中国科学院大学
University of Chinese Academy of Sciences

QCD@Work
International Workshop on QCD
Theory and Experiment

Spin alignment of vector mesons induced by spin density fluctuation

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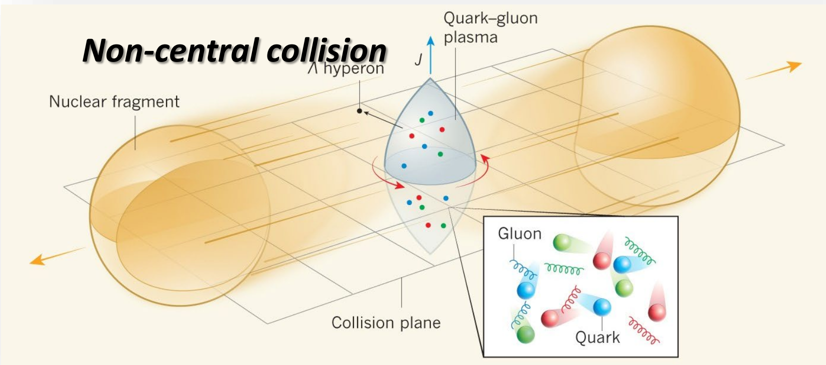
Kun Xu*, **Mei Huang,** paper in preparation…

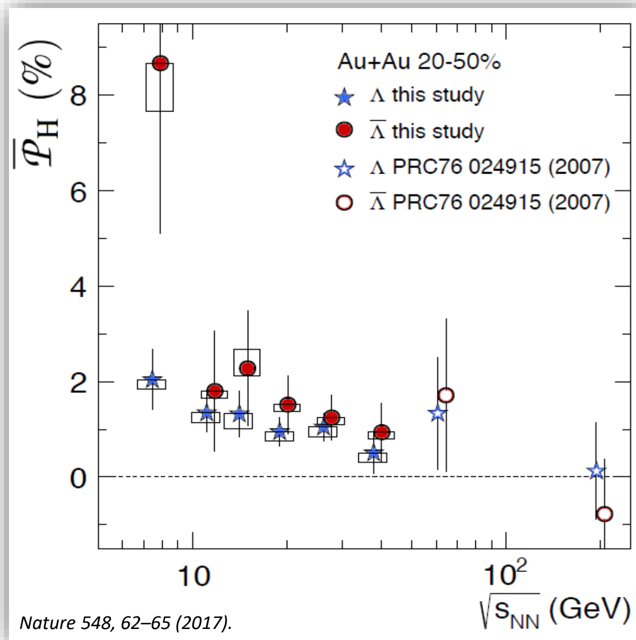


The fastest rotation: $\omega \sim 10^{21} s^{-1}$

The strongest magnetic field: $eB \sim 10^{8} G$

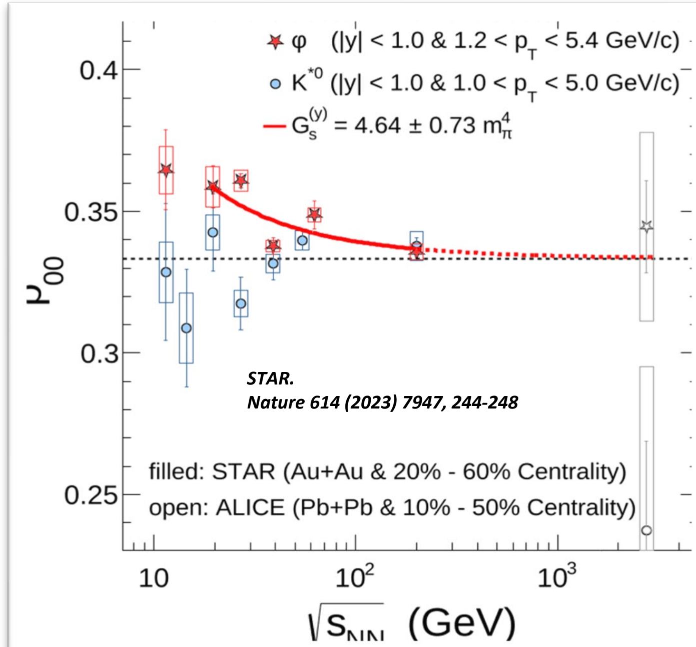
➔ Spin polarization of quarks

Spin polarization of Hyperon



Rotation &Magnetic field

Spin alignment of vector mesons



Rotation➔ $\rho_{00} < \frac{1}{3}$, Magnetic field ➔ $\rho_{00} > \frac{1}{3}$

Why spin alignment?

From Prof. Qun Wang's Talk

Lots of theoretical explanations

Possible contributions to ρ_{00}^{ϕ}

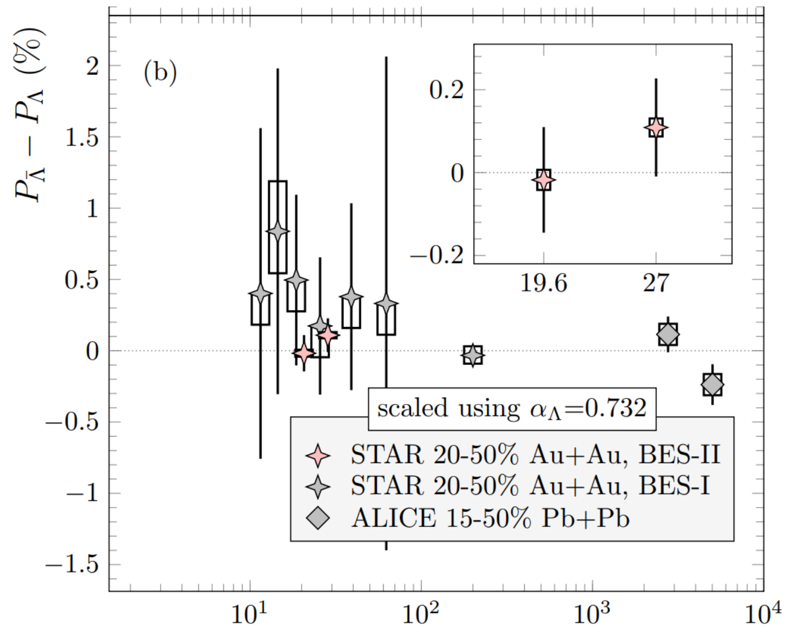
$$\rho_{00}^{\phi} = \frac{1}{3} + c_{\varepsilon} + c_{\omega} + c_E + c_B + c_F + c_A + c_L + c_{\phi}$$

- c_{ε}: E-part of vorticity tensor [1,2]
- c_{ω}: B-part of vorticity tensor [1,2]
- c_E: Electric field [1]
- c_B: Magnetic field [1,3]
- c_F: Frag. [4]
- c_A: Turbulent color field [5]
- c_L: Local+ Helicity [6,7]
- c_{ϕ}: ϕ field [1] our proposa

cannot explain large positive deviation from 1/3 (E-part of vorticity tensor through Local+Helicity)

[1] Sheng, Luica, QW (2019);
[2] Becattini, Csernai, Wang (2013);
[3] Yang, Fang, QW, Wang (2018);
[4] Liang, Wang (2005);
[5] Muller, Yang (2022);
[6] Xia, Li, Huang, Huang (2021);
[7] Gao (2021);
[8] Li, Liu (2022); Wanger, et. al. (2022)

Puzzle?



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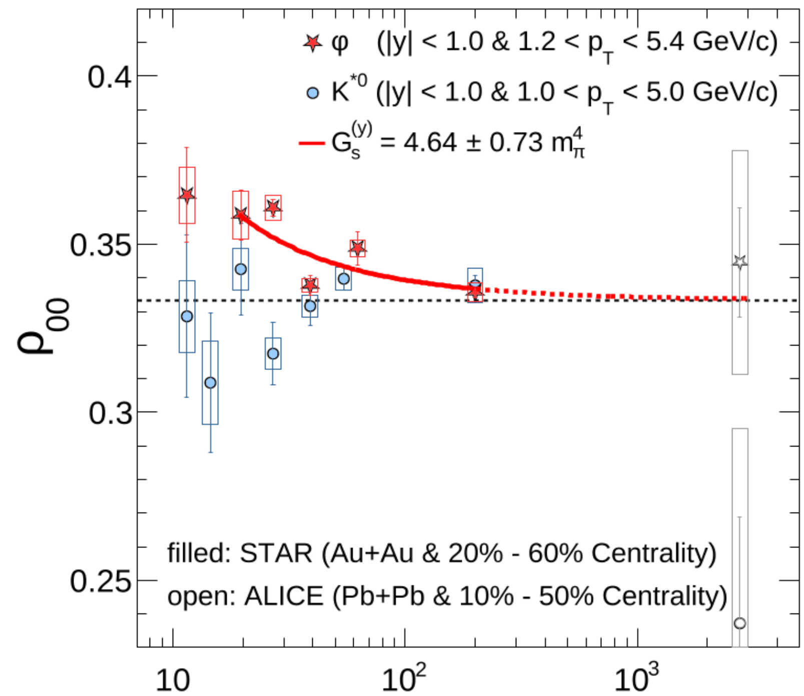
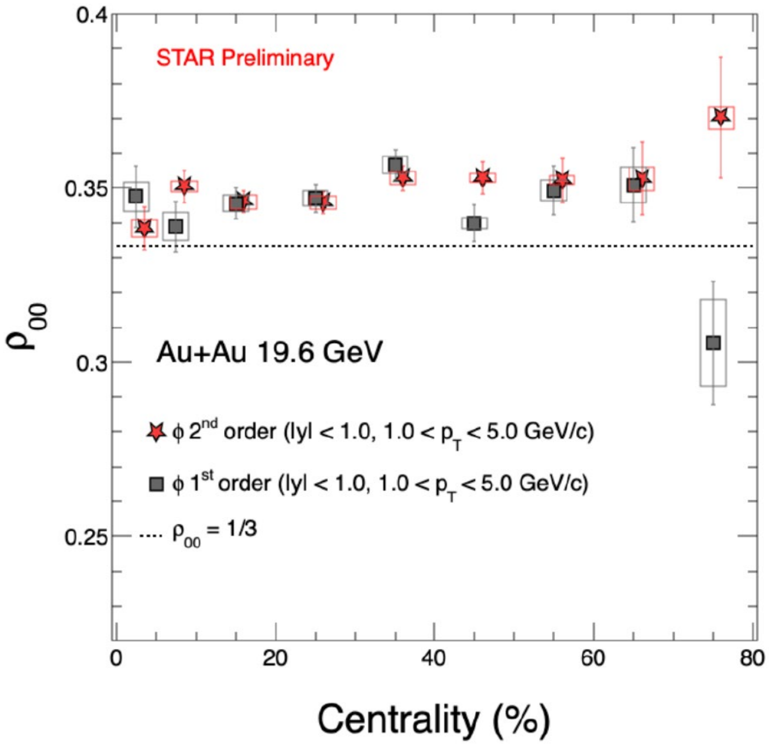


No splitting of P_Λ and $P_{\bar{\Lambda}}$ → No magnetic field

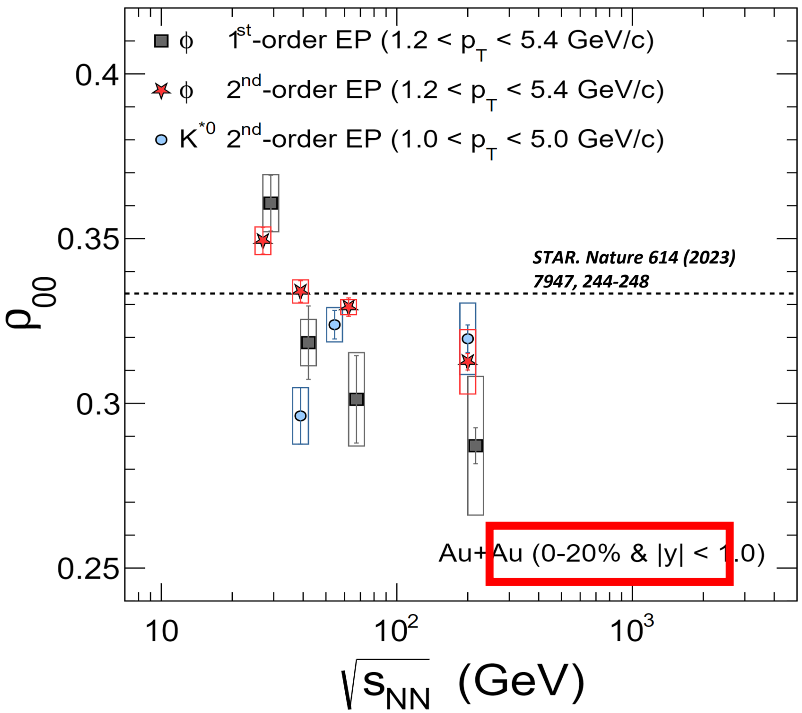
Magnetic field ➔ $\rho_{00} > \frac{1}{3}$

Magnetic field is not enough to induce such large ρ_{00}

Puzzle?



No obvious centrality-dependence



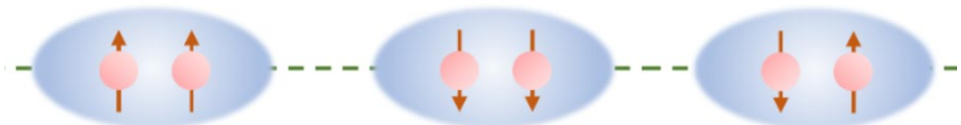
Most-central high energy collision:
No Rotation, **No** Magnetic field, **No** net baryon number,…

Can we obtain spin alignment without any rotation, magnetic field ……?

Spin alignment of vector meson ⟷ Local spin polarization of quarks

In a local region ,the possibilities to find ϕ of $s_z = 1,0,-1$:

$$P_1 = \frac{N_s^{\uparrow} N_{\bar{s}}^{\uparrow}}{N_s N_{\bar{s}}}, \quad P_{-1} = \frac{N_s^{\downarrow} N_{\bar{s}}^{\downarrow}}{N_s N_{\bar{s}}}, \quad P_0 = \frac{1}{2} \frac{N_s^{\uparrow} N_{\bar{s}}^{\downarrow} + N_s^{\downarrow} N_{\bar{s}}^{\uparrow}}{N_s N_{\bar{s}}}$$



Assume the small imbalance between spin-up and spin-down: $\delta N_s / N_s \ll 1$

$\delta N_s = N_s^{\uparrow} - N_s^{\downarrow}$ is the number difference between spin-up and spin-down

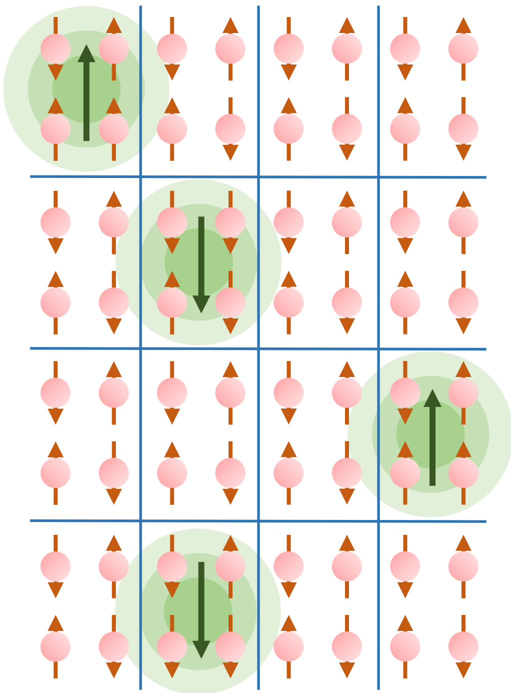
$N_s = N_s^{\uparrow} + N_s^{\downarrow}$ is the total number

Spin alignment of ϕ:
$$\rho_{00} = \frac{P_0}{P_1 + P_{-1} + P_0} \approx \frac{1}{3} - \frac{4}{9} \frac{\delta N_s}{N_s} \frac{\delta N_{\bar{s}}}{N_{\bar{s}}}$$

Average over all events or thermal states: $\langle\langle \delta N_s \delta N_{\bar{s}} \rangle\rangle$ Spin correlation

Similarly, spin alignment of K^{*0}:
$$\rho_{00} \approx \frac{1}{3} - \frac{4}{9} \frac{\delta N_d}{N_d} \frac{\delta N_{\bar{s}}}{N_{\bar{s}}}$$

Spin density fluctuation



Global polarization ×
Local polarization √

more spin-up than spin-down

$$\delta N_s > 0, \delta N_{\bar{s}} > 0 \Rightarrow \delta N_s \delta N_{\bar{s}} > 0$$

or more spin-down than spin-up

$$\delta N_s < 0, \delta N_{\bar{s}} < 0 \Rightarrow \delta N_s \delta N_{\bar{s}} > 0$$

Spin alignment of vector meson does not depend on the spin polarization direction of quark.

- the sign of $\delta N_s \delta N_{\bar{s}}$ doesn't change by the local spin fluctuation, thus $\langle\langle \delta N_s \delta N_{\bar{s}} \rangle\rangle$ survives after average.

Axial-Vector operator	Tensor operator
$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi$	$\bar{\psi}i\gamma^{1}\gamma^{2}\psi$
$\int d^3x\bar{\psi}(x)\gamma^{\mu}\gamma^{5}\psi(x)$ $\sim J_+^{\uparrow}-J_+^{\downarrow}+J_-^{\uparrow}-J_-^{\downarrow}\sim\delta J_s+\delta J_{\bar{s}}$	$\int d^3x\bar{\psi}(x)i\gamma^{1}\gamma^{2}\psi(x)$ $\sim M_+^{\uparrow}-M_+^{\downarrow}-(M_-^{\uparrow}-M_-^{\downarrow})\sim\delta M_s-\delta M_{\bar{s}}$
$J_+^{\uparrow}=\int\frac{d^3p}{(2\pi)^3}\frac{Ep_z^2+mp_\perp^2}{E\vec{p}^2}a_p^{\uparrow\dagger}a_p^{\uparrow}=cN_+^{\uparrow}$	$M_+^{\uparrow}=\int\frac{d^3p}{(2\pi)^3}\frac{Ep_\perp^2+mp_z^2}{E\vec{p}^2}a_p^{\uparrow\dagger}a_p^{\uparrow}=dN_+^{\uparrow}$
Spin Polarization(SP)	Magnetic Moment Polarization(MMP)

$$\ll\left(\int_x\bar{s}\gamma^3\gamma^5s\right)\left(\int_x\bar{s}\gamma^3\gamma^5s\right)\gg=\ll(\delta J_s+\delta J_{\bar{s}})(\delta J_s+\delta J_{\bar{s}})\gg=c^2(\ll\delta N_s^2\gg+\ll\delta N_{\bar{s}}^2\gg+2\ll\delta N_s\delta N_{\bar{s}}\gg)$$

$$\ll\left(\int_x\bar{s}i\gamma^1\gamma^2s\right)\left(\int_x\bar{s}i\gamma^1\gamma^2s\right)\gg=\ll(\delta M_s-\delta M_{\bar{s}})(\delta M_s-\delta M_{\bar{s}})\gg=d^2(\ll\delta N_s^2\gg+\ll\delta N_{\bar{s}}^2\gg-2\ll\delta N_s\delta N_{\bar{s}}\gg)$$

$$4\langle\langle\delta N_s\delta N_{\bar{s}}\rangle\rangle=\frac{1}{c^2}\langle\langle(\int_x\bar{s}\gamma^3\gamma^5s)^2\rangle\rangle-\frac{1}{d^2}\langle\langle(\int_x\bar{s}i\gamma^1\gamma^2s)^2\rangle\rangle$$

Local spin fluctuation induced by interactions

Lagrangian of NJL model with quark spin potential a and axial-vector interaction

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + a\gamma^3\gamma^5)\psi + G_A(\bar{\psi}\gamma^3\gamma^5\psi)^2$$

The grand potential: $$\Omega = -\frac{T}{V}\ln Z(a)$$

Consider second derivative: $$\mathcal{I} = \frac{\partial^2\Omega}{\partial a^2} = \mathcal{I}_1 + \mathcal{I}_2$$

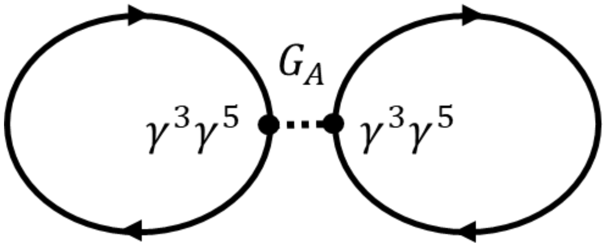
$$\mathcal{I}_1 = \frac{T}{V}\left(\int d^3x d\tau \langle\langle \bar{\psi}\gamma^3\gamma^5\psi \rangle\rangle\right)^2$$ **=0 for a→0**

$$\mathcal{I}_2 \;=\; -\frac{T}{V}\int d^3x d\tau d^3x' d\tau' \langle\langle (\bar{\psi}\gamma^3\gamma^5\psi)_{\tau,x}(\bar{\psi}\gamma^3\gamma^5\psi)_{\tau',x'} \rangle\rangle$$

≠0 for a→0

$$\Rightarrow \quad \langle\langle (\int_x \bar{s}\gamma^3\gamma^5 s)^2 \rangle\rangle = -VT\frac{\partial^2}{\partial a^2}\Omega(a)|_{a\to 0}$$

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m + a\gamma^3\gamma^5)\psi + G_A(\bar{\psi}\gamma^3\gamma^5\psi)^2$$



The grand potential, first-order expansion over G_A:

$$\Omega = -G_A N_c^2 \left(T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}[S(a)\gamma^3\gamma^5] \right)^2$$

Quark propagator:

$$S(a) = \frac{1}{\not{p} - m + a\gamma^3\gamma^5}$$

Together with $\langle\langle (\int_x \bar{s}\gamma^3\gamma^5 s)^2 \rangle\rangle = -VT \frac{\partial^2}{\partial a^2} \Omega(a)|_{a\to 0}$

$$\Rightarrow \quad \langle\langle (\int_x \bar{s}\gamma^3\gamma^5 s)^2 \rangle\rangle = 32 N_c^2 G_A V T L^2$$

where $L = \int \frac{d^3p}{(2\pi)^3} \frac{p_\perp^2}{2E^3} 2f(E)$

With similar process we can obtain $\frac{1}{d^2} \langle\langle (\int_x \bar{s}i\gamma^1\gamma^2 s)^2 \rangle\rangle$

The spin correlation can be obtained:

$$4\langle\langle \delta N_s \delta N_{\bar{s}} \rangle\rangle = \frac{1}{c^2}\langle\langle (\int_x \bar{s}\gamma^3\gamma^5 s)^2 \rangle\rangle - \frac{1}{d^2}\langle\langle (\int_x \bar{s} i\gamma^1\gamma^2 s)^2 \rangle\rangle$$

(1) Axial-vector interaction: $G_A(\bar{\psi}\gamma^3\gamma^5\psi)^2$

$$\ll \left(\int_x \bar{s}\gamma^3\gamma^5 s\right)\left(\int_x \bar{s}\gamma^3\gamma^5 s\right) \gg \neq 0 \qquad \ll \left(\int_x \bar{s}i\gamma^1\gamma^2 s\right)\left(\int_x \bar{s}i\gamma^1\gamma^2 s\right) \gg = 0$$

$$\delta\rho_{00}(\phi) \approx -\frac{4}{9}\frac{\langle\langle \delta N_s \delta N_{\bar{s}} \rangle\rangle}{N_s N_{\bar{s}}} = -\frac{1}{9c^2}\frac{\ll \left(\int_x \bar{s}\gamma^3\gamma^5 s\right)^2 \gg}{N_s N_{\bar{s}}} < 0$$

(2) Tensor interaction: $G_T(\bar{\psi}i\gamma^1\gamma^2\psi)^2$

$$\ll \left(\int_x \bar{s}\gamma^3\gamma^5 s\right)\left(\int_x \bar{s}\gamma^3\gamma^5 s\right) \gg = 0 \qquad \ll \left(\int_x \bar{s}i\gamma^1\gamma^2 s\right)\left(\int_x \bar{s}i\gamma^1\gamma^2 s\right) \gg \neq 0$$

$$\delta\rho_{00}(\phi) \approx -\frac{4}{9}\frac{\langle\langle \delta N_s \delta N_{\bar{s}} \rangle\rangle}{N_s N_{\bar{s}}} = \frac{1}{9d^2}\frac{\ll \left(\int_x \bar{s}i\gamma^1\gamma^2 s\right)^2 \gg}{N_s N_{\bar{s}}} > 0$$

Axial-vector interaction induce $\rho_{00} < \frac{1}{3}$ while tensor interaction induces $\rho_{00} > \frac{1}{3}$

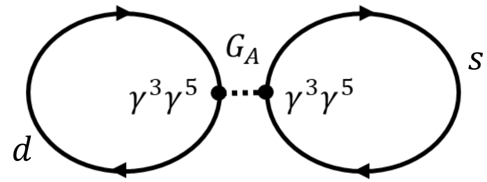
Spin alignment of K^{*0}: $$\rho_{00} \approx \frac{1}{3} - \frac{4}{9}\frac{\delta N_d}{N_d}\frac{\delta N_{\bar{s}}}{N_{\bar{s}}}$$

$$\langle\langle \delta N_d \delta N_{\bar{s}} \rangle\rangle = -VT \frac{\partial^2}{\partial a_d \partial a_s} \Omega(a)|_{a_d, a_s \to 0}$$

Spin potential matrix: $$\bar{\psi}\hat{a}\gamma^3\gamma^5\psi = a_d \bar{d}\gamma^3\gamma^5 d + a_s \bar{s}\gamma^3\gamma^5 s$$

where $$\hat{a} = \begin{pmatrix} a_d & 0 \\ 0 & a_s \end{pmatrix} \qquad \psi = (d, s)^T$$

Need flavor-mixing interaction: $$G_A^{mix}(\bar{s}\gamma^3\gamma^5 s)(\bar{d}\gamma_3\gamma^5 d)$$

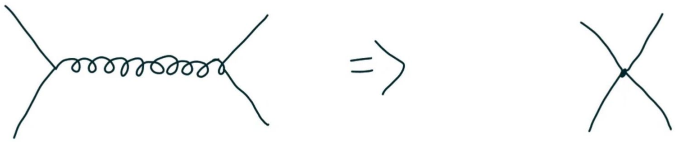


$$\langle\langle \delta N_d \delta N_{\bar{s}} \rangle\rangle = N_c^2 G_A^{mix} VTL(m_d)L(m_s)$$

Interaction induces spin alignment of K^{*0}meson

where is the interaction?

1. One-Gluon-Exchange(OGE)



$$J^\mu G_{\mu\nu} J^\nu \rightarrow G_V J^\mu g_{\mu\nu} J^\nu = G_V(\bar{\psi}\gamma^\mu\psi)^2$$

Gluon propagator with large gluon mass to an effective four-fermion contact interaction

Fierz transformation $\mathcal{F}(\gamma^\mu \times \gamma_\mu) = \mathbb{1} \times \mathbb{1} - \frac{1}{2}\gamma^\mu \times \gamma_\mu - \frac{1}{2}\gamma^\mu\gamma^5 \times \gamma_\mu\gamma^5 + \gamma^5 \times \gamma^5$

Axial-vector interaction appears!

2. Instanton-induced interaction

Phys.Rev.D 51 (1995) 1267-1281

Low temperature: Non-interacting $I\bar{I}$- Gas

$$\mathcal{L} = \int n(\rho)d\rho \frac{(2\pi\rho)^4}{8(N_c^2-1)}\left\{\frac{2N_c-1}{2N_c}[(\bar{\psi}\tau_a^-\psi)^2 + (\bar{\psi}\tau_a^-\gamma_5\psi)^2] - \frac{1}{4N_c}(\bar{\psi}\tau_a^-\sigma_{\mu\nu}\psi)^2\right\}$$

Tensor interaction

High temperature $T \gtrsim T_c$: $I\bar{I}$-Molecule

$$\mathcal{L}_{\mathrm{mol\,sym}} = G\left\{\frac{2}{N_c^2}[(\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}\tau^a\gamma_5\psi)^2] - \frac{1}{2N_c^2}[(\bar{\psi}\tau^a\gamma_\mu\psi)^2 + (\bar{\psi}\tau^a\gamma_\mu\gamma_5\psi)^2] + \frac{2}{N_c^2}(\bar{\psi}\gamma_\mu\gamma_5\psi)^2\right\} + \mathcal{L}_8$$

Axial-vector interaction

Flavor-mixed interaction: $(\bar{s}\gamma^\mu\gamma^5 s)(\bar{d}\gamma_\mu\gamma^5 d)$

Contributes to the spin alignment of K^{*0}

Summary

- Spin density fluctuation of quark can induce spin alignment of vector meson
- Axial-vector interaction induces $\rho_{00} < 1/3$, tensor interaction induces $\rho_{00} > 1/3$
- Instanton-induced interactions could be the source!

Thank you

BACKUP

Numerical example

Coupling constant as a function of chemical potential

$$G_A = \frac{1}{2} G_s - 8\mu_B$$
$$G_T = 40\mu_B$$

radius $r = 2fm$

Freeze-out line:

$$\mu_B = \frac{1.477}{1 + 0.343\sqrt{s}}$$

$$T = 0.158 - 0.14\mu_B^2 - 0.04\mu_B^4$$