Parity-odd CFT and Quantum Anomalies

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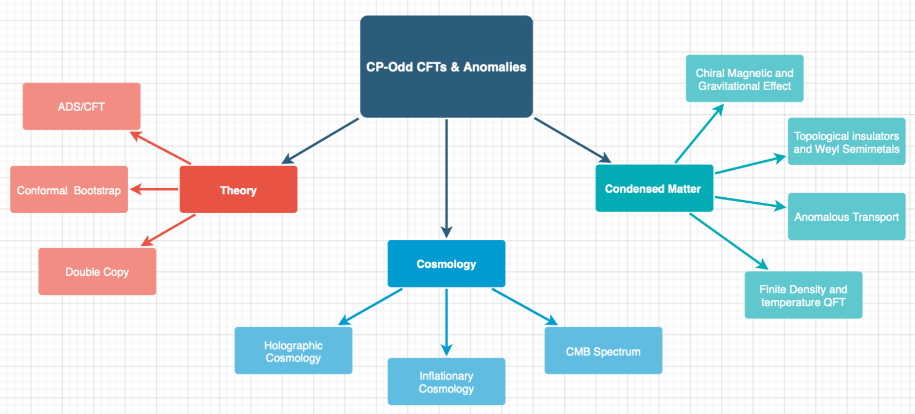
Outline of the presentation

- Introduction to CFTs and quantum anomalies
- Methodology: constraining correlators in momentum-space CFTs
- Results:
  - Chiral anomaly: the $\langle JJJ_5 \rangle$ and $\langle J_5 J_5 J_5 \rangle$ correlators
  - Chiral gravitational anomaly: the $\langle TTJ_5 \rangle$ correlator
  - Conformal anomaly: the $\langle TJJ \rangle_{odd}$ and $\langle TTT \rangle_{odd}$ correlators
- Conclusions

INTRODUCTION

Motivations and Applications



Conformal Field Theories

Conformal transformations consist of

(Translations)	$P_\mu = \partial_\mu,$
(Lorentz transformations)	$L_{\mu\nu} = x_\nu \partial_\mu - x_\mu \partial_\nu,$
(Dilations)	$D = x^\mu \partial_\mu,$
(Special conformal transformations)	$K_\mu = 2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu.$

Conformal invariance imposes strong constraints on correlations functions. Two-point and three-point correlation functions are completely determined up to a few constants

$$\langle \mathcal{O}_1 (x_1)\ \mathcal{O}_2 (x_2)\, \mathcal{O}_3 (x_3) \rangle = \frac{C_{123}}{(x_1 - x_2)^{\Delta_1 + \Delta_2 - \Delta_3} (x_2 - x_3)^{\Delta_2 + \Delta_3 - \Delta_1} (x_1 - x_3)^{\Delta_1 + \Delta_3 - \Delta_2}}$$

Momentum-space CFTs

Why do we study CFTs in momentum spaces?

- Most natural setting from the perspective of perturbative field theory
- The coordinate space analysis is not valid for correlator computed at the same points. The momentum space analysis doesn't have such problems. It allows us to understand the origin of quantum anomalies which occur in coordinate space at short distances.
- Applications to cosmology, condensed matter physics, holography and the conformal bootstrap program.

Quantum Anomalies

Chiral anomaly

$$\nabla_\mu J_5^\mu = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R^{\alpha}_{\beta\mu\nu} R^{\beta}_{\alpha\rho\sigma}$$

Conformal anomaly

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b_1\, E_4 + b_2\, C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + b_3\, \nabla^2 R + b_4\, F^{\mu\nu} F_{\mu\nu} + f_1\, \varepsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} + f_2\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}.$$

We can see the effect of the anomalies when looking at the longitudinal or trace part of the correlators. Taking the example of the $\langle AVV\rangle$ correlator, we have

$$p_{1\mu_1}\, \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3)\rangle = 0,$$
$$p_{2\mu_2}\, \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3)\rangle = 0$$
$$p_{3\mu_3}\, \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3)\rangle = -8\, a_1\, i\, \varepsilon^{p_1 p_2 \mu_1 \mu_2}$$

METHODOLOGY

Decomposition of the correlator

We decompose the correlator into a longitudinal and transverse-traceless part using the projector

$$\pi^{\mu}_{\alpha} = \delta^{\mu}_{\alpha} - \frac{p^{\mu} p_{\alpha}}{p^2}, \qquad \Pi^{\mu\nu}_{\alpha\beta} = \pi^{(\mu}_{\alpha}\pi^{\nu)}_{\beta} - \frac{1}{d-1}\pi^{\mu\nu}\pi_{\alpha\beta}$$

We can take the example of the $\langle AVV \rangle$ correlator. A current can be decomposed in the following way

$$J^{\mu}(p) = \frac{p^{\mu}}{p^2}\, p_{\alpha} J^{\alpha}(p) + \pi^{\mu}_{\alpha}(p)\, J^{\alpha}(p)$$

Therefore, we can write the correlator as

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3)\rangle = \\ -8\, i\, a_1\, \frac{p_{3\mu_3}}{p_3^2}\varepsilon^{p_1 p_2 \mu_1 \mu_2} + \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\, X^{\alpha_1\alpha_2\alpha_3}$$

Conformal Equations in Momentum Space

Dilatations are linear transformations $\delta x_\mu = \lambda x_\mu$ and lead to a first-order differential equation in momentum space

$$\left[\sum_{j=1}^{n} \Delta_j - (n-1)d - \sum_{j=1}^{n-1} p_j^\alpha \frac{\partial}{\partial p_j^\alpha} \right] \langle \ldots \rangle = 0$$

Special conformal transformations are non-linear $\delta x_\mu = b_\mu x^2 - 2x_\mu b \cdot x$ and lead to a second-order differential equation in momentum space

$$0 = \mathcal{K}^\kappa \langle \ldots \rangle$$

where

$$\mathcal{K}^\kappa = \sum_{j=1}^{n-1} \left(2\left(\Delta_j - d\right) \frac{\partial}{\partial p_j^\kappa} - 2 p_j^\alpha \frac{\partial}{\partial p_j^\alpha} \frac{\partial}{\partial p_j^\kappa} + \left(p_j\right)_\kappa \frac{\partial}{\partial p_j^\alpha} \frac{\partial}{\partial p_{j\alpha}} \right) + \mathcal{K}^\kappa_{spin}$$

The solution of the CWIs can be written in terms of the 3K integrals

$$I_{\alpha\{\beta_1\beta_2\beta_3\}}\left(p_1, p_2, p_3\right) = \int dx x^\alpha \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}\left(p_j x\right)$$

where K_ν is a modified Bessel function of the second kind

$$K_\nu(x) = \frac{\pi}{2} \frac{I_{-\nu}(x) - I_\nu(x)}{\sin(\nu\pi)}, \qquad I_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(\nu+1+k)} \left(\frac{x}{2}\right)^{2k}, \qquad \nu \notin \mathbb{Z}$$

RESULTS

The Chiral anomaly and the $\langle JJJ_5 \rangle$

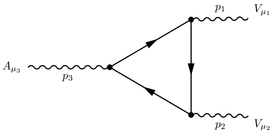
The chiral anomaly is given by

$$\nabla_\mu J_5^\mu = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R^{\alpha}_{\beta\mu\nu} R^{\beta}_{\alpha\rho\sigma}$$

The general structure of the $\langle AVV \rangle$ correlator in a CFT is

$$\langle J^{\mu_1}(p_1)\, J^{\mu_2}(p_2)\, J_5^{\mu_3}(p_3) \rangle = -8\, i\, a_1\, \frac{p_{3\mu_3}}{p_3^2} \varepsilon^{p_1 p_2 \mu_1 \mu_2} +$$
$$\pi^{\mu_1}_{\alpha_1}(p_1)\, \pi^{\mu_2}_{\alpha_2}(p_2)\, \pi^{\mu_3}_{\alpha_3}(p_3)\; 8\, i\, a_1 \left[I_{3\{1,0,1\}}\, p_2^2\, \varepsilon^{p_1 \alpha_1 \alpha_2 \alpha_3} - I_{3\{0,1,1\}}\, p_1^2\, \varepsilon^{p_2 \alpha_1 \alpha_2 \alpha_3} \right]$$

This solution is in accordings with the perturbative results



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The gravitational chiral anomaly and the $\langle TTJ_5 \rangle$

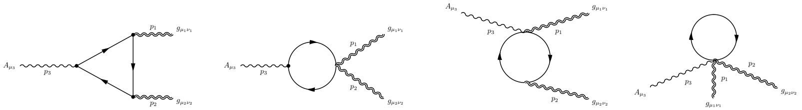
The chiral anomaly is given by

$$\nabla_\mu J_5^\mu = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\sigma}$$

The general structure of the $\langle TTJ_5 \rangle$ correlator in a CFT is

$$\langle T^{\mu_1\nu_1} T^{\mu_2\nu_2} J_5^{\mu_3} \rangle =$$
$$4ia_2 \frac{p_3^{\mu_3}}{p_3^2} (p_1 \cdot p_2) \left\{ \left[\varepsilon^{\nu_1\nu_2 p_1 p_2} \left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2} p_2^{\mu_1}}{p_1 \cdot p_2} \right) + (\mu_1 \leftrightarrow \nu_1) \right] + (\mu_2 \leftrightarrow \nu_2) \right\}$$
$$+ \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\, \Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\, \pi^{\mu_3}_{\alpha_3}(p_3)\, a_2 \left[\dots \right]$$

This solution is in accordings with the perturbative results



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The conformal anomaly and the $\langle TJJ \rangle_{odd}$

The conformal anomaly is given by

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b_1\, E_4 + b_2\, C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + b_3\, \nabla^2 R + b_4\, F^{\mu\nu} F_{\mu\nu}$$
$$+ f_1\, \varepsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} + f_2\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

The general structure of the $\langle JJT \rangle_{odd}$ correlator in a CFT is

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) T^{\mu_3\nu_3}(p_3)\rangle_{odd} = \frac{8}{3p_3^2} f_2 \left(p_3^2\, \delta^{\mu_3\nu_3} - p_3^{\mu_3} p_3^{\nu_3}\right) \varepsilon^{p_1 p_2 \mu_1 \mu_2}$$

Only the anomalous trace term is present in the correlator.

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What about the term in red? We need to analyze the $\langle TTT \rangle_{odd}$!

Conclusions and Outlook

Results:

- We have determined the structure of parity odd correlators that are affected by the chiral and conformal anomaly in a CFT. They are completely fixed!
- We have computed the correlators pertubatively and verified the agreement with the conformal results.

What about the future?

- We are currently investigating the $\langle TTT \rangle$ correlator
- We can also analyze odd-parity correlators in different scenarios: $d \neq 4$, non abelian fields, four-point correlators ecc.
- CFT correlators are very usufull tools for cosmological holography, condensed matter theory ecc. There are lot's of applications we can investigate!

Thank you for the attention