Non-perturbative Heavy Quark Effective Theory and B-physics

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Università di Roma "La Sapienza", June 1st 2011

Outline

- Introduction & motivations
- Non-perturbative Heavy Quark Effective Theory (HQET)
- Example: M_b in HQET including $O(1/m_{\rm h})$ corrections
- Computation of the $B_{d,s}$ meson B-parameters
- Non-perturbative renormalization of $\Delta B = 2$ HQET operators
- $\Delta B = 2$ transitions in general SUSY models

CP-violation in the SM

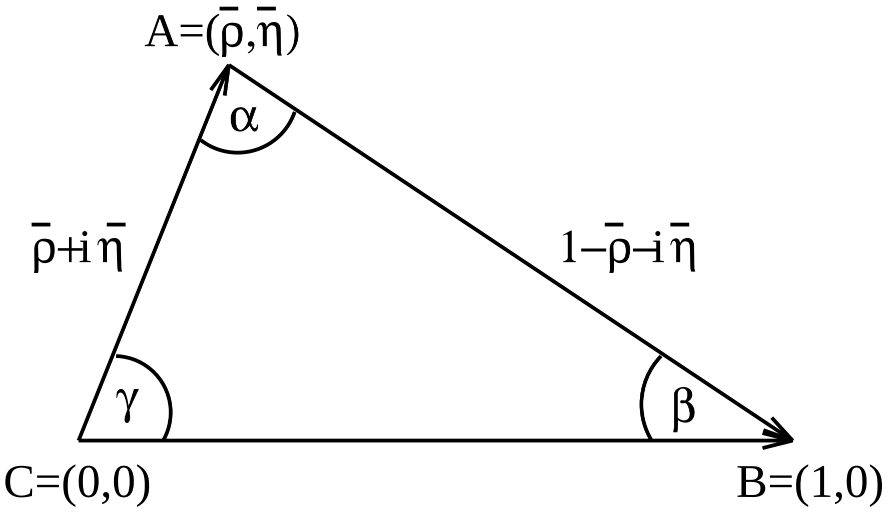
- CP-violation arises from the coupling of quark currents to charged gauge bosons:

$$W_\mu^+ \; (\bar{u} \;\; \bar{c} \;\; \bar{t}) \; \frac{1}{2}\gamma_\mu(1-\gamma_5) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + h.c.$$

- Unitarity of the CKM matrix leaves four free real parameters. In Wolfenstein parametrisation:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- CP-violation $\Leftrightarrow \eta \neq 0$. One can express the unitarity constraint on the CP-violation phase through the so-called unitarity triangle:



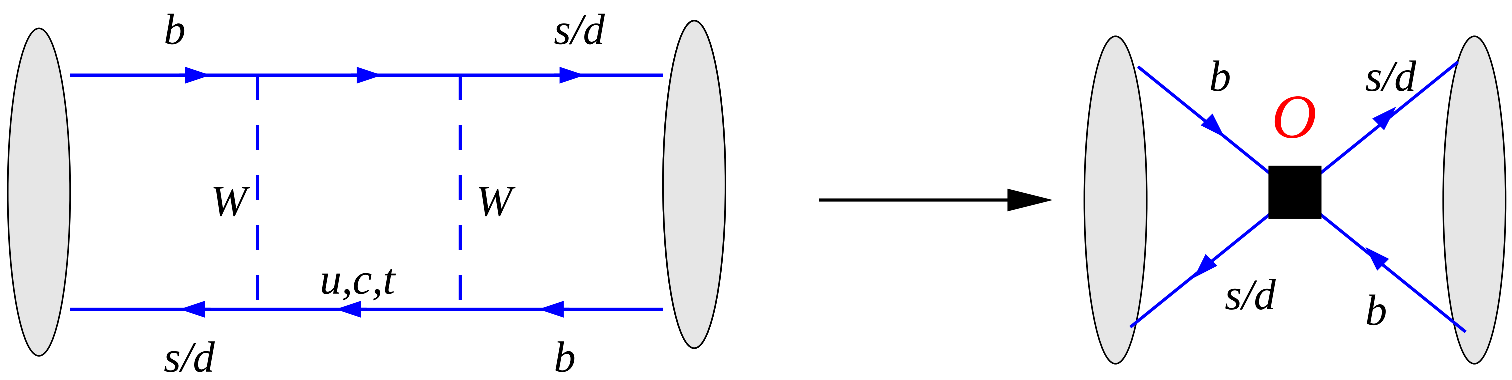
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\bar\rho = \rho(1-\lambda^2/2)$$

$$\bar\eta = \eta(1-\lambda^2/2)$$

Weak effective Hamiltonian $\mathcal{H}_{\rm eff}$

- Use OPE to integrate out high-energy electroweak physics



$$\mathcal{A}(i \to f) \approx \langle f|\mathcal{H}_{\rm eff}|i\rangle \qquad \mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\sum_k V^k_{CKM} C_k(\mu/M_W) Q_k(\mu)$$

1. $\mathcal{A}(i \to f)$ measured experimentally.

2. Wilson coefficients $C_k(\mu/M_W)$ $(\Lambda_{QCD} \ll \mu \ll M_W)$ $\Longleftrightarrow$ short-distance contribution, typically known at NLO in PT.

3. Weak matrix elements $\langle f|Q_k(\mu)|i\rangle$ $\Longleftrightarrow$ long-distance (QCD) contribution $\Rightarrow$ to be computed non-perturbatively, e.g. by using lattice QCD.

- 1 + 2 + 3 $\Rightarrow$ determination of V^k_{CKM}
- Examples of experimentally measured quantities (large amount of data collected [BaBar, Belle, D0, CDF II] & expected [LHC-b, SuperB factory,....]):
  - neutral B-meson mass differences $\Delta m_{d,s}$ (control the frequencies of $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ oscillations);
  - time-dependent CP asymmetry $a_{J/\psi K_s}(t)$ in $B_d \to J/\psi K_s$ decays.

CP-violation phenomenology in the SM

- $\Delta m_q = |\langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle| / m_{B_q} = C_B\, m_{B_q}\, f_{B_q}^2\, B_{B_q}\, |V_{tq}|^2$ with

$$B_{B_q}(\mu) = \frac{\langle \bar{B}_q^0 | \mathcal{O}_{LL}^{\Delta B=2}(\mu) | B_q^0 \rangle}{\langle \bar{B}_q^0 | \mathcal{O}_{LL}^{\Delta B=2} | B_q^0 \rangle_{VSA}} = \frac{\langle \bar{B}_q^0 | \bar{b}\gamma_\mu(1-\gamma_5)q\ \bar{b}\gamma_\mu(1-\gamma_5)q | B_q^0 \rangle}{\frac{8}{3} f_{B_q}^2 m_{B_q}^2}$$

- $\Delta m_d = C_B\, m_{B_d}\, f_{B_d}^2\, B_{B_d}\, A^2 \lambda^6 \left[(1-\bar{\rho})^2 + \bar{\eta}^2 \right]$

yields a ($f_{B_d}^2\, B_{B_d}$-dependent) circle centered in $(1, 0)$.

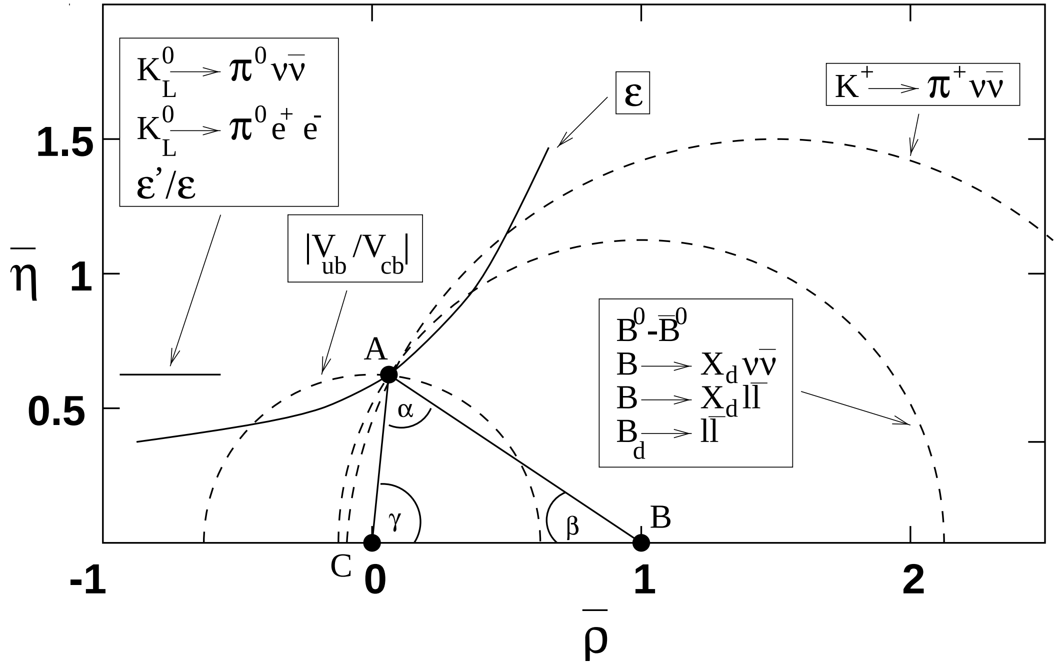
- $\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}\, f_{B_d}^2\, B_{B_d}}{m_{B_s}\, f_{B_s}^2\, B_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} \simeq \frac{m_{B_d}}{m_{B_s}} \xi^2 \frac{\lambda^2}{\left(1-\frac{\lambda^2}{2}\right)^2} \left[(1-\bar{\rho})^2 + \bar{\eta}^2\right]$

  yields a $\left(\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}\right.$-dependent$\left.\right)$ circle centered in $(1,0)$.

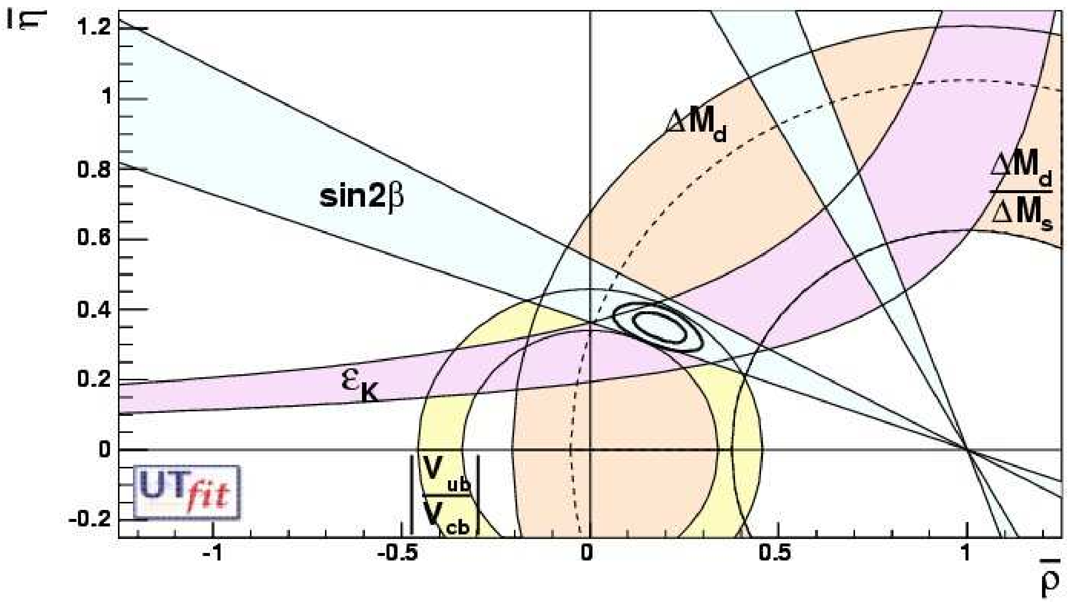
- $a_{J/\psi K_s}(t) \equiv a_{J/\psi K_s} \sin(\Delta m_d t) = \sin 2\beta \sin(\Delta m_d t) \Rightarrow$ yields $\sin 2\beta$

  (in the SM $2\beta = \arg\langle \bar{B}_d^0 | \mathcal{H}_{\rm eff}^{\Delta B=2} | B_d^0 \rangle$).

- In a world without uncertainties (within the SM):



- In the real world:



- Non-SM CP-violation could be detected as an inconsistency between the positions of the upper vertex as determined by different kinds of physics.

- uncertainties in the theoretical inputs (i.e. the hadronic parameters $m_{\rm b}$, $f_{B_{d,s}}$, $B_{B_{d,s}}$, B_K) plays a crucial role in the precise determination of the CKM parameters $\Rightarrow$ it's important to reduce them!

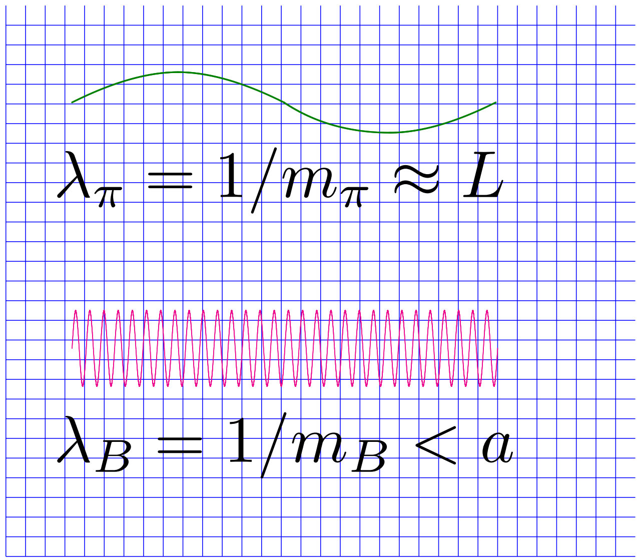
Lattice QCD: motivations

Lattice QCD is the only first-principles approach to study non-perturbative properties of QCD (hadron spectrum, matrix elements, . . .). Many systematics:

1. continuum limit extrapolation.

2. UV (lattice spacing a) and IR (volume V) cutoffs constrain quark masses $\Rightarrow$ extrapolations to the chiral/heavy quark regime.

3. dynamical light quark effects numerically expensive $\Rightarrow$ neglected in the past (quenched approx. $\Rightarrow$ useful to pin down systematics, develop new methods).



light quarks are too light $\Rightarrow$ extrapolate by matching with chiral effective theory.

b-quark is too heavy ($m_{\rm b} a > 1$) $\Rightarrow$ need an effective theory for the b quark: HQET (expansion in inverse powers of the heavy quark mass).

4 possible strategies:

1. Combine relativistic simulations with $m_{\rm h} \approx m_{\rm c}$ and the static limit of HQET to interpolate at $m_{\rm b}$.

2. Work in the static limit of HQET ($m_{\rm h} \to \infty$) and compute $1/m_{\rm h}^n$ corrections.[Heitger & Sommer, 2004]

3. Use finite size methods to relate relativistic observables computed on small volume at physical $m_{\rm b}$ to their value in infinite volume.[Guagnelli *et al.*, 2002]

4. Use appropriate ratios of observables which have an a priori known static limit. Evaluate them around $m_{\rm c}$ and interpolate to $m_{\rm b}$. Use recursive relation to obtain the obeservable at $m_{\rm b}$. [Blossier *et al.*[ETMC], 2009]

Non-perturbative HQET

heavy quark and anti-quark fields $\psi_{\rm h}$, $\psi_{\bar{\rm h}}$ are now independent and satisfy

$$P_+\psi_{\rm h} = \psi_{\rm h}\,, \quad \overline{\psi}_{\rm h}P_+ = \overline{\psi}_{\rm h}\,, \quad P_+ = \tfrac{1}{2}(1+\gamma_0)$$
$$P_-\psi_{\rm h} = \psi_{\rm h}\,, \quad \overline{\psi}_{\rm h}P_- = \overline{\psi}_{\rm h}\,, \quad P_- = \tfrac{1}{2}(1-\gamma_0)$$

HQET action on a lattice (scaling out $\exp(-m_{\rm bare}x_0)$ from $\overline{\psi}_{\rm h}(x)$):

$$S_{\rm HQET} = a^4 \sum_x \{\overline{\psi}_{\rm h}(x)D_0\psi_{\rm h}(x) + \omega_{\rm spin}\underbrace{\overline{\psi}_{\rm h}(-\sigma\cdot\mathbf{B})\psi_{\rm h}}_{\mathcal{O}_{\rm spin}} + \omega_{\rm kin}\underbrace{\overline{\psi}_{\rm h}(-\tfrac{1}{2}\nabla^2)\psi_{\rm h}}_{\mathcal{O}_{\rm kin}} + {\rm O}(1/m^2)\}$$

also the composite fields have a $1/m$ expansion in the effective theory

$$A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET}\overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm h}(x) + c_{\rm A}^{\rm HQET}\overline{\psi}_{\rm l}\gamma_j\overleftarrow{\nabla}_j\psi_{\rm h} + {\rm O}(1/m^2)$$

where $\quad \omega_{\rm kin} = {\rm O}(1/m)\,, \quad \omega_{\rm spin} = {\rm O}(1/m)\,, \quad c_{\rm A}^{\rm HQET} = {\rm O}(1/m)$

in the path integral: expand the action in powers of $1/m$

$$S_{\rm HQET} = a^4 \sum_x \left\{ \mathcal{L}_{\rm stat}(x) + \sum_{\nu=1}^{\infty} \mathcal{L}^{(\nu)}(x) \right\} \qquad \text{where} \qquad \mathcal{L}^{(\nu)} = \mathrm{O}(1/m^\nu)$$

and consider the higher orders $\mathcal{L}^{(\nu)}$, $\nu \geq 1$ only as operator insertions

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathrm{D}\phi \, \mathrm{e}^{-S_{\rm rel} - S_{\rm stat}} \mathcal{O} \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \right\}$$

at a given order in $1/m$ we have:

- renormalizability $\equiv$ existence of the continuum limit
- continuum asymptotic expansion in $1/m$

These properties are not automatic for an effective field theory defined non-perturbatively. E.g. NRQCD does not share them because $O(1/m)$ dimension-5 terms are kept in the action used to compute the path-integral.

Difference to ChPT: as $1/m \to 0$ interactions are not turned off $\Rightarrow$ need lattice formulation to evaluate it non-perturbatively in g^2.

Renormalization of HQET has to be done non-perturbatively: hard cut-off $(1/a)$

$$\Rightarrow \text{e.g.} \qquad \mathcal{O}_{\mathrm{R}}{}^{\mathrm{d=5}} = Z_{\mathcal{O}} \left[\mathcal{O}^{\mathrm{d=5}} + \sum_k c_k \mathcal{O}_k{}^{\mathrm{d=4}} \right] \qquad\qquad c_k = \frac{c_k^{(0)} + c_k^{(1)} g^2 + \dots}{a}$$

if c_k computed at a finite order l in g^2, there is no continuum limit!

$$\Delta c_k \sim \frac{g^{2(l+1)}}{a} \sim \frac{1}{a\,[\ln(a\Lambda)]^{l+1}} \xrightarrow{a \to 0} \infty$$

perturbative remainder of some parameters computed to l-loops with l arbitrary

Matching between QCD and HQET

1. Fix all the parameters of QCD (bare quark masses) by requiring a set of observables (e.g. hadron masses) to agree with experiment.

2. Determine the bare couplings of HQET at order n ($m_{\rm bare}$, $\omega_{\rm kin}$, $\omega_{\rm spin}$, $c_{\rm A}^{\rm HQET}$, $Z_{\rm A}^{\rm HQET}$, $\ldots$) by imposing

$$\Phi_k^{\rm QCD}(M) = \Phi_k^{\rm HQET}(M) + O\left(\tfrac{1}{M^{n+1}}\right) \,, \quad k = 1, 2, \ldots, N_n^{\rm HQET}$$

$\Phi_k^{\rm HQET}(M)$ could be determined phenomenologically from experimentally accessible observables $\Rightarrow$ predictive power of HQET reduced!

Compute $\Phi_k^{\rm QCD}$ in lattice QCD $\Rightarrow$ transfer of predictivity from QCD to HQET.

$\Rightarrow$ heavy quark must be treated relativistically: exactly the same problem in the motivations for using HQET!

Non-perturbative matching between QCD and HQET

[Heitger & Sommer, 2004]

Way out: use observables Φ_k defined in finite (small) volume and the fact that parameters of the QCD and HQET lagrangians are independent of the volume.

Φ_k defined on $L = L_1 \approx 0.4\,\mathrm{fm} \ll 2\,\mathrm{fm}$ $\Rightarrow$ simulate very fine a's where $m_\mathrm{b} a \ll 1$ and $1/(m_b L_1) \ll 1$ (well behaved $1/m$-expansion in finite volume).

HQET-parameters from QCD observables in small volume at small lattice spacing

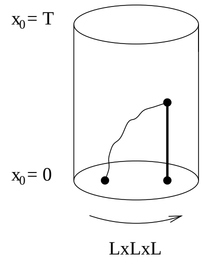
Physical observables (e.g. $B_{\mathrm{B_s}}$, $F_{\mathrm{B_s}}$) need a large volume, such that the B-meson fits comfortably: $L \approx 4L_1 \approx 1.6\,\mathrm{fm}$

Connection between L_1 and $4L_1$ achieved recursively in HQET: no relativistic b-quark any more $\Rightarrow$ no problems with the size of a.

Example: M_b static (at order $1/m^0$)

[Heitger & Sommer, 2004; Della Morte, Garron, Papinutto and Sommer, 2005-2006]

finite volume B-meson "mass":

$f_\mathrm{A} \sim$ 

$$\Gamma(L) = -\partial_0 \log\langle A_0(x_0) A_0(0)\rangle_{x_0=L/2}$$

In ∞ volume:

$$m_B \equiv \lim_{L\to\infty} \Gamma^{\mathrm{QCD}}(L, M_\mathrm{b}) = \lim_{L\to\infty} \Gamma^{\mathrm{stat}}(L) + m_\mathrm{bare} \equiv E_\mathrm{stat} + m_\mathrm{bare}$$

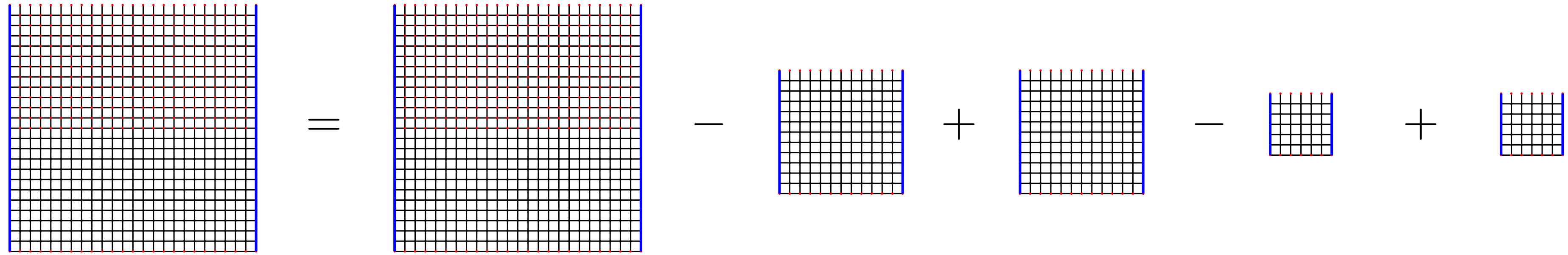
matching condition in finite volume: $\Gamma^{\mathrm{QCD}}(L_1, M_\mathrm{b}) = \Gamma^{\mathrm{stat}}(L_1) + m_\mathrm{bare}$

$\Rightarrow$ eliminate m_bare and insert $0 = \Gamma^{\mathrm{stat}}(2L_1) - \Gamma^{\mathrm{stat}}(2L_1)$

$$\underbrace{m_B}_{\mathrm{QCD}} = E_\mathrm{stat} - \Gamma^{\mathrm{stat}}(L_1) + \Gamma^{\mathrm{QCD}}(L_1, M_\mathrm{b})$$

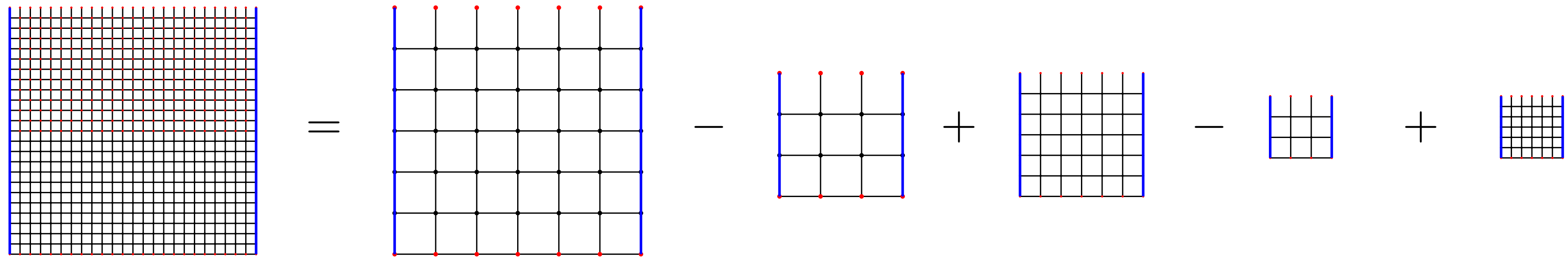
$$= \underbrace{E_\mathrm{stat} - \Gamma^{\mathrm{stat}}(2L_1)}_{\mathrm{HQET}} + \underbrace{\Gamma^{\mathrm{stat}}(2L_1) - \Gamma^{\mathrm{stat}}(L_1)}_{\mathrm{HQET}} + \underbrace{\Gamma^{\mathrm{QCD}}(L_1, M_\mathrm{b})}_{\mathrm{QCD}}$$

The identity:



$$\underbrace{m_B}_{\text{QCD}} \qquad \underbrace{E_{\rm stat}(a) \;-\; \Gamma^{\rm stat}(2L_1,a)}_{\text{HQET}} \quad \underbrace{\Gamma^{\rm stat}(2L_1,a)-\Gamma^{\rm stat}(L_1,a)}_{\text{HQET}} \quad \underbrace{\Gamma^{\rm QCD}(L_1,M_{\rm b},a)}_{\text{QCD}}$$

is equivalent to (up to lattice artifacts):



$$\underbrace{m_B}_{\text{QCD}} \qquad \underbrace{E_{\rm stat}(a'') \;-\; \Gamma^{\rm stat}(2L_1,a'')}_{\text{HQET}} \quad \underbrace{\Gamma^{\rm stat}(2L_1,a')-\Gamma^{\rm stat}(L_1,a')}_{\text{HQET}} \quad \underbrace{\Gamma^{\rm QCD}(L_1,M_{\rm b},a)}_{\text{QCD}}$$

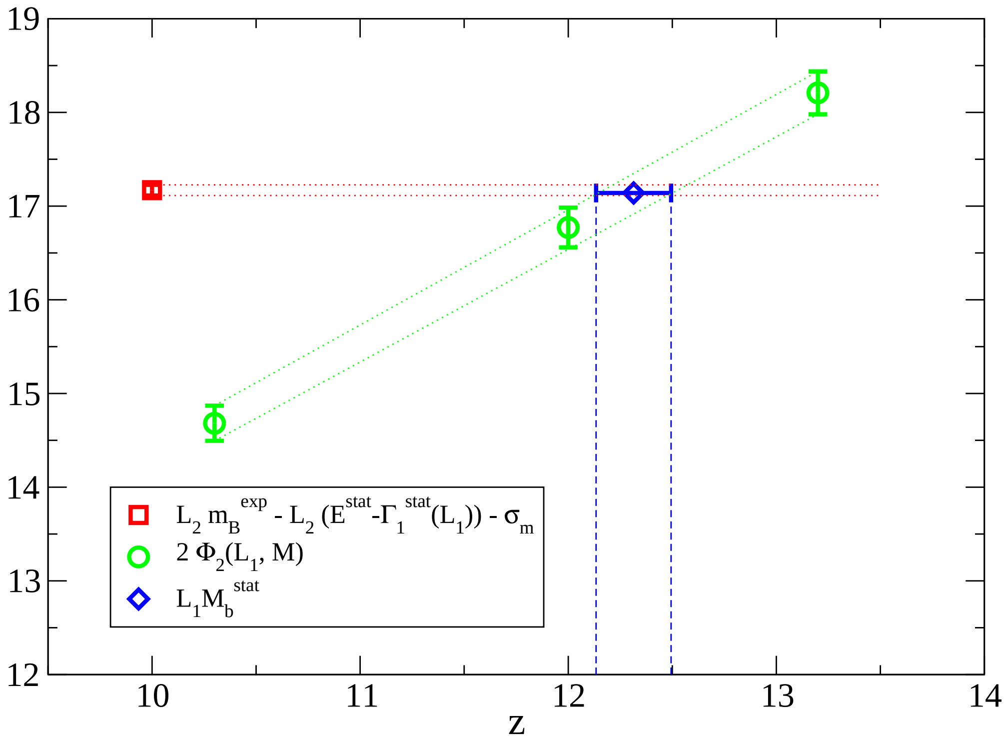
but now continuum limit is affordable in each $\underbrace{\qquad}$ of the r.h.s. independently.

$M_{\rm b}$ static

To determine $M_{\rm b}$ we set $\underbrace{m_B}_{\rm QCD}$ to $\underbrace{m_B}_{\rm exp.}$

$$\underbrace{m_{\rm B}}_{\rm exp.} - \underbrace{\left[E_{\rm stat}(a'') - \Gamma^{\rm stat}(2L_1, a'')\right]}_{a''\to 0 \text{ in HQET}} - \underbrace{\left[\Gamma^{\rm stat}(2L_1, a') - \Gamma^{\rm stat}(L_1, a')\right]}_{a'\to 0 \text{ in HQET}} = \underbrace{\Gamma^{\rm QCD}(L_1, M_{\rm b}, a)}_{a\to 0 \text{ for } M_{\rm b}L_1 \gg 1}$$

and we solve the above equation for $M_{\rm b}$ (the RGI b-quark mass)



$M_b^{\rm stat} = 6771 \pm 99$ MeV

$z = M_{\rm b} L_1$

$M_{\rm b}$ at order $1/m$ [Della Morte, Garron, Papinutto and Sommer, 2005-2006]

coefficients in the action:

$$O(1) \quad m_{\rm bare}$$

$$O(1/m) \quad \omega_{\rm kin} \quad \leftrightarrow \quad \overline{\psi}_{\rm h}(-\tfrac{1}{2}\nabla^2)\psi_{\rm h}$$

$$O(1/m) \quad \omega_{\rm spin} \quad \leftrightarrow \quad \overline{\psi}_{\rm h}(-\sigma\cdot\mathbf{B})\psi_{\rm h}$$

$\omega_{\rm spin}$ cancels in spin averaged quantities. Eliminate $m_{\rm bare}$, $\omega_{\rm kin}$:

∞ volume $\quad m_{\rm B} = E^{\rm stat} + m_{\rm bare} + \omega_{\rm kin}E^{\rm kin}$

Matching 1 $\quad \Gamma^{\rm QCD}(L,M_{\rm b}) = \Gamma^{\rm stat}(L) + m_{\rm bare} + \omega_{\rm kin}\Gamma^{\rm kin}(L)$

Matching 2 $\quad \Phi_1^{\rm QCD}(L,M_{\rm b}) = \omega_{\rm kin}R_1^{\rm kin}(L) = \Phi_1^{\rm HQET}(L)$

$$m_{\rm B} = \left[E^{\rm stat} - \Gamma^{\rm stat}(L)\right] + \Gamma^{\rm QCD}(L,M_{\rm b}) + \left[\frac{\Phi_1^{\rm QCD}(L,M_{\rm b})}{R_1^{\rm kin}(L)}(E^{\rm kin} - \Gamma^{\rm kin}(L))\right]$$

One obtains $m_{\rm B}^{\rm exp} = m_{\rm B}(M_{\rm b}^{\rm stat} + M_{\rm b}^{(1)})$ that has to be solved for $M_{\rm b}^{(1)}$.

The (quenched) result in the $\overline{\rm MS}$ scheme:

$$\begin{aligned} m_{\rm b}(m_{\rm b}) &= m_{\rm b}^{\rm stat} + m_{\rm b}^{(1)} = 4.35(5)\,{\rm GeV} \\ m_{\rm b}^{\rm stat} &= 4.37(5)\,{\rm GeV}\,, \quad m_{\rm b}^{(1)} = -0.027(22)\,{\rm GeV}\,. \end{aligned}$$

At this order, $M_{\rm b}$ affected by $O(\frac{1}{L_1^3 M_{\rm b}^2})$, $O(\frac{\Lambda_{QCD}}{L_1^2 M_{\rm b}^2})$, $O(\frac{\Lambda_{QCD}^2}{L_1 M_{\rm b}^2})$, $O(\frac{\Lambda_{QCD}^3}{M_{\rm b}^2})$ errors.

Checked by using different matching conditions (different correlators) $\Rightarrow$ final results agree up to (small) $O(1/m^2)$ corrections exactly of $O(\Lambda_{QCD}^3/M_{\rm b}^2)$

unquenched computation works in exactly the same way. A preliminary result with $N_f = 2$ dynamical flavours [B. Blossier *et al.* Lattice2010] gives

$$m_{\rm b}(m_{\rm b}) = 4.276(25)(50)\,{\rm GeV}$$

Other existing unquenched results: $m_{\rm b} = 4.164(23)\,{\rm GeV}$ (HPQCD coll. $N_f = 2+1$ from current-current correlators) and $m_{\rm b} = 4.63(27)\,{\rm GeV}$ (ETM coll. $N_f = 2$)

2nd strategy: B-parameters of $\Delta B = 2$ operators

[D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto, J. Reyes 2002]

- $\Delta B = 2$ four fermion operators:

$$
\begin{aligned}
Q_1 &= \bar{b}^i\gamma_\mu(1-\gamma_5)q^i\,\bar{b}^j\gamma_\mu(1-\gamma_5)q^j & O_1 &= \bar{\psi}_{\rm h}^i\gamma_\mu(1-\gamma_5)q^i\,\bar{\psi}_{\rm h}^j\gamma_\mu(1-\gamma_5)q^j \\
Q_2 &= \bar{b}^i(1-\gamma_5)q^i\,\bar{b}^j(1-\gamma_5)q^j & O_2 &= \bar{\psi}_{\rm h}^i(1-\gamma_5)q^i\,\bar{\psi}_{\rm h}^j(1-\gamma_5)q^j \\
Q_3 &= \bar{b}^i(1-\gamma_5)q^j\,\bar{b}^j(1-\gamma_5)q^i & O_3 &= -O_2 - \tfrac{1}{2}O_1 \\
Q_4 &= \bar{b}^i(1-\gamma_5)q^i\,\bar{b}^j(1+\gamma_5)q^j & O_4 &= \bar{\psi}_{\rm h}^i(1-\gamma_5)q^i\,\bar{\psi}_{\rm h}^j(1+\gamma_5)q^j \\
Q_5 &= \bar{b}^i(1-\gamma_5)q^j\,\bar{b}^j(1+\gamma_5)q^i & O_5 &= \bar{\psi}_{\rm h}^i(1-\gamma_5)q^j\,\bar{\psi}_{\rm h}^j(1+\gamma_5)q^i
\end{aligned}
$$

- Example $B_{B_q}(\mu) = \dfrac{\langle \bar{B}_q^0|\mathcal{O}_{\rm LL}^{\Delta B=2}(\mu)|B_q^0\rangle}{\frac{8}{3}f_{B_q}^2 m_{B_q}^2} = \dfrac{\langle \bar{B}_q^0|Q_1(\mu)|B_q^0\rangle}{\frac{8}{3}f_{B_q}^2 m_{B_q}^2}$:

$$
\begin{aligned}
\langle \bar{B}_q^0|Q_1(m_{\rm b})|B_q^0\rangle_{\rm QCD} &= C_1(m_{\rm b},\mu)\,\langle \bar{B}_q^0|O_1(\mu)|B_q^0\rangle_{\rm HQET} \\
&+ C_2(m_{\rm b},\mu)\,\langle \bar{B}_q^0|O_2(\mu)|B_q^0\rangle_{\rm HQET} + \mathrm{O}\left(1/m_{\rm b}\right)
\end{aligned}
$$

- We want to compute the B-parameters of the complete $\Delta B = 2$ basis at the physical b-quark mass. We explain the procedure for Q_1, Q_2, Q_3 which mix in the matching. Analogous procedure for Q_4 and Q_5.

$$
\begin{aligned}
\langle \overline{B}_q | Q_1(\mu) | B_q \rangle &\equiv \frac{8}{3} f_{B_q}^2 m_{B_q}^2 B_1(\mu) \equiv \frac{8}{3} f_{B_q}^2 m_{B_q}^2 B_{B_q}(\mu) \\
\langle \overline{B}_q | Q_2(\mu) | B_q \rangle &\equiv -\frac{5}{3} \left(\frac{f_{B_q} m_{B_q}^2}{m_b(\mu) + m_q(\mu)} \right)^2 B_2(\mu) \\
\langle \overline{B}_q | Q_3(\mu) | B_q \rangle &\equiv \frac{1}{3} \left(\frac{f_{B_q} m_{B_q}^2}{m_{\mathrm{b}}(\mu) + m_q(\mu)} \right)^2 B_3(\mu)
\end{aligned}
$$

- First step: QCD computation with $m_{\mathrm{h}} \approx m_{\mathrm{c}}$ (pseudoscalar meson mass m_P).

- Second step: computation in the static limit of HQET

- Third step: matching of QCD onto HQET and interpolation at m_{b}:

QCD at mass $m_{\rm h} \approx m_{\rm c}$ and renorm. scale μ
$\vec{B}_P(\mu) = [B_1(\mu), B_2(\mu), B_3(\mu)]^T$

↓

NLO QCD evolution
from μ to $m_{\rm h}$: $\vec{B}_P(m_{\rm h})$

↓

NLO matching onto HQET
$\vec{\Phi}(m_{\rm h}, m_P) \equiv C^{-1}(m_{\rm h})\vec{B}_P(m_{\rm h})$

static HQET at renorm. scale μ'
$\vec{B}^{\rm h}(\mu') = [B_1^{\rm h}(\mu'), B_2^{\rm h}(\mu'), B_3^{\rm h}(\mu')]^T$

↓

NLO HQET evolution from μ'
to $m_{\rm h}$: $\vec{B}^{\rm h}(m_{\rm h}) \equiv \vec{\Phi}(m_{\rm h}, \infty)$

← Interp. at $m_P = m_{B_q}$

$$\vec{\Phi}(m_{\rm h}, m_P) = \vec{\Phi}(m_{\rm h}, \infty) + \frac{\vec{a}_1(m_{\rm h})}{m_P} + \frac{\vec{a}_2(m_{\rm h})}{m_P^2}$$

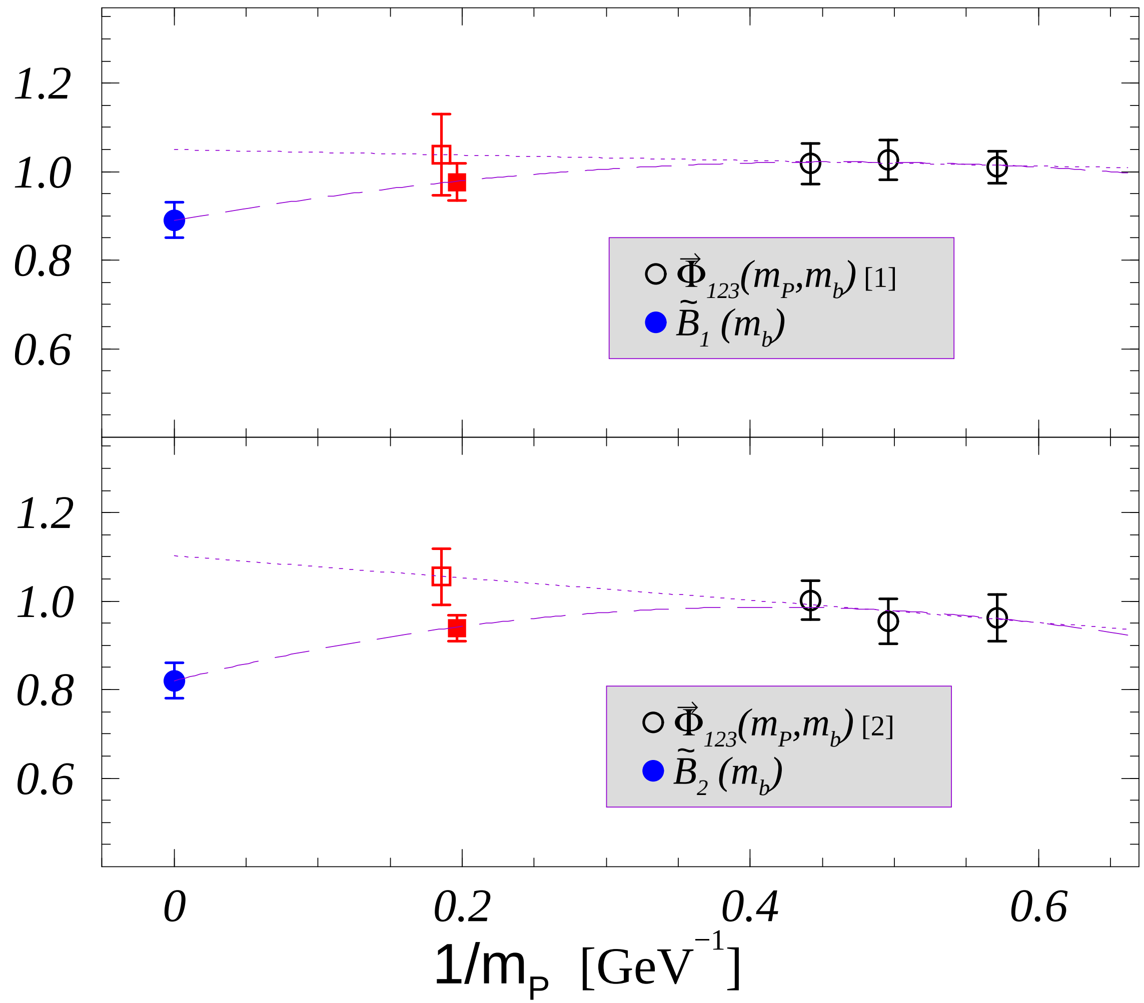
↓

$\vec{B}_{B_q}(m_{\rm h}) = C(m_{\rm h})\vec{\Phi}(m_{\rm h}, m_{B_q})$

↓

NLO QCD evolution from $m_{\rm h}$ to $m_{\rm b}$: $\vec{B}_{B_q}(m_{\rm b})$

- Combining HQET with QCD $\Rightarrow$ interpolation to $m_{\rm b}$



- Matching of 4-fermion HQET operators to QCD operators $C(\mu)$ computed at NLO in perturbation theory.

- Results in the $\overline{\mathrm{MS}}$(NDR) scheme at $\mu = m_{\mathrm{b}}$:

$$B_{B_d}(m_{\mathrm{b}}) = 0.87(3)(5) \qquad\qquad B_{B_s}(m_{\mathrm{b}}) = 0.87(2)(5)$$

- also B_2, B_3, B_4, B_5 (relavant for physics beyond the SM) have been computed.

- This was an exploratory study:

  1. quenched approximation
  2. no continuum limit extrapolation
  3. 1-loop perturbative renormalization of HQET operators

- Only existing $N_f = 2+1$ unquenched determination of $B_{B_d} = 0.84(7)$ and $B_{B_s} = 0.86(7)$ (HPQCD by using NRQCD). Difficult to asses the error due to power divergent subtractions of the operators computed in perturbation theory...

- two other $N_f = 2{+}1$ (HQET and FERMILAB action) calculations of $\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$ where the renormalization factors almost cancel out $\Rightarrow$ renormalisation of effective theories very difficult problem: need to be carried out non-perturbatively!

- No unquenched calculation for B_2, B_3, B_4,B_5 yet!

Non-perturbative renormalization of HQET operators

[Palombi,Papinutto,Pena,Wittig, 2005-2007 + Dimopoulos,Herdoiza,Palombi,Papinutto,Pena,Vladikas,Wittig 2008]

With Wilson-like fermions, using flavour exchange symmetry, parity, chiral symmetry, heavy quark spin symmetry and $H(3)$ spatial rotations, the mixing pattern is:

$$\begin{pmatrix} \mathcal{O}'_1 \\ \mathcal{O}'_2 \\ \mathcal{O}'_4 \\ \mathcal{O}'_5 \end{pmatrix}_{\rm R} = \begin{pmatrix} \mathcal{Z}_1 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_2 & 0 & 0 \\ 0 & 0 & \mathcal{Z}_4 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_5 \end{pmatrix} \left[\mathbb{1} + \begin{pmatrix} 0 & 0 & Д_1 & 0 \\ 0 & 0 & 0 & Д_2 \\ Д_4 & 0 & 0 & 0 \\ 0 & Д_5 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \mathcal{O}'_1 \\ \mathcal{O}'_2 \\ \mathcal{O}'_4 \\ \mathcal{O}'_5 \end{pmatrix}$$

time reversal $\Rightarrow Д_k = 0$ while $\Delta_k \neq 0$.

Strategy: use twisted mass QCD for the light quarks (with angle $\pi/2$). In this way $[O'_k]_{\rm R}^{\rm QCD} = \mathcal{Z}_k [\mathcal{O}'_k]^{\rm tmQCD}$ the operators renormalize multiplicatively.

We use the Schrödinger functional: 4 dim. box with periodic boundary conds. in space and Dirichlet boundary conds. in time $\rightarrow$ gap in the spectrum of the Dirac op.

$\rightarrow$ work directly at vanishing quark mass: naturally define a mass-independent renormalization scheme. $\mu = 1/L$, the inverse linear size of the box

We have computed non-perturbatively the renormalization constants $\mathcal{Z}_k(g, a\mu)$ with $N_f = 0$ and $N_f = 2$ dynamical flavors:

The computation requires the following ingredients:

1. perturbative calculation of the NLO anomalous dimension for the complete basis of static four-fermion operators in the SF scheme.

2. step-scaling function (SSF) $\sigma_k(u)$ in the continuum (in the SF scheme). Fixing the renormalized coupling $u \equiv \bar{g}^2(\mu)$ corresponds to fixing the renormalization scale $\mu = 1/L$.

   $$\sigma_k(u) = U_k(\mu, \mu/2) = \frac{\hat{c}_k(\mu/2)}{\hat{c}_k(\mu)} = \lim_{a\to 0} \frac{\mathcal{Z}_k(g_0, a\mu/2)}{\mathcal{Z}_k(g_0, a\mu)}\Big|^{m=0}_{u\equiv\bar{g}^2(\mu)}$$

   where $\hat{c}_k(\mu) = \left[\frac{\bar{g}^2(\mu)}{4\pi}\right]^{-\gamma_k^{(0)}/2b_0} \exp\left\{ -\int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma_k(g)}{\beta(g)} - \frac{\gamma_k^{(0)}}{b_0 g}\right)\right\}$

   together with $\sigma(u) = \bar{g}^2(\mu/2)$.

3. From the SSF we construct the non-perturbative RG evolution $U_k(\mu_{\rm pt}, \mu_{\rm had})$ with $\mu_{\rm pt} = 2^n \mu_{\rm had}$ $(n = 8)$ (from $\mu_{\rm had} \approx 270\,\mathrm{MeV}$ up to $\mu_{\rm pt} \approx 70\,\mathrm{GeV}$):

$$U_k(2^n\mu_{\rm had},\mu_{\rm had}) = \left\{\prod_{i=0}^{n-1}\sigma_k(u_i)\right\}^{-1}, \qquad u_i = \bar{g}^2(2^{(i+1)}\mu_{\rm had})$$

4. the matching factor $\mathcal{Z}_k(g_0, a\mu_{\rm had})$ at the low hadronic scale $\mu_{\rm had}$.

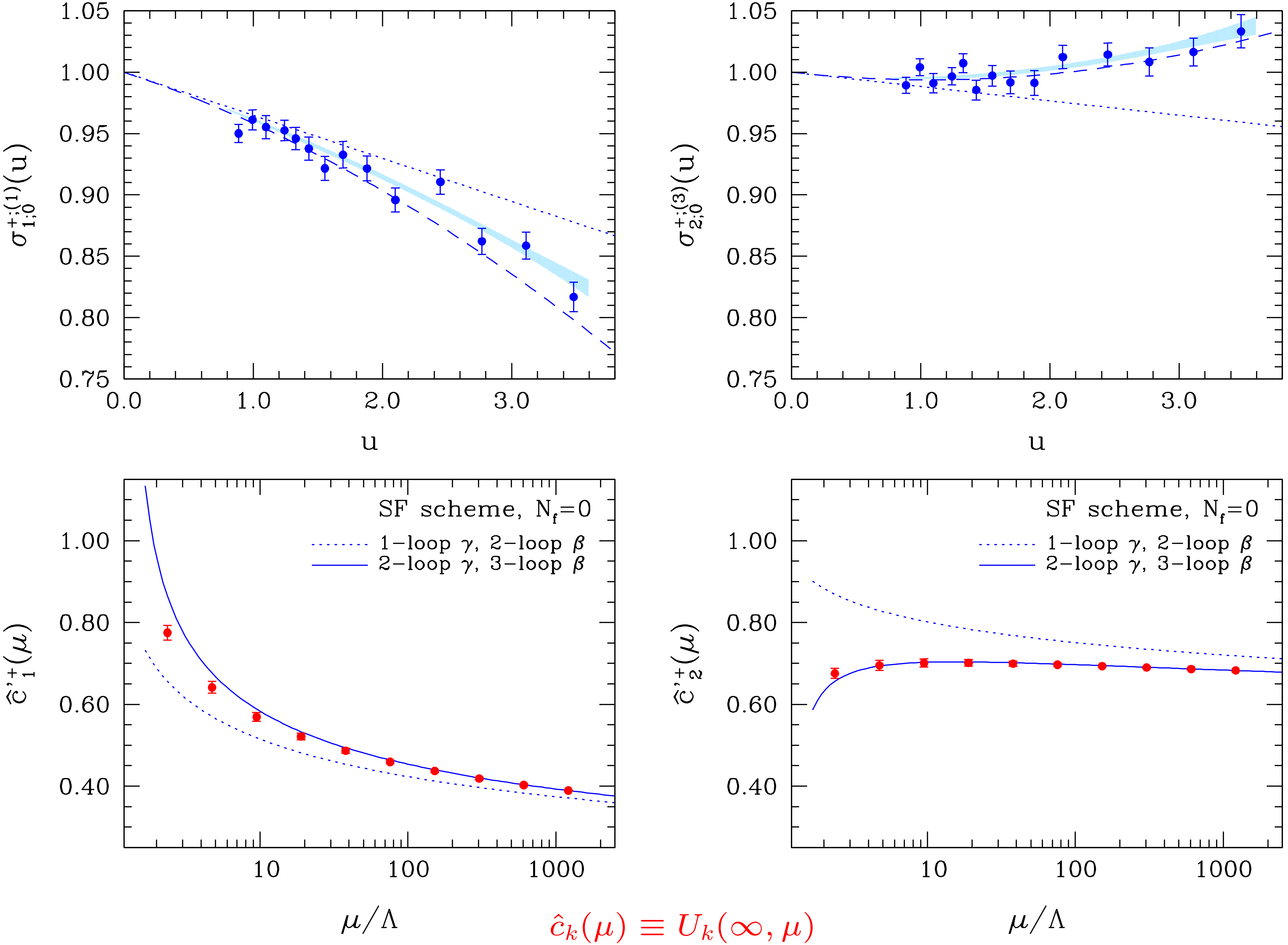
5. the RGI renormalization constant

$$\hat{Z}_{k,\rm RGI}(g_0) = \hat{c}_k(\mu_{\rm had})\mathcal{Z}_k(g_0,a\mu_{\rm had}) = \hat{c}\,_k(\mu_{\rm pt})U_k(\mu_{\rm pt},\mu_{\rm had})\mathcal{Z}_k(g_0,a\mu_{\rm had})$$

with $\hat{c}\,_k(\mu_{\rm pt})$ perturbative at NLO while $U_k(\mu_{\rm pt},\mu_{\rm had})$ non-perturbative.

Statistical + sytematic uncertainty $\leq 2\%$ for $N_f = 0$ and $\leq 5\%$ for $N_f = 2$.

This opens the door to a fully non-perturbative computation of HQET matrix elements.



$$\hat{c}_k(\mu) \equiv U_k(\infty, \mu)$$

$\Delta B = 2$ transitions in general SUSY model[Becirevic *et al.*, 2002]

- Most general effective Hamiltonian for $\Delta B = 2$ processes beyond the SM:

$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i$$

($\tilde{Q}_i$ obtained from Q_i by $(1-\gamma_5) \leftrightarrow (1+\gamma_5)$).

- Ingredients:

1. Wilson cofficient at the scale of "new physics" (that we call M_S).
2. Evolution from the scale M_S to the low energy scale (here we choose $m_{\rm b}$) including NLO QCD corrections.
3. matrix elements of the Q_i renormalized at the low energy scale $m_{\rm b}$.

- Evolution from M_S to m_b including NLO QCD corrections:

$$C_r(m_\mathrm{b}) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)}) \eta^{a_i} C_s(M_S)$$

with $M_S \sim m_{\tilde{g}} \sim m_{\tilde{q}}$ and $\eta = \alpha_s(M_S)/\alpha_s(m_t)$.

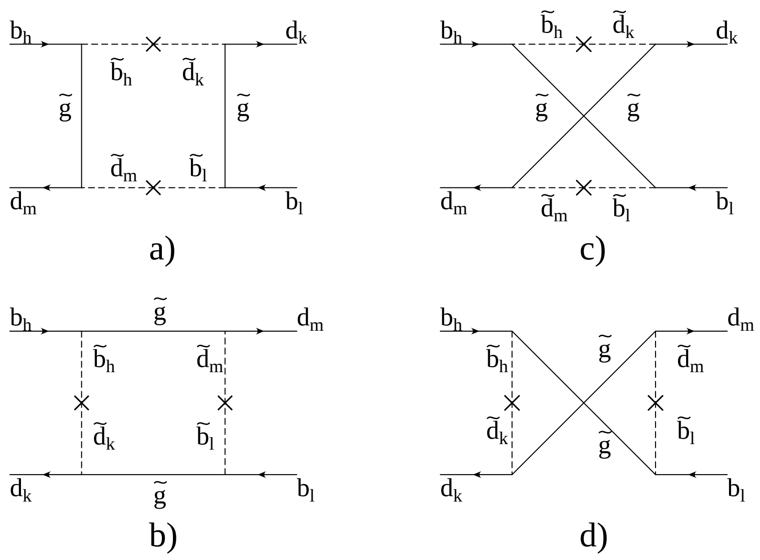
- Δm_d and β can be computed for any model of new physics in which the new contributions with respect to the SM originate from extra heavy particles.

- Just plug in the expression for $C_i(M_S)$ evaluated in such a model.

- When $O(\alpha_s)$ corrections to the $C_i(M_S)$ are not available, results contain a residual systematic uncertainty of order $\alpha(M_S)$.

- Example: MSSM with gluino-mediated FCNC. Except for specific models where charginos, stops and/or charged Higgs are relatively light and contribute significantly to FCNC, this should be sufficient to get the correct bulk of SUSY contributions.

- Computation performed in the mass insertion approximation. Super-CKM basis chosen so that genuine SUSY FC effects are exhibited by the non diagonal entries of the sfermion mass matrices Δ.



- Sfermion propagators expanded in powers of $\delta = \Delta^2/m_{\tilde{q}}^2 \ll 1$ and only the first term is kept. $m_{\tilde{q}} \in [0.25, 1]$TeV average sfermion mass and $m_{\tilde{g}}/m_{\tilde{q}} \in [0.5, 2]$.

- Experimental information about Δm_d and $a_{J/\psi K_s}$ + $\Delta B = 2$ matrix elements $\Rightarrow$ upper bounds on the δ_{13}s. Similar study for Δm_K and ε gives bounds for the δ_{12}s. [M. Ciuchini *et al.*, '98]

Conclusions and outlook

- Lattice QCD essential tool to determine CKM parameters and to compute contributions from new physics.

- Increasing statistical accuracy of exp. measurements $\Rightarrow$ better control over systematic uncertainties of theoretically predicted parameters very important:

1. fully non-perturbative formulation of HQET (including matching with QCD) order by order in the $1/m_{\rm h}$-expansion. Example: $M_{\rm b}$ at $O(1/m_{\rm h})$.

2. alternative strategy: interpolation between relativistic results at $m_{\rm h} \approx m_{\rm c}$ and HQET results at leading order.

3. non-perturbative renormalisation of $\Delta B = 2$ static operators performed $\Rightarrow$ fully non-perturbative computation of B_B static to be started.

4. dynamical simulation at small light quark mass (where contact with χPT can be made) are now available. Very important for F_{B_d} and B_{B_d}.

5. need to study further observables, e.g. $B \to \pi l \nu \Rightarrow$ extraction of $V_{\rm ub}$.