南華大學
UNIVERSITY OF SOUTH CHINA

# A holographic approach to heavy non $q\bar{q}$-states using configurational entropy

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Miguel Angel Martin Contreras
University of South China
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# Outline

# Hadrons in AdS/QCD

# Hadrons in holography

## From QCD:

- Hadrons are bounded states of quarks and gluons.
- Hadrons are direct evidence of confinement.
- Hadron masses are organized in taxonomic structures called *Regge trajectories.*
- Hadronic spectroscopy is non-perturbative.
- Non-holo. Approaches to hadron spectroscopy:
  - Relativistic and non-relativistic potential models.
  - Bethe-Salpeter equation.
  - Light-cone QCD.
  - Lattice QCD.

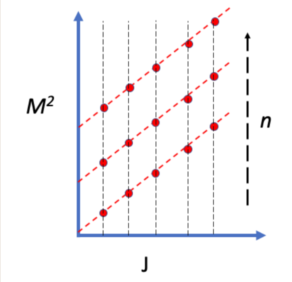



Figure: Linear Regge Trajectories

$$M_n^2 = a\,(n + J) + c$$

See Afonin 2006 and Masjuan et al., 2012.

# Hadrons in holography

## In AdS/CFT:

- Gauge/gravity duality: hadrons are non-perturbative boundary objects dual to bulk fields.

$$\mathcal{O}\;|0\rangle\;=\alpha\;|p\rangle$$

with operator creating hadrons $\mathcal{O}$ defined as

$$\begin{aligned}\mathcal{O} &= f(q,\bar{q},G_{\mu\nu},D_\mu)\\ \mathbf{dim}\,\mathcal{O} &= \Delta+L+\gamma\end{aligned}$$

$L=$ Orb. Ang. momentum, $\gamma=$ anom. Dim.

- Field Operator duality ($L=\gamma=0$):

$$\mathbf{dim}\,\mathcal{O}\Leftrightarrow\dim\psi(z,q)\equiv\Delta$$

Bulk field: $p$-form in $AdS_5$
$A_p(z,q)=A_p(q)\,\psi(z,q)$ with mass $M_5$.

$$\psi(z,q)|_{z\to 0}=\mathcal{C}\,z^{\Delta-p}$$

See Polchinski 2002.

- Hadronic identity: is defined by the bulk field mass $M_5=M_5(\Delta,L,\gamma)$.
- In this perspective, a hadron is *a bag of constituents* characterized by $M_5(\Delta)$.
- Hadrons in AdS/QCD are normalizable modes labeled by their bulk mass.

# Bottom-up approach: Static Non-quadratic dilaton

# Bottom-up semiclassical approach to hadron spectroscopy

## Bottom-up technology

- Bulk fields define a holographic potential.
- Example: $p-$form bulk fields

$$V(z) = \frac{1}{4} B'(z)^2 - \frac{1}{2} B''(z) + \frac{M_5^2 R^2}{z^2},$$

  with $B(z) = \Phi(z) + \beta \log \frac{R}{z}$, and $\beta = -(3 - 2p)$.
- Schrodinger-like eigenvalue eqn: $-u'' + V(z)u = M_n^2 u$.
- Regge Trajectories: $M_n^2$.
- Localized bulk modes $u(z)$ dual to hadron bndry. modes.

## Motivation

- Quadratic dilaton (Karch et al. 2005): $\Phi(z) = \kappa^2 z^2$ for light unflavored mesons.
  - linear Regge Trajectories.
  - associated with massless constituent quarks.
  - Not good for heavy mesons.
- Non-quadratic dilaton (M.A.M.C. & A. Vega, 2020): $\Phi(z) = (\kappa z)^{2-\alpha}$.
  - Describes non-linear RT: $M_n^2 = a(n+b)^\nu$.
  - From Bethe-Salpeter (J. K. Chen, 2018): Heavy quark mass breaks linearity, i.e., $M_n^2 \sim n^{2/3}$.
  - $(\kappa, \alpha)$ defines a *calibration curve* for heavy quark systems.

# Non-quadratic dilaton for isovector mesons

- Bulk action (see M.A.M.C. and A. Vega, 2020)

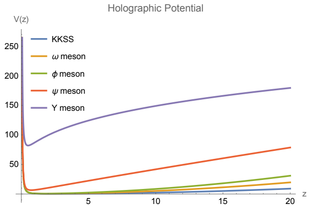
$$I_{\rm H} = -\frac{1}{2}\int d^5x\,\sqrt{-g}\,e^{-\phi(z)}\,\left(\frac{1}{2\,g_5^2}\,F_{mn}F^{mn} - M_5^2\,A_m\,A^m\right),$$

  Living in $\text{AdS}_5$, with the gauge fixing: $A_z = 0$ and $\Phi(z) = (\kappa\,z)^{2-\alpha}$.
- Bulk mass

$$M_5^2\,R^2 = (\Delta + L - 1)\,(\Delta_0 + L - 3)\,.$$

- Holographic potential for $s$-wave vector mesons ($M_5^2\,R^2 = 0$):

$$V_{q\,\bar{q}}(z,\kappa,\alpha) = \frac{3}{4z^2} - \frac{1}{2}\alpha^2\,\kappa^2\,(\kappa\,z)^{-\alpha} + \frac{1}{4}\alpha^2\,\kappa^2(\kappa\,z)^{2-2\alpha}$$
$$+\,\frac{3}{2}\alpha\,\kappa^2\,(\kappa\,z)^{-\alpha} - \kappa^2(\kappa\,z)^{-\alpha} - \alpha\,\kappa^2\,(\kappa\,z)^{2-2\alpha}$$
$$+\,\kappa^2\,(\kappa\,z)^{2-2\alpha} + \frac{\kappa}{z}(\kappa\,z)^{1-\alpha} - \frac{\alpha\,\kappa}{2\,z}(\kappa\,z)^{1-\alpha}.$$



# Non-quadratic dilaton for isovector mesons

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TABLE III. Summary of results for different families of isovector radial mesonic states considered in this work. All of the mass spectra displayed in this table are calculated with the parameters mentioned on each subtable header, using (15). The Regge trajectories are also presented in units of $\mathrm{GeV}^2$. The last column on each set of data is the relative error per state. Experimental results are read from PDG [30].

| $\omega$ with $\alpha = 0.04$ and $\kappa = 498$ MeV | | | | $\phi$ with $\alpha = 0.07$ and $\kappa = 585$ MeV | | | |
|---|---|---|---|---|---|---|---|
| n | $M_{\mathrm{Exp}}$ (MeV) | $M_{\mathrm{Th}}$ (MeV) | R. E. (%) | n | $M_{\mathrm{Exp}}$ (MeV) | $M_{\mathrm{Th}}$ (MeV) | R. E. (%) |
| 1 | $782.65 \pm 0.12$ | 981.43 | 25.4 | 1 | $1019.461 \pm 0.016$ | 1139.43 | 11.8 |
| 2 | 1400–1450 | 1374 | 3.6 | 2 | $1698 \pm 20$ | 1583 | 5.8 |
| 3 | $1670 \pm 30$ | 1674 | 0.25 | 3 | $2135 \pm 8 \pm 9$ | 1921 | 10 |
| 4 | $1960 \pm 25$ | 1967 | 1.7 | 4 | Not Seen | ... | ... |
| 5 | $2290 \pm 20$ | 2149 | 6.2 | 5 | Not Seen | ... | ... |
| $M^2 = 0.9514(0.012 + n)^{0.9798}$ with $R^2 = 0.999$ | | | | $M^2 = 1.268(0.0244 + n)^{0.9650}$ with $R^2 = 0.999$ | | | |

| $\psi$ with $\alpha = 0.54$ and $\kappa = 2150$ MeV | | | | $\Upsilon$ with $\alpha = 0.863$ and $\kappa = 11209$ MeV | | | |
|---|---|---|---|---|---|---|---|
| n | $M_{\mathrm{Exp}}$ (MeV) | $M_{\mathrm{Th}}$ (MeV) | R. E. (%) | n | $M_{\mathrm{Exp}}$ (MeV) | $M_{\mathrm{Th}}$ (MeV) | R. E. (%) |
| 1 | $3096.916 \pm 0.011$ | 3077.09 | 0.61 | 1 | $9460.3 \pm 0.26$ | 9438.5 | 0.23 |
| 2 | $3686.109 \pm 0.012$ | 3689.62 | 0.1 | 2 | $10023.26 \pm 0.32$ | 9923.32 | 0.78 |
| 3 | $4039 \pm 1$ | 4137.5 | 2.44 | 3 | $10355 \pm 0.5$ | 10277.2 | 0.75 |
| 4 | $4421 \pm 4$ | 4499.4 | 1.77 | 4 | $10579.4 \pm 1.2$ | 10558.6 | 0.19 |
| 5 | Not Seen | ... | ... | 5 | $10889.9^{+3.2}_{-2.6}$ | 10793.5 | 0.88 |
| 6 | Not Seen | ... | ... | 6 | $10992.9^{+10.0}_{-3.1}$ | 10995.7 | 0.03 |
| $M^2 = 8.07(0.287 + n)^{0.6315}$ with $R^2 = 0.999$ | | | | $M^2 = 76.511(0.901 + n)^{0.2369}$ with $R^2 = 0.999$ | | | |

# Non-$q\,\bar{q}$ states

# Non-quadratic dilaton for non-$q\bar{q}$ states

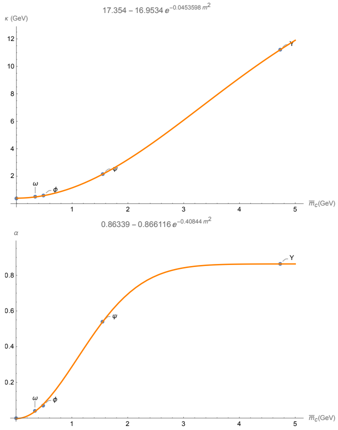



Figure: $(\kappa,\ \alpha)$ calibration curves in terms of $\bar{m}$.

$(\kappa,\ \alpha)$ define a *depend on* the average constituent mass $\bar{m}$, with

$$\bar{m} = \sum_{i=1}^{N} (P_i^{\text{quark}}\, \bar{m}_{q_i} + P_i^{\text{Gluon}}\, m_{G_i} + P_i^{\text{meson}}\, m_{\text{meson}_i})$$

Constraint condition

$$\sum_{i=1}^{N} (P_i^{\text{quark}} + P_i^{\text{Gluon}} + P_i^{\text{meson}}) = 1.$$

Using a proper choice for $\bar{m}$ sets $(\kappa,\ \alpha)$ univocally.

# Parametrizations for vector non-$q\bar{q}$ candidates

$\bar{m}$ motivated by non-holographic phenomenology (see N. Brambilla et al, 2019 and F. K. Guo et al, 2018)

- Diquark: $\bar{m}_{Dq,Z_c} = m_Q$.
- Hadroquarkonium: $\bar{m}_{HQc} = \frac{1}{2} m_{J/\psi} + \frac{1}{4}\left(\bar{m}_u + \bar{m}_d\right)$
- Hadronic Molecule:

$$
\begin{aligned}
\bar{m}_{HM,\,\psi} &= \frac{1}{3} m_{J/\psi} + \frac{2}{3} m_\rho, \\
\bar{m}_{HM,\,Z_c} &= 0.283\, m_{J/\psi} + 0.717\, m_\rho, \\
\bar{m}_{HM,\,Z_B} &= 0.458\, m_{\Upsilon(1S)} + 0.542\, m_\rho
\end{aligned}
$$

- Gluonic excitation:

$$
\begin{aligned}
\bar{m}_{Z_c} &= 0.98\, m_c + 0.2\, m_G = 1.533\,\text{GeV} \\
\bar{m}_{Z_b} &= 0.99\, m_b + 0.01\, m_G = 4.6897\,\text{GeV}
\end{aligned}
$$

Constituent masses (From PDG, constituent gluon from W.S. Hou et al, 2001):

$$
\begin{gathered}
m_u = 0.336 \text{ GeV}, \; m_d = 0.340 \text{ GeV} \\
m_c = 1.550 \text{ GeV}, \; m_b = 4.730 \text{ GeV} \\
m_G = 0.7 \text{ GeV}, \; m_{\rho(770)} = 0.775 \text{ GeV} \\
m_{J/\psi} = 3.097 \text{ GeV}, \; m_{\Upsilon(1S)} = 9.460 \text{ GeV}.
\end{gathered}
$$

Holographic Potential for non-$q\bar{q}$:

$$
V_{\text{non-}q\,\bar{q}}(z,\kappa,\,\Delta) = V_{q\,\bar{q}}(z,\kappa,\alpha) + \frac{M_5^2(\Delta)\,R^2}{z^2},
$$

Test subjects: heavy non-$q\bar{q}$ states

- $Z_c$ states.
- $Z_B$ states.
- $\psi$ (or $Y$) states.

# Non-$q\bar{q}$ masses

| $Z_c$ states | | | |
|---|---|---|---|
| **Non-$q\bar{q}$ state** | $Z_c(3900)$ | $Z_c(4200))$ | $Z_c(4430)$ |
| **Exp. Masses (MeV)** | $3887.1 \pm 3.6$ | $4296^{+35}_{-32}$ | $4478^{+13}_{-18}$ |
| **Diquark-Antidiquark (MeV)** $\bar{m}_{4Q} = 1550$ MeV $\kappa = 2151$ MeV and $\alpha = 0.5387$ | 4004.8 (3.0%) | 4384.9 (2.1%) | 4706.6 (5.1%) |
| **Hadronic Molecule (MeV)** $\bar{m}_{HM} = 1432.3$ MeV $\kappa = 1907$ MeV and $\alpha = 0.4887$ | 3817.1 (1.8%) | 4214.7 (1.9%) | 4552.1 (1.6%) |
| **Hybrid Meson (MeV)** $\bar{m}_{Z_c} = 1533$ MeV $\kappa = 2114.8$ MeV and $\alpha = 0.5317$ | 3721.9 (4.2%) | 4156.4 (3.2%) | 4513.2 (0.8%) |

| $\psi$ states | | | |
|---|---|---|---|
| **Non-$q\bar{q}$ state** | $\psi(4230)$ | $\psi(4360))$ | $\psi(4660)$ |
| **Exp. Masses (MeV)** | $4222.5 \pm 2.4$ | $4374 \pm 7$ | $4630 \pm 6$ |
| **Hadrocharmonium (MeV)** $\bar{m}_{HQc} = 1717$ MeV $\kappa = 2522.6$ MeV and $\alpha = 0.6036$ | 4222.5 (0.2%) | 4577.4 (4.6%) | 4871.8 (5.2%) |
| **Hadronic Molecule (MeV)** $\bar{m}_{HM} = 1549.1$ MeV $\kappa = 2149.2$ MeV and $\alpha = 0.5384$ | 4003.5 (5.2%) | 4383.7 (0.2%) | 4705.6 (1.6%) |

| $Z_b$ states | | |
|---|---|---|
| **Non-$q\bar{q}$ state** | $Z_b(10610)$ | $Z_b(10650))$ |
| **Exp. Masses (MeV)** | $10609 \pm 6$ | $10652.2 \pm 1.5$ |
| **Diquark-Antidiquark (MeV)** $\bar{m}_{HM} = 4753$ MeV $\kappa = 11269.5$ MeV and $\alpha = 0.8633$ | 10224.5 (3.6%) | 10517.6 (1.3%) |
| **Hybrid Meson (MeV)** $\bar{m}_{Zb} = 4689.7$ MeV $\kappa = 11102.3$ MeV and $\alpha = 0.8633$ | 10257.9 (3.3%) | 10512.5 (1.3%) |

- Each family is considered a *single* non-linear Regge trajectory.
- Bulk mass for vector non-$q\bar{q}$ states

$$M_5^2\, R^2 = (\Delta + L - 1)\,(\Delta + L - 3)\,.$$

- Multiquark states have $M_5^2\, R^2 = 15$.
- Gluonic excitations have:
  - $M_5^2\, R^2 = 8$ for one constituent gluon.
  - $M_5^2\, R^2 = 24$ for two constituent gluons.
- Preferred structures have the *smallest* RMS error.
  - $Z_c \to$ Hadronic Molecule.
  - $\psi \to$ Hadronic Molecule.
  - $Z_B \to$ Hybrid meson $(2\, G\, q\bar{q})$.

# Configurational entropy and Confinement

# Configurational Entropy in a nut-shell

## Main idea

Configurational Entropy is a *measure* of how well localized a function (e.o.m. solution) is in space.

- Localization is connected to confinement.
  - Confinement/deconfinement $\rightarrow$ localization/deslocalization.
  - Well-localized means living in a $\mathrm{fm}^3$ size region.
- A QCD-bounded state of constituent quarks and gluons has well spatially localized wave functions for these constituents. Example, See Van Royen-Weisskopf formula: $|\psi(0)|^2 \sim \Gamma_{l\bar{l}}$
- Configurational entropy is related to stability. See M. Gleiser & N. Jiang, 2015.
  - Well-localized $\rightarrow$ less configurational entropy $\rightarrow$ more stable.
  - Radial hadron excitations $M_n$ have higher configurational entropy.
  - Uncertainty principle: For hadron mass in a radial Regge trajectory, $M_n \sim \Gamma_n$

### 4 Experimental Data for Charmonium states

| State | $I^G J^{PC}$ | Meson | Mass (MeV) | $\Gamma_{Total}$ | $\Gamma_{V\to e^+e^-}$ (keV) | $f_n$ (MeV) | Source |
|---|---|---|---|---|---|---|---|
| $1^3S_1$ | $0^-(1^{--})$ | $J/\psi$ | $3096.916 \pm 0.011$ | $92.6 \pm 1.7$ keV | $5.547 \pm 0.14$ | $416.16 \pm 5.25$ | PDG |
| $2^3S_1$ | $0^-(1^{--})$ | $\psi(2S)$ | $3686.109 \pm 0.012$ | $293 \pm 9$ keV | $2.359 \pm 0.04$ | $296.08 \pm 2.51$ | PDG |
| $3^3S_1$ | $0^-(1^{--})$ | $\psi(4040)$ | $4040 \pm 4$ | $84 \pm 12$ MeV | $0.86 \pm 0.07$ | $187.13 \pm 7.61$ | PDG |
| $4^3S_1$ | $0^-(1^{--})$ | $\psi(4415)$ | $4415 \pm 5$ | $110 \pm 22$ MeV | $0.58 \pm 0.07$ | $160.78 \pm 9.70$ | PDG |

# Configurational Entropy and bottom-up holography

Confinement in bottom-up models is addressed by breaking conformal invariance in AdS space.

- Adding an energy scale→ emergence of bulk-bounded states → dual to hadrons at the Bndry.
- These bounded states are eigenvalues of a holographic potential $V(z)$ → Holographic RTs arise as eigenvalues of $V(z)$.
- Confinement in Bottom-up models is a *localization* process in the AdS bulk.
- For bottom-up spectroscopy, $S_{CE}$ is an increasing function of the hadron mass and the excitation number. See Bernardini & R. da Rocha, 2016, Braga et al, 2016, and Colangelo & Loparco, 2018.

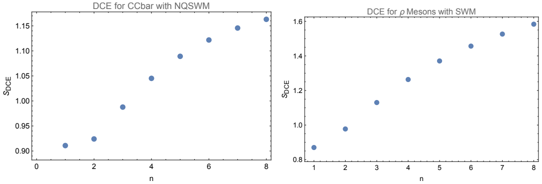



Figure: CE for vector mesons.

- A "good" bottom-up model should increase CE with the mass and excitation number for s-wave states.
- This defines a criterion to analyze which parameterization describes better a given non-$q\bar{q}$ state.

# Configurational Entropy and bottom-up holography

- A hadron defined by an operator $\mathcal{O}$ with dimension $\Delta$ at the boundary will be dual to a bulk field with mass $M_5$.
  - *A hadron at the holographic level is a bag of constituents.*
  - Any information about the hadronic inner structure is not present *ab initio*.
- Bottom-up approach: Normalizable bulk modes are dual to mesons at the boundary.
- From AdS/CFT dictionary: spatially localized wave functions at the boundary should be delocalized at the bulk because of the IR/UV behavior. Example: $\mathcal{N} = 4$ SYM, dual bulk states are unbounded, (see Hamilton et al, 2006).
- Lifting the bulk conformal invariance *reinterprets* the operator/field duality: extended and localized objects at the boundary are dual to localized objects at the bulk (See Bena, 2000).
- Thus, it is correct to infer that *locality at the bulk (involving lower configurational entropy) implies stability at the boundary.*

# Holographic Recipe I

- Define a geometrical background.

$$dS^2 = \frac{R^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]$$

- Define a set of bulk fields dual to the operators at the boundary.

$$I_{\text{Bulk}} = \int d^5x \sqrt{-g}\, e^{-\Phi(z)}\, \mathcal{L}_{\text{Bulk}}$$

- Compute bulk equations of motion.
- Compute the bulk energy-momentum tensor

$$T_{mn} = \frac{2}{\sqrt{-g}} \frac{\partial\left[\sqrt{-g}\, \mathcal{L}_{\text{Bulk}}\right]}{\partial\, g^{mn}}$$

# Holographic Recipe II

- Compute the on-shell energy density $\rho(z) \equiv T_{00}$.
- Fourier-transform the energy density

$$\bar{\rho}(k) = \int_0^\infty dz e^{-ik\,z} \rho(z).$$

- Compute modal fraction

$$f(k) = \frac{|\bar{\rho}(k)|^2}{\int dk |\bar{\rho}(k)|^2}$$

- Compute the *differential configurational entropy* (DCE)

$$S_{DCE} = -\int dk\, \tilde{f}(k) \log\, \tilde{f}(k)$$

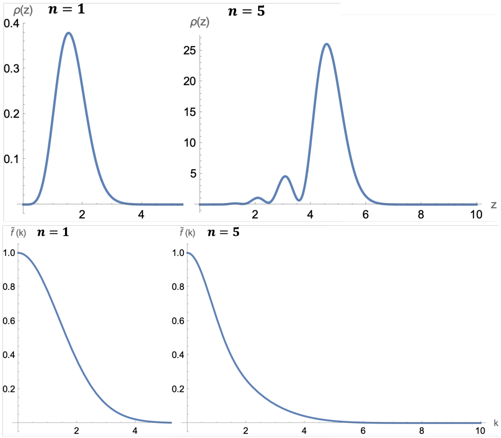
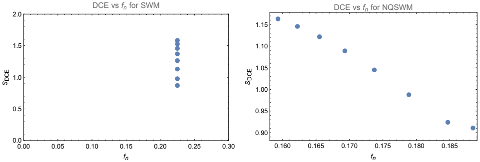



Figure: $\rho(z)$ and $\tilde{f}(k)$ for two different bulk modes.

# Stability

# Configurational entropy as a measure of Stability

- Stable hadronic modes have smaller configurational entropy.
- Decay constants decrease with the excitation number.




| $\Gamma_{V\to e^+e^-}$ (keV) | $f_n$ (MeV) | Source |
|---|---|---|
| $5.547 \pm 0.14$ | $416.16 \pm 5.25$ | PDG |
| $2.359 \pm 0.04$ | $296.08 \pm 2.51$ | PDG |
| $0.86 \pm 0.07$ | $187.13 \pm 7.61$ | PDG |
| $0.58 \pm 0.07$ | $160.78 \pm 9.70$ | PDG |

Figure: CE vs decay constants (up). Summary of EM decay properties vector charmonium. (down)

**Naive proof (see M.A.M.C., A.Vega & S. Diles, 2024):**

For vector meson hadronic states, we expect $M_n(DCE) \sim DCE^\tau \sim n^\gamma$, where $\tau$, $\gamma > 0$. Van Royen-Weisskopf (EM decays) and Segre formulas give us

$$\Gamma_{l\bar{l}} \sim |\psi(0)|^2 \sim E_n^{1/2}\,\frac{d\,E_n}{dn} \sim M_n^{1/2}\,\frac{d\,M_n}{dn} \sim \gamma\, n^{\frac{3}{2}\gamma-1}$$

With $M_n = M_1 + M_2 + E_n$.
Phenomenological bound: $0 \leq \gamma < 2/3$ (BSE)
On the other hand, for $M_n^2 = a(n+b)^\nu$ we have:

$$\Gamma_{l\bar{l}} \sim |\psi(0)|^2 \sim \nu\, a^{3/4}\, n^{\frac{3}{4}\nu-1}.$$

Notice that $\nu = 2\,\gamma$.
For $\nu = 1$, $\Gamma_{l\bar{l}} \sim n^{-1/4}$,

# Stability, Configurational entropy and non-$q\bar{q}$ states

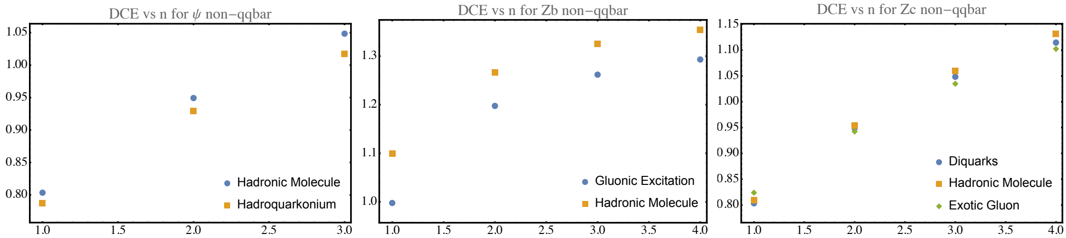



Figure: Configurational entropy vs excitation level for non-$q\bar{q}$ candidates.

- Premise: stable hadronic configuration should have smaller configurational entropy.
- Holographic prediction for heavy non-$q\,\bar{q}$ hadrons:
  - $\psi$ states could be *hadrocharmonium.*
  - $Z_b$ states could be *hybrid meson.*
  - $Z_c$ states could be *hybrid meson*

# Conclusions

# Summary and Conclusions

- Confinement, from the bottom-up holographic perspective, is equivalent to *localization* in the bulk.
- Configurational entropy is a measure of stability.
- For a consistent bottom-up model:
  - CE increases with hadron mass $M_n$.
  - CE increases with the excitation number $n$.
- Bottom-up models with increasing CE with $n$ and hadronic mass $M_n$ describe stable hadrons.

***Backup slides***

# Hadronic Identity

| Scalar hadrons | | Vector hadrons | | Spin 1/2 hadrons | |
|---|---|---|---|---|---|
| $(n\mathrm{Q})(m\mathrm{G})$ | $\Delta$ | $(n\mathrm{Q})(m\mathrm{G})$ | $\Delta$ | $(n\mathrm{Q})(m\mathrm{G})$ | $\Delta$ |
| (2Q) | 3 | (2Q) | 3 | (3Q) | 9/2 |
| (2G) | 4 | (2Q)(1G) | 5 | (1Q)(3G) or (3Q)(1G) | 13/2 |
| (2Q)(1G) | 5 | (4Q) or (3G) | 6 | (5Q) | 15/2 |
| (4Q) | 6 | (2Q)(2G) | 7 | (3Q)(2G) | 17/2 |
| (2Q)(2G) | 7 | (4Q)(1G) | 8 | (5Q)(1G) | 19/2 |
| (4Q)(1G) or (4G) | 8 | (6Q) or (2Q)(3G) | 9 | (3Q)(3G) or (7Q) | 21/2 |
| (6Q) or (2Q)(3G) | 9 | (5G) or (4Q)(2G) | 10 | (5Q)(2G) | 23/2 |

Table: Possible hadronic states composed by $n$ quarks (or antiquarks) and $m$ gluons and their conformal dimensions. To know the corresponding value of the associated bulk mass, solve the equation of motion for the bulk fields and compute the low $z$ limit to see how the fields scale. This scaling is the conformal dimension defined in terms of the bulk mass. Invert the relation, and you will find the expected $M_{d+2} = M_{d+2}(\Delta)$ relation.

# General Bottom-up Algorithm

## AdS-like Background

$$dS^2 = \frac{R^2}{z^2}\, e^{h(z)} \left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]$$

with $R$ defined as the AdS radius and $h(z)$ a geometric deformation.

## General Action with minimal coupling

$$I_{\text{SWM}} = \int d^5x\, \sqrt{-g}\, e^{-\Phi(z)}\, \mathcal{L}_{\text{Hadron}},$$

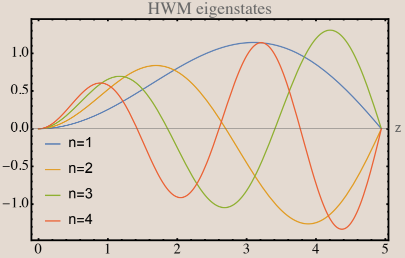
with $\Phi(z)$ defined as a static dilaton field, responsible for inducing the conformal symmetry breaking, i.e., confinement. The Lagrangian $\mathcal{L}_{\text{Hadron}}$ has the information about the bulk field dual to hadrons.

# Example: Hard Wall in a nutshell

## General Ideas

Holographic potential: $\beta = \{-3, -1\}$ for scalar and vector mesons.

$$V_{hw}(z) = \frac{4M_5^2 R^2 - 2\beta + \beta^2}{4z^2}, \quad 0 \leq z \leq z_{hw}.$$




- See Boschi-Filho, 2002), Braga et al. (2004), and Elrich et al. (2005).
- Confinement is understood as the emergence of bound states by geometric deformations in AdS space: a hard cutoff.
- Mesons are bound states of a *effective* bulk potential similar to a quantum infinite well.
- Mesons have $\Delta = 3 + l$ and $M_5^2 R^2 = (3 + l)(l - 1) \geq M_{BF}^2$, with mass spectrum given by $(l = 0)$

$$M_n^2 = \Lambda_{QCD}^2 \, \alpha_{1,n}^2 \text{ with } z_{\text{hw}} = \frac{1}{\Lambda_{QCD}}.$$

- $\alpha_{1,n} \propto n$ are the zeroes of the Bessel function of the first kind.

# General Bottom-up Algorithm

## Holographic Potential

The action written above defines a set of equations of motion for the bulk fields that, in general, has a Schrödinge-like form:

$$-\psi''(z) + V(z)\,\psi(z) = M_n^2\,\psi(z),$$

where $V(z)$ is the holographic potential written in terms of the deformation and the dilaton. In the case of $p-$form bulk fields as follows:

$$V(z) = \frac{1}{4}B'(z)^2 - \frac{1}{2}\,B''(z) + \frac{M_5^2\,R^2}{z^2}e^{h(z)}, \tag{1}$$

with $B(z) = \Phi(z) + \beta\,[\log\frac{R}{z} + \frac{1}{2}h(z)]$, $M_5$ is the bulk mass associated to $\psi(z)$ and $\beta = -(3-2\,p)$. Later, we will connect $\beta$ with the hadronic (integer) spin.

# General Bottom-up Algorithm

Since we want to deal with no geometric deformations in our particular case, we will fix $h(z) = 0$. This choice is the case of the so-called softwall-like models (Karch *et al.* 2006).

## Holographic Regge Trajectories

Regge trajectories will emerge as the eigenvalue spectrum associated with the Sturm-Liouville problem defined by $V(z)$:

$$M_n^2 = A\,(n + B)^{\nu},$$

where $A$ is an energy scale defined by the dilaton and/or deformation, $B$ carries information about the hadronic angular momentum and $\nu$ measures linearity. If the deformations and dilatons are quadratic at the high-$z$ limit, the out-coming trajectory will be linear.

# Why $\nu$? Non-linearity Hypothesis

## Hypothesis

Linearity is connected with the hadron constituent mass: when constituent mass rises, linearity ceases. The linear case appears when constituent quark masses are supposed to be zero, i.e., small enough compared with the meson mass.

From Bethe-Salpeter (Afonin and Pusenkov, 2014; J. K. Chen, 2018) we can write the trajectory as

$$(M_n - m_{q_1} - m_{q_2})^2 = a(n + b).$$

When the limit $m_{q_{1,2} \to \infty}$ (as in heavy quarkonium), the trajectory acquires a generic non-linear form:

$$M_n^2 \propto n^{2/3}.$$

Also, in heavy-light systems, non-linearity is expected (J. K. Chen, 2018).

# Light unflavored mesons: $\nu = 1$

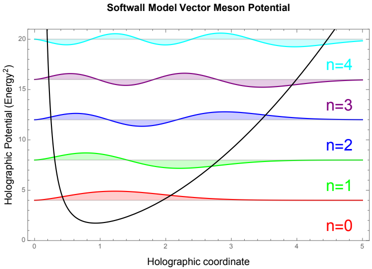



Figure: Holographic potential .

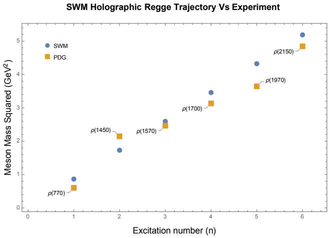



Figure: $\rho$ meson Regge Trajectory.

- Ex: Softwall model: $\Phi(z) = \kappa^2 z^2$.
- Regge trajectories emerge as eigenvalue spectrum of $V(z)$.

- Linearity is controlled by the high-$z$ behavior of $V(z)$. **Example:**
  - Softwall model (see KKSS, 2006): $M_n^2 \propto n$

# Heavy Mesons: $0 < \nu < 1$

Main idea: quark masses (as in BSE) deviate RT from linearity. Holographically, the dilaton profile at large $z$ should behave as

$$\Phi(z) = (\kappa\, z)^{2-\alpha}$$

$\alpha$ carries information of constituent masses. See (Martin and Vega, 2021.)

| $\psi$ **with** $\alpha = 0.54$ **and** $\kappa = 2150$ MeV | | | |
|---|---|---|---|
| **n** | $M_{\mathbf{Exp}}$ **(MeV)** | $M_{\mathbf{Th}}$ **(MeV)** | **R. E. (%)** |
| 1 | $3096.916 \pm 0.011$ | 3077.09 | 0.61 |
| 2 | $3686.109 \pm 0.012$ | 3689.62 | 0.1 |
| 3 | $4039 \pm 1$ | 4137.5 | 2.44 |
| 4 | $4421 \pm 4$ | 4499.4 | 1.77 |
| $M_n^2 = 8.07(0.287 + n)^{0.6315}$ with $R^2 = 0.999$ | | | |

Table: Vector Charmomium. Experimental results are read from PDG.