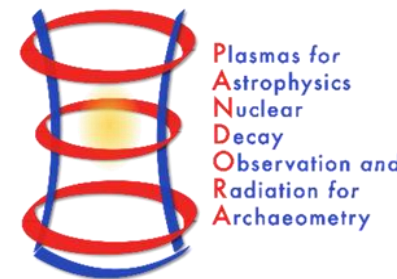


Modification of ${}^7\text{Be}$ β -Decay Rates in Laboratory Magnetoplasma and Perspectives for PANDORA

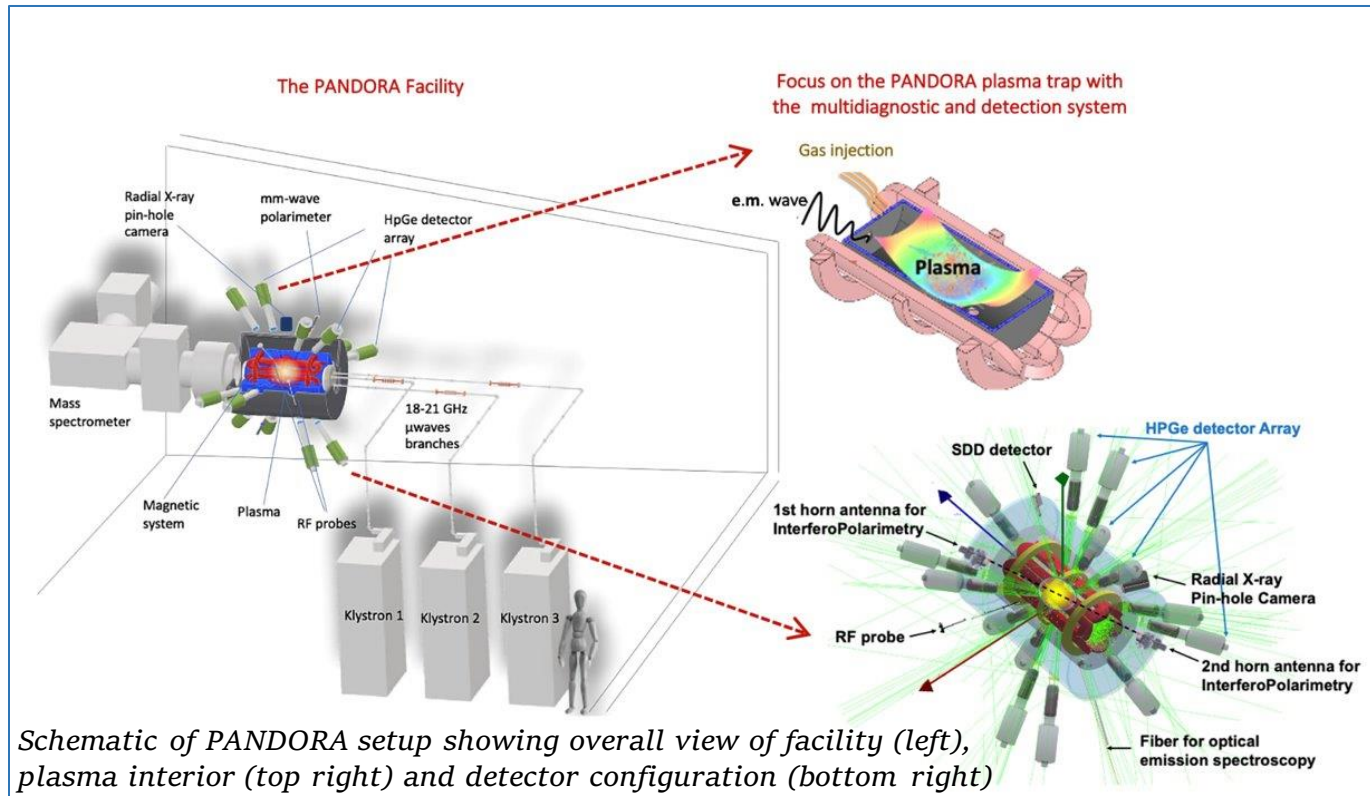
B. Mishra, A. Pidatella, A. Galatà, S. Taioli, S. Simonucci and D. Mascali,
for the PANDORA collaboration

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INFN – LNS, Catania



PANDORA and In-Plasma Decay



Plasmas for Astrophysics, Nuclear Decay Observations and Radiation for Archaeometry is an upcoming facility at INFN – LNS

Objective: Investigate properties of nuclei and atoms *inside* a high energy density plasma for application to nucleosynthesis [1,2]

First tasks:

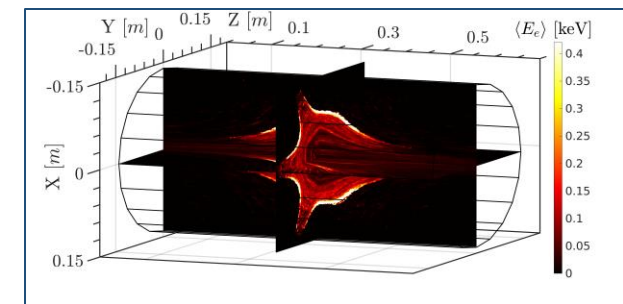
- Measuring light element opacity in-plasma for modelling blue kilonova spectra (*r*-process) [3]
- **Measuring in-plasma β -decay rates of radio-isotopes (*s*-process)**

Experimental Methodology:

- Generate plasma in ECR magnetoplasma trap [$n_e \sim 10^{11-12} \text{ cm}^{-3}$, $k_B T_e \sim 10 \text{ eV} - 100 \text{ keV}$]
- Inject radio-isotopes like ^{176}Lu , ^{134}Cs and ^{94}Nb into plasma and allow ionisation/excitation
- Measure count rate of secondary- γ emitted from decay and correlate γ -emission rate with in-plasma decay rate

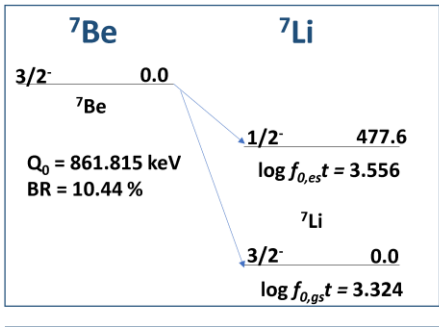
Challenges:

- ECR plasmas are similar but not identical to stellar plasmas – **non-uniform, non-local** and in **NLTE**



Plasma-Decay Model and PIC Simulations

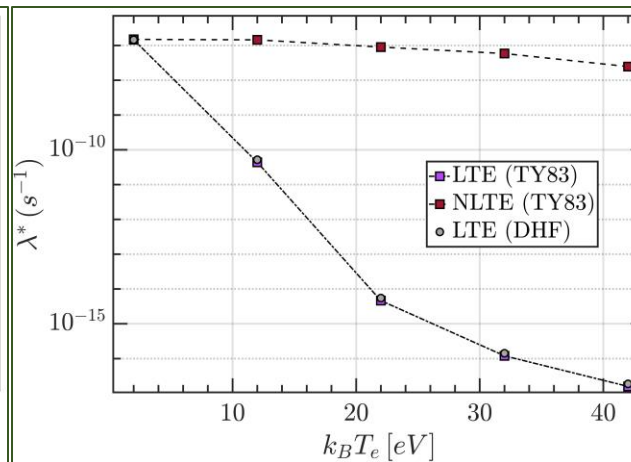
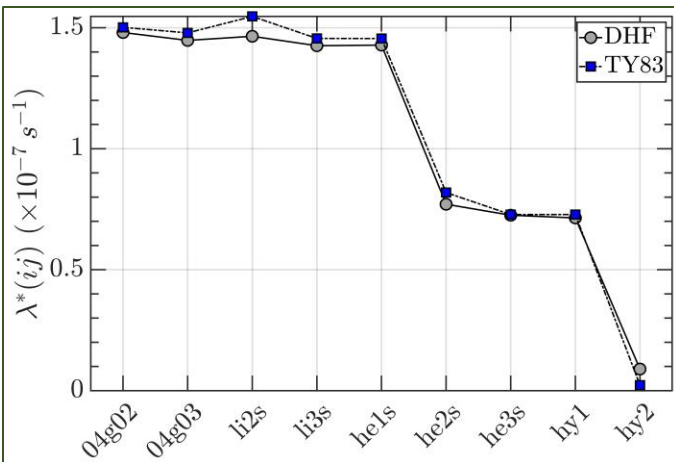
**Step 1: Develop a general model of in-plasma β -decay
[Generalised TY83 Model]**



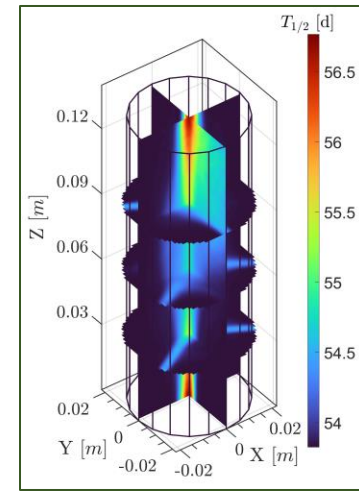
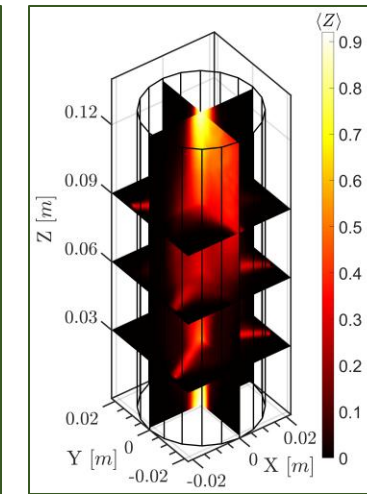
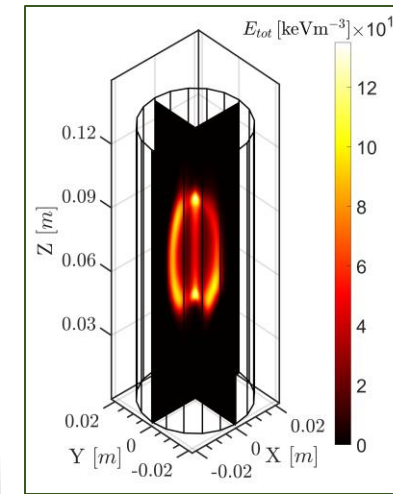
Calculate decay rate according to TY83 [4] model with atomic data from HULLAC database

$$f_m^* = \sum_{ij} p_{ij} \sum_{x(ij)} \sigma_x \frac{\pi}{2} [f_x \text{ or } g_x]^2 (Q_{ij}/m_e c^2)^2 S_{(m)x}$$

$$\lambda^* = \ln 2 \left(\frac{f_{gs}^*}{f_{0,gs} t} + \frac{f_{es}^*}{f_{0,es} t} \right)$$



**Step 2: Couple with Particle-in-Cell (PIC) codes
[3D Generalised TY83 Model]**



Electron PIC codes for calculating 3D n_e and E_e in ECR plasma trap [5]

Ion PIC codes for calculating 3D CSD and LPD [6]

3D plasma-decay model predicts $T_{1/2}$ in ECR plasma trap

Self-consistent PIC simulations can produce maps of plasma density, energy and ionisation state. Plasma-decay models show that for ${}^7\text{Be}$, regions of high $\langle Z \rangle$ are correlated with lower decay rates.

Conclusions and Perspectives

Measurement of in-plasma decay rates in PANDORA will be useful for improving s-process models

PIC-simulations coupled with generalised plasma-decay models can simplify complexity of ECR plasmas and predict spatial gradients in $T_{1/2}$ of radio-isotopes

Simulations show that large modification of decay rates can be achieved by increasing plasma energy content and through efficient injection of isotopes into the magnetic trap

The generalised plasma-decay models are necessary to bridge the gap between low n_e , NLTE laboratory plasma and high n_e , LTE stellar plasma

The PANDORA trap will operate at high power (\sim kW) and use an efficient injection system customised according to the isotope to optimise "plasmisation" and gradients in $T_{1/2}$

Once model-predicted decay rates are benchmarked with experimental results, the model will be applied to the stellar interior for improved decay rates

${}^7\text{Be}$ in itself will be an important measurement:

- Allow investigating effectiveness of charge-breeder techniques for light elements
- Demonstrate effect of hyperfine splitting of atomic orbitals
- Can have possible impact on calculating abundance of ${}^7\text{Li}$



David Mascali Domenico Santonocito
Angelo Pidotella Eugenia Naselli
Giuseppe Torrisi Giorgio Mauro



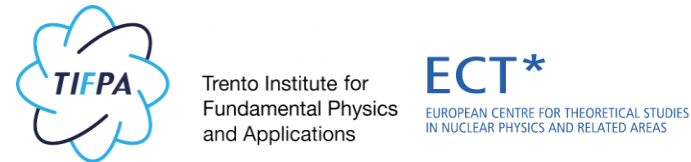
Alessio Galatà



Alberto Mengoni



Stefano Simonucci
Sara Palmerini
Maurizio Busso



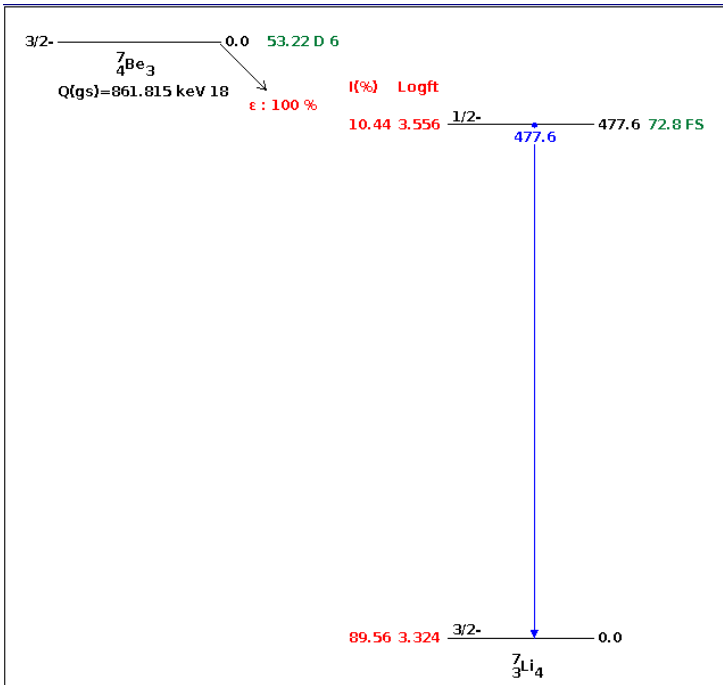
Simone Taioli

...and the PANDORA collaboration

**THANK YOU
FOR YOUR
ATTENTION!**

Additional Slides: Plasma-Decay Model

Step 1: Select isotope and transitions



Step 2: Calculate lepton phase volume

Continuum decay

$$f_{IF(m)}^* = \sum_{ij} p_{ij} \int_1^{W_{max}(ij)} (W^2 - 1)^{1/2} W (W_{max}(ij) - W)^2 F_0 S_{(m)}(ij) f_d(ij) dW$$

Continuum capture

$$f_{IF(m)}^* = \sum_{ij} p_{ij} \int_{W_{min}(ij)}^{\infty} (W^2 - 1)^{1/2} W (Q(ij)/m_e c^2)^2 F_0 S_{(m)}(ij) f_c(ij) dW$$

Bound decay/ Bound capture

$$f_{IF(m)}^* = \sum_{ij} p_{ij} \sum_{x(ij)} \sigma_x \frac{\pi}{2} [g_x \text{ or } f_x]^2 (Q(ij)/m_e c^2)^2 S_{(m)x}(ij)$$

Step 3: Calculate decay rates

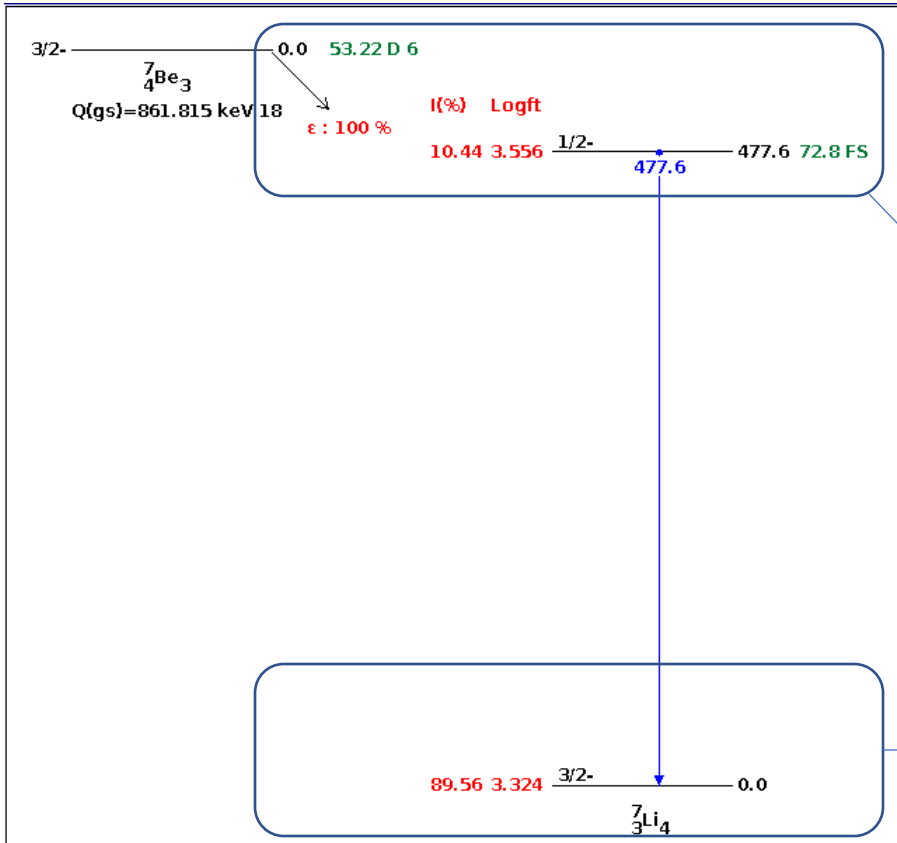
$$\lambda_{tot} = \sum \frac{\ln 2}{f_{IF(m)} t_{1/2}} f_{IF(m)}^*$$

Summation over all possible decay channels

${}^7\text{Be}$ undergoes electron capture to ${}^7\text{Li}$
 $3/2^- \rightarrow 3/2^-$ (gs \rightarrow gs transition, allowed)
 $3/2^- \rightarrow 1/2^-$ (gs \rightarrow es transition, allowed)

BR = 10.44 %

Additional Slides: Plasma-Decay Model



${}^7\text{Be}$ undergoes electron capture to ${}^7\text{Li}$

Contribution from both continuum and bound state electron capture

Information needed: $\log ft$ values
 Correlated with the strength of the decay and the nuclear matrix element

$3/2^- \rightarrow 1/2^-$ (gs \rightarrow es transition)
 Allowed transition
 $\log ft = 3.556$

$3/2^- \rightarrow 3/2^-$ (gs \rightarrow gs transition)
 Allowed transition
 $\log ft = 3.324$

NO OTHER DECAY CHANNEL CONSIDERED

Additional Slides: Plasma-Decay Model

The lepton phase volume quantifies the number of ways a decay can occur. The phase volume changes with variations in atomic configuration, depending on type of decay

Information needed: level probability distribution (LPD), orbital occupancy, orbital electron wavefunction, decay energy and shape factor

$$f_{IF(m)}^* = \sum_{ij} P_{ij} \sum_{x(ij)} \sigma_x \frac{\pi}{2} [g_x \text{ or } f_x]^2 (Q(ij)/m_e c^2)^2 S_{(m)x(ij)}$$

(e) Probability distribution of various charge states and levels of isotope

(d) Occupancy of orbital contributing to certain decay within relevant ionic configuration

(c) Larger of squared radial component of electron wavefunction evaluated on nuclear surface

(a) Q-value of decay when isotope in (i,j) charge state and level configuration

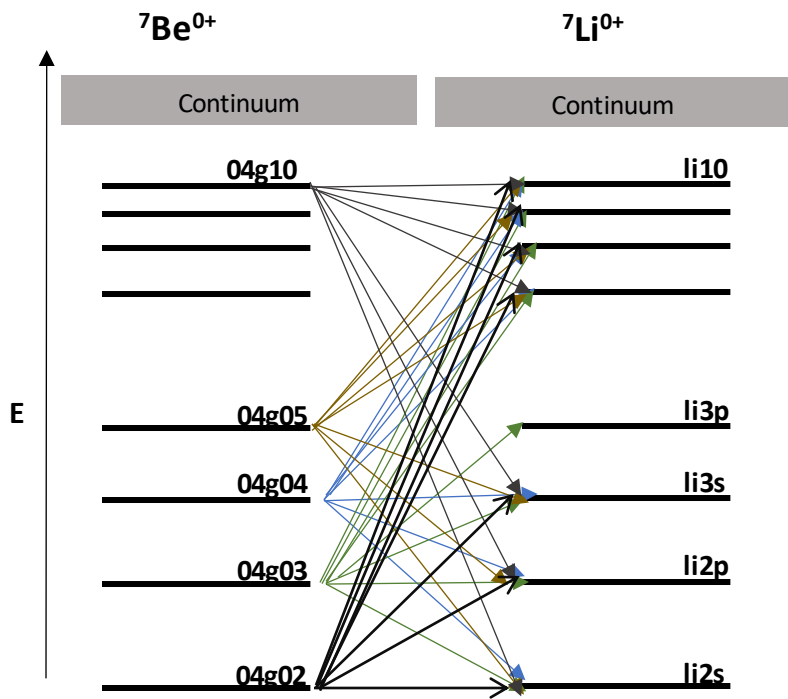
(b) Shape factor describing compatibility between nuclear and lepton wavefunction

Additional Slides: Plasma-Decay Model

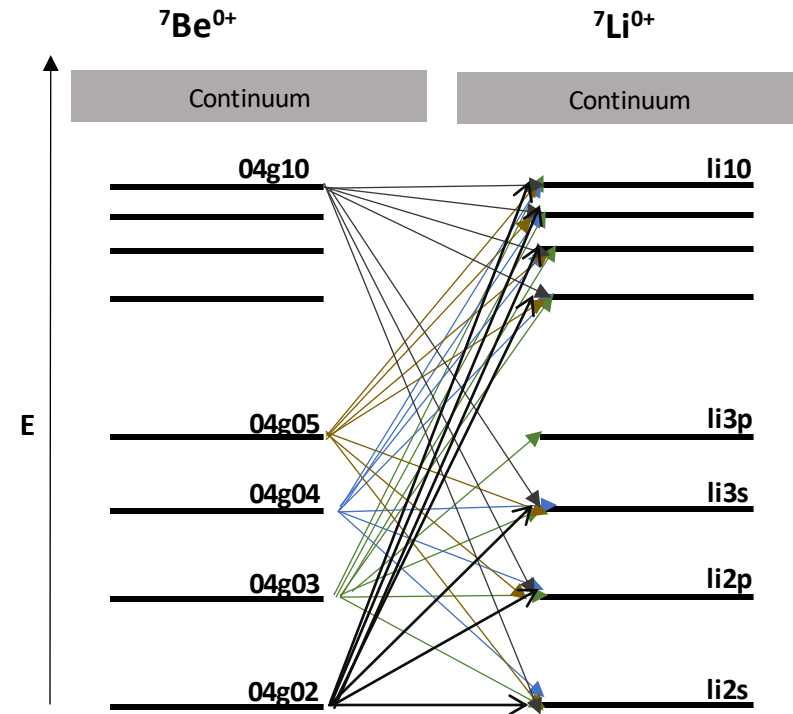
$$Q = Q_0 + (E_{X,K}^* - E_{Y,K'}^*) + (\epsilon^{i,j} - \epsilon^{i',j'}) + (\Delta_X - \Delta_Y)$$

The decay energy depends on not just the difference in nuclear masses, but on the overall system energy which includes atomic/ionic energy

Information needed: energy of different atomic configurations of parent system and coupling with daughter system



${}^7\text{Be} - {}^7\text{Li}$ level coupling schematic for K- and L-shell capture (neutral ion)



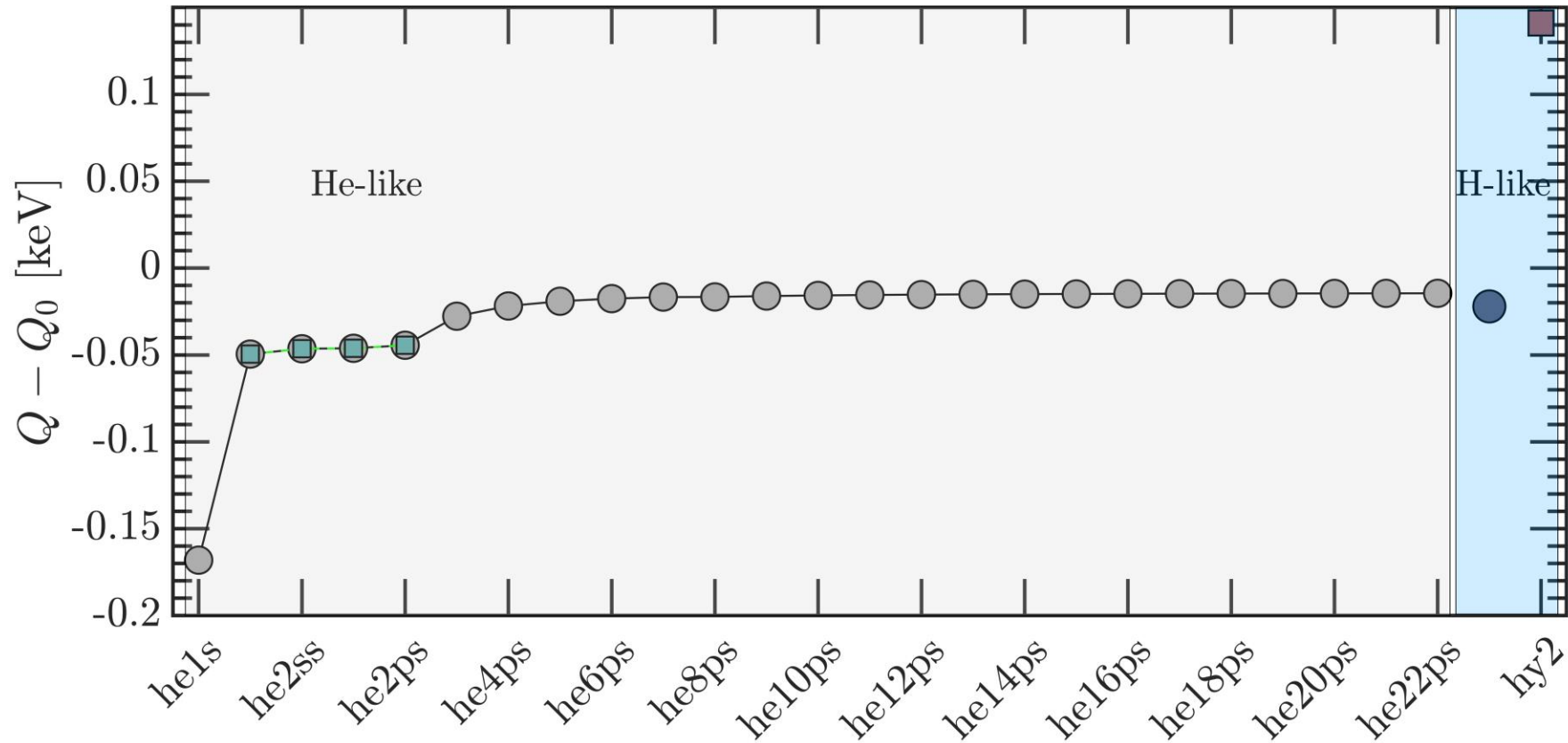
All configurations should have at least one K-shell electron

No selection on daughter configuration
K-shell vacant states autoionising

All configurations should have at least one L-shell electron

No selection on daughter configuration
L-shell vacant states autoionising

$$Q = Q_0 + (E_{X,K}^* - E_{Y,K'}^*) + (\epsilon^{i,j} - \epsilon^{i',j'}) + (\Delta_X - \Delta_Y)$$



The decay energy changes according to the configuration of the ion.
Previously blocked channels can open up and vice versa

Additional Slides: Plasma-Decay Model

Conservation of total angular momentum implies that only certain electron orbitals can interact with the nucleus, depending on the spin and parity of the decay

Information needed: spin-parity of electron orbitals and decay transition

$$S_{(m)x} = \begin{cases} 1 & \text{for } m = a, nu \text{ and } x = ns_{1/2}, np_{1/2} \\ q^2 & \text{for } m = u \text{ and } x = ns_{1/2}, np_{1/2} \\ 9/R^2 & \text{for } m = u \text{ and } x = np_{3/2}, nd_{3/2} \\ 0 & \text{otherwise.} \end{cases}$$

Additional Slides: Plasma-Decay Model

The probability of electron capture from bound states depends on the square of the radial component of the orbital wavefunction evaluated on the nuclear surface

Information needed: formalism for radial wavefunctions of different orbitals

Radial component of Dirac equation – Coupled differential equations

$$\frac{dP(r)}{dr} = -\frac{\kappa}{r}P(r) - \left(2c + \frac{V - \epsilon}{c}\right)Q(r) \quad \frac{dQ(r)}{dr} = \frac{\kappa}{r}Q(r) + \left(\frac{V - \epsilon}{c}\right)P(r)$$

For $V = Z/r$ (in atomic units)

$$P(r) = \left(1 - \frac{\epsilon}{c^2}\right)^{1/2} \xi \left(\frac{\rho}{N}\right)^\gamma e^{-\rho/2N} [-n_r F_1 + (N - \kappa) F_2]$$

$$Q(r) = \left(\frac{\epsilon}{c^2}\right)^{1/2} \xi \left(\frac{\rho}{N}\right)^\gamma e^{-\rho/2N} [n_r F_1 + (N - \kappa) F_2]$$

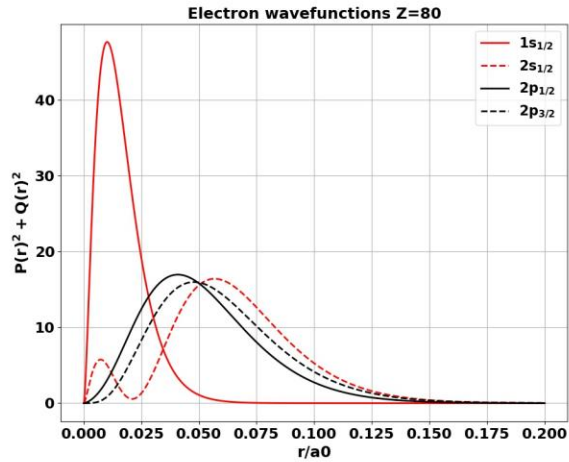
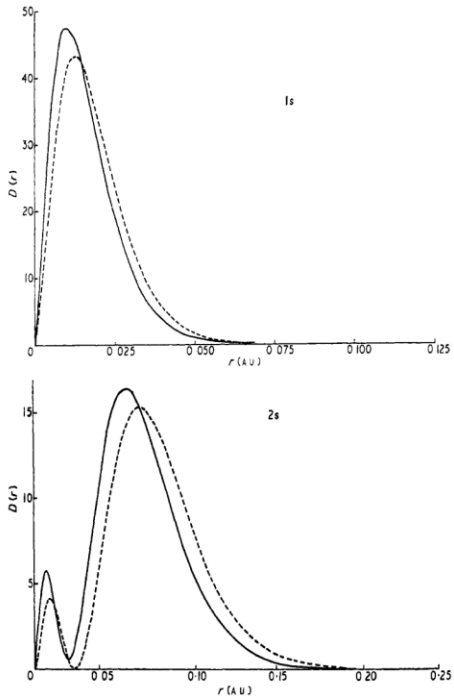
- $N =$ *apparent principal quantum number*
- $n_r =$ *number of nodes in orbital spatial distribution*
- $\xi =$ *normalisation*
- $\rho = 2Zr =$ *radial function*
- $\epsilon =$ *quantised electron energy*
- $\kappa = -(j+1/2)a, a = \pm 1$
- $F_1, F_2 =$ *confluent hypergeometric functions*

Additional Slides: Plasma-Decay Model

Physics verification

$$D(r) = P(r)^2 + Q(r)^2$$

Radial probability density

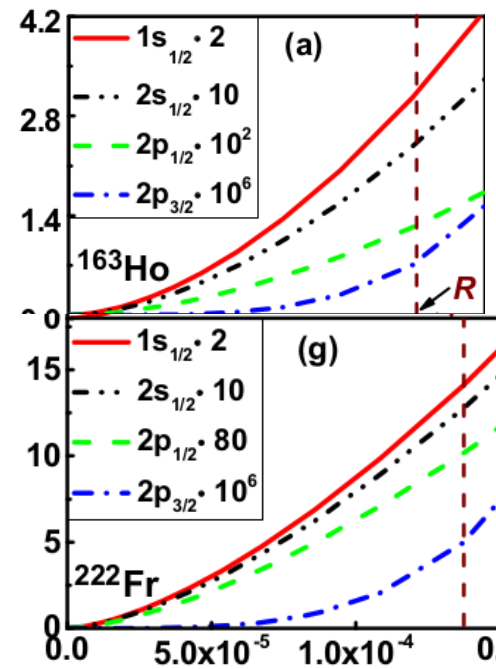


Radial probability density of 1s, 2s and 2p orbitals for Z = 80 as calculated in [5] and as calculated here

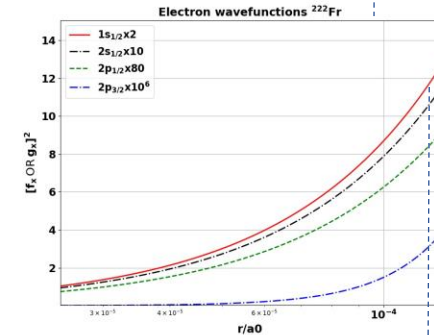
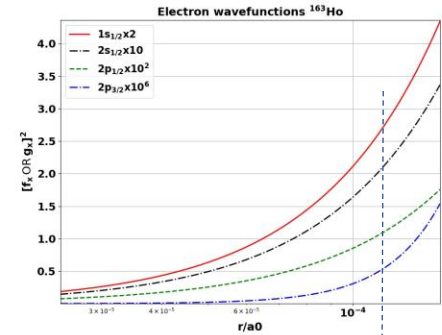
Literature benchmark

$$f_x = \left(\frac{\bar{\lambda}_c}{\hbar}\right)^{3/2} \frac{P(r)}{r} \quad g_x = \left(\frac{\bar{\lambda}_c}{\hbar}\right)^{3/2} \frac{Q(r)}{r}$$

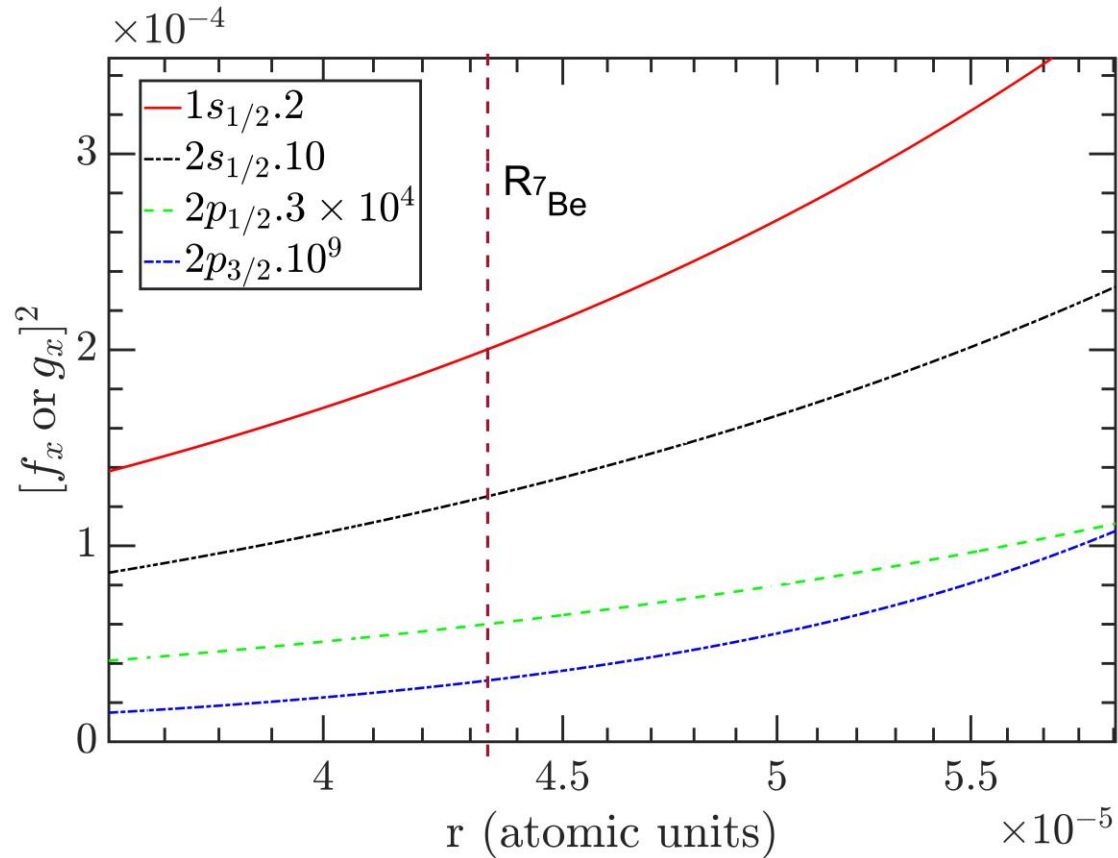
Effective radial wavefunction



^{163}Ho and ^{222}Fr radial wavefunctions as calculated with RADIAL in [6] compared with analytical formalism



Additional Slides: Plasma-Decay Model



Larger of f_x^2 or g_x^2 as calculated for ${}^7\text{Be}$ taking $R = R_0 A^{1/3}$

Wavefunctions evaluated on nuclear surface have small values on account of small nuclear size

Only 1s and 2s contributions may be considered

Additional Slides: Plasma-Decay Model

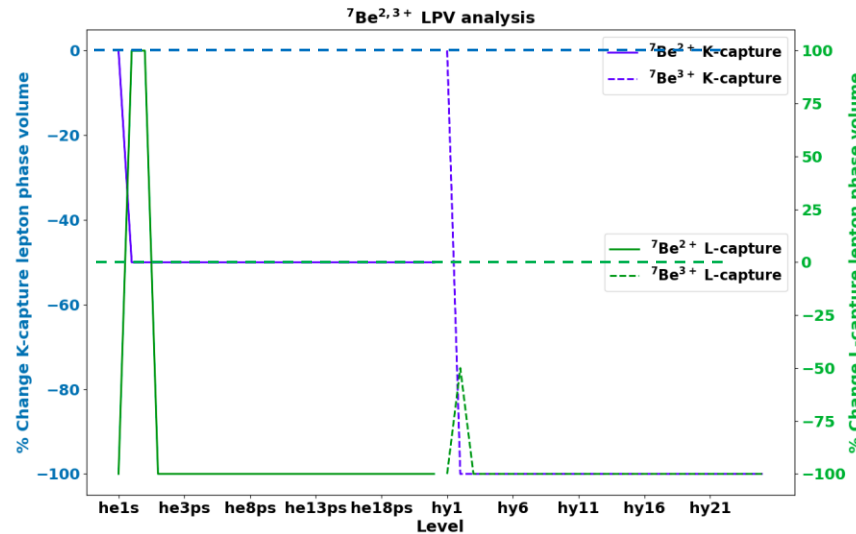
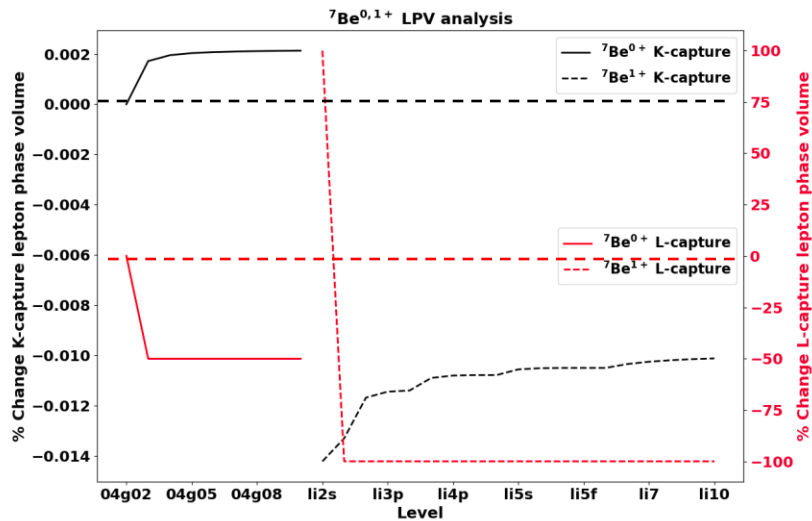
The probability of electron capture from bound states also depends on the occupancy of the relevant orbital

Information needed: Occupancy of relevant orbitals as a function of the level configuration and ion charge state

li2s level in ${}^7\text{Be}^{1+}$ - 2e⁻ in 1s, 1e⁻ in 2s
 1s_{1/2} occupancy = 1
 2s_{1/2} occupancy = 0.5
 2p_{1/2} occupancy = 2p_{3/2} occupancy = 0

Suppressed L-capture

li2p level in ${}^7\text{Be}^{1+}$ - 2e⁻ in 1s, 1e⁻ in 2p
 1s_{1/2} occupancy = 1
 2s_{1/2} occupancy = 0
 2p_{1/2} occupancy = 2p_{3/2} occupancy = 0.167



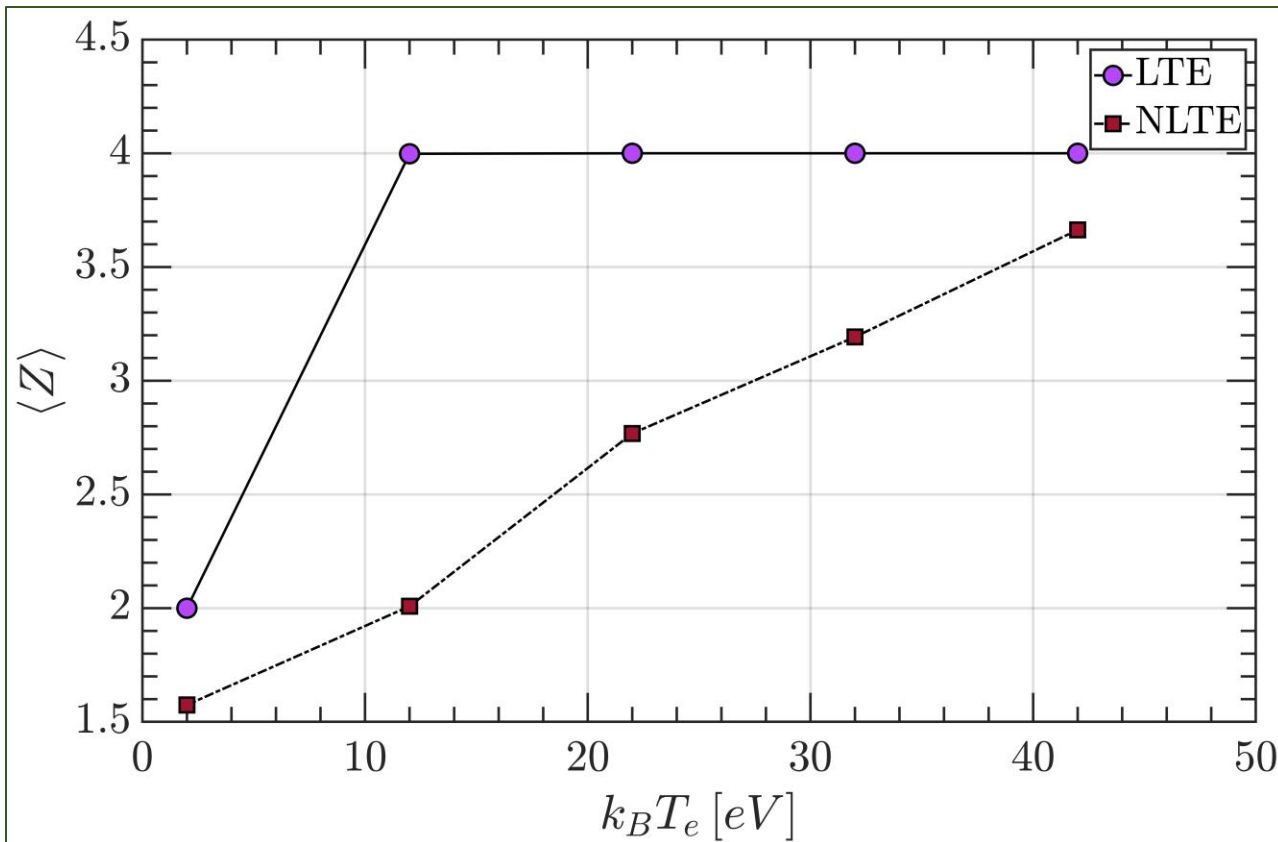
% change in lepton phase volume can be quite pronounced for some configurations which are accessible by plasma

Levels can be combined into super-groups based on similarity in lepton phase volume – computational advantage

Additional Slides: Plasma-Decay Model

The ion CSD and LPD strongly depends on electron density and temperature

Information needed: CSD and LPD of ${}^7\text{Be}$ for various n_e and T_e
(calculated using FLYCHK)

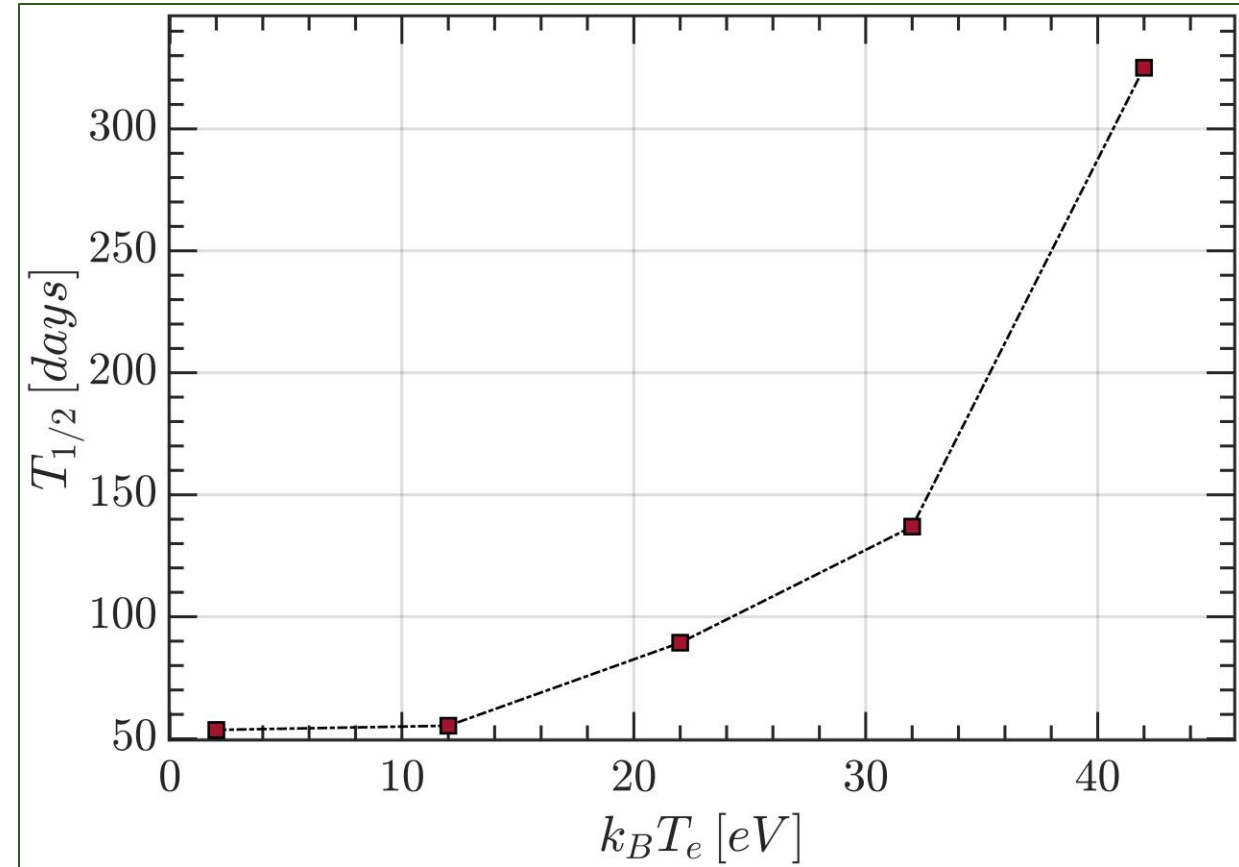
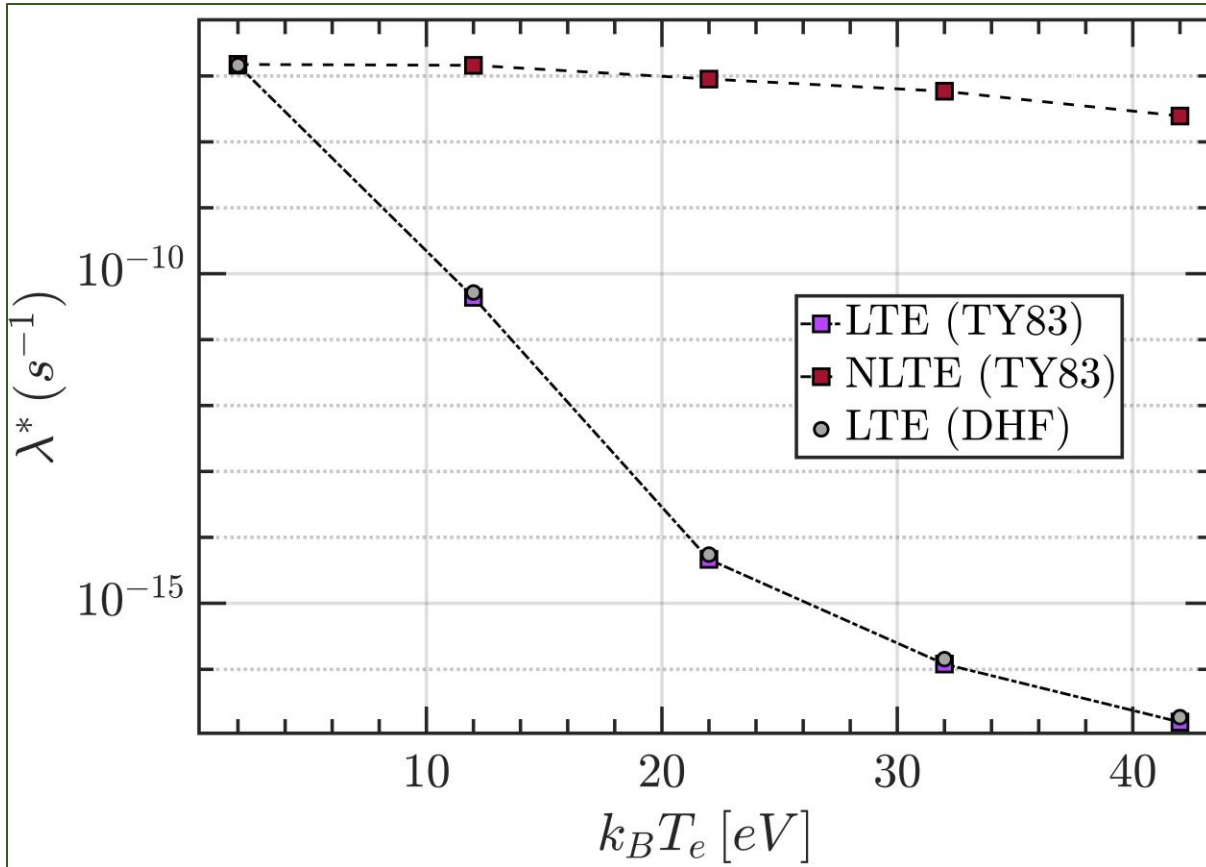


Mean charge of plasma, and consequently ion CSD and LPD show vastly different trends in LTE and NLTE conditions

TY83 cannot be directly applied to laboratory plasmas which are necessarily NLTE

The generalised plasma-decay model can solve this issue

Additional Slides: Plasma-Decay Model



^7Be ground state (neutral and 04g02 level config)

$T_{1/2}$ calculated = 53.44 days
Branching ratio = 10.4 %

$T_{1/2}$ measured (ENSDF) = 53.22 ± 6 days
Branching ratio = $10.44 \pm 0.2\%$

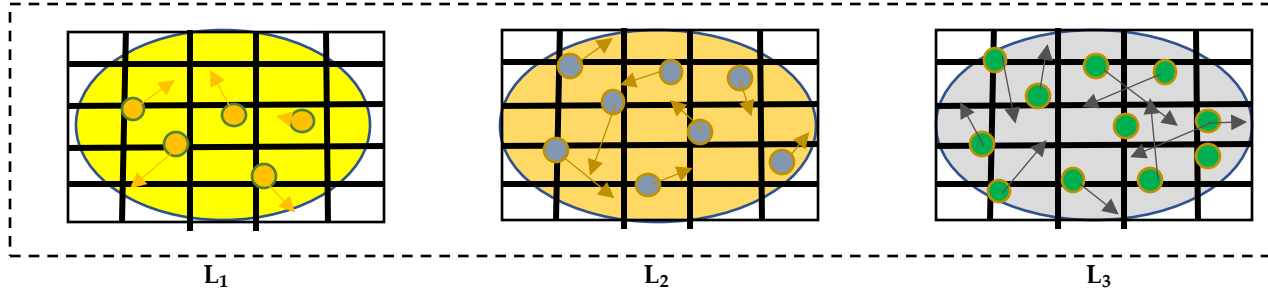
Additional Slides: PIC-Simulations

BUFFER IONS BALANCE EQUATION:

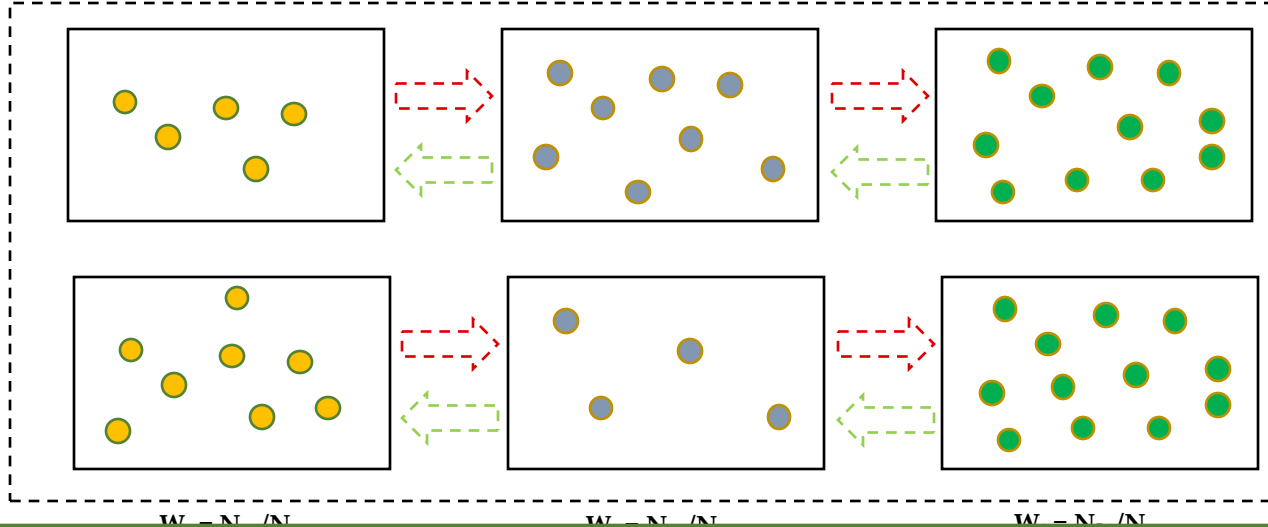
$$\frac{dn_i}{dt} = n_{i-1}n_e\gamma_{i-1,i} - n_in_e\gamma_{i,i+1} + n_{i+1}n_0E_{i+1,i} - n_in_0E_{i,i-1} - \frac{n_i}{\tau_i}$$

KEYWORD(S): Steady state

The contribution from each process can be evaluated independently to construct CSD



Particle transport under EM fields with collision to generate 3D spatial distribution (occupation maps)



Constant monitoring of loss/gain of macroparticles between various levels to calculate relative weight of each level

Forward reactions – ionisation, excitation, absorption
Backward reactions – charge exchange, de-excitation, spontaneous emission

N macroparticles
Initial r, v , reaction parameters
B field
ION, CEX and OCC maps

Set T_{span} and T_{step}

Begin iteration $t=0$

Ionisation → Update ION
Exchange → Update CEX
Neither → Update OCC

$t = \tau_{p-2}$

$n_i \rightarrow \rho_i \rightarrow \Delta\rho \rightarrow$ Calculate E_{DL}

Ionisation → Update ION
Exchange → Update CEX
Neither → Update OCC

$t = \tau_{p-2}$

End iteration $t=T_{span}$

Save Occ, Ion, CEX maps and transfer coefficients

