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EOB@Work23

5PN and 6PN completion of the 4PM EOB Hamiltonian

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PM-based EOB Hamiltonians

Standard EOB Hamiltonian, built on PN results: $\hat{H}_{\rm EOB} \equiv \frac{1}{\nu}\sqrt{1+2\nu\left(\hat{H}_{\rm eff}-1\right)}$

$$\left(\hat{H}_{\rm eff}^{\rm PN}\right)^2 \equiv A_{\rm PN}(u)\left[1+p_\varphi^2 u^2 + \frac{1}{B_{\rm PN}(u)}p_r^2 + Q_{\rm PN}(p_r,u)\right] \qquad \left(D_{\rm PN} = A_{\rm PN}\,B_{\rm PN}\right)$$

DJS gauge, no dependence in p_φ

Alternatives devised to use instead PM information:

$$\hat{H}_S^2 \equiv A_S\left(1+p_\varphi^2 u^2 + A_S\,p_r^2\right)$$

PS Hamiltonian

$A = A_S \equiv 1-2u, \quad B = B_S \equiv A_S^{-1},$

$Q = Q_{\rm PM} \equiv u^2 q_{2\rm PM} + u^3 q_{3\rm PM} + u^4 q_{4\rm PM} + \mathcal{O}(u^5)$

$$\left(\hat{H}_{\rm eff}^{\rm PS}\right)^2 \equiv A_S\left(1+p_\varphi^2 u^2 + A_S\,p_r^2 + Q_{\rm PM}\right) = \hat{H}_S^2 + A_S\,Q_{\rm PM}$$

PS* Hamiltonian

$A = A_S + A_{\rm PM},$

$A_{\rm PM} \equiv u^2 a_{2\rm PM} + u^3 a_{3\rm PM} + u^4 a_{4\rm PM} + \mathcal{O}(u^5),$

$B = B_S \equiv A_S^{-1}, \quad Q = 0$

$$\left(\hat{H}_{\rm eff}^{\rm PS^*}\right)^2 \equiv \left(A_S + A_{\rm PM}\right)\left(1+p_\varphi^2 u^2 + A_S\,p_r^2\right) = \hat{H}_S^2 + A_{\rm PM}\left(1+p_\varphi^2 u^2 + A_S\,p_r^2\right) = \hat{H}_S^2\left(1+A_{\rm PM}/A_S\right)$$

Here $(q_{n\rm PM}, a_{n\rm PM})$ are PM-informed coefficients

PM-based EOB Hamiltonians

The coefficients $(q_{n\mathrm{PM}}, a_{n\mathrm{PM}})$ are determined from the **PM expansion of the scattering angle**:

$$\chi_{\mathrm{eff}}\left(\hat{H}_{\mathrm{eff}}^{\mathrm{PS}}\right) = \chi_{\mathrm{PM}} \quad \rightarrow \quad q_{n\mathrm{PM}} = q_{n\mathrm{PM}}(\gamma)$$

$$\chi_{\mathrm{eff}}\left(\hat{H}_{\mathrm{eff}}^{\mathrm{PS}*}\right) = \chi_{\mathrm{PM}} \quad \rightarrow \quad a_{n\mathrm{PM}} = a_{n\mathrm{PM}}(\gamma)$$

$$\gamma \equiv E_{\mathrm{eff}}/\mu$$

[Damour 2017]: up to 2PM

[Khalil et al. 2022]: up to 4PM

For instance:

$$q_{2\mathrm{PM}}(\gamma) = \frac{3\left(5\gamma^2 - 1\right)(\Gamma - 1)}{2\Gamma}, \qquad a_{2\mathrm{PM}}(\gamma) = \frac{3\left(5\gamma^2 - 1\right)(\Gamma - 1)}{2\Gamma\gamma^2}, \qquad \Gamma \equiv \sqrt{1 + 2\nu\left(\gamma - 1\right)}$$

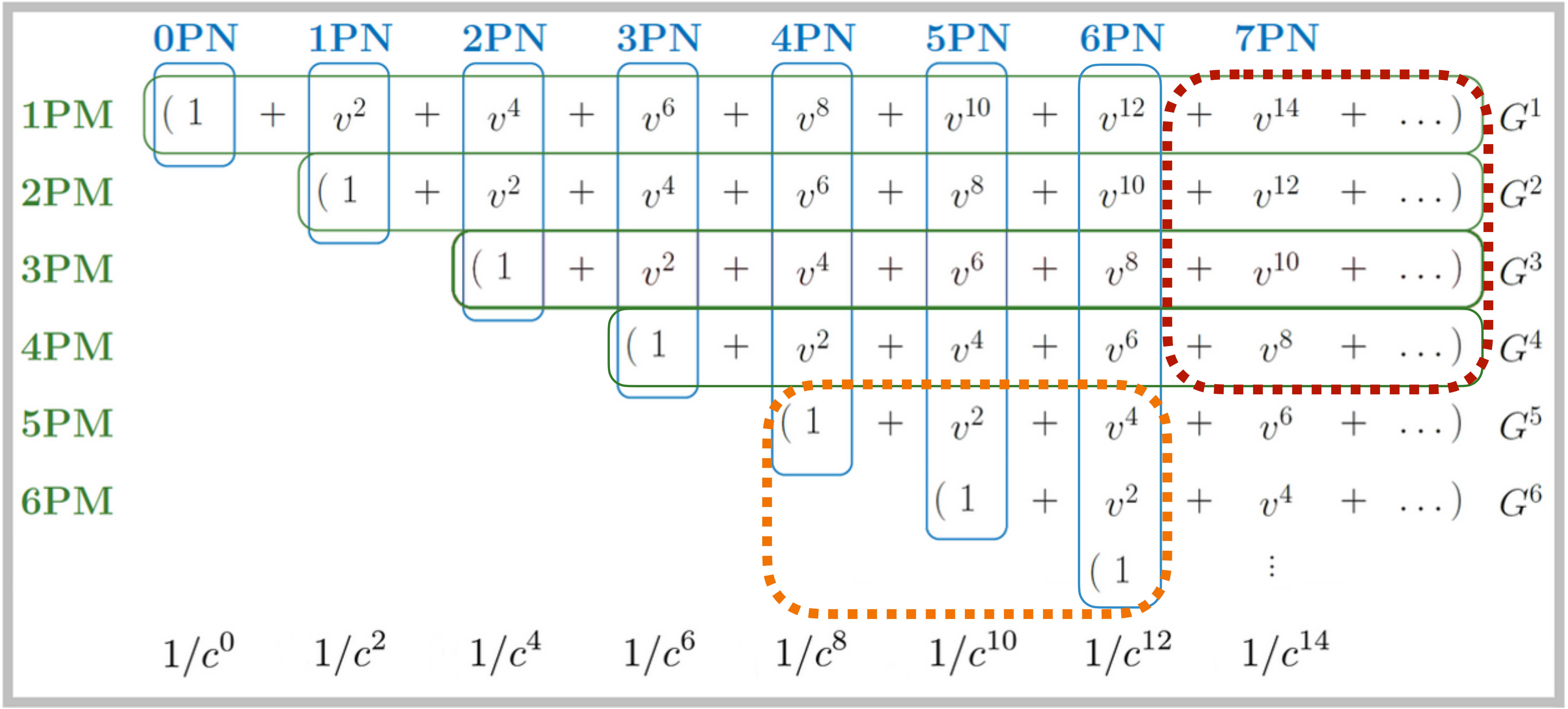
Here the dependence on the effective energy must be understood **perturbatively in the PM-sense**. At 4PM accuracy:

- $q_{4\mathrm{PM}}(\gamma),\ q_{3\mathrm{PM}}(\gamma),\ a_{4\mathrm{PM}}(\gamma),\ a_{3\mathrm{PM}}(\gamma) \rightarrow \gamma = \hat{H}_{\mathrm{eff}}^{\mathrm{PS}}\Big|_{1\mathrm{PM}} = \hat{H}_S$
- $q_{2\mathrm{PM}}(\gamma) \rightarrow \gamma = \hat{H}_{\mathrm{eff}}^{\mathrm{PS}}\Big|_{2\mathrm{PM}} = \sqrt{\hat{H}_S^2 + (1 - 2u)\, u^2 q_{2\mathrm{PM}}\left(\hat{H}_S\right)}$
- $a_{2\mathrm{PM}}(\gamma) \rightarrow \gamma = \hat{H}_{\mathrm{eff}}^{\mathrm{PS}*}\Big|_{2\mathrm{PM}} = \hat{H}_S\sqrt{1 + \frac{u^2\, a_{2\mathrm{PM}}\left(\hat{H}_S\right)}{1 - 2u}}$

Need for a PN completion

	0PN		1PN		2PN		3PN		4PN		5PN		6PN		7PN			
1PM	$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	v^{12}	$+$	v^{14}	$+$	$\dots)$	G^1
2PM			$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	v^{12}	$+$	$\dots)$	G^2
3PM					$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	v^{10}	$+$	$\dots)$	G^3
4PM							$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	v^8	$+$	$\dots)$	G^4
5PM									$(1$	$+$	v^2	$+$	v^4	$+$	v^6	$+$	$\dots)$	G^5
6PM											$(1$	$+$	v^2	$+$	v^4	$+$	$\dots)$	G^6
													$(1$		$\vdots$			
	$1/c^0$		$1/c^2$		$1/c^4$		$1/c^6$		$1/c^8$		$1/c^{10}$		$1/c^{12}$		$1/c^{14}$			

Need for a PN completion



W.r.t. to the usual 6PN Hamiltonian, the 4PM Hamiltonian make us **gain** new analytical information but also **lose** some

How to gain with no pain?

We compute extra **PN complementary terms** for our 4PM Hamiltonian, going beyond the original 4PN result of [Khalil et al. 2022], up to 6PN

Computation outline

1. 6PN-accurate transformation from DJS to energy gauge
2. 6PN effective Hamiltonian in the energy gauge, $\hat{H}_{\text{eff}}^{6\text{PN}}(H_S, u)$
3. General ansatz for the PN complementary parts of $\hat{H}_{\text{eff}}^{\text{PS}*}$
4. 6PN expansion of the ansatz-dependent $\hat{H}_{\text{eff}}^{\text{PS}*}$
5. Matching the two Hamiltonians at 4PN, 5PN, and 6PN
6. 4PN, 5PN and 6PN complementary terms for $\hat{H}_{\text{eff}}^{\text{PS}*}$

6PN gauge transformation: DJS to E

General generating function: $(p^2 = p_r^2 + p_\varphi^2 u^2)$

$$g_{\rm E}(r, p_r, p) = \frac{r\,p_r}{r^2}\left\{\frac{f_1}{c^4} + \frac{1}{c^6}\left(g_1\,p^2 + g_2\,p_r^2 + \frac{g_3}{r}\right) + \frac{1}{c^8}\left(h_1\,p^4 + h_2\,p^2 p_r^2 + h_3\,\frac{p^2}{r} + h_4\,p_r^4 + h_5\,\frac{p_r^2}{r} + \frac{h_6}{r^2} + \frac{\log r}{r^2}\,h_{1,\log}\right) + \frac{1}{c^{10}}\left[p^6\,n_1 + \cdots + \frac{n_{10}}{r^3} + \frac{\log r}{r^2}\left(n_{1,\log}\,p^2 + n_{2,\log}\,p_r^2 + \frac{n_{3,\log}}{r}\right)\right] + \frac{1}{c^{12}}\left[p^8\,w_1 + \cdots + \frac{w_{15}}{r^4} + \frac{\log r}{r^2}\left(w_{1,\log}\,p^4 + \cdots + \frac{w_{6,\log}}{r^2}\right)\right]\right.$$

Corresponding (parameter-dependent) gauge transformation:

$$r \to r - \frac{\partial g_E}{\partial p_r} + \frac{\partial g_E}{\partial p_r}\frac{\partial^2 g_E}{\partial r\,\partial p_r} - \left[\frac{\partial g_E}{\partial p_r}\left(\frac{\partial^2 g_E}{\partial r\,\partial p_r}\right)^2 + \frac{1}{2}\left(\frac{\partial g_E}{\partial p_r}\right)^2\frac{\partial^3 g_E}{\partial r^2\,\partial p_r}\right]$$

$$p_r \to p_r + \frac{\partial g_E}{\partial r} - \frac{\partial g_E}{\partial p_r}\frac{\partial^2 g_E}{\partial r^2} + \left[\frac{\partial g_E}{\partial p_r}\frac{\partial^2 g_E}{\partial r\,\partial p_r}\frac{\partial^2 g_E}{\partial r^2} + \frac{1}{2}\left(\frac{\partial g_E}{\partial p_r}\right)^2\frac{\partial^3 g_E}{\partial r^3}\right]$$

6PN gauge transformation: DJS to E

In the energy gauge, the Hamiltonian must be writable as a function of **only** $\varepsilon = \hat{H}_S^2 - 1$ and u, both of order $1/c^2$

$$\varepsilon = p^2(1-2u) + 2\left(2u^2 - 3u + 1\right) p_r^2 - 2u \implies p_r^2 = \frac{\varepsilon - p^2(1-2u) + 2u}{2 - 6u + 4u^2}$$

If we use this to replace p_r^2 in the energy gauge Hamiltonian, the dependence on p^2 should also disappear

⬇

Procedure to fix the ansatz parameters in the generating function:

- We use the ansatz-dependent transformation on the DJS $\hat{H}_{\rm eff}^{\rm 6PN}(p^2, p_r^2, u)$
- We replace p_r^2 with ε using the relation above, re-expanding at 6PN
- We find the expressions of the parameters that cancel out each term that still depends on p^2

This completely determines the transformation and allows us to obtain $\hat{H}_{\rm eff}^{\rm 6PN}(\varepsilon, u)$

General PN complementary parts in $\hat{H}^{\text{PS}*}_{\text{eff}}$

$$\left(\hat{H}^{\text{PS}*}_{\text{eff}}\right)^2 = (1+\varepsilon)\left[1 + \frac{A_{4\text{PM}}(\varepsilon, u)}{1-2u}\right]$$

PN completion:

$$A_{4\text{PM}} \to A^{6\text{PN}}_{4\text{PM}} \equiv u^2 a_{2\text{PM}} + u^3 a_{3\text{PM}} + u^4 a_{4\text{PM}} + \Delta^{\text{gen}}_{4\text{PN}} + \Delta^{\text{gen}}_{5\text{PN}} + \Delta^{\text{gen}}_{6\text{PN}}$$

where:

$$\Delta^{\text{gen}}_{4\text{PN}} = \sum_{n=2}^{5} \alpha_n\, u^n \varepsilon^{5-n} + \log u \sum_{n=0}^{1} \alpha_{n+4,\log u}\, u^{n+4} \epsilon^{1-n} + \alpha_{4,\log\varepsilon}\, u^4 \varepsilon \log\varepsilon\,,$$

$$\Delta^{\text{gen}}_{5\text{PN}} = \sum_{n=2}^{6} \beta_n\, u^n \varepsilon^{6-n} + \log u \sum_{n=0}^{2} \beta_{n+4,\log u}\, u^{n+4} \epsilon^{2-n} + \beta_{4,\log\varepsilon}\, u^4 \varepsilon^2 \log\varepsilon\,,$$

$$\Delta^{\text{gen}}_{6\text{PN}} = \sum_{n=2}^{7} \gamma_n\, u^n \varepsilon^{7-n} + \log u \sum_{n=0}^{3} \gamma_{n+4,\log u}\, u^{n+4} \epsilon^{3-n} + \gamma_{4,\log\varepsilon}\, u^4 \varepsilon^3 \log\varepsilon$$

Hamiltonian matching

$$\hat{H}^{6PN}_{\rm eff}(\varepsilon, u) = \hat{H}^{PS*}_{\rm eff}\left(\varepsilon, u, \Delta^{\rm gen}_{4PN}, \Delta^{\rm gen}_{5PN}, \Delta^{\rm gen}_{6PN}\right)$$

After the PN expansion (simultaneous expansion in ε and u) **the coefficients of each combination of (ε, u) must coincide**

PN complementary terms completely fixed

$$\Delta_{4PN} = u^2 \epsilon^3 \nu \left(-\frac{1027}{12} - \frac{147432 \,\mathrm{Log}[2]}{5} + \frac{1399437 \,\mathrm{Log}[3]}{160} + \frac{1953125 \,\mathrm{Log}[5]}{288}\right) +$$
$$u^4 \epsilon \nu \left(-\frac{27139}{75} + \frac{296 \,\mathrm{EulerGamma}}{15} - \frac{9766576 \,\mathrm{Log}[2]}{225} + \frac{1182681 \,\mathrm{Log}[3]}{100} + \frac{390625 \,\mathrm{Log}[5]}{36}\right) +$$
$$u^3 \epsilon^2 \nu \left(-\frac{78917}{300} - \frac{14099512 \,\mathrm{Log}[2]}{225} + \frac{14336271 \,\mathrm{Log}[3]}{800} + \frac{4296875 \,\mathrm{Log}[5]}{288}\right) +$$
$$u^5 \left(\left(-\frac{2051}{24} + \frac{205 \pi^2}{64}\right) \nu^2 + \frac{9 \nu^3}{4} + \right.$$
$$\left.\nu \left(-\frac{418499}{1800} + \frac{136 \,\mathrm{EulerGamma}}{3} + \frac{1571 \pi^2}{6144} - \frac{254936 \,\mathrm{Log}[2]}{25} + \frac{1061181 \,\mathrm{Log}[3]}{400} + \frac{390625 \,\mathrm{Log}[5]}{144}\right)\right) +$$
$$\left(\frac{68 u^5 \nu}{3} + \frac{148}{15} u^4 \epsilon \nu\right) \mathrm{Log}[u] - \frac{148}{15} u^4 \epsilon \nu \,\mathrm{Log}[\epsilon]$$

Agrees with the result of [Khalil et al. 2022]

Resulting $\Delta_{5\mathrm{PN}}$

$$
\begin{aligned}
& u^2 \epsilon^4 \nu \left(\frac{1027}{12} + \frac{147\,432\,\mathrm{Log}[2]}{5} - \frac{1\,399\,437\,\mathrm{Log}[3]}{160} - \frac{1\,953\,125\,\mathrm{Log}[5]}{288}\right) + \\
& u^3 \epsilon^3 \left(\nu \left(\frac{8\,564\,039}{14\,700} + \frac{8\,558\,268\,728\,\mathrm{Log}[2]}{33\,075} - \frac{1\,305\,576\,549\,\mathrm{Log}[3]}{39\,200} - \frac{3\,770\,796\,875\,\mathrm{Log}[5]}{42\,336}\right) + \right. \\
& \quad \left. \nu^2 \left(\frac{160\,124}{525} - \frac{4\,998\,308\,864\,\mathrm{Log}[2]}{11\,025} - \frac{45\,409\,167\,\mathrm{Log}[3]}{2450} + \frac{26\,171\,875\,\mathrm{Log}[5]}{126}\right)\right) + \\
& u^4 \epsilon^2 \left(\nu \left(\frac{11\,900\,489}{7350} - \frac{143\,\mathrm{EulerGamma}}{21} + \frac{17\,414\,088\,142\,\mathrm{Log}[2]}{33\,075} - \frac{814\,089\,123\,\mathrm{Log}[3]}{15\,680} - \frac{16\,285\,015\,625\,\mathrm{Log}[5]}{84\,672}\right) + \right. \\
& \quad \nu^2 \left(\frac{52\,969}{50} - \frac{148\,\mathrm{EulerGamma}}{5} - \frac{11\,429\,082\,232\,\mathrm{Log}[2]}{11\,025} - \frac{135\,831\,411\,\mathrm{Log}[3]}{2450} + \frac{10\,156\,250\,\mathrm{Log}[5]}{21}\right) + \\
& \quad \left. \left(-\frac{143\,\nu}{42} - \frac{74\,\nu^2}{5}\right) \mathrm{Log}[u] + \left(\frac{143\,\nu}{42} + \frac{74\,\nu^2}{5}\right) \mathrm{Log}[\epsilon]\right) + \\
& u^5 \epsilon \left(\left(\frac{4453}{24} - \frac{1763\,\pi^2}{256}\right) \nu^3 - \frac{93\,\nu^4}{16} + \nu \left(\frac{634\,391\,981}{294\,000} - \frac{3391\,\mathrm{EulerGamma}}{25} - \frac{420\,839\,\pi^2}{20\,480} + \frac{4\,319\,707\,969\,\mathrm{Log}[2]}{11\,025} - \right.\right. \\
& \quad \left. \frac{2\,549\,915\,397\,\mathrm{Log}[3]}{78\,400} - \frac{4\,174\,140\,625\,\mathrm{Log}[5]}{28\,224}\right) + \left(-\frac{3391\,\nu}{50} - \frac{1078\,\nu^2}{15}\right) \mathrm{Log}[u] + \\
& \quad \left. \nu^2 \left(\frac{12\,210\,733}{8400} - \frac{2156\,\mathrm{EulerGamma}}{15} + \frac{124\,633\,\pi^2}{20\,480} - \frac{2\,870\,039\,404\,\mathrm{Log}[2]}{3675} - \frac{131\,432\,139\,\mathrm{Log}[3]}{2450} + \frac{7\,812\,500\,\mathrm{Log}[5]}{21} + \frac{\bar{d}_5^{\nu^2}}{5}\right)\right) + \\
& u^6 \left(\left(\frac{299}{3} - \frac{287\,\pi^2}{64}\right) \nu^3 - \frac{11\,\nu^4}{8} + \nu \left(\frac{17\,809\,268}{55\,125} - \frac{21\,148\,\mathrm{EulerGamma}}{75} + \frac{2\,064\,989\,\pi^2}{30\,720} + \frac{3\,334\,690\,612\,\mathrm{Log}[2]}{33\,075} - \right.\right. \\
& \quad \left. \frac{72\,082\,791\,\mathrm{Log}[3]}{9800} - \frac{414\,953\,125\,\mathrm{Log}[5]}{10\,584}\right) + \left(-\frac{10\,574\,\nu}{75} - \frac{1288\,\nu^2}{15}\right) \mathrm{Log}[u] + \\
& \quad \left. \nu^2 \left(\frac{461\,789}{420} - \frac{2576\,\mathrm{EulerGamma}}{15} - \frac{47\,743\,\pi^2}{5120} - \frac{2\,179\,378\,864\,\mathrm{Log}[2]}{11\,025} - \frac{8\,270\,019\,\mathrm{Log}[3]}{490} + \frac{12\,109\,375\,\mathrm{Log}[5]}{126} + a_6^{\nu^2} + \frac{\bar{d}_5^{\nu^2}}{5}\right)\right)
\end{aligned}
$$

Resulting $\Delta_{5\text{PN}}$

$$
\begin{aligned}
& u^2 \epsilon^4 \nu \left(\frac{1027}{12} + \frac{147432\,\mathrm{Log}[2]}{5} - \frac{1399437\,\mathrm{Log}[3]}{160} - \frac{1953125\,\mathrm{Log}[5]}{288} \right) + \\
& u^3 \epsilon^3 \left(\nu \left(\frac{8564039}{14700} + \frac{8558268728\,\mathrm{Log}[2]}{33075} - \frac{1305576549\,\mathrm{Log}[3]}{39200} - \frac{3770796875\,\mathrm{Log}[5]}{42336} \right) + \right. \\
& \quad \left. \nu^2 \left(\frac{160124}{525} - \frac{4998308864\,\mathrm{Log}[2]}{11025} - \frac{45409167\,\mathrm{Log}[3]}{2450} + \frac{26171875\,\mathrm{Log}[5]}{126} \right) \right) + \\
& u^4 \epsilon^2 \left(\nu \left(\frac{11900489}{7350} - \frac{143\,\mathrm{EulerGamma}}{21} + \frac{17414088142\,\mathrm{Log}[2]}{33075} - \frac{814089123\,\mathrm{Log}[3]}{15680} - \frac{16285015625\,\mathrm{Log}[5]}{84672} \right) + \right. \\
& \quad \nu^2 \left(\frac{52969}{50} - \frac{148\,\mathrm{EulerGamma}}{5} - \frac{11429082232\,\mathrm{Log}[2]}{11025} - \frac{135831411\,\mathrm{Log}[3]}{2450} + \frac{10156250\,\mathrm{Log}[5]}{21} \right) + \\
& \quad \left. \left(-\frac{143\,\nu}{42} - \frac{74\,\nu^2}{5} \right) \mathrm{Log}[u] + \left(\frac{143\,\nu}{42} + \frac{74\,\nu^2}{5} \right) \mathrm{Log}[\epsilon] \right) + \\
& u^5 \epsilon \left(\left(\frac{4453}{24} - \frac{1763\,\pi^2}{256} \right) \nu^3 - \frac{93\,\nu^4}{16} + \nu \left(\frac{634391981}{294000} - \frac{3391\,\mathrm{EulerGamma}}{25} - \frac{420839\,\pi^2}{20480} + \frac{4319707969\,\mathrm{Log}[2]}{11025} - \right. \right. \\
& \quad \left. \frac{2549915397\,\mathrm{Log}[3]}{78400} - \frac{4174140625\,\mathrm{Log}[5]}{28224} \right) + \left(-\frac{3391\,\nu}{50} - \frac{1078\,\nu^2}{15} \right) \mathrm{Log}[u] + \\
& \quad \left. \nu^2 \left(\frac{12210733}{8400} - \frac{2156\,\mathrm{EulerGamma}}{15} + \frac{124633\,\pi^2}{20480} - \frac{2870039404\,\mathrm{Log}[2]}{3675} - \frac{131432139\,\mathrm{Log}[3]}{2450} + \frac{7812500\,\mathrm{Log}[5]}{21} + \boxed{\frac{\bar{d}_5^{\nu^2}}{5}} \right) \right) + \\
& u^6 \left(\left(\frac{299}{3} - \frac{287\,\pi^2}{64} \right) \nu^3 - \frac{11\,\nu^4}{8} + \nu \left(\frac{17809268}{55125} - \frac{21148\,\mathrm{EulerGamma}}{75} + \frac{2064989\,\pi^2}{30720} + \frac{3334690612\,\mathrm{Log}[2]}{33075} - \right. \right. \\
& \quad \left. \frac{72082791\,\mathrm{Log}[3]}{9800} - \frac{414953125\,\mathrm{Log}[5]}{10584} \right) + \left(-\frac{10574\,\nu}{75} - \frac{1288\,\nu^2}{15} \right) \mathrm{Log}[u] + \\
& \quad \left. \nu^2 \left(\frac{461789}{420} - \frac{2576\,\mathrm{EulerGamma}}{15} - \frac{47743\,\pi^2}{5120} - \frac{2179378864\,\mathrm{Log}[2]}{11025} - \frac{8270019\,\mathrm{Log}[3]}{490} + \frac{12109375\,\mathrm{Log}[5]}{126} + \boxed{a_6^{\nu^2} + \frac{\bar{d}_5^{\nu^2}}{5}} \right) \right)
\end{aligned}
$$

2 unknown parameters of the 5PN Hamiltonian

Resulting $\Delta_{6\text{PN}}$

$$u^2 \epsilon^5 \nu \left(-\frac{1027}{12} - \frac{147432\,\mathrm{Log}[2]}{5} + \frac{1399437\,\mathrm{Log}[3]}{160} + \frac{1953125\,\mathrm{Log}[5]}{288}\right) + u^3 \epsilon^4 \left(\nu^3 \left(-\frac{154862}{189} + \frac{57604236136064\,\mathrm{Log}[2]}{893025} + \frac{1163064811149\,\mathrm{Log}[3]}{39200} - \frac{73366198046875\,\mathrm{Log}[5]}{3429216} - \frac{7709596970957\,\mathrm{Log}[7]}{349920}\right) +\right.$$

$$\nu \left(-\frac{350720677}{297675} + \frac{127828149103496\,\mathrm{Log}[2]}{8037225} + \frac{1956626615421\,\mathrm{Log}[3]}{313600} - \frac{63072125155625\,\mathrm{Log}[5]}{9144576} - \frac{29247366220639\,\mathrm{Log}[7]}{8398080}\right) +$$

$$\left.\nu^2 \left(-\frac{9692209}{3150} - \frac{175841085685856\,\mathrm{Log}[2]}{2679075} - \frac{4854840976431\,\mathrm{Log}[3]}{156800} + \frac{363599976015625\,\mathrm{Log}[5]}{13716864} + \frac{26506549233199\,\mathrm{Log}[7]}{1399680}\right)\right) +$$

$$u^4 \epsilon^3 \left(\nu^3 \left(-\frac{95339269}{30240} + 37\,\mathrm{EulerGamma} + \frac{164772232514312\,\mathrm{Log}[2]}{893025} + \frac{106829150283279\,\mathrm{Log}[3]}{1254400} - \frac{6635329272265625\,\mathrm{Log}[5]}{109734912} - \frac{714307308582007\,\mathrm{Log}[7]}{11197440}\right) +\right.$$

$$\nu \left(-\frac{240657345013}{76204800} + \frac{5219\,\mathrm{EulerGamma}}{180} + \frac{294862220042503\,\mathrm{Log}[2]}{6429780} + \frac{361386357950943\,\mathrm{Log}[3]}{20070400} - \frac{1288508836466875\,\mathrm{Log}[5]}{65028096} - \frac{5434377439512973\,\mathrm{Log}[7]}{537477120}\right) +$$

$$\nu^2 \left(-\frac{1416824663}{120960} + \frac{1233\,\mathrm{EulerGamma}}{70} - \frac{1009842624606643\,\mathrm{Log}[2]}{5358150} - \frac{894724321337613\,\mathrm{Log}[3]}{10035200} + \frac{66372895640546875\,\mathrm{Log}[5]}{877879296} + \frac{4921844662861693\,\mathrm{Log}[7]}{89579520}\right) +$$

$$\left.\left(\frac{5219\,\nu}{360} + \frac{1233\,\nu^2}{140} + \frac{37\,\nu^3}{2}\right)\mathrm{Log}[u] + \left(-\frac{5219\,\nu}{360} - \frac{1233\,\nu^2}{140} - \frac{37\,\nu^3}{2}\right)\mathrm{Log}[\epsilon]\right) + u^5 \epsilon^2 \left(\left(-\frac{36401}{128} + \frac{5535\,\pi^2}{512}\right)\nu^4 + \frac{657\,\nu^5}{64} +\right.$$

$$\nu \left(-\frac{1137294330443}{296352000} + \frac{75421\,\mathrm{EulerGamma}}{2940} + \frac{52420636459\,\pi^2}{440401920} + \frac{44384392527823\,\mathrm{Log}[2]}{893025} + \frac{393968239260003\,\mathrm{Log}[3]}{20070400} - \frac{4170604493078125\,\mathrm{Log}[5]}{195084288} - \frac{663055887182801\,\mathrm{Log}[7]}{59719680}\right) +$$

$$\left(\frac{75421\,\nu}{5880} + \frac{293203\,\nu^2}{2100} + \frac{3743\,\nu^3}{30}\right)\mathrm{Log}[u] +$$

$$\nu^3 \left(-\frac{924170593}{176400} + \frac{3743\,\mathrm{EulerGamma}}{15} - \frac{3302207\,\pi^2}{245760} + \frac{790301920439\,\mathrm{Log}[2]}{3969} + \frac{115626711539763\,\mathrm{Log}[3]}{1254400} - \frac{783616805078125\,\mathrm{Log}[5]}{12192768} - \frac{17370815437255\,\mathrm{Log}[7]}{248832} - \frac{2\,\bar{d}_5^{\nu^2}}{5}\right) +$$

$$\left.\nu^2 \left(-\frac{28907281327}{1411200} + \frac{293203\,\mathrm{EulerGamma}}{1050} + \frac{11129071\,\pi^2}{573440} - \frac{60705182892502\,\mathrm{Log}[2]}{297675} - \frac{19819428299433\,\mathrm{Log}[3]}{204800} + \frac{7902284224484375\,\mathrm{Log}[5]}{97542144} + \frac{600017096471681\,\mathrm{Log}[7]}{9953280} + \frac{3\,q_{45}^{\nu^2}}{35} - \frac{\bar{d}_5^{\nu^2}}{5}\right)\right) +$$

$$u^7 \left(\left(-\frac{2845}{32} + \frac{615\,\pi^2}{128}\right)\nu^4 + \frac{13\,\nu^5}{16} + \nu \left(-\frac{833629606433}{133358400} - \frac{1569433\,\mathrm{EulerGamma}}{19845} + \frac{186839270477\,\pi^2}{198180864} - \frac{5556443\,\pi^4}{524288} + \frac{7258348963574\,\mathrm{Log}[2]}{1607445} +\right.\right.$$

$$\left.\frac{365447489481\,\mathrm{Log}[3]}{200704} - \frac{280433823078125\,\mathrm{Log}[5]}{146313216} - \frac{28250793542167\,\mathrm{Log}[7]}{26873856}\right) + \left(-\frac{1569433\,\nu}{39690} + \frac{30431\,\nu^2}{35} + 174\,\nu^3\right)\mathrm{Log}[u] +$$

$$\nu^3 \left(-\frac{1005117919}{317520} + 348\,\mathrm{EulerGamma} + \frac{593483\,\pi^2}{12288} + \frac{3241904436308\,\mathrm{Log}[2]}{178605} + \frac{527833234161\,\mathrm{Log}[3]}{62720} - \frac{154715013671875\,\mathrm{Log}[5]}{27433728} - \frac{3667941108265\,\mathrm{Log}[7]}{559872} - \frac{5\,a_6^{\nu^2}}{2} + a_7^{\nu^3} - \frac{\bar{d}_5^{\nu^2}}{2}\right) +$$

$$\left.\nu^2 \left(-\frac{1771607989}{352800} + \frac{60862\,\mathrm{EulerGamma}}{35} - \frac{17131\,\pi^2}{224} - \frac{49767352960384\,\mathrm{Log}[2]}{2679075} - \frac{22250579674419\,\mathrm{Log}[3]}{2508800} + \frac{1574692344453125\,\mathrm{Log}[5]}{219469824} + \frac{25509976554727\,\mathrm{Log}[7]}{4478976} + \frac{a_6^{\nu^2}}{6} + a_7^{\nu^2} + \frac{q_{45}^{\nu^2}}{14} - \frac{5\,\bar{d}_5^{\nu^2}}{6} + \frac{\bar{d}_6^{\nu^2}}{6}\right)\right) +$$

$$u^6 \epsilon \left(\left(-\frac{26099}{96} + \frac{205\,\pi^2}{16}\right)\nu^4 + \frac{73\,\nu^5}{16} + \nu \left(-\frac{206675932697}{33339600} - \frac{4392103\,\mathrm{EulerGamma}}{33075} + \frac{114018862003\,\pi^2}{165150720} + \frac{45303\,\pi^4}{524288} + \frac{195261895301761\,\mathrm{Log}[2]}{8037225} +\right.\right.$$

$$\left.\frac{4856923744821\,\mathrm{Log}[3]}{501760} - \frac{760035683163125\,\mathrm{Log}[5]}{73156608} - \frac{371246606762059\,\mathrm{Log}[7]}{67184640}\right) + \left(-\frac{4392103\,\nu}{66150} + \frac{1034546\,\nu^2}{1575} + \frac{3964\,\nu^3}{15}\right)\mathrm{Log}[u] +$$

$$\nu^3 \left(-\frac{2190504443}{396900} + \frac{7928\,\mathrm{EulerGamma}}{15} + \frac{552001\,\pi^2}{30720} + \frac{86858343124312\,\mathrm{Log}[2]}{893025} + \frac{7071646351059\,\mathrm{Log}[3]}{156800} - \frac{422975045703125\,\mathrm{Log}[5]}{13716864} - \frac{48430663916299\,\mathrm{Log}[7]}{1399680} - \frac{5\,a_6^{\nu^2}}{2} - \frac{9\,\bar{d}_5^{\nu^2}}{10}\right) +$$

$$\left.\nu^2 \left(-\frac{18339327269}{1058400} + \frac{2069092\,\mathrm{EulerGamma}}{1575} - \frac{865969\,\pi^2}{28672} - \frac{267134124074056\,\mathrm{Log}[2]}{2679075} - \frac{11905972315983\,\mathrm{Log}[3]}{250880} + \frac{4291995809078125\,\mathrm{Log}[5]}{109734912} + \frac{335615985925339\,\mathrm{Log}[7]}{11197440} + \frac{a_6^{\nu^2}}{6} + \frac{11\,q_{45}^{\nu^2}}{70} - \frac{5\,\bar{d}_5^{\nu^2}}{6} + \frac{\bar{d}_6^{\nu^2}}{6}\right)\right)$$

Resulting $\Delta_{6\mathrm{PN}}$

$$u^2 \epsilon^5 \nu \left(-\frac{1027}{12}-\frac{147432\,\text{Log}[2]}{5}+\frac{1399437\,\text{Log}[3]}{160}+\frac{1953125\,\text{Log}[5]}{288}\right)+u^3 \epsilon^4 \left(\nu^3\left(-\frac{154862}{189}+\frac{57604236136064\,\text{Log}[2]}{893025}+\frac{1163064811149\,\text{Log}[3]}{39200}-\frac{73366198046875\,\text{Log}[5]}{3429216}-\frac{7709596970957\,\text{Log}[7]}{349920}\right)+\right.$$

$$\nu\left(-\frac{350720677}{297675}+\frac{127828149103496\,\text{Log}[2]}{8037225}+\frac{1956626615421\,\text{Log}[3]}{313600}-\frac{63072125155625\,\text{Log}[5]}{9144576}-\frac{29247366220639\,\text{Log}[7]}{8398080}\right)+$$

$$\left.\nu^2\left(-\frac{9692209}{3150}-\frac{175841085685856\,\text{Log}[2]}{2679075}-\frac{4854840976431\,\text{Log}[3]}{156800}+\frac{363599976015625\,\text{Log}[5]}{13716864}+\frac{26506549233199\,\text{Log}[7]}{1399680}\right)\right)+$$

$$u^4 \epsilon^3 \left(\nu^3\left(-\frac{95339269}{30240}+37\,\text{EulerGamma}+\frac{164772232514312\,\text{Log}[2]}{893025}+\frac{106829150283279\,\text{Log}[3]}{1254400}-\frac{6635329272265625\,\text{Log}[5]}{109734912}-\frac{714307308582007\,\text{Log}[7]}{11197440}\right)+\right.$$

$$\nu\left(-\frac{240657345013}{76204800}+\frac{5219\,\text{EulerGamma}}{180}+\frac{294862220042503\,\text{Log}[2]}{6429780}+\frac{361386357950943\,\text{Log}[3]}{20070400}-\frac{1288508836466875\,\text{Log}[5]}{65028096}-\frac{5434377439512973\,\text{Log}[7]}{537477120}\right)+$$

$$\nu^2\left(-\frac{1416824663}{120960}+\frac{1233\,\text{EulerGamma}}{70}-\frac{1009842624606643\,\text{Log}[2]}{5358150}-\frac{894724321337613\,\text{Log}[3]}{10035200}+\frac{66372895640546875\,\text{Log}[5]}{877879296}+\frac{4921844662861693\,\text{Log}[7]}{89579520}\right)+$$

$$\left.\left(\frac{5219\nu}{360}+\frac{1233\nu^2}{140}+\frac{37\nu^3}{2}\right)\text{Log}[u]+\left(-\frac{5219\nu}{360}-\frac{1233\nu^2}{140}-\frac{37\nu^3}{2}\right)\text{Log}[\epsilon]\right)+u^5\epsilon^2\left(\left(-\frac{36401}{128}+\frac{5535\pi^2}{512}\right)\nu^4+\frac{657\nu^5}{64}+\right.$$

$$\nu\left(-\frac{1137294330443}{296352000}+\frac{75421\,\text{EulerGamma}}{2940}+\frac{52420636459\,\pi^2}{440401920}+\frac{44384392527823\,\text{Log}[2]}{893025}+\frac{393968239260003\,\text{Log}[3]}{20070400}-\frac{4170604493078125\,\text{Log}[5]}{195084288}-\frac{663055887182801\,\text{Log}[7]}{59719680}\right)+$$

$$\left(\frac{75421\nu}{5880}+\frac{293203\nu^2}{2100}+\frac{3743\nu^3}{30}\right)\text{Log}[u]+$$

$$\nu^3\left(-\frac{924170593}{176400}+\frac{3743\,\text{EulerGamma}}{15}-\frac{3302207\pi^2}{245760}+\frac{790301920439\,\text{Log}[2]}{3969}+\frac{115626711539763\,\text{Log}[3]}{1254400}-\frac{783616805078125\,\text{Log}[5]}{12192768}-\frac{17370815437255\,\text{Log}[7]}{248832}-\boxed{\frac{2\bar{d}_5^{\nu^2}}{5}}\right)+$$

$$\left.\nu^2\left(-\frac{28907281327}{1411200}+\frac{293203\,\text{EulerGamma}}{1050}+\frac{11129071\pi^2}{573440}-\frac{60705182892502\,\text{Log}[2]}{297675}-\frac{19819428299433\,\text{Log}[3]}{204800}+\frac{7902284224484375\,\text{Log}[5]}{97542144}+\frac{600017096471681\,\text{Log}[7]}{9953280}+\boxed{\frac{3q_{45}^{\nu^2}}{35}-\frac{\bar{d}_5^{\nu^2}}{5}}\right)\right)+$$

$$u^7\left(\left(-\frac{2845}{32}+\frac{615\pi^2}{128}\right)\nu^4+\frac{13\nu^5}{16}+\nu\left(-\frac{833629606433}{133358400}-\frac{1569433\,\text{EulerGamma}}{19845}+\frac{186839270477\pi^2}{198180864}-\frac{5556443\pi^4}{524288}+\frac{7258348963574\,\text{Log}[2]}{1607445}+\right.\right.$$

$$\left.\frac{365447489481\,\text{Log}[3]}{200704}-\frac{280433823078125\,\text{Log}[5]}{146313216}-\frac{28250793542167\,\text{Log}[7]}{26873856}\right)+\left(-\frac{1569433\nu}{39690}+\frac{30431\nu^2}{35}+174\nu^3\right)\text{Log}[u]+$$

$$\nu^3\left(-\frac{1005117919}{317520}+348\,\text{EulerGamma}+\frac{593483\pi^2}{12288}+\frac{3241904436308\,\text{Log}[2]}{178605}+\frac{527833234161\,\text{Log}[3]}{62720}-\frac{154715013671875\,\text{Log}[5]}{27433728}-\frac{3667941108265\,\text{Log}[7]}{559872}-\boxed{\frac{5a_6^{\nu^2}}{2}+a_7^{\nu^3}-\frac{\bar{d}_5^{\nu^2}}{2}}\right)+$$

$$\left.\nu^2\left(-\frac{1771607989}{352800}+\frac{60862\,\text{EulerGamma}}{35}-\frac{17131\pi^2}{224}-\frac{49767352960384\,\text{Log}[2]}{2679075}-\frac{22250579674419\,\text{Log}[3]}{2508800}+\frac{1574692344453125\,\text{Log}[5]}{219469824}+\frac{25509976554727\,\text{Log}[7]}{4478976}+\boxed{\frac{a_6^{\nu^2}}{6}+a_7^{\nu^2}+\frac{q_{45}^{\nu^2}}{14}-\frac{5\bar{d}_5^{\nu^2}}{6}+\frac{\bar{d}_6^{\nu^2}}{6}}\right)\right)+$$

$$u^6\epsilon\left(\left(-\frac{26099}{96}+\frac{205\pi^2}{16}\right)\nu^4+\frac{73\nu^5}{16}+\nu\left(-\frac{206675932697}{33339600}-\frac{4392103\,\text{EulerGamma}}{33075}+\frac{114018862003\pi^2}{165150720}+\frac{45303\pi^4}{524288}+\frac{195261895301761\,\text{Log}[2]}{8037225}+\right.\right.$$

$$\left.\frac{4856923744821\,\text{Log}[3]}{501760}-\frac{760035683163125\,\text{Log}[5]}{73156608}-\frac{371246606762059\,\text{Log}[7]}{67184640}\right)+\left(-\frac{4392103\nu}{66150}+\frac{1034546\nu^2}{1575}+\frac{3964\nu^3}{15}\right)\text{Log}[u]+$$

$$\nu^3\left(-\frac{2190504443}{396900}+\frac{7928\,\text{EulerGamma}}{15}+\frac{552001\pi^2}{30720}+\frac{86858343124312\,\text{Log}[2]}{893025}+\frac{7071646351059\,\text{Log}[3]}{156800}-\frac{422975045703125\,\text{Log}[5]}{13716864}-\frac{48430663916299\,\text{Log}[7]}{1399680}-\boxed{\frac{5a_6^{\nu^2}}{2}-\frac{9\bar{d}_5^{\nu^2}}{10}}\right)+$$

$$\left.\nu^2\left(-\frac{18339327269}{1058400}+\frac{2069092\,\text{EulerGamma}}{1575}-\frac{865969\pi^2}{28672}-\frac{267134124074056\,\text{Log}[2]}{2679075}-\frac{11905972315983\,\text{Log}[3]}{250880}+\frac{4291995809078125\,\text{Log}[5]}{109734912}+\frac{335615985925339\,\text{Log}[7]}{11197440}+\boxed{\frac{a_6^{\nu^2}}{6}+\frac{11q_{45}^{\nu^2}}{70}-\frac{5\bar{d}_5^{\nu^2}}{6}+\frac{\bar{d}_6^{\nu^2}}{6}}\right)\right)$$

2+4 unknown parameters of the 6PN Hamiltonian

Is there a better PM structure for $\hat{H}_{\text{eff}}$?

Issue with the PS Hamiltonian:

$$\left(\hat{H}_{\text{eff}}^{\text{PS}}\right)^2 = (1-2u)\left[1 + p_\varphi^2 u^2 + (1-2u)\,p_r^2 + Q_{\text{PM}}\right]$$

→ The effective horizon is locked to its Schwarzschild value, $u_{\text{EH}} = 1/2$

Issue with the PS* Hamiltonian:

$$\left(\hat{H}_{\text{eff}}^{\text{PS}*}\right)^2 \equiv \left(1 - 2u + A_{\text{PM}}\right)\left(1 + p_\varphi^2 u^2 + (1-2u)\,p_r^2\right)$$

OK effective horizon but in the circular limit we have singularities for $u \to 1/2$ (i.e. $\hat{H}_S \to 0$) in every PM coefficient $a_{n\text{PM}}$

For instance:

$$\hat{H}_S^2 \equiv (1-2u)\left[1 + p_\varphi^2 u^2 + (1-2u)\,p_r^2\right]$$

$$a_{2\text{PM}}^{\text{circ}}\left(\hat{H}_S\right) = \frac{3\left(5\hat{H}_S^2 - 1\right)\left[\sqrt{2\nu(\hat{H}_S - 1) + 1} - 1\right]}{2\hat{H}_S^2\sqrt{2\nu(\hat{H}_S - 1) + 1}} \xrightarrow[\hat{H}_S \to 0]{} \infty$$

BONUS SLIDES

Explicit $a_{3\mathrm{PM}}$ and $a_{4\mathrm{PM}}$

```
a3PM /. Hs → γ // Simplify
```

$$\frac{8\gamma\left(-25+11\gamma^2+14\gamma^4\right)\nu+9\left(5-31\gamma^2+30\gamma^4\right)\sqrt{1+2(-1+\gamma)\nu}-9\left(5-31\gamma^2+30\gamma^4\right)(1+2(-1+\gamma)\nu)+48\sqrt{-1+\gamma^2}\left(-3-12\gamma^2+4\gamma^4\right)\nu\,\mathrm{ArcSinh}\left[\frac{\sqrt{-1+\gamma}}{\sqrt{2}}\right]}{6\gamma^2\left(-1+\gamma^2\right)(1+2(-1+\gamma)\nu)}$$

```
a4PM /. Hs → γ // Simplify
```

$$\frac{7\left(169+380\gamma^2\right)\nu\,\mathrm{EllipticE}\left[\frac{-1+\gamma}{1+\gamma}\right]^2}{8(-1+\gamma)\gamma^2(1+2(-1+\gamma)\nu)^{3/2}}-\frac{\left(1183+2929\gamma+2660\gamma^2+1200\gamma^3\right)\nu\,\mathrm{EllipticE}\left[\frac{-1+\gamma}{1+\gamma}\right]\times\mathrm{EllipticK}\left[\frac{-1+\gamma}{1+\gamma}\right]}{4\gamma^2\left(-1+\gamma^2\right)(1+2(-1+\gamma)\nu)^{3/2}}+$$

$$\frac{\left(834+2095\gamma+1200\gamma^2\right)\nu\,\mathrm{EllipticK}\left[\frac{-1+\gamma}{1+\gamma}\right]^2}{4\gamma^2\left(-1+\gamma^2\right)(1+2(-1+\gamma)\nu)^{3/2}}+\frac{2\left(-852-283\gamma^2-140\gamma^4+75\gamma^6\right)\nu\,\mathrm{Log}[\gamma]}{3\gamma\left(-1+\gamma^2\right)(1+2(-1+\gamma)\nu)^{3/2}}+\frac{1}{144\gamma^9\left(-1+\gamma^2\right)^2(1+2(-1+\gamma)\nu)^{3/2}}$$

$$\Big(-648\gamma^5+10035\gamma^7-50913\gamma^9+85833\gamma^{11}-46035\gamma^{13}-45\nu+207\gamma^2\nu-1471\gamma^4\nu+1296\gamma^5\nu+12053\gamma^6\nu-57575\gamma^7\nu-288\pi^2\gamma^7\nu+83864\gamma^8\nu+1824\pi^2\gamma^8\nu+57679\gamma^9\nu-2520\pi^2\gamma^9\nu-$$
$$104912\gamma^{10}\nu-2208\pi^2\gamma^{10}\nu-164681\gamma^{11}\nu+6624\pi^2\gamma^{11}\nu+167488\gamma^{12}\nu-1056\pi^2\gamma^{12}\nu+104529\gamma^{13}\nu-5136\pi^2\gamma^{13}\nu-106640\gamma^{14}\nu+1440\pi^2\gamma^{14}\nu+4320\gamma^{15}\nu+1920\pi^2\gamma^{15}\nu+$$
$$3600\gamma^{16}\nu-600\pi^2\gamma^{17}\nu+648\gamma^5\sqrt{1+2(-1+\gamma)\nu}-10035\gamma^7\sqrt{1+2(-1+\gamma)\nu}+50913\gamma^9\sqrt{1+2(-1+\gamma)\nu}-85833\gamma^{11}\sqrt{1+2(-1+\gamma)\nu}+46035\gamma^{13}\sqrt{1+2(-1+\gamma)\nu}-$$
$$648\gamma^5\nu\sqrt{1+2(-1+\gamma)\nu}+648\gamma^6\nu\sqrt{1+2(-1+\gamma)\nu}+11970\gamma^7\nu\sqrt{1+2(-1+\gamma)\nu}-35970\gamma^8\nu\sqrt{1+2(-1+\gamma)\nu}-68454\gamma^9\nu\sqrt{1+2(-1+\gamma)\nu}+112614\gamma^{10}\nu\sqrt{1+2(-1+\gamma)\nu}+$$
$$121446\gamma^{11}\nu\sqrt{1+2(-1+\gamma)\nu}-122790\gamma^{12}\nu\sqrt{1+2(-1+\gamma)\nu}-67770\gamma^{13}\nu\sqrt{1+2(-1+\gamma)\nu}+48954\gamma^{14}\nu\sqrt{1+2(-1+\gamma)\nu}+8640\gamma^7\sqrt{-1+\gamma^2}\,\nu\sqrt{1+2(-1+\gamma)\nu}\,\mathrm{Log}[2]+$$
$$22464\gamma^9\sqrt{-1+\gamma^2}\,\nu\sqrt{1+2(-1+\gamma)\nu}\,\mathrm{Log}[2]-59904\gamma^{11}\sqrt{-1+\gamma^2}\,\nu\sqrt{1+2(-1+\gamma)\nu}\,\mathrm{Log}[2]+16128\gamma^{13}\sqrt{-1+\gamma^2}\,\nu\sqrt{1+2(-1+\gamma)\nu}\,\mathrm{Log}[2]-$$
$$36\gamma^7\left(-1+\gamma^2\right)^2\left(-42-228\gamma-51\gamma^2-180\gamma^3-100\gamma^4+25\gamma^6\right)\nu\,\mathrm{Log}\left[\frac{1+\gamma}{2}\right]^2+24\gamma^7\left(-1151+3336\gamma-1997\gamma^2-2424\gamma^3+2809\gamma^4-360\gamma^5+129\gamma^6-552\gamma^7+210\gamma^8\right)\nu\,\mathrm{Log}\left[\frac{1}{2}\sqrt{-1+\gamma^2}\right]-$$
$$48\gamma^7\left(-1+\gamma^2\right)\nu\,\mathrm{Log}\left[\frac{1+\gamma}{2}\right]\left(-152-2520\gamma-136\gamma^2-739\gamma^3-612\gamma^4-416\gamma^5+75\gamma^7+3\left(5-76\gamma+145\gamma^2+16\gamma^3-185\gamma^4+60\gamma^5+35\gamma^6\right)\mathrm{Log}\left[\frac{1}{2}\sqrt{-1+\gamma^2}\right]\right)\Big)-$$

$$\frac{8\left(15+39\gamma^2-104\gamma^4+28\gamma^6\right)\nu\,\mathrm{Log}\left[\sqrt{-1+\gamma}+\sqrt{1+\gamma}\right]}{\gamma^2\left(-1+\gamma^2\right)^{3/2}(1+2(-1+\gamma)\nu)}+$$

$$\frac{\left(-3+2\gamma^2\right)\nu\left(-1151+3504\gamma-3148\gamma^2+576\gamma^3-339\gamma^4+720\gamma^5-210\gamma^6+\left(96+624\gamma^2-720\gamma^4\right)\mathrm{Log}\left[\frac{1+\gamma}{2}\right]+6\left(-11+41\gamma^2-65\gamma^4+35\gamma^6\right)\mathrm{Log}\left[\frac{1}{2}\sqrt{-1+\gamma^2}\right]\right)\mathrm{Log}\left[\gamma+\sqrt{-1+\gamma}\sqrt{1+\gamma}\right]}{12\gamma\left(-1+\gamma^2\right)^{5/2}(1+2(-1+\gamma)\nu)^{3/2}}-$$

$$\frac{\left(3-2\gamma^2\right)^2\left(11-30\gamma^2+35\gamma^4\right)\nu\,\mathrm{Log}\left[\gamma+\sqrt{-1+\gamma}\sqrt{1+\gamma}\right]^2}{8\left(-1+\gamma^2\right)^3(1+2(-1+\gamma)\nu)^{3/2}}-\frac{(1+\gamma)\left(12+64\gamma+65\gamma^2-5\gamma^3-25\gamma^4+25\gamma^5\right)\nu\,\mathrm{PolyLog}\left[2,\frac{1-\gamma}{2}\right]}{2\gamma^2(1+2(-1+\gamma)\nu)^{3/2}}+$$

$$\frac{2\left(12-57\gamma+82\gamma^2-7\gamma^3-60\gamma^4+30\gamma^5\right)\nu\,\mathrm{PolyLog}\left[2,-\sqrt{\frac{-1+\gamma}{1+\gamma}}\right]}{\gamma\left(-1+\gamma^2\right)^{3/2}(1+2(-1+\gamma)\nu)^{3/2}}-$$

$$\frac{2\left(12-57\gamma+82\gamma^2-7\gamma^3-60\gamma^4+30\gamma^5\right)\nu\,\mathrm{PolyLog}\left[2,\sqrt{\frac{-1+\gamma}{1+\gamma}}\right]}{\gamma\left(-1+\gamma^2\right)^{3/2}(1+2(-1+\gamma)\nu)^{3/2}}+\frac{\left(20+111\gamma^2+30\gamma^4-25\gamma^6\right)\nu\,\mathrm{PolyLog}\left[2,\frac{1-\gamma}{1+\gamma}\right]}{\gamma^2(1+2(-1+\gamma)\nu)^{3/2}}$$