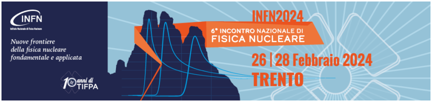

Momentum dependent nucleon-nucleon contact interactions and their effect on p-d scattering observables

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Summary

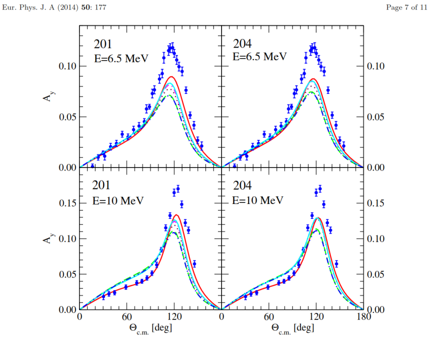
Motivations

- A more sophisticated version of the three-body force is needed to
  - Equation of state for neutron matter ⇒ Neutron stars ⇒ Gravitational waves
  - Structure of nuclei
  - Calculations on the fusion between polarized nuclei (d-d, p-7Li, p-11B) [Lecce-Pisa group]

  - Long-standing discrepancies: The N-d A_y puzzle

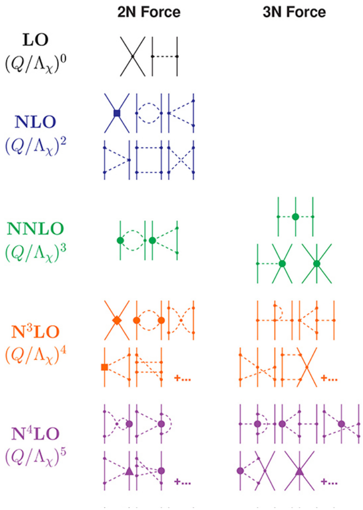
$$A_y \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

$\sigma_\uparrow$, $\sigma_\downarrow$ = differential cross section with the spin of the incoming nucleon normal to the scattering plane

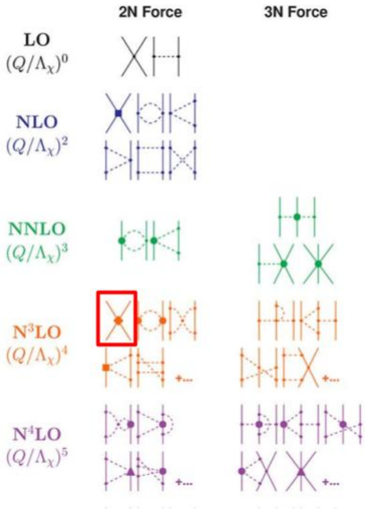


[J. Golak et al. Eur. Phys. J. A 50 (2014) 177]

Chiral EFT potentials



Chiral EFT potentials



N3LO 2N Contact Interactions from Poincaré symmetry

From Poincaré algebra constrains:

Generators:

$$\boldsymbol{P} = \boldsymbol{P}_0,\ \boldsymbol{J} = \boldsymbol{J}_0,\ H = H_0 + V,\ \boldsymbol{K} = \boldsymbol{K}_0 + \boldsymbol{W},$$

P = total momentum, J = total angular momentum, K = boost generator.

Commutation relations:

$$[P_i, P_j] = 0, \qquad [J_i, P_j] = i\epsilon_{ijk} P_k,$$
$$[P_i, H] = 0, \qquad [J_i, J_j] = i\epsilon_{ijk} J_k,$$
$$[J_i, H] = 0, \qquad [J_i, K_j] = i\epsilon_{ijk} K_k,$$
$$[H, K_i] = -iP_i,$$
$$[K_i, K_j] = -i\epsilon_{ijk} J_k / c^2,$$
$$[P_i, K_j] = -i\delta_{ij} H / c^2$$

Expanding in powers of $(\frac{1}{m^2})$ as

$$H = Mc^2 + H^{(0)} + H^{(1)} + \ldots,$$
$$\boldsymbol{K} = \boldsymbol{K}^{(0)} + \boldsymbol{K}^{(1)} + \ldots,$$

and solving the commutation relation at each order, relativistic correction can be found.

From Non-Relativistic expansion of:

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
$1 \otimes \gamma$	$\tilde{O}_{7-9} = \frac{i}{2m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\psi\bar{\psi}\gamma_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$1 \otimes \gamma\gamma_5$	$\tilde{O}_{10-12} = \frac{-1}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\overleftrightarrow{\partial}_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \gamma_5$	$\tilde{O}_{13-15} = -\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \sigma$	$\tilde{O}_{16} = \frac{1}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_5\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\mu}\psi\partial_{\nu}\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi}\gamma_{\mu}\psi\bar{\psi}\gamma^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{20-22} = \frac{-1}{4m^2}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\gamma^{\nu}\overleftrightarrow{\partial}^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma \otimes \gamma\gamma_5$	$\tilde{O}_{23} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\nu}\psi\partial_{\alpha}\bar{\psi}\gamma_{\gamma}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{24} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma^{\gamma}\overleftrightarrow{\partial}_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{25-27} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\overleftrightarrow{\partial}_{\nu}\psi\partial_{\alpha}\bar{\psi}\gamma_{\beta}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{28-30} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\psi\partial_{\nu}\bar{\psi}\gamma_{\alpha}\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi}\gamma^{\mu}\gamma_5\psi\bar{\psi}\gamma_{\mu}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
	$\tilde{O}_{37-39} = \frac{-1}{4m^2}\bar{\psi}\gamma^{\mu}\gamma_5\overleftrightarrow{\partial}^{\nu}\psi\bar{\psi}\gamma_{\nu}\gamma_5\overleftrightarrow{\partial}_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \sigma$	$\tilde{O}_{40-42} = \frac{-i}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\overleftrightarrow{\partial}_{\nu}\overleftrightarrow{\partial}^{\gamma}\psi\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{43-45} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\psi\bar{\psi}\sigma_{\nu\alpha}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{46-48} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_{\mu}\gamma_5\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\sigma_{\alpha\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\sigma \otimes \sigma$	$\tilde{O}_{49-51} = \bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma^{\mu\nu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{52-54} = \frac{-1}{4m^2}\bar{\psi}\sigma^{\mu\alpha}\overleftrightarrow{\partial}^{\beta}\psi\bar{\psi}\sigma_{\mu\beta}\overleftrightarrow{\partial}_{\alpha}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$

A complete non-minimal set of relativistic contact operators. Here $\overleftrightarrow{d}_X = \overleftrightarrow{\partial}_X/(2m)$ $(X = A, B)$ and $d = \partial/(2m)$.

NR expansions of relativistic operators can be expressed via a complete basis of NR operators.

[E. Filandri and L. Girlanda, Phys. Lett. B (2023)]

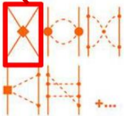
$\Longrightarrow$ In the end, we find:

- 26 independent combinations:
  - 2 LO
  - 7 NLO
  - 17 N3LO

- Existence of 2 free LECs which parameterize an interaction that depends on P

$$
\begin{aligned}
V_{NN}^{(4)} = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\boldsymbol{k} \times \boldsymbol{Q})^2 \\
& + \left[D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\boldsymbol{k} \times \boldsymbol{Q})^2\right](\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\
& + \frac{i}{2} (D_9 k^2 + D_{10} Q^2)\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{Q} \times \boldsymbol{k}) \\
& + (D_{11} k^2 + D_{12} Q^2)\, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \\
& + (D_{13} k^2 + D_{14} Q^2)\, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{Q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{Q}) \\
& + D_{15}\, \boldsymbol{\sigma}_1 \cdot (\boldsymbol{k} \times \boldsymbol{Q})\, \boldsymbol{\sigma}_2 \cdot (\boldsymbol{k} \times \boldsymbol{Q}) \\
& + i D_{16}\, \boldsymbol{k} \cdot \boldsymbol{Q}\; \boldsymbol{Q} \times \boldsymbol{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \\
& + D_{17}\, \boldsymbol{k} \cdot \boldsymbol{Q}\, (\boldsymbol{k} \times \boldsymbol{P}) \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)]
\end{aligned}
$$

N^3LO
$(Q/\Lambda_\chi)^4$



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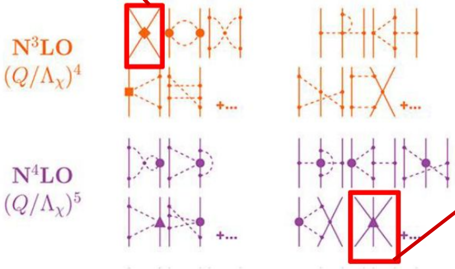
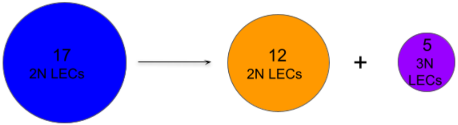
& + \left[D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right] (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\
& + \frac{i}{2} \left(D_9 k^2 + D_{10} Q^2 \right) (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k}) \\
& + \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
& + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) \\
& + D_{15}\, \boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q})\, \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q}) \\
& + i D_{16}\, \mathbf{k} \cdot \mathbf{Q}\, \mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \\
& + D_{17}\, \mathbf{k} \cdot \mathbf{Q}\, (\mathbf{k} \times \mathbf{P}) \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)]
\end{aligned}
$$

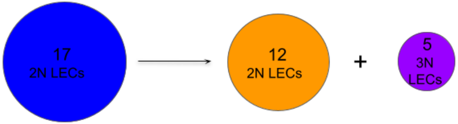


$$
\begin{aligned}
V_{3N}^{(2)} = \sum_{ijk} \Big(& -E_1\, \mathbf{k}_i^2 - E_2\, \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
& - E_3\, \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4\, \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
& - E_5 \left(3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \\
& - E_6 \left(3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
& + \frac{i}{2} E_7\, \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \\
& + \frac{i}{2} E_8\, \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k \\
& - E_9\, \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - E_{10}\, \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
& - E_{11}\, \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i - E_{12}\, \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
& - E_{13}\, \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \Big)
\end{aligned}
$$

Redundancy at N3LO



Redundancy at N3LO



$$U = e^{\alpha_i T_i}$$

$$T_1 = \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i \left(N^\dagger N\right)$$

$$T_2 = \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i \left(N^\dagger \sigma^j N\right)$$

$$T_3 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

[P. Reinert et al., Eur. Phys. J. A (2018)]

$$T_4 = i\epsilon^{ijk} \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N$$

$$T_5 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

[L. Girlanda et al., Phys. Rev. C (2020)]

$$U^\dagger H_0 U = H_0 + \alpha_i [H_0, T_i] + \cdots \equiv H_0 + \alpha_i \delta_i H_0 + \cdots \Rightarrow$$

$$U^\dagger H_{C_S/C_T}\left(\right.$$

$$\left.\right) U = H_{C_S/C_T} + \alpha_i [H_{C_S/C_T}, T_i] + \cdots \equiv H_{C_S/C_T} + \alpha_i \delta_i H_{C_S/C_T} + \cdots \Rightarrow$$

α_i are related with the D_i and E_i LECs

$$\delta E_1 = \alpha_1 (C_S + C_T) + \alpha_2 (C_S - 2C_T)$$
$$\delta E_2 = 3\alpha_2 C_T + 2\alpha_3 C_T - 8\alpha_4 C_T + 2\alpha_5 C_T$$
$$\delta E_3 = 2\alpha_1 C_T + \alpha_2 (2C_S - C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T$$
$$\delta E_4 = \frac{2}{3}\alpha_1 C_T + \frac{1}{3}\alpha_2 (2C_S - 7C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T$$
$$\delta E_5 = 2\alpha_1 C_T + 2\alpha_2 (C_S - 2C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T$$
$$\delta E_6 = \frac{2}{3}\alpha_1 C_T + \frac{2}{3}\alpha_2 (C_S - 2C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T,$$
$$\delta E_7 = 24\alpha_4 C_T$$
$$\delta E_8 = \frac{1}{3}\delta E_7$$
$$\delta E_9 = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) - \alpha_4 (C_S - 11C_T) + 2\alpha_5 (C_S - 2C_T)$$
$$\delta E_{10} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) - \frac{1}{3}\alpha_4 (3C_S - 15C_T)$$
$$\delta E_{11} = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) + \alpha_4 (C_S - 11C_T) - 2\alpha_5 (C_S - 2C_T)$$
$$\delta E_{12} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) + \frac{1}{3}\alpha_4 (3C_S - 15C_T)$$
$$\delta E_{13} = -16\alpha_4 C_T + 4\alpha_5 C_T$$

$$\alpha_1 = \frac{m}{16}(16D_1 + D_2 + 4D_3)$$
$$\alpha_2 = \frac{m}{16}(16D_5 + D_6 + 4D_7)$$
$$\alpha_3 = \frac{m}{32}(D_{14} + 16D_{11} + 4D_{12} + 4D_{13})$$
$$\alpha_4 = \frac{m}{2}D_{16}$$
$$\alpha_5 = \frac{m}{16}(8D_{17} - D_{14} - 16D_{11} - 4D_{12} - 4D_{13})$$

Therefore:

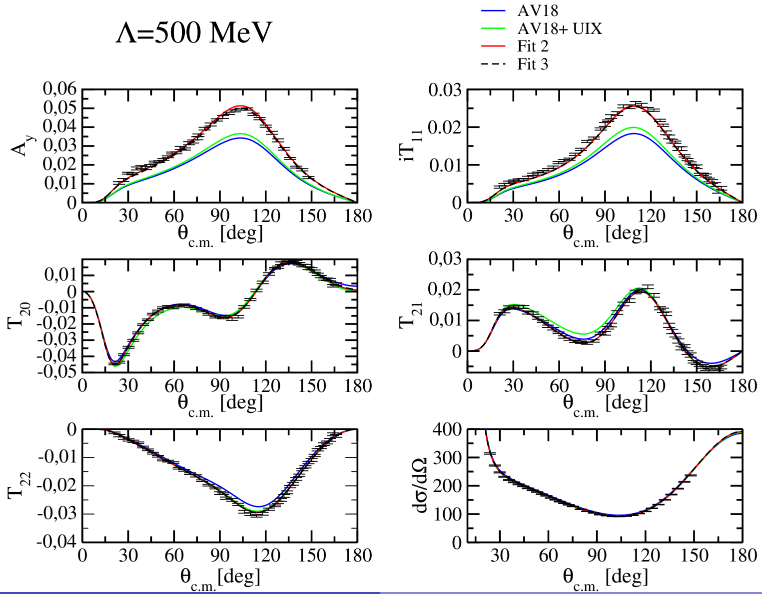
- The five LECs parametrizing the N3LO NN off-shell interaction can be fitted to observables of the 3N system and interpreted as a 3N interaction

$$\Downarrow$$

To see the effect of D_{16} and D_{17} free LECs, we tried to fit the α_4 and α_5 parameters to the p-d scattering observables

As a first step, we used and hybrid model with a phenomenological two body potential (the Av18) plus the α_4 and α_5 contributions

Fit on p-d observables



Fitting procedure	2-param.	3-param.
χ^2/d.o.f.	2.1	1.9
e_0	-	0.459
$\tilde{\alpha}_4 C_S$	1.751	1.894
$\tilde{\alpha}_5 C_S$	-0.495	-1.175
$^2a_{nd}$ (fm)	0.573	0.599

Conclusions

- Preliminary investigations show that the N-d Ay problem could be solved including the two free LECs in the 3N force at N3LO

.. and Outlook

- Use of unitary transformations in the full N3LO Chiral potential
- Calculation of scattering observables also exploring the energy dependence and quantitative error estimation
- Analysing the effect of D_{16} and D_{17} in systems with $A \geq 3$

Backup slides

Choosing [Reinert, et al., Eur. Phys. J. A 54, 86 (2018)]

$$\alpha_1 = m(D_1+D_2+4D_3),\ \alpha_2 = m(16D_5+D_6+4D_7),\ \alpha_3 = m(8D_{11}+2D_{12}+2D_{13}+D_{14}/2),\ \alpha_4 = -\frac{m}{2}D_{16},\ \alpha_5 = -\frac{m}{2}D_{17}$$

it is possible to transform the N3LO two body contact potential

$$\begin{aligned} V^{(4)} =& D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\boldsymbol{k}\times\boldsymbol{Q})^2 + \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\boldsymbol{k}\times\boldsymbol{Q})^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\ &+ \frac{i}{2}\left(D_9 k^2 + D_{10} Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{Q}\times\boldsymbol{k}) + \left(D_{11} k^2 + D_{12} Q^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) + \left(D_{13} k^2 + D_{14} Q^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{Q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{Q}) \\ &+ D_{15}\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}\times\boldsymbol{Q})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}\times\boldsymbol{Q}) + iD_{16}\boldsymbol{k}\cdot\boldsymbol{Q}\boldsymbol{Q}\times\boldsymbol{P}\cdot(\sigma_1-\sigma_2) + D_{17}\boldsymbol{k}\cdot\boldsymbol{Q}(\boldsymbol{k}\times\boldsymbol{P})\cdot(\boldsymbol{\sigma_1}\times\boldsymbol{\sigma_2}) \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} V^{(4)} =& D'_1\left(k^4 - 4(\boldsymbol{Q}\cdot\boldsymbol{k})^2\right) + D'_2\left(Q^4 - \frac{1}{4}(\boldsymbol{Q}\cdot\boldsymbol{k})^2\right) + D'_3(\boldsymbol{k}\times\boldsymbol{Q})^2 \\ &+ \left(D'_4\left(k^4 - 4(\boldsymbol{Q}\cdot\boldsymbol{k})^2\right) + D'_5\left(Q^4 - \frac{1}{4}(\boldsymbol{Q}\cdot\boldsymbol{k})^2\right) + D'_6(\boldsymbol{k}\times\boldsymbol{Q})^2\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \frac{i}{2}\left(D'_7 k^2 + D'_8 Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{Q}\times\boldsymbol{k}) \\ &+ D'_9\left(-\frac{1}{4}k^2\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k} + 4Q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{Q}\boldsymbol{\sigma}_2\cdot\boldsymbol{Q}\right) + D'_{10}Q^2\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k} - 4\boldsymbol{\sigma}_1\cdot\boldsymbol{Q}\boldsymbol{\sigma}_2\cdot\boldsymbol{Q}\right) + D'_{11}\left(k^2 - 4Q^2\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{Q}\boldsymbol{\sigma}_2\cdot\boldsymbol{Q} \\ &+ D'_{12}\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}\times\boldsymbol{Q})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}\times\boldsymbol{Q}) \end{aligned}$$

$$\text{with}\ \ k = p' - p,\quad Q = \frac{p'+p}{2},\quad P = p_1 + p_2$$

As long as we promote 5 terms of the N4LO 3B potential to N3LO

Bound and scattering wave functions

The ^3H wave function is written as an expansion over spin-isospin-Hyperspherical Harmonics (HH) states times hyperradial functions, which are themselves expanded on a basis of Laguerre polynomials,

$$\Psi = \sum_{\mu} c_{\mu} \phi_{\mu} \quad (1)$$

where μ denotes collectively the quantum numbers specifying the combination ϕ_{μ} of spin-isospin-HH states.

The Rayleigh-Ritz variational principle,

$$\delta \langle \Psi | H - E | \Psi \rangle = 0$$

is used to determine the expansion coefficients c_{μ} and bound state energy E

To describe $\mathrm{N-d}$ scattering states below the deuteron breakup threshold the w.f. is taken as

$$\Psi = \Psi_C + \Omega^R + \sum_{\mu} \mathcal{R}_{\mu} \Omega^I_{\mu}$$

Ψ_C describes configurations in which all the particles of the system are close to each other and is decomposed as Eq (1), $\Omega^{\lambda=I,R}$ are functions describing the asymptotic region. $\mathcal{R}_{\mu}$ are the $\mathcal{R}$-matrix elements.

The Ψ_C coefficients c_{μ} and $\mathcal{R}_{\mu}$ are determined by using the Kohn variational principle which guarantees that the $\mathcal{R}$-matrix elements, considered as functionals of the w.f., are stationary with respect to variations of all the trial parameters