Forward Higgs production: a challenge in high-energy perturbative QCD

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based on M. Fucilla, M.A. Nefedov, A. Papa, JHEP 04 (2024) 078

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Outline

Introduction and Motivation

- The importance of exploring the Higgs sector of the Standard Model can hardly be overestimated; opening new channels of theoretical investigation or improving the reliability of known ones can only be beneficial.

- An interesting new option in this respect is the inclusive production at the LHC (and future colliders) of a forward Higgs, possibly in association with a backward jet or hadron.

- It belongs to the class of semihard processes, where the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD} \;, \quad Q \text{ a hard scale } (Q = m_{H\perp} \text{ in the Higgs case}) \;,$$

  holds, making fixed-order perturbative calculations insufficient, due to $\alpha_s(Q) \log s \sim 1$, and calling for an all-order resummation of energy logarithms.

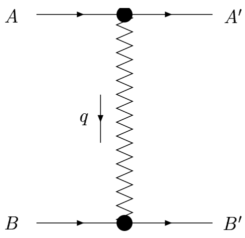
- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach provides a general framework for the large-s / high-energy resummation: I will briefly review the theoretical basis of the BFKL approach and present its (challenging) application to the forward Higgs production case.

Outline

Gluon Reggeization in perturbative QCD

Elastic scattering process $A + B \longrightarrow A' + B'$

- gluon quantum numbers in the t-channel: octet color representation, negative signature
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s)
- all-order resummation:
  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



$$\left(\mathcal{A}_8^-\right)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t) \; , \qquad j(0) = 1$$

$\omega(t)$ – Reggeized gluon trajectory

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A}$$

T^c fundamental (quarks) or adjoint (gluons)

Gluon Reggeization in perturbative QCD

Interlude: Sudakov decomposition

$$p = \beta p_1 + \alpha p_2 + p_\perp \ , \qquad p_\perp^2 = -\vec{p}^{\,2}$$

(p_1, p_2) light-cone basis of the initial particle momenta plane

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \ , \qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \ , \qquad 2\, p_1 \cdot p_2 = s \qquad \blacksquare$$

The gluon Reggeization has been first verified in fixed order calculations, then rigorously proved

- in the LLA [Ya.Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma_{A'A}^{(0)} = \delta_{\lambda_{A'}\lambda_A} \ , \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

$$D = 4 + 2\epsilon \ , \qquad t = q^2 \simeq q_\perp^2$$

- in the NLA [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]

$$\Gamma_{A'A}^{(1)} = \delta_{\lambda_{A'}\lambda_A} \Gamma_{AA}^{(+)} + \delta_{\lambda_{A'},-\lambda_A} \Gamma_{AA}^{(-)} \ , \qquad \omega^{(2)}(t)$$

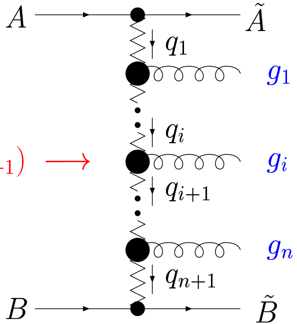
Outline

BFKL in leading accuracy

Inelastic scattering process $A + B \;\to\; \tilde{A} + \tilde{B} + n$ in the LLA

multi-Regge kinematics

$\gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \longrightarrow$



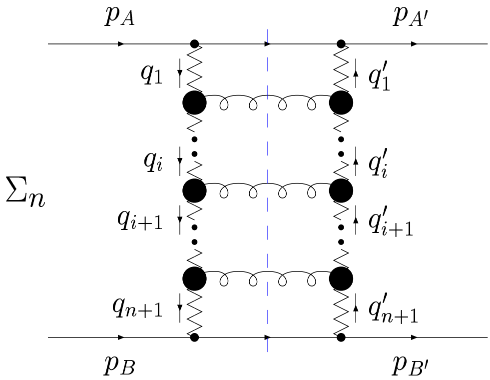
$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_i invariant mass of the $\{g_{i-1}, g_i\}$ system, proportional to s

s_R energy scale, irrelevant in the LLA

BFKL in leading accuracy

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity

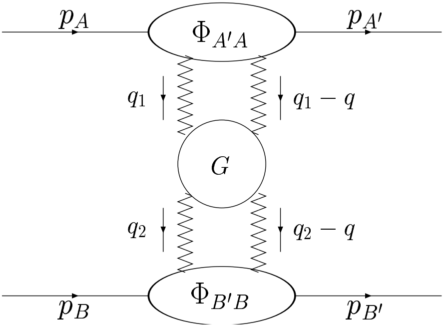


$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = 1 \text{ (singlet)}, 8^- \text{ (octet)}, \ldots$$

The singlet color representation appears in the t-channel
$\longrightarrow$ colliding colorless objects $\longrightarrow$ phenomenology

BFKL in leading accuracy

Structure of the amplitude:



$$\mathrm{Im}_s (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1; \vec{q})$$

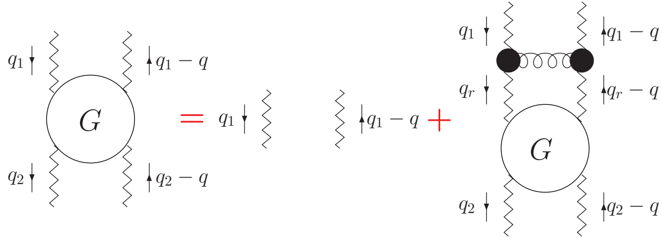
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2; -\vec{q})$$

BFKL in leading accuracy

- $G_\omega^{(\mathcal{R})}$ – Mellin transform of the Green's functions for Reggeon-Reggeon scattering

$$
\begin{aligned}
\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right) &= \vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^{\,2}\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right) \\
&+ \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r-\vec{q})^{\,2}}\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_r;\vec{q}\right) G_\omega^{(\mathcal{R})}\left(\vec{q}_r,\vec{q}_2;\vec{q}\right)
\end{aligned}
$$

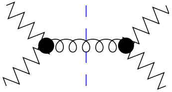
BFKL equation: $t=0$ and singlet color representation
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]



BFKL in leading accuracy

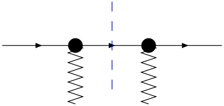
$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-(\vec{q}_1 - \vec{q})^{\,2}\right)\right] \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q})$$

In the LLA: $\omega(t) = \omega^{(1)}(t), \quad \mathcal{K}_r = \mathcal{K}_{RRG}^{(B)}$



- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ – impact factors in the t-channel color state $(\mathcal{R}, \nu)$

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^c_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$



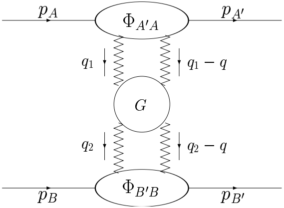
constant in the LLA

Optical theorem:
the high-energy behavior of pQCD cross sections is completely determined by the properties of the BFKL kernel.

BFKL factorization

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

$\Rightarrow$ BFKL factorization for $\mathrm{Im}_s\mathcal{A}$:
convolution of a Green's function with the impact factors of the colliding particles.



Valid both in
LLA (resummation of all terms $(\alpha_s \ln s)^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

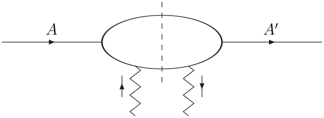
$$\mathrm{Im}_s\mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_{AA'}(\vec{q}_1, \vec{q}; s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_{BB'}(-\vec{q}_2, -\vec{q}; s_0)$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

The Green's function is process-independent and is determined through the BFKL equation. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)$$

Outline

Impact factors are process-dependent;
only very few of them known in the NLA ...



- At parton level

  $A = A'$=quark, $A = A'$=gluon [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

- For exclusive processes

  $A = \gamma_L^*$, $A' = V_L$, with $V = \rho^0, \omega, \phi$ (forward)
  [D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]

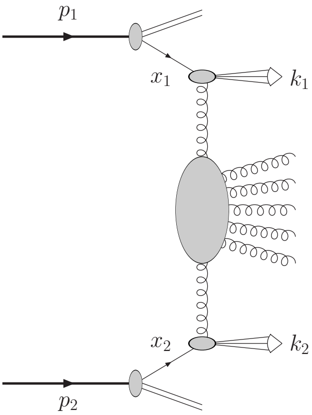
- For total cross sections (via the optical theorem)

  $A = A' = \gamma^*$ (forward) [J. Bartels, S. Gieseke, C.F. Qiao (2001)]
  [J. Bartels, S. Gieseke, A. Kyrieleis (2002)]
  [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
  [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
  [J. Bartels, A. Kyrieleis (2004)]
  [I. Balitsky, G.A. Chirilli (2013)] [G.A. Chirilli, Yu.V. Kovchegov (2014)]

Inclusive processes

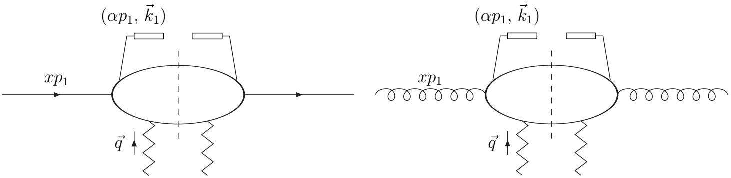
A lot more possibilities open for inclusive processes, with jets or identified particles in the final state, produced in the fragmentation regions...

... and if the fragmentation subprocess is hybridized with collinear factorization.



Identified 'object' (jet, hadron, Higgs, ...) with momentum k_1 (k_2) in the forward (backward) region; all the rest undetected.

Straightforward adaptation of the BFKL factorization: just restrict the summation over final states entering the definition of impact factors.



quark vertex   gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one (or more) parton(s) to generate a jet or a Higgs or one parton to fragment into a given hadron

- use QCD collinear factorization

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes (D_a^h \;/\; S_a^J) \;/\; H) \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes (D_g^h \;/\; S_g^J \;/\; H)$$

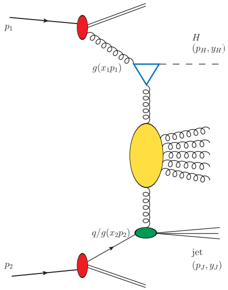
$f_{a,g}$: unpolarized collinear PDFs, $D^h_{a,g}$: unpolarized collinear FFs,

$S^J_{a,g}$: jet selection functions, H: Higgs vertex

Higgs plus jet as a paradigm

Inclusive **Higgs plus jet** production in p-p collisions [V. Del Duca, C.R. Schmidt (1994)]
(+ Sudakov logs) [B. Xiao, F. Yuan (2018)]

- Full NLA Green function + partially NLO impact factor (full m_t-dependence) [F.G. Celiberto, D. Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]
- Same process in HEJ framework (full m_t,m_b-dep.) [J. Andersen *et al.* (2022)]



$$\frac{d\sigma_{\rm pp}}{dy_H dy_J d|\vec{p}_H| d|\vec{p}_J| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \left(\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1)\right)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_H x_J s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_p \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_p(x_2)\right)$$

Hadronic cross section expanded in azimuthal coefficients

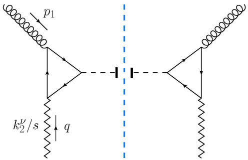
$$\frac{d\sigma_{\rm pp}}{dy_H dy_J d|\vec{p}_H| d|\vec{p}_J| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + 2\sum_{n=1}^{\infty}\cos(n\phi)\mathcal{C}_n\right] \qquad \phi = \phi_1 - \phi_2 - \pi$$

Outline

LO Higgs impact factor

Gluon-Reggeon $\rightarrow$ Higgs
(through the top quark loop)

Off-shell t-channel gluon with effective $\frac{k_2^\nu}{s}$ polarization



- LO impact factor

[V. Del Duca, C.R. Schmidt (1994)]

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2} \frac{\vec{q}^{\,2}|\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2\sqrt{N^2-1}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

$$\xrightarrow{m_t\to\infty} \frac{\alpha_s^2}{v^2} \frac{\vec{q}^{\,2}}{72\pi^2\sqrt{N^2-1}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

- NLO impact factor for $m_t \to \infty$

[M. Nefedov (2019)] [M. Hentschinski, K. Kutak, A. van Hameren (2020)]
[F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2022)]

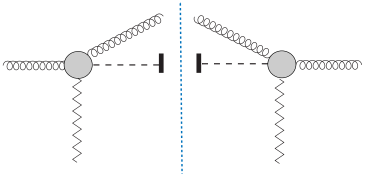
Effective Lagrangian for gluon-Higgs interaction

$$\mathcal{L}_{ggH} = -\frac{1}{4} g_H F_{\mu\nu}^a F^{\mu\nu,a} H \,, \qquad g_H = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

Outline

NLO Higgs impact factor: Real corrections

Gluon-initiated subprocess



NLO Higgs impact factor: Real corrections

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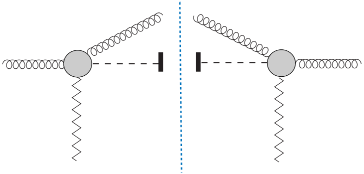
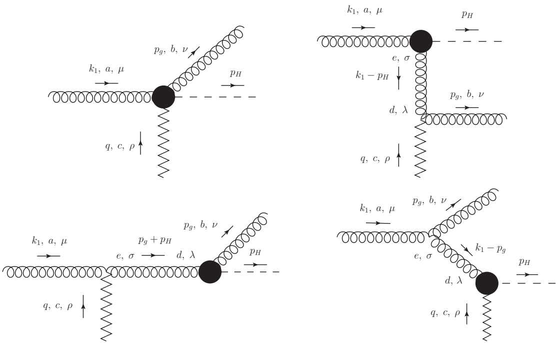
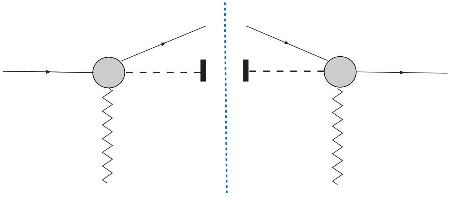




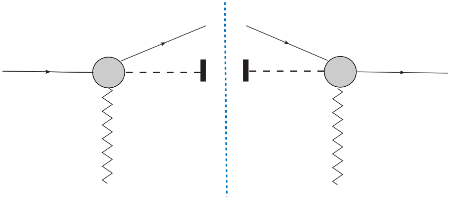
NLO Higgs impact factor: Real corrections

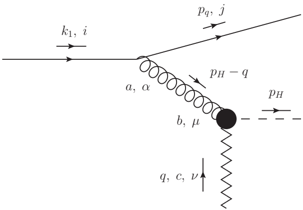
Quark-initiated subprocess



NLO Higgs impact factor: Real corrections

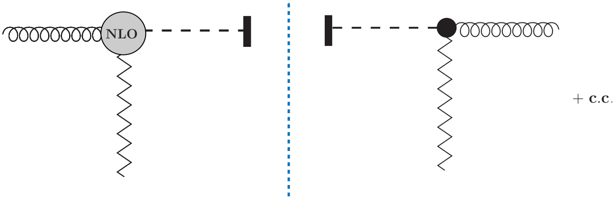
Quark-initiated subprocess





Outline

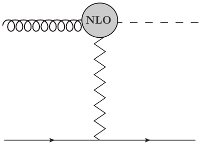
NLO Higgs impact factor: Virtual corrections



NLO Higgs impact factor: Virtual corrections

Target: 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \underbrace{\Gamma^{ac(0)}_{\{H\}g}(q)}_{\frac{g_H \delta^{ac} (\varepsilon_\perp \cdot q_\perp)}{2}} \; [1 + \delta_{\rm NLO}]$$



(reference process: g+q $\to$ H + q)

Strategy: compare a suitable high-energy amplitude with the Regge form

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g} \frac{s}{t} \left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)} \right] \Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g} \frac{2s}{t} \Gamma^{c(0)}_{qq}$$

$$+ \Gamma^{ac(0)}_{\{H\}g} \frac{s}{t} \omega^{(1)}(t) \left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right) \right] \Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g} \frac{2s}{t} \Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g} \frac{2s}{t} \Gamma^{c(0)}_{qq}$$

Virtual corrections to the impact factor:

$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H} \frac{\bar{\alpha}_s}{2\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} \right.$$

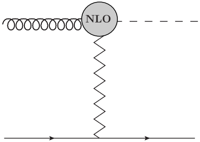
$$\left. - \frac{C_A}{\epsilon} \ln\left(\frac{\vec{q}^{\,2}}{s_0}\right) - \frac{5n_f}{9} + C_A\left(2\,\Re\left(\mathrm{Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11 \right]$$

Agreement with [M. Nefedov (2019)] (tool: Lipatov high-energy effective theory)

NLO Higgs impact factor: Virtual corrections

Target: 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \underbrace{\Gamma^{ac(0)}_{\{H\}g}(q)}_{\frac{g_H \delta^{ac}(\varepsilon_\perp \cdot q_\perp)}{2}} \; [1 + \delta_{\rm NLO}]$$



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$$+\Gamma^{ac(0)}_{\{H\}g} \frac{s}{t} \omega^{(1)}(t) \left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right) \right] \Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g} \frac{2s}{t} \Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g} \frac{2s}{t} \Gamma^{c(0)}_{qq}$$

Virtual corrections to the impact factor:

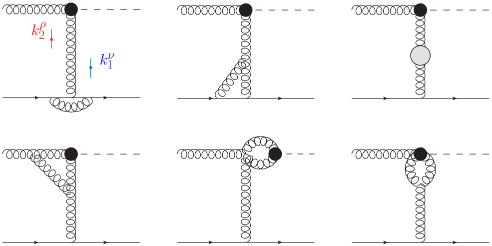
$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H} \frac{\bar{\alpha}_s}{2\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} \right.$$

$$\left. -\frac{C_A}{\epsilon} \ln\left(\frac{\vec{q}^{\,2}}{s_0}\right) - \frac{5n_f}{9} + C_A \left(2\,\Re\left(\mathrm{Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11 \right]$$

Agreement with [M. Nefedov (2019)] (tool: Lipatov high-energy effective theory)

Anatomy of the calculation: “non-Gribov” terms

- Single-gluon in the t-channel



Gribov's prescription (eikonal approximation): $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s}$

- Two gluons in the t-channel

Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ Gribov's trick modification

$$g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s} + g_{\perp\perp}^{\rho\nu}$$

Anatomy of the calculation: “non-Gribov” terms

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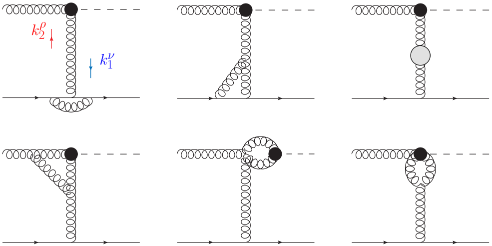

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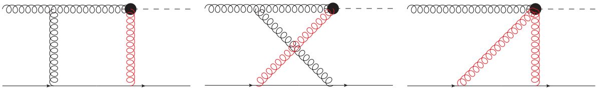
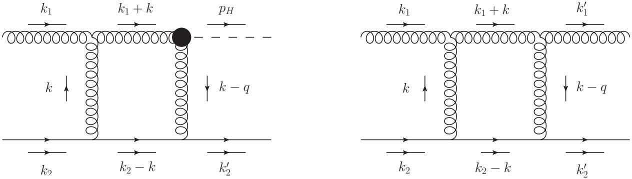


Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ Gribov’s trick modification

$$g^{\rho\nu} = g^{\rho\nu}_{\perp\perp} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s} + g^{\rho\nu}_{\perp\perp}$$

Non-Gribov terms: comparison with pure QCD



- Non-Gribov term in $\mathcal{A}_{gq \to Hq}$

$$-s\, \bar{u}(k_2')\gamma_{\perp,\nu} u(k_2)\, \varepsilon_\perp^\nu(k_1)\, H_\nu{}^\sigma(-k_1 - k, k - q)$$

$$H^{\nu\sigma}(p_1, p_2) = g^{\nu\sigma}(p_1 \cdot p_2) - p_1^\nu p_2^\sigma$$

$\longrightarrow$ order s after integration over loop momentum k

- Non-Gribov term in $\mathcal{A}_{gq \to gq}$

$$-s\, \bar{u}(k_2')\gamma_{\perp,\sigma} u(k_2)\, \varepsilon_\perp^\nu(k_1) \left(\varepsilon^*_{\perp,\beta}(k_1') - \frac{\varepsilon^*_\perp(k_1') \cdot k'_{1,\perp}}{k_{1'} \cdot k_2} k_{2,\beta} \right) A_\nu{}^{\sigma\beta}(k - q, k_1 + k)$$

$$A^{\nu\sigma\beta}(k - q, -k_1 - k) = g^{\sigma\beta}(q - k_1 - 2k)^\nu + g^{\nu\sigma}(k - 2q - k_1)^\beta + g^{\nu\beta}(2k_1 + k + q)$$

$\longrightarrow$ order s^0 after integration over loop momentum k

Non-Gribov terms: impact on Regge form of amplitude

- Born helicity structure:

$$\mathcal{H}_{\rm Born} \equiv (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\frac{\hat{k}_1}{s}u(k_2)$$

$$= (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\frac{\hat{q}_\perp}{|q_\perp|^2}u(k_2) \quad \text{by Sudakov decomposition of } q$$

Basis $n_q^\mu = \frac{q_\perp^\mu}{|q_\perp|}$, $\quad n_{\tilde{q}}^\mu = \epsilon^{\mu\nu+-}\frac{q_\perp^\nu}{|q_\perp|}$

$$\mathcal{H}_{\rm Born} = (\varepsilon_\perp(k_1)\cdot n_q)\,\bar{u}(k_2-q)\hat{n}_q u(k_2)$$

- Helicity structure of a non-Gribov term

$$\bar{u}(k_2-q)\hat{\varepsilon}_\perp(k_1)u(k_2) = -\bar{u}(k_2-q)\gamma_\mu u(k_2)\left(n_q^\mu n_q^\nu + n_{\tilde{q}}^\mu n_{\tilde{q}}^\nu\right)\varepsilon_{\perp,\nu} = -\mathcal{H}_{\rm Born} - \mathcal{H}_{\rm anomalous}$$

Interference between $\mathcal{H}_{\rm Born}$ and $\mathcal{H}_{\rm anomalous}$ + spin sum gives zero

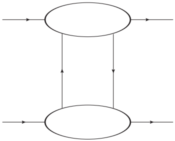
The anomalous helicity structure vanishes at amplitude level

- Nonetheless, non-Gribov terms give a total contribution

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

Gribov part: strategy of rapidity regions, QCD case

Strategy for the calculation of two-gluon t-channel diagrams
[V.S. Fadin, A.D. Martin (1999)] [V.S. Fadin, R. Fiore (2001)]



- Feynman gauge and Gribov's trick

$$g^{\mu\nu} = g^{\mu\nu}_{\perp\perp} + 2\frac{k_2^\mu k_1^\nu + k_2^\mu k_1^\nu}{s} \longrightarrow \frac{2k_2^\mu k_1^\nu}{s}$$

- Loop momentum k decomposed à la Sudakov: $k = \beta k_1 + \alpha k_2 + k_\perp$

$$\begin{cases} \textbf{Central region} & |\alpha| \lesssim \alpha_0 \;,\; |\beta| \lesssim \beta_0 \;, \\ \textbf{Region A} & |\alpha| \lesssim \alpha_0 \;,\; |\beta| > \beta_0 \;, \\ \textbf{Region B} & |\alpha| > \alpha_0 \;,\; |\beta| \lesssim \beta_0 \;, \\ \textbf{Region C} & |\alpha| > \alpha_0 \;,\; |\beta| > \beta_0 \;, \end{cases}$$

$$\alpha_0 \ll 1 \;, \qquad \beta_0 \ll 1 \;, \qquad s\alpha_0\beta_0 \gg |t|$$

- Factorization of vertices:
  in the region $|\alpha| \ll 1 \longrightarrow \Gamma^{(0)}_{B'B}$ in the region $|\beta| \ll 1 \longrightarrow \Gamma^{(0)}_{A'A}$

Gribov part: strategy of rapidity regions, QCD case

- Region C is suppressed by a factor $|t|/\alpha_0\beta_0 s \ll 1$
- Central region: box + crossed diagram

$$\mathcal{A}^{(8,-)}_{\rm Central} = \Gamma^{(0)}_{A'A}\frac{2s}{t}\Gamma^{(0)}_{B'B}\,\omega^{(1)}(t)\left[\frac{1}{2}\ln\left(\frac{-s}{t}\right)+\frac{1}{2}\ln\left(\frac{-s}{-t}\right)+\phi(\alpha_0)+\phi(\beta_0)\right]$$

$$\phi(z) = \ln z + \frac{1}{2}\left(-\frac{1}{\epsilon} - \psi(1) + \psi(1+\epsilon) - 2\psi(1-\epsilon) + 2\psi(1-2\epsilon)\right)$$

Correction to the upper and lower effective vertex from the central region

$$\Gamma^{(\rm Central)}_{A'A} = \Gamma^{(0)}_{A'A}\,\omega^{(1)}(t)\phi(\beta_0) \qquad \Gamma^{(\rm Central)}_{B'B} = \Gamma^{(0)}_{B'B}\,\omega^{(1)}(t)\phi(\alpha_0)$$

- Region A

$$\Gamma^{(\rm A)}_{A'A} = \Gamma^{(0)}_{A'A}\delta^{(\rm A)}_{\rm NLO} = \Gamma^{(0)}_{A'A}\left[-\omega(t)\ln\beta_0 + \tilde{\delta}^{(\rm A)}_{\rm NLO}\right]$$

- Region B

$$\Gamma^{(\rm B)}_{B'B} = \Gamma^{(0)}_{B'B}\,\delta^{(\rm B)}_{\rm NLO} = \Gamma^{(0)}_{B'B}\left[-\omega(t)\ln\alpha_0 + \tilde{\delta}^{(\rm B)}_{\rm NLO}\right]$$

Gribov part: strategy of rapidity regions, with Higgs

Reference amplitude: gluon + quark $\to$ Higgs + quark

- Region C is suppressed by a factor $|t|/\alpha_0\beta_0 s \ll 1$
- Central region

$$\mathcal{A}_{\text{Box,Central}} = \Gamma^{c(0)}_{q'q}\left(\frac{2s}{t}\right) g_H \epsilon_\mu(k_1)\delta^{ac}\left(-\frac{g^2 C_A st}{2}\right)\frac{s}{2}\int_{-\alpha_0}^{\alpha_0} d\alpha \int_{-\beta_0}^{\beta_0} d\beta$$

$$\times \int \frac{d^{D-2}k_\perp}{(2\pi)^D i}\frac{q_\perp^\mu - k_\perp^\mu}{(\alpha\beta s + k_\perp^2 + i0)(\alpha\beta s + (q-k)_\perp^2 + i0)(-\beta s + i0)(\alpha s + i0)}$$

Apparently, no factorization of the $\Gamma^{ac(0)}_{gH}$ vertex.

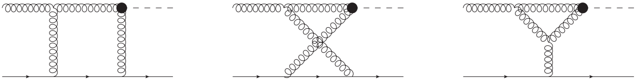
But, the change of variables $k_\perp \to q_\perp - k_\perp$ implies $q_\perp \to \frac{1}{2} q_\perp$ in the numerator. Then

$$\mathcal{A}^{(8,-)}_{\text{Central}} = \Gamma^{ac(0)}_{gH}\frac{2s}{t}\Gamma^{c(0)}_{q'q}\,\omega^{(1)}(t)\left[\frac{1}{2}\ln\left(\frac{-s}{t}\right) + \frac{1}{2}\ln\left(\frac{-s}{-t}\right) + \phi(\alpha_0) + \phi(\beta_0)\right]$$

Correct factorization!

Gribov part: strategy of rapidity regions, with Higgs

- Region A + triangular diagram



Proper factorization of the upper and lower vertex and

$$\delta_{\rm NLO}^{({\rm Tri+A})} = -\omega^{(1)}(t)\ln\beta_0 + \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\left\{\frac{7}{3}\frac{C_A}{\epsilon} + \frac{85}{18}C_A + \frac{1}{6}C_A\ln\left(-\frac{m_H^2}{\vec{q}^{\,2}}\right) + 2C_A\left(\frac{\pi^2}{6} + {\rm Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right)\right\} + \mathcal{O}(\epsilon)$$

Gribov part: strategy of rapidity regions, with Higgs

- Region B

  Expectation: factorization of the upper and lower vertices and extraction of the NLO correction to the lower (quark) one.

  Instead,

$$\mathcal{A}_{\rm B} = \Gamma^{c(0)}_{qq'} \left(\frac{2s}{t}\right) \frac{\epsilon_\mu(k_1)\delta^{ac} g_H}{2} g^2 C_A t \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{(q-k)_\perp^\mu}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

$$\frac{\mathcal{A}_B}{\Gamma^{ac(0)}_{gH}\left(\frac{2s}{t}\right)\Gamma^{c(0)}_{qq'}} = \frac{g^2 C_A t}{2} \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 (1+\alpha) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

  The "anomalous" term can only be assigned to the Higgs vertex:

$$\delta^{(\rm B)}_{\rm NLO} = \frac{\bar\alpha_s}{4\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\left[\frac{2C_A}{\epsilon} + 4C_A\right]$$

Compare with non-Gribov contribution ...

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar\alpha_s}{4\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

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$$\frac{\mathcal{A}_B}{\Gamma^{ac(0)}_{gH} \left(\frac{2s}{t}\right) \Gamma^{c(0)}_{qq'}} = \frac{g^2 C_A t}{2} \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 (1+\alpha) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - (1-\alpha) q_\perp)^2}$$

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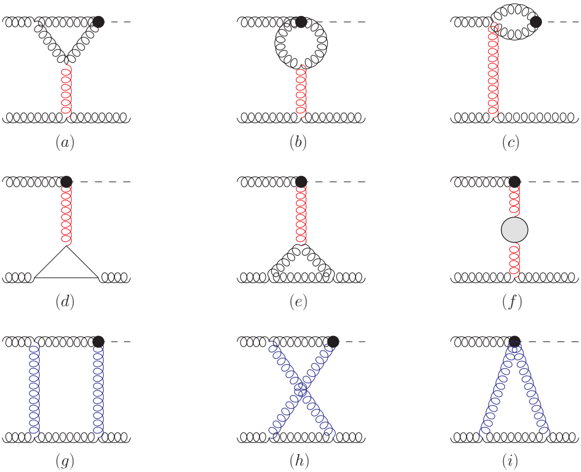
Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

Diffusion of a gluon off a gluon to produce a Higgs plus a gluon



Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

- Compare with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\rm NLO} \simeq \frac{\bar\alpha_s}{4\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A\left(2{\rm Li}_2\left(1+\frac{m_H^2}{\vec q^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18}\right)\right\}$$

- Non-Gribov contributions ...

$$\delta^{\rm n.G.}_{gg\to Hg} = \frac{\bar\alpha_s}{4\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 9 - \zeta(2)\right] + \mathcal{O}(\epsilon)$$

... exactly compensate the "anomalous" term from the region B

$$\delta^{\rm (B)}_{\rm NLO} = \frac{\bar\alpha_s}{4\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[-\frac{1}{\epsilon^2} + \frac{5}{\epsilon} + 9 + \zeta(2)\right] + \mathcal{O}(\epsilon)$$

Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

- Compare with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\rm NLO} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A\left(2\mathrm{Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18}\right)\right\}$$

- Non-Gribov contributions ...

$$\delta^{\rm n.G.}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 9 - \zeta(2)\right] + \mathcal{O}(\epsilon)$$

... exactly compensate the “anomalous” term from the region B

$$\delta^{\rm (B)}_{\rm NLO} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[-\frac{1}{\epsilon^2} + \frac{5}{\epsilon} + 9 + \zeta(2)\right] + \mathcal{O}(\epsilon)$$

Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

- Compare with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)}+\left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg}+\Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg}+\Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\mathrm{NLO}} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2}+\frac{11C_A-2n_f}{6\epsilon}-\frac{5n_f}{9}+C_A\left(2\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)+\frac{\pi^2}{3}+\frac{67}{18}\right)\right\}$$

- Non-Gribov contributions ...

$$\delta^{\mathrm{n.G.}}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2}-\frac{5}{\epsilon}-9-\zeta(2)\right]+\mathcal{O}(\epsilon)$$

... exactly compensate the “anomalous” term from the region B

$$\delta^{(\mathrm{B})}_{\mathrm{NLO}} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[-\frac{1}{\epsilon^2}+\frac{5}{\epsilon}+9+\zeta(2)\right]+\mathcal{O}(\epsilon)$$

Conclusions and outlook

- The calculation of the impact factor for forward Higgs production in the infinite-top-mass limit turned out to be very challenging, due to the presence of a dimension-5 non-renormalizable effective interaction

- Unexpected terms beyond eikonal approximation (“non-Gribov” terms) appear, potentially jeopardizing the Regge form of one-loop amplitudes

  In fact, terms not in accordance with the Regge form in different diagrams finally cancel out

  Reggeization outweighs renormalizability?

- Unexpected terms arise also within eikonal approximation (“Gribov” part), in rapidity regions where all next-to-leading order corrections would naively be attributed to the impact factor of the backward produced particle

- Relieving cancellation among the two unexpected effects.

- Prospects:

  - restore physical top mass

    [F.G. Celiberto, L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in progress)]

  - applications to phenomenology

https://indico.cern.ch/e/difflowx2024

Backup

BFKL in leading accuracy: Pomeron channel

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

Redefinition: $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q_r \mathcal{K}(\vec{q}_1, \vec{q}_r)\, G_\omega(\vec{q}_r, \vec{q}_2)$$

$$\mathcal{K}(\vec{q}_1, \vec{q}_2) = 2\omega(-\vec{q}_1^{\,2})\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r(\vec{q}_1, \vec{q}_2)$$

Infrared divergences cancel in the singlet kernel

$\mathcal{K}(\vec{q}_1, \vec{q}_2)$ is scale-invariant $\longrightarrow$ its eigenfunctions are powers of $\vec{q}_2^{\,2}$:

$$\int d^{D-2}q_2 \mathcal{K}(\vec{q}_1, \vec{q}_2)\,(\vec{q}_2^{\,2})^{\gamma-1} = \frac{N\alpha_s}{\pi}\chi(\gamma)\,(\vec{q}_1^{\,2})^{\gamma-1}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\,, \quad \psi(\gamma) = \frac{\Gamma'(\gamma)}{\Gamma(\gamma)}$$

The set of functions $(\vec{q}_2^{\,2})^{\gamma-1}$, with $\gamma = 1/2 + i\nu$, $\nu \in (-\infty, +\infty)$ is complete.

BFKL in leading accuracy

Total cross section for the process $A + B \to$ all

$$\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$$

$$= \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_A}{\vec{q}_A^{\,2}} \Phi_A(\vec{q}_A) \int \frac{d^{D-2}\vec{q}_B}{\vec{q}_B^{\,2}} \Phi_B(-\vec{q}_B) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_A, \vec{q}_B)$$

Using the complete set of kernel eigenfunctions, the BFKL equation and $D = 4$

$$\sigma_{AB}(s) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \int_{-\infty}^{+\infty} \frac{d\nu}{\omega - \frac{N\alpha_s}{\pi}\chi(1/2 + i\nu)}$$

$$\times \int \frac{d^2\vec{q}_A}{2\pi} \int \frac{d^2\vec{q}_B}{2\pi} \left(\frac{s}{s_0}\right)^{\omega} \Phi_A(\vec{q}_A) \frac{(\vec{q}_A^{\,2})^{-i\nu-3/2}}{\pi\sqrt{2}} \Phi_B(-\vec{q}_B) \frac{(\vec{q}_B^{\,2})^{i\nu-3/2}}{\pi\sqrt{2}}$$

Infrared finiteness guaranteed for colorless colliding particles

[V.S. Fadin, A.D. Martin (1999)]

BFKL in leading accuracy

Contour integration over ω

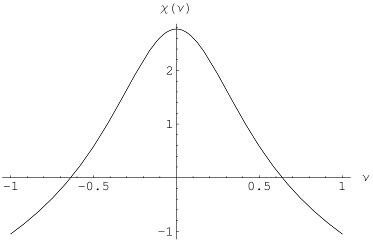
$$\sigma_{AB}(s) = \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi^2} \int \frac{d^2\vec{q}_A}{2\pi} \int \frac{d^2\vec{q}_B}{2\pi} \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s \chi(\nu)} \Phi_A(\vec{q}_A)(\vec{q}_A^{\,2})^{-i\nu-3/2} \Phi_B(-\vec{q}_B)(\vec{q}_B^{\,2})^{i\nu-3/2}$$

$$\bar{\alpha}_s \equiv \frac{N\alpha_s}{\pi}\ , \qquad \chi(\nu) \equiv \chi(1/2 + i\nu)$$

Saddle point approximation:
$\chi(\nu) = 4\ln 2 - 14\zeta(3)\nu^2 + O(\nu^4)$

$$\sigma_{AB}(s) \sim \frac{s^{4\bar{\alpha}_s \ln 2}}{\sqrt{\ln s}}$$

$\omega_P = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ for $\alpha_s = 0.15$



- unitarity is violated; BFKL cannot be applied at asymptotically high energies
- the scale of s and the argument of the running coupling constant are not fixed in the LLA $\longrightarrow$ NLA

BFKL in next-to-leading accuracy

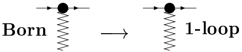
Production amplitudes keep the simple factorized form

$$\mathrm{Re}\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

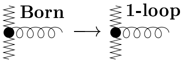
but, with respect to the LLA case, one replacement is allowed among the following:

multi-Regge kinematics

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\,\text{(Born)}} \longrightarrow \Gamma_{P'P}^{c\,\text{(1-loop)}}$



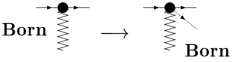
- $\gamma_{c_i c_{i+1}}^{G_i\text{(Born)}} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i\text{(1-loop)}}$



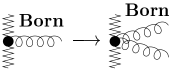
BFKL in next-to-leading accuracy

quasi-multi-Regge kinematics

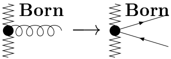
- $\Gamma^{c\,(\text{Born})}_{P'P} \longrightarrow \Gamma^{c\,(\text{Born})}_{\{f\}P}$



- $\gamma^{G_i(\text{Born})}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}(\text{Born})}_{c_i c_{i+1}}$



- $\gamma^{G_i(\text{Born})}_{c_i c_{i+1}} \longrightarrow \gamma^{GG(\text{Born})}_{c_i c_{i+1}}$



This is the program of calculation of radiative corrections to the LLA BFKL

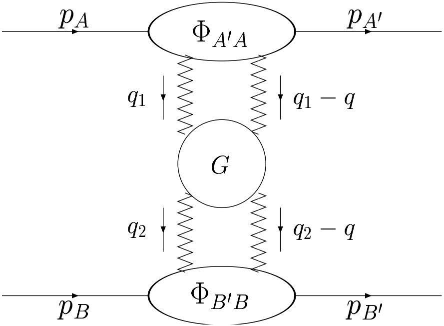
[V.S. Fadin, L.N. Lipatov (1989)]

BFKL in next-to-leading accuracy

- $\omega^{(2)}(t)$ [V.S. Fadin (1995)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)] [V.S. Fadin, R. Fiore, A. Quartarolo (1996)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)] [V.S. Fadin, M.I. Kotsky (1996)]
- $\gamma^{G_i}_{c_i c_{i+1}}$ (1-loop) [V.S. Fadin, L.N. Lipatov (1993)] [V.S. Fadin, R. Fiore, A. Quartarolo (1994)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)] [V.S. Fadin, R. Fiore, A. P. (2001)]
- $\Gamma^{c}_{P'P}$ (1-loop) [V.S. Fadin, R. Fiore (1992)] [V.S. Fadin, L.N. Lipatov (1993)] [V.S. Fadin, R. Fiore, A. Quartarolo (1994)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]
- $\gamma^{Q\overline{Q}}_{c_i c_{i+1}}$ (Born) [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)] [S. Catani, M. Ciafaloni, F. Hautmann (1990)] [G. Camici, M. Ciafaloni (1996)]
- $\gamma^{GG}_{c_i c_{i+1}}$(Born) [V.S. Fadin, L.N. Lipatov (1996)] [V.S. Fadin, M.I. Kotsky, L.N. Lipatov (1997)]

BFKL in next-to-leading accuracy

Structure of the amplitude:



$$\mathrm{Im}_s(\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1; \vec{q}; s_0)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2; -\vec{q}; s_0)$$

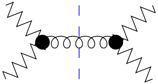
BFKL in next-to-leading accuracy

- $G_\omega^{(\mathcal{R})}$ – Mellin transform of the Green's functions for Reggeon-Reggeon scattering

$$\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right) = \vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^{\,2}\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r-\vec{q})^{\,2}}\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_r;\vec{q}\right) G_\omega^{(\mathcal{R})}\left(\vec{q}_r,\vec{q}_2;\vec{q}\right)$$

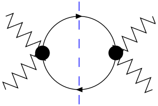
$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right)+\omega\left(-(\vec{q}_1-\vec{q})^{\,2}\right)\right]\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right)+\mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$$

In the NLA: $\omega(t)=\omega^{(1)}(t)+\omega^{(2)}(t)$, $\quad \mathcal{K}_r=\mathcal{K}_{RRG}^{(B)}+\mathcal{K}_{RRG}^{(1)}+\mathcal{K}_{RRQ\overline{Q}}^{(B)}+\mathcal{K}_{RRGG}^{(B)}$



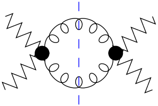
$\mathcal{K}_{RRG}^{(1)}$

$t=0$: [V.S. Fadin, L.N. Lipatov (1993)]
[V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
[V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (2001)]



$\mathcal{K}_{RRQ\overline{Q}}^{(B)}$

$t=0$: [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (1999)]



$\mathcal{K}_{RRGG}^{(B)}$

$t=0$: [V.S. Fadin, L.N. Lipatov, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, D.A. Gorbachev (2000)]
[V.S. Fadin, R. Fiore (2005)]

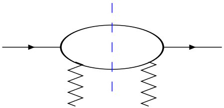
− **counterterm**

BFKL in next-to-leading accuracy

- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ – impact factors in the t-channel color state $(\mathcal{R},\nu)$

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^{c}_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$

$$\times \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{\omega(-\vec{q}_1^{\,2})}{2}} \left(\frac{s_0}{(\vec{q}_1 - \vec{q})^2}\right)^{\frac{\omega(-(\vec{q}_1-\vec{q})^2)}{2}}$$



$-$ **counterterm**

non-trivial momentum and scale-dependence

BFKL in next-to-leading accuracy

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \; , \; \gamma = \frac{1}{2} + i\nu \right)$$

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) \left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \frac{N\alpha_s(\vec{q}_1^{\,2})}{\pi} \left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^{\,2})}{\pi} \chi^{(1)}(\gamma) \right) \left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

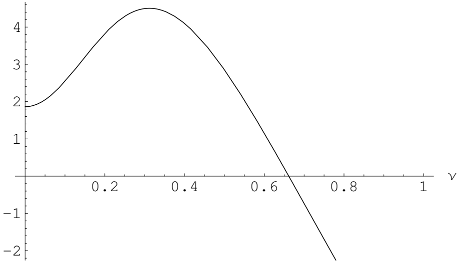
- broken scale invariance
- large corrections: $\quad -\left.\frac{\chi^{(1)}(\gamma)}{\chi(\gamma)}\right|_{\gamma=1/2} \simeq 6.46 + 0.05\,\frac{n_f}{N} + 0.96\,\frac{n_f}{N^3}$

[V.S. Fadin, L.N. Lipatov (1998)] [G. Camici, M. Ciafaloni (1998)]

BFKL in next-to-leading accuracy

$\chi(\nu) + \bar{\alpha}_s(\vec{q}_1^{\,2})\chi^{(1)}(\nu)$ *vs* ν

$\bar{\alpha}_s(\vec{q}_1^{\,2}) \equiv \frac{\alpha_s(\vec{q}_1^{\,2})N}{\pi} = 0.15$



Double maxima $\longrightarrow$ oscillations in momentum space after ν-integration

Ways out:

- rapidity veto [C.R. Schmidt (1999)] [J.R. Forshaw, D.A. Ross, A. Sabio Vera (1999)]
- collinear improvement [G. Salam (1998)] [M. Ciafaloni, D. Colferai (1999)]
- renormalization with a physical scheme [S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov (1999)]
- ...

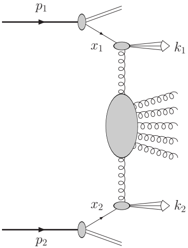
Phenomenology

A few more impact factors became available in the hybrid collinear/high-energy factorization in the NLA ...

- jet vertex [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
  [D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)
  [D. Colferai, A. Niccoli (2015)]
- hadron vertex [D.Yu. Ivanov, A.P. (2012)]
- Higgs vertex
  [M.A. Nefedov (2019)] [M. Hentschinski, K. Kutak, A. van Hameren (2020)]
  [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2022)]

... to be added to several LA ones:

- J/Ψ vertex [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair vertex [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- heavy-quark production vertex
  [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2019)]



$$\text{proton}(p_1) + \text{proton}(p_2) \longrightarrow \text{object}_1(k_1) + X + \text{object}_2(k_2)$$

Taking also into account the large variety of available FFs, a plethora of predictions were recently produced for the inclusive production at the LHC of a forward and a backward identified 'objects'.

Full-NLA analyses (for LHC and possibly FCC):

- jet + jet (Mueller-Navelet) [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)] [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)] [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)] [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2015,2016)] [F.G. Celiberto, A.P. (2022)]
- light hadron + light hadron [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2016,2017)]
- light hadron + jet [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2018)]
- $\Lambda + \Lambda$, Λ + jet [F.G. Celiberto, D.Yu. Ivanov, A.P. (2020)]
- $\Lambda_c + \Lambda_c$, Λ_c + jet [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2020)]
- b-hadron + b-hadron, b-hadron + jet [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]
- $B_c(B_c^*)$ + b-hadron, $B_c(B_c^*)$ + jet [F.G. Celiberto (2022)]
- J/ψ or Υ + jet [F.G. Celiberto, M. Fucilla (2022)]
- heavy-light tetraquark + b, c-hadron or jet [F.G. Celiberto, A. Papa (2023)]

Partial-NLA analyses:

- J/Ψ + jet [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair + jet [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- Higgs + jet [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2021)]
- light jet + heavy jet [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2021)]
- Higgs + c-hadron [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]

Observables

BFKL-factorized form, with "differential" impact factors:

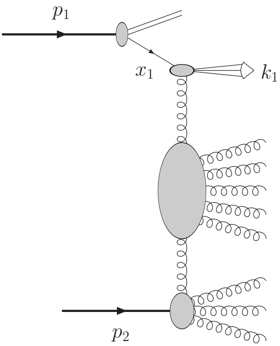
$$\frac{d\sigma}{dy_1 dy_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\phi_1 d\phi_2} = \frac{d\Phi_1}{dy_1\, d|\vec{k}_1| d\phi_1} \otimes G \otimes \frac{d\Phi_2}{dy_2\, d|\vec{k}_2| d\phi_2}$$

$$= \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right], \quad \phi = \phi_1 - \phi_2 - \pi$$

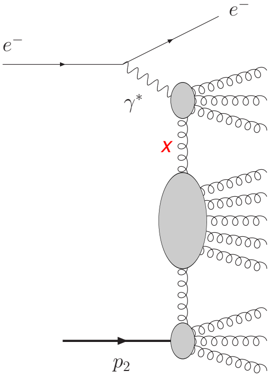
- azimuthal correlations, $\langle\cos(n\phi)\rangle = \mathcal{C}_n/\mathcal{C}_0$, and ratios between them [A. Sabio Vera, F. Schwennsen (2007)]
- cross sections differential in $Y \equiv y_1 - y_2$
- ...

Inclusive processes: single forward

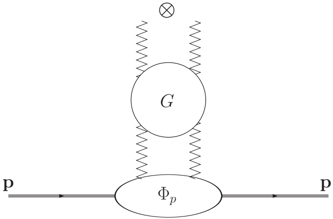
Another important class of inclusive processes arises when there is just one identified object (jet, hadron, Higgs boson, ...), produced in the fragmentation region of one of the colliding particles.



Single-forward inclusive production of an identified object with momentum k_1



Small-x
deep inelastic scattering



$$\underbrace{G \otimes \Phi_p}_{\text{UGD}}$$

Straightforward emergence of the "unintegrated gluon distribution (UGD)":

- valid both in the LLA and in the NLA
- not unambiguous: normalization, s_0-scale setting, etc., follow from the definition of the impact factor to be convoluted with
- non-perturbative ...
  - the proton impact factor Φ_p is intrinsically non-perturbative
  - unavoidable diffusion in the infrared of the transverse momentum integration
- ... but $(\ln x)$-resummation automatically encoded.

BFKL-inspired UGD models adopted for deep inelastic scattering at HERA

[G. Chachamis, M. Déak, M. Hentschinski, G. Rodrigo, A. Sabio Vera (2013)]

and several (exclusive) electroproduction processes

[G. Chachamis, M. Déak, M. Hentschinski, G. Rodrigo, A. Sabio Vera (2015)]

[I. Bautista, A. Fernandez Tellez, M. Hentschinski (2016)]

[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A.P. (2018)]

[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A.P., W. Schäfer (2021)]