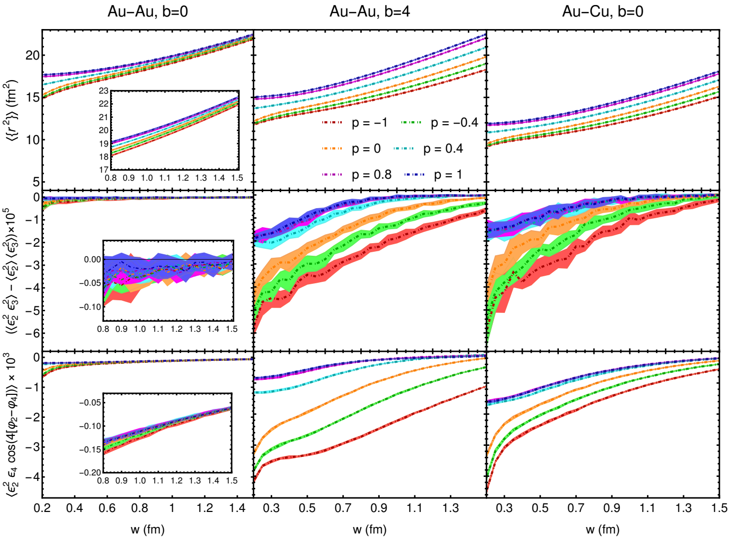
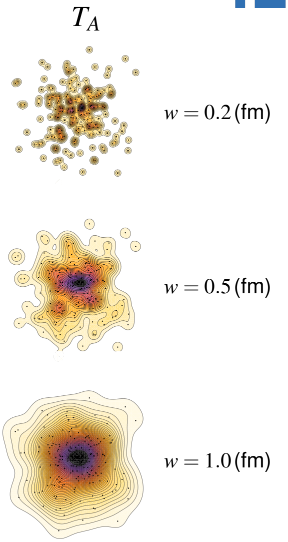
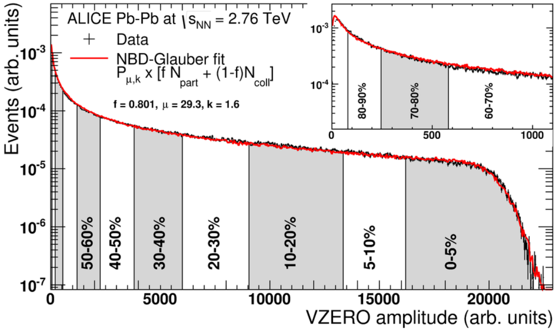


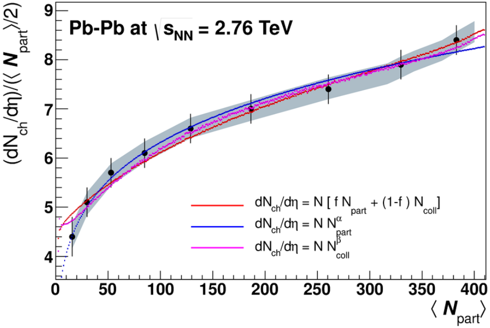


Charge multiplicity and function $f(N_A, N_B)$

$$N_{part} = N_A + N_B, \qquad N_{col} = N_A N_B$$

[ALICE collaboration, Phys.Rev.Lett. 106 (2011) 032301], [ALICE collaboration, Phys.Rev.C 88 (2013) 4, 044909]





Model	Normalization	Error	Fit Par.	Error	χ^2/NDF
$f \cdot N_{part} + (1-f) \cdot N_{coll}$	2.441	0.281	f = 0.788	0.021	0.347
${N_{part}}^{\alpha}$	1.317	0.116	α = 1.190	0.017	0.182
${N_{coll}}^{\beta}$	4.102	0.297	β = 0.803	0.012	0.225

Functionality of $f(T_A, T_B)$

- $f_{\text{Glauber}}(T_A, T_B) = (T_A + T_B) + \alpha T_A T_B$
- U-U collisions disfavor binary term **[Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901]**

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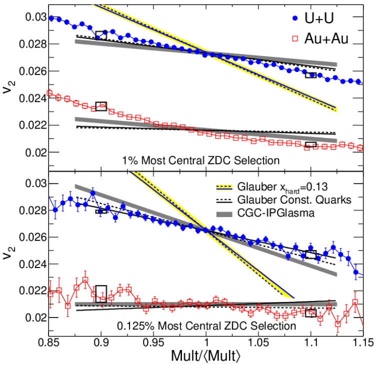

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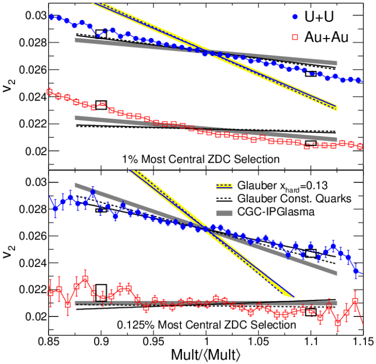

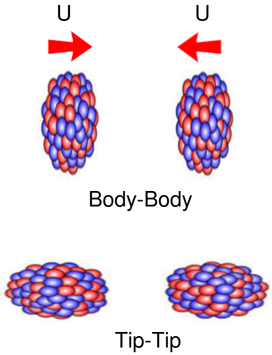

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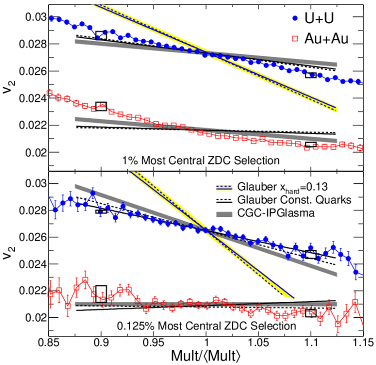

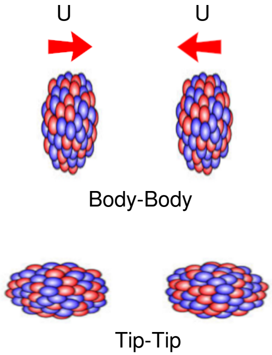

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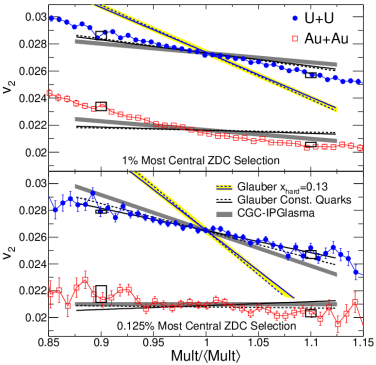
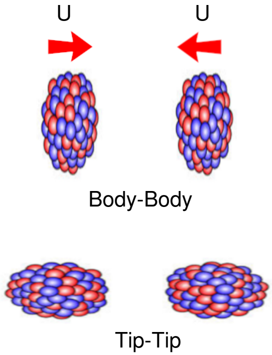


$$f(cT_A, cT_B) \approx cf(T_A, T_B)$$



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- T$_R$ENTo event generator:

$$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

Ultracentral symmetric heavy-ion collisions

Assuming scaling invariance, $f(cT_A, cT_B) = cf(T_A, T_B)$*:*

For the **ultracentral symmetric** A–A collisions with **large nucleon size**:

- The participants thickness function A and B are "similar" to each other,

$$T_A \approx T_B$$

- $f(cT_A, cT_A) = cf(T_A, T_A) \to g(cT_A) = c\,g(T_A) \to g(x)$ **is linear**!

For the most central symmetric A–A collisions:

observables should have small dependence on functionality $f(T_A, T_B)$*.*

Analytical result

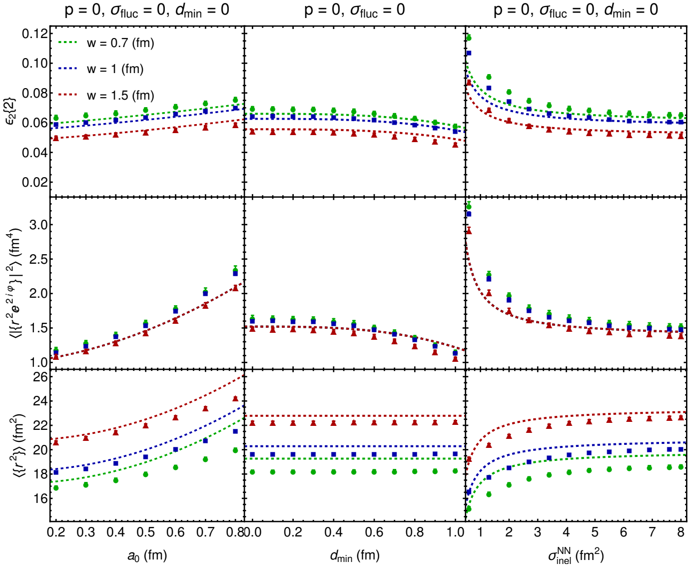
$$\langle\{r^2\}\rangle^2 = \frac{4R_0^4\,\omega^2}{25}\left[\omega^2 + \alpha_r\left(\frac{a_0}{R_0}\right)^2 + s_r\left(\frac{a_0}{R_0}\right)\left(\frac{\pi R_0^2/A}{\sigma_{\text{inel}}^{\text{NN}}}\right)\right.$$
$$\left. + b_{2,r}\beta_2^2 + b_{3,r}\beta_3^2 + \cdots\right]$$

$$\left\langle\left|\{r^2 e^{2i\varphi}\}\right|^2\right\rangle = \frac{1}{A}\frac{4R_0^4}{35}\left[1 + \alpha_\varphi\left(\frac{a_0}{R_0}\right)^2 + s_\varphi\left(\frac{a_0}{R_0}\right)\left(\frac{\pi R_0^2/A}{\sigma_{\text{inel}}^{\text{NN}}}\right)\right.$$
$$\left. + d_\varphi\left(\frac{d_{\text{min}}}{R_0}\right)^3 + v_\varphi\frac{\sigma_{\text{fluc}}^2}{n_c} + b_{2,\varphi}\beta_2^2 + \cdots\right]$$

$$\varepsilon_2^2\{2\} = \frac{5}{7}\frac{1}{A\,\omega^4}\left[1 + \alpha_\varepsilon\left(\frac{a_0}{R_0}\right)^2 + s_\varepsilon\left(\frac{a_0}{R_0}\right)\left(\frac{s_\perp}{\sigma_{\text{inel}}^{\text{NN}}}\right) + d_\varphi\left(\frac{d_{\text{min}}}{R_0}\right)^3\right.$$
$$\left. + v_\varphi\frac{\sigma_{\text{fluc}}^2}{n_c} + b_{2,\varepsilon}\beta_2^2 + b_{3,\varepsilon}\beta_3^2 + \cdots\right]$$

coefficient	value
ω^2	$1+5\,(w/R_0)^2$
α_r	$14\pi^2/3$
α_φ	$6\pi^2$
s_r	-6
s_φ	$51/4$
d'_φ	$-A+\alpha_\varphi A\,(a_0/R_0)^2$
d_φ	$d'_\varphi-\alpha_r A\,(a_0/R_0)^2$
v_φ	$1+10\,(w^2-v^2)\,/R_0^2+\alpha_\varphi\,(a_0/R_0)^2$
$b_{2,r}$	$29/8\pi$
$b_{3,r}$	$7/2\pi$
$b_{2,\varphi}$	$21A/20\pi$
α_ε	$\alpha_\varphi-\alpha_r/\omega^2$
s_ε	$s_\varphi-s_r/\omega^2$
$b_{2,\varepsilon}$	$b_{2,\varphi}-b_{2,r}\omega^2$
$b_{3,\varepsilon}$	$-b_{3,r}/\omega^2$

Analytical result



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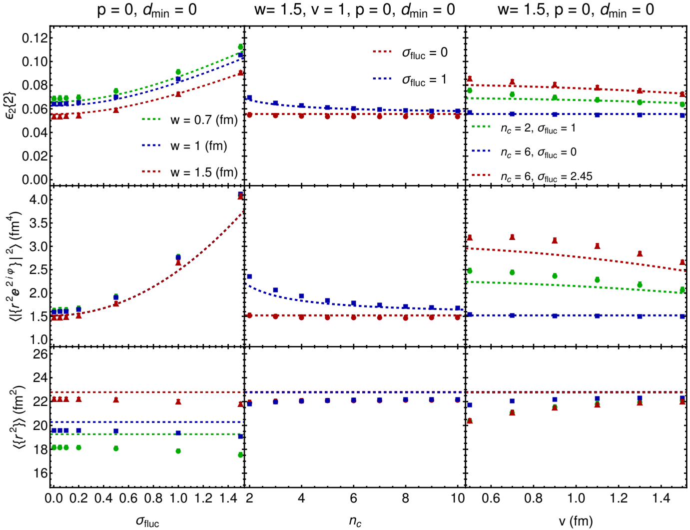

Generating random sources with two-body corelation

Our analytical analysis has revealed a problem in creating sources with correlations.

To implement the minimum distance:

Generating random sources with two-body corelation

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To implement the minimum distance:

1. **Iterative** (used in T$_{\mathrm{R}}$ENTo [1]): Generate the nth source based on WS and compare its distance to all other $(n-1)$th sources. If there is any distance smaller than d_{min} reject the nth source and generate a new one.

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2. **Shifting position** [2]: Generate A sources at once and shift positions symmetrically for pairs with distance less than d_{min}.
3. **Direct**: Generate A sources at once and compare all mutual distances. If there is any pair with distance smaller than d_{min} reject all A sources and generate a new set.

[1] Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901
[2] Luzum, Hippert, Ollitrault, Eur.Phys.J.A 59 (2023) 5, 110, Eur.Phys.J.A 59 (2023) 110

Comparing “Direct” method, iterative method and shifting method