Recent theory developments on the physics of quark gluon plasma

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Sesto Incontro Nazionale di Fisica Nucleare, INFN-TIFPA

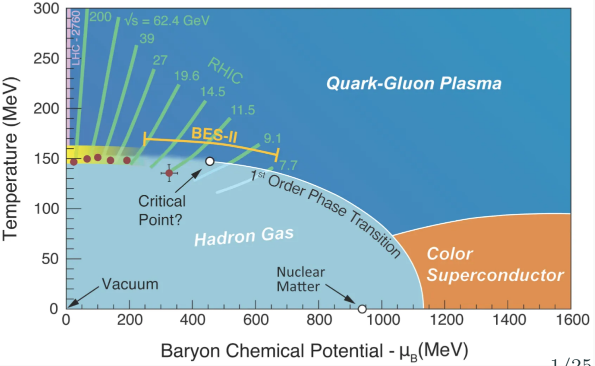
The quark gluon plasma and the QCD phase diagram

What: deconfined and chirally restored medium

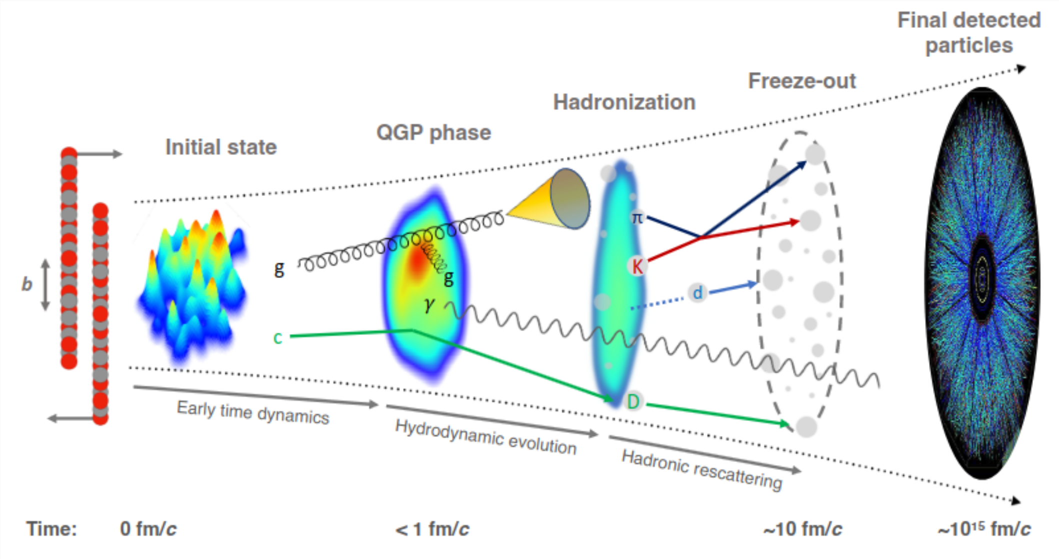
Where/when: early Universe down to $\sim 10^{-6} - 10^{-5}$ s after the Big Bang, AND iven sufficient energy, we can re-create it in relativistic heavy-ion collisions

Summarize our knowledge in the **QCD phase diagram**, with evidence from experiment + theory

- Hadron phase at low T/μ, QGP at high T/μ
- Crossover at zero density at $T \simeq 160\,\text{MeV}$
- **Heavy-ion collisions** probe high T, varying density with energy scans
- Ordinary nuclear matter at $T \simeq 0$ and $\mu_B \simeq 922\,\text{MeV}$
- Critical point? Exotic phases?



Heavy-ion collisions: the “standard model”



Arslandok *et al.*, 2303.17254

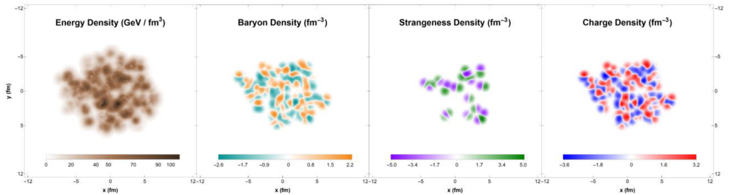
Hydrodynamic simulations

The bulk evolution of the system is described by **relativistic viscous hydrodynamics**:

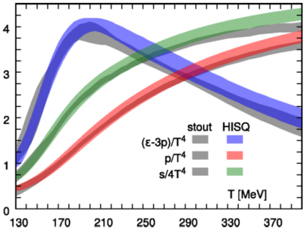
$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu N^\mu = 0$$

$\rightarrow$ evolution from conservation equations

Properties of the medium is encoded in **transport coefficients**: shear viscosity η/s, bulk viscosity ζ/s, ...



Plumberg *et al.*, 2312.07415



Borsanyi *et al.*, PLB 730 (2014) 99

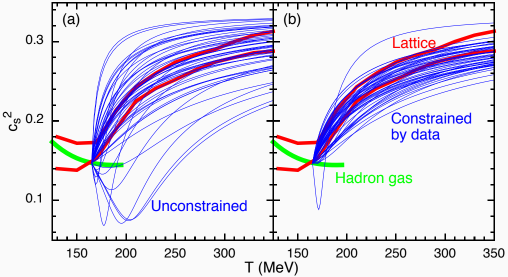
Input: initial conditions, equation of state (to close the set of equations)

Output: after hadronization and an afterburner, particle spectra $\rightarrow R_{AA}$, v_2, ...

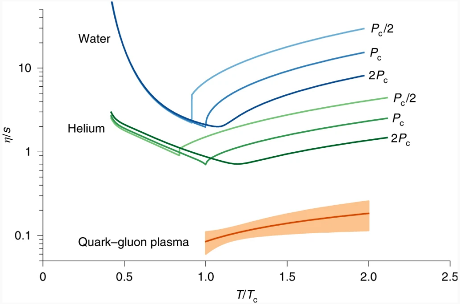
Hydrodynamic simulations

Hydrodynamic simulations can reproduce experimental data and constrain the physics

Bayesian analyses have been used to constrain e.g. the **equation of state** and the **viscosity**



Pratt *et al.*, PRL 114 (2015) 202301

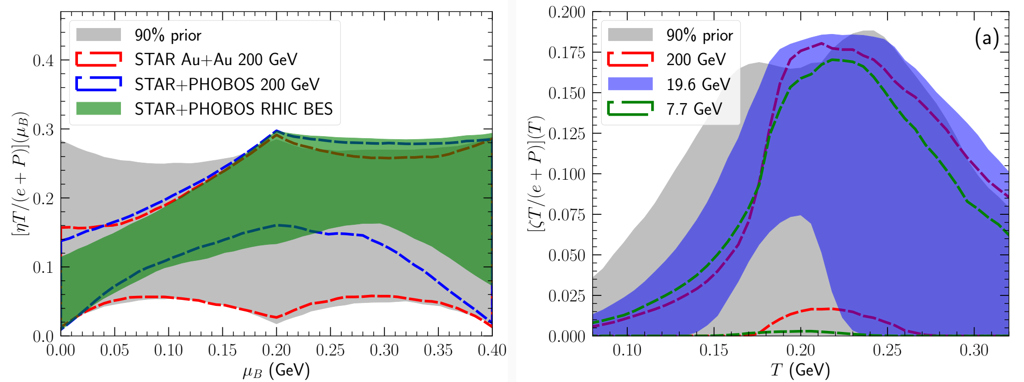


Bernhard *et al.*, Nature Phys. 15 (2019) 1113

- Posterior equation of state in agreement with theroetical calculations
- QGP is the most perfect fluid know to humanity!

Hydrodynamic simulations: shear and bulk viscosity

Large scale Bayesian analysis on Au+Au RHIC data @ $\sqrt{s}$ =200, 19.6, 7.7 GeV

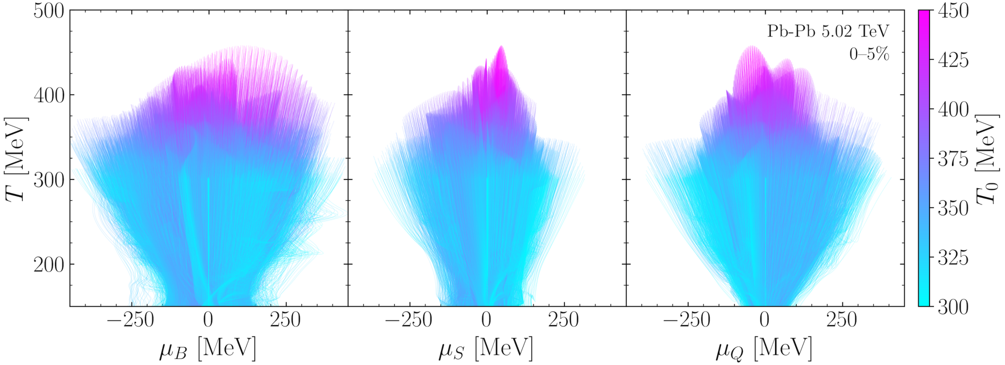


Very tight constraints on the chemical potential dependence of shear and bulk viscosity

Shen *et al.*, PRL 132 (2024) 072301

Hydrodynamic simulations: B,Q,S conserved charges

Full trajectories of hydro cells until hadronization. We can have a picture of density fluctuations in all B, Q, S, as a function of T



Plumberg *et al.*, 2312.07415

Results obtained with a 4D lattice QCD-based equation of state. Big challenge for eos, very large μ needed even at LHC, where $\langle \mu_i \rangle = 0$

Thermodynamic description of QCD

The thermodynamics of QCD is of fundamental interest, in itself and as input for a number of applications

- **Transition line** necessary for models, information on phase structure

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 + \kappa_{\mathbf{2}} \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \kappa_{\mathbf{4}} \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 + \mathcal{O}(\mu_B^6)$$

- **Equation of state**: p, s, n_i, ϵ, input for hydro and models, from zero to large density

- **Fluctuations of conserved charges**

$$\chi_{ijk}^{BQS}(T) = \left.\frac{\partial^{i+j+k}\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}\right|_{\mu=0}$$

  - Connection to experiment (cumulants of net-proton, net-charge, etc.)
  - Signatures for the QCD critical point

Lattice formulation of QCD

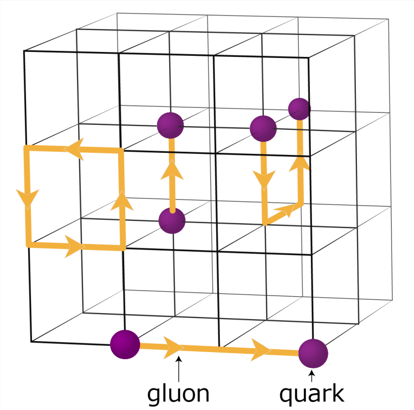
In the non-perturbative regime where the coupling g_s is not small, lattice QCD is the major tool of investigation of equilibrium properties of QCD

- The theory is defined on a discretized 3+1d spacetime. Quark fields on the sites, gauge fields on the links.
- The partition function is given by a finite number of integrals:

$$Z[U,\bar{\psi},\psi] = \int \mathcal{D}U\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{-S_G[U,\bar{\psi},\psi]-S_F[U,\bar{\psi},\psi]}$$
$$= \int \mathcal{D}U\ \det M[U]\, e^{-S_G[U]}$$

- Observables $\hat{O}$ can then be calculated as:

$$\left\langle \hat{O} \right\rangle = \frac{1}{Z} \int \mathcal{D}U\, \hat{O}\ \det M[U]\, e^{-S_G[U]}$$

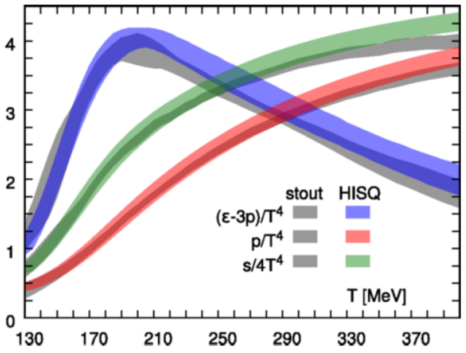


The equation of state of QCD

- Lattice QCD is the most robust tool to determine QCD thermodynamics
- Known at $\mu_B = 0$ to high precision for a few years now (continuum limit, physical quark masses) $\longrightarrow$ Agreement between different calculations (2013-2014)

From grancanonical partition function $\mathcal{Z}$

* **Pressure**: $p = -k_B T \frac{\partial \ln \mathcal{Z}}{\partial V}$
* **Entropy density**: $s = \left(\frac{\partial p}{\partial T}\right)_{\mu_i}$
* **Charge densities**: $n_i = \left(\frac{\partial p}{\partial \mu_i}\right)_{T,\mu_{j\neq i}}$
* **Energy density**: $\epsilon = Ts - p + \sum_i \mu_i n_i$
* **Speed of sound**: $c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{s/n_B}$



Borsányi *et al.*, PLB 730 (2014) 99
Bazavov *et al.*, PRD 90 (2014) 094503

Finite density: the sign/complex action problem

Statistical weight becomes complex if μ_B is real $\Rightarrow$ sampling algorithms break down

$$Z(V,T,\mu) = \int \mathcal{D}U \, \det M(U,\mu) e^{-S_G(U)}$$

Not if $\mu = 0$, and not for imaginary chemical potential ($\mu^2 < 0$).

Several alternatives nowadays:

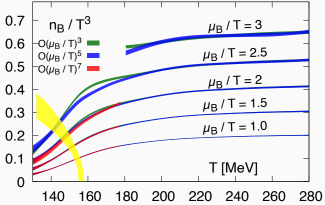
- Taylor expansion:

$$\frac{p(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n} , \qquad c_n(T) = \frac{1}{n!}\chi_n^B(T,\mu_B=0)$$

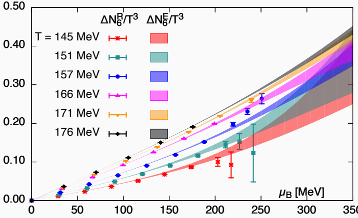
- Analytical continuation from imaginary μ_B
- Reweighting: single parameter, multi-parameter, phase quenched, sign quenched

Taylor, analytic continuation, reweighting

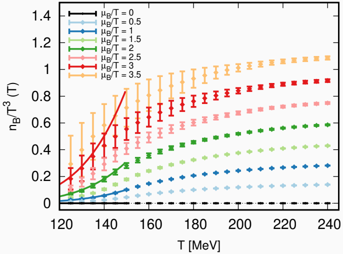
Taylor expansion Bollweg+ ’22



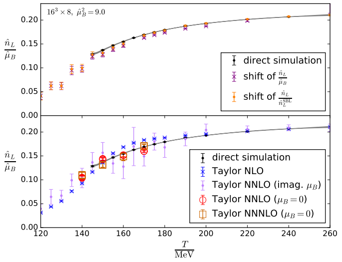
Approximate reweighting Mondal+ ’21



Alternative expansion Borsanyi+ ’21, ’22



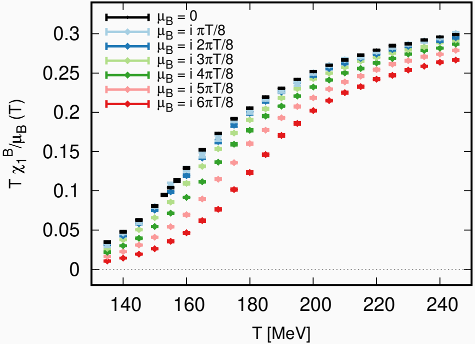
Reweighting Borsanyi+ ’22

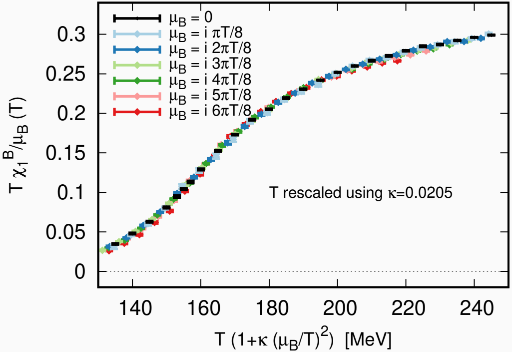


Finite density: an alternative expansion scheme

One *observes* in imaginary μ_B simulations that $\chi_1^B(T,\,\hat{\mu}_B)$ differs from $\chi_2^B(T,0)$ only by **a redefinition of** T:

$$\frac{\chi_1^B(T,\,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)\ , \qquad \text{with} \quad T' = T\left(1+\kappa\,\hat{\mu}_B^2\right)$$





Borsányi, PP *et al.*, PRL 126 (2021) 232001, PRD 105 (2022) 114504

Equation of state at finite $\hat{\mu}_B$

- Allow for more than $\mathcal{O}(\hat{\mu}^2)$ expansion of T' and let the coefficients be $T-$dependent:

$$\frac{\chi_1^B(T,\,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)\ , \qquad\qquad \text{with} \quad T' = T(1+\kappa_2(T)\,\hat{\mu}_B^2 + \kappa_4(T)\,\hat{\mu}_B^4)$$

(we are simply re-organizing the Taylor expansion via an expansion in $\Delta T = T - T'$)

- Determine $\kappa_2(T)$, $\kappa_4(T)$ (from $\mu_B = 0$ or imaginary)

- Thermodynamics at finite μ_B is reconstruted from the same ansazt:

$$\frac{n_B(T,\,\hat{\mu}_B)}{T^3} = \hat{\mu}_B \chi_2^B(T',0)$$

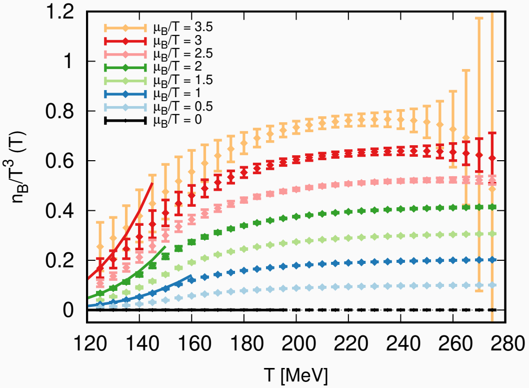
From the baryon density n_B one finds the pressure, then all other quantities

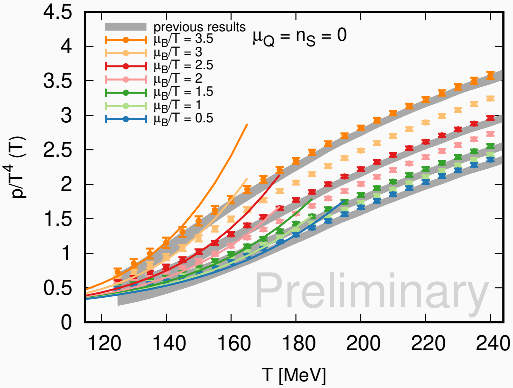
$$\frac{p(T,\,\hat{\mu}_B)}{T^4} = \frac{p(T,0)}{T^4} + \int_0^{\hat{\mu}_B} \mathrm{d}\hat{\mu}_B' \frac{n_B(T,\,\hat{\mu}_B')}{T^3} \qquad \longrightarrow \qquad s, \epsilon, c_s^2, \cdots$$

Equation of state at finite $\hat{\mu}_B$

Thermodynamic quantities have uncertainties well under control up to $\hat{\mu}_B \simeq 3.5$

No pathological (non-monotonic) behavior typical of other expansions





Note: recently reduced errors by improving precision at $\mu_B = 0$

Borsányi, PP *et al.*, PRL 126 (2021) 232001, PRD 105 (2022) 114504, in preparation

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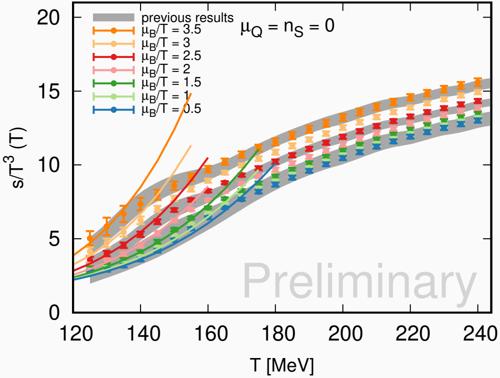

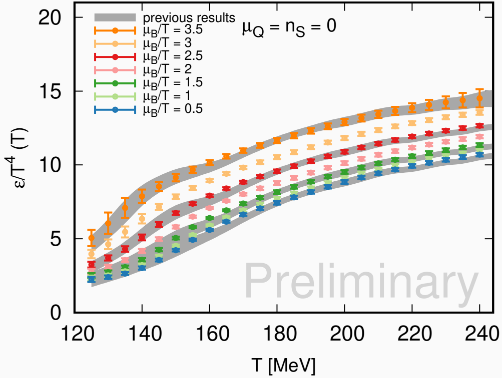

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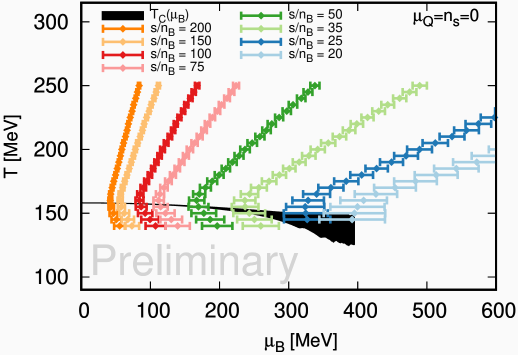
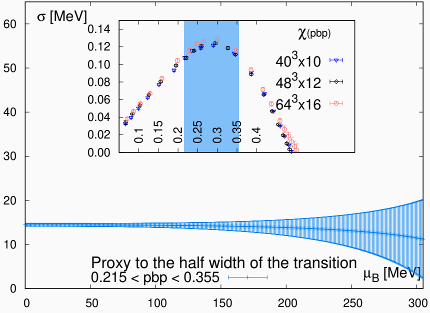
Borsányi, PP *et al.*, PRL 126 (2021) 232001, PRD 105 (2022) 114504, in preparation

Equation of state at finite $\hat{\mu}_B$

Very broad coverage in μ_B with small errors, no sign of critical lensing





Borsányi *et al.*, PRL 125 (2020) 052001

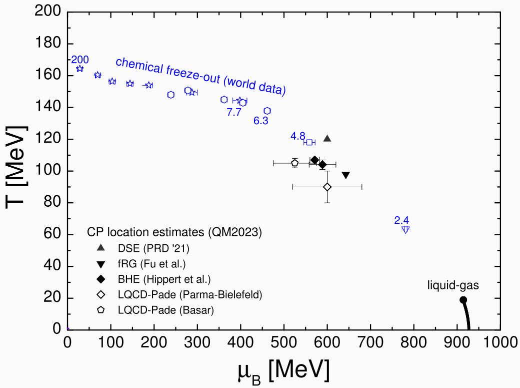
Compatible with results on the width of crossover transition at finite μ_B

Hydro with BQS has shown the need to extend to 4D eos Abuali, PP *et al.*, in preparation

The QCD critical point

The crossover is “expected” to turn first order at larger μ_B

Critical point would be in the same universality class as 3D Ising model

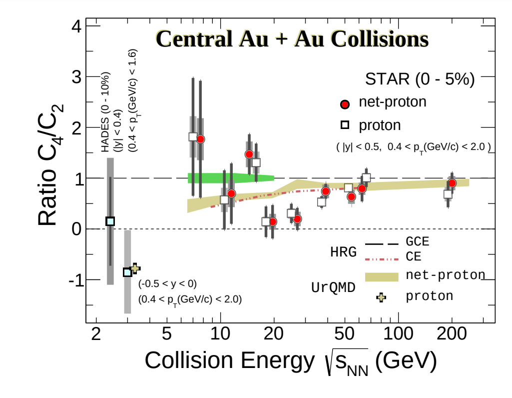


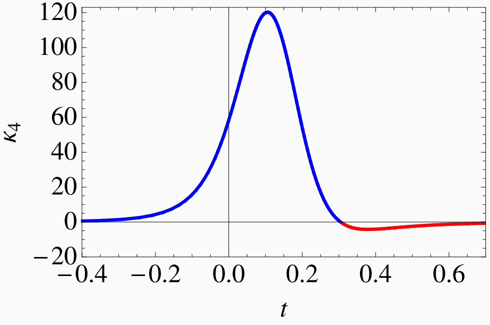
Vovchenko, 2312.09528

→ recent estimates seem to “converge”

Looking for critical behavior: experiment

Baryon fluctuations diverge at the critical point with increasing powers of the correlation length → higher order net-proton fluctuations are most promising





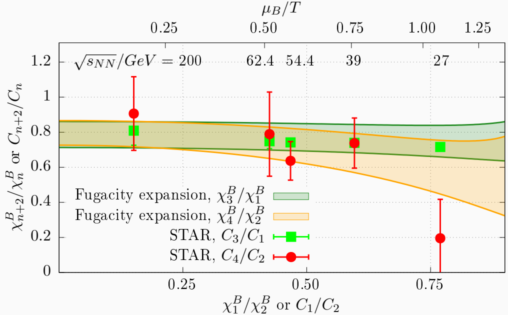
Suggestive behaviour, though errors are still large → STAR data coming soon

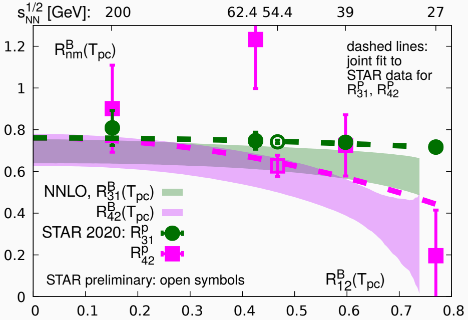
STAR, PRL 128 (2022), 202303; Stephanov, PRL 107 (2011) 052301

Looking for critical behavior: extrapolations

Extrapolations from lattice can be made for fluctuations too, e.g.:

$$\chi_2^B(T,\mu_B) = \chi_2^B(T) + \frac{1}{2}\chi_4^B(T) + \frac{1}{24}\chi_6^B(T) + \cdots$$





Lattice results similar to experiment, hard to draw conclusions for now

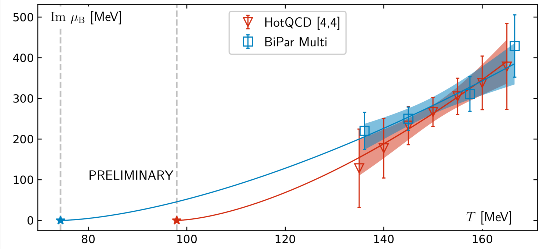
Borsányi *et al.*, PRD 104 (2021) 094508Bazavov *et al.*, PRD101 (2020), 074502

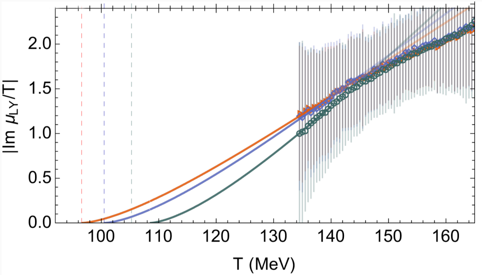
Looking for critical behavior: Lee-Yang edge singularities

Complex zeroes of partition function (LYE edges) mode to real μ at critical point

Zeroes from Padé fits to lattice QCD fluctuations, then universality-motivated T−fit

$$\mathrm{Im}(\mu_{LY})(T) = A(T - T_c)^{\beta\delta}$$





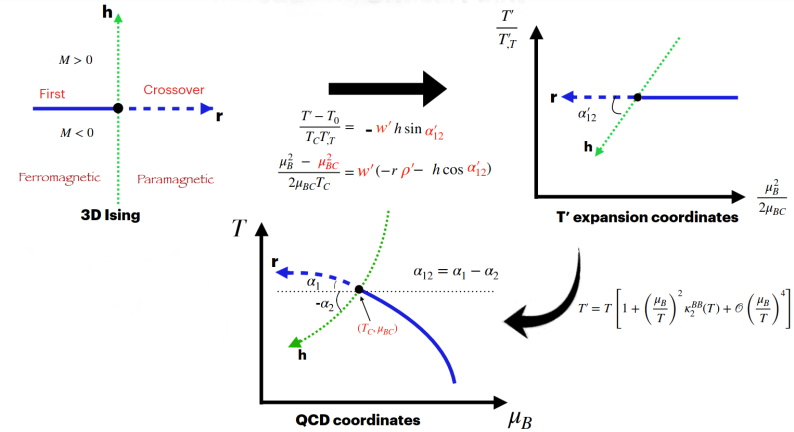
$$\Rightarrow \qquad T_c \sim 100 \pm 30\,\mathrm{MeV} \qquad \mu_{BC} \sim 550 \pm 50\,\mathrm{MeV}$$

- Assumes critical point exists and we are close to it
- Errors are large and hard to constrain

Clarke *et al.*, 2401.08820; Basar, 2312.06952

Looking for critical behavior: critical equation of state

Combine lattice QCD alternative expansion scheme with 3D Ising critical behaviour

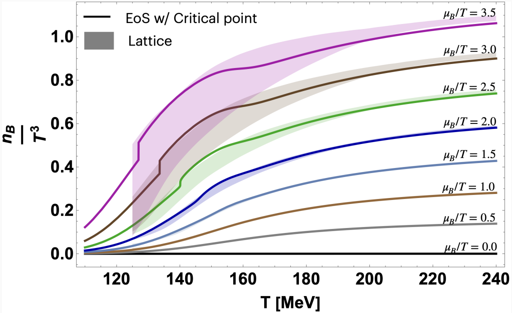


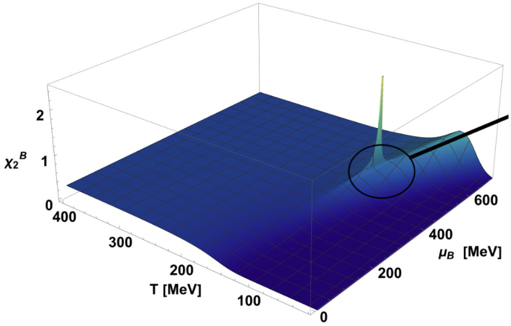
Family of equations of state for $T = 10 - 800\,\text{MeV}$, $\mu_B = 0 - 700\,\text{MeV}$

Kahangirwe, PP *et al.*, 2402.08636

Looking for critical behavior: critical equation of state

By construction, the equations of state agree with lattice QCD at low μ_B, AND have the correct critical behavior near the CP





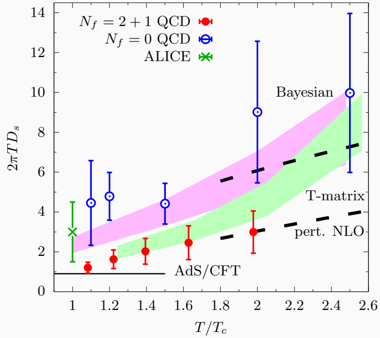
- **Thermodynamic consistency** constrains parameters, including location of CP
- As **input to hydro simulations** → comparison with data provides further constraints

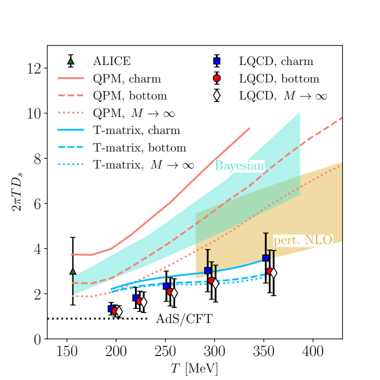
Kahangirwe, PP *et al.*, 2402.08636

A touch on heavy-flavour: heavy quark diffusion

Influence of the meduim on heavy quarkonium encoded in **transport coefficients**

First dynamical lattice results for spatial diffusion coefficient D_s





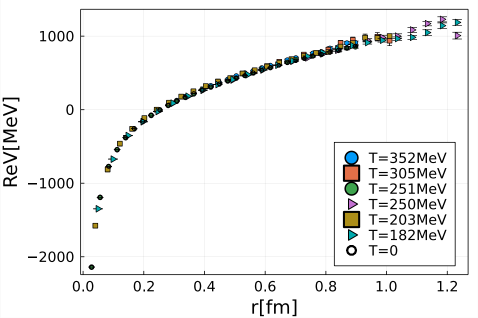
Even smaller than previous (quenched) calculations and than phenomenological estimates
⇒ **very fast thermalization** of heavy quarks in the medium

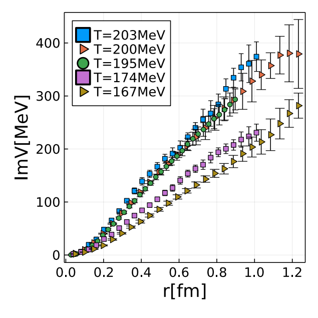
Altenkort *et al.*, PRL 130 (2023) 231902, PRL 132 (2024) 051902

Heavy $Q\bar{Q}$ potential

Quarkonium is suppressed in the medium compared to vacuum (pp) events

Standard picture (Matsui, Satz) is that of dissolution by screening





New results on heavy quark-antiquark potential show no screening in the real part of the potential

The imaginary part of the potential still accounts for dissolution

Bazavov *et al.*, 2308.16587

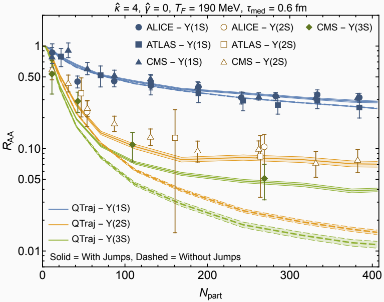
Quarkonium: open quantum systems

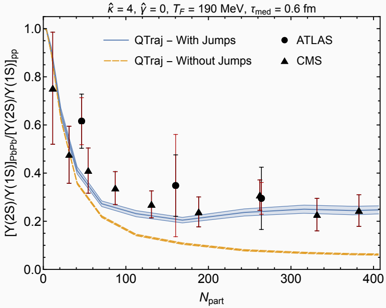
Quarkonium is treated as an open quantum system in contact with the environment

Evolution of the density matrix ρ encodes all the information on the system. Lindblad equation with jump operators L_n that encode the non-unitary evolution of the system (imaginary potential)

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \sum_n \left(L_n \rho L_n^\dagger - \frac{1}{2}\left\{ L_n^\dagger L_n, \rho \right\} \right)$$

Inclusion of jumps needed for quantitative agreement with data

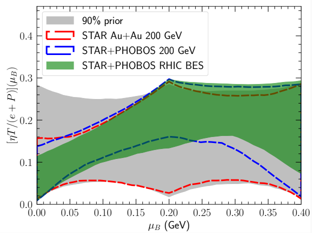


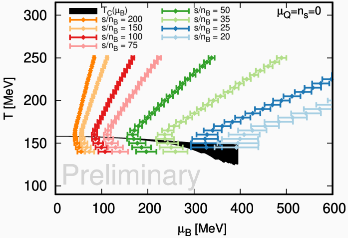


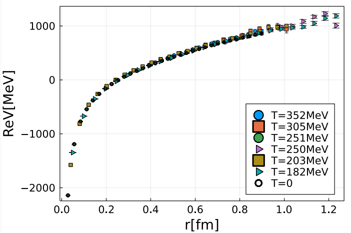
Brambilla *et al.*, PRD 108 (2023) L011502

Summary

New exciting results in all areas of QGP theory!







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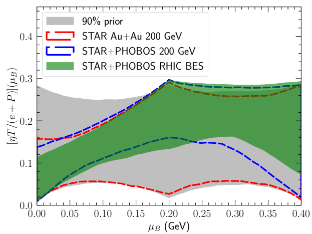
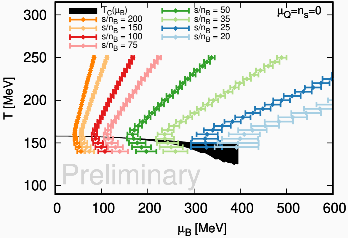
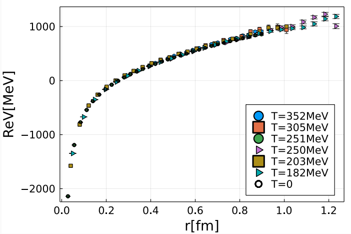




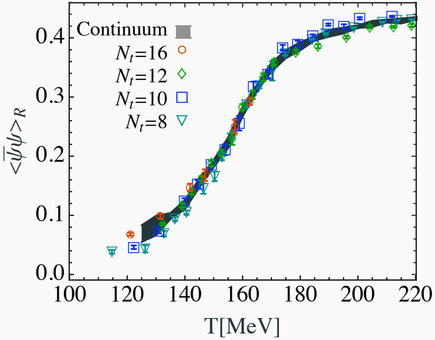


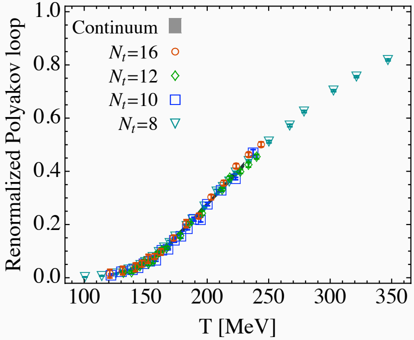
THANK YOU!

BACKUP

Confinement and chiral symmetry breaking

At low temperature and density, quarks and gluons are **confined** inside hadrons. The approximate **chiral symmetry** of QCD is **spontaneously broken**





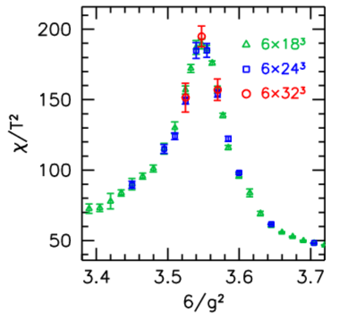
At large temperature and/or density, a **deconfined medium** is formed, with quasi-free quarks and gluons, with **effectively restored chiral symmetry**

Borsányi *et al.*, JHEP 1009:073 (2010)

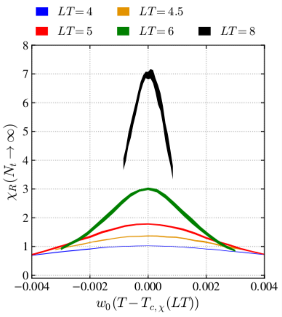
The QCD transition: crossover vs. first order

On the lattice we study the volume scaling of certain quantities to determine the order of the transition

Left: physical masses



Right: infinite masses (pure gauge)

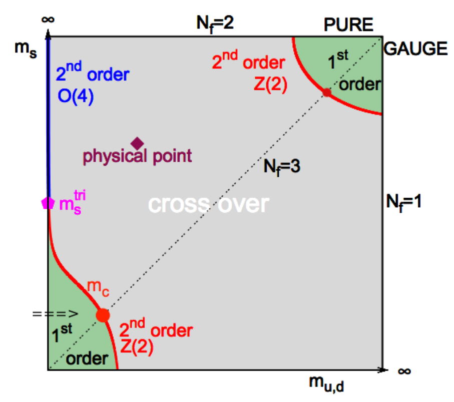


- For a crossover (left), **the peak height is independent of the volume**
- For a first order transition, **it scales linearly with the volume**

Aoki *et al.* Nature 443 (2006), Borsányi, PP *et al.*, PRD 105 (2022)

The QCD transition: Columbia plot

As a function of the light (u,d) and strange quark masses, the order of the transition changes



- At the physical point $m_s/m_{ud} \simeq 27$, the transition is a smooth crossover!
- In the heavy-quark limit (pure gauge), the transition is first order

Fluctuations of conserved charges

- **Theory**: grand canonical fluctuations are derivatives of the free energy:

$$Z(V,T,\mu_B,\mu_Q,\mu_S) = \sum_{B,Q,S} e^{B\mu_B} e^{Q\mu_Q} e^{S\mu_S} Z_C(V,T,B,Q,S)$$

wrt the associated chemical potentials:

$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \frac{1}{VT^3} \frac{\partial^{i+j+k} \ln Z\left(T,\mu_B,\mu_Q,\mu_S\right)}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}$$

- **Experiment**: moments/cumulants $\langle(\Delta N)^n\rangle_{\text{events}}$ of net-particle distributions:

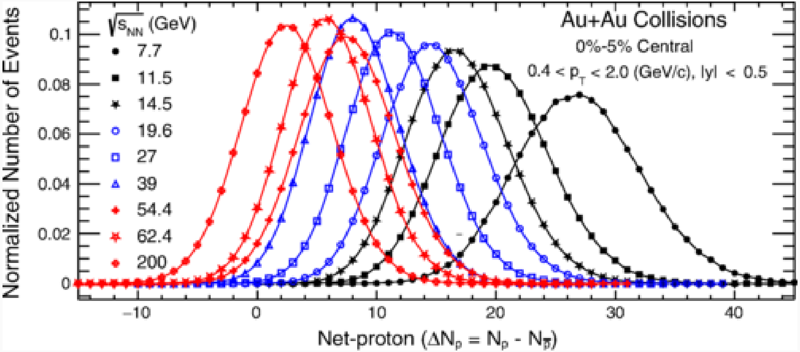
$$\langle B\rangle = \frac{1}{VT^3} \frac{\partial \ln Z\left(T,\mu_B,\mu_Q,\mu_S\right)}{\partial\left(\mu_B/T\right)} = \chi_1^B$$

$$\left\langle B^2\right\rangle - \langle B\rangle^2 = \frac{1}{VT^3} \frac{\partial^2 \ln Z\left(T,\mu_B,\mu_Q,\mu_S\right)}{\partial\left(\mu_B/T\right)^2} = \chi_2^B$$

$$\langle BS\rangle - \langle B\rangle\langle S\rangle = \frac{1}{VT^3} \frac{\partial^2 \ln Z\left(T,\mu_B,\mu_Q,\mu_S\right)}{\partial\left(\mu_B/T\right)\partial\left(\mu_S/T\right)} = \chi_{11}^{BS}$$

Heavy-ion collisions: event-by-event fluctuations

- Conserved charges in QCD are all quark numbers
  $\longrightarrow$ B (baryon number), Q (electric charge), S (strangeness)
- Weak effects are not considered (time's too short)
- Charm is ignored (might not thermalize)
- Conserved charges are conserved only on average in experiment

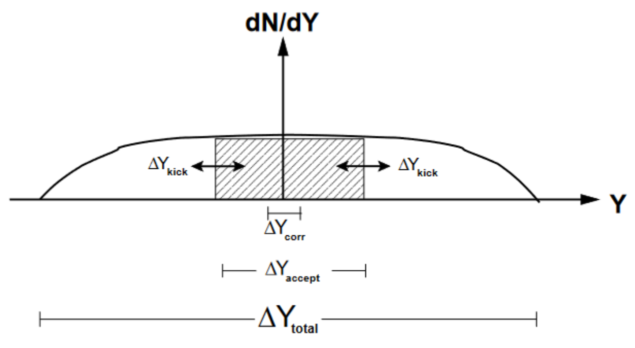


STAR Collaboration: PRL 112 (2014) 032302

Fluctuations of conserved charges

How can CONSERVED CHARGES fluctuate?

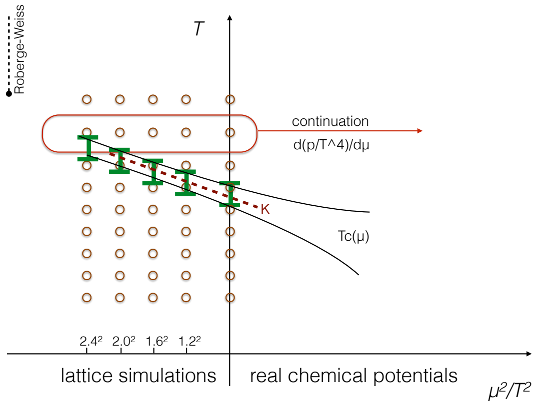
- If we could measure ALL particles in a collision, they would not
- If we look at a small enough subsystem, fluctuations occur and become meaningful



Simulations at imaginary chemical potential

- While for real chemical potential ($\mu^2 > 0$) $\det M(U)$ is complex, for **imaginary** chemical potential ($\mu^2 < 0$) $\det M(U)$ is real
- We perform simulations at imaginary chemical potentials:

$$\hat{\mu}_B = i\frac{j\pi}{8} \quad j = 0, 1, 2, \ldots$$



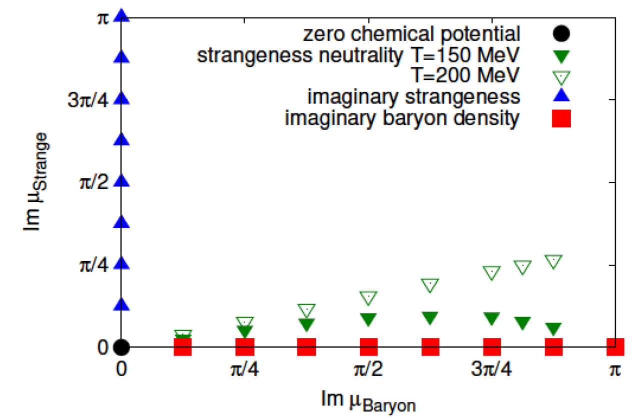
We then analytically continue to $\mu^2 > 0$ by means of suitable extrapolation schemes

Simulations at imaginary chemical potential

Strangeness neutrality (or not)

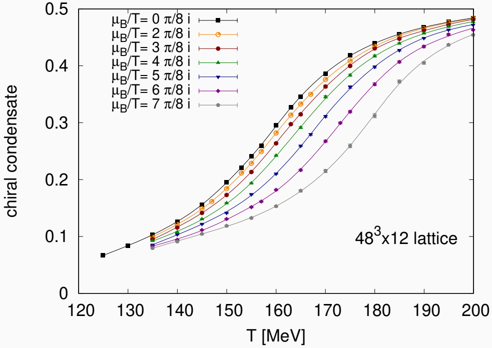
Set the chemical potentials for heavy-ion collisions scenario, or simpler setup:

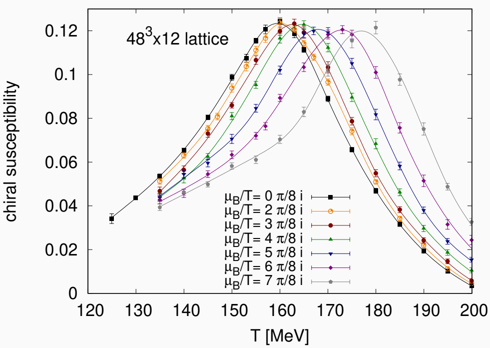
$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle \qquad \text{or} \qquad \mu_Q = \mu_S = 0$$



Chiral observables at imaginary μ_B

Chiral condensate and chiral susceptibility at imaginary chemical potential





Borsányi, PP *et al.* PRL 125 (2020), 052001

Lattice QCD at finite μ_B - Taylor coefficients

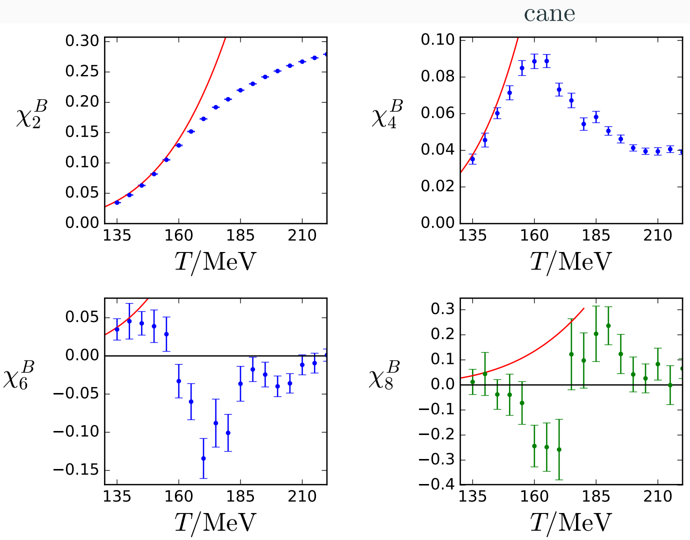
cane

- Fluctuations of baryon number are the Taylor expansion coefficients of the pressure

$$\chi_{ijk}^{BQS}(T) = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$$

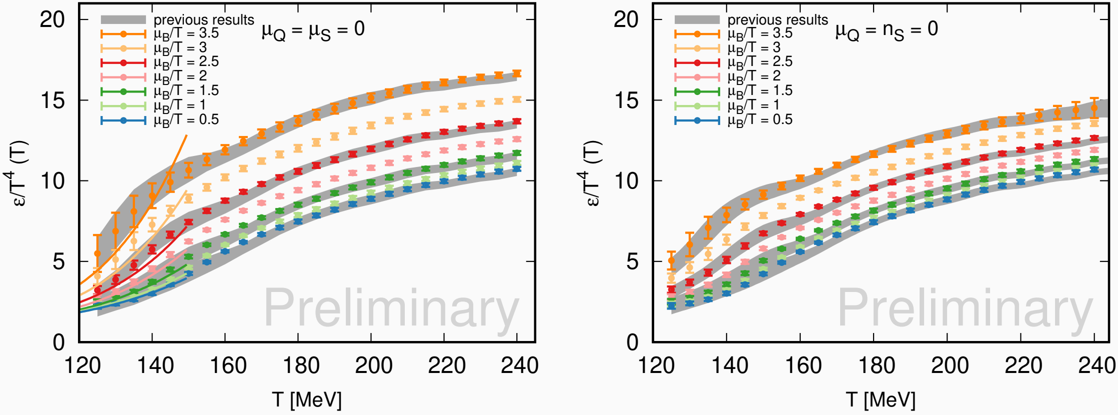
- Signal extraction is increasingly difficult with higher orders, especially in the transition region

- Higher order coefficients present a more complicated structure



Equation of state at finite $\hat{\mu}_B$

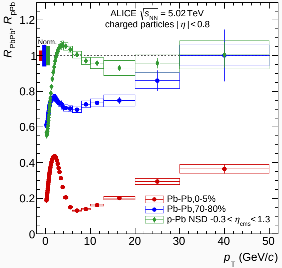
Energy density up to $\hat{\mu}_B = 3.5$ with $\mu_S = 0$ and $n_S = 0$



Observables

Nuclear modification factor

$$R_{AA} = \frac{1}{N_{\text{coll}}} \frac{\frac{dN^{AA}}{dp_T}}{\frac{dN^{pp}}{dp_T}} \quad (= 1 \text{ if no medium})$$

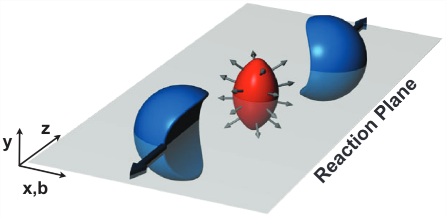


ALICE, JHEP 11 (2018) 013

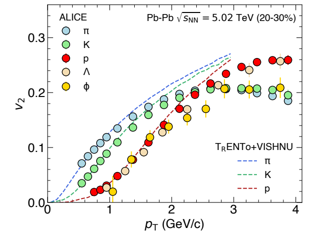
Anisotropic flow

Fourier coefficients of azimuthal distribution

$$\rho(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_n v_n \cos(n(\phi - \psi_s))\right] \quad (= 0 \text{ if no medium})$$



Snellings, New J. Phys. 13 055008

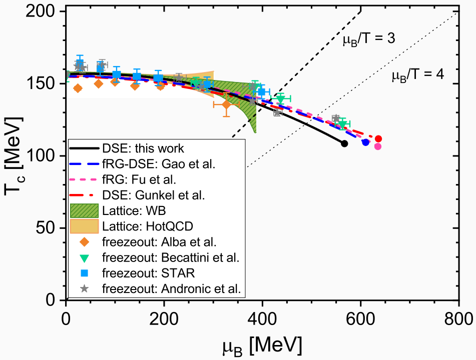


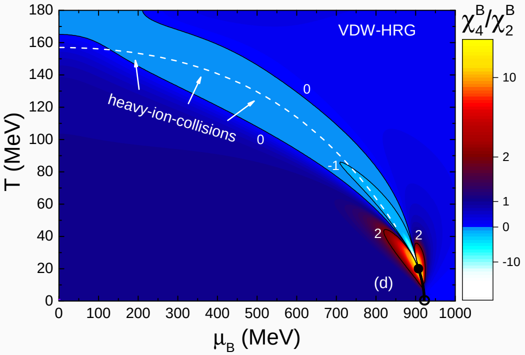
Arslandok *et al.*, 2303.17254

QCD critical point: an additional problem

Critical bmore ehaviour in QCD is present independently of this “high-T” critical point

Critical point at the liquid-gas transition $T \simeq 20\,\mathrm{MeV}$, $\mu_B \simeq 900\,\mathrm{MeV}$

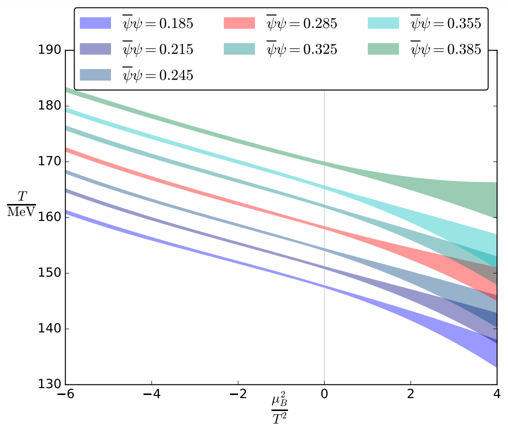


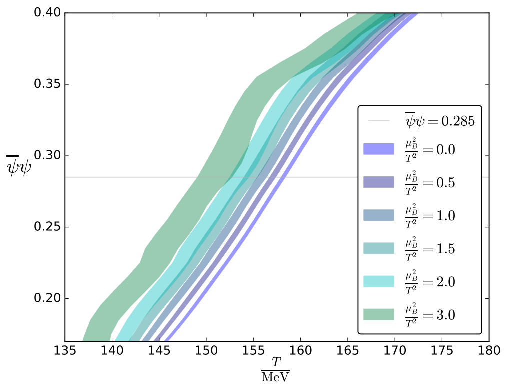


Additional (different) critical behaviour in the limit $m_q \to 0$ at $\mu_B = 0$

The width of the transition at finite chemical potential

We can extrapolate our results for $\langle\bar{\psi}\psi\rangle$ along contours of constant $\langle\bar{\psi}\psi\rangle$ (left) or constant μ_B/T (right)





The extrapolated $\langle\bar{\psi}\psi\rangle$ at finite μ_B is quite precise for $\mu_B < 3T$

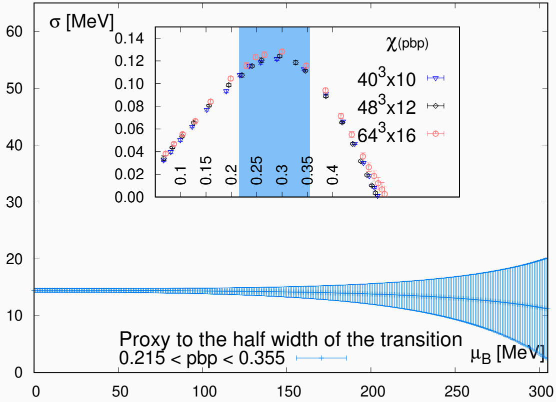
The width of the transition at finite chemical potential

As a side product of our analysis, we get at $\mu_B = 0$ the most accurate determination of T_c:

$$T_c(LT = 4, \mu_B = 0) = 158.0 \pm 0.6 \text{ MeV}$$

while for the width we obtain

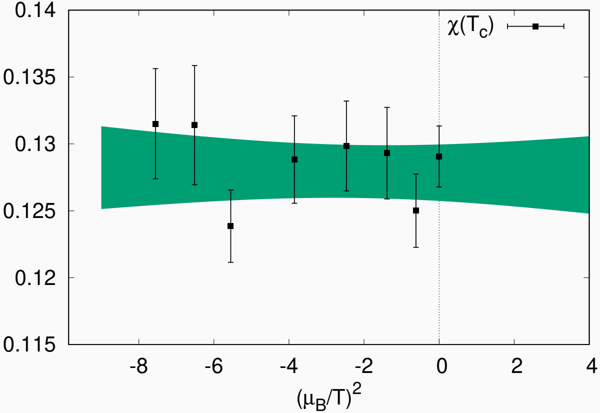
$$\Delta T(LT = 4, \mu_B = 0) = 15.0 \pm 1.0 \text{ MeV}$$



We also note that the width of the transition has a very mild chemical potential dependence

The strength of the transition at finite chemical potential

- We can look at the strength of the transition by looking at the peak value of $\chi(T)$
- In the case of a true transition, this peak value would diverge for $V \to \infty$
- We see again an extremely mild dependence on $\hat{\mu}_B$, suggesting that the strength of the transition does not change



Equation of state from the lattice

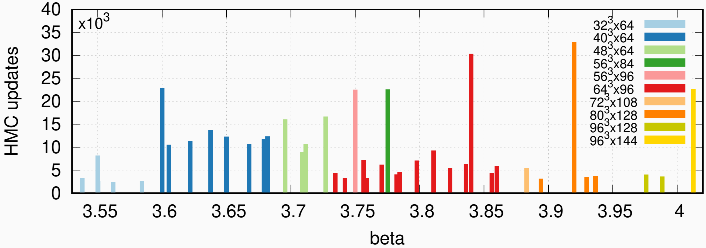
The pressure cannot be determined directly (not a derivative of $\ln Z$ wrt to a parameter), but via an integral of the trace anomaly $I(T)$:

$$\boxed{\frac{p(T)}{T^4} = \frac{p(T_0)}{T_0^4} + \int_{T_0}^{T} \frac{dT'}{T'} \frac{I(T')}{T'^4}}$$

where the **trace anomaly** $I(T)$ *can* be determined directly on the lattice:

$$\frac{I(T)}{T^4} = N_\tau^4 \left(\boxed{T > 0} - \boxed{T = 0} \right)$$

but **needs renormalization**, which means (a lot of) **simulations at $T = 0$ are needed**



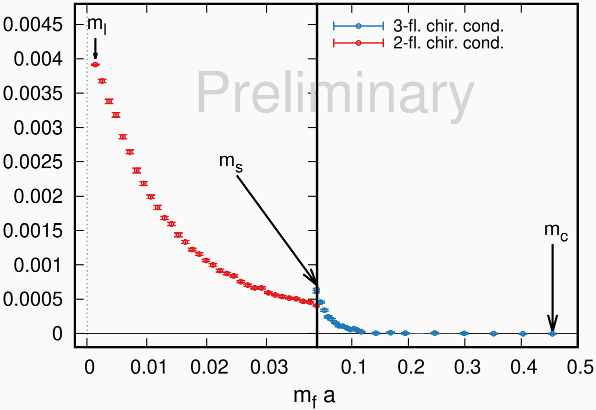
I. Pressure constant

We calculate the integration constant $\frac{p(T_0)}{T_0^4}$ at a chosen $T_0 = 185\,\mathrm{MeV}$.

The pressure is determined as an integral in the quark masses down from infinity (where $p = 0$):

$$\frac{p(T_0)}{T_0^4} = \int_{m_s}^{m_l} dm_2 \left\langle \bar{\psi}\psi \right\rangle_{R,2}(m_2) + \int_{\infty}^{m_s} dm_3 \left\langle \bar{\psi}\psi \right\rangle_{R,3}(m_3)$$

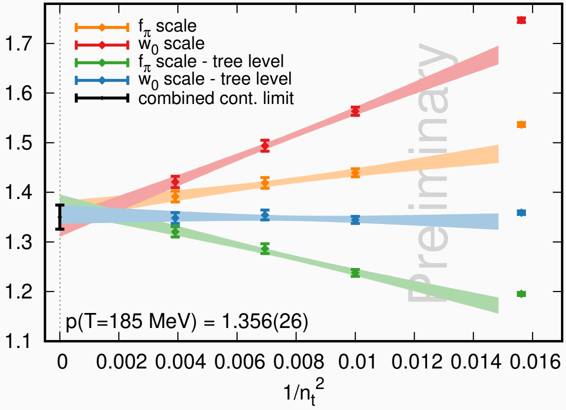
We first integrate in the two light flavours up to m_s, then the three flavours up to infinity by fitting an exponential.



Note: the chiral condensates $\left\langle \bar{\psi}\psi \right\rangle_{R,i}$ are the renormalized ones

I. Pressure constant

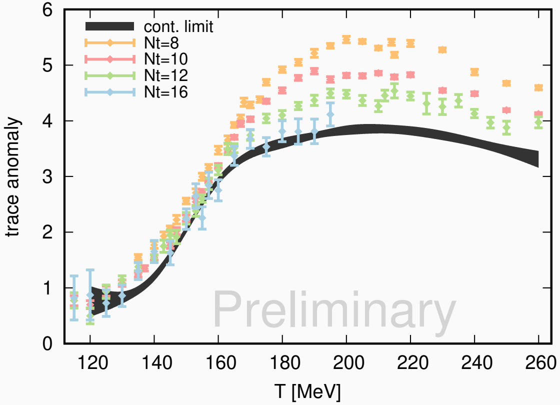
We calculate the integration constant with two settings of the scale, with and without applying the tree level improvement on the observables → 4x systematics



For the first time, we have results for up to $N_t = 16$ which allow us to discard $N_t = 8$ in the continuum extrapolation. ⇒ <u>∼ 2x improvement in uncertainty</u>

II. Trace anomaly

We determine on our lattices $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$, $64^3 \times 16$ the trace anomaly:



then perform a global continuum extrapolation + spline fit in T.

Equation of state at $\hat{\mu}_B = 0$

Now we have both ingredients to determine the equation of state at $\mu_B = 0$, as shown previously:

$$\boxed{\frac{p(T)}{T^4} = \frac{p(T_0)}{T_0^4} + \int_{T_0}^{T} \frac{dT'}{T'} \frac{I(T')}{T'^4}}$$

From the pressure, the other quantities follow. At $\mu_B = 0$, normalized quantities $\hat{O}(T)$:

$$\hat{s} = 4\hat{p}(T) + T\frac{d\hat{p}(T)}{dT}$$

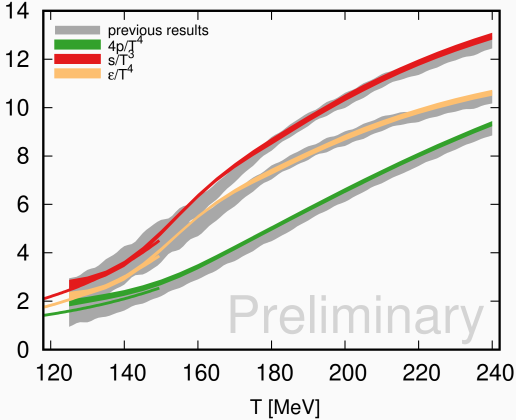
$$\hat{\epsilon}(T) = \hat{s}(T) - \hat{p}(T)$$

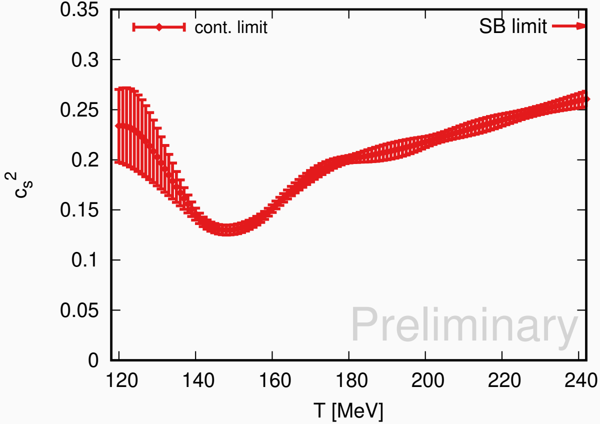
$$c_s^2(T) = \frac{\hat{I}(T) + 4\hat{p}(T)}{7\hat{I}(T) + 12\hat{p}(T) + T\frac{d\hat{I}(T)}{dT}}$$

Equation of state at $\hat{\mu}_B = 0$

We can compare the resulting equation of state at $\mu_B = 0$ to our previous result from 2014

Borsányi+ '14





Note: full systematics analysis still in the making, more statistics coming at $N_\tau = 16$