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Collision integrals for first-order phase transitions

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based on: JHEP 03 (2022) 163, JHEP 05 (2023) 194, JHEP 05 (2024)

Thermal History of the Universe

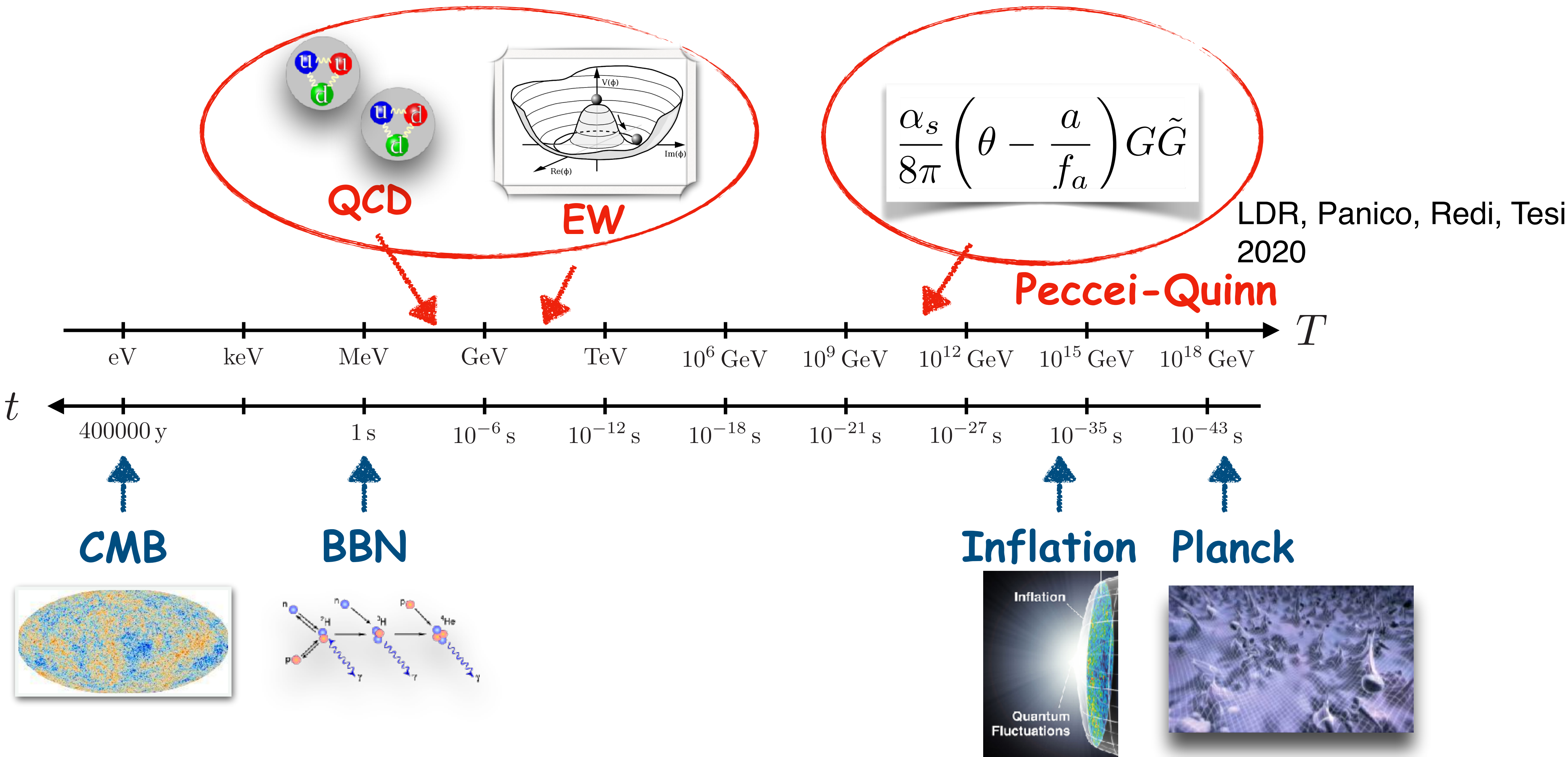
Phase transitions are crucial events in the evolution of the Universe

- the SM predicts two of them *(the two phases are smoothly connected (cross over))*

  no strong breaking of thermal equilibrium

  no distinctive experimental signatures

- New physics may change the nature of these PhTs, or add new ones



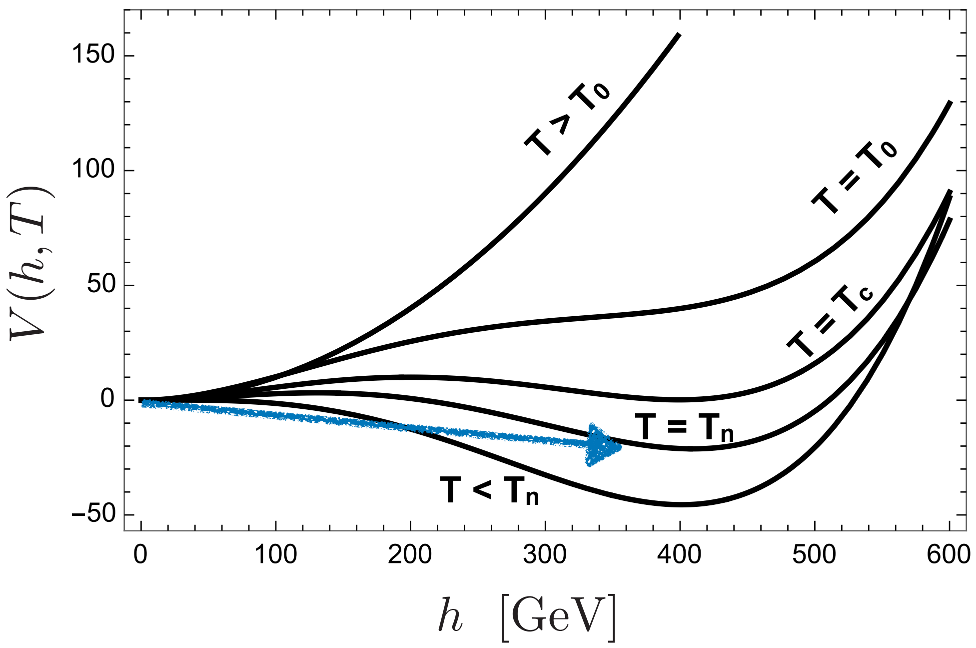
First-order EWPhT

New physics may provide **first order** phase transitions

- a barrier in the potential may be generated from tree-level deformations, thermal or quantum effects

- the field tunnels from false to true minimum at $T = T_n < T_c$

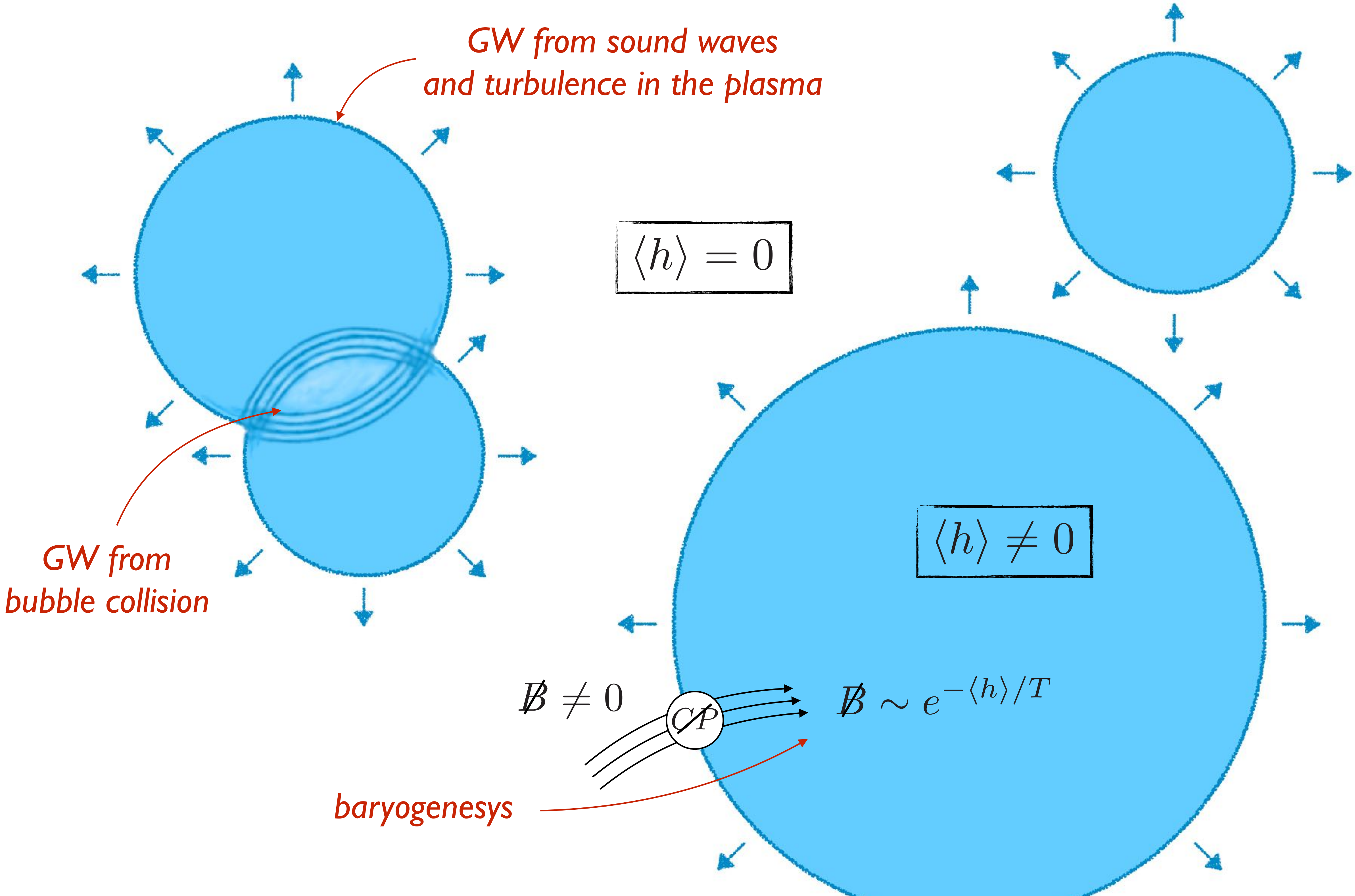
- the transition proceeds through bubble nucleation



- significant breaking of thermal equilibrium (relevant for baryogenesis)
- interesting experimental signatures (eg. gravitational waves)

Bubble nucleation

Bubble dynamics can produce **gravitational waves** and **baryogenesys**



Key features of a first-order PhT

- the nucleation temperature T_n
- the strength α
- the (inverse) time duration of the transition β/H

} *equilibrium quantities*

- the speed of the bubble wall v_w
- the thickness of the bubble wall L_w

} *non-equilibrium quantities*

Gravitational waves and the efficiency of the EW-baryogenesis crucially depend on them

EWBG is typically efficient for slowly-moving walls. Recent results show efficiency also for fast-moving walls [Dorsch, Huber, Konstandin, 2021]

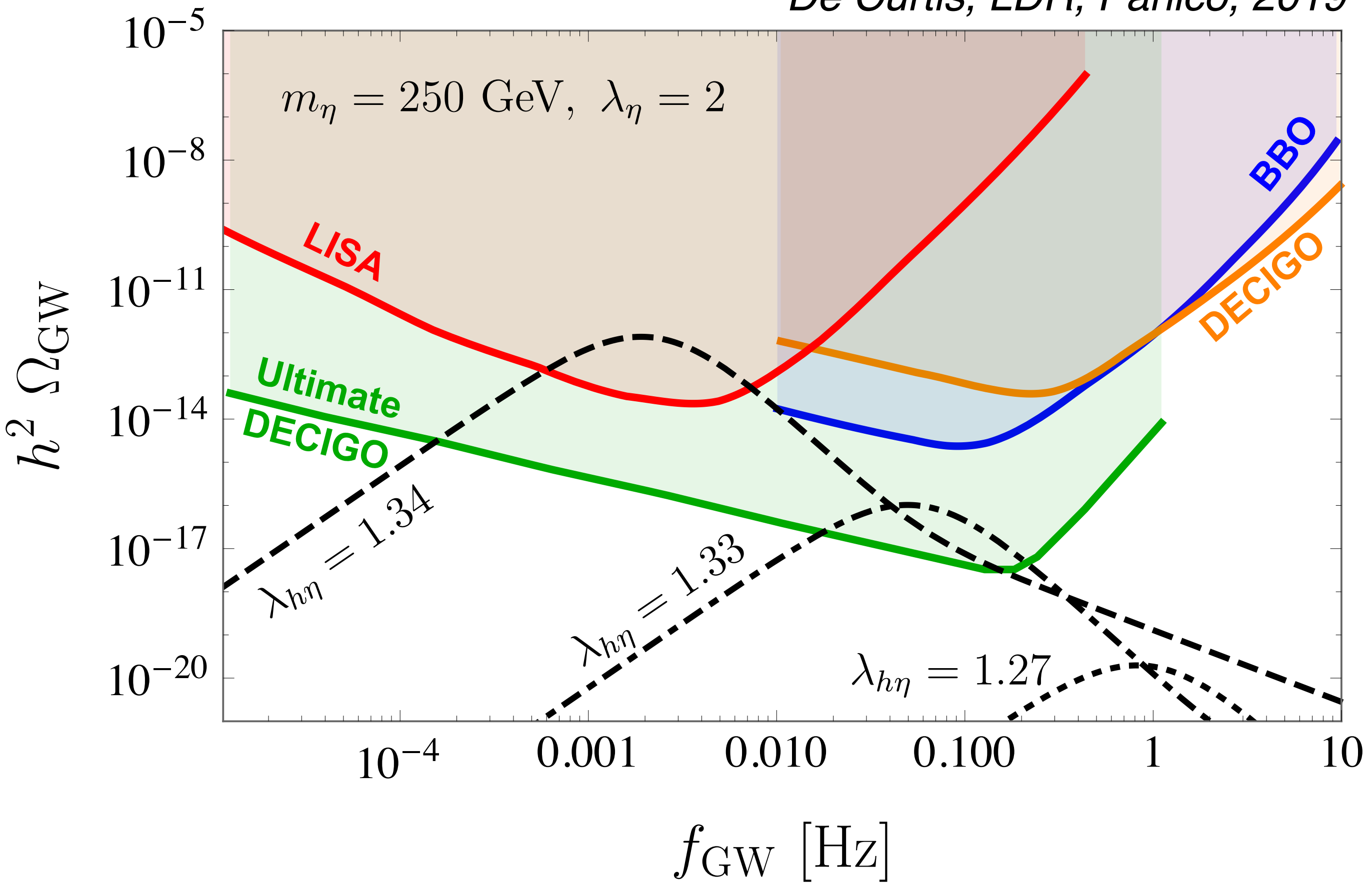
BG at the EW scale also possibile with $v_w \sim 1$ [Azatov, Vanvlasselaer, Yin, 2021]

GWs are maximised for fast-moving walls

GW from a first-order PhT

First-order PhTs produce stochastic background of gravitational waves

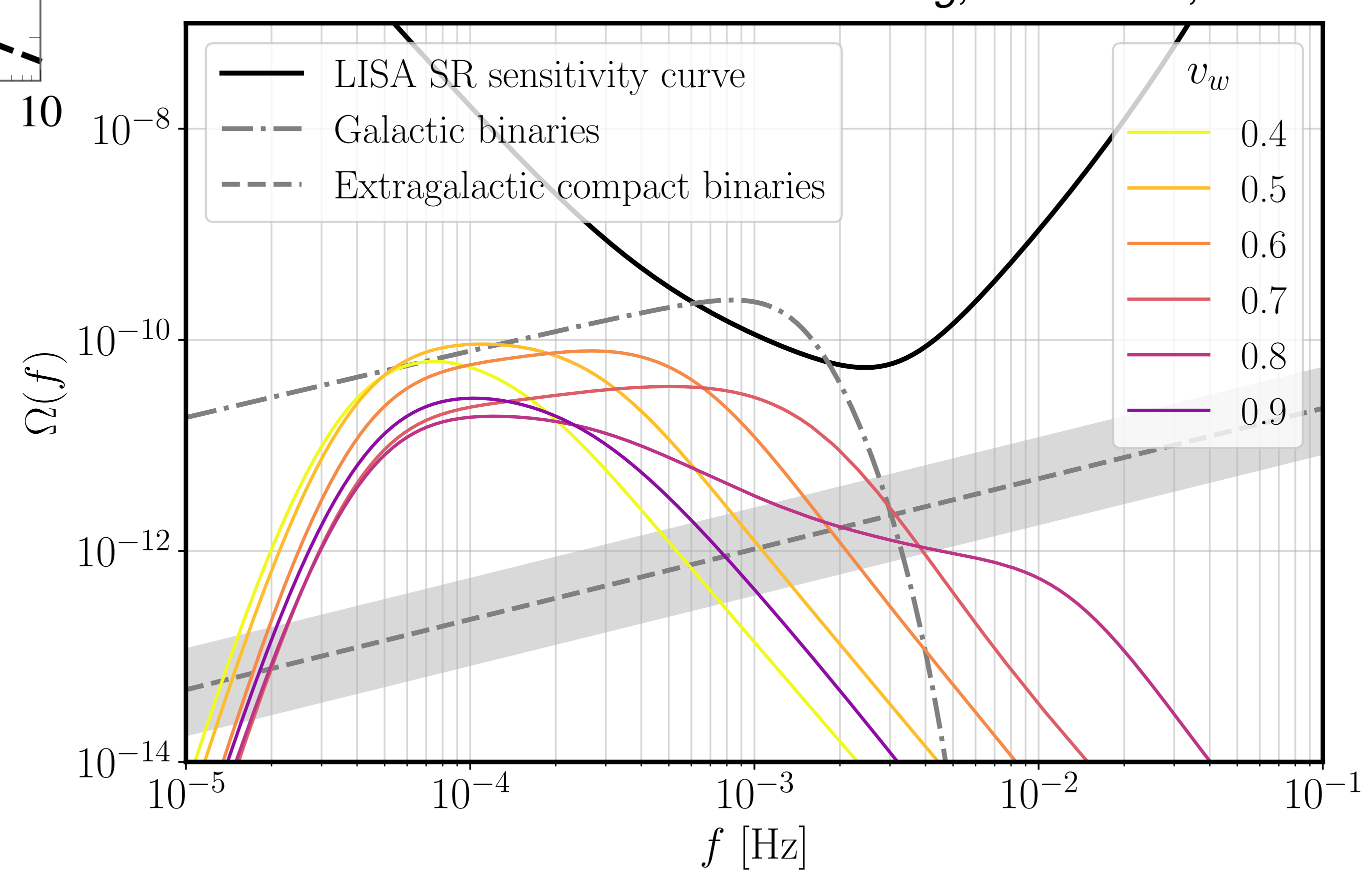
De Curtis, LDR, Panico, 2019



for the EWPhT the peak frequency is within the range of future experiments

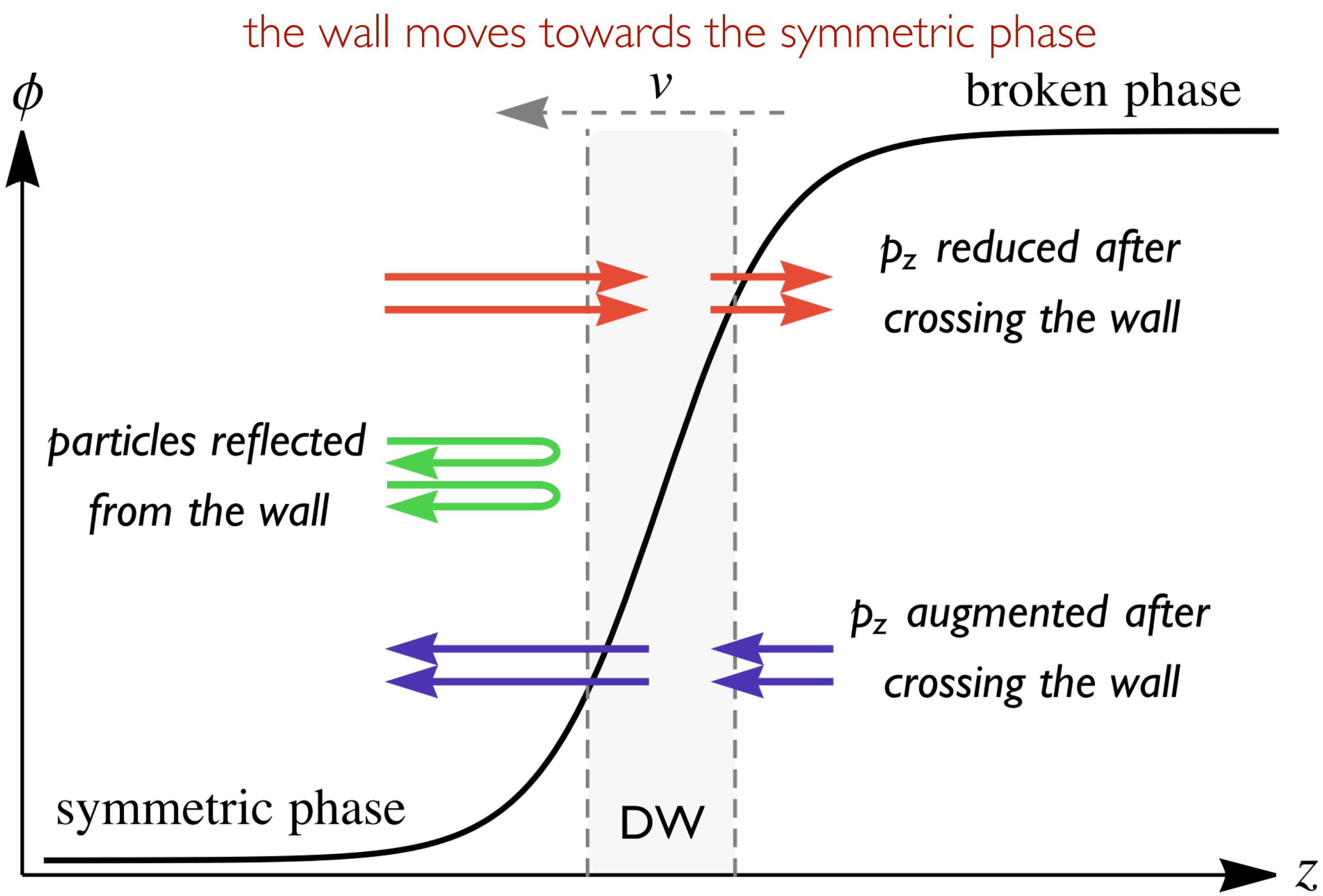
- wall speed has a strong effect on the shape of the power spectrum
- wall speed will be the best determined parameter

Gowling, Hindmarsh, 2019



Dynamics of the bubble wall

System setup: scalar field + plasma



- The bubble wall drives plasma out of equilibrium
- Interactions between plasma and wall front produce a friction
- If the friction and pressure inside the bubble balance we can realise a steady state regime (terminal velocity reached)

Dynamics of the bubble wall

For each particle species in the plasma

out-of-equilibrium

$$f(p,z) = f_v(p,z) + \delta f(p,z)$$

equilibrium

1. Scalar field equation $$\phi' \Box \phi - V_T' = \sum N_i \frac{dm^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_p} \delta f(p)$$

2. Boltzmann equation $$\left(\frac{p_z}{E} \partial_z - \frac{(m^2)'}{2E} \partial_{p_z} \right) (f_v + \delta f) = -\mathcal{C}[f_v + \delta f]$$

two competing effects:
$(m^2)' \sim (\phi^2)'$ *and* C

we assume a planar wall and a steady state regime

Approaches to the Boltzmann equation

To deal with the collision term, previous approaches made assumptions on the *shape* of $\delta f(p, z)$ in momentum space

- Fluid approximation [1]
- Extended fluid approximation [2]
- New formalism [3]

[1] Moore, Prokopec, 1995
[2] Dorsch, Huber, Konstandin, 2022
[3] Laurent, Cline, 2020

[1] and [2] dubbed "old formalism" (OF) in the following

1!!! the $\partial_{p_z} \delta f$ term neglected

2!!! Boltzmann equation integrated with a set of (*not unique*) weights

Alternative methods

- Expansion of δf in a polynomial basis [4]
- Holographic approach [5]

[4] Laurent, Cline, 2022
[5] Bigazzi, Caddeo, Canneti, Cotrone

Full solution to the Boltzmann equation

- We propose a new method to solve the Boltzmann equation **without imposing any ansatz for** δf *De Curtis, LDR, Guiggiani, Gil Muyor, Panico, 2022*

- We developed an algorithm to solve the coupled system of *bubble wall* and *Boltzmann* equations, thus getting v_w, L_w, etc.

De Curtis, LDR, Guiggiani, Gil Muyor, Panico, 2023

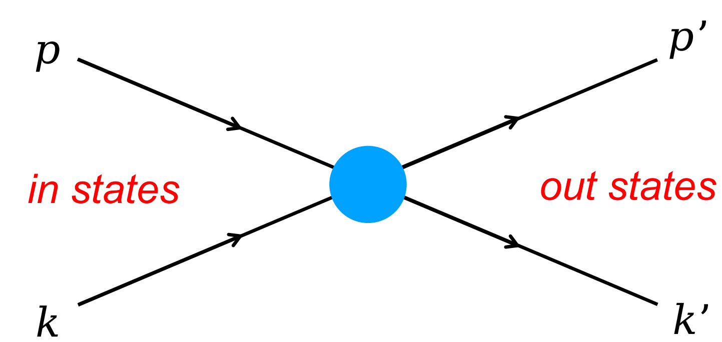
Key features

- New approach (*spectral decomposition*) to deal with collision integrals
- No term in the Boltzmann equation is neglected
- Iterative routine where convergence is achieved in few steps

Structure of the collision integral

$$\mathcal{C}[f] = \frac{1}{2N_pE_p}\sum_i \int \frac{d^3k d^3p' d^3k'}{8(2\pi)^5 E_k E_{p'} E_{k'}} |\mathcal{M}_i|^2 \mathcal{P}[f] \delta^4(p+k-p'-k')$$

$$\mathcal{P}[f] = f(p)f(k)(1 \pm f(p'))(1 \pm f(k')) - f(p')f(k')(1 \pm f(p))(1 \pm f(k))$$



The collision integral yields two classes of terms:

$$C[f] = \mathcal{Q}(p)\frac{\delta f(p)}{f_\nu'(p)} + f_\nu(p)\langle \delta f \rangle$$

- perturbations do not appear inside the integral: *easy to handle*

- perturbations are integrated (brackets): *very challenging*

Structure of the collision integral

$$\langle \delta f \rangle = \mathcal{O}[\delta f] \sim \int \mathcal{D}k \, \mathcal{K}(p, \cos\theta_p, k, \cos\theta_k) \delta f(k, \cos\theta_k, z)$$

Brackets can be seen as the application of Hermitian operators $\mathcal{O}$ on perturbations

Main idea: decompose the operators on the basis of their eigenfunctions

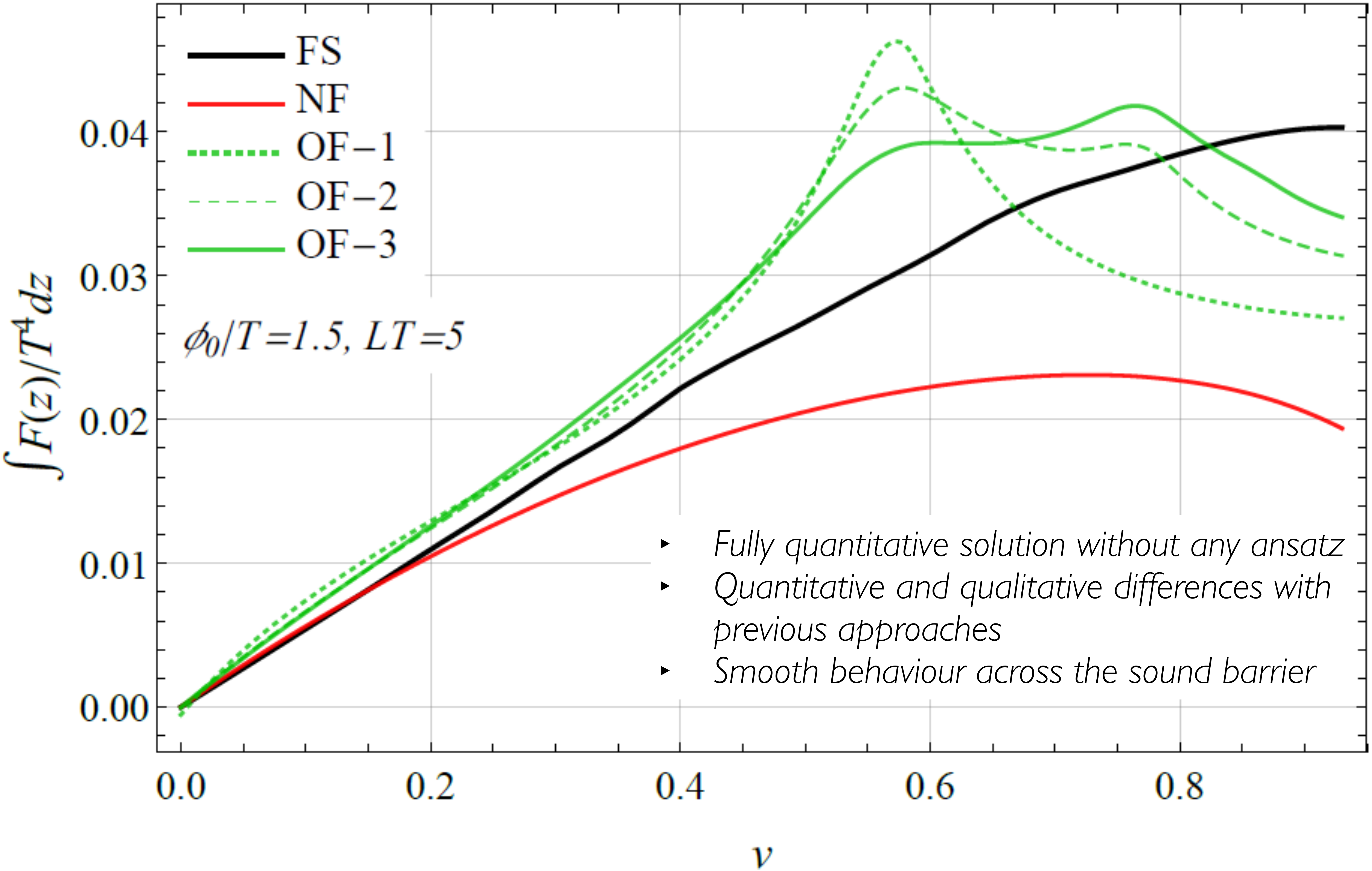
Exploit rotational invariance of the collision integral:
the kernel is block diagonal on the angular momentum basis

$$\mathcal{K}(p, \cos\theta_p, k, \cos\theta_k) = \sum_l \frac{2l+1}{2} P_l(\cos\theta_p) P_l(\cos\theta_k) \mathcal{G}_l(p, k)$$

$$\mathcal{G}_l(p, k) = \sum_i \lambda_i^{(l)} \varphi_i(p) \varphi_i(k)$$

kernels can be (numerically) evaluated only once
huge improvement in time performance (~ 2 orders of magnitude)

Integrated friction

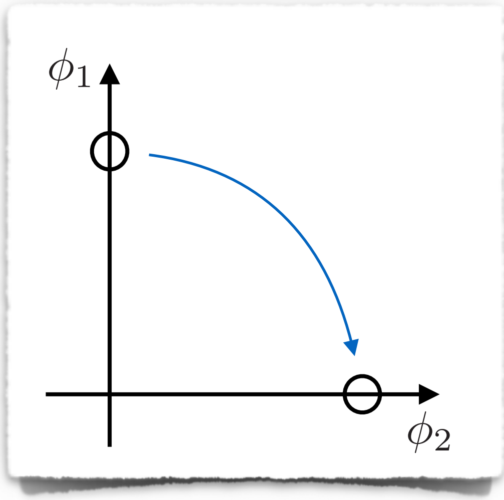


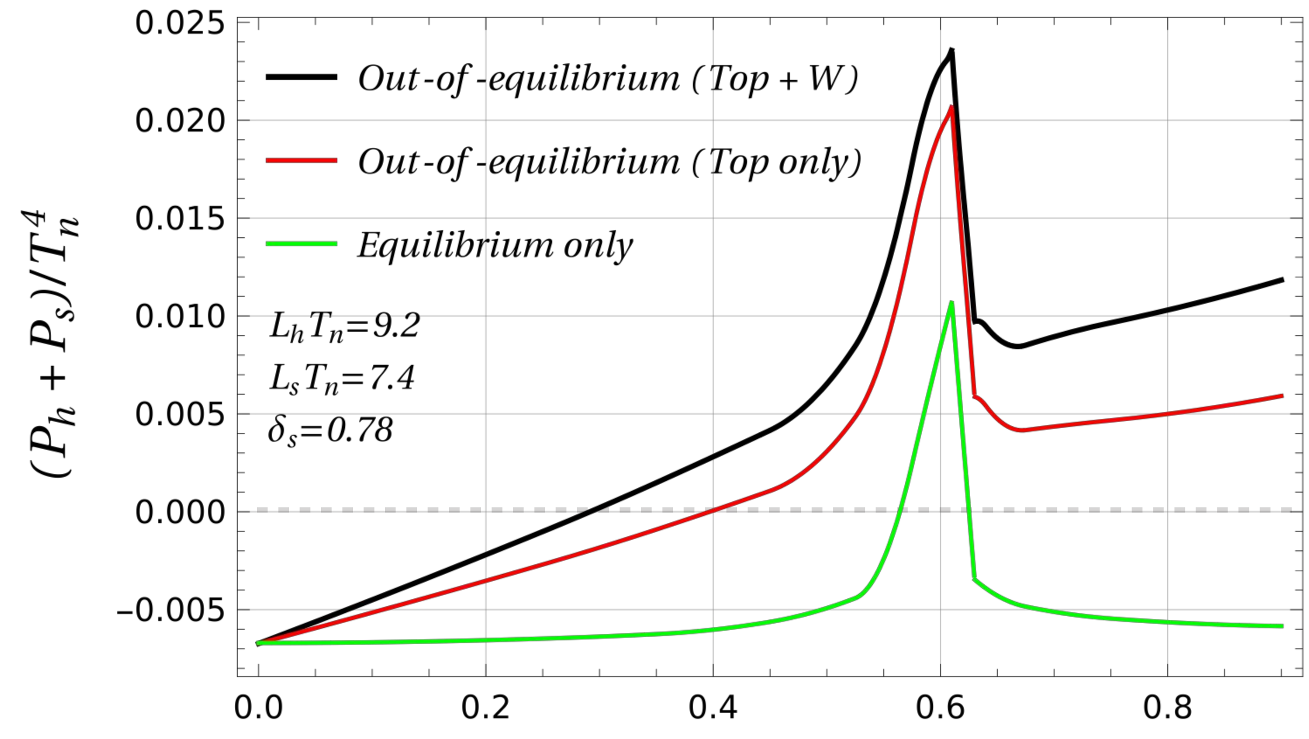
- *Fully quantitative solution without any ansatz*
- *Quantitative and qualitative differences with previous approaches*
- *Smooth behaviour across the sound barrier*

De Curtis, LDR, Guiggiani, Gil Muyor, Panico, 2022

Determination of the wall speed

Benchmark scenario: SM + singlet





	m_s (GeV)	λ_{hs}	λ_s	T_n (GeV)	T_c (GeV)	T_+ (GeV)	T_- (GeV)
BP1	103.8	0.72	1	129.9	132.5	130.1	129.9

	v_w	δ_s	$L_h T_n$	$L_s T_n$
BP1	0.28 [0.39] (0.57)	0.78 [0.79] (0.75)	9.2 [9.7] (8.1)	7.4 [7.7] (6.7)

- *Important corrections from out-of-equilibrium perturbations*
- *Sizeable corrections given by the W bosons*
- *Peak corresponding to the Jouguet velocity*

De Curtis, LDR, Guiggiani, Gil Muyor, Panico, 2024

Conclusions and outlook

Conclusions:

- Fully quantitative solution to the Boltzmann equation
- New spectral method based on multipole decomposition of the collision integral
- Computation of v_w
- Quantitative and qualitative differences with previous approaches
- Important impact of out-of-equilibrium friction

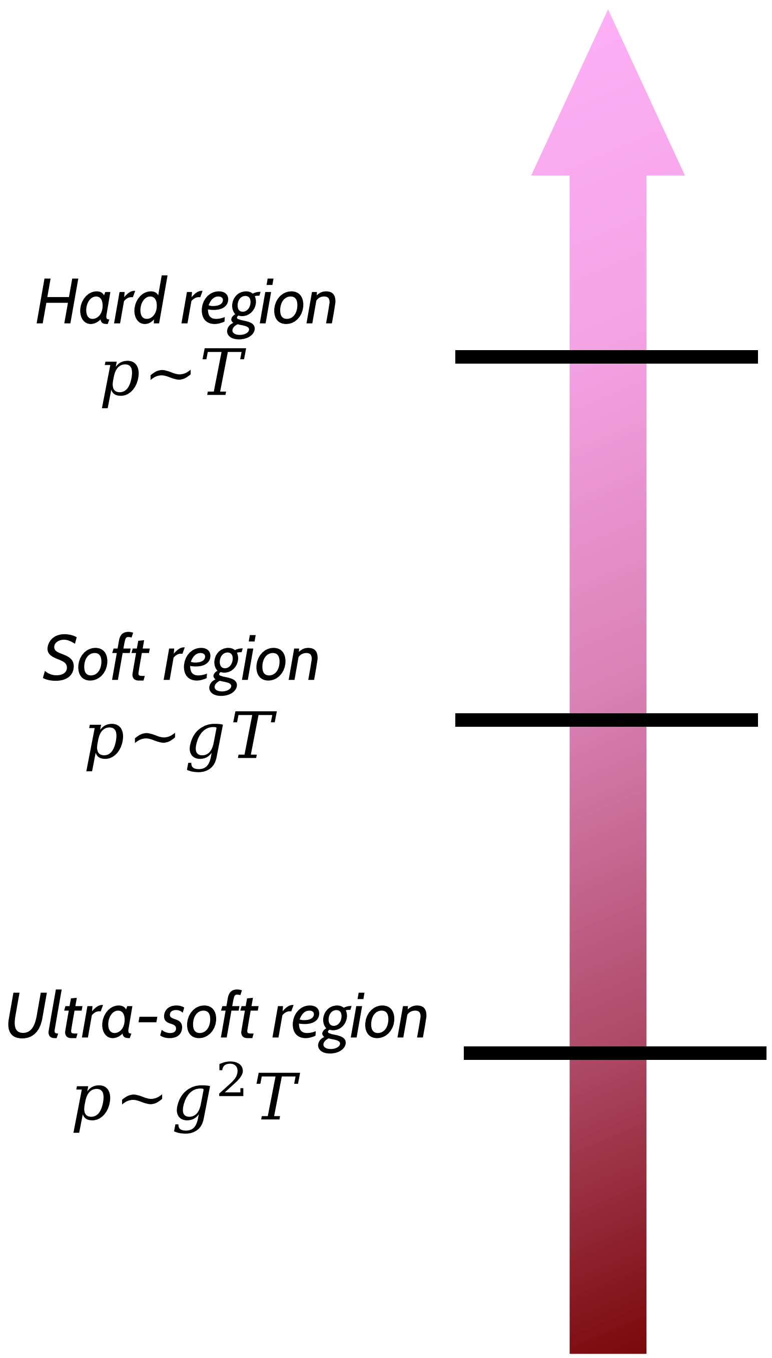
Work in progress:

- inclusion of $1 \to 2$ and $2 \to 1$ plasma processes in the collision integrals
- improving the description of W bosons in the ultra-soft regime

Backup slides

The **effective kinetic theory** depends on the **momentum** of the particles in the plasma

Hot plasma of **weakly** interacting particles (separation of scales)



- Point-like particles interacting through local collisions
- Screening and damping effects are important
- Quantum corrections are relevant