PV DIS Beyond Leading Twist

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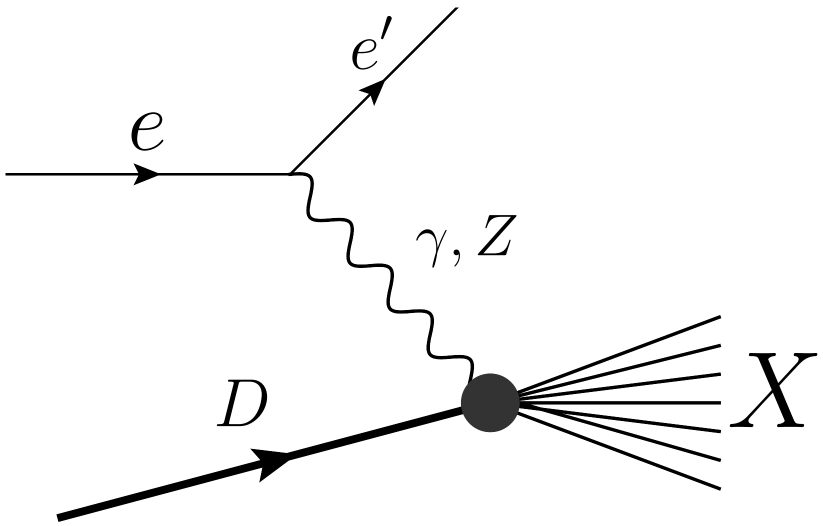
"From Parity Violation to Hadronic Structure and more..."

PAVI, 2011, Rome, September 5th-9th

Electron-Deuteron Asymmetry



- Parity Violating asymmetry

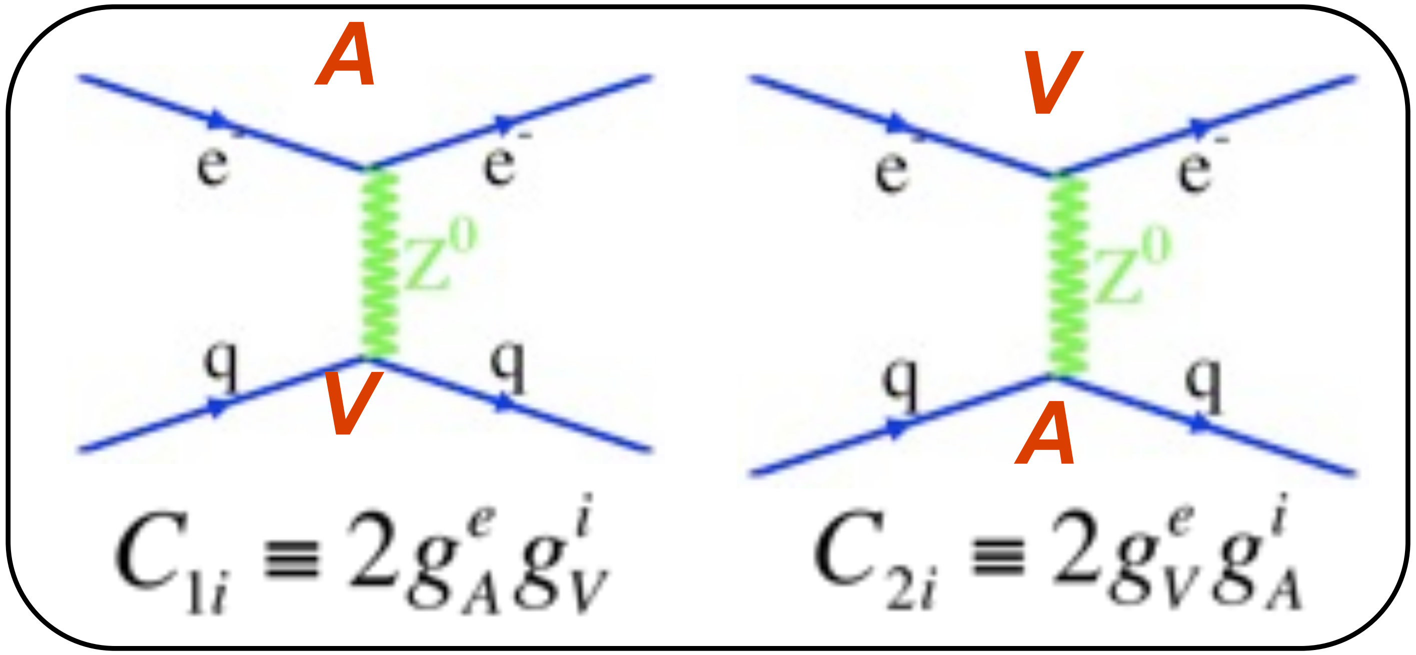
$$A_{RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- Probe of the parity-violating Weak Neutral Current (WNC) in the Standard Model (SM).

- Led to a spectacular confirmation of WNC theory of the SM in 1978 (SLAC).

- Gave one of the first precise measurements of the weak mixing angle to within 10%.

Electron-Deuteron Asymmetry



- Probe of parity-violating interactions in the Standard Model.

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\Big[\bar{e}\gamma^\mu\gamma_5 e\big(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d\big) + \bar{e}\gamma^\mu e\big(C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d\big)\Big]$$

- Led to one of the first measurements of the weak mixing angle

$$C_{1u}^{\text{tree}} = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W, \qquad C_{1d}^{\text{tree}} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W,$$
$$C_{2u}^{\text{tree}} = -\frac{1}{2} + 2\sin^2\theta_W, \qquad C_{2d}^{\text{tree}} = \frac{1}{2} - 2\sin^2\theta_W \quad .$$

Electron-Deuteron Asymmetry

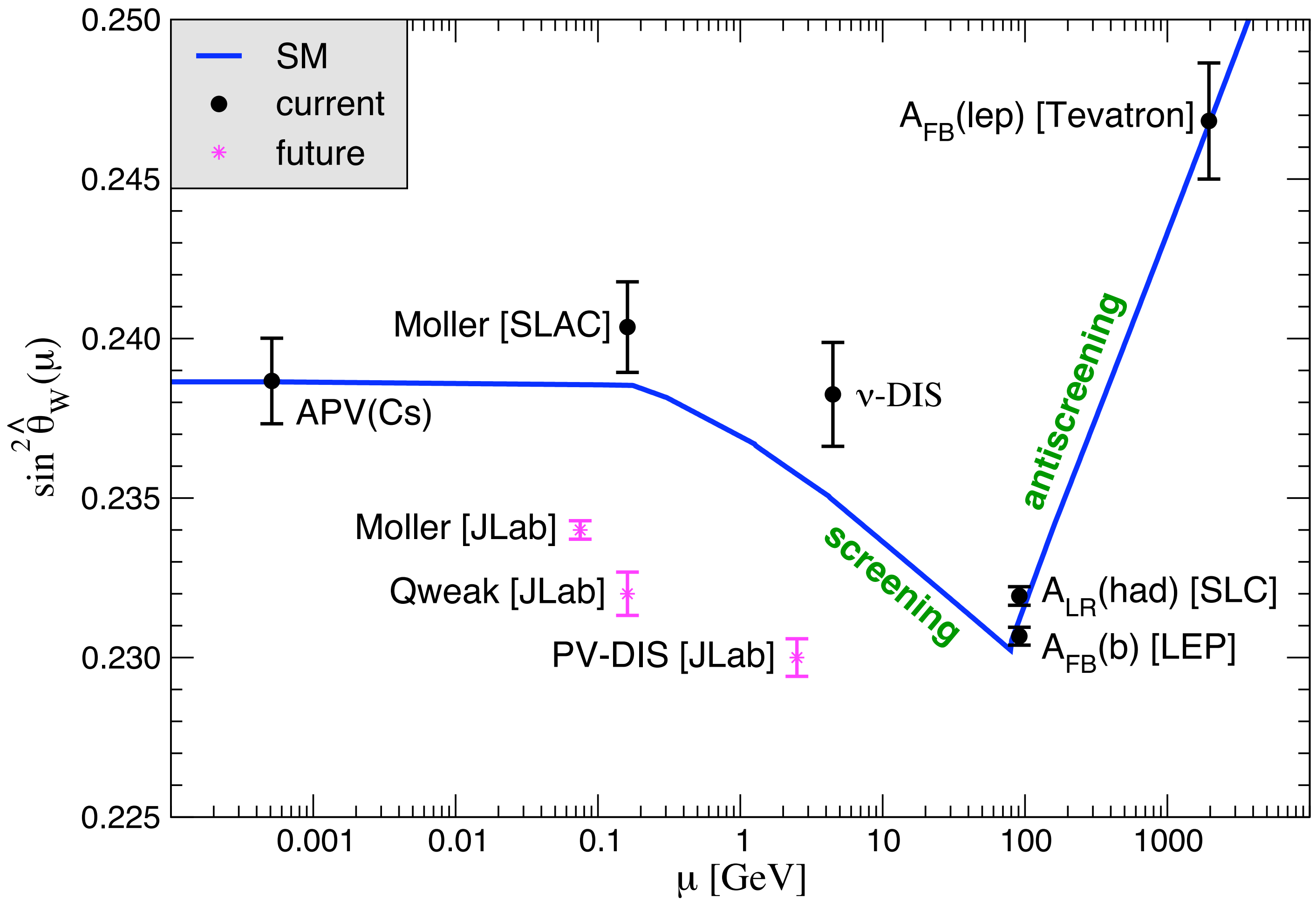
- All hadronic effects cancel in the asymmetry to first approximation; Cahn-Gilman (CG) formula:

$$A_{\rm CG}^{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left[\left(1-\frac{20}{9}\sin^2\theta_W\right)+\left(1-4\sin^2\theta_W\right)\frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

All hadronic effects cancel! → Clean probe of WNC

- Hadronic effects appear as small corrections to the CG formula.

Precision Era



(J. Erler, M. Ramsey-Musolf)

- Qweak
  - Weinberg angle to within 0.3%
- 12 GeV program at JLab to begin 2014:
  - Moller
  - SoLID, 6 GeV, and 12, GeV experiments
- The focus has shifted from the SM WNC theory to detecting hints of physics beyond the SM.

Corrections to Cahn-Gilman

- In the precision era, all corrections to CG must be under control

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left[\tilde{a}_1 + \tilde{a}_2\frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

$$\tilde{a}_j = -\frac{2}{3}(2C_{ju} - C_{jd})\left[1 + R_j(\mathrm{new}) + R_j(\mathrm{sea}) + R_j(\mathrm{CSV}) + R_j(\mathrm{TMC}) + R_j(\mathrm{HT})\right]$$

New physics

Sea quarks

Charge symmetry violation

Target mass

Higher twist

- Hadronic and electroweak effects must be well understood before any claim for evidence of new physics can be made.

Asymmetry as a Probe of Hadronic Physics

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left[\tilde{a}_1 + \tilde{a}_2 \frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

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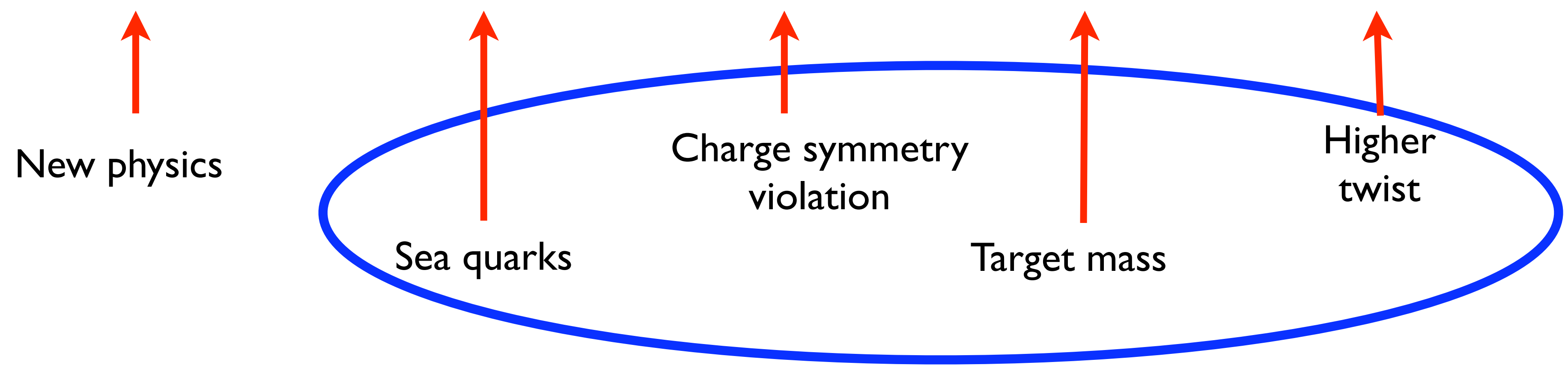
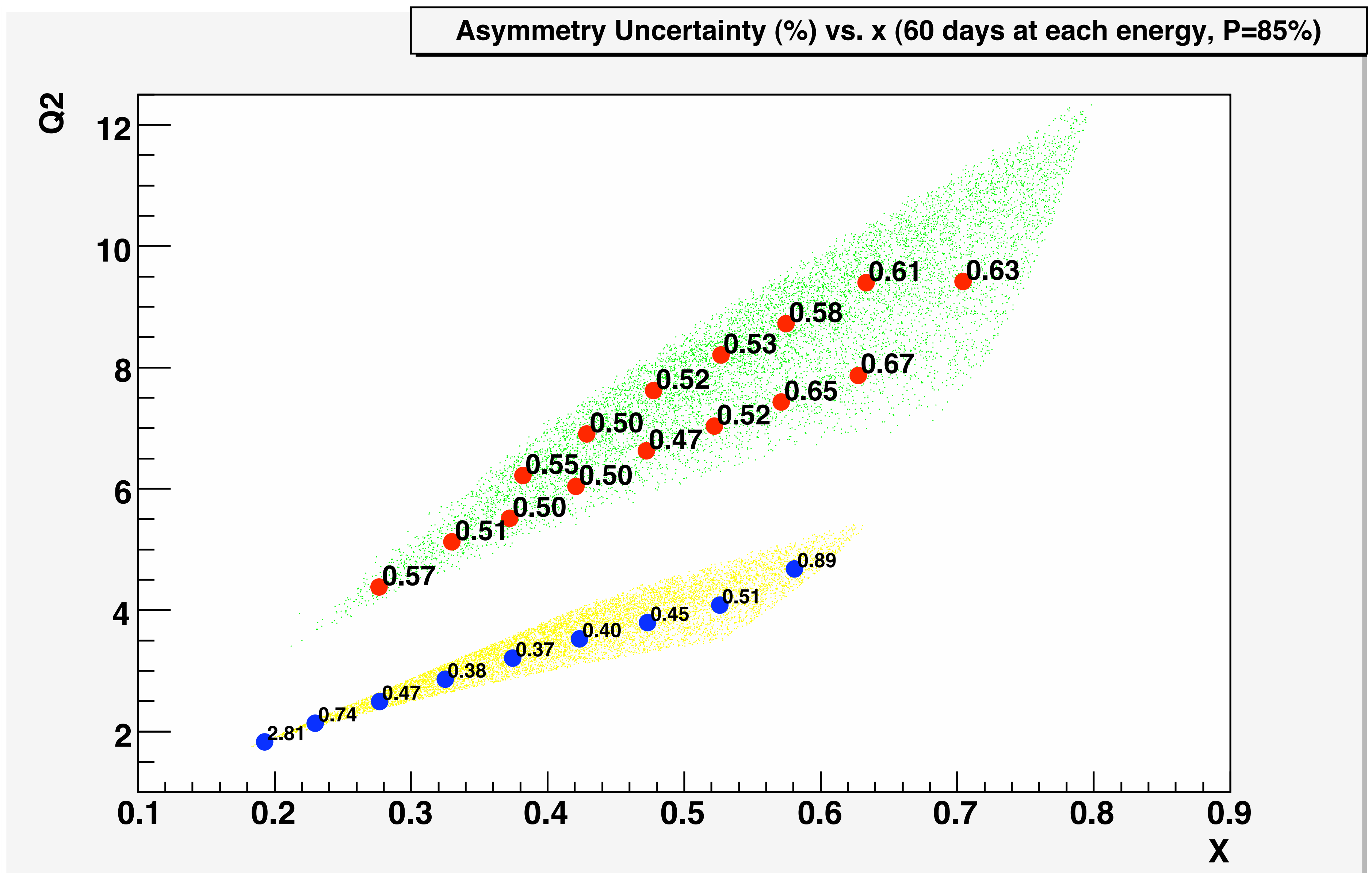


- Alternatively, precision PV DIS can be viewed as a probe of hadronic physics.

- Precision measurements over wide kinematic range can disentangle various effects.

SOLID

- SOLID plans to measure the asymmetry at the percent level over a wide kinematic range:



Projected data with errors for SOLID
(K.Kumar, P. Souder)

Asymmetry as a Probe of Higher Twist

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left[\tilde{a}_1 + \tilde{a}_2 \frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

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- Precision PV DIS can be a probe of higher twist correlations.

Asymmetry as a Probe of Higher Twist

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left[\tilde{a}_1 + \tilde{a}_2 \frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

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New physics

Sea quarks

Charge symmetry violation

Target mass

Higher twist

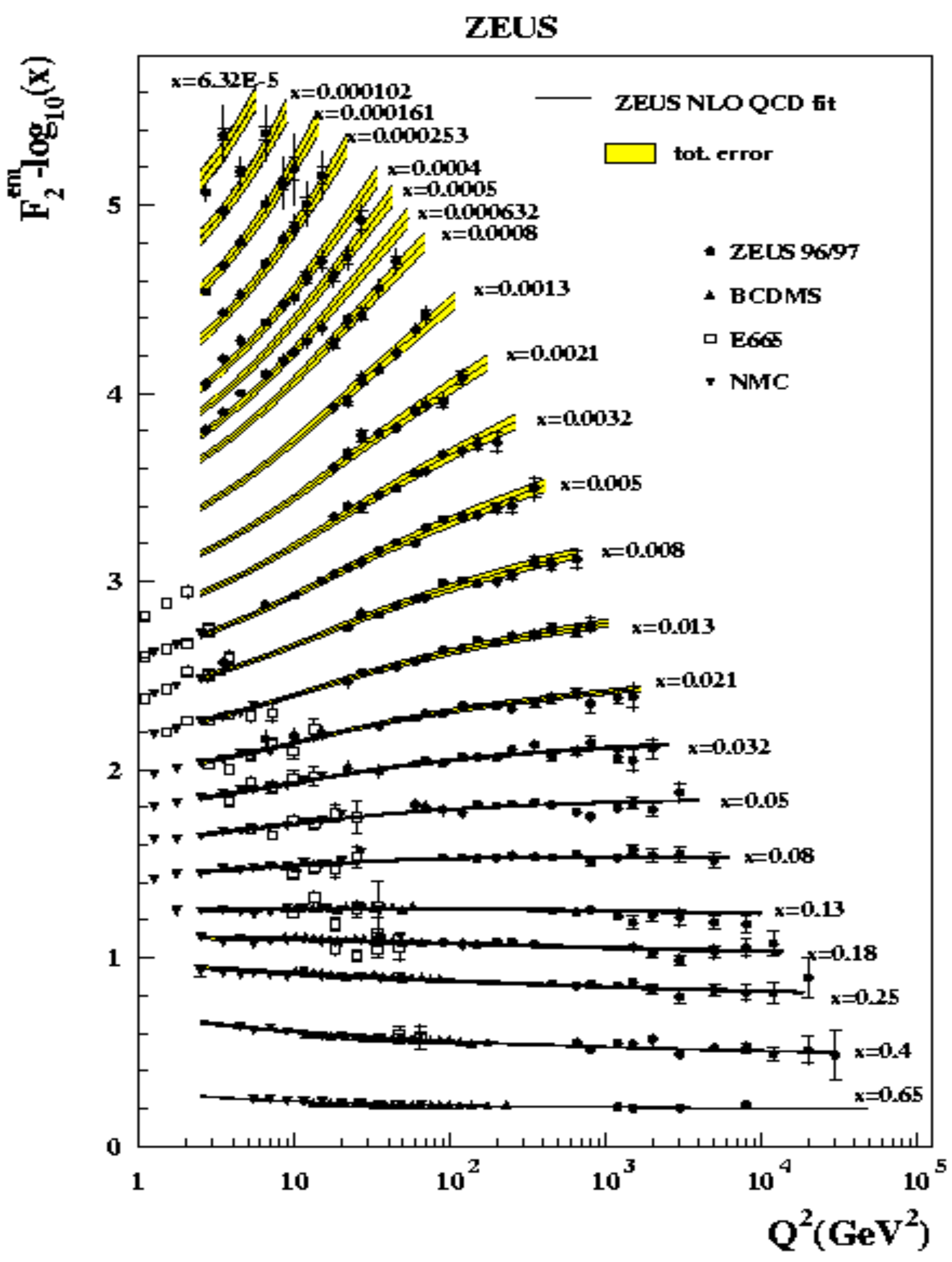
- Precision PV DIS can be a probe of higher twist correlations.

- Can probe a single four-quark matrix element which encodes quark-quark correlations. (Bjorken, Wolfenstein)

Higher Twist

Parton Model and Bjorken Scaling

- Bjorken limit: high Q^2 at fixed Bjorken-x.



- Structure functions independent of Q^2 up to logarithmic deviations.

- Nucleon is a collection of "almost free quarks".

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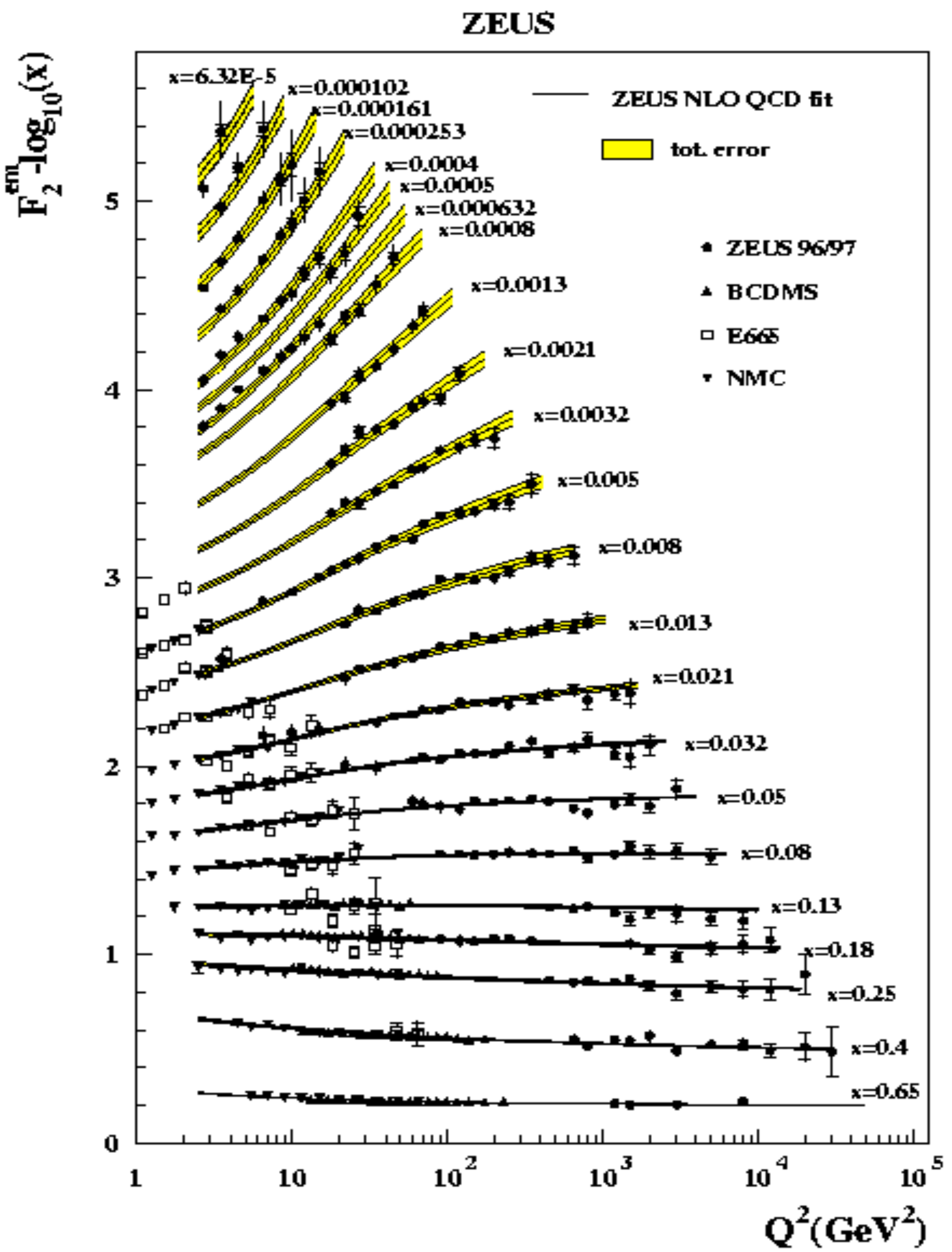

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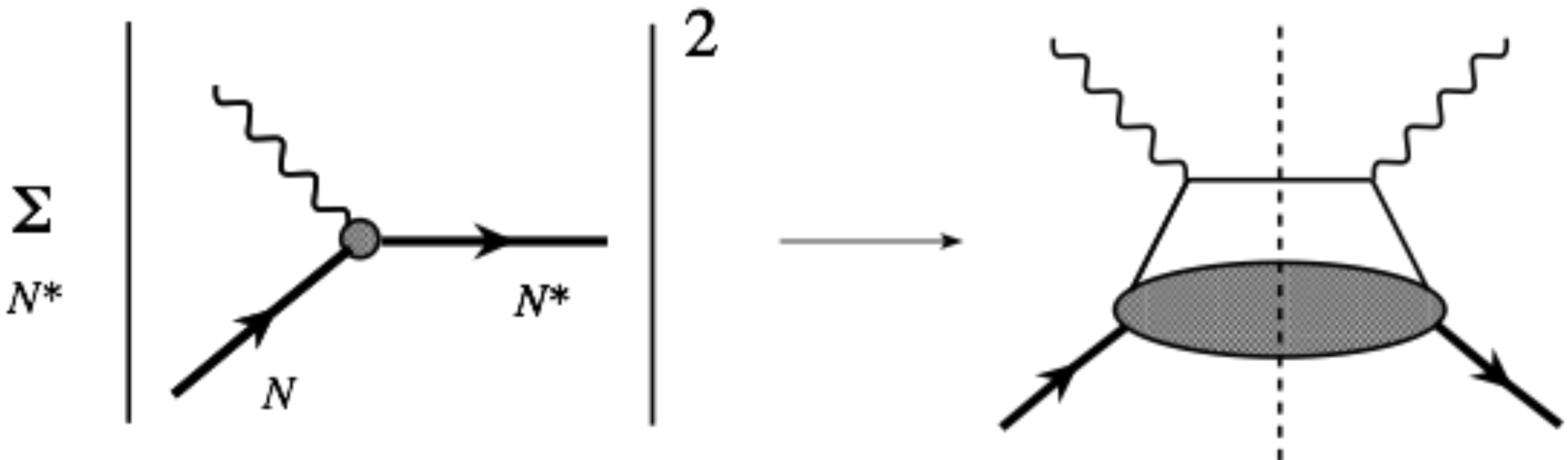
Long range quark and gluon correlations?

Operator Product expansion



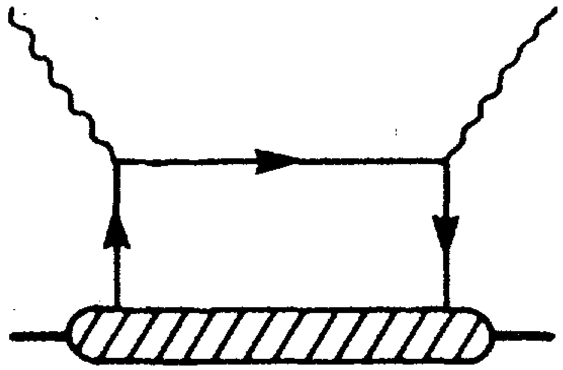
Optical Theorem

- Twist expansion:

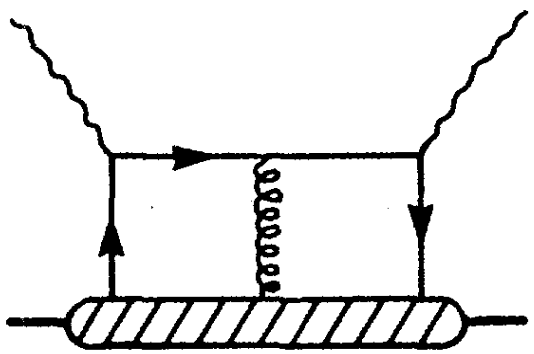
$$i \int d^4x \, e^{iq\cdot x} \, J(x)J(0) \sim \sum_{n=0}^{\infty} \sum_{i,j} \frac{C_i^n(Q^2,\mu^2)}{(Q^2)^{\frac{\tau-2}{2}}} \langle \mathcal{O}^{\tau,i}_{\mu_1\ldots\mu_n}(0)\rangle q^{\mu_1}\ldots q^{\mu_n}$$

- Leading twist gives the parton model.
- Correlation matrix elements beyond parton model suppressed by powers of 1/Q^2: Higher Twist Terms

Operator Product Expansion

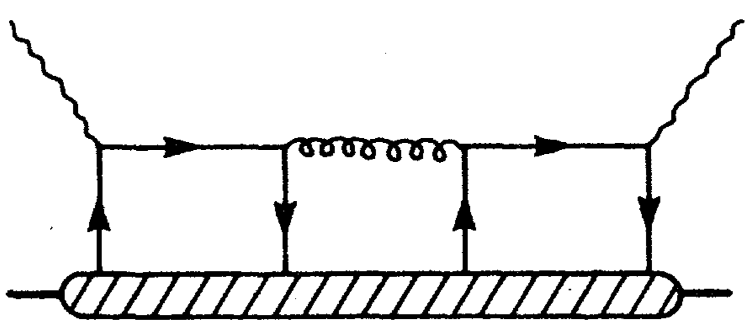


Twist-2 → Bjorken scaling



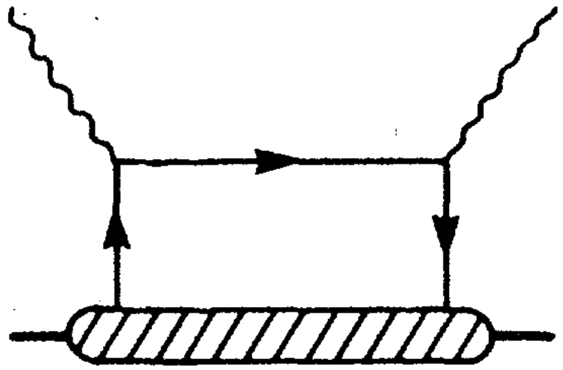
Quark-gluon correlation (Twist-4)

Power corrections

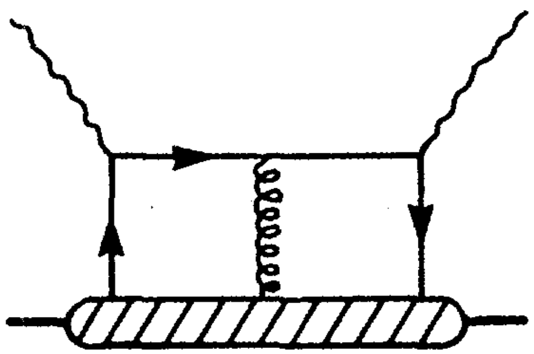


Quark-quark correlation(Twist-4)

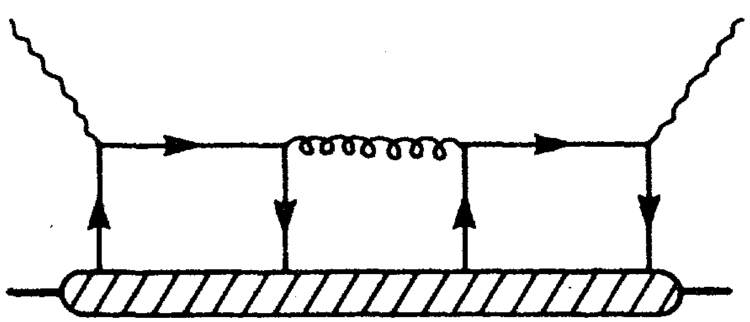
Operator Product Expansion



Twist-2



Quark-gluon correlation (Twist-4)



Quark-quark correlation(Twist-4)

Isolated in e-D asymmetry

$$\mathcal{O}_{ud}^{\mu\nu}(x) = \frac{1}{2}[\bar{u}(x)\gamma^{\mu}u(x)d(0)\gamma^{\nu}d(0) + (u \leftrightarrow d)]$$

Key features of the Asymmetry Terms

- Asymmetry can be brought into the form:

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

- Dominant term in asymmetry
- Can in principle be kinematically distinguished from second term (independent of y)
- Can be sensitive to only quark-quark correlations
- A single twist-4 matrix element determines quark-quark correlations.

- suppressed by small electron vector coupling
- Can be kinematically distinguished from second term(dependent on y)
- Can be sensitive to quark-quark and quark-gluon correlations
- Multiple twist-4 matrix elements determine correlations
- Can be extracted from neutrino scattering data

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Focus of this talk

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

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Some Definitions and Notation

- Asymmetry can be brought into the form:

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

- The Y_1 factor has the form:

$$Y_1 = \left(\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right) \frac{1+(1-y)^2 - y^2\left[1-r^2/(1+R^{\gamma Z})\right] - 2xyM/E}{1+(1-y)^2 - y^2\left[1-r^2/(1+R^{\gamma})\right] - 2xyM/E}$$

- The Y_3 factor has the form:

$$Y_3 = \left(\frac{r^2}{1+R^{\gamma}}\right) \frac{1-(1-y)^2}{1+(1-y)^2 - y^2\left[1-r^2/(1+R^{\gamma})\right] - 2xyM/E}$$

- We have used the definitions:

$$R^{\gamma(\gamma Z)} \equiv \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1, \qquad r^2 = 1 + \frac{4M^2x^2}{Q^2}$$

Cahn-Gilman Limit

- Cahn-Gilman limit:

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

$$R^{\gamma} = R^{\gamma Z} = r^2 - 1,$$
$$Y_1 = 1$$

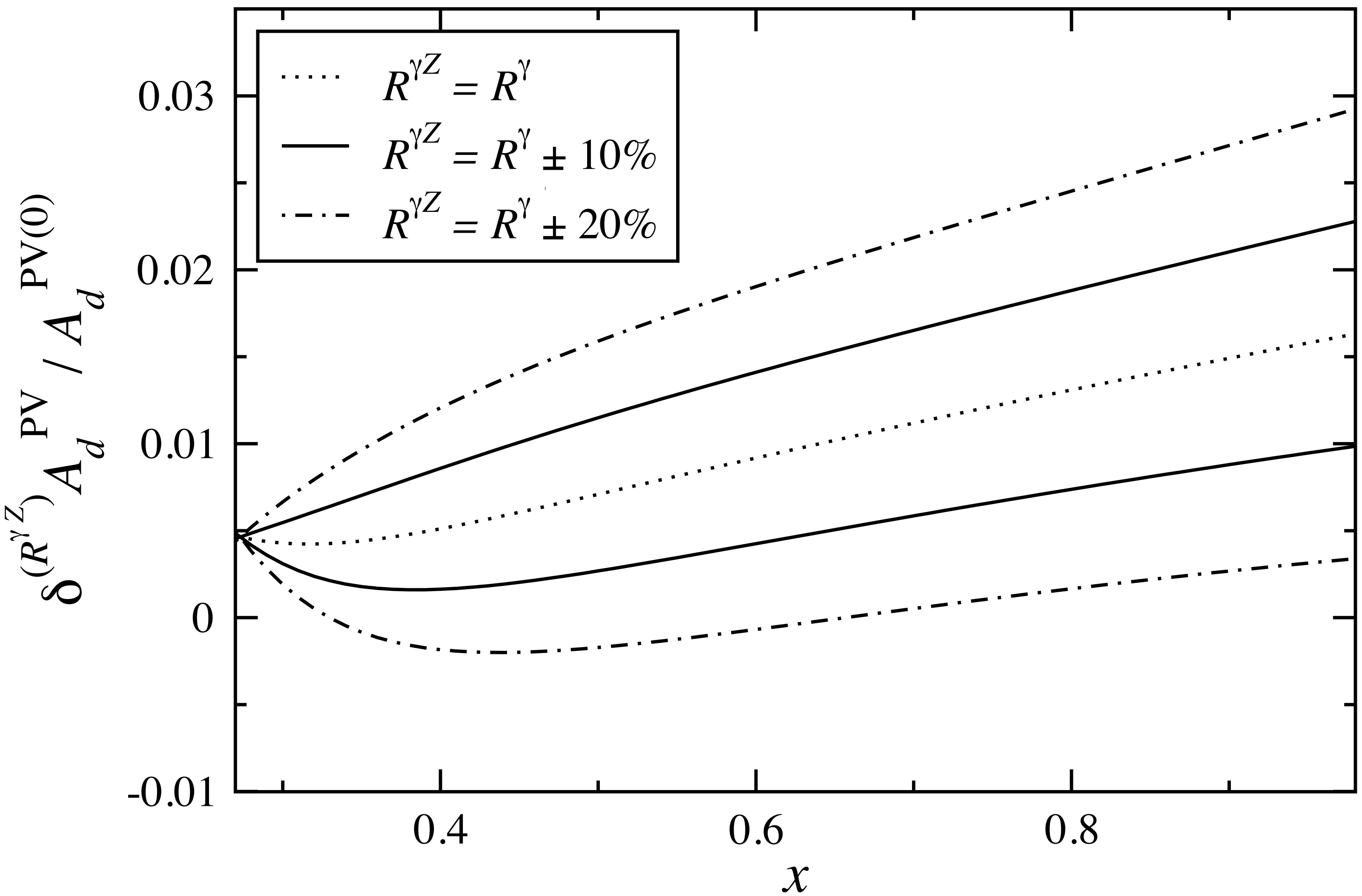
$$\left[\frac{F_1^{\gamma Z}}{F_1^{\gamma}}\right]_{\mathrm{CG}} = \frac{9}{10}\left(1 - \frac{20}{9}\sin^2\theta_W\right)$$

- Higher twist effects can modify these relations.

Hobbs/Melnitchouck Analysis

- Considered the possibility that higher twist effects arise entirely through the relation:

$$R^{\gamma} \neq R^{\gamma Z}$$



- Concluded that:

-20% difference gives a 1% effect in asymmetry
-Could interfere with extraction of CSV effects

More Recent Analysis

(SM, M.Ramsey-Musolf, G.Sacco)

- Our conclusions based on the Bjorken/Wolfenstein argument:

-Twist-4 effects in vector WNC term come only from quark-quark correlations.
-A single 4-quark twist-4 matrix element contributes to the vector WNC term.
- The relation $R^{\gamma Z} = R^{\gamma}$ holds true at twist-4 up to perturbative corrections.

Only quark-quark correlations given by a single matrix element

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

$$Y_1 = 1$$

Twist-4 effects reside in ratio of form factors

Bjorken-Wolfenstein Argument

- Isospin decomposition of electromagnetic and vector neutral currents:

$$J^\mu_\gamma \;=\; v_\mu + \frac{1}{3}s_\mu - \frac{1}{3}\lambda_\mu, \quad J^{V\mu}_Z \;=\; 2\big[(1-2\sin^2\theta)v_\mu - \frac{2}{3}\sin^2\theta s_\mu - (\frac{1}{2}-\frac{2}{3}\sin^2\theta)\lambda_\mu\big]$$

$$v_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \qquad s_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d), \qquad \lambda_\mu = \bar{s}\gamma_\mu s$$

- Isospin decomposition of electromagnetic and interference hadronic tensors

$$W^\gamma_{\mu\nu} \;=\; W^{vv}_{\mu\nu} + \frac{1}{9}W^{ss}_{\mu\nu}\,.$$

$$W^{V;\gamma Z}_{\mu\nu} \;=\; 2(1-2\sin^2\theta)W^{vv}_{\mu\nu} - \frac{4}{9}\sin^2\theta W^{ss}_{\mu\nu}$$

Bjorken-Wolfenstein Argument

- Isospin decomposed hadronic tensors:

$$W_{\mu\nu}^{vv} = \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|v_\mu(x)v_\nu(0)|D(P)\rangle$$
$$W_{\mu\nu}^{ss} = \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|s_\mu(x)s_\nu(0)|D(P)\rangle$$

- Twist-4 quark-quark correlation hadronic tensor:

$$\begin{aligned} W_{\mu\nu}^{du} &= W_{\mu\nu}^{ss} - W_{\mu\nu}^{vv}, \\ &= \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|\frac{1}{2}\{\bar{d}(x)\gamma_\mu d(x)\ \bar{u}(0)\gamma_\nu u(0) + (u \leftrightarrow d)\}|D(P)\rangle \end{aligned}$$

Bjorken-Wolfenstein Argument

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$$W_{\mu\nu}^{ss} = \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|s_\mu(x)s_\nu(0)|D(P)\rangle$$

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- Structure Function definitions

$$W_{\mu\nu}^{vv,ss,du} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{F_1^{vv,ss,du}}{M} + \left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\frac{F_2^{vv,ss,du}}{MP\cdot q}$$

Bjorken-Wolfenstein Argument

- Isospin decomposed hadronic tensors:

$$W^{vv}_{\mu\nu} = \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|v_\mu(x)v_\nu(0)|D(P)\rangle$$

$$W^{ss}_{\mu\nu} = \frac{1}{2\pi M}\int d^4x\, e^{iq\cdot x}\langle D(P)|s_\mu(x)s_\nu(0)|D(P)\rangle$$

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$$F^{vv}_{1,2} = F^{ss}_{1,2} - F^{du}_{1,2}$$

Bjorken-Wolfenstein Argument

- Original structure functions can be written as:

$$F_{1,2}^{\gamma} = \frac{10}{9} F_{1,2}^{ss} - F_{1,2}^{du}$$

$$F_{1,2}^{\gamma Z} = 2\left(1 - \frac{20}{9}\sin^2\theta\right) F_{1,2}^{ss} - 2(1 - 2\sin^2\theta) F_{1,2}^{du}$$

- Asymmetry in terms of original structure functions:

$$A_{RL} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e Y_3 \frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

- Y1 term:

$$A_{RL}^{V} = -\frac{9}{10}\left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) g_A^e \left\{\left(1 - \frac{20}{9}\sin^2\theta\right) - \frac{1}{10}\frac{F_1^{du}}{F_{1;LT}^{ss}} + \cdots\right\}$$

Form of twist-4 correction

- Y1 term:

$$A_{RL}^{V} = -\frac{9}{10}\left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) g_A^e \left\{ \left(1 - \frac{20}{9}\sin^2\theta\right) - \frac{1}{10}\frac{F_1^{du}}{F_{1;LT}^{ss}} + \cdots \right\}$$

- Twist-4 correction given by:

$$R_1(\mathrm{HT}) = \left[\frac{-4}{5(1 - \frac{20}{9}\sin^2\theta_W)}\right] \frac{F_1^{du}}{u_p(x) + d_p(x)}.$$

$$R^{\gamma Z} = R^{\gamma}$$

- Quark-quark correlation twist-4 operator matrix element:

twist-4 structure functions

$$\frac{1+R^{\gamma Z}}{1+R^{\gamma}} = 1 + \frac{(F_2^{du} - 2xF_1^{du})}{2xF_{1;LT}^{ss}}\left[\frac{9}{10} - \frac{1-2\sin^2\theta}{1-\frac{20}{9}\sin^2\theta}\right]$$

twist-2 structure function

- Using the Callan-Gross relation at tree level we get:

$$F_2^{du} = 2xF_1^{du} \longrightarrow \frac{1+R^{\gamma Z}}{1+R^{\gamma}} = 1$$

(R.Ellis,W.Furmanski,R.Petronzio; X.Ji; J.Qiu)

- We also give an effective field theory (SCET) argument

(SM, M. Ramsey-Musolf, G.Sacco)

$$R^{\gamma Z} = R^{\gamma}$$

$$\frac{1 + R^{\gamma Z}}{1 + R^{\gamma}} = 1$$

- Using the Callan-Gross relation at tree level we get:

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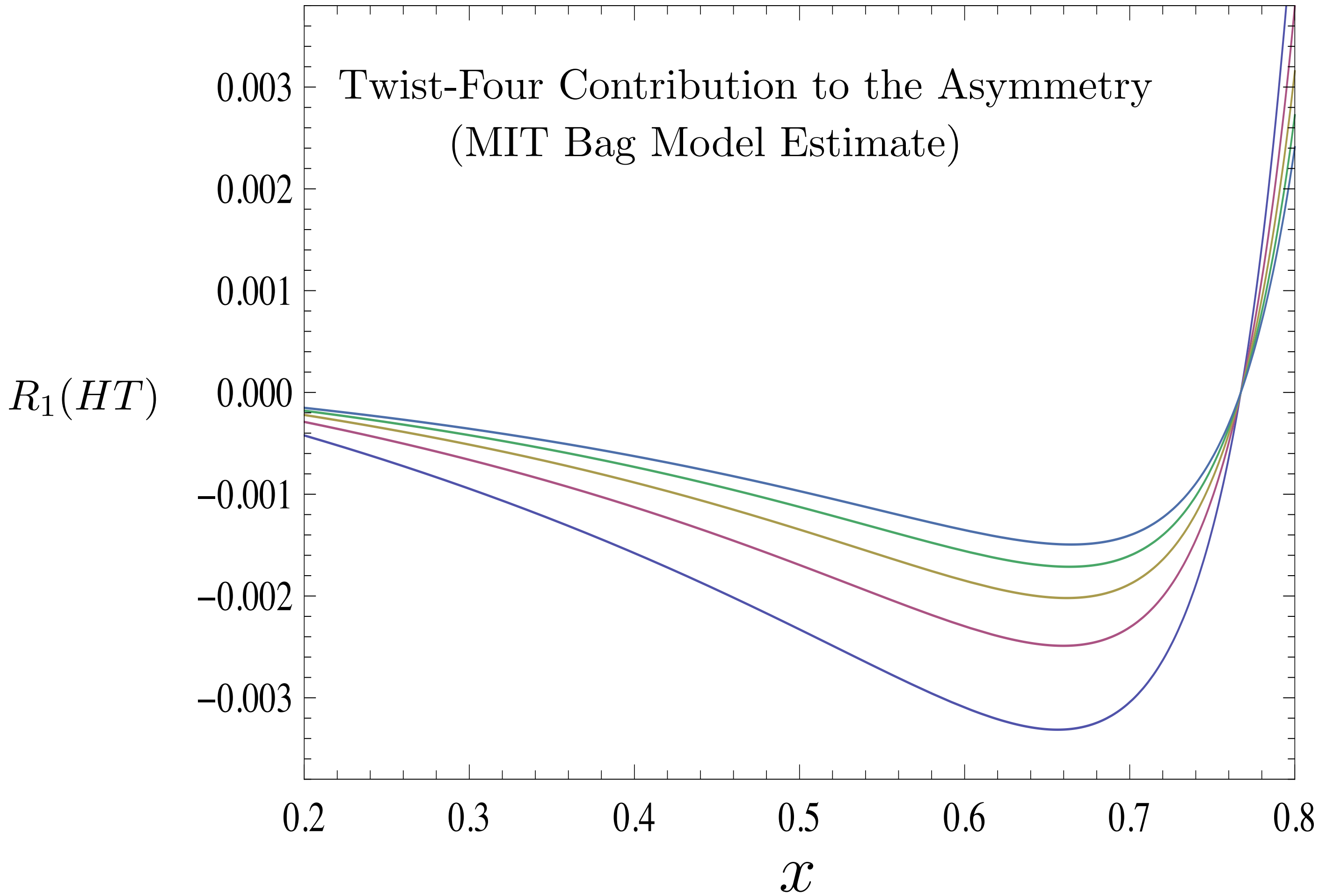
free of twist-4 effects

twist-4 quark-quark correlations reside in ratio of structure functions

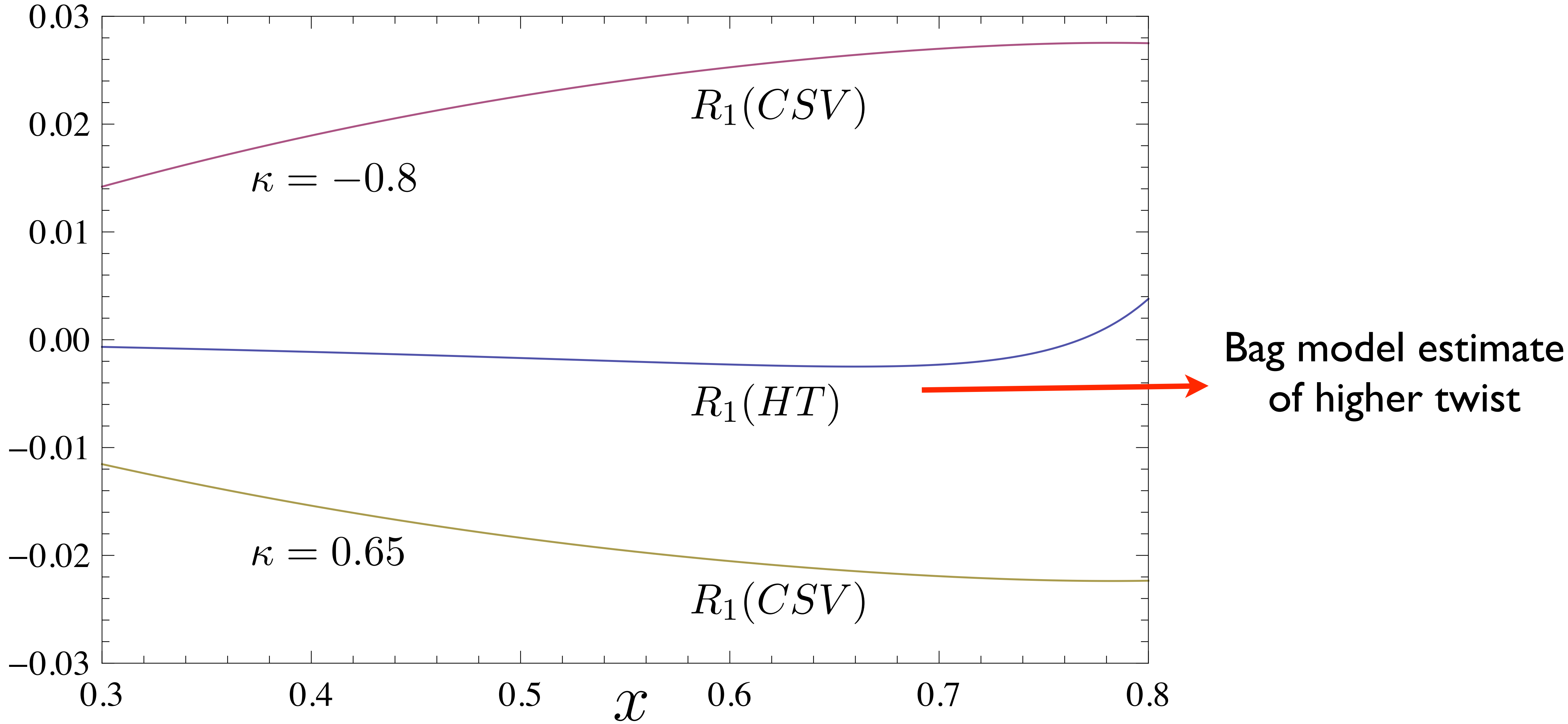
Form of twist-4 correction



$$R_1(\mathrm{HT}) = \left[\frac{-4}{5(1-\frac{20}{9}\sin^2\theta_W)}\right]\frac{F_1^{du}}{u_p(x)+d_p(x)}.$$

- Bag model estimate of quark-quark correlation is below the half-percent level.
- Estimates using multi-parton nucleon light-cone wave functions, found an effect twice as big. (Belitsky, Manashov, Schafer)

CSV vs Higher Twist



$$\delta u - \delta d = 2\kappa f(x), \qquad f(x) = x^{-1/2}(1-x)^4(x - 0.0909)$$

- Negligible higher twist effects can allow for a cleaner extraction of CSV or new physics effects.

Conclusions

- PV DIS can be a powerful probe of hadronic physics beyond the parton model.

- The precision and wide kinematic reach of 12 GeV Upgrade at JLAB can in principle disentangle various hadronic effects such as sea quarks, CSV, and higher twist.

- PV DIS can probe a `single' twist-4 quark-quark correlation matrix element and is the only known observable with this property.

- Uncertainties in R-gamma-Z appear to have only a small effect on higher twist effects.