The Three-Nucleon Correlation function

A. Kievsky

Istituto Nazionale di Fisica Nucleare, Sezione di Pisa

Sesto Incontro Nazionale di Fisica Nucleare
INFN2024, Trento, 26 - 28 February 2024

Collaboration ALICE-Pisa Nuclear Theory Group

- Theory
  A. Kievsky (INFN)
  M. Viviani (INFN-Pisa)
  L.E. Marcucci (Universita' di Pisa and INFN-Pisa)
  S. König (UNC)
  E. Garrido (CSIC-Madrid)
  M. Gattobigio (Université Côte d'Azur and CNRS)

- Experiment
  L. Fabbietti (TUM)
  B. Singh (TUM)
  O. Vázquez Doce (INFN-Frascati)
  R. Del Grande (TUM)
  L. Šerkšnytė (TUM)
  Dmytro Melnichenko (TUM)
  ...

Introduction

- When a high-energy pp or p–nucleus collision occurs, particles are produced and emitted at relative distances of the order of the nuclear force
- The effect of the mutual interaction between hadrons is reflected as a correlation signal in the momentum distributions of the detected particles which can be studied using correlation functions
- The correlation function incorporate information on the emission process as well as on the final state interaction of the emitted pairs
- By measuring correlated particle pairs or triplets at low relative energies and comparing the yields to theoretical predictions, it is possible to study the hadron dynamics.

The two-particle correlation function

- The two-particle correlation function is defined as the ratio of the yield of a particle pair to the product of the single-particle yields.

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{\mathcal{P}\left(\vec{p}_1, \vec{p}_2\right)}{\mathcal{P}\left(\vec{p}_1\right)\mathcal{P}\left(\vec{p}_2\right)}$$

- $\mathcal{P}(\vec{p}_1, \vec{p}_2)$ is the probability of finding a pair with momenta $\vec{p}_1$ and $\vec{p}_2$
- $\mathcal{P}(\vec{p}_i)$ is the probability of finding each particle with momentum $\vec{p}_i$.
- In absence of correlations, the two-particle probability factorizes, $\mathcal{P}(\vec{p}_1, \vec{p}_2) = \mathcal{P}(\vec{p}_1)\mathcal{P}(\vec{p}_2)$, and the correlation function is equal to unity.

The two-particle correlation function

- The correlation between the pair is related to the particle emission and the subsequent interaction of the pair

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{1}{\Gamma} \sum_{m_1, m_2} \int d^3 r_1\, d^3 r_2 S_1\left(r_1\right) S_1\left(r_2\right) \times \left|\Psi_{m_1, m_2}(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2)\right|^2$$

- $S_1(r)$ describes the spatial shape of the source for single-particle emissions. It can be approximated as a Gaussian probability distribution with a width R_M

$$S_1(r) = \frac{1}{(2\pi R_M^2)^{\frac{3}{2}}} e^{-r^2/2R_M^2}$$

- The integration on the CM coordinates leads to the Koonin-Pratt relation for two-particle correlation function

$$C(k) = \frac{1}{\Gamma} \int d^3 r\, S(r) |\psi_k\left(\vec{r}\right)|^2$$

The pp correlation function

- $S(r)$ is the two-particle emission source, given by

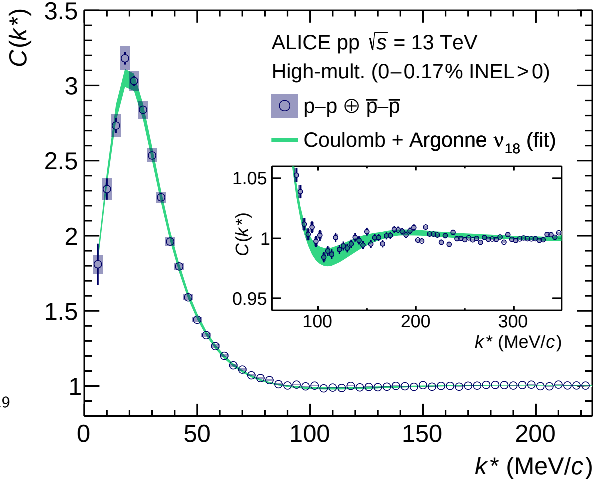
$$S(r) = \left(\frac{1}{4\pi R_{\rm M}^2}\right)^{3/2} e^{-\frac{r^2}{4R_{\rm M}^2}}$$

- $\psi_k(\vec{r})$ is the two-particle scattering wave function at $E = \hbar^2 k^2/2\mu$
- The scattering wave function is expanded in partial waves

$$\psi_k = 4\pi \sum_{[\ell S]} i^\ell (kr)^{-1} u_\ell(kr) \mathcal{Y}_{[\ell S]}(\hat{r}) \mathcal{Y}^*_{[\ell S]}(\hat{k})$$

- In the case of two protons $u_\ell(kr) \to F_\ell(\eta, kr) + T_{\ell\ell}\mathcal{O}_\ell(kr)$

The pp Correlation Function



ALICE collaboration
Phys. Lett. B 805, (2020) 135419

The pd Correlation Function

- We now consider the pd correlation function:

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2, m_1} \int d^3 r_1 d^3 r_2 d^3 r_3 \; S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2, m_1}|^2$$

- the probability of deuteron formation

$$A_d = \frac{1}{3} \sum_{m_2} \int d^3 r_1 d^3 r_2 \; S_1(r_1) S_1(r_2) |\phi_{m_2}|^2$$

- the single particle source function

$$S_1(r) = \frac{1}{(2\pi R_{\rm M}^2)^{\frac{3}{2}}} e^{-r^2/2R_{\rm M}^2}$$

The pd Correlation Function

- the pd correlation function results

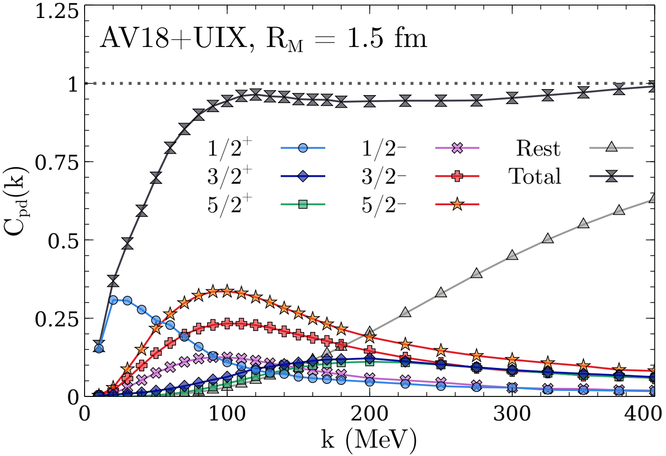
$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi_{m_2,m_1}|^2$$

$$\Psi_{m_2,m_1} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 \mid S J_z)(L 0 S J_z \mid J J_z) \Psi_{LSJJ_z}$$

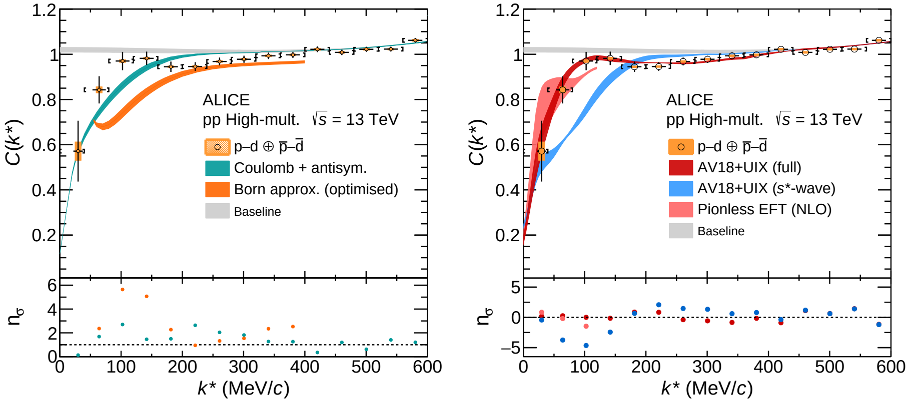
- the Jacobi coordinates: $\mathbf{x}_\ell = \mathbf{r}_j - \mathbf{r}_i$, $\quad \mathbf{y}_\ell = \mathbf{r}_\ell - \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$
- the hyperspherical coordinates $\rho = \sqrt{x_1^2 + (4/3) y_1^2}$, $\Omega \equiv [\alpha_1, \hat{x}_1, \hat{y}_1]$
- The scattering wave function is expanded in partial waves using the HH basis

$$\Psi_{LSJJ_z} = \rho^{-5/2} \sum_{[K]} u_{[K]}(\rho) \mathcal{Y}_{[K]}^{LSJJ_z}(\Omega)$$

The pd Correlation Function: partial-wave contributions



The pd Correlation Function: comparison to experiment



M. Viviani, S. König, A. Kievsky, L.E. Marcucci, B. Singh, O. Vázquez Doce, Phys. Rev. C 108, 064002 (2023)
ALICE collaboration, arXiv:2308.16120 [nucl-ex]

The ppp correlation function

- Now we consider the ppp correlation function:

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega \; S_{\rho_0}(\rho)|\Psi_{ppp}|^2$$

with Q the hyper-momentum, S_{ρ_0} the source function defined as

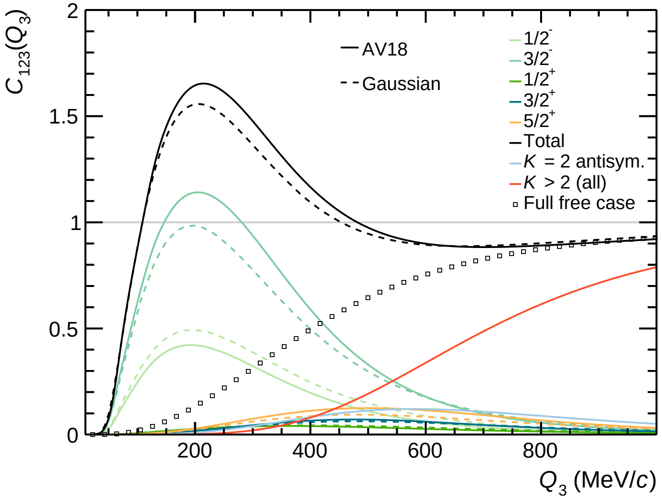
$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Ψ_{ppp} is the ppp scattering wave function

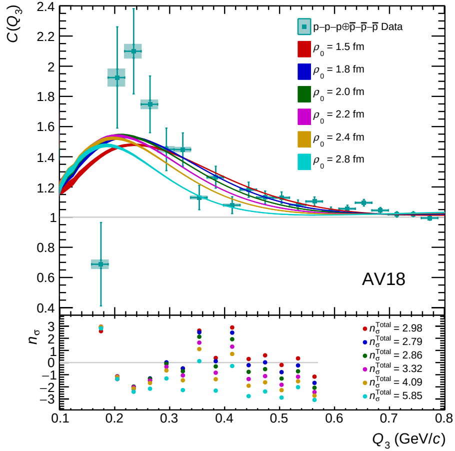
$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho)\mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,[K]}^{\overline{J},\overline{K}} \Psi^J_{[K]}$$

To be noticed that Ψ^0 is not well known. In $\Psi^J_{[K]}$ the interaction has been considered up to $\overline{J}$ and $\overline{K}$

The ppp correlation function



Comparison to data (work in progress)



A. Kievsky, E. Garrido, M.Viviani, L.E. Marcucci, L. Šerkšnytė, R. Del Grande, Phys. Rev. C, in press

Some remarks

- To compare the experimental and the theoretical correlation functions some corrections have been considered
- For the *pp* case the corrected correlation function is defiend as

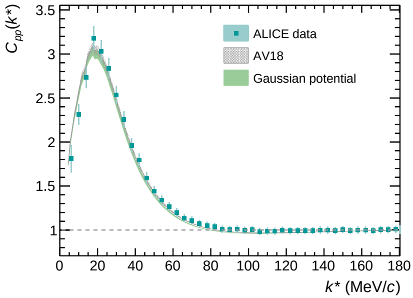
$$C(k) = \lambda_{pp} C_{pp}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(k)$$

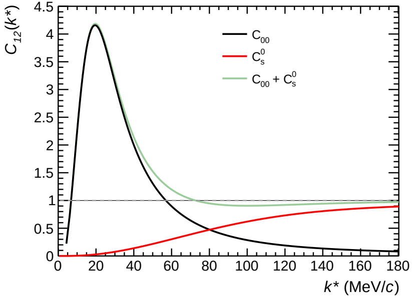
- primary protons $\lambda_{pp} = 0.67$, secondary protons produced mainly in the decay of the Λ, $\lambda_{pp_\Lambda} = 0.203$, misidentification contributions $\lambda_x = 0.127$
- For the *ppp* case the corrected correlation function is defiend as

$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{ppp_\Lambda} C_{ppp_\Lambda}(Q_3) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(Q_3)$$

- primary protons $\lambda_{ppp} = 0.618$, secondary protons produced mainly in the decay of the Λ, $\lambda_{ppp_\Lambda} = 0.196$, misidentification contributions $\lambda_x = 0.186$

The pp correlation function





$$C_{12}(k) = C_s^0 + C_{00} = \int d\mathbf{r} S_{12}(r) \left[|\Psi_s^0|^2_\Omega - \frac{1}{2}\left(\frac{F_0(\eta, kr)}{kr}\right)^2 + \frac{1}{2}\left(\frac{u_0(kr)}{kr}\right)^2 \right]$$

$$C_{pp}(k) = \lambda_{pp} C_{12}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\rm X} C_{\rm X}(k)$$

The $p\Lambda$ and $pp\Lambda$ correlation functions

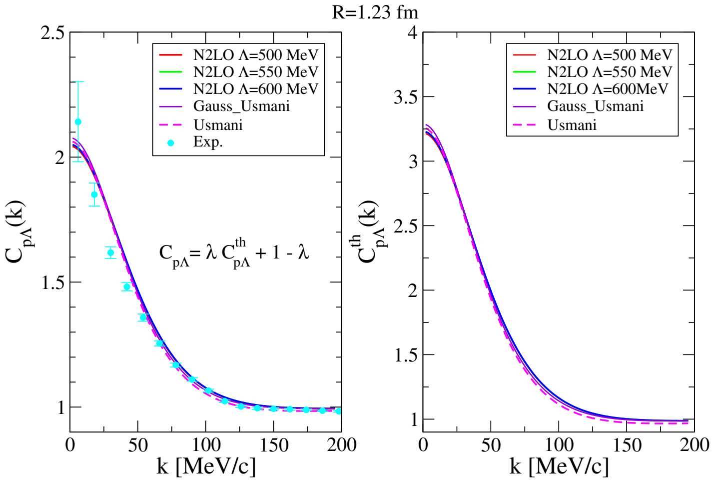
- The $p\Lambda$ correlation function is defined as

$$C(k) = \int d^3r\, S(r) |\psi_{p\Lambda}(\vec{r})|^2$$

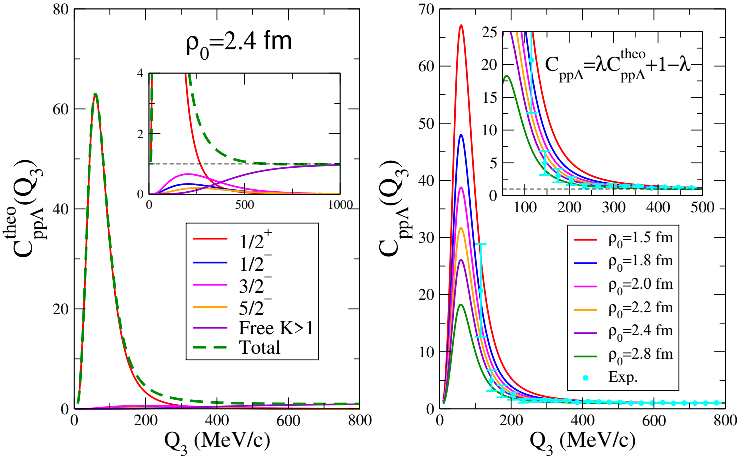
- $\psi_{p\Lambda}$ is the scattering $p\Lambda$ wave function. It is governed by the $p\Lambda$ interaction which is not very well kown
- The few $p\Lambda$ scattering data can be described in the context of the EFT at different orders
- At different cutoffs different sets of low-energy scattering parameters appear

	NLO13						NLO19				SMS N2LO		
C(MeV)	450	500	550	600	650	700	500	550	600	650	500	550	600
a_0 (fm)	-2.90	-2.91	-2.91	-2.91	-2.90	-2.90	-2.91	-2.90	-2.91	-2.90	-2.80	-2.79	-2.80
r_e^0 (fm)	2.64	2.86	2.84	2.78	2.65	2.56	3.10	2.93	2.78	2.65	2.82	2.89	2.68
a_1 (fm)	-1.70	-1.61	-1.52	-1.54	-1.51	-1.48	-1.52	-1.46	-1.41	-1.40	-1.56	-1.58	-1.56
r_e^1 (fm)	3.44	3.05	2.83	2.72	2.64	2.62	2.62	2.61	2.53	2.59	3.16	3.09	3.17

The $p\Lambda$ correlation function



The $pp\Lambda$ correlation function (preliminary)



- A $NN\Lambda$ three-body force is included fixed to describe the $B(^3_\Lambda\mathrm{H})$

Summary

- Although its apparent simplicity, the three-nucleon problem is of great complexity
- Measurements of the correlation function allow for new tests of the NN and NNN interactions
- In the ppp case the Coulomb interaction couples the asymptotic dynamics increasing the difficulties of the numerical treatment
- The corrections of the computed pp and ppp correlation functions needs the kwonledge of the $p\Lambda$ and $pp\Lambda$ correlation functions
- However the $p\Lambda$ interaction is not well known

- Studies on the $p\Lambda$ and $pp\Lambda$ correlation functions have been started
- The $pp\Lambda$ correlation functions could be sensitive to the $NN\Lambda$ three-body force, an important ingredient in the studies of compact systems