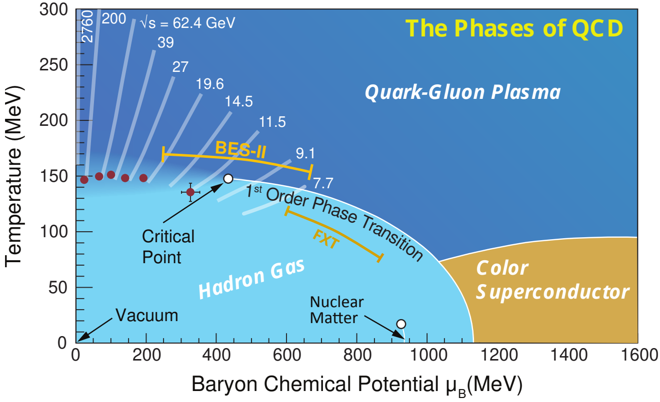
QCD critical point, fluctuations and hydrodynamics

M. Stephanov

QCD critical point

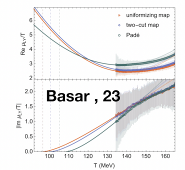
Where on the QCD phase boundary is the CP?



Motivation for BES at RHIC and BEST topical collaboration.

Latest theory developments on locating CP

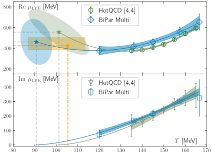
From Maneesha Pradeep's talk at CPOD 2024:



G. Basar, Fri, 11:40 am

Extrapolations of Lee-Yang edge singularities to real axis

$$(\mu_{BC}, T_c) \approx (580, 100)\ \mathrm{MeV}$$



C. Schmidt, Tue, 11:00 am

Clarke et al., 24

$$(\mu_{BC}, T_c) = (422^{+80}_{-35}, 105^{+8}_{-18})\ \mathrm{MeV}$$

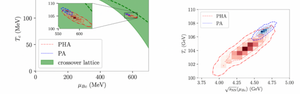
Bayesian Holography + Lattice input at $\mu = 0$

Hippert et al, e-Print: 2309.00579 [nucl-th]

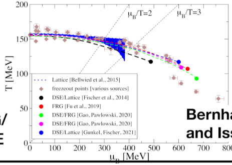
Predict CEP (95% confidence level):

$$T_c = 101 - 108\ \mathrm{MeV} \quad \mu_c = 560 - 625\ \mathrm{MeV}$$

$$\sqrt{s} = 4.0 - 4.8\ \mathrm{GeV}$$



J. Noronha, Tue, 11:40 am



C. Fischer, Tue, 11:20 am

FRG/ DSE

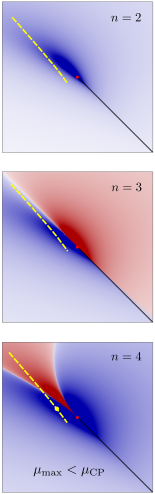
Bernhardt, Fischer and Isserstedt, 23

$$(\mu_{BC}, T_c) = (495 - 654, 108 - 119)\ \mathrm{MeV}$$

4

Theory vs BES-II data

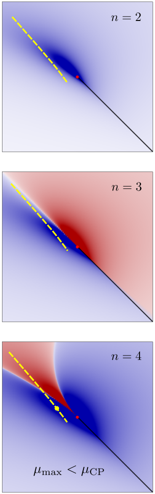
(universal EOS) critical χ_n:



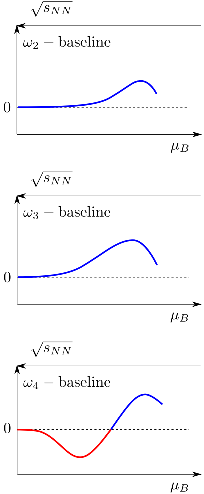
Bzdak et al review 1906.00936

Theory vs BES-II data

(universal EOS) critical χ_n:



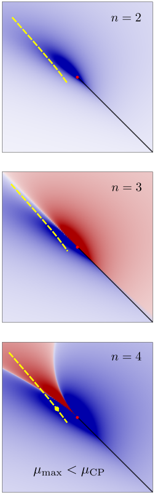
(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$



Bzdak et al review 1906.00936

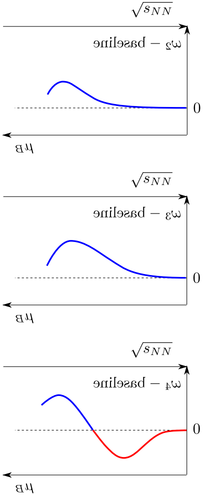
Theory vs BES-II data

(universal EOS) critical χ_n:



Bzdak et al review 1906.00936

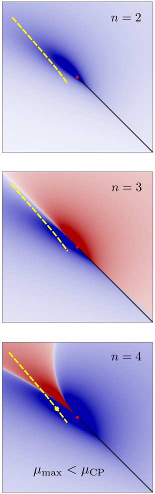
(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$



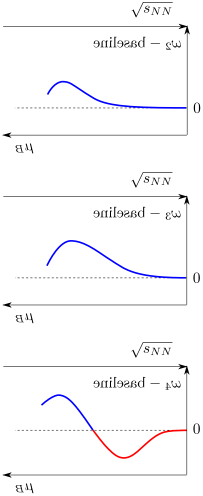
Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4

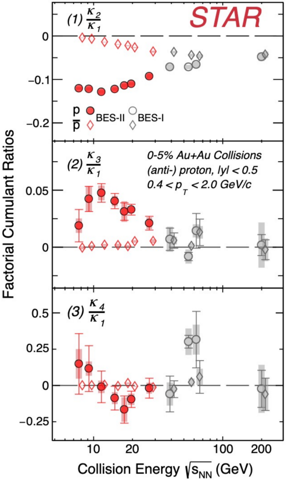
Theory vs BES-II data

(universal EOS) critical χ_n:



(irreducible correlations) $\mathrm{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \mathrm{FC}_n/\mathrm{FC}_1$





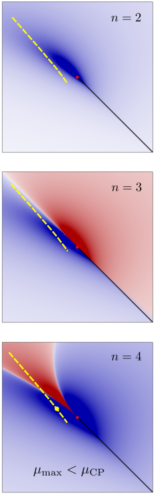
Bzdak et al review 1906.00936

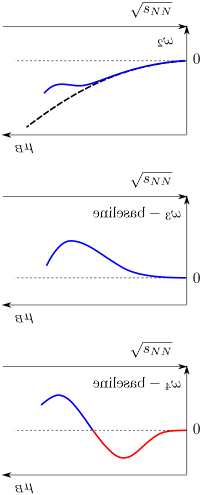
Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

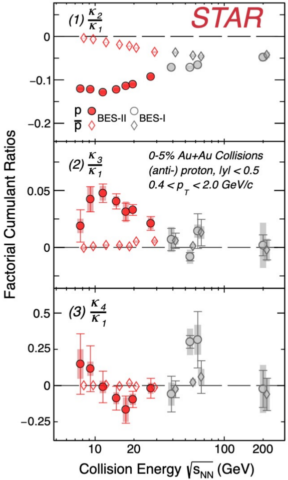
Theory vs BES-II data

(universal EOS) critical χ_n:

(irreducible correlations) FC$_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv$ FC$_n$/FC$_1$







Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

BEST Framework

The goal of BES theory: connect observables to QCD phase diagram.

BEST framework: An et al (40+ authors, 100+ pp, 369 refs) 2108.13867

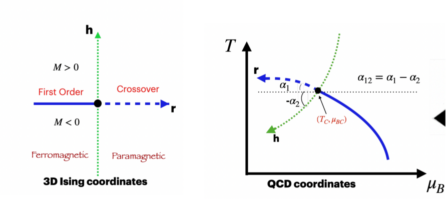
BES theory review: Du, Sorensen, MS 2402.10183

- Lattice EOS + CP $\rightarrow$ parametric EOS
- EOS $\rightarrow$ Hydrodynamics with (non-gaussian) fluctuations.
- Freezeout, including fluctuations. *reviewed in 2403.03255*
- Comparison with experiment. Bayesian analysis (ML). Determine/constrain EOS, critical point parameters.

Parametric EOS (now with T'-expansion)

From Maneesha Pradeep's review talk at CPOD 2024:

$$P_{\mathrm{QCD}}(\mu, T) = P_{\mathrm{BG}}(\mu, T) + A\, G(r(\mu, T), h(\mu, T))$$



Kahangirwe et al., 24

Independent & non-universal parameters

$$\mu_C\,,\ \alpha_{12}\,,\ \rho\,,\ w$$

Weakly constrained in the chiral limit

MP, Stephanov, 19

Range of Validity improved

$$0 \le \mu_B \le 700\,\mathrm{MeV}\,,\ 25\,\mathrm{MeV} \le T \le 800\,\mathrm{MeV}$$

The new construction is causal and stable for a larger range of ρ and w

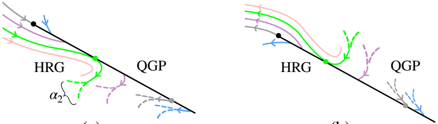
M. Kahangirwe, Wed, 12:10 pm

Parotto et al 1805.05249 PRC
Kahangirwe et al 2402.08636 PRD

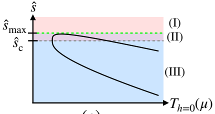
Critical point and non-trivial hydro trajectories

Pradeep, Sogabe, MS, Yee 2402.09519, PRC

- $\hat{s} \equiv s/n$ is non-monotonic along coexistence (1st order) line
- non-trivial deformation of trajectories



(a) (b)

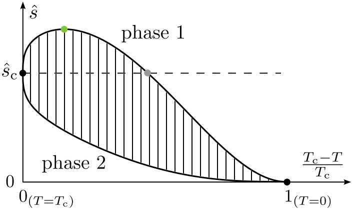


(c)

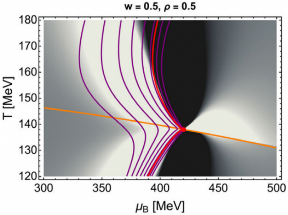
	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

depending on $(\partial P/\partial T)_n$ at CP



explains "lensing", "cusp"



Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

Deterministic approach to non-Gaussian fluctuations

non-Gaussian fluctuations are non-trivial and sensitive signatures of the critical point

- *Infinite* hierarchy of coupled equations *An et al 2009.10742 PRL*

  for connected hydro correlators $H_n \equiv \langle \underbrace{\delta\breve{\psi} \ldots \delta\breve{\psi}}_{n} \rangle^{\text{connected}}$:

$$\partial_t \psi = -\nabla \cdot \mathsf{Flux}[\psi, H, H_3, H_4, \ldots];$$

$$\partial_t H = \mathsf{F}[\psi, H, H_3, H_4, \ldots];$$

$$\partial_t H_3 = \mathsf{F}_3[\psi, H, H_3, H_4, \ldots];$$

$$\vdots$$

Controlled perturbation theory

An et al 2009.10742 PRL

- *Small* fluctuations are *almost* Gaussian
- Introduce expansion parameter ε, so that $\delta\breve{\psi} \sim \sqrt{\varepsilon}$.

  Then $H_n \equiv \varepsilon^{n-1}$ and to leading order in ε:

$$\partial_t \psi = -\nabla \cdot (\text{Flux}[\psi] + \mathcal{O}(\varepsilon));$$

$$\partial_t H = -2\Gamma(H - \bar{H}[\psi]) + \mathcal{O}(\varepsilon^2);$$

$$\vdots$$

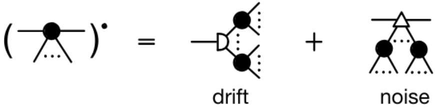
$$\partial_t H_n = -n\Gamma(H_n - \bar{H}_n[\psi, H, \dots, H^{n-1}]) + \mathcal{O}(\varepsilon^n);$$

  To leading order, the equations are iterative and "linear".
- In hydrodynamics the small parameter is $(q/\Lambda)^3$, i.e., fluctuation wavelength $1/q \gg$ size of hydro cell $1/\Lambda$ (UV cutoff).

Diagrammatic representation

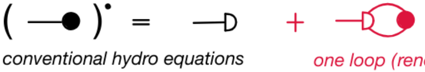
An et al 2009.10742, 2212.14029, An's talk at CPOD 2024

- Leading order in ε $\Leftrightarrow$ tree diagrams.



all *combinatorial* configurations of trees

- Loops describe feedback of fluctuations (renormalization and long-time tails).

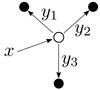


1-pt equation including leading loop

Generalizing Wigner transform

An et al 2009.10742 PRL

- Definition:

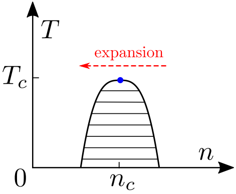


$$W_n(\boldsymbol{x}; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n) \equiv \int d\boldsymbol{y}_1^3 \ldots \int d\boldsymbol{y}_n^3 \, H_n\left(\boldsymbol{x} + \boldsymbol{y}_1, \ldots, \boldsymbol{x} + \boldsymbol{y}_n\right) \delta^{(3)}\left(\frac{\boldsymbol{y}_1 + \ldots + \boldsymbol{y}_n}{n}\right) e^{-i(\boldsymbol{q}_1 \cdot \boldsymbol{y}_1 + \ldots + \boldsymbol{q}_n \cdot \boldsymbol{y}_n)};$$

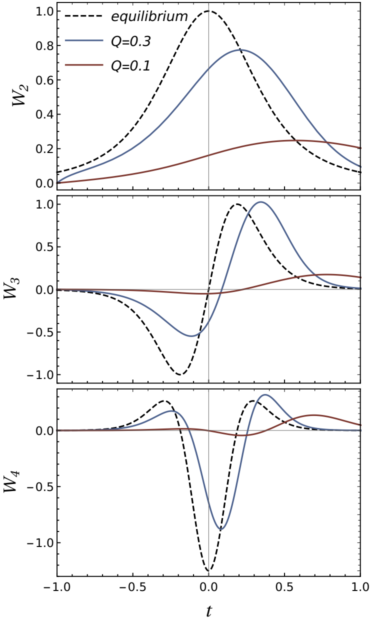
- W_n's quantify magnitude and non-gaussianity of fluctuation harmonics with wave-vectors $\boldsymbol{q}_i$.

Example: expansion through a critical region

An et al [2009.10742](), PRL



- Two main features:
  - Lag, ”memory”.
  - Smaller Q – slower evolution. Conservation laws.



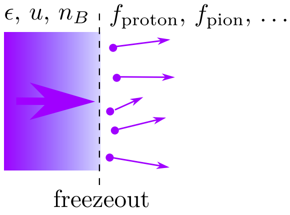
Freezeout of fluctuations

- Freezeout: translation of correlators of hydrodynamic fluctuations ($\psi = \epsilon, n_B, u$)

  $\langle \delta\psi \dots \delta\psi \rangle = H_n(\boldsymbol{x}_1, \dots, \boldsymbol{x}_n)$

  to particle correlators

  $\langle \delta f \dots \delta f \rangle = G_n(\boldsymbol{x}_1, \boldsymbol{p}_1, \dots, \boldsymbol{x}_n, \boldsymbol{p}_n)$.



- *Conservation laws* relate $\boldsymbol{p}$ integrals of G_n to H_n.

- But the $\boldsymbol{p}$ dependence in G_n is not constrained.

  There are ∞ many possibilities/solutions (G_n) satisfying conservation laws.

Maximum entropy freezeout

Pradeep, MS, 2211.09142, PRL

- There is a unique solution which maximizes the entropy!
  - for $n = 1$ equivalent to Cooper-Frye
  - for critical fluctuations similar to the σ field coupling
  - but applies much more generally
  - model independent, i.e., determined by QCD EOS

$$\underbrace{\hat{\Delta} G_{ABC}}_{\text{irreducible particle correlations (FC)}} = \underbrace{\hat{\Delta} H_{abc}}_{\text{hydrodynamic correlations}} \underbrace{(\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B (\bar{H}^{-1} P \bar{G})^c_C}_{\text{kinematic factors}}$$

- Work in progress – implement in a hydro model and estimate *nonequilibrium* expectations for multiplicity cumulants in BES

Karthein, Pradeep, MS, Rajagopal, Yin

Summary

- BES-II data is in.

  Qualitatively agrees with non-monotonic expectations from CP.

  Not only in $n = 4$ factorial cumulant, but in $n = 3$ and $n = 2$.

- To produce such signatures the CP has to be at $\mu_B > 420$ MeV.

  Agreement with recent theory estimates by different approaches.

- To convert these qualitative statements into quantitative ones, i.e., provide constraints on the QCD EOS from BES-II data more work is needed and is underway.

More

Factorial Cumulants are better experimental measures

Three reasons:

- Normal cumulants (NC) measure non-gaussianity;
  Factorial cumulants (FC) measure non-poissonianity,
  (irreducible particle correlations).

  NCs are for densities (continuous);
  FCs are for multiplicities (discrete).

Factorial Cumulants are better experimental measures

Three reasons:

- Normal cumulants (NC) measure non-gaussianity;
  Factorial cumulants (FC) measure non-poissonianity,
  (irreducible particle correlations).

  NCs are for densities (continuous);
  FCs are for multiplicities (discrete).

- Acceptance dependence:

  FCs are powers of Δy for small Δy; NCs are polynomials.

Factorial Cumulants are better experimental measures

Three reasons:

- Normal cumulants (NC) measure non-gaussianity;
  Factorial cumulants (FC) measure non-poissonianity, (irreducible particle correlations).

  NCs are for densities (continuous);
  FCs are for multiplicities (discrete).

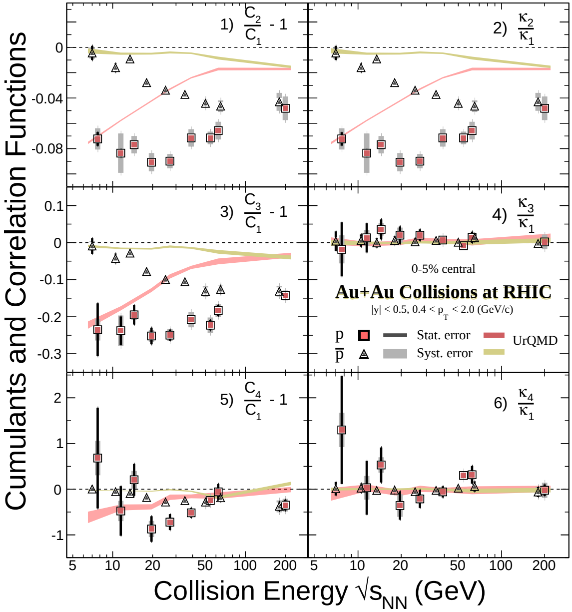
- Acceptance dependence:

  FCs are powers of Δy for small Δy; NCs are polynomials.

- Maximum Entropy freezeout *(Pradeep, MS 2211.09142)*:

  FCs of multiplicities are directly related to hydrodynamic correlators (or susceptibilities in thermodynamics).

BES-I data



Cumulants and Correlation Functions
1) $\frac{C_2}{C_1} - 1$
2) $\frac{\kappa_2}{\kappa_1}$
3) $\frac{C_3}{C_1} - 1$
4) $\frac{\kappa_3}{\kappa_1}$
5) $\frac{C_4}{C_1} - 1$
6) $\frac{\kappa_4}{\kappa_1}$
0-5% central
Au+Au Collisions at RHIC
|y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)
p
$\bar{p}$
Stat. error
Syst. error
UrQMD
Collision Energy $\sqrt{s_{NN}}$ (GeV)