Measuring eccentricity in binary black hole mergers

Alice Bonino, Patricia Schmidt, Geraint Pratten

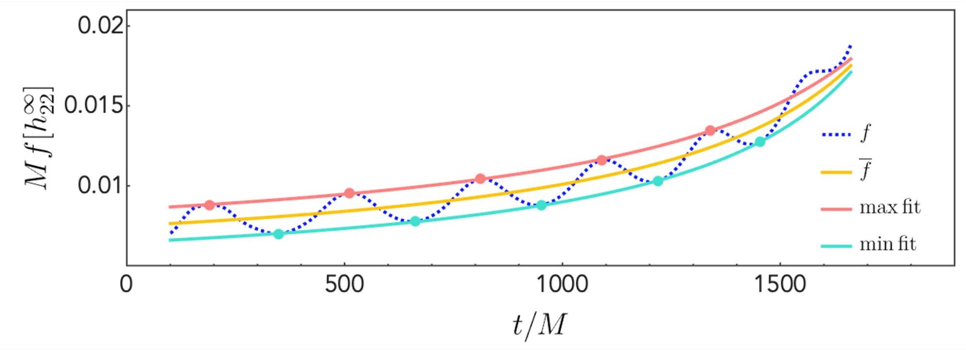
University of Birmingham

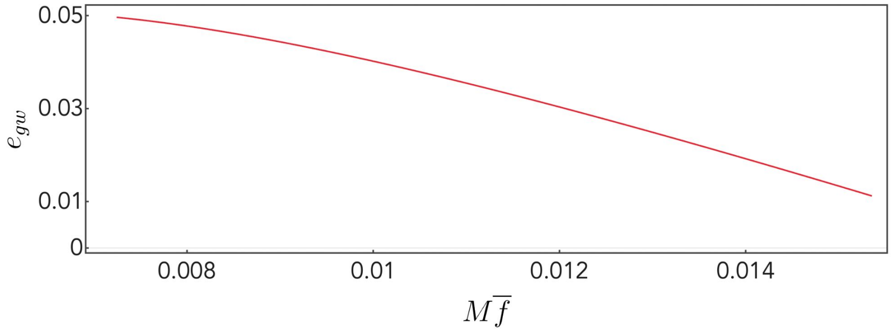
Eccentricity estimator

- Eccentricity estimator: $e_{\rm gw}(t) = \cos(\Psi/3) - \sqrt{3}\sin(\Psi/3)$ arXiv:2209.03390

  [from GW frequency] $\Psi = \arctan\left(\frac{1-e_f^2}{2e_f}\right)$ $e_f(t) = \frac{\sqrt{f_{\rm per}} - \sqrt{f_{\rm ap}}}{\sqrt{f_{\rm per}} + \sqrt{f_{\rm ap}}}$

- Average frequency: $\bar{f}(t) = \frac{1}{2}(f_{\rm per}(t) + f_{\rm ap}(t))$ (can use other definitions, important is to be consistent) arXiv:2207.10474
- Impact of data conditioning and numerical errors: junk radiation, waveform extraction (dependence on the Fixed Frequency Integration cut-off), extrapolation, noise, where the estimator breaks down (how many peaks we need)...





NR vs TEOBResumS: TEOB initial conditions

- We calculate the eccentricity from NR simulations $(\bar{f}_0, e_0)$ corresponding to average frequency and eccentricity at first apastron ($t_0^{\rm NR} = t_{\rm first\ ap}^{\rm NR}$)
- Generate TEOB waveforms using as initial conditions $(\bar{f}_0, e_0)$
- Read off the initial times (corresponding to the times of first apastron) of NR and TEOB: the initial times do not correspond!

⟶ Take the difference between the two times $\Delta t^{\rm NRTEOB} = t_0^{\rm NR} - t_0^{\rm TEOB}$

⟶ From the eccentricity curve read off average frequency and eccentricity at $t^{\rm NR}_{t_0+\Delta t} = t_0^{\rm NR} + \Delta t^{\rm NRTEOB}$

⟶ Re-generate TEOB waveforms using as initial conditions the average frequency and the eccentricity at $t^{\rm NR}_{t_0+\Delta t}$ i.e. $(\bar{f}_0^{\rm TEOB}, e_0^{\rm TEOB})$. The initial time is now ~ $t_0^{\rm NR}$:

⇨ The NR initial conditions and the TEOB initial conditions correspond now to the same reference time!

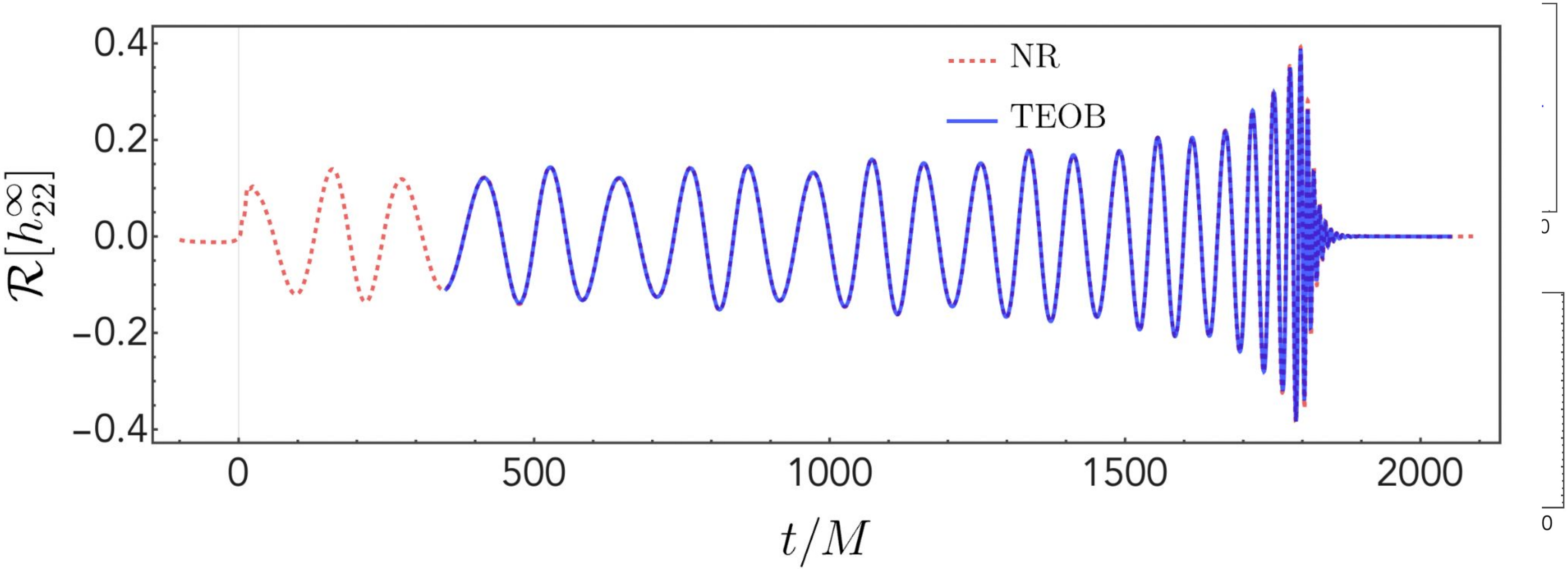
⟶ Perform a phase shift in order to have $\phi^{\rm TEOB}(t_0^{\rm TEOB}) = \phi^{\rm NR}(t_0^{\rm TEOB})$ where $t_0^{\rm TEOB} \simeq t_0^{\rm NR}$
(this is equivalent to put both phases to zero at the time of first apastron)

NR vs TEOBResumS: Preliminary results

Einstein Toolkit simulations

$q = 1, e_0 = 0.2, a_1 = a_2 = 0.5$

$q = 1, e_0 = 0.1, a_1 = a_2 = 0.5$



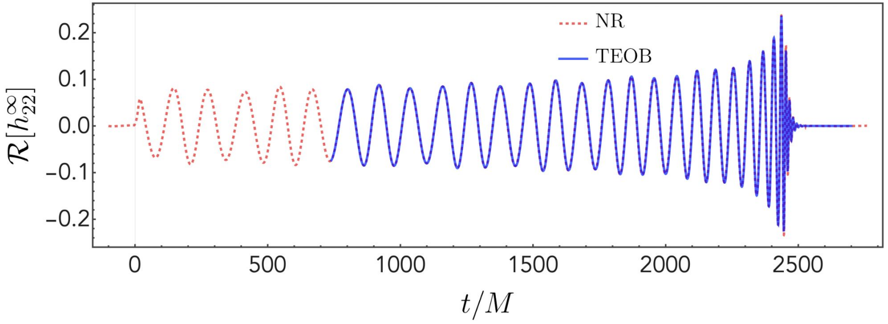
$\Delta\phi^{\rm EOBNR}_{\rm merger} = -0.014$

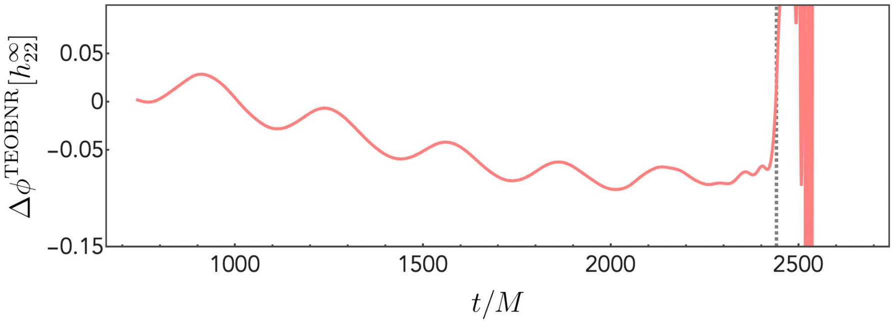
$\Delta\phi^{\rm EOBNR}_{\rm merger} = -0.057$

NR vs TEOBResumS: Preliminary results

Einstein Toolkit simulations

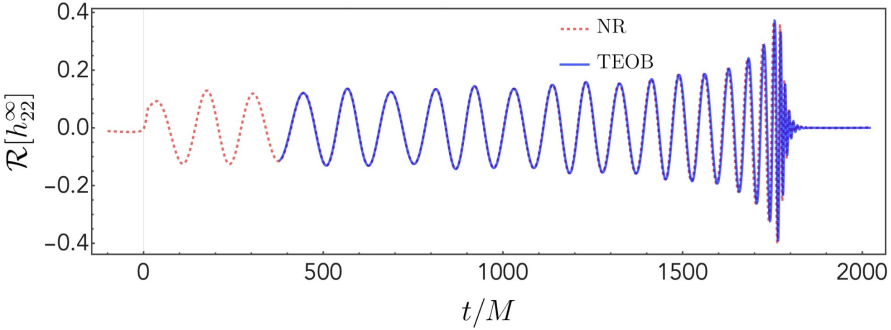
$q = 4, e_0 = 0.05, a_1 = a_2 = 0$

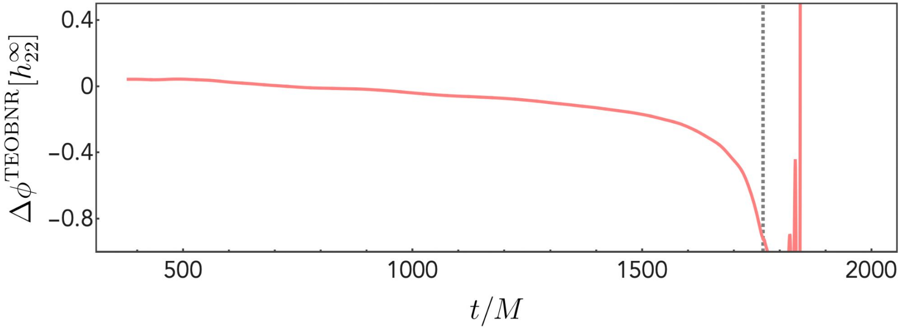




$$\Delta\phi^{\mathrm{EOBNR}}_{\mathrm{merger}} = 0.032$$

$q = 1, e_0 = 0.05, a_1 = a_2 = 0$



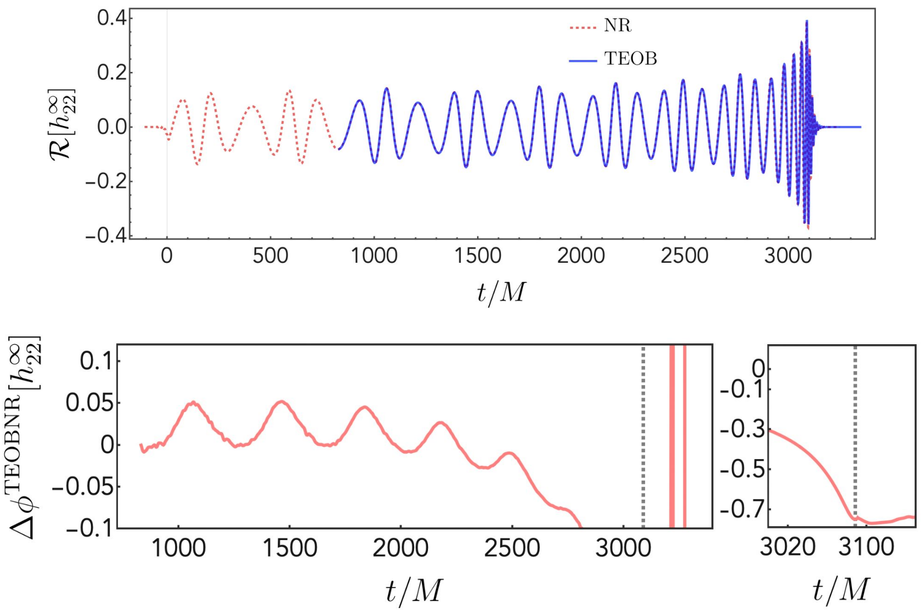


$$\Delta\phi^{\mathrm{EOBNR}}_{\mathrm{merger}} = -0.968$$

NR vs TEOBResumS-Dali: Preliminary results

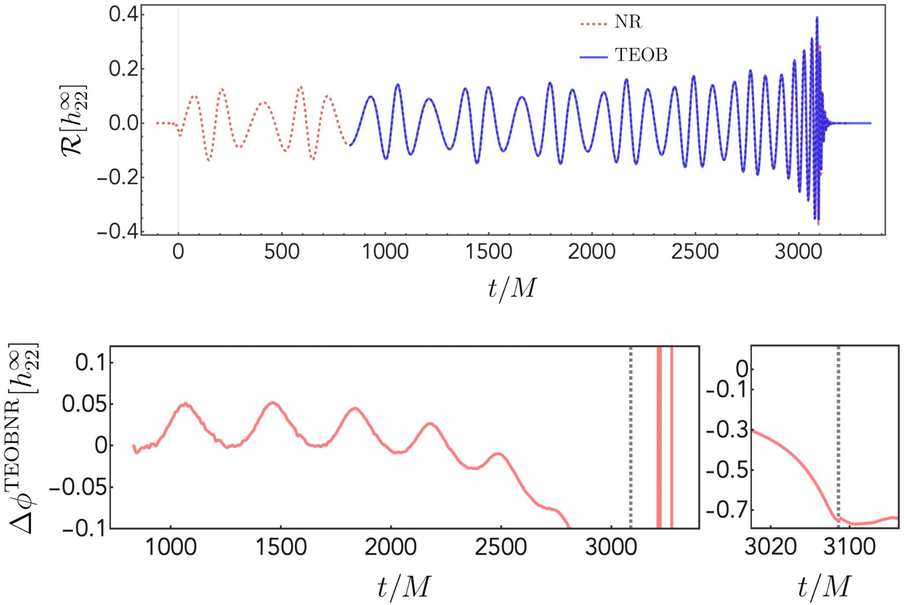
SXS simulations

SXS_BBH_1359



$$\Delta\phi^{\mathrm{EOBNR}}_{\mathrm{merger}} = -0.747$$

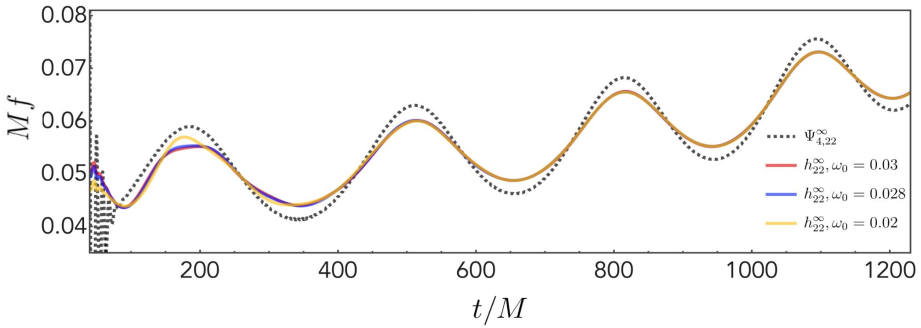
SXS_BBH_1360



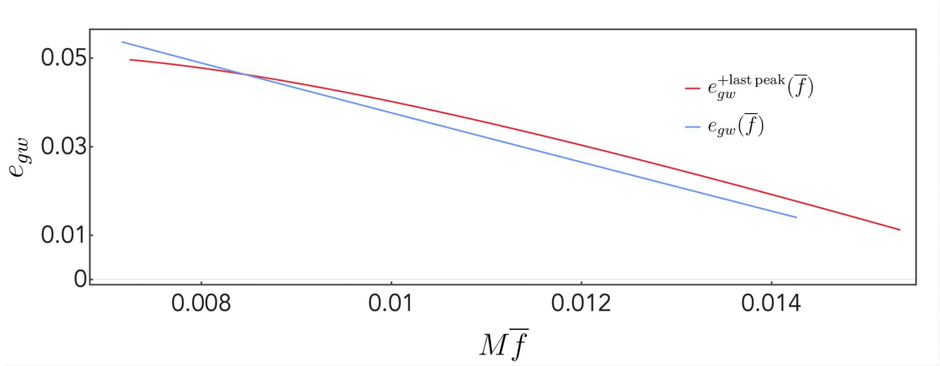
$$\Delta\phi^{\mathrm{EOBNR}}_{\mathrm{merger}} = -0.747$$

Additional plots

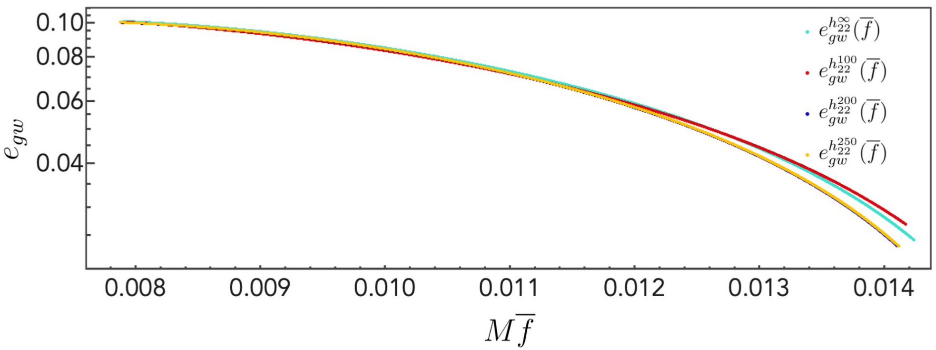
Study of the first peak:



Study of the last peak:



Different extraction radii:



Preliminary table of ETK simulations

Simulation			DALI		$\Delta\phi^{\mathrm{EOBNR}}_{22\,\mathrm{merger}}$
ID	$\bar{f}_{\mathrm{NR}}$	e_{NR}	$\bar{f}_{\mathrm{DALI}}$	e_{DALI}	
BBH_q1_e0.1_a0.5_a0.5	0.00807595	0.0876158	0.00811859	0.0873097	0.057
BBH_q1_e0.05_N80	0.0076203	0.0510486	0.0076187	0.0510577	0.968
BBH_q1_e1_N80_v2	0.00853042	0.0980399	0.00851807	0.0981181	0.872
BBH_q1_e0.1_a+0.5_a-0.5	0.00853016	0.0993008	0.00853016	0.0993008	0.933
BBH_q1_e0.2_a0_a0	0.0113294	0.186742	0.0111475	0.192398	0.757
BBH_q1_e0.2_a0.5_a0.5	0.00987994	0.170253	0.00998431	0.168399	0.014
BBH_q1_e0.2_a+0.5_a-0.5	0.0113196	0.1871	0.0111526	0.192687	0.582
BBH_q1_e0.1_a-0.5_a-0.5	0.00949267	0.116386	0.00937812	0.119773	0.552
BBH_q1_e0.2_a-0.5_a-0.5	N/A	N/A	N/A	N/A	N/A
BBH_q1_e3_N80	N/A	N/A	N/A	N/A	N/A
BBH_q2_e0.05_a0.5_a0.5 (psi4)	0.00738382	0.0460373	0.00733199	0.0462486	−0.349
BBH_q2_e0.05_a-0.5_a-0.5	0.00799565	0.0613589	0.00794926	0.0622875	0.377
BBH_q2_e0.1_a0_a0	0.00844699	0.10109	0.00845138	0.101055	0.485
BBH_q2_e0.1_a0.5_a0.5	0.00801989	0.0871244	0.00804491	0.0869572	0.442
BBH_q2_e0.1_a-0.5_a-0.5	0.00936667	0.176516	0.00926028	0.121797	0.296
q4_e0p05_D12p39 (second apastron)	0.00802277	0.05008	0.00799697	0.0502306	−0.032
q4_e0p05_D12p39 (first apastron)	0.00753472	0.0541705	0.00752044	0.054268	0.119
q6_a1_0p0_a2_0p0_e_0p1_D12p83	0.00781405	0.105506	0.00777654	0.105947	0.190
BBH_q3_e1_N80	0.00834383	0.102726	0.00829156	0.103095	0.190
BBH_q3_e3_N80	N/A	N/A	N/A	N/A	N/A
q2_e0p05_a1_0p0	0.00791276	0.053303	0.00793732	0.0531424	0.527
q3_a1_0p0_a2_0p0_e0p05_D12p379_vf (first apastron)	0.00757658	0.0547304	0.00756841	0.0546278	0.722
q3_a1_0p0_a2_0p0_e0p05_D12p379_vf (second apastron)	0.00817619	0.0499211	0.00814253	0.0501046	0.655
q3_a1_0p5_a2_0p5_e0p05_D12p379_vf	0.00741632	0.0477771	0.00738953	0.0479334	−0.356
q3_a1_m0p5_a2_m0p5_e0p05_D12p379_vf	0.00800259	0.0617299	0.00797244	0.0620075	0.618