PV DIS Beyond Leading Twist

Sonny Mantry
Northwestern University/Argonne National Lab

In Collaboration with M.J. Ramsey-Musolf and G.F. Sacco

“From Parity Violation to Hadronic Structure and more...”

PAVI, 2011, Rome, September 5th-9th
Electron-Deuteron Asymmetry

- Parity Violating asymmetry
  \[ A_{RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \]

- Probe of the parity-violating Weak Neutral Current (WNC) in the Standard Model (SM).

- Led to a spectacular confirmation of WNC theory of the SM in 1978 (SLAC).

- Gave one of the first precise measurements of the weak mixing angle to within 10%.
Electron-Deuteron Asymmetry

- Probe of parity-violating interactions in the Standard Model.

\[ \mathcal{L} = \frac{G_F}{\sqrt{2}} \left[ \bar{e}\gamma^\mu\gamma_5 e \left( C_{1u} \bar{u} \gamma_\mu u + C_{1d} \bar{d} \gamma_\mu d \right) + \bar{e}\gamma^\mu e \left( C_{2u} \bar{u} \gamma_\mu \gamma_5 u + C_{2d} \bar{d} \gamma_\mu \gamma_5 d \right) \right] \]

- Led to one of the first measurements of the weak mixing angle

\[
\begin{align*}
C_{1u}^{\text{tree}} &= -\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W, & C_{1d}^{\text{tree}} &= \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, \\
C_{2u}^{\text{tree}} &= -\frac{1}{2} + 2 \sin^2 \theta_W, & C_{2d}^{\text{tree}} &= \frac{1}{2} - 2 \sin^2 \theta_W.
\end{align*}
\]
Electron-Deuteron Asymmetry

- All hadronic effects cancel in the asymmetry to first approximation; Cahn-Gilman (CG) formula:

\[
A^{RL}_{CG} = -\frac{G_F Q^2}{2\sqrt{2\pi\alpha}} \frac{9}{10} \left[ \left( 1 - \frac{20}{9} \sin^2 \theta_W \right) + \left( 1 - 4 \sin^2 \theta_W \right) \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right]
\]

- Hadronic effects appear as small corrections to the CG formula.

All hadronic effects cancel!

Clean probe of WNC
• **Qweak**
  - Weinberg angle to within 0.3%

• **12 GeV program at JLab to begin 2014:**
  - Moller
  - SoLID, 6 GeV, and 12, GeV experiments

• The focus has shifted from the SM WNC theory to detecting hints of physics beyond the SM.
Corrections to Cahn-Gilman

- In the precision era, all corrections to CG must be under control

\[
A_{RL} = - \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{9}{10} \left[ \tilde{a}_1 + \tilde{a}_2 \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right]
\]

\[
\tilde{a}_j = -\frac{2}{3} (2C_{ju} - C_{jd}) \left[ 1 + R_j(\text{new}) + R_j(\text{sea}) + R_j(\text{CSV}) + R_j(\text{TMC}) + R_j(\text{HT}) \right]
\]

- Hadronic and electroweak effects must be well understood before any claim for evidence of new physics can be made.
Asymmetry as a Probe of Hadronic Physics

\[ A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi \alpha} \frac{9}{10} \left[ \tilde{a}_1 + \tilde{a}_2 \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right] \]

\[ \tilde{a}_j = -\frac{2}{3} (2C_{ju} - C_{jd}) \left[ 1 + R_j(\text{new}) + R_j(\text{sea}) + R_j(\text{CSV}) + R_j(\text{TMC}) + R_j(\text{HT}) \right] \]

- Alternatively, precision PV DIS can be viewed as a probe of hadronic physics.

- Precision measurements over wide kinematic range can disentangle various effects.
• SOLID plans to measure the asymmetry at the percent level over a wide kinematic range:

Projected data with errors for SOLID
(K.Kumar, P.Souder)
Asymmetry as a Probe of Higher Twist

\[ A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{9}{10} \left[ \tilde{a}_1 + \tilde{a}_2 \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right] \]

\[ \tilde{a}_j = -\frac{2}{3} (2C_{ju} - C_{jd}) \left[ 1 + R_j(\text{new}) + R_j(\text{sea}) + R_j(\text{CSV}) + R_j(\text{TMC}) + R_j(\text{HT}) \right] \]

- Precision PV DIS can be a probe of higher twist correlations.
Asymmetry as a Probe of Higher Twist

\[ A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{9}{10} \left( \tilde{a}_1 + \tilde{a}_2 \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right) \]

\[ \tilde{a}_j = -\frac{2}{3} (2C_{ju} - C_{jd}) \left[ 1 + R_j(\text{new}) + R_j(\text{sea}) + R_j(\text{CSV}) + R_j(\text{TMC}) + R_j(\text{HT}) \right] \]

- Precision PV DIS can be a probe of higher twist correlations.

- Can probe a single four-quark matrix element which encodes quark-quark correlations. (Bjorken, Wolfenstein)
Higher Twist
Parton Model and Bjorken Scaling

- Bjorken limit: high $Q^2$ at fixed Bjorken-x.

- Structure functions independent of $Q^2$ up to logarithmic deviations.

- Nucleon is a collection of “almost free quarks”.

![Graph showing $F_2(x,Q^2)$ for different values of $x$ and $Q^2$]
Parton Model and Bjorken Scaling

- Bjorken limit: high \( Q^2 \) at fixed Bjorken-\( x \).

- Structure functions independent of \( Q^2 \) up to logarithmic deviations.

- Nucleon is a collection of “almost free quarks”.

Long range quark and gluon correlations?
Operator Product expansion

\[ i \int d^4x \ e^{i q \cdot x} \ J(x) J(0) \sim \sum_{n=0}^{\infty} \sum_{i,j} C_{i}^{n}(Q^2, \mu^2) (Q^2)^{\frac{\tau-2}{2}} \langle O_{\mu_1 \ldots \mu_n}^{\tau,i}(0) \rangle q^{\mu_1} \ldots q^{\mu_n}. \]

- Twist expansion:

- Leading twist gives the parton model.

- Correlation matrix elements beyond parton model suppressed by powers of $1/Q^2$: Higher Twist Terms
Operator Product Expansion

Twist-2 $\rightarrow$ Bjorken scaling

Quark-gluon correlation (Twist-4)

Power corrections

Quark-quark correlation (Twist-4)
Operator Product Expansion

Twist-2

Quark-gluon correlation (Twist-4)

Quark-quark correlation (Twist-4)

\[ \mathcal{O}_{ud}^{\mu\nu}(x) = \frac{1}{2} \left[ \bar{u}(x)\gamma^\mu u(x)d(0)\gamma^\nu d(0) + (u \leftrightarrow d) \right] \]
Key features of the Asymmetry Terms

- Asymmetry can be brought into the form:

\[ A_{RL} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left[ g_A^e Y_1 \frac{F_{1\gamma}^Z}{F_1} + g_V^e Y_3 \frac{F_{3\gamma}^Z}{F_1} \right] \]

- Dominant term in asymmetry
- Can in principle be
  kinematically distinguished from second term (independent of y)
- Can be sensitive to only quark-quark correlations
- A single twist-4 matrix element determines quark-quark correlations.

- suppressed by small electron vector coupling
- Can be kinematically distinguished from second term (dependent on y)
- Can be sensitive to quark-quark and quark-gluon correlations
- Multiple twist-4 matrix elements determine correlations
- Can be extracted from neutrino scattering data
Key features of the Asymmetry Terms

- Asymmetry can be brought into the form:

\[ A_{RL} = -\left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left[ g_A^e Y_1 \frac{F_1^\gamma Z}{F_1^\gamma} + g_V^e Y_3 \frac{F_3^\gamma Z}{F_1^\gamma} \right] \]

- Dominant term in asymmetry
- Can in principle be kinematically distinguished from second term (independent of y)
- Can be sensitive to only quark-quark correlations
- A single twist-4 matrix element determines quark-quark correlations.

Focus of this talk
- suppressed by small electron vector coupling
- Can be kinematically distinguished from second term (dependent on y)
- Can be sensitive to quark-quark and quark-gluon correlations
- Multiple twist-4 matrix elements determine correlations
- Can be extracted from neutrino scattering data
Some Definitions and Notation

- Asymmetry can be brought into the form:

\[
A_{RL} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi \alpha} \right) \left[ g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^\gamma} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^\gamma} \right]
\]

- The \( Y_1 \) factor has the form:

\[
Y_1 = \left( \frac{1 + R^{\gamma Z}}{1 + R^\gamma} \right) \frac{1 + (1 - y)^2 - y^2 \left[ 1 - r^2/(1 + R^{\gamma Z}) \right] - 2xyM/E}{1 + (1 - y)^2 - y^2 \left[ 1 - r^2/(1 + R^\gamma) \right] - 2xyM/E}
\]

- The \( Y_3 \) factor has the form:

\[
Y_3 = \left( \frac{r^2}{1 + R^\gamma} \right) \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - y^2 \left[ 1 - r^2/(1 + R^\gamma) \right] - 2xyM/E}
\]

- We have used the definitions:

\[
R^{\gamma(\gamma Z)} = \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1, \quad r^2 = 1 + \frac{4M^2 x^2}{Q^2}
\]
Cahn-Gilman Limit

- Cahn-Gilman limit:

\[
A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right) \left[ g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^\gamma} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^\gamma} \right]
\]

\[
R^\gamma = R^{\gamma Z} = r^2 - 1,
\]

\[
Y_1 = 1
\]

\[
\left[ \frac{F_1^{\gamma Z}}{F_1^\gamma} \right]_{CG} = \frac{9}{10} \left(1 - \frac{20}{9} \sin^2 \theta_W \right)
\]

- Higher twist effects can modify these relations.
Concluded that:

-20% difference gives a 1% effect in asymmetry
-Could interfere with extraction of CSV effects

\[ R^\gamma \neq R^{\gamma Z} \]
More Recent Analysis

(SM, M. Ramsey-Musolf, G. Sacco)

- Our conclusions based on the Bjorken/Wolfenstein argument:
  - Twist-4 effects in vector WNC term come only from quark-quark correlations.
  - A single 4-quark twist-4 matrix element contributes to the vector WNC term.
  - The relation $R_{Y^Z} = R_{Y^\gamma}$ holds true at twist-4 up to perturbative corrections.

Only quark-quark correlations given by a single matrix element

$$A_{RL} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \right) \left[ g_A^e Y_1 \frac{F_{1}^{\gamma Z}}{F_{1}^{\gamma}} + g_V^e Y_3 \frac{F_{3}^{\gamma Z}}{F_{1}^{\gamma}} \right]$$

$Y_1 = 1$

Twist-4 effects reside in ratio of form factors
Bjorken-Wolfenstein Argument

- Isospin decomposition of electromagnetic and vector neutral currents:

\[ J^\mu_\gamma = v_\mu + \frac{1}{3}s_\mu - \frac{1}{3}\lambda_\mu, \quad J^\mu_Z = 2\left[ (1 - 2\sin^2\theta)v_\mu - \frac{2}{3}\sin^2\theta s_\mu - \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta\right)\lambda_\mu \right] \]

\[ v_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \quad s_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d), \quad \lambda_\mu = \bar{s}\gamma_\mu s \]

- Isospin decomposition of electromagnetic and interference hadronic tensors

\[ W^{\gamma}_{\mu\nu} = W^\nu_{\mu\nu} + \frac{1}{9}W^{ss}_{\mu\nu} \]

\[ W^{V;\gamma Z}_{\mu\nu} = 2(1 - 2\sin^2\theta)W^{\nu\nu}_{\mu\nu} - \frac{4}{9}\sin^2\theta W^{ss}_{\mu\nu} \]
Bjorken-Wolfenstein Argument

- Isospin decomposed hadronic tensors:

\[
W_{\mu\nu}^{vv} = \frac{1}{2\pi M} \int d^4 x \, e^{iq \cdot x} \langle D(P) | v_\mu(x) v_\nu(0) | D(P) \rangle
\]

\[
W_{\mu\nu}^{ss} = \frac{1}{2\pi M} \int d^4 x \, e^{iq \cdot x} \langle D(P) | s_\mu(x) s_\nu(0) | D(P) \rangle
\]

- Twist-4 quark-quark correlation hadronic tensor:

\[
W_{\mu\nu}^{du} = W_{\mu\nu}^{ss} - W_{\mu\nu}^{vv},
\]

\[
= \frac{1}{2\pi M} \int d^4 x \, e^{iq \cdot x} \langle D(P) | \frac{1}{2} \{ \bar{d}(x) \gamma_\mu d(x) \, \bar{u}(0) \gamma_\nu u(0) + (u \leftrightarrow d) \} | D(P) \rangle
\]
**Bjorken-Wolfenstein Argument**

- **Isospin decomposed hadronic tensors:**
  \[
  W_{\mu\nu}^{vv} = \frac{1}{2\pi M} \int d^4x \ e^{iq\cdot x} \langle D(P) | \nu_\mu(x) \nu_\nu(0) | D(P) \rangle \\
  W_{\mu\nu}^{ss} = \frac{1}{2\pi M} \int d^4x \ e^{iq\cdot x} \langle D(P) | s_\mu(x) s_\nu(0) | D(P) \rangle 
  \]

- **Twist-4 quark-quark correlation hadronic tensor:**
  \[
  W_{\mu\nu}^{du} = W_{\mu\nu}^{ss} - W_{\mu\nu}^{vv}, \\
  = \frac{1}{2\pi M} \int d^4x \ e^{iq\cdot x} \langle D(P) | \frac{1}{2} \{ \bar{d}(x) \gamma_\mu d(x) \bar{u}(0) \gamma_\nu u(0) + (u \leftrightarrow d) \} | D(P) \rangle 
  \]

- **Structure Function definitions**
  \[
  W_{\mu\nu}^{vv,ss,du} = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{F_{1}^{vv,ss,du}}{M} + \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \frac{F_{2}^{vv,ss,du}}{MP \cdot q}. 
  \]
Bjorken-Wolfenstein Argument

- **Isospin decomposed hadronic tensors:**
  \[
  W_{\mu\nu}^{vv} = \frac{1}{2\pi M} \int d^4x \ e^{iq \cdot x} \langle D(P) | v_\mu(x)v_\nu(0) | D(P) \rangle
  \]
  \[
  W_{\mu\nu}^{ss} = \frac{1}{2\pi M} \int d^4x \ e^{iq \cdot x} \langle D(P) | s_\mu(x)s_\nu(0) | D(P) \rangle
  \]

- **Twist-4 quark-quark correlation hadronic tensor:**
  \[
  W_{\mu\nu}^{du} = W_{\mu\nu}^{ss} - W_{\mu\nu}^{vv},
  \]
  \[
  = \frac{1}{2\pi M} \int d^4x \ e^{iq \cdot x} \langle D(P) | \frac{1}{2} \{ \bar{d}(x)\gamma_\mu d(x) \bar{u}(0)\gamma_\nu u(0) + (u \leftrightarrow d) \} | D(P) \rangle
  \]

- **Structure Function definitions**
  \[
  W_{\mu\nu}^{vv,ss,du} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \frac{F_{1}^{vv,ss,du}}{M} + \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \frac{F_{2}^{vv,ss,du}}{MP \cdot q}
  \]
  \[
  F^{vv,ss,du}_{1,2} = F^{ss}_{1,2} - F^{du}_{1,2}
  \]
Bjorken-Wolfenstein Argument

- Original structure functions can be written as:
  \[
  F_{1,2}^{\gamma} = \frac{10}{9} F_{1,2}^{ss} - F_{1,2}^{du}
  \]
  \[
  F_{1,2}^{\gamma Z} = 2(1 - \frac{20}{9} \sin^2 \theta) F_{1,2}^{ss} - 2(1 - 2 \sin^2 \theta) F_{1,2}^{du}
  \]

- Asymmetry in terms of original structure functions:
  \[
  A_{RL} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \right) \left[ g^e_A Y_1 \frac{F_{1}^{\gamma Z}}{F_{1}^{\gamma}} + g^e_V Y_3 \frac{F_{3}^{\gamma Z}}{F_{1}^{\gamma}} \right]
  \]

- Y1 term:
  \[
  A_{RL}^V = - \frac{9}{10} \left( \frac{G_F Q^2}{2 \sqrt{2} \pi \alpha} \right) g^e_A \left\{ \left( 1 - \frac{20}{9} \sin^2 \theta \right) - \frac{1}{10} \frac{F_{1}^{du}}{F_{1;LT}^{ss}} \right\}
  \]
Form of twist-4 correction

- $Y_1$ term:

$$A^V_{RL} = -\frac{9}{10} \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) g^e_A \left\{ \left( 1 - \frac{20}{9} \sin^2 \theta \right) - \frac{1}{10} \frac{F_1^{du}}{F_{1;LT}^{ss}} + \cdots \right\}$$

- Twist-4 correction given by:

$$R_1(HT) = \left[ \frac{-4}{5(1 - \frac{20}{9} \sin^2 \theta_W)} \right] \frac{F_1^{du}}{u_p(x) + d_p(x)}.$$
\[
R^{\gamma Z} = R^{\gamma}
\]

- **Quark-quark correlation twist-4 operator matrix element:**

  twist-4 structure
  functions

  \[
  \frac{1 + R^{\gamma Z}}{1 + R^{\gamma}} = 1 + \left( \frac{F_2^{du} - 2xF_1^{du}}{2xF_1^{ss,LT}} \right) \left[ \frac{9}{10} - \frac{1 - 2\sin^2\theta}{1 - \frac{20}{9}\sin^2\theta} \right]
  \]

  twist-2 structure
  function

- Using the Callan-Gross relation at tree level we get:

  \[
  F_2^{du} = 2xF_1^{du}
  \]

  \[
  \frac{1 + R^{\gamma Z}}{1 + R^{\gamma}} = 1
  \]

  (R.Ellis, W.Furmanski, R.Petronzio; X.Ji; J.Qiu)

- We also give an effective field theory (SCET) argument

  (SM, M. Ramsey-Musolf, G. Sacco)
\[ R^{\gamma Z} = R^\gamma \]

\[
\frac{1 + R^{\gamma Z}}{1 + R^\gamma} = 1
\]

- Using the Callan-Gross relation at tree level we get:

\[
A_{RL} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left[ g_A^e Y_1 \frac{F^{\gamma Z}_1}{F_1^\gamma} + g_V^e Y_3 \frac{F^{\gamma Z}_3}{F_1^\gamma} \right]
\]

- free of twist-4 effects

- twist-4 quark-quark correlations reside in ratio of structure functions
Form of twist-4 correction

\[ R_1(HT) = \left[ -\frac{4}{5(1 - \frac{20}{9}\sin^2 \theta_W)} \right] \frac{F_1^{du}}{u_p(x) + d_p(x)}. \]

- Bag model estimate of quark-quark correlation is below the half-percent level.
- Estimates using multi-parton nucleon light-cone wave functions, found an effect twice as big. (Belitsky, Manashov, Schafer)
\[ \delta u - \delta d = 2\kappa f(x), \quad f(x) = x^{-1/2}(1 - x)^4(x - 0.0909) \]

- Negligible higher twist effects can allow for a cleaner extraction of CSV or new physics effects.
Conclusions

• PV DIS can be a powerful probe of hadronic physics beyond the parton model.

• The precision and wide kinematic reach of 12 GeV Upgrade at JLAB can in principle disentangle various hadronic effects such as sea quarks, CSV, and higher twist.

• PV DIS can probe a `single’ twist-4 quark-quark correlation matrix element and is the only known observable with this property.

• Uncertainties in R-gamma-Z appear to have only a small effect on higher twist effects.