

Radiative Corrections to PREX and QWEAK



Implications of PREX for:

- **Theory** and chiral EFT three neutron forces.
- **Astrophysics** X-ray obs of neutron star radii.
- **Nuclear structure** and dipole polarizability.
- **Heavy ion collisions** and symmetry energy.

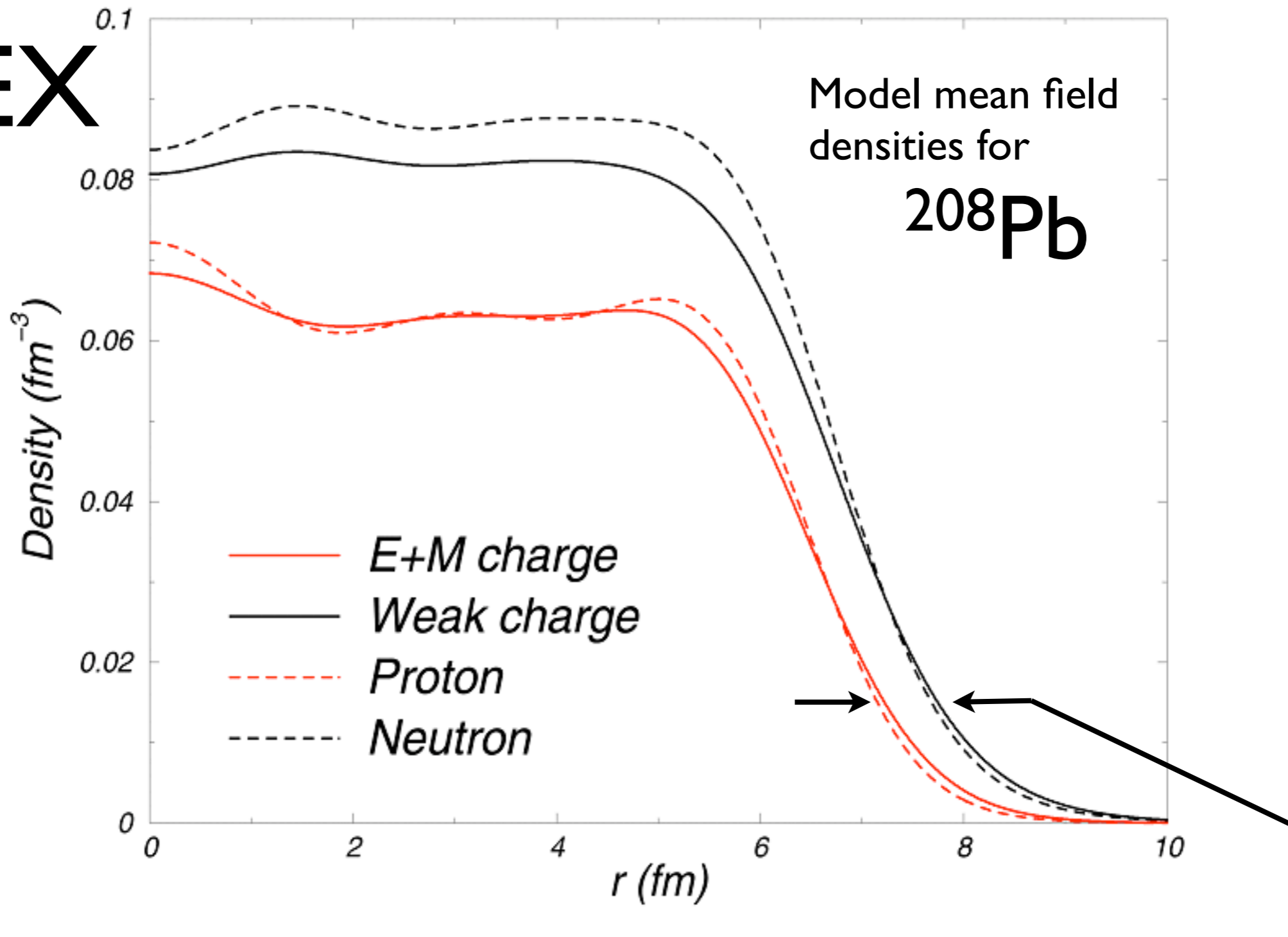
Coulomb distortion, dispersion Corrections for:

- PREX
- Transverse asymmetry
- QWEAK

PAVI11, Rome, Sept. 2011

C. J. Horowitz, Indiana University

PREX

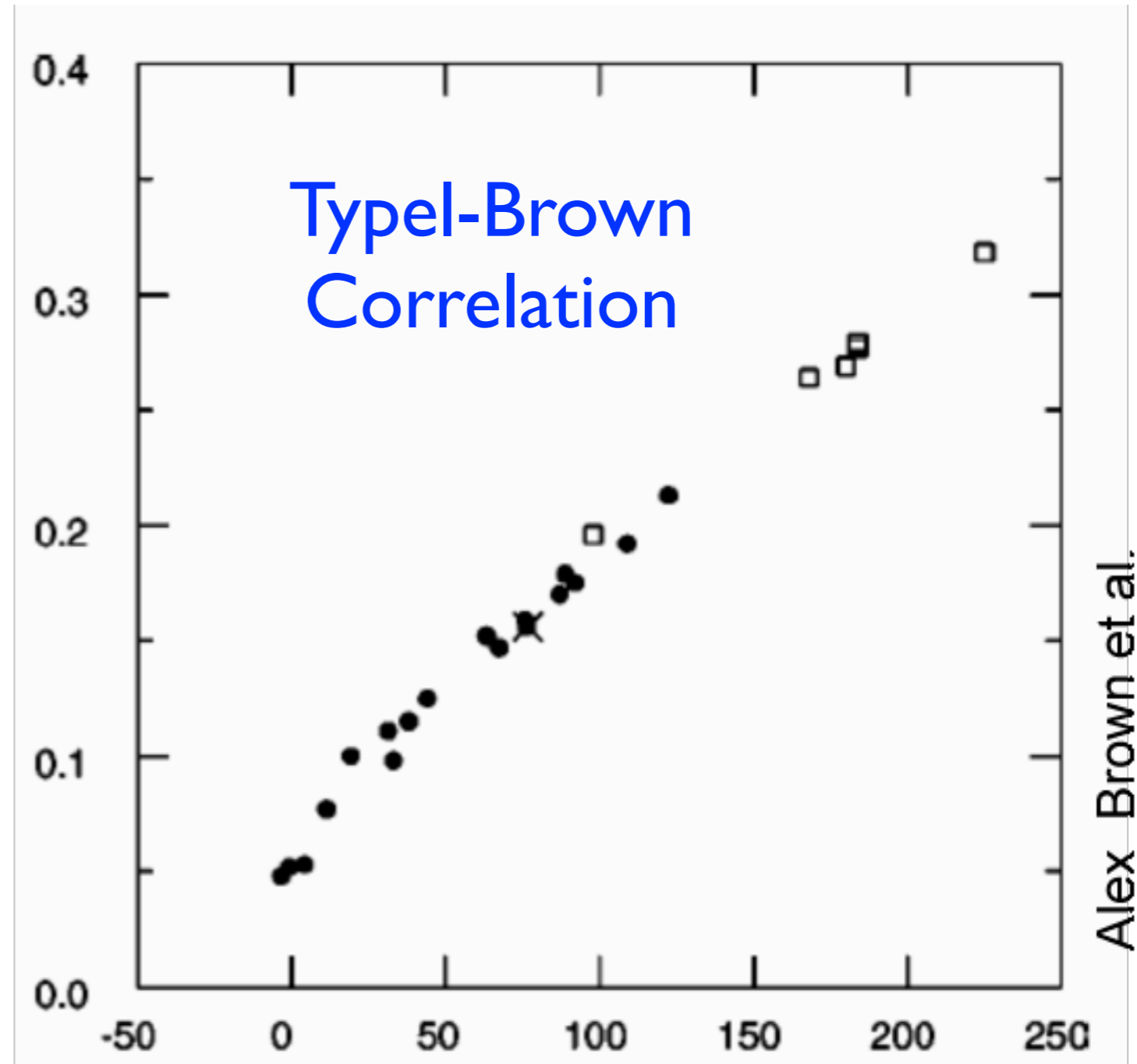


- PREX measures how much neutrons stick out past protons (neutron skin).
- First measurement of parity violating asymmetry for elastic electron scattering from ^{208}Pb at 1050 MeV and about 5 degrees.
- Interpretation of electroweak reaction is model independent.

^{208}Pb radius and Equation of State

- Pressure of neutron matter forces neutrons out against surface tension. A large pressure gives a large neutron skin.
- Measuring R_n in ^{208}Pb constrains the pressure of neutron matter at $\sim 2/3\rho_0 = 0.1 \text{ fm}^{-3}$.

$R_n - R_p$ (fm) for ^{208}Pb

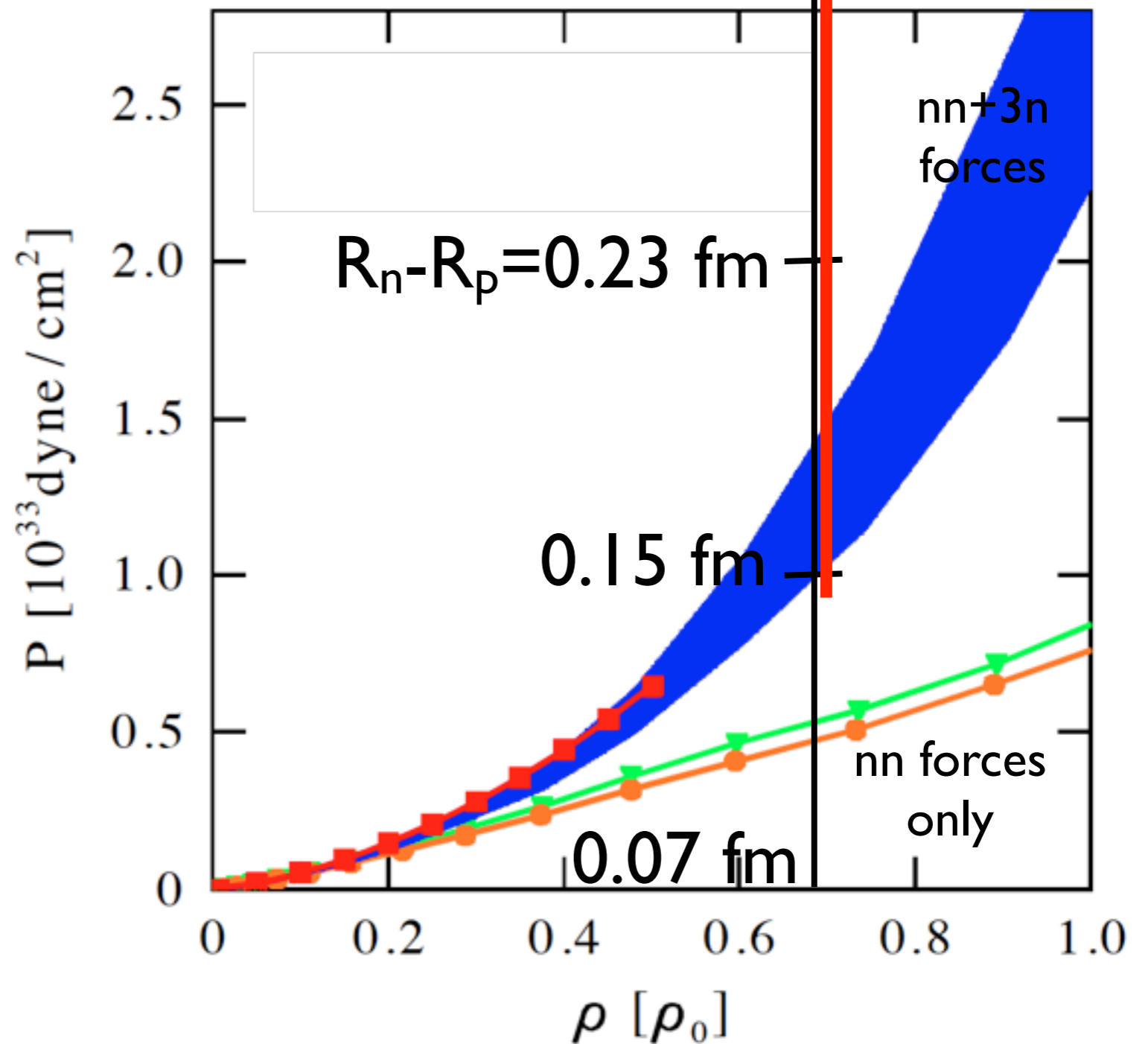


Neutron matter P (MeV/fm³)
x 100 at a density of 0.1 fm⁻³.

Chiral Effective Field Theory Calculations of Pressure of Neutron Matter vs Density

Preliminary
PREX

- Chiral EFT calc. of pressure P of neutron matter by Hebeler et al. including three *neutron* forces (blue band)
PRL **105**, 161102 (2010)
- Their calculated P and Typel-Brown correlation
--> $R_n - R_p = 0.14$ to 0.2 fm
- PREX agrees with results including $3n$ forces. Three *neutron* forces are very interesting, unconstrained. Some information on 3 nucleon forces in ${}^3\text{H}$, ${}^3\text{He}$...



A Neutron Star is Newton's 10 km Apple



- In astrophysics and in the laboratory it is the same neutrons, the same strong interactions, the same neutron rich matter, and the same equation of state. A measurement in one domain has important implications in the other domain.

Pb Radius vs Neutron Star Radius

- The ^{208}Pb radius constrains the pressure of neutron matter at subnuclear densities.
- The NS radius depends on the pressure at nuclear density and above. Central density of NS few to 10 x nuclear density.
- If Pb radius is relatively large: EOS at low density is stiff with high P. If NS radius is small than high density EOS soft.
 - This softening of EOS with density could strongly suggest a transition to an exotic high density phase such as quark matter, strange matter, color super-conductor...

J. Piekarewicz, CJH

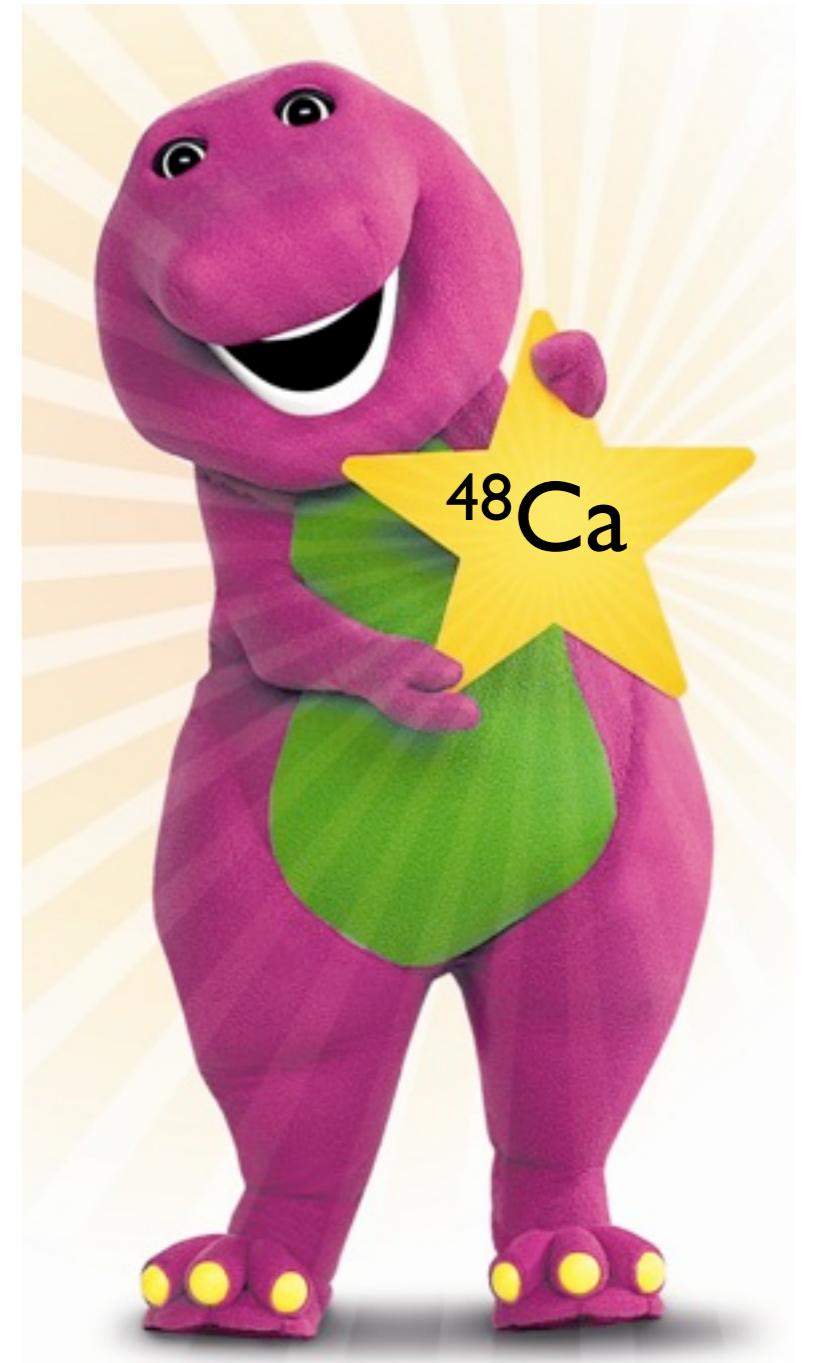
- Measure area of NS from luminosity, temp. from X-ray spectrum.

$$L_\gamma = 4\pi R^2 \sigma_{\text{SB}} T^4$$

- Complications:
 - Non-blackbody corrections from atmosphere models.
 - Curvature of space: measure combination of radius and mass.
- Steiner, Lattimer, Brown [ArXiv: 1005.0811] combine observations of NS in X-Ray bursts and globular clusters and deduce radii + EOS.
 - EOS + Typel-Brown correlation -> Predict ^{208}Pb neutron skin:
 $R_n - R_p = 0.15 \pm 0.02 \text{ fm}$.
 - Model dependent. F. Ozel et al. get smaller radii for NS and softer EOS.

PREX next Steps

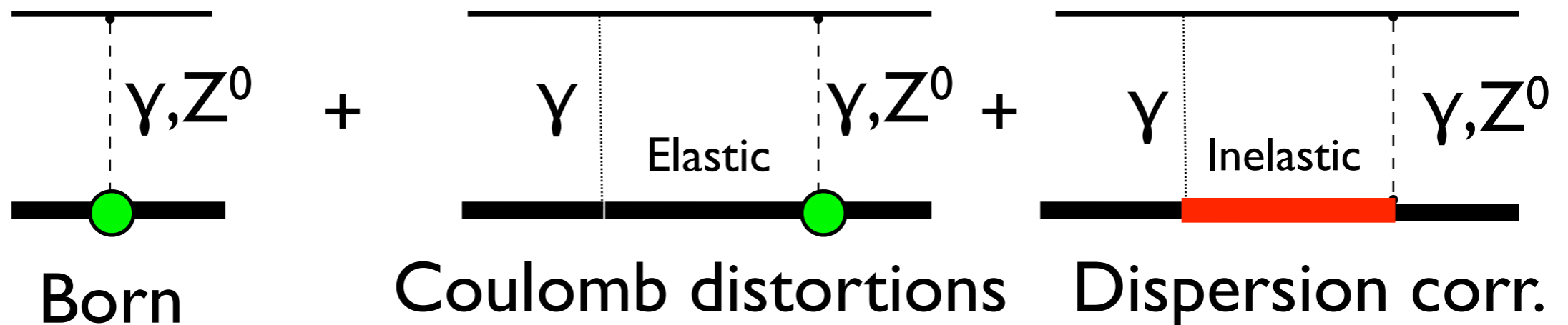
- Important to run again with ^{208}Pb to reach 1% goal on R_n (3% for A). Provides sharp test of several theoretical, astrophysical, and nuclear structure predictions. Approved by JLAB PAC.
- Very attractive to also measure R_n in ^{48}Ca . Smaller nucleus can be measured at higher Q^2 (and beam energy) where experimental figure of merit is higher. Microscopic coupled cluster and no-core shell model calculations can directly relate R_n (^{48}Ca) to three neutron forces.



Radiative Corrections



Radiative corrections



- Coulomb distortions are coherent, order $Z\alpha$. Important for PREX (Pb has $Z=82$).
- Dispersion corrections order α (not $Z\alpha$). Important for QWEAK because correction is order $\alpha/Q_w \sim 10\%$ relative to small Born term (Q_w).
- Both Coulomb distortion and dispersion cor. can be important for Transverse Beam Asymmetry A_n for ^{208}Pb . Note Born term gives zero by time reversal symmetry.

Coulomb Distortions for PREX

- We sum elastic intermediate states to all orders in $Z\alpha$ by solving Dirac equ. for e moving in coulomb V and weak axial A potentials.

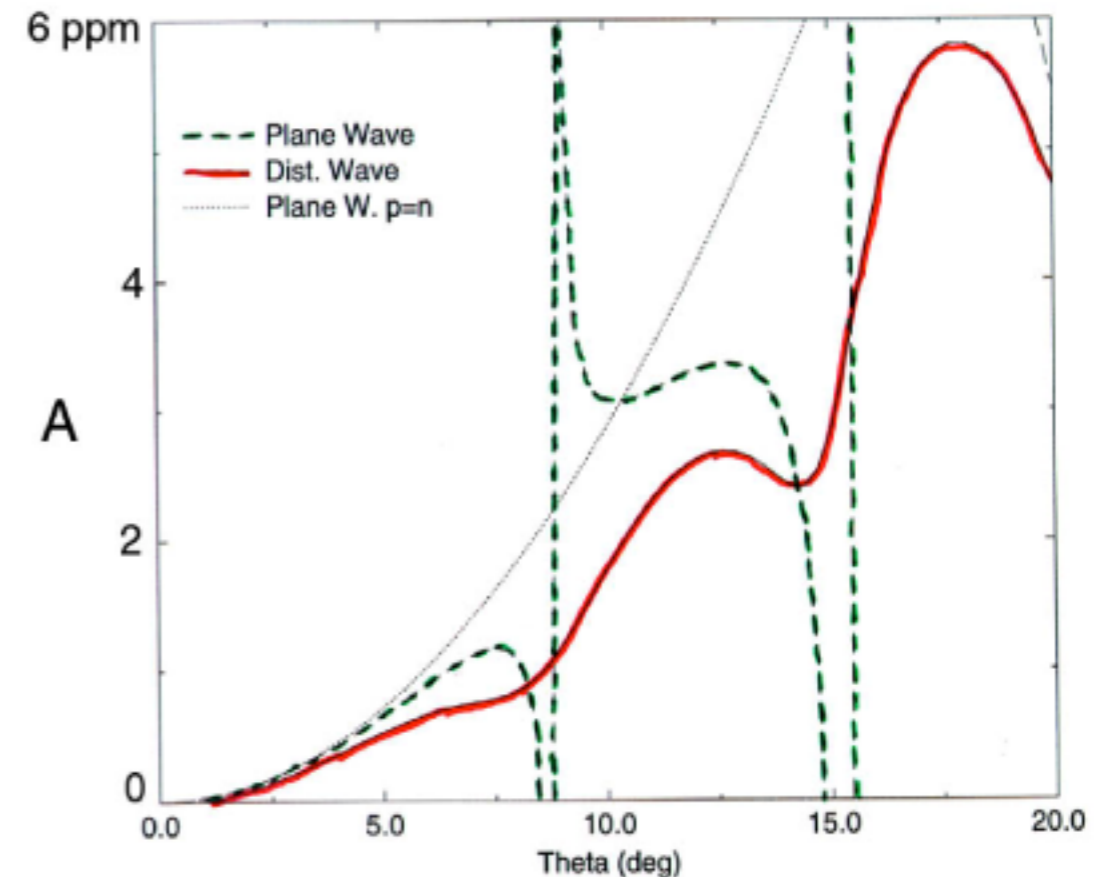
$$A \propto G_F \rho_W(r) \approx 10 \text{ eV} \quad V(r) \approx 25 \text{ MeV}$$

- Right handed e sees $V+A$, left handed $V-A$

$$A_{pv} = [d\sigma/d\Omega|_{V+A} - d\sigma/d\Omega|_{V-A}] / 2d\sigma/d\Omega$$

- Coulomb distortions reduce A_{pv} by $\sim 30\%$, but they are accurately calculated. Q^2 shared between “hard” weak, and soft interactions so weak amplitude $G_F Q^2$ reduced.

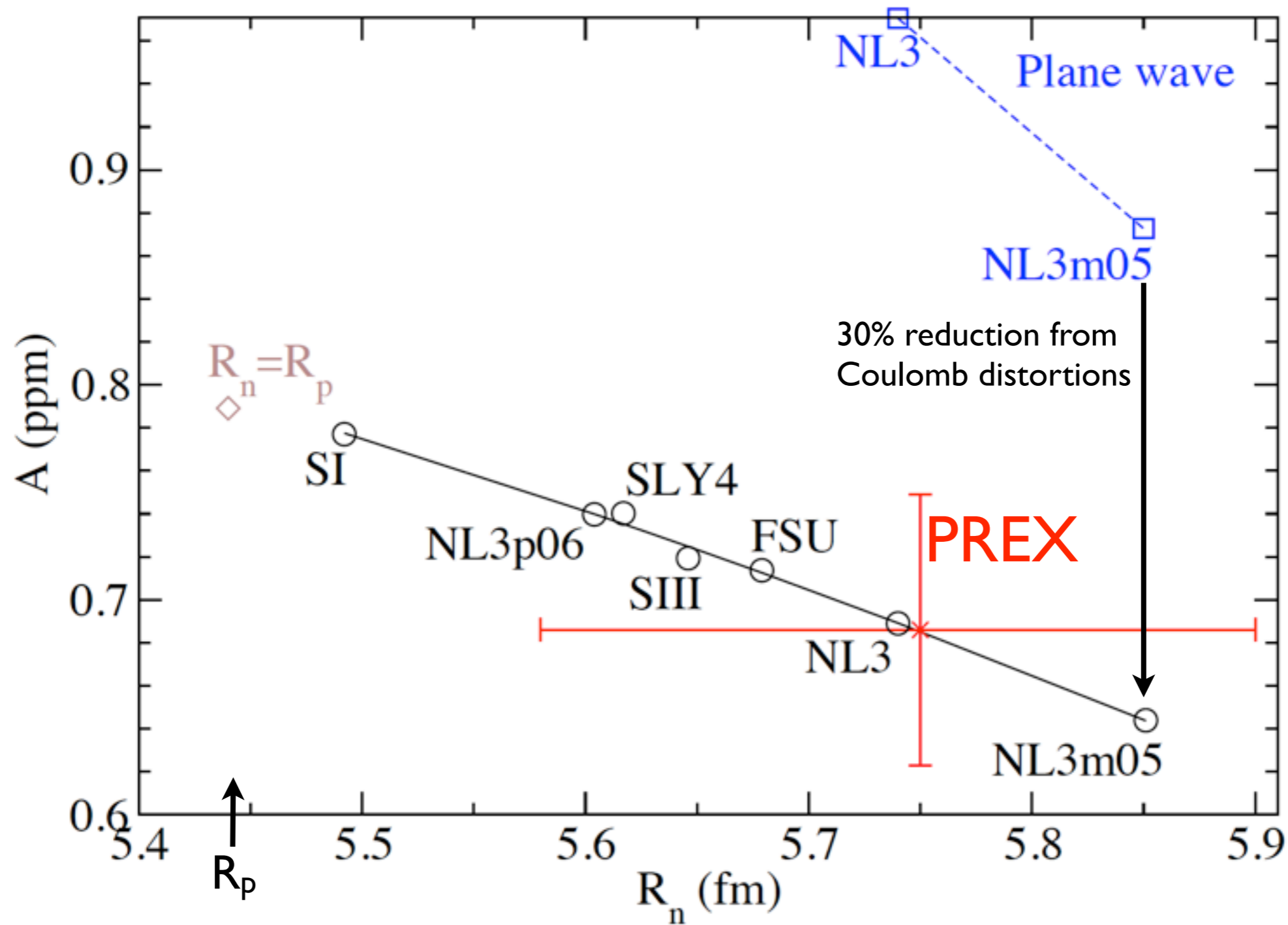
^{208}Pb at 850 MeV



$$A_{pv} = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}} \frac{F_W(Q^2)}{F_{ch}(Q^2)}$$

--- With E.D. Cooper!

Preliminary PREX result



- Need final acceptance to compare theory/ exper.

Dispersion correction to QWEAK

$$A^{PV} = \frac{G_F t}{4\sqrt{2}\pi\alpha_{em}} \left[(1 + \Delta\rho + \Delta_e)(1 - 4\sin^2 \hat{\theta}_W(0) + \Delta'_e) + \square_{WW} + \square_{ZZ} + \square_{\gamma Z} \right] + \dots,$$

$$\text{Im}\square_{\gamma Z_A}(\nu) = \alpha_{em} g_A^e \int_{W_\pi^2}^s \frac{dW^2}{(s - M^2)^2} \int_0^{Q_{max}^2} \frac{dQ^2}{1 + \frac{Q^2}{M_Z^2}} \left[F_1^{\gamma Z} + \frac{s(Q_{max}^2 - Q^2)}{Q^2(W^2 - M^2 + Q^2)} F_2^{\gamma Z} \right]$$

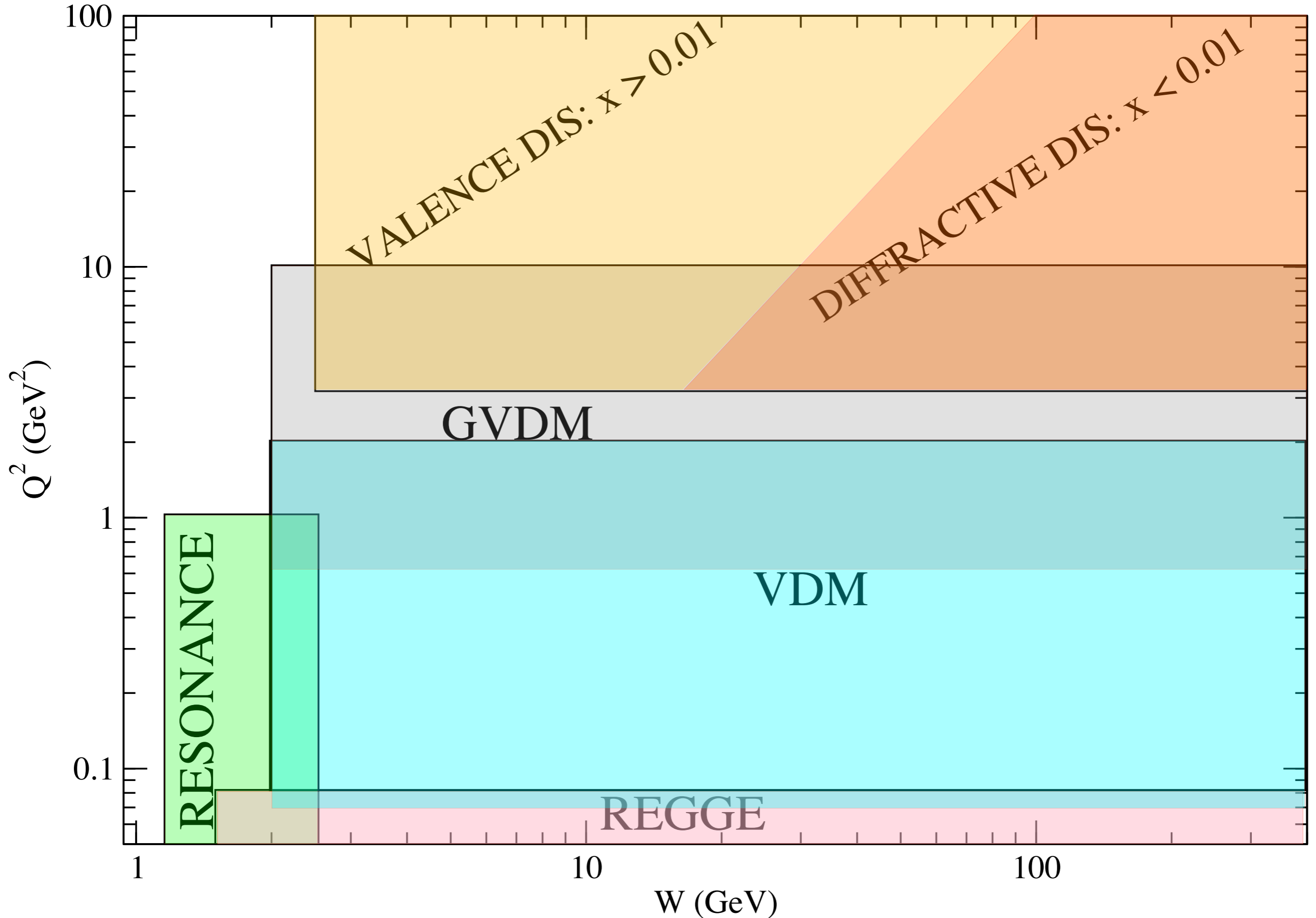
$$\text{Re}\square_{\gamma Z_A}(\nu) = \frac{2\nu}{\pi} \int_{\nu_\pi}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \text{Im}\square_{\gamma Z_A}(\nu')$$

* Short ranged WW, ZZ, and γZ box contributions renormalized. Important energy dependent γ -Z box correction of intermediate range and size $\alpha/Q_W=10\%$!

* Need interference structure functions $F_i^{\gamma Z}(x, Q^2)$. In principle can use PVDIS (and not so DIS) data.

* Instead use real+virtual photo-absorption data and isospin rotation $F_i^{\gamma\gamma} \rightarrow F_i^{\gamma Z}$.

Kinematics: Resonance and VDM regions important



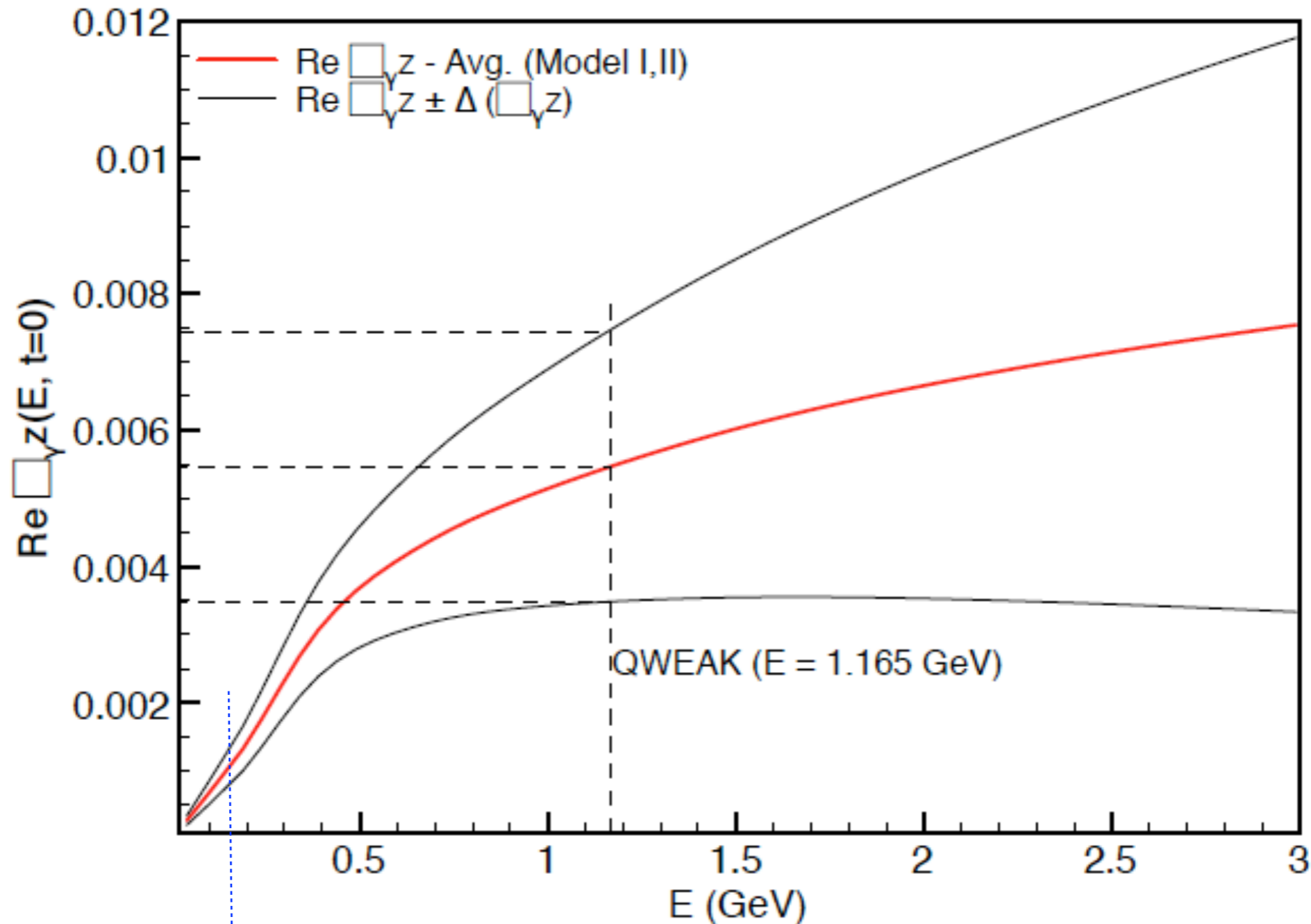
Result at QWEAK kinematics

$$\text{Re} \square_{\gamma Z_A}(E = 1.165 \text{ GeV}, t = -0.03 \text{ GeV}^2) = \left[5.39 \pm 0.27 \text{ (mod. avg.)} \pm 1.88 \text{ (backgr.)}^{+0.58}_{-0.49} \text{ (res.)} \pm 0.07 \text{ (} t \text{ - dep.)} \right] \times 10^{-3}$$

$$Q_W^p = 0.0713 \pm 0.0008 \quad \frac{\text{Re} \square_{\gamma Z_A}}{Q_W^p} = (7.6 \pm 2.8)\%$$

- Error is dominated by uncertainty in isospin rotation of background. [Example in simple VD model photon goes to omega, rho, and phi mesons of known isospin.]
- Can measure isospin of background with PV(not so deep) inelastic scattering at low to moderate Q^2 .
- Compared to small weak charge we predict a 7.6 +/- 2.8 % correction.

Energy dependence for Mainz



Mainz: at 180 MeV correction is smaller with a six times smaller uncertainty than for QWEAK.

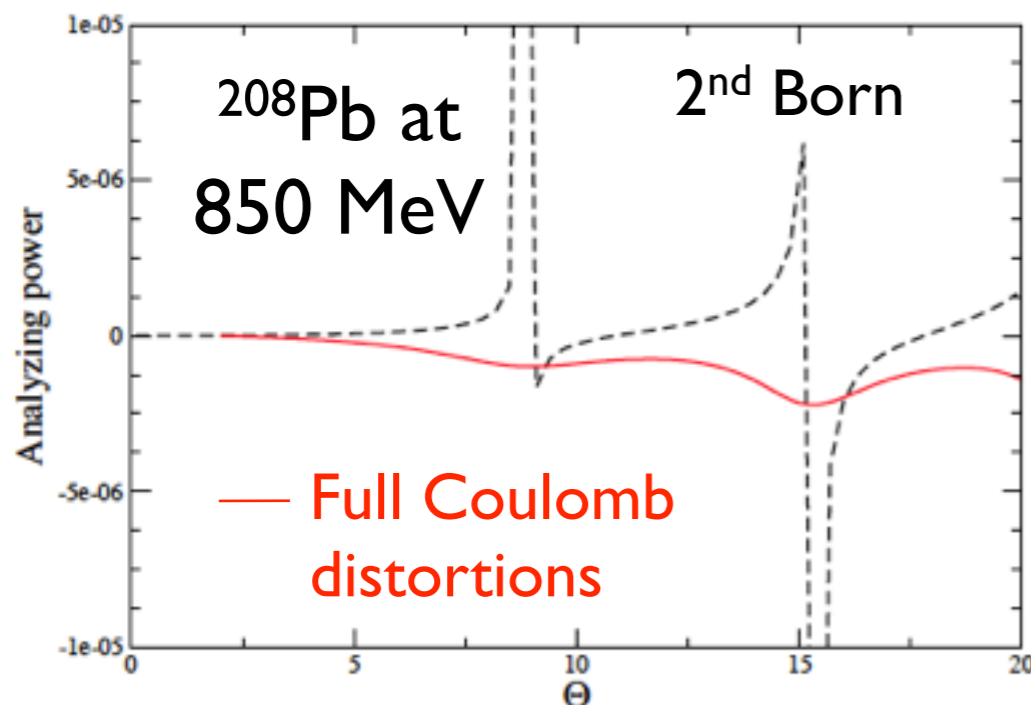
$$\text{Re } \alpha_{\gamma Z}(E = 0.180 \text{ GeV}, t = 0) = [1.32 \pm 0.05 (\text{mod. avg.}) \pm 0.27 (\text{backgr.})_{-0.08}^{+0.11} (\text{res.})] \times 10^{-3}$$

Transverse Beam Asymmetry A_n for PREX (^{208}Pb)

- Left / Right cross section asymmetry for electrons with transverse polarization.
- Potential systematic error for PV from small trans components of beam polarization.
- Relativistic effects make A_n of order m/E .
- A_n vanishes in Born approx (time reversal) --> Sensitive probe of 2 or more photon effects.

Coulomb Distortions

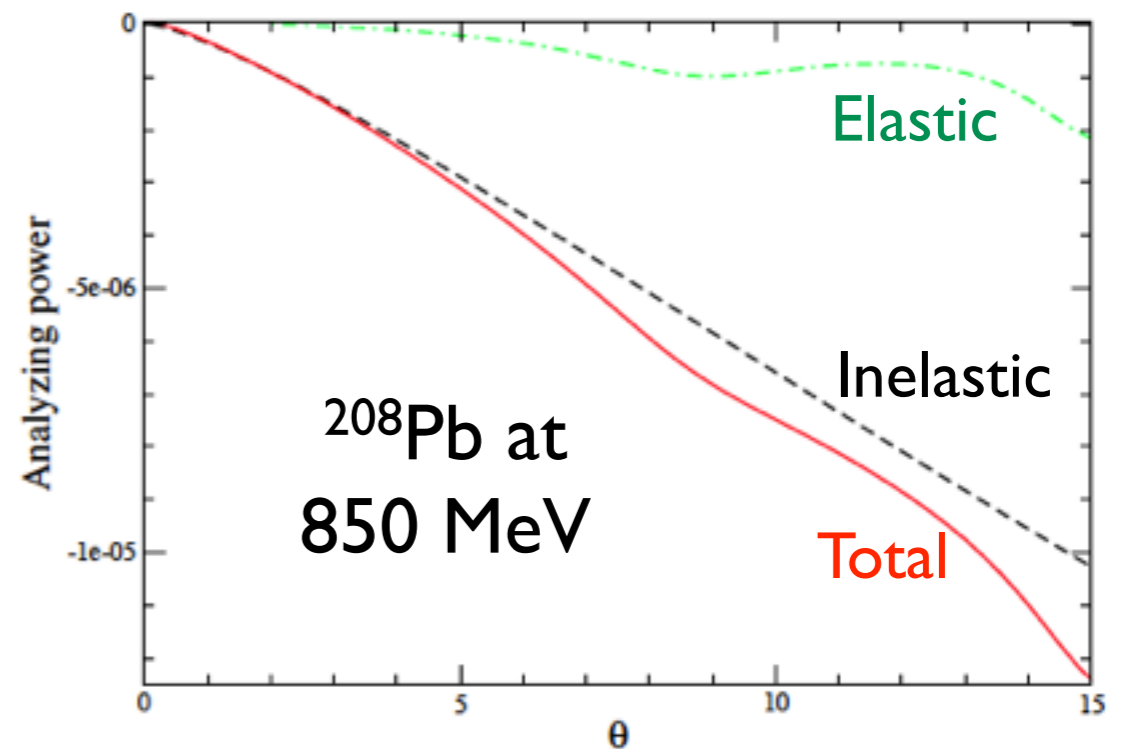
- Only keep elastic intermediate states.
- Just solve Dirac Eq. for electron of mass m in Coulomb potential (very hard numerically at high energies).
- Coulomb distortion contribution to A_n very sensitive to Z of target and small for small Z .
- 2nd Born bad for large Z .



Dispersion contributions

- Sum over excited states in 2nd Born approximation with dispersion relation.

$$A_n^{inelast} \approx -\frac{1}{4\pi^2} \frac{m_e}{E_{lab}} \frac{M}{\sqrt{s}} \frac{A}{Z} \frac{g_N(Q^2)}{F_N(Q^2)} \tan \frac{\theta_{c.m.}}{2} \times \int_0^{E_{lab}} d\omega \omega \sigma_{\gamma p}(\omega) \ln \left[\frac{Q^2}{m^2} \left(\frac{E_{lab}}{\omega} - 1 \right)^2 \right]$$



Note: 2nd Born is probably not good for inelastic but that is all that has been calculated.

A_n for different nuclei

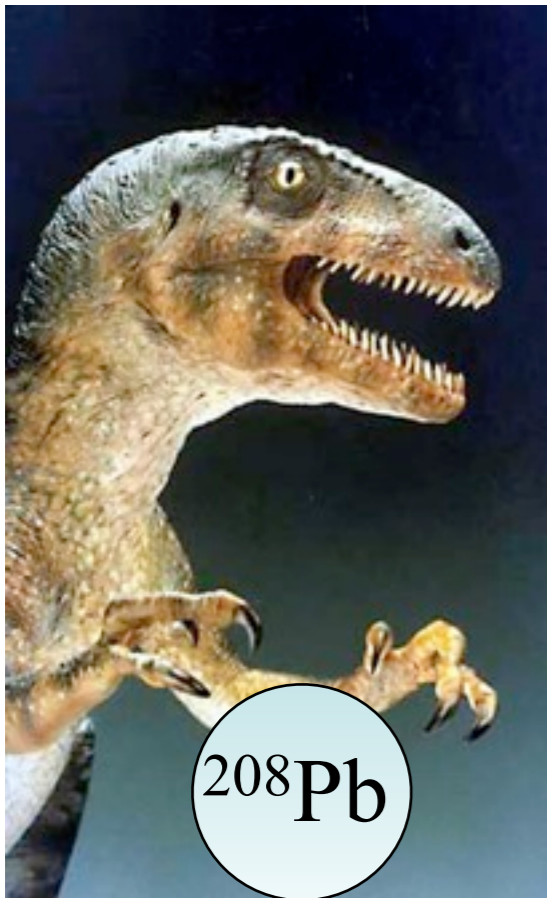
- Dispersion correction calculations (to order α^2) suggest that A_n should scale roughly as A/Z and should only weakly depend on energy.
- This qualitatively agrees for H, ^4He , and ^{12}C but not ^{208}Pb .
- Preliminary PREX results (see R. Michaels talk)
 - $A_n(^{12}\text{C}) = -6.52 \pm 0.36 \pm 0.35$ ppm
 - $A_n(^{208}\text{Pb}) = 0.13 \pm 0.19 \pm 0.36$ ppm
- The large difference for ^{208}Pb is likely from Coulomb distortions. Measure A_n for different nuclei between C and Pb. Measure A_n vs Q^2 near diffraction minimum. Calculate dispersion corrections with Coulomb distortions.

Three Photon Observables



- The large difference between A_n for ^{208}Pb and ^{12}C is an observable that likely requires the exchange of three (or more) photons.
- Full 2 photon exchange calculations predict similar A_n for C and Pb. Difference is likely due to Coulomb distortions in Pb that involve the exchange of additional photons.

Radiative Corrections to PREX and QWEAK

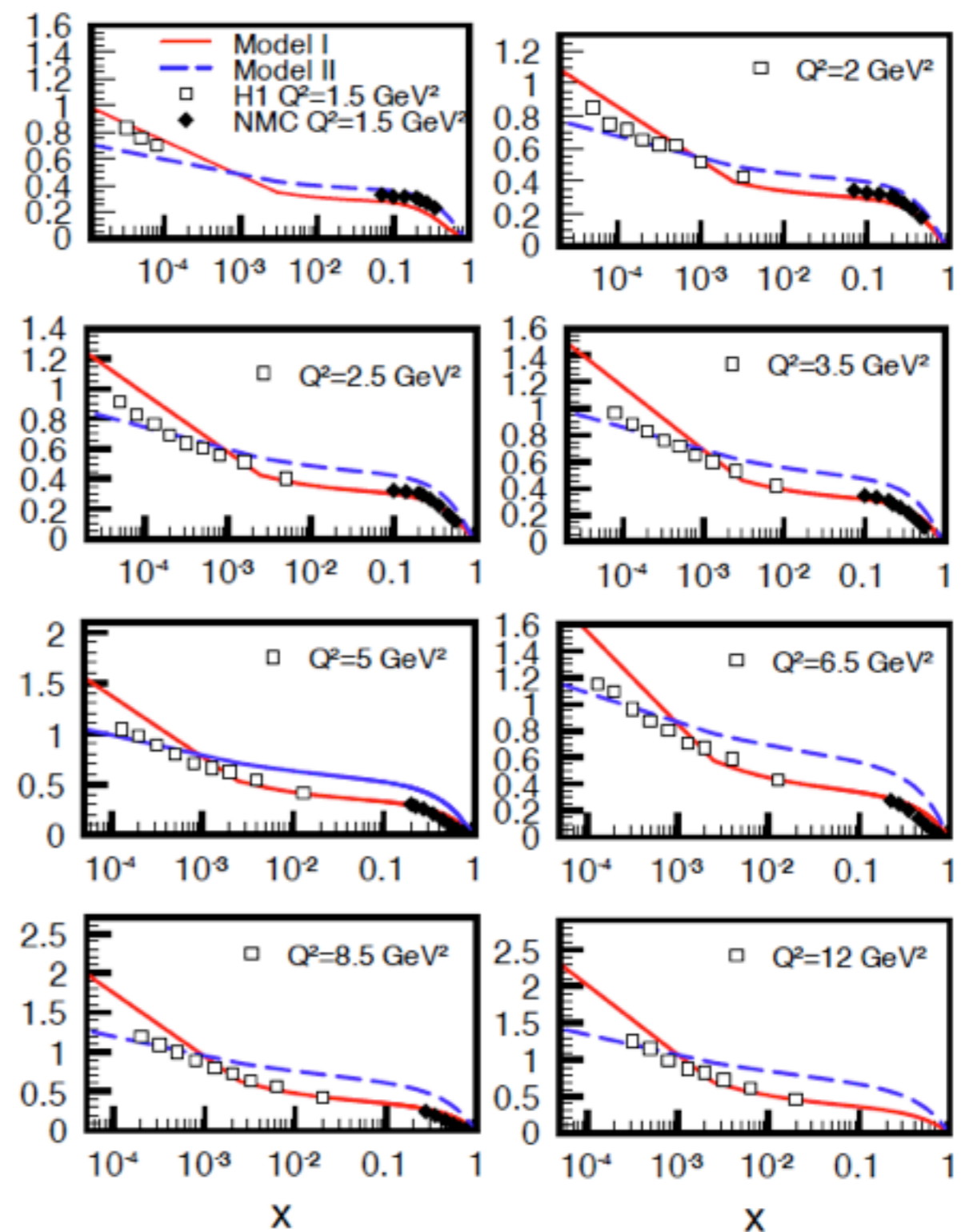
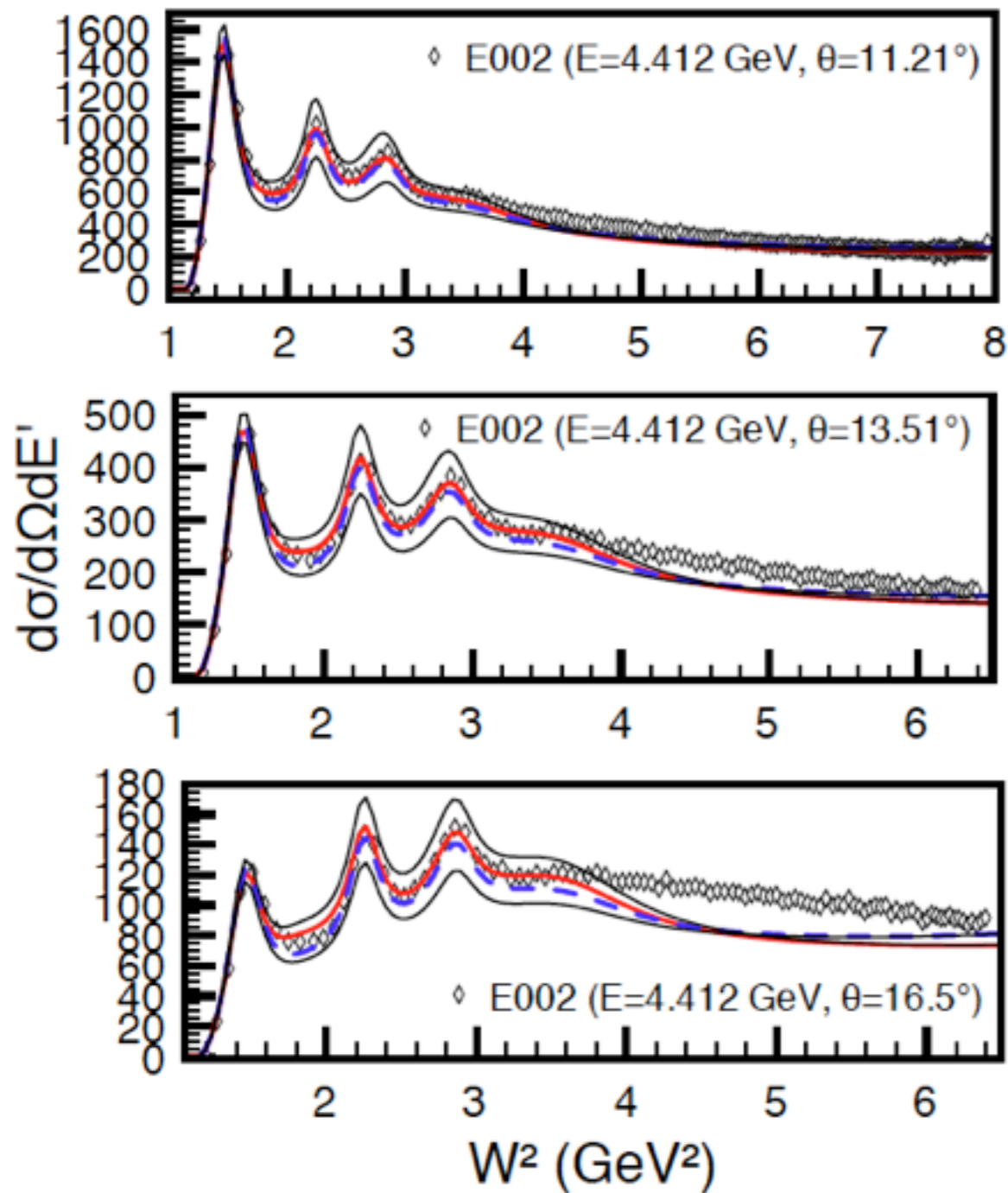


- Collaborators: *Mikhail Gorchtein*, M. Ramsey-Musolf, Shufang Ban
- PREX spokespersons: Krishna Kumar, Robert Michaels, Kent Paschke, Paul Souder, Guido Urciuoli
- Supported in part by DOE

C. J. Horowitz, Indiana University,
PAVI11, Rome, Sept. 2011.

Models

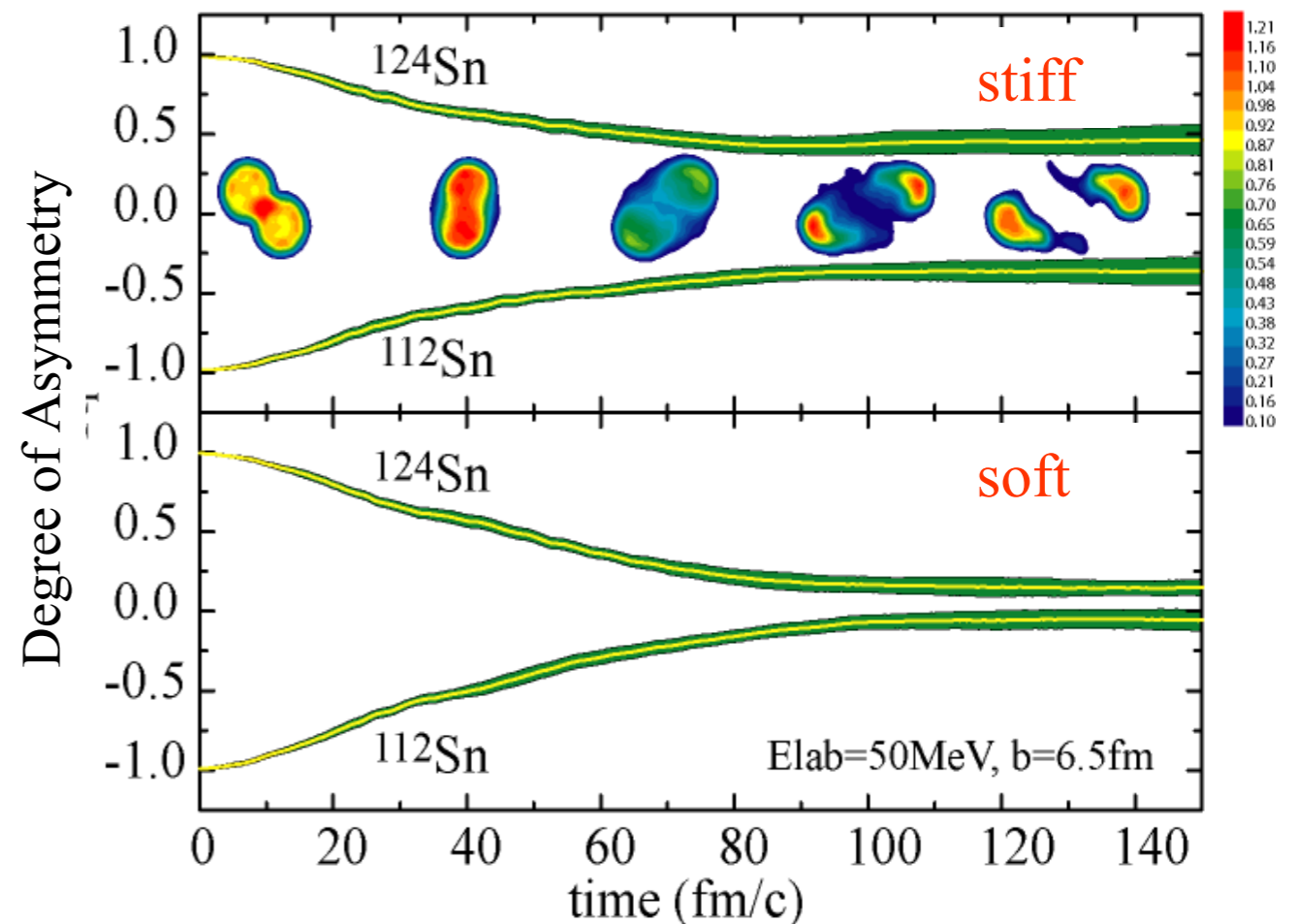
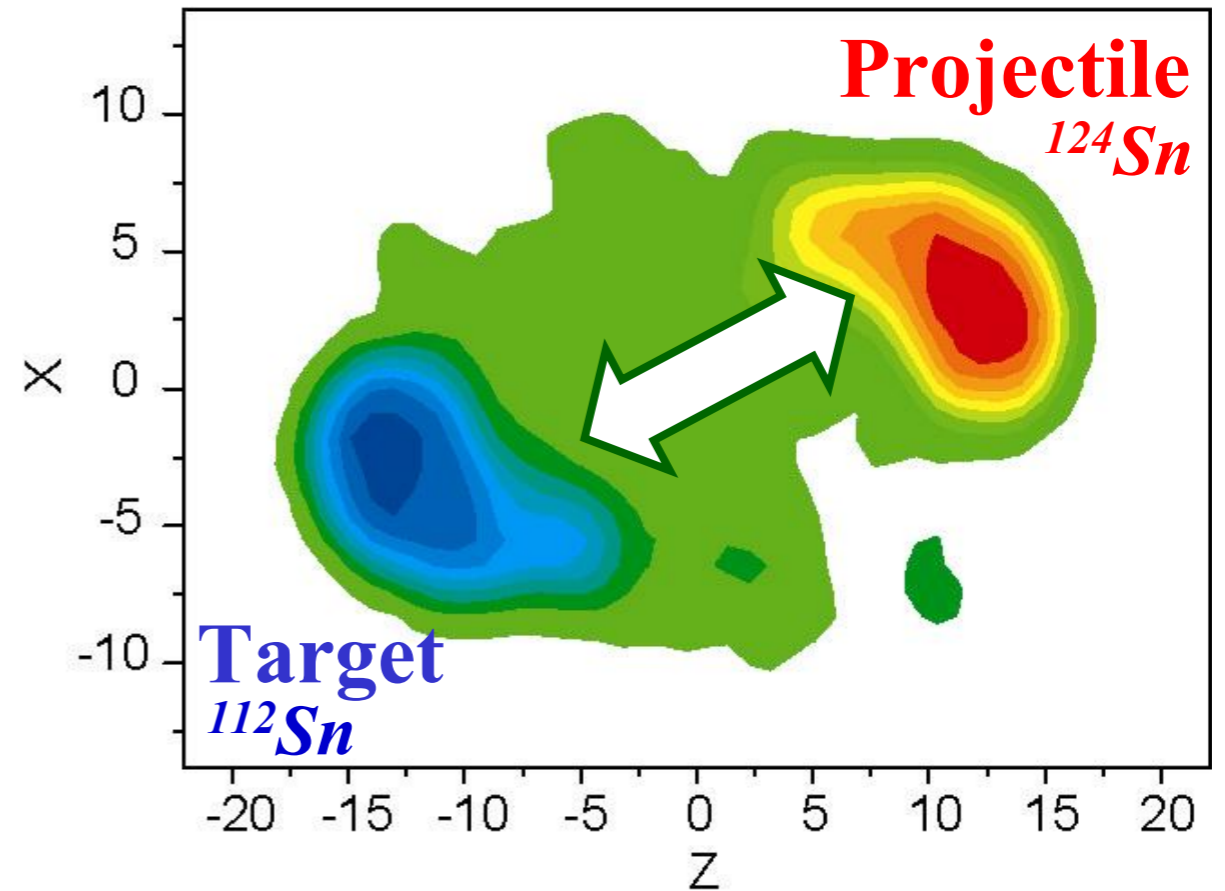
- Model I:
 - Resonances fit to electro-production data [Cristy+Bosted, PRC81 (2010) 055213, (background adjusted for our high W form)]
 - High W part from hybrid GVD/color dipole approach [Cvetič et al, Eur. Phys. J C13(2000) 301.]
- Model II:
 - High W part from “naive” GVD model [Alwall et al, Phys Lett B596 (2004) 77.]



- Resonances left, and DIS F_2 structure function right. Model I is red, model II is blue dashed.

Isospin Diffusion in Heavy Ion Collisions

- *Probe the symmetry energy at subsaturation densities in peripheral collisions, e.g. $^{124}\text{Sn} + ^{112}\text{Sn}$*
- *Isospin “diffuse” through low-density neck region*
- *Symmetry energy drives system towards equilibrium.*
 - *stiff EOS \rightarrow small diffusion*
 - *soft EOS \rightarrow fast equilibrium*
- *Require measurements of collisions of nuclei with no isospin asymmetries for scaling due to existence of non-isospin diffusion effects:*
 - *Pre-equilibrium emissions*
 - *Sequential decays*
 - *Coulomb effects*



From sym. E \rightarrow ^{208}Pb skin

Can measure sym. E at high densities!

B. Tsang

Observing Neutron Star Radii, Masses

- Deduce surface area from luminosity, temperature from X-ray spectrum.

$$L_{\gamma} = 4\pi R^2 \sigma_{\text{SB}} T^4$$

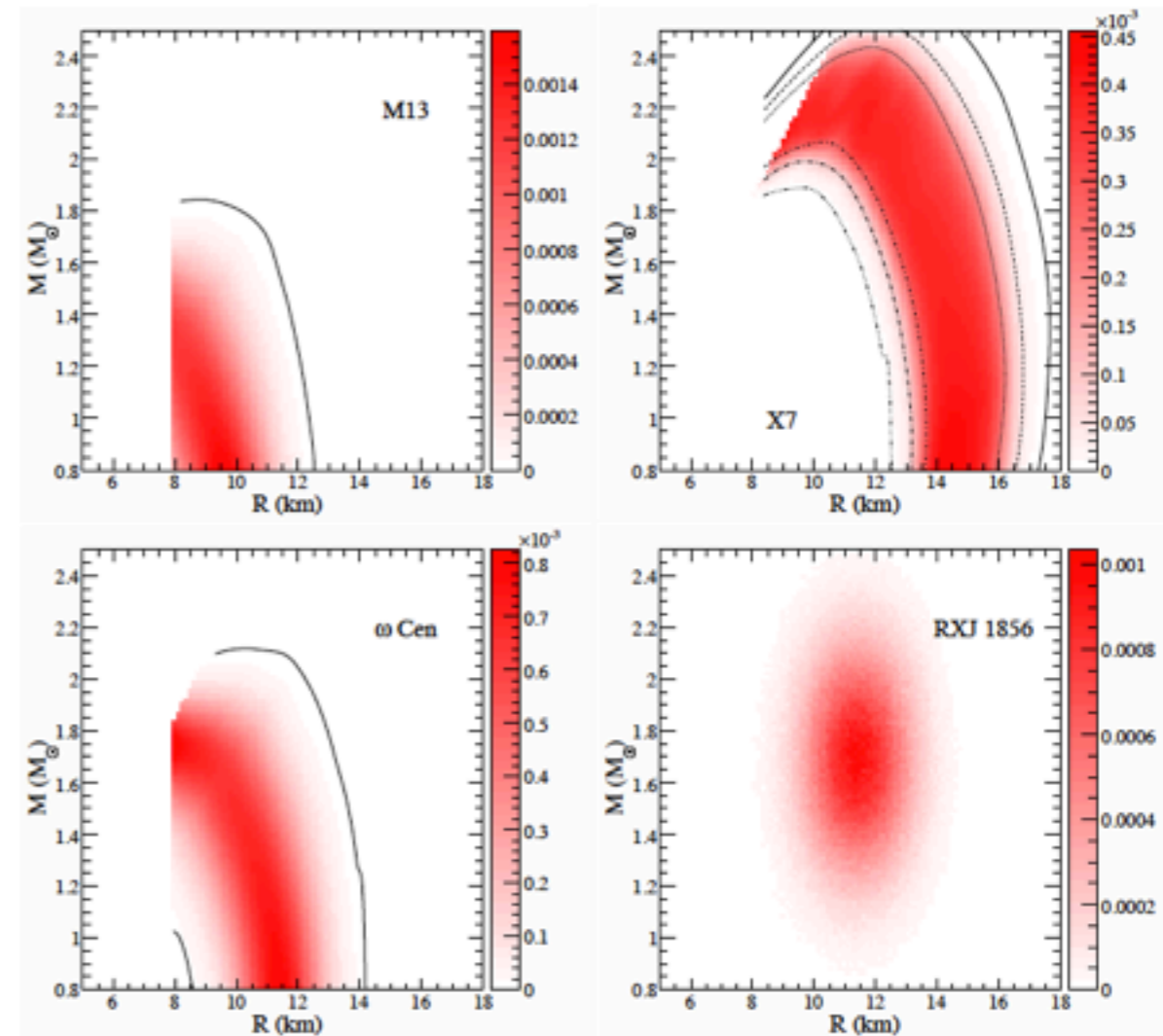
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 - Non-blackbody corrections from atmosphere models can depend on composition and B field.
 - Curvature of space: measure combination of radius and mass.
- Steiner, Lattimer, Brown [ArXiv: 1005.0811] combine observations of NS in X-Ray bursts and globular clusters and deduce
 - EOS is soft at low density so 1.4 M_{sun} star has 12 km radius.
 - EOS + Typel-Brown correlation -> Predict ^{208}Pb neutron skin:
 $R_n - R_p = 0.15 \pm 0.02 \text{ fm}$.

- Important to test this model dependent extraction of EOS with accurate PREX measurement.

- Depends on assumptions about X-ray bursts.
- F. Ozel et al. get smaller radii.
- Radio observations of PSR J1614 find $M = 1.97 \pm 0.04 M_{\text{sun}}$! From binary with $0.5 M_{\text{sun}}$ WD, see relativistic Shapiro delay.
-- P. Demorest et al., Nature **467** (2010) 1081.
- All soft high density EOS including many with exotic high density phases are ruled out.
- Real progress on EOS of cold dense matter is being made with astrophysical observations.

Observations of NS Radii and Masses

- A. Steiner, E. Brown, J. Lattimer, ArXiv:1005.0811, combine observations of 7 NS in 3 classes: X-Ray bursts (important modeling uncertainties), NS in globular clusters, and an isolated nearby NS.
 - Observations favor stiff high density EOS with $\sim 2 M_{\text{sun}}$ maximum NS mass.
 - EOS is soft at low densities so $1.4 M_{\text{sun}}$ star has ~ 12 km radius.
- Predict neutron skin in ^{208}Pb : $R_n - R_p = 0.15 \pm 0.02$ fm.
- If true, important implications for cold dense QCD.



Error bands for masses and radii of 4 neutron stars.

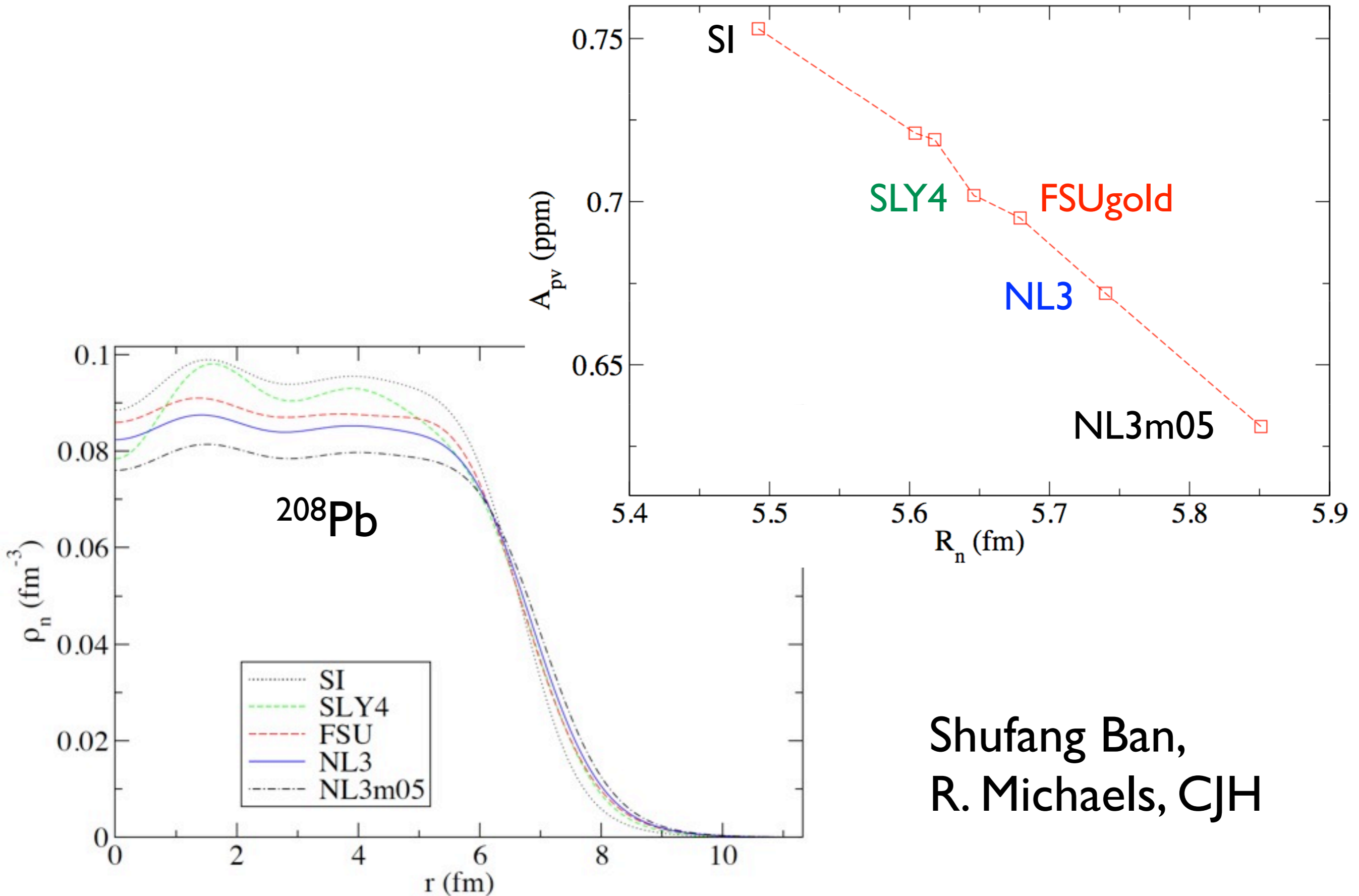
Statistical error at JLAB Hall A: assuming 100μA, 5° for 30 days

	E	Rate	A_{pv}	R_n	$t(1\%)$	a
	(GeV)	MHZ	ppm	%	days	%
^{208}Pb	1.05	1700	0.72	0.66	13	
	1.8	53	2.1			8.0
^{120}Sn	1.2	1080	1.06	0.56	9.4	
^{48}Ca	1.7	270	2.2	0.43	5.5	
	2.1	21	2.8			3.0

Surface thickness a for $\rho(r) = \rho_0/[1 + e^{(r-R_0)/a}]$

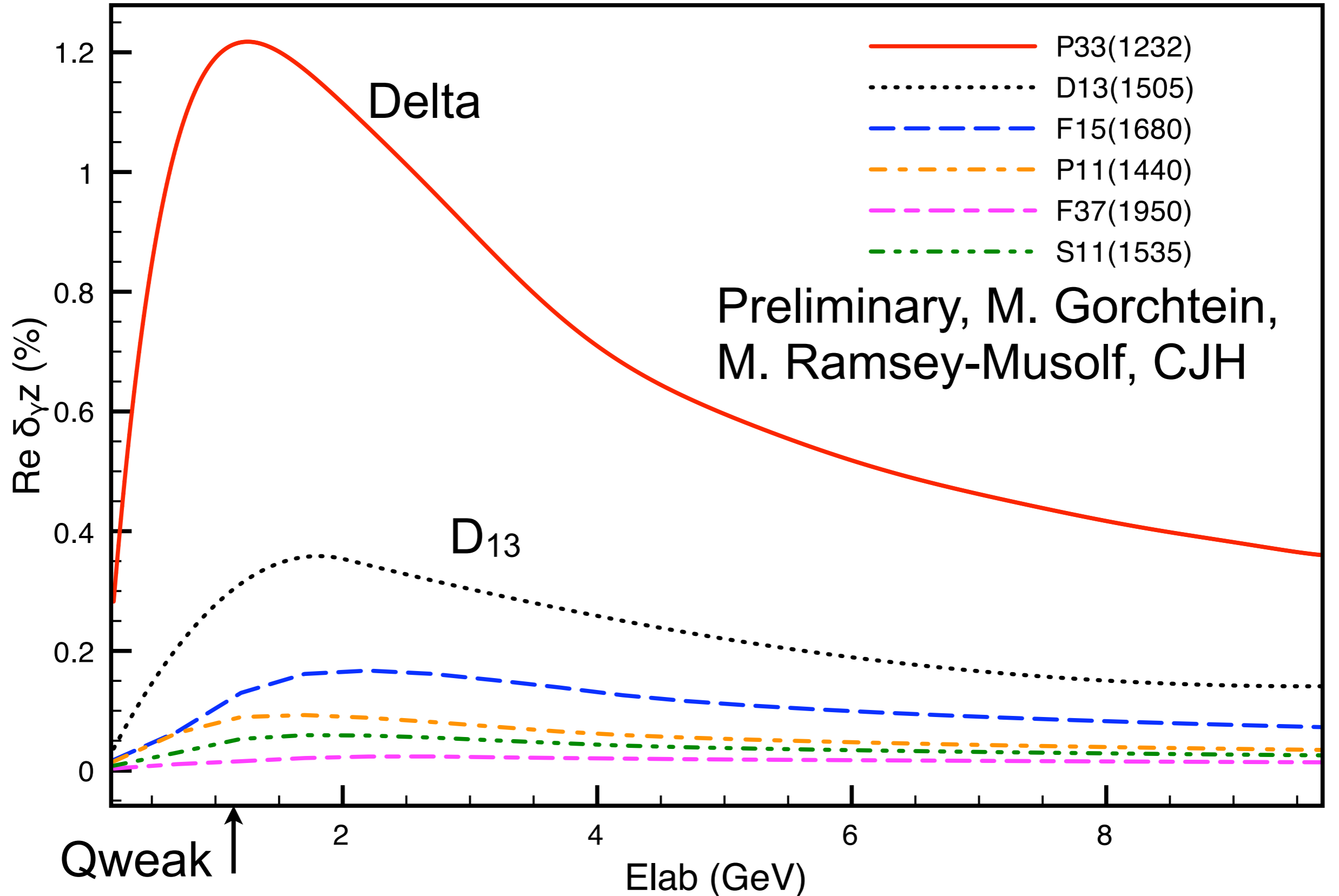
--- Shufang Ban, Bob Michaels, CJH

Parity violating asym. vs neutron radius



Shufang Ban,
R. Michaels, CJH

Resonance contributions, $\sim 1.8\%$



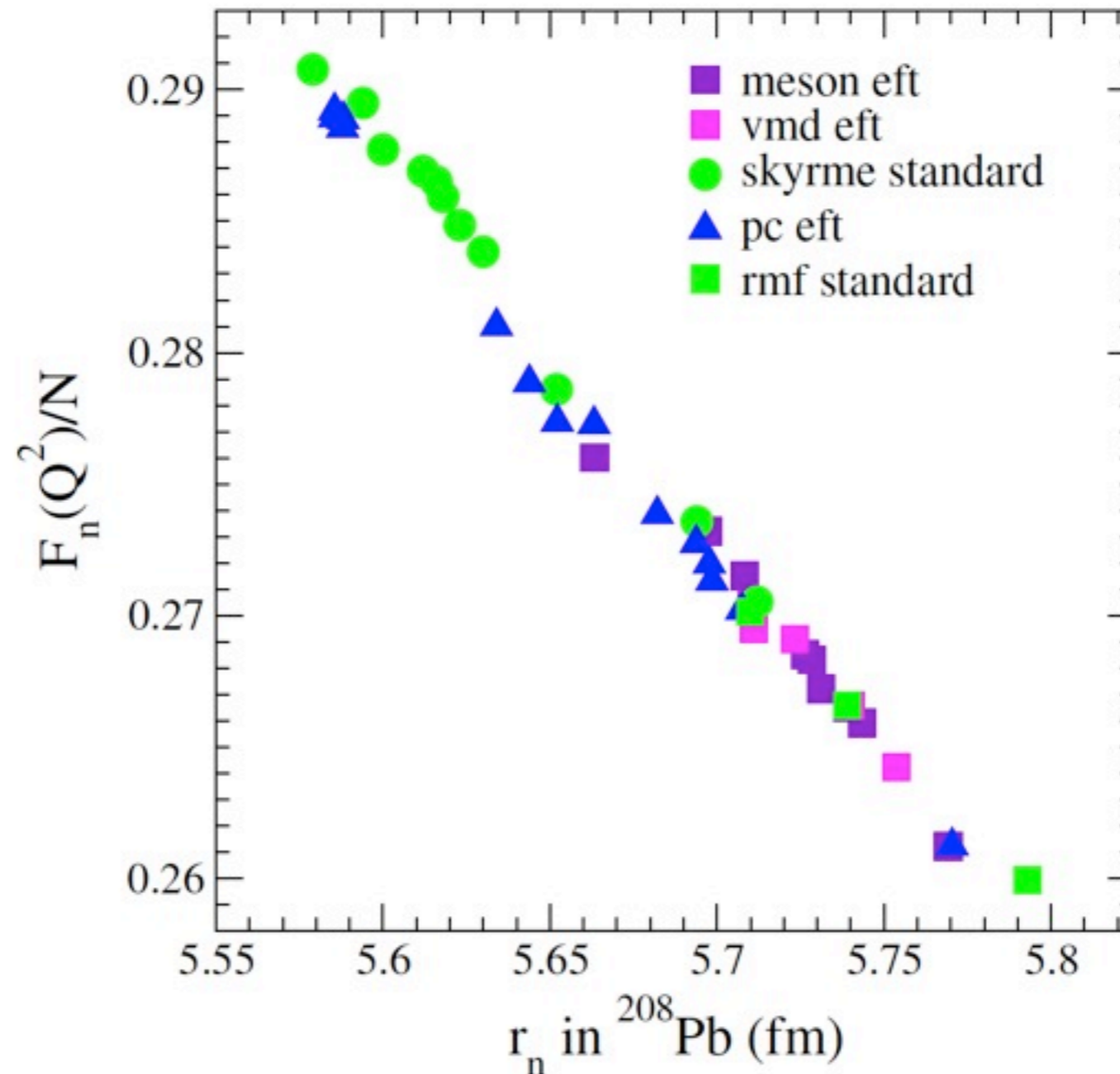
Preliminary, M. Gorchtein,
M. Ramsey-Musolf, CJH

Total 1.8% dominated by Delta but D_{13} also

Calcium 48

- Calcium 48 with 20 protons and 28 neutrons is a closed shell neutron rich nucleus “far” from ^{208}Pb .
- New microscopic coupled cluster and no core shell model calculations are now feasible for ^{48}Ca but not ^{208}Pb . Relate measured neutron density to two nucleon and three nucleon forces (including 3 neutron forces).
- Calcium 48 is an important double beta decay nucleus that allows a test if neutrinos are their own antiparticles. Understanding structure of ^{48}Ca important for calculations of decay matrix element.
- PV exp needs a target involving about a gram of ^{48}Ca .

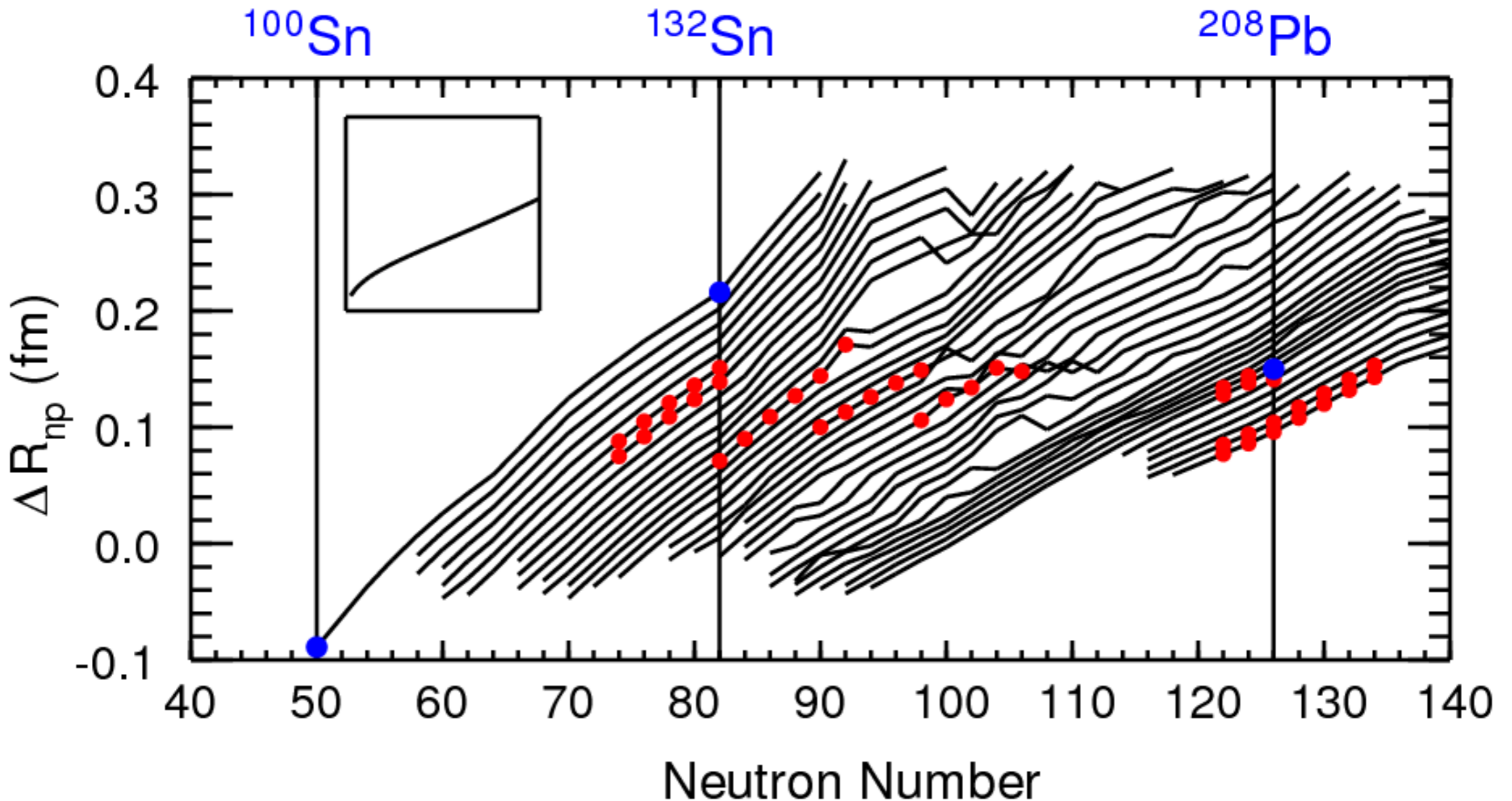
Form Factor at Low Q^2 vs. Neutron Radius in Pb



- How are the low momentum form factor and r_n correlated?
 - Form factor: $F_n(Q^2) = \int d^3r \rho_n(r) j_0(qr)$
 - PREX: $q \equiv (Q^2)^{1/2} = 0.45 \text{ fm}^{-1}$

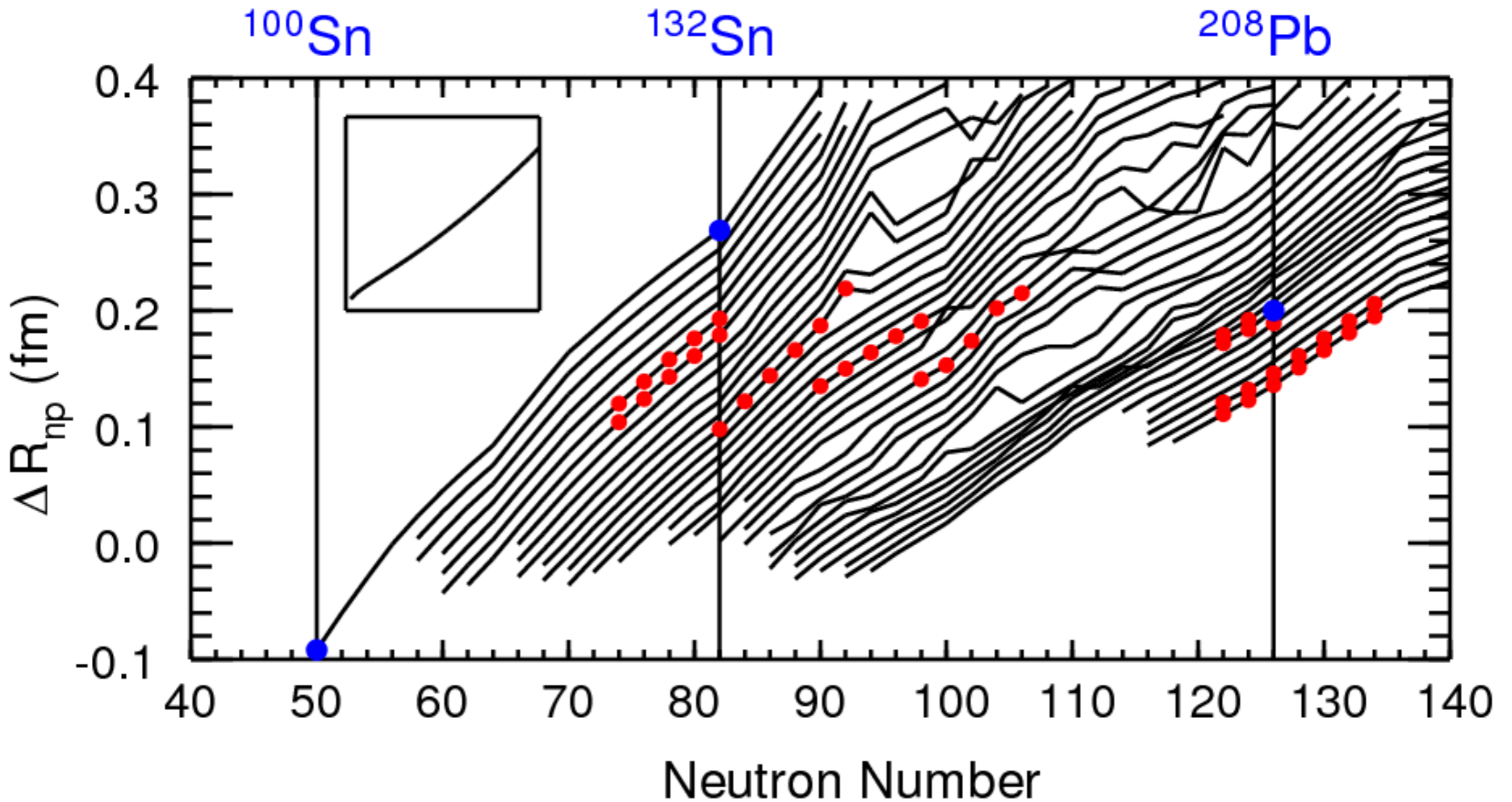
R. Furnstahl

Neutron Skins for Atomic PNC



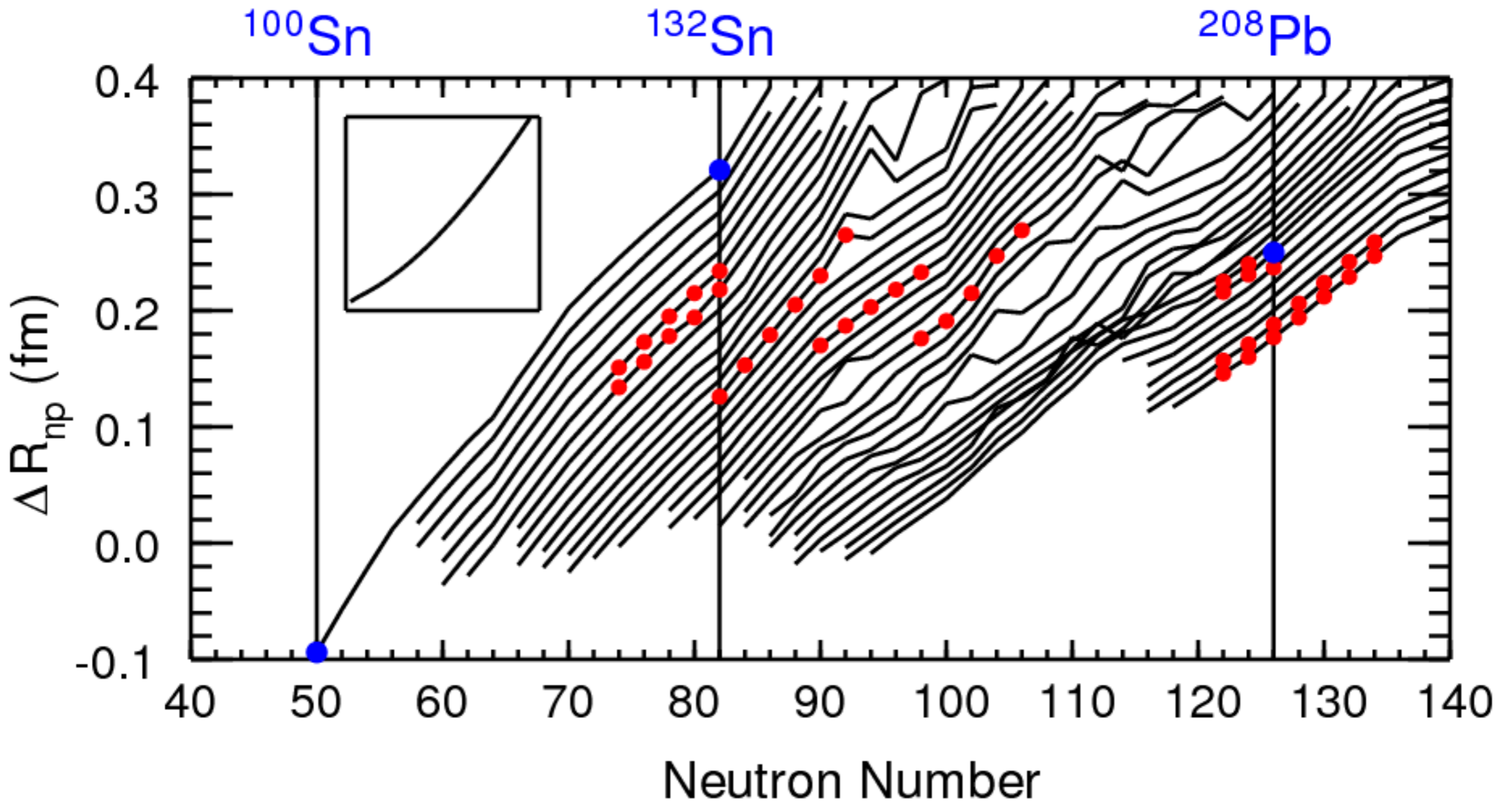
- A. Brown claims n skins of APNC isotopes track ^{208}Pb . PREX constrains them.

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Neutron Skins for Atomic PNC



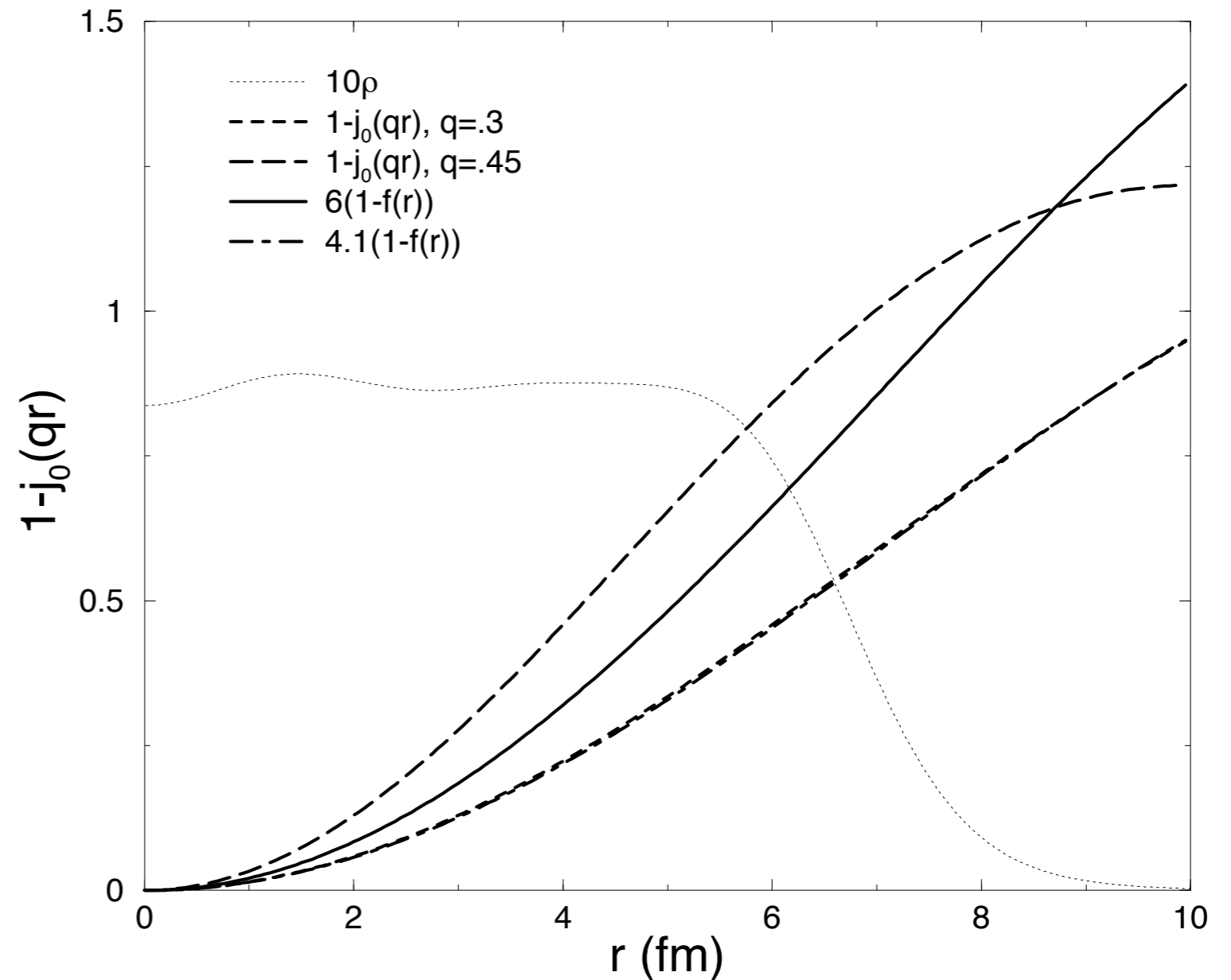
- A. Brown claims n skins of APNC isotopes track ^{208}Pb . PREX constrains them.

Atomic Parity Overlap

Atomic PV depends on overlap of elec. axial transition matrix element with nuclear weak density.

$$f(r) \approx \psi_p^\dagger(r) \gamma_5 \psi_s(r)$$

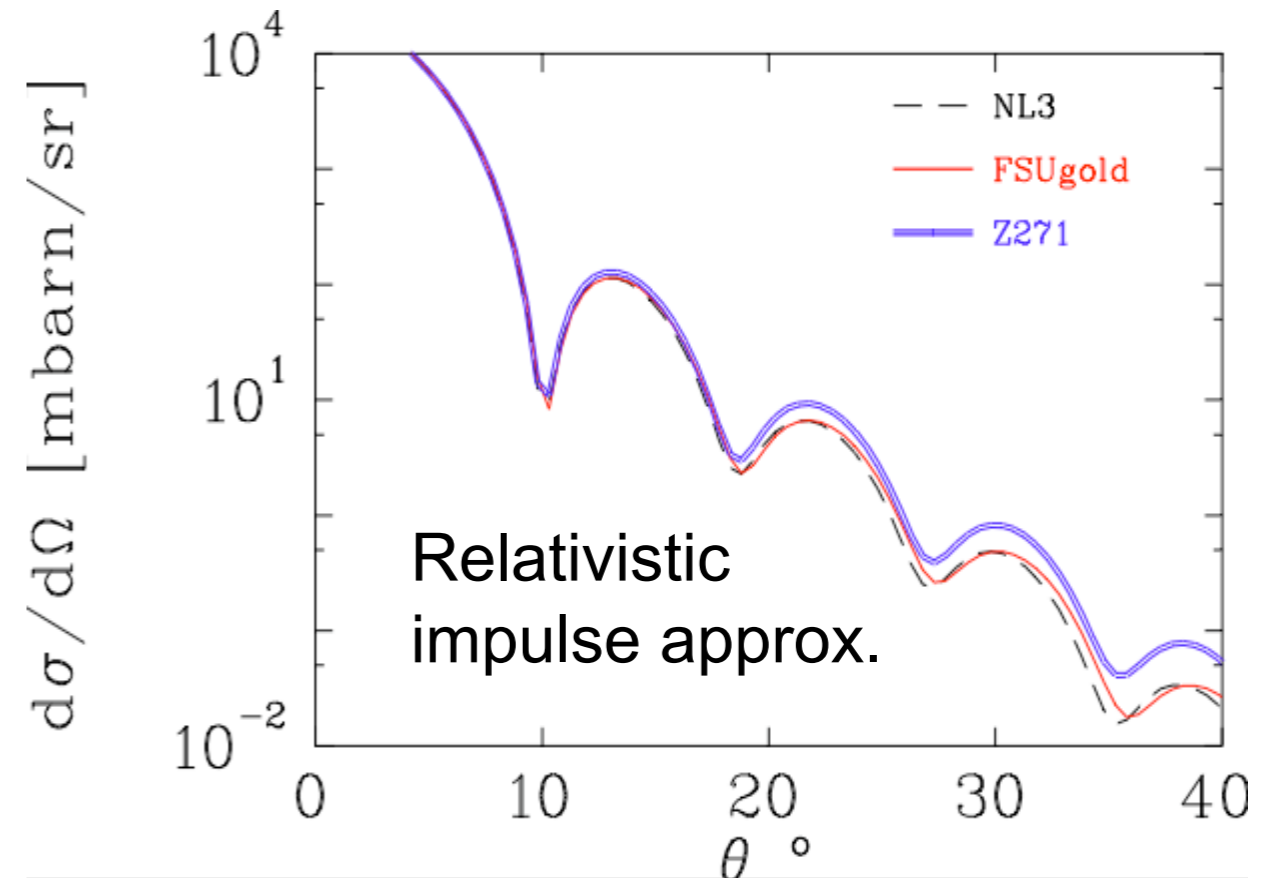
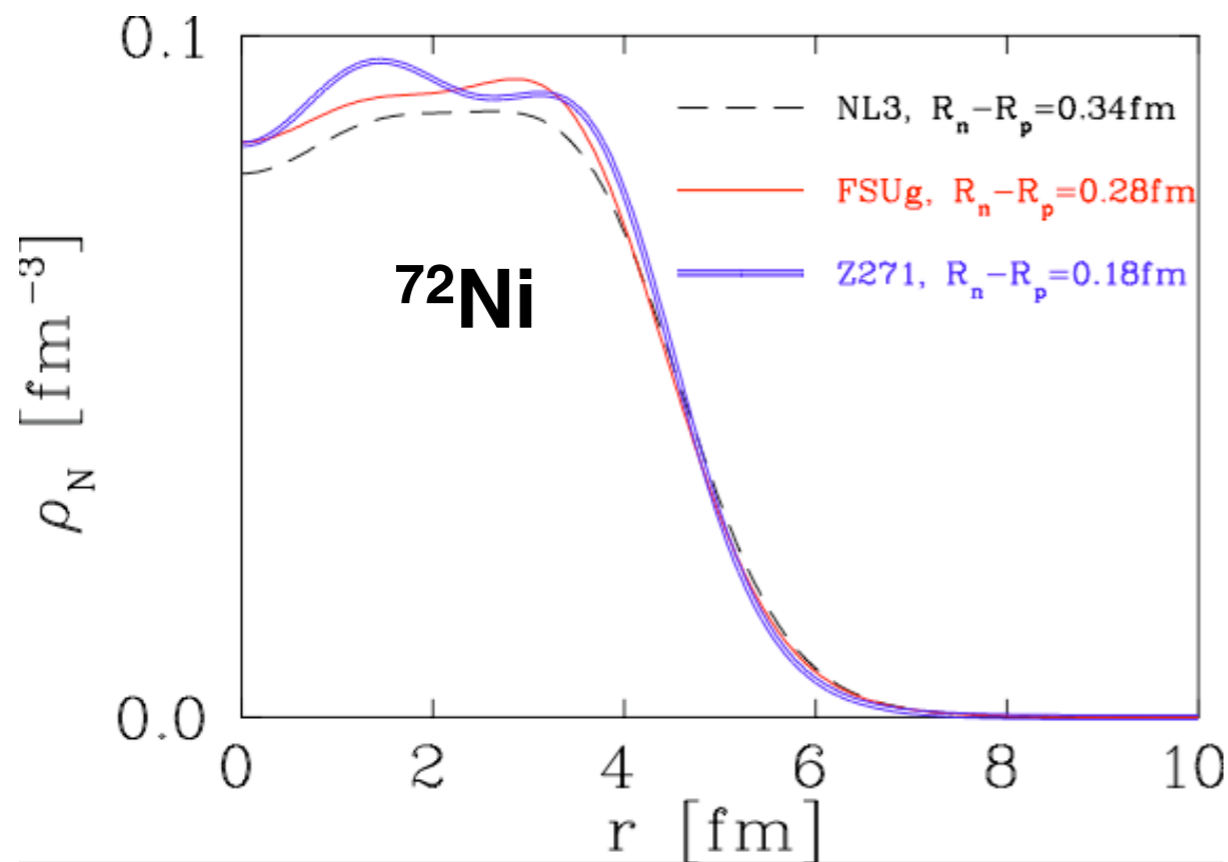
For Pb, $f(r)$ looks like $j_0(qr)$ for $q \sim 0.3 \text{ fm}^{-1}$



Hadronic Probes of Neutron Density

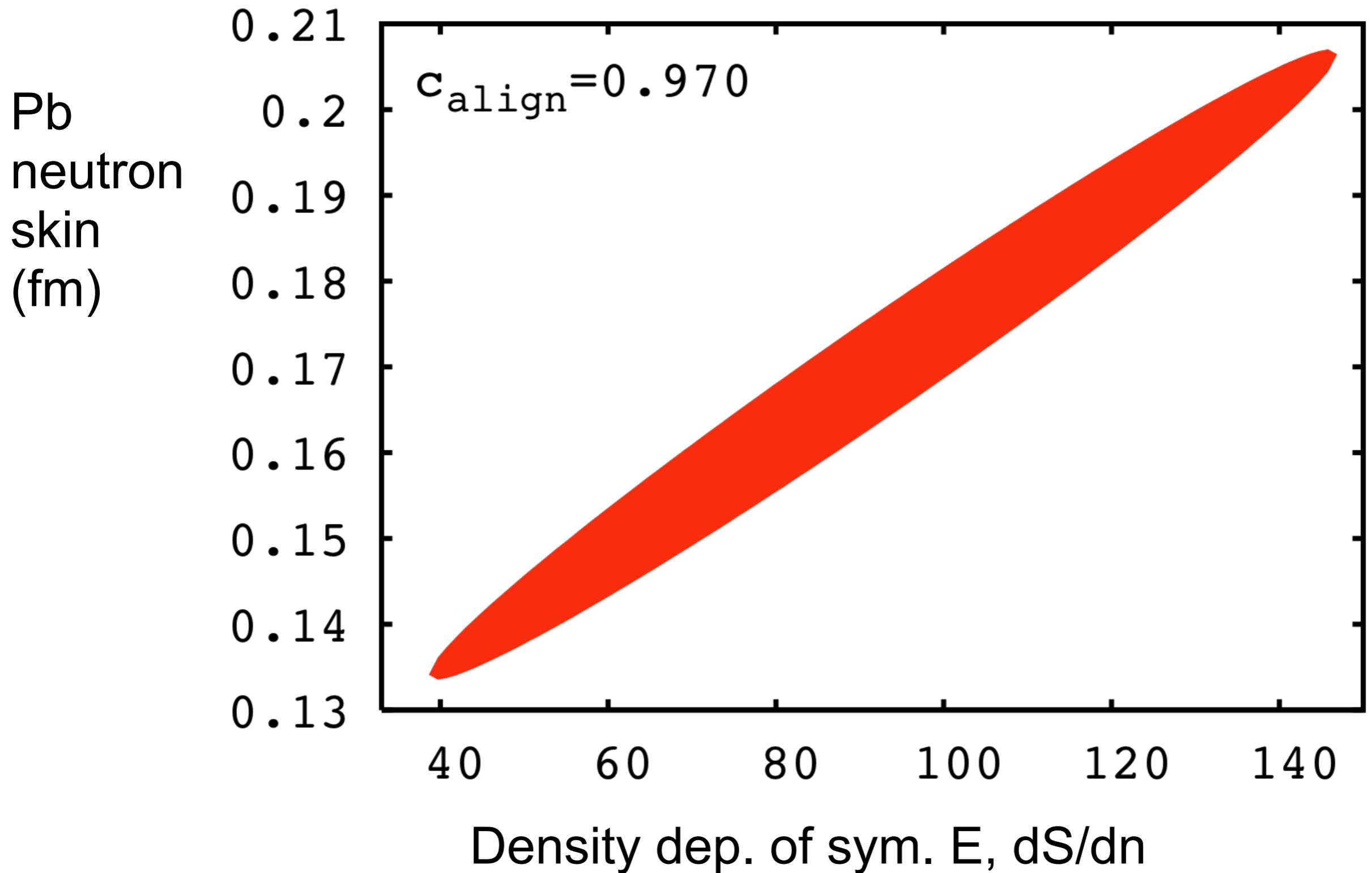
- Anti-protons are sensitive to low density tail, not rms radius. [Model dependent to fit wood-saxon ... to tail and use it to calculate rms radius.]
- Proton and or alpha elastic scattering can measure neutron density.
 - What are systematic errors of rxn mechanism?
 - Calibrate with PREX ^{208}Pb result.
 - Measure n densities of rare isotopes in inverse kinematics.

$^{72}\text{Ni}+p$ elastic scattering at 400MeV/A



- Calibrate proton-nucleus elastic scattering reaction model by reproducing PREX neutron radius with p - ^{208}Pb scattering.
- Measure neutron radii of exotic nuclei with p elastic scattering using radioactive beams in inverse kinematics.
- Example GSI experiment with ^{72}Ni beam on solid H target.

Helber Dussan



Correlation between Pb skin and dS/dn from full error matrix of Skyrme Fit --Witold Nazarewicz